The effect of inflation on wage inequality: A North-South monetary model of endogenous growth with international trade*

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Abstract

This study analyzes the effects of inflation on intra- and inter-county wage inequality, specialization, and growth, using a North-South endogenous growth model with international trade and money. The relationship between inflation and intra-country wage inequality depends on firms' credit constraints and on the inflation levels. Our results indicate that inflation decreases specialization in skilled-production and increases intra-country wage inequality. Moreover, increasing inflation in the South increases the wage inequality gap between countries. Theoretical results are confirmed through calibration and match with existing empirical evidence.

Keywords: Inflation; Wage inequality; North-South trade; CIA constraints; Technological-knowledge bias.

JEL classification Codes: E41, F16, F43, O31, O33, O40.

1 Introduction

The relationship between inflation and economic growth has been studied since the fundamental work of Tobin (1965). Since then and especially during the last 10 years this relationship has been explored through capital accumulation and innovation, mostly in closed economy models. However, the seminal work by Acemoglu (2002) showed that skill-biased technical change tended to increase the income gap between rich and poor countries and then explored why international trade may

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induce these results. Therefore, to fully capture the effects of inflation on the macroeconomy, it is important to explore the effects on variables other than economic growth such as inequality and trade specialization. Moreover, it is important to do that via skill-biased technical change. This study follows this path.

Intuitively, the influence of different cash-in-advance constraints in different sectors (which were almost ignored in previous studies) and different inflation rates in more and less advanced countries may act through the skill-bias to influence wage inequality. These effects can be important both qualitatively and quantitatively and differ among countries. It is noteworthy that due to the relationship among countries, the monetary policy conducted by each one affects the international trade, penalizes the worldwide economic growth, and penalizes both the technological-knowledge bias and the skill premium if there is a worsening of the cash-in-advance constraint on the production of skilled intermediate goods and R&D activities.

We seek to contribute to the understanding of the effect of monetary policy in international trade specialization and inequality. To that end, we devise a North-South endogenous growth model with a monetary sector in order to analyze the effects of both monetary and trade-related technological diffusion shocks on intra- and inter-country wage inequality. There is some recent and robust evidence on the influence of inflation on wage inequality (e.g., Chu et al. 2019b).[1]

Our results highlight that inflation and trade shocks have opposite effects on wage inequality and specialization: while trade tends to decrease wage inequality in the South, inflation tends to increase it; while trade tends to increase the number of different intermediate goods produced with unskilled technology in the South; inflation acts the other way around. Depending on cash-in-advance constraints in different sectors and different Northern and Southern monetary policies, different theoretical steady-state relationships between inflation, wage inequality (or skill premium), and technological specialization can arise.[2] However, the negative effect of inflation on economic growth is obtained for all possible combinations of parameters. In a quantitative analysis of the long-run equilibrium, for the more relevant case in which unskilled-intensive firms are more financially constrained than skilled-intensive firms, we obtain that inflation increases wage inequality and decreases specialization in the production of skilled-intensive goods. Interestingly, if inflation increases in both countries in the same way, then inter-country wage inequality tends to shrink, but when the inflation rises more in the South, wage inequality patterns tend to diverge: the South tends to become relatively more unequal when compared to the North.

The study of the relationship between monetary policy and growth has seen an exponential surge in recent years (e.g., Alogoskoufis and Ploeg 1994, López-Villavicencio and Mignon 2011, Arawatari et al. 2018, Chu et al. 2019a, Chu and Cozzi 2014, Chu et al. 2015, Ho et al. 2007, Hori 2020, Okawa and Ueda 2018). Few authors have studied the relationship between inflation and

[1] In Appendix A, we show our own results that confirm a robust linear conditional relationship between inflation and wage inequality, which is stronger in the less advanced countries.

[2] Those possible different relationships are consistent with recent surveys (e.g., Cigno et al. 2018).

other variables (other than economic growth rate), with the exceptions of Gil and Iglésias (2020) and Chu et al. (2019b), who also study the influence of inflation on income inequality. Chu et al. (2019b) discovered that the theoretical relationship between inflation and income inequality features an inverted U-shape, meaning that inequality rises until a certain level of inflation and then decreases above that level. However, they did not analyze the influence of inflation in an open economies setting and its interactions with trade.

The focus on the explanation of technological-knowledge bias, intra- and inter-country wage inequality, and trade specialization is clearly our novel contribution to the literature. In fact, there are quite a few examples of works that study an open economy monetary endogenous growth model. The exceptions are Chen et al. (2020), Chu et al. (2015), and Chu et al. (2019b). The first devises a small open economy growth model analyzing if monetary policy changes the possibility of macroeconomic instability in the model. The second studies a two-country Schumpeterian growth model with cash-in-advance constraints on consumption and R&D investment, focusing on the influence of both domestic and foreign inflation in R&D investment and economic growth. The third, which is the closest to ours, devises a quality-ladder R&D North-South endogenous growth model and analyzes the effect of inflation in the North and in the South in R&D investment, economic growth, and north-south wage gap. Major differences between Chu et al. (2019b) and our approach should be noted. First, while the former included only labor and thus studied the influence of wage differentials between North and South, we consider both unskilled and skilled labor enabling the study of wage inequality both intra- and inter-country. Second, while the former included CIA constraints just in the the R&D sector, we consider that those constraints can affect all sectors in the economy. Third, while we explicitly analyze (trade) specialization effects and the effect of openness to trade in the Northern and Southern country (allowing monetary and trade shocks to be analyzed together), Chu et al. (2019b) analyze the effect of technology transfer to the South in the reallocation of labor in the South between production and adaptative R&D.

Taking into account the close relationship between technological-knowledge bias and wages emphasized by the directed technical change (DTC) literature (Acemoglu 2002, Afonso 2012), our North-South dynamic general equilibrium endogenous directed technical change (DTC) growth model with international trade intends to contribute to the explanation for the influence of monetary policy on income inequality. In this sense, we also wish to contribute to the literature that explains the different levels and paths of the skill premium across different countries and periods (e.g., Acemoglu 2002, 2003). In fact, our growth model also predicts such a positive relationship depending on the fact that skilled production firms are less financially constrained than unskilled production firms, which is a well documented empirical regularity (e.g., Popov 2013, Cao and Leung 2016, Frank and Yang 2018, Gómez 2018, Feng et al. 2020).

The remainder of this study proceeds as follows. The theoretical setup is developed in Section 2, including consumers' decisions, monetary authority, the production and price decisions, R&D activity, and the international trade. Section 3 analyzes the dynamic general equilibrium, looking at equilibrium R&D and at the steady state analysis. Section 4 takes the model to data, through calibration, and presents quantitative effects of the inflation on wage inequality, specialization, and growth. Section 5 concludes the paper.

2 A North-South Monetary DTC Growth Model

We consider that in each country an aggregate final good is used for consumption and investment. This is an aggregate of the output of skilled and unskilled sectors. In each of these two sectors, numerous competitive firms use either unskilled labor or skilled labor plus a continuum of non-durable quality-adjusted intermediate goods to produce a continuum of unskilled or skilled final goods. As both countries have skilled and unskilled labor, both produce skilled and unskilled final goods. Each intermediate-good sector consists of a continuum of monopolistic producers, each one using a specific design. A new (quality-adjusted) intermediate good is thus introduced in the North, but can also be imitated in the South. Monetary policy – implemented by a monetary authority (the only form of government in the model) – affects the production and technology side of the economy through country-specific CIA constraints faced by intermediate-good producers and R&D firms in each country. That is, the CIA constraints imply the existence of a mechanism that, through the different levels of the nominal interest rate in the two countries, imparts different inflation costs that distort incentives and the use of economic resources, which, in our context, are spread around the world, affecting R&D activities and, thereby, the global economic growth rate, the technological-knowledge bias, and the intra- and inter-country wage inequality. Infinitely-lived households inelastically supply labor, skilled and unskilled, maximize utility obtained with the consumption of the homogeneous final good, and earn income from labor and from investments in financial assets and money balances. This is a dynamic general-equilibrium endogenous growth model in which the homogeneous final good can thus be used in consumption and investment (production of intermediate goods and R&D), and the dynamic general equilibrium implies that firms and households are rational and solve their problems, free-entry R&D conditions are met, and markets clear.

2.1 Consumers

The economies are populated by a fixed number of infinitely-lived households that consume and collect income from investments in financial assets and in money balances, and from labor. In each country, North and South, there is a representative household, which inelastically supplies unskilled labor, L, or skilled labor, H, to final-good firms. We assume that consumers have perfect foresight concerning the technological change over time and choose the path of final-good aggregate consumption $\{C(t), t \geq 0\}$ to maximize discounted lifetime utility. With a constant intertemporal elasticity of substitution (CIES) instantaneous utility function, the infinite horizon lifetime utility is $U = \int_0^\infty \left(\frac{C(t)^{1-\theta}-1}{1-\theta} \right) e^{-\rho t} dt$, where $\rho > 0$ is the subjective discount rate and $\theta > 0$ is the inverse of the intertemporal elasticity of substitution. The maximization is subject to the flow budget constraint:

$$\dot{a}(t) + \dot{m}(t) = r(t) \cdot a(t) + w_L(t) \cdot L + w_H(t) \cdot H - C(t) + \tau(t) - \pi(t) \cdot m(t) + i(t) \cdot b(t), \quad (2.1)$$

where: $a(t)$ denotes the household's real financial assets; $m(t)$ is the household's real money balances; $r(t)$ is the real interest rate; w_L and w_H are the wages paid to L and H, respectively; $\tau(t)$ denotes a lump-sum transfer/tax from the monetary authority; $\pi(t)$ is the inflation rate, which determines the cost of holding money; and $b(t)$ is the amount of money lent by households to firms (final-good firms, intermediate-good firms, and R&D investments) and its return is $i(t)$. Thus, the CIA constraints imply that $b(t) \le m(t)$.[3] From standard dynamic optimization, we derive a no-arbitrage condition between real money balances and real financial assets (this is equivalent to the well-known Fisher equation) and the optimal path of consumption (the households' Euler equation),[4]

$$i(t) = r(t) + \pi(t), \quad (2.2)$$

$$\dot{C}(t) = \frac{1}{\theta} \cdot (r(t) - \rho) \cdot C(t). \quad (2.3)$$

The transversality conditions are $\lim_{t \to +\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot a(t) = 0$ and $\lim_{t \to +\infty} e^{-\rho t} \cdot C(t)^{-\theta} \cdot m(t) = 0$.

2.2 Monetary authority

The monetary sector in each country is considered as in, e.g., Chu and Cozzi (2014). The nominal money supply in each country is denoted by $M(t)$ and its growth rate is $\mu(t) \equiv \frac{\dot{M}(t)}{M(t)}$. Real money balances are then $m(t) = \frac{M(t)}{P(t)}$, where $P(t)$ is the nominal price of the final good. Since the growth rate of $P(t)$ is the inflation rate, $\pi(t) \equiv \frac{\dot{P}(t)}{P(t)}$, the growth rate of $m(t)$ is $\frac{\dot{m}(t)}{m(t)} = \mu(t) - \pi(t)$. We consider that the monetary authority adopts an inflation targeting framework, in which the monetary policy instrument is the nominal interest rate. In this context, we follow the literature and assume that the nominal interest rate is exogenously chosen by the monetary authority (e.g., Chu and Cozzi 2014, Chu and Ji 2016, Chu et al. 2017, and Chu et al. 2019a), so that $i(t) = i$, and thus $\pi(t)$ is endogenously determined according to the Fisher equation (2.2), for each $r(t)$: $\pi(t) = i - r(t)$. Then, given $\pi(t)$, the growth rate of the nominal money supply will be endogenously determined according to $\mu(t) = \frac{\dot{m}(t)}{m(t)} + \pi(t)$. That is, the monetary authority will endogenously adjust the money growth rate to whatever level is needed for the interest rate i to prevail. As isusual in the literature, we consider that to balance its budget, the monetary authority returns

[3] We do not consider the possibility of CIA constraint on consumption (on the contrary to Chu and Cozzi 2014). As we focus on inelastic labor supply to preserve the analytical tractability of the model, this additional CIA constraint would have no effect on the equilibrium allocations, with the exception of the real money balances.

[4] As will be shown below, in equilibrium the real interest rate, r, will be the same worldwide.

the seigniorage revenues to households as a lump-sum transfer, i.e., $\tau(t) = \frac{\dot{M}(t)}{P(t)} = \frac{(m(t)\dot{\cdot}P(t))}{P(t)} = \frac{\dot{m}(t)\cdot P(t)+\dot{P}(t)\cdot m(t)}{P(t)} = \dot{m}(t) + \pi(t)\cdot m(t)$.[5]

2.3 Production and price decisions

Final-goods sector. Following Acemoglu and Zilibotti (2001) and Afonso (2012), in each country, North and South, each final good, indexed by $n \in [0,1]$, is produced by one of two technologies. The L-technology uses L complemented with a continuum of L-specific intermediate goods indexed by $j \in [0,J]$. The H-technology's inputs are H complemented with a continuum of H-specific intermediate goods indexed by $j \in [J,1]$. Both productions are affected by a scaling variable A, common to both technologies, representing the productivity level dependent on the country's domestic institutions. The constant returns to scale production function at time t is:

$$Y_n(t) = \begin{cases} A\left[\int_0^J z_n(j,t)^{1-\alpha}dj\right]\left[(1-n)\cdot l\cdot L_n\right]^{\alpha} & , if\ n \leq \overline{n}(t) \\ A\left[\int_J^1 z_n(j,t)^{1-\alpha}dj\right](n\cdot h\cdot H_n)^{\alpha} & , if\ n > \overline{n}(t) \end{cases}. \tag{2.4}$$

By considering $z_n(j,t) = q^{k(j,t)}x_n(j,t)$ in (2.4), the integral terms are the contributions to production of quality-adjusted intermediate goods. The size of each quality upgrade obtained with each success in R&D is $q>1$. The rungs of the quality ladder are indexed by k, with higher ks denoting higher quality. At time 0 the top quality good in each intermediate good has a quality index $k = 0$. At period t the highest quality good produced by j has a quality index $k(j,t)$, which is used due to profit maximizing limit pricing by the monopolist producers of intermediate goods. The quantity $x_n(j,t)$ of j is used, together with its specific labor, to produce $Y_n(t)$. The term $(1-\alpha)$ is the intermediate-goods input share, and $\alpha \in (0,1)$ is the labor share. In (2.4), the labor terms include the quantities employed in the production of the n^{th} final good, L_n and H_n, and two corrective, but important, factors accounting for productivity differentials. An absolute productivity advantage of skilled over unskilled labor is accounted for by assuming $h > l \geq 1$. A relative productivity advantage of either labor type is captured by the adjustment terms n and $(1-n)$. These adjustment terms transform the index n into an ordering index, meaning that final goods indexed by larger ns are relatively more intensive in skilled labor. Since $n \in [0,1]$, there is a threshold final good, $\overline{n}(t)$, endogenously determined, at which the switch from unskilled to skilled technology becomes advantageous. The production function (2.4) combines complementarity between inputs in each technology, L and H, and substitutability between the

[5]The long-run equilibrium relationships will reveal a relationship between the inflation rate, π, and the nominal interest rate, i, implying that we can extend all the comparative-statics results pertaining to shifts in i also to shifts in the steady-state inflation rate, π^*. Therefore, one can consider the inflation rate as the policy variable directly controlled by the monetary authority. The consideration of the nominal interest rate as the policy instrument, however, simplifies the analytical derivation of the steady-state equilibrium of the model without changing the comparative-statics results and has become standard in the previous literature on the influence of inflation on growth.

two technologies since optimally only the L-technology is used to produce final goods indexed by $n \leq \overline{n}(t)$, and only the H-technology is used to produce goods with $n > \overline{n}(t)$ – e.g., Afonso (2012). That is, $\overline{n}(t)$ defines the structure (or specialization) of final-goods production: at each time t, there are $\overline{n}(t)$ final goods produced with the L-technology and $1 - \overline{n}(t)$ final goods produced with the H-technology. Hence, in production function (2.4), $H_n(t) = x_n(j,t) = 0$, for $0 \leq j \leq J$, $\forall 0 \leq n \leq \overline{n}(t)$ and $L_n(t) = x_n(j,t) = 0$, for $J < j \leq 1$, $\forall \overline{n}(t) \leq n \leq 1$, and from the competitive profit maximization conditions by the representative producer of n^{th} final good, $\pi_n = p_n \cdot Y_n - \int_0^J p(j) \cdot x_n(j)dj - \int_0^J p(j) \cdot x_n(j)dj - w_L \cdot L_n - w_H \cdot H_n$, the demand for each intermediate good j by this producer is $x_n(j,t) = (1-n) \cdot l \cdot L_n \left[\frac{Ap_n(t)\cdot(1-\alpha)}{p(j,t)|_{0<j\leq J}}\right]^{\frac{1}{\alpha}} q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}$ if $0 < j \leq J$, $\forall\ 0 \leq n \leq \overline{n}(t)$, and $x_n(j,t) = n \cdot h \cdot H_n \left[\frac{Ap_n(t)\cdot(1-\alpha)}{p(j,t)|_{J<j\leq 1}}\right]^{\frac{1}{\alpha}} q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}$ if $J < j \leq 1$, $\forall\ \overline{n}(t) \leq n \leq 1$, where $p_n(t)$ is the real price of final good n and $p(j,t)$ is the real price of intermediate good j (prices given for the perfectly competitive producers of final goods).[6] The higher the $\overline{n}$, the higher the number of varieties produced with the unskilled technologies and, since L is exogenous, the lower the average quantity produced of each L variety.

Intermediate-goods sector without trade. Firms in the intermediate-goods sector use one unit of aggregate output to produce one unit of j whereby its marginal cost is 1. However, for intermediate goods used in L-technology and in H-technology, a CIA constraint is introduced on the production by assuming that firms use money, borrowed from households subject to the nominal interest rate $i(t)$, to pay for a fraction $\Omega_o \in [0,1]$,[7] where $\mathcal{L} = L$ or $\mathcal{L} = H$, of the input. Since firms cannot repay this amount to households until they earn revenue from production, households are effectively providing credit to these firms (e.g., Feenstra 1986, Gil and Iglésias 2020). Hence, the cost of intermediate good j has the following operational and financial component $(1 - \Omega_{\mathcal{L}}) \cdot 1 + \Omega_{\mathcal{L}} \cdot (1 + i(t)) \cdot 1$ if j is used in the $\mathcal{L}$-technology, and thus the cost functions are $(1 + \Omega_{\mathcal{L}} \cdot i(t))$.[8]

Each quality of j is exclusively produced by the owner of its patent and, at time t, this monopolist obtains the profit flow $\pi(j,t)|_{0<j\leq J} = \left[p(j,t)|_{0<j\leq J} - (1+\Omega_L \cdot i(t))\right] X(j,t)|_{0<j\leq J}$ or $\pi(j,t)|_{J<j\leq 1} = \left[p(j,t)|_{J<j\leq 1} - (1+\Omega_H \cdot i(t))\right] X(j,t)|_{J<j\leq 1}$, where $X(j,t)|_{0<j\leq J} = \int_0^{\overline{n}(t)} x_n(j,t)\,dn$ and $X(j,t)|_{J<j\leq 1} = \int_{\overline{n}(t)}^1 x_n(j,t)\,dn$ represent the aggregate demand for the top quality, obtained from the demand by the respective final-goods producers at each t. Since intermediate goods, bought by the producers

[6] Prices of output in each sector are relative to the price of the output (as in Chu et al. 2019b). As in Chu et al. (2015), the law of one price holds such that $P_{Y,S}^{Nominal}(t) = e(t) \cdot P_{Y,N}^{Nominal}(t)$, where e is the nominal exchange.

[7] Firms borrow domestic currency from domestic households. There is no incentive for firms to borrow foreign currency and convert it into domestic currency even when the nominal interest rates differ across regions because uncovered interest rate parity holds in equilibrium. To see this, remember our assumption of the law of one price, which implies that $\frac{\dot{e}(t)}{e(t)} = \pi_S(t) - \pi_N(t)$. Together with our result (to be shown below) that, in equilibrium, the real interest rate, r, is the same worldwide, it is thus ensured that, under an equilibrium of rational expectations, $i_N = i_S - \frac{\dot{e}(t)}{e(t)}$.

[8] In other words, $\Omega_{\mathcal{L}}$ measures the intensity of the CIA constraint on intermediate-goods production used by the $\mathcal{L}$-technology, respectively.

of final goods, fully depreciate at the end of each t, the monopolist faces no dynamic constraints and every time$t$ chooses $p(j,t)$ in order to maximize π, obtaining: $p(j,t)|_{0<j\leq J} = \frac{1+\Omega_L \cdot i(t)}{1-\alpha}$ or $p(j,t)|_{J<j\leq 1} = \frac{1+\Omega_H \cdot i(t)}{1-\alpha}$, which, in any case, is a mark-up over the marginal cost since $0<\alpha<1$. Hence, in each range of j, each mark-up is constant across t, k, and j. As the leader is the only one legally allowed to produce the highest quality, it will use pricing to wipe out sales of lower quality. Following Grossman and Helpman (1991, chs. 4 and 12) limit pricing by each leading monopolist is optimal such that $q \equiv \frac{1}{1-\alpha}$ and to capture the entire market (Barro and Sala-i-Martin 2004, ch. 7), $p(j,t)|_{0<j\leq J} = q\left[1+\Omega_L \cdot i(t)\right]$ or $p(j,t)|_{J<j\leq 1} = q\left[1+\Omega_H \cdot i(t)\right]$.

Economic structure given the inputs. The optimal choice of L- or H-technology is thus reflected in $\overline{n}(t)$, obtained from profit maximization (by perfectly competitive final-goods producers and by intermediate-goods monopolists) and full-employment equilibrium in factor markets, given the labor supply and the current state of technological knowledge,

$$\overline{n}(t) = \left\{ 1 + \left[G(t) \left(\frac{h \cdot H}{l \cdot L} \right) \left(\frac{1+\Omega_L \cdot i}{1+\Omega_H \cdot i} \right)^{\left(\frac{1-\alpha}{\alpha}\right)} \right]^{\frac{1}{2}} \right\}^{-1}, \tag{2.5}$$

$$\text{where: } G(t) \equiv \frac{Q_H(t)}{Q_L(t)},\ Q_L(t) \equiv \int_0^J q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}\, dj,\ Q_H(t) \equiv \int_J^1 q^{k(j,t)\left[\frac{1-\alpha}{\alpha}\right]}\, dj, \tag{2.6}$$

i.e., Q_L and Q_L are aggregate quality indexes of the technological-knowledge stocks, and the ratio $G \equiv \frac{Q_H}{Q_L}$ is the appropriate measure of the technological-knowledge bias.[9] The threshold final good $\overline{n}(t)$ can be implicitly expressed in terms of price indexes, which is achieved by considering that in the production of the threshold $n = \overline{n}(t)$ a firm that uses L-technology and a firm that uses H-technology should break even, resulting in the following ratio of index prices of goods produced with H and L technologies:

$$\frac{p_H(t)}{p_L(t)} = \left(\frac{\overline{n}(t)}{1-\overline{n}(t)} \right)^{\alpha}. \tag{2.7}$$

The relative price of final goods produced with the H-technology, $\frac{p_H}{p_L}$, is low when the threshold final good, $\overline{n}$, is small. In this case, the demand for H-intermediate goods is low, which, as we see below, discourages R&D activities aimed at improving their quality by the *price channel.*

The composite final good, Y, is produced by a continuum of firms, indexed by $n \in [0,1]$, such that $Y(t) = \int_0^1 p_n(t) \cdot Y_n(t)\, dn$, where $p_n(t)$ and $Y_n(t)$ are, respectively, the price and the output of

[9] From (2.5), the threshold final good, $\overline{n}(t)$, is small, meaning that the fraction of final goods using the H-technology in (2.4) is large, when the technological knowledge, G, is highly H-biased, the relative supply of H, $\frac{H}{L}$, is large, the absolute advantage of the skilled labor, $\frac{h}{l}$, is strong, and the relative intensity of the CIA constraints on intermediate-goods used by the H-technology, $\frac{\Omega_H}{\Omega_L}$, is smaller.

the final good n. Plugging the demand functions $x_n(j,t)$ into (2.4) the supply of n is obtained,

$$Y_n = A^{\frac{1}{\alpha}} \left[\frac{p_n \cdot (1-\alpha)}{q}\right]^{\frac{1-\alpha}{\alpha}} \left\{(1+\Omega_L \cdot i)^{\left(\frac{\alpha-1}{\alpha}\right)} \cdot (1-n) \cdot l \cdot L_n \cdot Q_L + (1+\Omega_H \cdot i)^{\left(\frac{\alpha-1}{\alpha}\right)} \cdot n \cdot h \cdot H_n \cdot Q_H\right\}, \tag{2.8}$$

so that the appropriate real price of the aggregate final good is 1 at each time t, $P_Y = \exp\int_0^1 \ln p_n(t)\, dn = 1$, and thus bearing also in mind (2.5) and (2.7), the real price-indexes of L- and H-final goods are, respectively, $p_L(t) = p_n\,(1-n)^{\alpha} = \exp(-\alpha)\overline{n}(t)^{-\alpha}$ and $p_H(t) = p_n n^{\alpha} = \exp(-\alpha)\left[1-\overline{n}(t)\right]^{-\alpha}$; therefore,

$$Y = \exp(-1)\left[\frac{(1-\alpha)}{q}\right]^{\frac{1-\alpha}{\alpha}} \left\{\left[(1+\Omega_L \cdot i)^{\frac{\alpha-1}{\alpha}} \cdot l \cdot L \cdot Q_L\right]^{\frac{1}{2}} + \left[(1+\Omega_H \cdot i)^{\frac{\alpha-1}{\alpha}} \cdot h \cdot H \cdot Q_H\right]^{\frac{1}{2}}\right\}^2, \tag{2.9}$$

which clearly shows how final-production growth – the economic growth rate – is driven by the technological-knowledge progress. From the profit maximization conditions of final-goods production full employment in the labor market is guaranteed, which is also implicit in $\overline{n}$, and results that the marginal productivity of each labor type equals its cost. The equilibrium skilled premium, measuring intra-country wage inequality, yields:

$$\frac{w_H(t)}{w_L(t)} = \left[G(t)\left(\frac{h}{l}\frac{L}{H}\right)\left(\frac{1+\Omega_L \cdot i}{1+\Omega_H \cdot i}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}. \tag{2.10}$$

From (2.10), the skill premium, $\frac{w_H}{w_L}$, is greater when the technological knowledge, $G \equiv \frac{Q_H}{Q_L}$, is more skill-biased, the absolute advantage of the skilled labor, $\frac{h}{l}$, is strong, skilled labor, $\frac{H}{L}$, is relatively scarcer, and the relative intensity of the CIA constraints on intermediate-goods used by the H-technology, $\frac{\Omega_H}{\Omega_L}$, is smaller. The wage ratio depends on the nominal interest rate positively if and only if financial restrictions affecting the unskilled production are greater than those affecting the skilled production, $\Omega_L > \Omega_H$.

Equations (2.5), (2.7), and (2.10) are useful in foreseeing the operation of the *price (of final goods) channel* from the stocks (of labor, technological knowledge, and CIA constraints) to the flows of resources used in R&D and to wage inequality. For example, in a country relatively H-abundant and (or) with a large technological-knowledge bias and (or) with a strong CIA constraint in intermediate goods used by the L-technology, $\overline{n}(t)$ is small, i.e., many final goods are produced with the H-technology and thus final goods produced with the H-technology are sold at a relatively low price. Profit opportunities in the production of intermediate-goods used by the relatively high-priced L-technology final goods induce a change in the direction of R&D against the technological-knowledge bias and in favor of unskilled wages, i.e., there are stronger incentives to develop technologies when the final goods produced by these technologies command higher prices. The overall effect on the technological-knowledge bias thus depends on the magnitude of the two contradictory channels – *price channel* and *market-size channel*.[10]

2.4 Research and development sector

Research and development drives the North and South economic growth. It results in innovative designs for the manufacture of intermediate goods, increasing their quality, in the North and in imitation of Northern designs in the South. Designs are (only) domestically patented and the leader firm in each intermediate-goods industry – the one that produces according to the latest patent – uses limit pricing to assure monopoly. In turn, the innovation process is stochastic.[11] In fact, the probability of quality improvement can be interpreted as the probability of innovation in the case of $o = N$ and of imitation if $o = S$, which are defined, respectively, in the following way:

$$I_N(j,t) = y_N(j,t) \cdot \beta_N q^{k_N(j,t)} \cdot \zeta_N^{-1} q^{-\alpha^{-1} k(j,t)} \cdot (\mathcal{L}_N + \mathcal{L}_S)^{-\xi_N}, \tag{2.11}$$

$$I_S(j,t) = \frac{y_S(j,t) \cdot \beta_S q^{k_S(j,t)} \cdot \zeta_S^{-1} q^{-\alpha^{-1} k(j,t)} \cdot (\mathcal{L}_N + \mathcal{L}_S)^{-\xi_S}}{B_D(j,t) \cdot B_T(j,t) \cdot f(\widetilde{Q}_{\mathcal{L}}(t), d)^{-\sigma + \widetilde{Q}_{\mathcal{L}}(t)}}, \tag{2.12}$$

where: (i) $y_o(j,t)$ $(o = N, S)$ is the flow of domestic final-good resources devoted to R&D in intermediate good j, which defines our framework as a lab equipment model (Rivera-Batiz and Romer 1991); (ii) $\beta_0\ q^{k_o(j,t)}$, $\beta_N > 0$, $0 < \beta_S < \beta_N$, $k_S \leq k$ represents learning-by-past domestic R&D, as a positive learning effect of accumulated public knowledge from past successful R&D (e.g., Romer 1990), considering that the learning-by-past imitations is lower than the learning-by-past innovations; (iii) $\zeta_o^{-1} q^{-\alpha^{-1} k(j,t)}$, $\zeta_N > \zeta_S > 0$, is the adverse effect – cost of complexity – caused by the increasing complexity of quality improvements (e.g., Kortum 1997), assuming also that the complexity cost of imitation is lower than the innovation's in line with Mansfield et al. (1981) and Teece (1977);[12] (iv) $(\mathcal{L}_N + \mathcal{L}_S)^{-\xi_o}$, $\mathcal{L} = L$ when $0 \leq j \leq J$ and $\mathcal{L} = H$ when $J < j \leq 1$, $\xi_o > 0$, is the adverse effect of market size, capturing the idea that the difficulty of introducing new quality intermediate goods and replacing old ones is proportional to the size of the market measured by the respective labor. That is, for reasons of simplicity, we reflect in R&D the costs of scale due to coordination among agents, processing of ideas, informational, organizational, marketing, and transportation costs (as in e.g., Dinopoulos and Segerstrom 1999, and Dinopoulos and Thompson 1999); (v) $B_D(j,t) \cdot B_T(j,t) \cdot f(\widetilde{Q}_{\mathcal{L}}(t), d)^{-\sigma + \widetilde{Q}_{\mathcal{L}}(t)}$, $0 < \widetilde{Q}_{\mathcal{L}}(t) < 1$, $\sigma > 0$; this is a catching-up term, specific to the South, which sums up positive effects of imitation capacity and backwardness. Terms $B_D(j,t)$ and $B_T(j,t)$ are positive exogenous variables, which capture, respectively, other domestic and external determinants of imitation capacity.[13]

[10] This *price channel* appears e.g., in Acemoglu (2002), although is always dominated by the market-size channel. Our model removes the *cost-of-the-market size* – see below the equilibrium R&D.

[11] See Online Appendix OB.

[12] This complexity cost, together with the positive learning effect (ii), exactly offsets the positive influence of the quality rung on the profits – see e.g., Barro and Sala-i-Martin (2004, ch. 7).

[13] Details on the Backward Function $f(\widetilde{Q}_{\mathcal{L}}(t), d)^{-\sigma + \widetilde{Q}_{\mathcal{L}}(t)}$ are given in Appendix B.

2.5 International trade

Under international trade, the state-of-the-art intermediate goods, available internationally, embody the North's technological knowledge – Q_H and Q_L. Assuming that endowments of labor are such that the North is relatively H-abundant, i.e.,

$$\frac{H_N}{L_N} > \frac{H_S}{L_S}, \tag{2.13}$$

and also that:

$$\frac{H_N}{L_N}\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\frac{1-\alpha}{\alpha}} > \frac{H_S}{L_S}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\frac{1-\alpha}{\alpha}}, \tag{2.14}$$

the comparison of inter-country threshold final goods – see (OA.1) in Online Appendix OA – shows that $\overline{n}_S > \overline{n}_N$. In other words, since Northern and Southern producers have access to the same state-of-the-art intermediate goods under trade, and differences in the structure of final-goods production is determined by differences in domestic labor endowments and in CIA constraints on the production of intermediate goods, it implies that, under international trade, the North produces more H-technology final goods than the South. Notice that through the operation of the *price channel*, the $\overline{n}_S$ is larger than in pre-trade. This is because, as discussed above, labor endowments influence the direction of R&D in such a way that there are stronger incentives to improve technological knowledge that saves the relatively scarce type of labor. Since the South is H-scarce, its pre-trade technological-knowledge bias is $\frac{Q_{H,S}}{Q_{L,S}} > \frac{Q_H}{Q_L} \Longleftrightarrow G_S > G$.

Concerning the level effect on wages, the access to more productive intermediate goods shifts upwards the demand for both labor types in the South. The resulting absolute (and relative to the North) benefit to both Southern labor types is not balanced. Indeed, the level effect reduces intra-South wage inequality (the skilled-labor premium), as shown by plugging the technological-knowledge bias implied by the assumed relative labor endowments (2.13) into (2.10),

$$\frac{w_{H,S}(t_0)}{w_{L,S}(t_0)} = \left[G(t_0)\left(\frac{h}{l}\frac{L_S}{H_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}} < \left.\frac{w_{H,S}(t_0)}{w_{L,S}(t_0)}\right|_{pre-trade} = \left[G_S(t_0)\left(\frac{h}{l}\frac{L_S}{H_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}. \tag{2.15}$$

It is important to note that a shock on inflation depends on the relative unskilled-skilled financial constraints and on the change in the technological-knowledge bias. Thus it is straightforward to see that for a given $G(t_0)$ – which is verified in the short run –, an increase in the inflation rate increases (decreases) wage inequality if the unskilled (skilled) sector is more financially constrained than the skilled (unskilled) sector, which happens both in the North and in the South. We summarize the result as follows.

Remark. for a given $G(t_0)$: (a) After an inflation shock that raises wage inequality due to a more constrained unskilled sector, openness to trade may counter-influence the effect of that shock; and also (b) if $\frac{\Omega_{L,S}}{\Omega_{H,S}} > \frac{\Omega_{L,N}}{\Omega_{H,S}}$, a monetary shock that increases inflation, leads wage inequality to increase more in the South than in the North; the effect of trade is opposite to the effect of a monetary shock leading to converging wage inequalities.

3 General equilibrium

As the countries' economic structure has been characterized for given state of technological knowledge, we now include the general equilibrium dynamics of technological knowledge.

3.1 Intermediate Goods Pricing and Equilibrium R&D

3.1.1 International Limit pricing

We turn now to the production and international pricing of intermediate goods. Since, by assumption, the production of intermediate goods and R&D are financed by the resources saved after consumption of the composite final good, we consider that, in each country, the production function of intermediate goods is identical to the production function of the composite final good specified by equations (2.4) and (2.9).[14] Thus, the marginal cost of producing an intermediate good equals the marginal cost of producing the composite final good, MC, which, due to perfect competition in the final-goods sector, equals the price of the composite final good. In consequence, in each country the marginal cost of producing an intermediate good is independent of its quality level and is identical across all domestic industries. Regarding inter-country differences, we assume that the marginal cost of producing the composite final good in the South is less than in the North, in order to allow for the entry of the South's intermediate goods in international markets. Normalizing to 1 the marginal cost in the North yields $0 < MC_S < MC_N = 1$, allowing the Southern producer of the same quality rung, k, to under-price its Northern competitor.

Even without international protection of patents, the current producer of each intermediate good enjoys some international monopoly power: for example, if the current producer is from the North, thus being challenged by either another Northerner or a Southerner imitator, monopoly is temporarily assured by IPRs in the North and costly imitation in the South. Notice that the length and magnitude (measured by the mark-up) of the monopoly power are shortened by international competition as without international trade of intermediate goods the current producer in the North is challenged only by another Northerner and not by a Southern imitator with lower marginal cost. In general, there are three possible sequences of successful R&D outcomes and their limit pricing consequences, at time t, for given quality k at time $t-dt$: (i) first in $t-dt$, N produces and exports quality k and then in t, N produces and exports quality $k+1$;(ii) in $t-dt$, N produces and exports quality k and then in t, S produces and exports quality k;in $t-dt$, S produces and

[14] Or, equivalently, as in Barro and Sala-i-Martin (2004, ch. 8).

exports quality k and then in t, N produces and exports quality $k+1$.[15]

Following Afonso (2012), let: (i) $\Phi_{\mathcal{L}}$ and $(1-\Phi_{\mathcal{L}})$ be the proportion of intermediate goods of $\mathcal{L}$-type with production in the North and in the South, respectively; (ii) $\Psi_{\mathcal{L}}$ be the proportion of intermediate goods of $\mathcal{L}$-type produced in the North having overcome imitator competition; (iii) $(1-\Psi_{\mathcal{L}})$ be the proportion of intermediate goods of m-type produced in the North having overcome innovator competition. The proportion of intermediate goods produced in the North increases with the probability of innovation and decreases with the probability of imitation. It is possible to define a price index for the m-type intermediate goods – at each moment in time – as a weighted average of the limit prices in Table 2:

$$\overline{p}_{\mathcal{L}} = \Phi_{\mathcal{L}} \cdot q \cdot [1 - \Psi_{\mathcal{L}}\ (1 - MC_S)] \cdot [1 + \Omega_{\mathcal{L},N} \cdot i_N] + (1-\Phi_{\mathcal{L}}) \cdot [1 + \Omega_{\mathcal{L},S} \cdot i_S]\,, \tag{3.1}$$

and thus price index in equation (3.1) is affected by the costs imposed by the CIA constraints.

3.1.2 Free-entry and non-arbitrage conditions in R&D

The value of the leading-edge patent for the producer of an intermediate good j with quality level k at time t is the present value of flow of profits given by the following equation:

$$V_S(j,k,t,T(k)) = \int_t^{t+T(k)} \Pi(j,s) \exp\left(-\int_t^s r(w)dw\right) ds,$$

where $T(k)$ is the duration of the patent during which there is no innovation in the quality level of intermediate good j by another entrant.[16]

Given the functional forms (2.11) and (2.12) of the probabilities of success in R&D, which rely on the resources – composite final goods – allocated to it, free-entry equilibrium is defined by the equality between expected revenue and resources spent. We assume that the financing of R&D costs also requires money borrowed from households, such that a CIA constraint on R&D activities also exists alongside that on manufacturing of intermediate goods. Therefore, the R&D cost has an operational and a financial component, that is, $y\,(j,t)+\Upsilon_{\mathcal{L}} \cdot i(t) \cdot y\,(j,t)$, where $\Upsilon_{\mathcal{L}} \in [0,1]$, $\mathcal{L}=L$ or $\mathcal{L}=H$, is the share of the R&D cost that requires the borrowing of money from households. By considering free entry in R&D activities, free access to the R&D technology, and a proportional relationship between successful R&D and the share of R&D effort, the R&D spending aimed at,

[15] We follow Grossman and Helpman (1991, ch. 12), assuming that limit pricing by each leading monopolist is optimal. Depending on whether $q(1-\alpha)$ is greater or less than MC, the leader of each industry would, respectively, use the monopoly pricing $p = \frac{MC}{1-\alpha}$ or the limit pricing $p = q\ MC$ to capture the entire domestic market (Barro and Sala-i-Martin 2004, ch. 7). In order to rule out monopoly pricing, we assume that the size of each quality improvement, q, is not large enough. Table 2 in Appendix C summarizes all the possibilities.

[16] For a complete derivation and explanation of the value of the patent, see Online Appendix OB and references therein (Aghion and Howitt 1992, Barro and Sala-i-Martin 2004, Gil et al. 2013).

for example, imitating j should equal the expected payoff generated by the imitation; i.e.,

$$I_S(j,t)\ V_S(j,t) = y_S(j,t) \cdot \left(1 + \Upsilon_{\mathcal{L},S} \cdot i_S\right), \tag{3.2}$$

where $V_S(k,j,t)$ is the expected current value of the flow of profits to the monopolist producer of intermediate good j, the market value of the patent, or the value of the monopolist firm owned by domestic consumers.

Bearing in mind (2.12), we remove market-size scale effects ($\xi_S = 1$) (following Jones, 1995 and the literature that followed that seminal paper), taking into account the amount of profits, Π_S, at time t, for the monopolist producer of intermediate good j, using an imitation of quality k, which depends on the marginal cost, the mark-up, the world demand for the intermediate good j by final-goods producers, and the price in the second sequence in Table 2, then plugging (OB.2) into (3.2), and solving for I_N, the equilibrium probability of a successful innovation in a H-specific intermediate good – given the interest rate and the price indexes of final goods – is

$$\begin{aligned} I_{H,N}(t) = & \underbrace{\beta_S \cdot \zeta_S^{-1} \cdot B_D \cdot B_T \cdot h \cdot (1-\alpha)^{\alpha^{-1}} \cdot (1 - MC_S)}_{Technology\ channel} \cdot \underbrace{D_H(t)}_{Price\ channel} \\ & \underbrace{f(\widetilde{Q}_H(t), d)^{-\sigma + \widetilde{Q}_H(t)} \cdot \widetilde{Q}_H(t)}_{backwardness\ channel} \cdot \underbrace{\frac{(1 + \Omega_{H,S} \cdot i_S)^{\left(\frac{\alpha-1}{\alpha}\right)}}{1 + \Upsilon_{H,S} \cdot i_S}}_{CIA-constraint\ channel} - r_S(t) \end{aligned}, \tag{3.3}$$

where: $D_H(t) = \frac{H_S}{H_S+H_N}\left[A_S \cdot p_{H,S}(t)\right]^{\alpha^{-1}} + \frac{H_N}{H_S+H_N}\left[A_N \cdot p_{H,N}(t)\right]^{\alpha^{-1}}$, where $A_N > A_S$and $H_N > H_S$ measuring respectively the institutional quality and skilled-labor levels in the North and in the South.[17]

Since the probability of successful innovation determines the speed of technological-knowledge progress, equilibrium can be translated into the path of Northern technological knowledge, from which free trade in intermediate goods allows the South to benefit as well. The relationship turns out to yield the expression, where (3.3) is plugged in, for the equilibrium rate of growth of, for example, H-specific technological knowledge:

$$\frac{\dot{Q}_H(t)}{Q_H(t)} = \left[I_{H,N}(t) - r_S(t)\right]\left[q^{(1-\alpha)\alpha^{-1}} - 1\right]. \tag{3.4}$$

In (3.3) we can identify four channels: (i) the *technology channel* related with parameters from the productive side of the economy, (ii) the *backwardness channel* related with the North-South technological-knowledge gap, (iii) the *price channel* through which the trade and the inter-sectors,

[17] The equilibrium $\mathcal{L}$-specific $I_{\mathcal{L},N}$ in (3.3) is independent of j and k due to two reasons: the first and most substantial one is the removal of scale of technological-knowledge effects – see the exponents of q in the demand of intermediate goods above, which impacts the expression of profits, and in equation (2.12); the second is the simplifying assumption according to whichB_D and B_T in equation (2.12), are not specific to each intermediate good.

L and H, and CIA constraints on the production of intermediate goods operate, and the intra-sectors, L or H, (iv) *CIA-constraint channel* that is also affected by trade. It should also be noted that the CIA constraint channel tends to decrease the probability of innovation both when constraints affect the manufacturing sector through $\Omega_{H,S}$, and through the R&D sector, $\Upsilon_{H,S}$. However, given the fact that the CIA constraints linked with R&D are (empirically) higher and are not affected by the relative labor shares, this means that that CIA constraints linked with R&D have greater influence in the probability of innovation. It is clear in (3.4) that there are trade feedback effects from imitation to innovation: the access to the state-of-the-art intermediate goods increases production and, thus, the resources available to imitation R&D – feeds back into the innovator, affecting the Northern technological knowledge through creative destruction.

Due to the technological complementarity in the production of final goods, the rate of growth of $\mathcal{L}$-specific technological knowledge – equation (3.4) for the South and $\mathcal{L} = H$ – translates into the growth of demand for $\mathcal{L}$-type labor interrelated with the dynamics of the price indexes of final and intermediate goods ($p_{\mathcal{L},S}$ and $\overline{p}_{\mathcal{L}}$, respectively), such that

$$\frac{\dot{w}_{\mathcal{L},S}(t)}{w_{\mathcal{L},S}(t)} = \frac{1}{\alpha} \cdot \frac{\dot{p}_{\mathcal{L},S}(t)}{p_{\mathcal{L},S}(t)} + \frac{\alpha - 1}{\alpha} \cdot \frac{\dot{\overline{p}}_{\mathcal{L}}(t)}{p_{\mathcal{L}}(t)} + \frac{\dot{Q}_{\mathcal{L}}(t)}{Q_{\mathcal{L}}(t)}. \tag{3.5}$$

Thus, the path of $\mathcal{L}$-wages in each country relies on the path of domestic demand for $\mathcal{L}$-type labor, which, in turn, relies on the path of: (i) the domestic range of the $\mathcal{L}$-technology, established by threshold $\overline{n}$, which determines prices of (non-tradable) final goods; (ii) the world demand for $\mathcal{L}$-specific intermediate goods, reflected in international prices and driven by technological knowledge.

3.2 Steady state

Since by assumption both countries have access through free trade to the same state-of-the-art intermediate goods and the same technology of production of final goods,[18] the steady-state growth rate must be the same as well. This implies, through the Euler equation (2.3), that interest rates are also equalized between countries in steady state. As for the sectorial growth rates, we note first that the instantaneous aggregate resources constraint in the North and South, $o = S, N$, is $Y_o(t) = C_o(t) + X_o(t) + R_o(t)$,[19] where: (i) $Y_o(t)$ is total resources, the composite final good; (ii) $C_o(t) = \int_0^1 c_o(i,t)di$ is aggregate consumption; (iii) $X_o(t) = \int_0^1 \int_0^1 x_{n,o}(j,t)dndj$ is aggregate intermediate goods; (iv) $R_o(t) = \int_0^1 y_o(j,t)dj$ is total resources spent on R&D. In other words, the aggregate final good is used for consumption and savings, which, in turn, are allocated between production of intermediate goods and R&D.[20] This implies that the steady-state growth rate of

[18]Except for the levels of exogenous productivity, A, and labor, $\mathcal{L}$, in production function (2.4), implying differences in the levels, but not in the growth rates.

[19]For details see Online Appendix OC.

[20]Net exports are always zero since, by assumption, international trade is balanced.

each of these variables is equal to the North's growth rate of technological knowledge. Since the composite final-good production is constant returns to scale in the inputs from equations (2.8) and (2.9), the constant, common to both countries, unique steady-state growth rate, designated by g^*, is

$$\left(\frac{\dot{Q}_H}{Q_H}\right)^* = \left(\frac{\dot{Q}_L}{Q_L}\right)^* = \left(\frac{\dot{Y}}{Y}\right)^* = \left(\frac{\dot{X}}{X}\right)^* = \left(\frac{\dot{R}}{R}\right)^* = \left(\frac{\dot{C}}{C}\right)^* = \left(\frac{\dot{c}}{c}\right)^* = \theta^{-1}\left(r^* - \rho\right) = g^*, \quad (3.6)$$

implying constant steady-state levels of threshold final goods, final and intermediate goods price indexes, wage premia, and North-South gaps in both technological-knowledge types. Although levels remain different (due to international immobility of labor and differences in exogenous productivity and marginal costs), the steady-state growth of wages is equalized between countries as derived by plugging in constant steady-state prices in (3.5), which is a Schumpeterian dynamic result equivalent to the static factor-price equalization Samuelson's result.

From (3.4) and (3.3) for H- and L-technology, as well as from (2.5) and (2.7), $\frac{\dot{Q}_H}{Q_H}$ and $\frac{\dot{Q}_L}{Q_L}$ rise at the same rate if

$$\frac{\left(1+\left[G^*\frac{h}{l}\frac{H_S}{L_S}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2}\right)\left[G^*\frac{h}{l}\frac{H_N}{L_N}\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2}}{\left(1+\left[G^*\frac{h}{l}\frac{H_N}{L_N}\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2}\right)\left[G^*\frac{h}{l}\frac{H_S}{L_S}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2}} -$$
$$- \frac{\frac{H_N A_N^{\alpha-1}}{H_S+H_N} - \frac{f(\widetilde{Q}_L^*,d)^{-\sigma+\widetilde{Q}_L^*}\cdot\widetilde{Q}_L^*}{f(\widetilde{Q}_H^*,d)^{-\sigma+\widetilde{Q}_H^*}\cdot\widetilde{Q}_H^*}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\left(\frac{1+\Upsilon_{L,S}\cdot i_S}{1+\Upsilon_{H,S}\cdot i_S}\right)\frac{L_N A_N^{\alpha-1}}{L_S+L_N}\left[G^*\frac{h}{l}\frac{H_N}{L_N}\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2}}{\frac{f(\widetilde{Q}_L^*,d)^{-\sigma+\widetilde{Q}_L^*}\cdot\widetilde{Q}_L^*}{f(\widetilde{Q}_H^*,d)^{-\sigma+\widetilde{Q}_H^*}\cdot\widetilde{Q}_H^*}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\left(\frac{1+\Upsilon_{L,S}\cdot i_S}{1+\Upsilon_{H,S}\cdot i_S}\right)\frac{L_S A_S^{\alpha-1}}{L_S+L_N}\left[G^*\frac{h}{l}\frac{H_S}{L_S}\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{1/2} - \frac{H_S A_S^{\alpha-1}}{H_S+H_N}} = 0.$$

(3.7)

Equation (3.7) can be found since $\frac{\dot{\widetilde{Q}}_H^*}{\widetilde{Q}_H^*} = \frac{\dot{\widetilde{Q}}_L^*}{\widetilde{Q}_L^*} = 0$. In steady state, the stable and unique endogenous technological-knowledge bias, G^*, depends (in this case, implicitly) on h, l, A_N, A_S, H_N, H_S, L_N, L_S, $\Omega_{L,S}$, $\Omega_{H,S}$, $\Omega_{L,N}$, $\Omega_{H,N}$, $\Upsilon_{L,N}$, $\Upsilon_{H,N}$, and i_o. For simplification and since our main interest is in the effect of credit constraints and nominal interest rate, we rewrite (3.7) as $F(G^*, i_N, i_S) = F_1(G^*, i_N, i_S) - F_2(G^*, i_N, i_S) = 0$. It is straightforward to see that $F_1(G^*, i_N, i_S) > 0$ and, then, for (3.7) to hold, $F_2(G^*, i_N, i_S) > 0$. Thus, the numerator and the denominator of $F_2(G^*, i_N, i_S)$ are both positive or both negative. Note that $\Omega_{L,o} > \Omega_{H,o}$ and $\Upsilon_{L,o} > \Upsilon_{H,o}$, means that credit requirements by unskilled production firms are higher than those of the skilled production firms.

Proposition 1. Let $\Omega_{L,o} > \Omega_{H,o}$ ($\Omega_{L,o} < \Omega_{H,o}$) and $\Upsilon_{L,o} > \Upsilon_{H,o}$ ($\Upsilon_{L,o} < \Upsilon_{H,o}$); *(a) an increase in the nominal interest rate, with* $i_N = i_S$, *– and thus in the inflation rate, with* $\pi_N = \pi_S$ *– decreases (increases)*G^*; *(b) an increase in the nominal interest rate in the South,* i_S, *– and thus in the southern inflation rate,*π_S*– decreases (increases)* G^*.

Proof. In Appendix C. □

As can be seen by the results of Proposition 1, CIA constraints and differences existing in those in N and S are crucial to explain the effects of the monetary policy (or inflation). As can also be obtained from the (realistic) assumption that the North has lower inflation than the South should imply the same qualitative effects of rising inflation, but may differ in quantitative effects, which we will analyze below.[21]

By removing scale effects, the *price channel* dominates the *market-size channel*. However, since $\frac{p_{H,N}}{p_{L,N}}$ remains always lower than $\frac{p_{H,S}}{p_{L,S}}$ due to relative labor endowments, the North-South average (the one that becomes relevant under international trade) relative price of H-technology final goods is higher than the one prevailing in the North alone. As a result, the price channel – discussed above in 2.3 – enhances the relative demand for H-specific new designs, biasing innovation R&D in that direction; i.e., G^* increases and this bias increases the world supply of H-specific intermediate goods; an increase in Ω_L (Ω_H) raises costs in the L-technology (H-technology), thereby diverting, directly or indirectly, resources from R&D activities directed to the L-technology (H-technology) and, thus, improving (penalizing) the steady-state technological-knowledge bias. This also happens with $\frac{h}{l}$, which *ceteris paribus*, increases the relative cost of the H-technology and, in turn, decreases the technological-knowledge bias.

Moreover, from (2.5) and (2.14).[22]

$$\overline{n}_S^* = \left\{1 + \left[G^* \left(\frac{h}{l}\frac{H_S}{L_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1} > \overline{n}_N^* = \left\{1 + \left[G^* \left(\frac{h}{l}\frac{H_N}{L_N}\right)\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1}, \tag{3.8}$$

in steady state. The stable and unique endogenous threshold final good, $\overline{n}^*$ in each country, for a given value of the endogenous technological-knowledge bias, G^*, relies on the (structural) level of the nominal interest rate, i_o, on parameters and variables related to the technology, l, h, L, H, and α, and on the CIA constraints, Ω_L and Ω_H. An increase in l, L, and Ω_H or a decrease in h, H, α, and Ω_L increases $\overline{n}^*$. The sign of the effect of i on $\overline{n}^*$ depends on the values assumed by Ω_L and Ω_H. Note that the *difference* between the threshold final good in the North and in the South is not especially dependent on the technological-knowledge bias, G^*. Proposition 2 highlights the determinants of the steady-state threshold final good, $\overline{n}_o^*$, and particularly the influence of inflation on it.

Proposition 2. *The effect of nominal interest rate – and thus inflation – on the threshold final good depends on the relative strength of the direct effect and the effect through the technological-knowledge bias. Thus, if the technological-knowledge bias is stronger,* $\frac{\partial \overline{n}_o^*}{\partial i_o} \gtreqless 0$ *for* $\Omega_{L,o} \gtreqless \Omega_{H,o}$ *and* $\Upsilon_{L,o} \gtreqless \Upsilon_{H,o}$. *If the direct effect is stronger* $\frac{\partial \overline{n}_o^*}{\partial i_o} \gtreqless 0$ *for* $\Omega_{L,o} \lesseqgtr \Omega_{H,o}$ *and* $\Upsilon_{L,o} \lesseqgtr \Upsilon_{H,o}$, $o = S, N$.

[21] The theoretical study of the influence of other variables on the technological-knowledge bias is beyond the scope of this article as our focus is on credit constraints and inflation.

[22] See also Online Appendix OA.

Proof. In Appendix C. □

For a constant technological-knowledge bias this is straightforward to see, because $\frac{\partial n_o}{\partial i_o} \lesseqgtr 0$ for $\Omega_{L,o} \gtreqless \Omega_{H,o}$. However, the effect can be reversed through the effect of the skill-biased technological knowledge, as stated in the Proposition 2. This proposition highlights that expansionary monetary policies in the South (when compared to the North), together with the most realistic assumption, empirically validated,[23] according to which $\Omega_{L,S} > \Omega_{H,S}$, leading to even higher inflation in the South may lead to a process of convergence in the Southern specialization in skilled-intensive goods.[24]

From (2.15), since $\frac{L_S}{H_S} > \frac{L_N}{H_N}$, the steady-state skill premium in each country is:

$$\left(\frac{w_{H,S}}{w_{L,S}}\right)^* = \left[G^* \left(\frac{h}{l}\frac{L_S}{H_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}} > \left(\frac{w_{H,N}}{w_{L,N}}\right)^* = \left[G^* \left(\frac{h}{l}\frac{L_N}{H_N}\right)\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}, \tag{3.9}$$

i.e., the stable and unique steady-state endogenous skill premium depends on G^*, $\frac{h}{l}$, $\frac{H}{L}$, Ω_L, Ω_H, and i. If the (unskilled-) relative financial constraints in the South are higher than in the North, this contributes to obtaining a higher skill premium in the South than in the North. This also implies a relationship in inter-country wage ratio such as $\left(\frac{w_{H,N}}{w_{H,S}}\right) < \left(\frac{w_{L,N}}{w_{L,S}}\right)$. The greater the difference between (unskilled-) relative financial constraints in the South when compared to the North, the greater the difference in inter-country wage ratios by skills. It is worth noting that the greater the difference between nominal interest rate rates and inflation between the South, S, and the North, N, for a given technological-knowledge bias, the higher the wage inequality in the South relative to the wage inequality in the North for the more realistic case of $\Omega_{L,S} > \Omega_{H,S}$. Proposition 3 highlights the determinants of the steady-state skill premium, $\left(\frac{w_{H,o}}{w_{L,o}}\right)^*$, and particularly the influence of the inflation on it.

Proposition 3. *An increase in the unskilled financial costs in the intermediate goods production, $\Omega_{L,o}$, and in the R&D activities, $\Upsilon_{L,o}$, increases $\left(\frac{w_{H,o}}{w_{L,o}}\right)^*$, whereas an increase in the skilled financial costs in the intermediate-goods production, $\Omega_{H,o}$, and in the R&D activities, $\Upsilon_{H,o}$, decreases $\left(\frac{w_{H,o}}{w_{L,o}}\right)^*$. An increase in the nominal interest rate – and thus in inflation – increases (decreases) wage inequality, $\left(\frac{w_{H,o}}{w_{L,o}}\right)^*$, if the direct effect is stronger than the indirect effect through the technological-knowledge bias for $\Omega_{L,o} \gtrless \Omega_{H,o}$ (and $\Upsilon_{L,o} \gtrless \Upsilon_{H,o}$). Conversely, an increase in the nominal interest rate – and thus in inflation – increases (decreases) wage inequality, $\left(\frac{w_{H,o}}{w_{L,o}}\right)^*$, if the indirect effect is stronger for $\Omega_{L,o} \lessgtr \Omega_{H,o}$ (and $\Upsilon_{L,o} \lessgtr \Upsilon_{H,o}$).*

Proof. In Appendix C. □

[23]See the discussion above in the Introduction.

[24]For instance, the Chinese inflation rate (1987-2018) of about 5.10% (when compared to the US rate of nearly 2.6% in the same period) may have contributed to a convergence of China in skilled-intensive production of intermediate goods.

It is worth noting that if, as is expected, skilled producers will be less financially constrained than unskilled producers, $\Omega_{H,o} < \Omega_{L,o}$, then inflation increases wage inequality as the empirical evidence highlighted (see e.g., Appendix A). This happens when the direct effect is stronger than the effect through the technological-knowledge bias.

Additionally, the model predicts that if the nominal interest rate and inflation are higher in the South and skilled producers are less financially constrained than unskilled ones, then wage inequality tends to rise more in the South. In fact, consistently with recent surveys such as Cigno et al. (2018) and Crozet and Orefice (2017), our model shows that results depend on certain conditions relating to CIA constraints. Note also that $\left(\frac{\dot{w_H}}{w_H}\right)^* - \left(\frac{\dot{w_L}}{w_L}\right)^* = 0$ and wages rise steadily in line with the technological-knowledge progress; i.e., $\left(\frac{\dot{w_H}}{w_H}\right)^* = \left(\frac{\dot{w_L}}{w_L}\right)^* = \left(\frac{\dot{Q_H}}{Q_H}\right)^* = \left(\frac{\dot{Q_L}}{Q_L}\right)^*$. From the previous analysis, for example, an increase in β as well as a decrease in θ, ρ, and ζ increases g^* and has no impact on $\left(\frac{w_H}{w_L}\right)^*$. Hence, any change in these parameters in the sense referred to implies that all workers will earn higher wages in the new steady state (i.e., welfare gains emerge).

As follows from, for example, (2.9) and (3.6), R&D drives steady-state endogenous growth. The intensity of the driving force is, in turn, influenced by international trade and CIA constraints. In order to look at the steady-state effects of international trade in a context with CIA constraints we must investigate g^* further. To this end, since g^* results directly from plugging r^* into the Euler equation (2.3), it is sufficient to compare the steady-state interest rate:

$$
\begin{aligned}
r^* = &\left\{\left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right]\ \theta + 1\right\}^{-1} \left\{\beta_S \cdot \zeta_S^{-1} \cdot B_D \cdot B_T \cdot f(\widetilde{Q}_H^*, d)^{-\sigma + \widetilde{Q}_{H,}^*} \cdot \widetilde{Q}_H^* \cdot h \right. \\
&\left. (1-\alpha)^{\alpha^{-1}} \cdot (1 - MC_S) \cdot \frac{D_H^*}{(1+\Omega_{H,S} \cdot i_S)^{\left(\frac{1-\alpha}{\alpha}\right)} (1+\Upsilon_{H,S} \cdot i_S)} \ \left[q^{\left(\frac{1-\alpha}{\alpha}\right)} - 1\right]\ \theta + \rho \right\} \quad ; \qquad (3.10)
\end{aligned}
$$

obtained by setting the growth rate of consumption in (2.3) equal to the growth rate of Northern technological knowledge in (3.4) with the one that would prevail in a pre-trade steady state and where $D_H(t) = \frac{H_S}{H_S + H_N} \left[A_S \cdot p_{H,S}(t)\right]^{\alpha^{-1}} + \frac{H_N}{H_S + H_N} \left[A_N \cdot p_{H,N}(t)\right]^{\alpha^{-1}}$, and prices are given by $p_H(t) = p_n n^\alpha = \exp(-\alpha)\left[1 - \overline{n}(t)\right]^{-\alpha}$. The long-run real interest rate, r^*, and economic growth rate, g^*, depend on preferences parameters, on the (structural) level of the nominal interest rate, on parameters related to the technology, and on the CIA constraints.

Proposition 4. *The real interest rate, r^*, and, consequently, the economic growth rate, g^*, decrease when Υ_H or Υ_L (Ω_H or Ω_L – acting through p_H) increase. The sign of the effect of i_o on r^* and g^* is negative.*

Proof. In Appendix C. □

Therefore, our theoretical results are consistent with the recent empirical evidence suggesting that both the economic growth rate and the real interest rate are negatively related to long-run inflation (e.g., Valdovinos 2003, Chu et al. 2015, Akinsola and Odhiambo 2017), and that those effects can be nonlinear and differ greatly across countries (e.g., López-Villavicencio and Mignon

2011). These effects will be evaluated through a calibration exercise below. Moreover, it can be noted that international trade and CIA constraints influence economic growth in opposite directions.[25]

Finally, we can extend all the above comparative-statics results pertaining to shifts in i_o also to shifts in the long-run inflation rate, π_o^*. The long-run inflation rate, π_o^*, is an increasing function of the exogenous monetary-policy variable, i_o. Bearing in mind the Fisher equation (2.2) and the equilibrium relationship $\pi_o(t) = \pi_o^*$, with $\pi_o^* = i_o - r^*$, since r^* is a decreasing function of i_o, then, for a given exogenous shift in i_o, Δi_o, implies that $\text{sgn}\,(\Delta\pi_o^*) = \text{sgn}\,(\Delta i_0)$ and $\Delta\pi_o^* > \Delta i_o$.

4 Quantitative effects

4.1 Calibration

Most of the parameters have been used in the macroeconomics growth literature and are summarized in Table 3 in Appendix D. In those cases we refer to that literature to justify the values used. For the labor share we use $\alpha = 0.6$, for the discount rate we consider $\rho = 0.03$, for the intertemporal elasticity of substitution we take $\theta = 1.05$ – values that are similar to those used in, e.g., Jones (1995) and Jones and Williams (2000). Spillovers or the standing-on-the-shoulders effects, β, take the value 1.6 for the North and 1.0 for the South,[26] the complexity effects, ζ, take the value 4 for the North and 2.5 for the South, consistent with a higher complexity for more developed countries as argued, e.g., by Mansfield et al. (1981) and Sequeira et al. (2018). The other parameters of the technology channel, B_D and B_T, and those of the backwardness channel, σ and d, as well as variables $\widetilde{Q}_L(t_0)$ and $\widetilde{Q}_H(t_0)$, in equation (3.3) are taken from Afonso (2012). The same is done considering the relative productivity advantage of the skilled over unskilled workers $\frac{h}{l} = 1.2$.

Some other parameters reflect empirical facts such as A_N and A_S. In this case we set $A_N = 1.4$. If we consider that the North (say the USA) is seven times richer that the South (say China),[27] considering GDP *per capita*, this is roughly consistent with the Hall and Jones (1999) finding that a 1% increase in institutional quality implies an increase in 5% in GDP *per capita;* i.e. $\frac{7}{5} = 1.4$. It can be shown that some reasonable changes in these values do not affect our main results. Furthermore, we adjust the parameters related to the backwardness function, σ and d, such that we replicate reasonable values for both the skill premium (near 2.5 in the North with zero inflation) and the economic growth rate (near 3% with zero inflation).

Skilled and unskilled labor endowments as well as CIA constraints are essential elements in

[25] A detailed explanation is included in Appendix C.

[26] This means that the North has a spillover 0.6 higher than the South and are in line with estimates from Neves and Sequeira (2018).

[27] For this comparison we considered differences in GDP per capita, averaged between 1990 and 2017, from the PWT 9.1. If this comparison would be between the USA and India, the USA would be around 11 times richer than India, considering the period after 2000. In this case a higher $\frac{A_N}{A_S}$ would be appropriate.

our model. First, for skilled and unskilled labor we use the Barro and Lee education attainment dataset (updated June 2018) – Barro and Lee (2013) – considering for unskilled labor the number of people in population aged 15 or above with total primary and secondary education and for skilled labor the number of people in population aged 15 or above with total tertiary education. As Northern countries, we consider those that are leading in research.[28] As Southern countries we included those that imitate or adopt Northern technologies.[29] This yields a skilled-unskilled labor of 0.57 in the North and 0.12 in the South, also yielding a much higher skill premium in the South than in the North, consistently with empirical evidence. For CIA constraints in the North, we follow Gómez (2018) and consider $\Omega_{H,N} = 0.2$. As expected, unskilled production (less productive) firms are more financially constrained than the skilled production (more productive) firms (e.g., Popov 2013; Cao and Leung 2016; Frank and Yang 2018; Gómez 2018; Feng et al. 2020). It is worth noting that in his calculations Gómez (2018) does not consider the extensive margin, i.e., the firms that can exit the markets once hit by a financial constraint. Also, Beck et al. (2005) present results according to which smaller firms are more financially constrained than big ones. Cao and Leung (2016), Frank and Yang (2018), and Feng et al. (2020) show the strong association between higher productivity of firms and lower debt constraints.[30] It is, thus, natural to assume that unskilled production firms are smaller and more prone to exit than skilled production firms when hit by a financial constraint. This leads us to assume that $\Omega_{L,N} = 0.4$, taking the average estimate in Popov (2013). For the financial constraints in the South, we use the higher value in Popov (2013) for setting $\Omega_{L,S} = 0.8$ and an intermediate value for setting $\Omega_{H,S} = 0.6$. Finally, R&D firms – and by the same reasons also firms that adapt technologies in the South – need higher cash-flow for payments than intermediate goods firms (e.g., Chu and Cozzi 2014). Thus, we assume $\Upsilon_{L,S} = 0.9$ and $\Upsilon_{H,S} = 0.7$.[31]

4.2 Comparative steady-state effects of inflation

After using equation (3.7) to calculate the endogenous technological-knowledge bias, we can use equation (3.8) to calculate the unskilled threshold variety for the North and the South, equation (3.9) to calculate wage inequality in the North and in the South, and equation (3.10) to calculate economic growth.

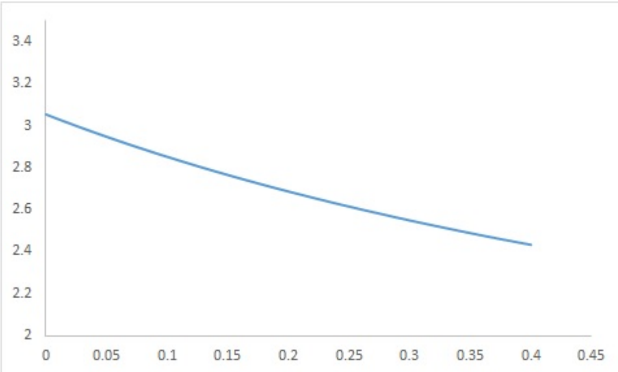
First, we plot the endogenous technological-knowledge bias (on the Y-axis of Figure 1), G^*, which is negatively related to the nominal interest (panel (a), on the left side) and inflation rates

[28] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the USA.

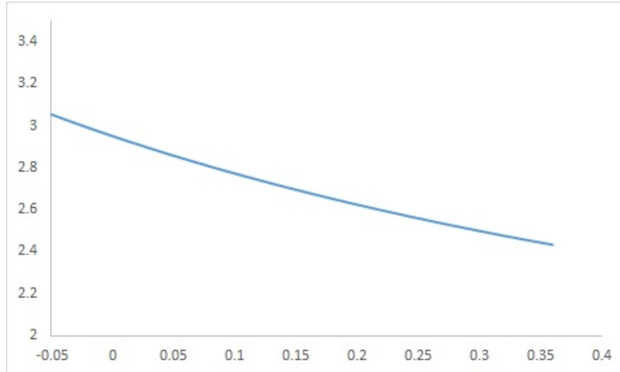
[29] China, India, Indonesia, Russian Federation, Brazil, and South Africa.

[30] We take these empirical results as indirect or suggestive evidence backing up the assumption in our model that firms dealing with skilled labor technologies, being more productive firms *ceteris paribus*, tend to require less liquidity and external financing, and thereby face a lower intensity of the CIA constraint, via-à-vis the unskilled complementary sectors.

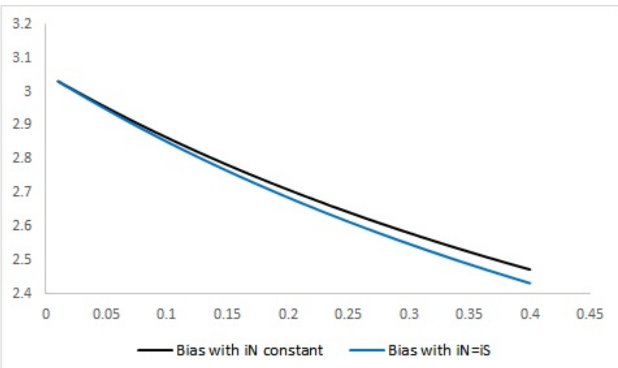
[31] Table 3 in Appendix C summarizes the calibrated values for parameters and predetermined variables.



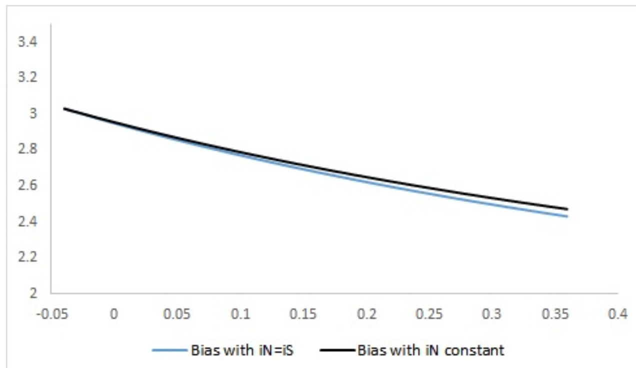
(a) G^* for different values of $i_S = i_N$



(b) G^* for different values of $\pi_S^* = \pi_N^*$



(c) G^* with $i_N = i_S$ and G^* by rising i_S, maintaining i_N

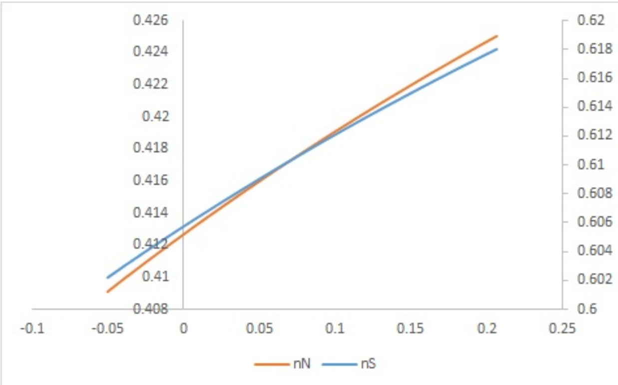


(d) G^* with $\pi_N^* = \pi_S^*$ and G^* by rising π_S^*, maintaining π_N^*

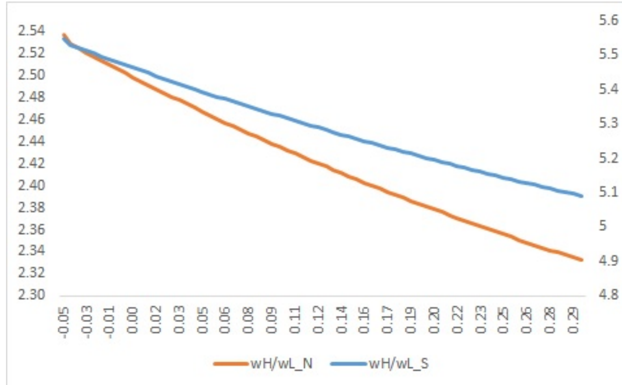
Figure 1: Changes in the endogenous technological-knowledge bias for different nominal interest rates (panels a and b) and inflation rates (panels c and d)

(panel (b), on the right side), as stated by Proposition 2. In panel (c) and (d) of Figure 1, we plot a comparison of G^* with $i_N = i_S$ (the baseline analysis) with G^* against a rising i_S, maintaining nominal interest rates and inflation constant in the North.

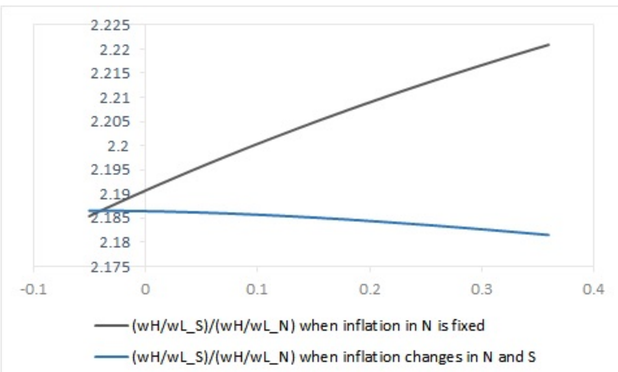
Next, we plot the threshold final good for the North and the South (Figure 2: panel (a), on the left side), the skill premium in the North and in the South (Figure 2: panel (b), on the right side), the skill premium in the South related to the skill premium in the North (Figure 2: panel (c), on the left side), and economic growth (Figure 2: panel (d), on the right side). There are several results that can be highlighted by this quantitative exercise, even to compare them with available empirical evidence. First, as expected, the number of varieties produced with unskilled technologies is greater in the South than in the North (compare the right- with the left-hand scale in panel (a), Figure 2). However, due to the effect of inflation, the number of varieties produced with skilled technologies decreases in both the South and the North. Despite the quantitative effects being relatively small, effects in the North seem to be higher. In Figure 2, panel (b) we observe the skill premia for the North and the South. As expected, both theoretically and empirically, the skill premium is higher in the South than in the North. Both increase with inflation as Proposition 4



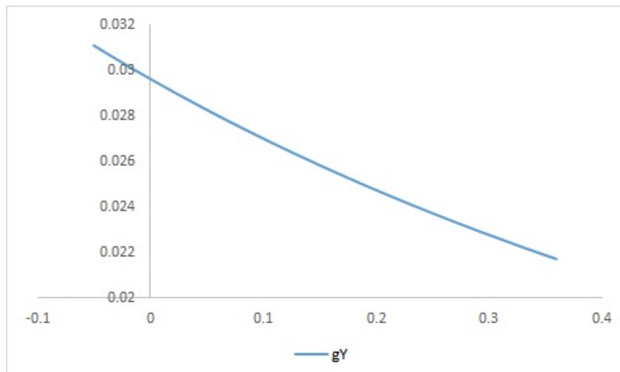
(a) Threshold final good for different inflation rates



(b) Skill premium for different inflation rates



(c) Relative South-North skill premium for different inflation rates



(d) Economic growth rate for different inflation rates

Figure 2: Changes in the endogenous threshold final good, panel (a), skill premium, panel (b), relative South-North skill premium, panel (c), and economic growth rate, panel (d), for different inflation rates

predicted. However, the magnitude of the effects is greater in the South, where, quantitatively, the skill premium is also higher, which is again consistent with empirical evidence – see Appendix A. However, the distances between skill premia regarding skill premia in the North and in the South seem to shrink with increasing inflation, but just for the case that $i_N = i_S$, i.e., when both countries also face rising inflation of the same amount. In the realistic case that just the South experiences rising inflation, this yields the interesting result that more inflation in the South implies rising wage inequalities in the South relative to the North – see panel (c) in Figure 2. Finally, panel (d) in Figure 2 shows the responsiveness of economic growth to inflation rate. As pointed out by the majority of the previous contributions (e.g., Gillman and Kejak 2005, for a survey), there is a negative effect of (non-hyper) inflation rates on economic growth that tends to become slightly smoother for higher levels of inflation – note the slightly convex negatively sloped curve.

We also note that an increase in the inflation rate decreases the real interest rates, which, in fact, can be observed by the path in the economic growth rate in Figure 2, panel (c) given the Euler condition. This would imply the verification of the Tobin effects mentioned in Gillman and Kejak (2005), i.e., an increase in the investment and an increase in capital-labor ratios, as also a decrease in the consumption share in output.[32]

5 Concluding remarks

Given the recent interest in studying the real effects of monetary policy, several contributions have been made in the past few years concerning the effects of inflation on growth both empirically and theoretically. In this paper we focus on an almost overlooked issue about the inflation-development nexus: the effect of inflation on wage inequality and on trade specialization. To that end, we devise a North-South model of endogenous growth in which there is international trade between both regions. We study the theoretical effect of inflation in different macroeconomic variables, namely, the wage inequality, the specialization pattern, and economic growth.

An important result of the model is that inflation and trade shocks have opposite *short-run* effects on wage inequality: while trade tends to decrease wage inequality in the South, inflation tends to increase it. This counter-effect of inflation (in comparison to trade) in wage inequality in the developing South could indicate that the monetary policy may have a role in counterbalancing the negative effects of a potential decrease in international trade due to escalating protectionism.

For plausible relationships between financial constraints of the different sectors in the economy, we obtain that an increase in the nominal interest rate (and also in the inflation rate) has the following *long-run* effects: (i) it decreases the technological-knowledge bias; (ii) it decreases the relative specialization in skilled-intensive goods; and (iii) it decreases wage inequality. Thus, the model confirms the empirical prediction according to which inflation tends to increase inequality. Also, inflation undoubtedly decreases economic growth, in which the model also follows the existing empirical evidence. All the results are confirmed quantitatively. Furthermore, we show that in the long run inflation decreases the difference of wage inequality levels between the South and the North, when both countries have the same inflation rate. This means that more inflation shrinks the difference between the skill premium in the North relatively to the skill premium in the South. However, when inflation rises in the South, but is kept constant (or rises, but at a lower pace) in the North, then the skill premium in the South rises more than in the North, indicating that expansionary monetary policies in developing countries contribute to increased inequality in those countries, relative to inequality in more developed countries.

Our paper contributes to two strands of literature inside the endogenous growth theory. First, it is the first theoretical model linking inflation to inequality in a North-South open economies

[32] In the Online Appendix OD, Figure 3, we show a similar Figure for the threshold final good, for skill premium and for economic growth rate for the case in which only inflation in the South rises. The inter-country wage inequality in the case that only Southern inflation rises is (again) plotted in Subfigure 4c, although it has been also plotted in Subfigure 3c against the inter-country wage inequality in the case of $i_S = i_N$. We can observe similar qualitative, but different quantitative patterns. As may be expected in this case, we observe smoother effects in the North, both in the rising pattern of the threshold final good as well as in the decreasing pattern of the skill premium.

framework that allows to analyzing the effect of rising inflation and rising inflation differentials in wage inequality and production specialization across countries. As consequence, it also contributes to the literature that is devoted to explaining different wage inequality patterns in the sense that it introduces inflation variations and differentials as an additional justification to (the widely documented) different wage inequality levels and paths across different countries. In both these contributions, the model highlighted effects that have a sound empirical validation.

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Appendix A Empirical Evidence on the relationship between wage inequality and inflation

In this Section we reassess the evidence concerning the determinants of income inequality (with special emphasis on wage inequality) and show that inflation appears to be an additional robust determinant of inequality, which has never been considered. Since Kuznets (1955) presented empirical evidence according to which income inequality has an inverted U-shaped relationship with GDP per capita, several contributions have appeared to highlight empirical determinants of income inequality. Barro (2000) presents cross-country evidence on several determinants of inequality (resembling the well-known Barro growth regressions, but for inequality). Despite the use of several institutional variables and fixed-effects, Barro does not include inflation or monetary-policy variables as possible determinants of inequality. Milanovic (2000), Rodriguez-Pose and Tsellios (2009) and Jaumotte et al. (2013) considered other determinants of inequality, but without considering monetary variables. Thus, although the above-mentioned papers discovered some association with financial and credit institutions or measures with income inequality, none considered inflation – as a direct target of the monetary policy – as determinant of different levels of income inequality, as we do in this paper. From all the empirical literature pertaining to the search for determinants of inequality, four sets of variables, apart from controls, have been taken into account and have shown significant results – e.g., Chusseau et al. (2008), McAdam and Willman (2018), and Song et al. (2019): human capital, skill ratios or correlates, technological-knowledge level or correlates, and openness or globalization measures. In this Section we regress inter-quantile wage ratios (in OECD countries) – this is the measure that best matches our theoretical approach – as well as Gini coefficients (in a worldwide database) – on human capital or skill ratios and inflation – shown in

Table 1, as well as in other controls such as TFP and Openness. Table 1 summarizes the results.[33]

Regression number:	1	2	3	4	5	6
Dependent variable:	i-d 90-10	i-d 90-50	i-d 90-10	Gini	Gini	Gini
Skilled ratio or human capital	0.966***	0.085	1.434***	-0.322**	-0.299***	-0.217
	(0.309)	(0.225)	(0.343)	(0.044)	(0.046)	(0.161)
Inflation	0.971*	0.237**	1.196**	0.0021**	0.0029**	0.0031**
	(0.564)	(0.118)	(0.543)	(0.001)	(0.001)	(0.001)
R^2	0.95	0.11	0.94	0.85	0.88	0.85
Number observations	328	328	245	3367	2986	2156
Time dummies	yes	yes	yes	yes	yes	yes
Country dummies	yes	no	yes	yes	yes	yes
Clustered standard-errors	no	yes	no	yes	yes	yes
Limited Sample	no	no	excludes leaders	no	no	excludes leaders

Table 1: Empirical Evidence on the influence of Inflation on Inequality. Notes: ***, **, * indicate statistical significance at the 1%, 5%, and 10% levels respectively; robust or clustered (by country) standard-errors are presented in parentheses below the coefficients, according to information given in the "Clustered standard-errors" line; inter-decile wage ratios (I-d) and skilled-unskilled ratios are from OECD. Gini coefficients (Gini) are net measures from SWIID (Solt, 2009) and are inserted in logs as dependent variables in regressions of columns (3) to (6); Human capital, TFP and Openness (inserted in logs) are from PWT 8.0 and Inflation (consumer prices) is from the World Bank; TFP and Openness are introduced as controls in regressions presented in columns (5) and (6) where they are statistically significant, where the dependent variable is the Gini coefficient; in the regression of column (3) excluded technological leaders are France, Germany, Norway, United Kingdom, United States, Sweden, and Japan; in the last regression (column 6) excluded leaders are the countries in the first quartile of GDP *per capita*.

Table 1 shows that despite the relationship with human capital (quantities) and other correlates, inflation rates tend to be positively associated with income or wage inequality. This result is robust to the introduction of time dummies, which proxies (unobserved) common time-specific shocks that affects both inflation and inequality, as well as to the introduction of country dummies (fixed-effects), which proxies institutional country-specific (unobserved) factors that may affect both inflation and inequality. These results seem to be a good reason to include inflation as a determinant of wage inequality when considered together with other already studied determinants of inequality, such as technological-knowledge and international trade. Additionally, the fact that this relationship seems to be preserved when the richest or the most technologically advanced countries are excluded,[34] also motivates us to study this relationship in an open economy model that considers trade between the advanced North and the more technologically backward Southern countries. It is also worth noting that the effect of inflation on inequality is greater in less rich

[33] We cannot obtain the nonlinear inverted-U relationship between income inequality and inflation shown in Chu et al. (2019b) in our specifications, either in the case in which human capital or skills ratio are included in regressions and when they are not.

[34] The positive relationship is also kept when the dependent variable is the Inter-decile wage ratio 90-50 and technological leaders are excluded from the sample, although the regression is not shown in Table 1.

countries (compare coefficients in column (3) and (6) that excludes richer countries with columns (1) and (3) that includes them).

Appendix B The Backwardness Function

In order to capture the benefits of relative backwardness, function $f(\widetilde{Q}_{\mathcal{L}}(t), d)$ – similar to Papageorgiou (2002) and Afonso (2012) – is

$$f(\widetilde{Q}_{\mathcal{L}}(t), d) = \begin{cases} 0 & \text{, if } 0 < \widetilde{Q}_{\mathcal{L}}(t) \leq d \\ -\widetilde{Q}_{\mathcal{L}}(t)^2 + (1+d) \cdot \widetilde{Q}_{\mathcal{L}}(t) - d & \text{, if } d < \widetilde{Q}_{\mathcal{L}}(t) < 1 \end{cases}, \tag{B.1}$$

where $\widetilde{Q}_{\mathcal{L}}(t) \equiv \frac{Q_{\mathcal{L},S}(t)}{Q_{\mathcal{L}}(t)}$ is the relative technological-knowledge level of the South's $\mathcal{L}$-specific intermediate good. Thus, we assume that the probability of successful imitation in intermediate good j is state dependent on all past successful R&D in all intermediate goods of their type in both countries, contrary to the probability of successful innovation, which is state dependent only on the stock of past successful R&D in intermediate good j in the North. Provided that the gap is not large – i.e., if $\widetilde{Q}_{\mathcal{L}}(t)$ is above threshold d – then the country can benefit from an advantage of backwardness, as in Barro and Sala-i-Martin (2004, ch. 8). When the gap is wider – so that $\widetilde{Q}_{\mathcal{L}}(t)$ is below threshold d – backwardness is no longer an advantage (in line with Verspagen 1993, and Papageorgiou 2002). Function $f(\widetilde{Q}_{\mathcal{L}}(t), d)$ is quadratic over the range of main interest, and, once affected by the exponent function $(-\sigma + \widetilde{Q}_{\mathcal{L}})$ in equation (2.12)-(see description of component (v) above), yields an increasing (in the technological-knowledge gap) advantage of backwardness – where the size of σ affects how quickly the probability of successful imitation falls as the technological-knowledge gap falls.

Appendix C Proofs and other steady-state results

C.1. Proofs of Propositions

Proof. (*Proposition 1*) Define (3.7) implicitly as $F(G^*, i_N, i_S) = 0$ and, thus, $\frac{\partial G^*}{\partial i_o} = -\frac{\frac{\partial F}{\partial i_o}}{\frac{\partial F}{\partial G^*}} = -\frac{\frac{\partial (F_1(G^*, i_N, i_S) - F_2(G^*, i_N, i_S))}{\partial i_o}}{\frac{\partial (F_1(G^*, i_N, i_S) - F_2(G^*, i_N, i_S))}{\partial G^*}}$, $o = N, S$. (a) Let $\Omega_{L,o} > \Omega_{H,o}$ and $\Upsilon_{L,o} > \Upsilon_{H,o}$; it is straightforward to see $sign\left(\frac{\partial F}{\partial G^*}\right) = sign\left(\frac{\partial F}{\partial i_o}\right)$, which implies that $\frac{\partial G^*}{\partial i_o} < 0$. Now, let $\Omega_{L,o} < \Omega_{H,o}$ and $\Upsilon_{L,o} < \Upsilon_{H,o}$; in this case, we have $sign\left(\frac{\partial F}{\partial G^*}\right) \neq sign\left(\frac{\partial F}{\partial i_o}\right)$ and so $\frac{\partial G^*}{\partial i_o} > 0$. (b) Assume that $i_S > i_N$ and $\Omega_{L,S} > \Omega_{H,S}$ and $\Upsilon_{L,S} > \Upsilon_{H,S}$; in this case $sign\left(\frac{\partial F}{\partial G^*}\right) = sign\left(\frac{\partial F}{\partial i_S}\right)$, which implies that $\frac{\partial G^*}{\partial i_S} < 0$. Now, let $i_S > i_N$ and $\Omega_{L,S} < \Omega_{H,S}$ and $\Upsilon_{L,S} < \Upsilon_{H,S}$; in this case, we have $sign\left(\frac{\partial F}{\partial G^*}\right) \neq sign\left(\frac{\partial F}{\partial i_S}\right)$ and so $\frac{\partial G^*}{\partial i_S} > 0$. □

Proof. (*Proposition 2*) Let $\overline{n}_o^*$ be given by (3.8). From Proposition 2 $\frac{\partial G^*}{\partial i_o} < 0$ $\left(\frac{\partial G^*}{\partial i_o} > 0\right)$ when $\Omega_{L,o} > \Omega_{H,o}$ ($\Omega_{L,o} < \Omega_{H,o}$) and $\Upsilon_{L,o} > \Upsilon_{H,o}$ ($\Upsilon_{L,o} < \Upsilon_{H,o}$). Moreover, $\frac{\partial\left(\frac{h}{l}\frac{H_o}{L_o}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}}{\partial i_o} > 0$ $\left(\frac{\partial\left(\frac{h}{l}\frac{H_o}{L_o}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}}{\partial i_o} < 0\right)$. Thus, $\frac{\partial \overline{n}_o^*}{\partial i_o} \gtreqless 0$ if and only if $\mid \frac{\partial G^*}{\partial i_o} \mid \gtrless \mid \frac{\partial\partial\left(\frac{h}{l}\frac{H_o}{L_o}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}}{\partial i_o} \mid$ for $\Omega_{L,o} > \Omega_{H,o}$ and $\Upsilon_{L,o} > \Upsilon_{H,o}$, while $\frac{\partial \overline{n}_o^*}{\partial i_o} \lesseqgtr 0$ if and only if $\mid \frac{\partial G^*}{\partial i_o} \mid \gtrless \mid \frac{\partial\partial\left(\frac{h}{l}\frac{H_o}{L_o}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}}{\partial i_o} \mid$ for $\Omega_{L,o} < \Omega_{H,o}$ and $\Upsilon_{L,o} < \Upsilon_{H,o}$. □

Proof. (*Proposition 3*) Partially derive (3.9) in order to $\Omega_{L,S}$, $\Omega_{H,S}$, $\Omega_{L,N}$, $\Omega_{H,N}$, $\Upsilon_{L,N}$, and $\Upsilon_{H,N}$, then evaluate the sign of the derivatives, taking into account (3.7). For the effect of the nominal interest rate and inflation note that $\frac{\partial\left(\frac{w_{\dot{H},o}}{w_{L,o}}\right)^*}{\partial i_o} = \frac{1}{2}\left[\frac{\partial G^*}{\partial i_o} + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-2\alpha}{\alpha}\right)} \frac{\partial\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)}{\partial i_o}\right]^{-\frac{1}{2}}$; if $\mid \frac{\partial G^*}{\partial i_o} \mid > \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-2\alpha}{\alpha}\right)} \mid \frac{\partial\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)}{\partial i_o} \mid$, then $\frac{\partial\left(\frac{w_{\dot{H},o}}{w_{L,o}}\right)^*}{\partial i_o} \lesseqgtr 0$ for $\Omega_{L,o} \gtreqless \Omega_{H,o}$ (and $\Upsilon_{L,o} \gtreqless \Upsilon_{H,o}$), while if $\mid \frac{\partial G^*}{\partial i_o} \mid < \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1+\Omega_{L,o}\cdot i_o}{1+\Omega_{H,o}\cdot i_o}\right)^{\left(\frac{1-2\alpha}{\alpha}\right)} \mid \frac{\partial\left(\frac{1+\Omega_{L,N}\cdot i_N}{1+\Omega_{H,N}\cdot i_N}\right)}{\partial i_o} \mid$ then $\frac{\partial\left(\frac{w_{\dot{H},o}}{w_{L,o}}\right)^*}{\partial i_o} \gtreqless 0$ for $\Omega_{L,o} \gtreqless \Omega_{H,o}$ (and $\Upsilon_{L,o} \gtreqless \Upsilon_{H,o}$). □

Proof. (*Proposition 4*) Partially derive (3.10) in order to $\Omega_{L,S}$, $\Omega_{H,S}$, $\Omega_{L,N}$, $\Omega_{H,N}$, $\Upsilon_{L,N}$, $\Upsilon_{H,N}$, taking the expression for p_H into account. For the effect of i_o, taking D_H^* as given, it is directly seen that $\left.\frac{\partial r^*}{\partial i_S}\right|_{D_H^*} < 0$. Additionally, note that (a) $\frac{\partial D_H^*}{\partial i_S} < 0$ as $sign\left(\frac{\partial D_H}{\partial i_S}\right) = sign\left(\frac{\partial p_{H,j}}{\partial i_S}\right) = sign\left(\frac{\partial \bar{n}}{\partial i_S}\right) < 0$ and $\frac{\partial D_H^*}{\partial i_N} < 0$ as $sign\left(\frac{\partial D_H}{\partial i_N}\right) = sign\left(\frac{\partial p_{H,j}}{\partial i_N}\right) = sign\left(\frac{\partial \bar{n}}{\partial i_N}\right) < 0$. □

C.2. Table summarizing the three possibilities of limit-pricing

The first mark-up is the highest: the Northern entrant (N) competes with a Northern incumbent (N) at the same marginal cost but with better quality. The second one is smaller: the Southern entrant (S), with lower marginal cost, competes in the same quality rung with a Northern incumbent (N). Compared with the first, the third mark-up is again smaller, but due to a different reason: the Northern entrant improves quality as in the first case, but competes with an incumbent with lower marginal cost.

C.3 On the opposite effects of CIA constraints and trade on steady-state growth

Considering that in the absence of international trade, the advantages of backwardness and openness terms vanish from the probability of successful imitation (2.12) and that the relevant market

$t-dt$	t	Share in intermediate goods production at t	$p(j)$
N produces and exports quality k	N produces and exports quality $k+1$	$\Phi_{\mathcal{L}} \cdot (1-\Psi_{\mathcal{L}})$	$p_{\mathcal{L},N-N}(j) = q \cdot MC_N \cdot [1+\Omega_{\mathcal{L},N} \cdot i_N]$
N produces and exports quality k	S produces and exports quality k	$1-\Phi_{\mathcal{L}}$	$p_{\mathcal{L},S-N}(j) = MC_N \cdot [1+\Omega_{\mathcal{L},S} \cdot i_S]$
S produces and exports quality k	N produces and exports quality $k+1$	$\Phi_{\mathcal{L}} \cdot \Psi_{\mathcal{L}}$	$p_{\mathcal{L},N-S}(j) = q \cdot MC_S \cdot [1+\Omega_{\mathcal{L},N} \cdot i_N]$

Table 2: Limit pricing of each intermediate good

size in each country is its own domestic labor, the increment in the steady-state interest rate from pre-trade to international trade in intermediate goods relies on the difference

$$\begin{aligned} & B_T \cdot f(\widetilde{Q}_H^*, d)^{-\sigma+\widetilde{Q}_{H,}^*} \cdot \widetilde{Q}_H^* \cdot (1-MC_S) \cdot D_H^* - \\ & - \left(\frac{q-1}{q}\right) \left[A_S \cdot p_{H,S}^*\big|_{pre-trade}\right]^{\alpha^{-1}} \cdot MC_S^{(\alpha-1)\alpha^{-1}} \end{aligned} . \tag{C.1}$$

While evaluation of equation (C.1) requires solving for transitional dynamics through calibration and simulation, we can, however, emphasize five ways, in addition to the level effects, through which international trade and CIA constraints influence in opposite directions, steady-state growth. It should be stressed that, although CIA constraints are not explicit in (C.1), they work through D_H^* and $P_{H,S}^*$. The first way in which international trade influences steady-state growth is the positive catching-up effect on the probability of successful imitation. Imitation capacity increases with the degree of openness, which is captured by B_T, and the advantages of backwardness are obtained only in the presence of international trade. Through the feedback effect described above, the probability of successful innovation, and thus the steady-state growth rate, are also affected – see equations (3.3) and (3.4). The second way is the positive spillovers from North to South. Each innovation in the North tends to lower the cost of Southern imitation because the backwardness advantage is strengthened with each improvement of the technological-knowledge frontier. The third – counteracting – channel reflects the effect of CIA constraints on the production of intermediate goods. Since $D_H^* = \frac{H_S}{H_S+H_N}\left[A_S \cdot p_{H,S}^*\right]^{\alpha^{-1}} + \frac{H_N}{H_S+H_N}\left[A_N \cdot p_{H,N}^*\right]^{\alpha^{-1}}$ any change in $\Omega_{L;S}$, $\Omega_{H,S}$, or i_o that affects $p_{H,S}^*$ has negative influence on (C.1). Indeed, under international trade the cost of introducing new qualities of intermediate goods also in the South has to be considered, which feeds back into the North by making the R&D innovative activity more difficult. The fourth – counteracting as well – channel is the monopolistic competition mark-up. The Northern monopolist loses profits with the entry into international trade: the average mark-up between the first and third situations in Table 2 above is smaller than $(q-1)$, which is the pre-trade mark-up. The reason for this is that in pre-trade successful innovators are protected from international competition. Once engaged in international trade, and imitation becomes profitable (provided that the technological-knowledge threshold d is overcome), profit margins in the North are reduced, which discourages R&D activities.[35] The fifth – counteracting as well – way through which trade influences steady-state growth, is that Southern firms have to support the R&D imitative cost of state-of-the-art intermediate goods, possibly several quality rungs above (and thus more complex) their own experience level in pre-trade. This is captured by the presence of the technological-knowledge ratio, $\widetilde{Q}_H^*$, in (C.1).

The effect of trade on the steady-state growth rate is, thus, ambiguous. However, the comparative statics (numerically computed based on the calibration in Table 3) are not affected by such ambiguity because changes in g^* refer to steady-state growth under trade. This rate is affected by the levels of exogenous variables and parameters, which is to be expected in an endogenous growth model. In particular, both countries' exogenous levels of productivity (A_N and A_S) and parameters of R&D technology (β, B_D and B_T) improve the common growth rate through their positive effect on the profitability of R&D, as (3.3) demonstrates. The impact on steady-state growth of an increase in the Southern marginal cost of final-goods production, MC_S, results from the combination of typical Schumpeterian R&D effects: (i) by reducing productivity, it reduces resources available to R&D, and, consequently, both imitation and innovation (feedback effect); it also implies a smaller mark-up for the intermediate-goods producers in the South, thereby (ii) discouraging imitative R&D and (iii) encouraging innovative R&D; in our numerical calculations, the effects (i) and (ii) clearly dominate (iii).

C.4. Stability of the Steady State

To prove that the steady state is stable, let us consider that the economy is initially out of the steady state whereby, for example, $I_{H,N} > I_{L,N}$. In particular, this implies that $\frac{P_H}{P_L} > \left(\frac{P_H}{P_L}\right)^*$; i.e., that $\overline{n} > \overline{n}^*$, meaning that $\frac{\dot{Q}_H}{Q_H} > \frac{\dot{Q}_L}{Q_L}$ and, since from (2.7) $\frac{P_H}{P_L} = \left\{ G^* \left(\frac{h \cdot H}{l \cdot L}\right) \left(\frac{1+\Omega_{L,N} \cdot i_N}{1+\Omega_{H,N} \cdot i_N}\right)^{\left(\frac{1-\alpha}{\alpha}\right)} \right\}^{-\frac{\alpha}{2}}$, $\frac{\dot{P}_H}{P_H} - \frac{\dot{P}_L}{P_L} < 0$. Thus, $\frac{P_H}{P_L}$ (or $\overline{n}$) is decreasing toward $\frac{P_H^*}{P_L^*}$ (or $\overline{n}^*$). Notice that the decrease in $\frac{P_H}{P_L}$ (or $\overline{n}$) attenuates the rate at which the technological-knowledge bias is increasing. Thus, due to market incentives, while $\frac{\dot{Q}_H}{Q_H} > \frac{\dot{Q}_L}{Q_L}$, $\frac{\dot{Q}_H}{Q_H} - \frac{\dot{Q}_L}{Q_L}$ is decreasing until the unique and stable steady state is achieved, at which $\left(\frac{\dot{Q}_H}{Q_H}\right)^* - \left(\frac{\dot{Q}_L}{Q_L}\right)^* = 0$. The argument to show that the economy starting with $I_{H,N} < I_{L,N}$ converges to $\left(\frac{P_H}{P_L}\right)^*$ is identical. Hence, the economy starting out at the steady state converges to this state and, without any exogenous disturbance, it remains there.

[35] Contrary to the previous models in which the reduction of margins is offset by market enlargement, e.g., Rivera-Batiz and Romer (1991), we have removed the scale effect, as explained above.

Appendix D Calibration Values

Parameter	Value		Parameter	Value		Variables	Value
α	0.66		B_D	1.28		A_N	1.60
h	1.20		B_T	1.85		A_S	1.00
MC_S	0.80		σ	1.70		$\frac{H_N}{L_N}$	0.57
β_N	1.60		d	0.10		$\frac{H_S}{L_S}$	0.12
β_S	1.00		q	1.10		$\frac{Q_H(t_0)}{Q_L(t_0)}$	1.00
ζ_N	4.00		θ	1.05		$\widetilde{Q}_H(t_0)$	0.35
ζ_S	2.50		ρ	0.02		$\widetilde{Q}_L(t_0)$	0.30
$\Omega_{L,N}$	0.4		$\Omega_{H,N}$	0.2		$\Omega_{L,S}$	0.8
$\Omega_{H,S}$	0.6		$\Upsilon_{L,S}$	0.9		$\Upsilon_{H,S}$	0.7

Table 3: Baseline parameter and initial values

Appendix OA More on trade and level effects in the South

In the absence of international trade, the South mimics the R&D process of the North, but less efficiently, i.e., with $k_S \leq k$ in expression (2.11). Since the South is less developed, but not too backward, we assume that there are some intermediate goods j, but not all, for which $k_S < k$, implying that even in the absence of trade there are some state-of-the-art intermediate goods produced in both countries (i.e., for which $k_S = k$). Once the South has access to all the best quality intermediate goods through international trade, it becomes an imitator, improving the probability of successful R&D.

In addition to the direct effect of openness on the capacity of imitation, the level effect of entry into international trade also involves immediate changes in the allocation of resources to R&D. In particular, the amount of Southern resources devoted to R&D increases for two reasons. On the one hand, incentives to imitation increase through the positive effect of openness on the probability of successful imitation – see description of component (v) associated with equation (2.12); and, on the other hand, access to enlarged markets requires more resources due to the adverse effect of market size on the probability of successful imitation – see description of component (iv) associated with equation (2.12).[36]

Some empirical studies provide strong evidence that imports of intermediate goods improve productivity in developing countries (Amiti and Konings 2007, Goldberg et al. 2010). Thus, in order to emphasize the diffusion of technological knowledge embodied in intermediate goods, we assume that only these goods are internationally traded, while final goods and assets are

[36]Resources devoted to R&D immediately increase in the North as well, but only for the second reason, i.e., the adverse effect of market size on the probability of successful innovation – see description of component (iv) associated with equation (2.11). Northern resources are reallocated at the expense of current consumption, differently from the South, where consumption increases with the immediate increase in income.

internationally immobile.[37] Each intermediate good in the international market is produced either in the North or in the South. In the former case, it embodies the latest innovation, while in the latter it results from the imitation, at a lower cost, of the latest innovation. In either case, internationally traded intermediate goods embody the state-of-the-art technological knowledge accumulated in the North, which is summarized in $Q_H(t)$ and $Q_L(t)$.

When compared with a pre-trade situation, the improvement in the level of technological knowledge available to the South – through access to the state-of-the-art intermediate goods – is a static benefit of international trade. Indeed, the technological-knowledge gap is always favorable to the North in either specific knowledge, i.e., $Q_{\mathcal{L}} > Q_{\mathcal{L},S}$, since even under trade, at each t not all innovations have been imitated yet and, thus, the South enjoys an immediate absolute and relative (to the North) benefit in terms of aggregate product and factor prices. In fact, both the level of the composite final good – see (2.9) – and the marginal productivity of H and L increase with $Q_{\mathcal{L}}$.

The structure of final-goods production in the South is also affected, as the North's technological-knowledge bias, $\frac{Q_H}{Q_L}$, is transmitted to the South. In fact, comparing the threshold final good in the South – given, in general, by (2.5) – immediately before and immediately after entry into trade at time t_0,

$$\begin{array}{c} \overline{n}_S(t_0)|_{pre-trade} = \left\{1 + \left[G_S(t_0)\left(\frac{h}{l}\frac{H_S}{L_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1} < \overline{n}_S(t_0) = \left\{1 + \left[G(t_0)\left(\frac{h}{l}\frac{H_S}{L_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1} \\ versus \\ \overline{n}_S(t_0) = \left\{1 + \left[G(t_0)\left(\frac{h}{l}\frac{H_S}{L_S}\right)\left(\frac{1+\Omega_{L,S}\cdot i_S}{1+\Omega_{H,S}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1} > \overline{n}_N(t_0) = \left\{1 + \left[G(t_0)\left(\frac{h}{l}\frac{H_N}{L_N}\right)\left(\frac{1+\Omega_{L,N}\cdot i_S}{1+\Omega_{H,N}\cdot i_S}\right)^{\left(\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{2}}\right\}^{-1} \end{array}, \quad \text{(OA.1)}$$

where $G_S(t_0) \equiv \frac{Q_{H,S}(t_0)}{Q_{L,S}(t_0)}$ is the South's technological-knowledge bias at time t_0 and $G(t_0) \equiv \frac{Q_H(t_0)}{Q_L(t_0)}$ is the North's technological-knowledge bias at time t_0. That is, while before trade the level of technological knowledge available to the South is just the domestic technological knowledge – $Q_{H,S}$ and $Q_{L,S}$ –, under trade the state-of-the-art intermediate goods available internationally embody the North's technological knowledge – Q_H and Q_L.

Appendix OB Derivation of the patent value

Each moment in time in country $o = N, S$ there is a probability $I_o(j,t)dt$ that the quality level improves by 1, *i.e.*, $k(j,t+dt) - k(j,t) = 1$, and a probability $(1 - I_o(j,t))\,dt$ that there is no improvement in the quality level, i.e., $k(j,t+dt) - k(j,t) = 0$. Bearing this in mind, if we consider each moment in time as a random experiment that can result in a success with probability $I_o(j,t)$, we can characterize the time derivative of $k(j,t)$ as a random variable that follows a Binomial Distribution with an expected value of $I_o(j,t)$, i.e., $\dot{k}(j,t) \sim B(1, I_o(j,t))$. Therefore, although

[37] Thus, in order to import some intermediate goods, the South has to be able to export other intermediate goods, since we consider balanced trade.

$k(j,t)$ assumes only integer values, $k(j,t)$ and all the variables that that depend on it can be differentiated in relation to time but, as a result of the derivative being stochastic, they are also random variables.

Since $k(j,t)$ is a random variable, $T(k)$ is also a random variable with a probability distribution that is equal to $P_S(T(k)=\tau) = \left(1 - \int_0^\tau P(T(k)=z)dz\right) \cdot I_N(j,t+\tau)$. The intuition behind this formula is that the probability of no quality improvement of an intermediate good j with quality level k being exactly equal to τ since time t, the time in which the monopoly was initiated, is the probability of no improvement occurring before $t+\tau$, times the probability of a successful innovation at time $t+\tau$ (Barro and Sala-i-Martin 2004) occurring in the North, which is the potential challenger. In the case of the value of a patented innovation, V_N, the challenge comes from both a new Northern innovation and a Southern imitation. Assuming that $I_N(j,t+\tau) = I_N(j,t)$, and the $P_S(T(k)=0)=0$, we have that $P_S(T(k)=\tau) = I_N(j,t) \cdot \exp\left(-I_N(j,t)\cdot\tau\right)$. Since $V_S(j,k,t,T(k))$ depends on $T(k)$, this is also a random variable with the same probability density function of $T(k)$, $P_S(T(k)=\tau)$. Assuming that the investors are risk-neutral implies that they only care about the expected value of $V_S(j,k,t,T(k))$ (Gil et al. 2013), which is equal to the following expression:

$$V_S(j,k,t) = \int_0^\infty \Pi(j,s) \exp\left(-\left(\int_t^s r(w) + I_N(j,t)\right) dw\right) ds \tag{OB.1}$$

Assuming that all the prices and quantities are fixed during the time in which there is no quality improvements (e.g., Aghion and Howitt 1992, Barro and Sala-i-Martin 2004, Gil et al. 2013), then we have that:

$$V_S(j,k,t) = \frac{\Pi_S(j,k,t)}{r_S(t) + I_N(j,t)}. \tag{OB.2}$$

Appendix OC Aggregation

To analyze the consistency of our dynamic general-equilibrium endogenous growth model, we now show that the aggregate final good, Y_o, is used in consumption, C_o, and investment, $X_o + R_o$, considering the South as example, but omitting for the sake of simplicity the country index whenever it proves to be justified. In this process, firms and households are rational and solve their problems, free-entry R&D conditions are met, and markets clear. We start by stating that the equilibrium country-specific probability of successful-R&D is independent of the quality level k – see (3.3); i.e., $I_N(j,k,t) = I_N(j,t)$, $\forall k \in \mathbb{N}$ – and from the definition of market value of a firm $V_{\mathcal{L},S}(j,k,t) = \frac{\Pi_{\mathcal{L},S}(j,k,t)}{r(t)+I_N(j,k-1,t)}$. Thus,

$$r(t) \cdot V_{\mathcal{L}}(j,k,t) = (q-1)\cdot(1+\Omega_S \cdot i)\cdot x_{\mathcal{L}}(j,k,t) - q^{\frac{\alpha-1}{\alpha}} \cdot y_{\mathcal{L}}(j,k,t), \tag{OC.1}$$

where: $x_L(j,k,t) = \int_0^{\overline{n}} x_n(j,k,t)\cdot dn$, $x_H(j,k,t) = \int_{\overline{n}}^1 x_n(j,k,t)\cdot dn$, $\Pi_{\mathcal{L}}(j,k,t) = (q-1)(1+\Omega_{\mathcal{L}}\cdot i)\cdot x_{\mathcal{L}}(j,k,t)$, and $I_{\mathcal{L}}(j,t)\cdot V_{\mathcal{L}}(j,k,t) = q^{\frac{\alpha-1}{\alpha}}\cdot y_{\mathcal{L}}(j,k,t)$.[38] By integrating (OC.1) over j,[39] we have: $r\cdot a_{\mathcal{L}} = p_{\mathcal{L}}\cdot(1-\alpha)\cdot Y_{\mathcal{L}} - (1+\Omega_{\mathcal{L}}\cdot i)\cdot X_{\mathcal{L}} - q^{\left(\frac{\alpha-1}{\alpha}\right)}\cdot(1+\Upsilon_{\mathcal{L}}\cdot i)\cdot R_{\mathcal{L}}$, where, by definition, $a_L = \int_0^J V_L(j,k)\cdot dj$ and $a_H = \int_J^1 V_H(j,k)\cdot dj$ are the market value of all the firms that produce intermediate goods j at time t complementary with L and H, respectively, $X_L = \int_0^J x_{L,S}(j,k)\cdot dj$, $X_H = \int_J^1 x_{H,S}(j,k)\cdot dj$, $R_L = \int_0^J y_L(j,k)\cdot dj$, $R_H = \int_J^1 y_H(j,k)\cdot dj$, and $q\cdot(1+\Omega_{\mathcal{L}}\cdot i)\cdot X_{\mathcal{L}} = (1-\alpha)\cdot p_{\mathcal{L}}\cdot Y_{\mathcal{L}}$. In turn, since $a = \sum_{\mathcal{L}=L,H} a_{\mathcal{L}}$ results

$$r\cdot a = (1-\alpha)\cdot Y - X + \sum_{\mathcal{L}=L,H}(-\Omega_{\mathcal{L}}\cdot i)\cdot X_S - q^{\left(\frac{\alpha-1}{\alpha}\right)}\sum_{\mathcal{L}=L,H}(1+\Upsilon_{\mathcal{L}}\cdot i)\cdot R_{\mathcal{L}}, \tag{OC.2}$$

where $Y = \sum_{\mathcal{L}=L,H} p_{\mathcal{L}} Y_{\mathcal{L}}$ and $X = \sum_{\mathcal{L}=L,H} X_{\mathcal{L}}$. Still from the definition of market value of a firm, $V_{\mathcal{L}}(j,k,t) = \frac{\Pi_{\mathcal{L}}(j,k,t)}{r(t)+I_N(j,t)}$, but given the expression for profits, $\Pi_{\mathcal{L}}(j,k,t)$, and the expression for the equilibrium probability, $I_N(j,t)$, (e.g., in large brackets of (3.3)), an expression for $V_{\mathcal{L}}(j,k,t)$ results and thus for $a_L = \int_0^J V_L(j,k)\cdot dj$ and $a_H = \int_J^1 V_H(j,k)\cdot dj$. Then, taking into account time derivatives, we have $\dot{a}_{\mathcal{L}} = (1+\Upsilon_{\mathcal{L}}\cdot i)\cdot\left(1-q^{\frac{1-\alpha}{\alpha}}\right)\cdot R_{\mathcal{L}}$ since $\dot{Q}_{\mathcal{L}} = I_N(j,t)\cdot\left(q^{\frac{1-\alpha}{\alpha}}-1\right)\cdot Q_N$; thus,

$$\dot{a} = \left(1-q^{\frac{1-\alpha}{\alpha}}\right)\sum_{\mathcal{L}=L,H}(1+\Upsilon_{\mathcal{L}}\cdot i)\cdot R_{\mathcal{L}}, \tag{OC.3}$$

where: $\dot{a} = \sum_{m=L,H}\dot{a}_m$. On the other hand, since $\tau(t) = \dot{m}(t) + \pi(t)\cdot m(t)$, we can write from (2.1) that

$$\dot{a} = r\cdot a + \sum_{\mathcal{L}=L,H} w_{\mathcal{L}}\cdot\mathcal{L} - C + b\cdot i, \tag{OC.4}$$

where: $a = \sum_{\mathcal{L}=L,H} a_{\mathcal{L}}$. Replacing (OC.2) and (OC.3) in (OC.4), and since $\sum_{\mathcal{L}=L,H} w_{\mathcal{L}}\cdot\mathcal{L} = \alpha\cdot Y$ since $w_{\mathcal{L}} = \frac{\alpha\cdot p_{\mathcal{L}}\cdot Y_{\mathcal{L}}}{\mathcal{L}}$, and the money lent by households $b = \sum_{\mathcal{L}=L,H}\Omega_{\mathcal{L}}\cdot X_{\mathcal{L}} + \sum_{\mathcal{L}=L,H}\Upsilon_{\mathcal{L}}\cdot R_{\mathcal{L}}$, we have:

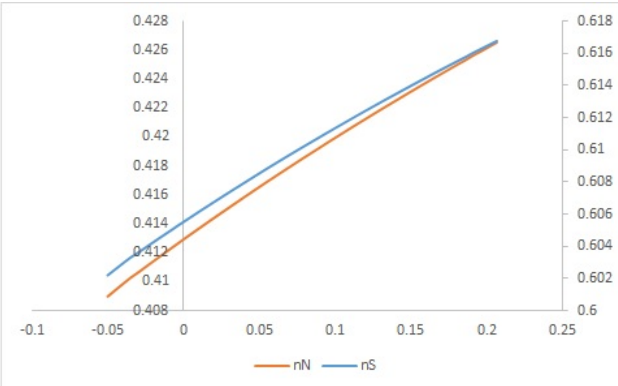
$$Y = C + X + R. \tag{OC.5}$$

That is, resources, Y, that are not consumed, C, are indeed used in the production of intermediates, X, and in the R&D sector, R.

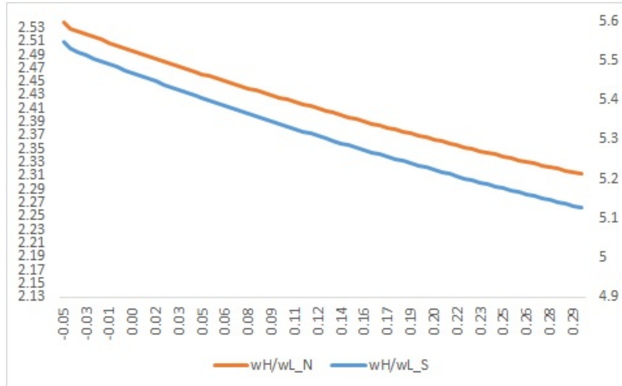
[38] From the free-entry condition $I_{\mathcal{L}}(j,t)\cdot V_{\mathcal{L}}(j,k+1,t) = (1+\Upsilon_{\mathcal{L}}\cdot i)\cdot y_{\mathcal{L}}(j,k,t)$ and thus $I_{\mathcal{L}}(j,t)\cdot V_{\mathcal{L}}(j,k,t) = (1+\Upsilon_{\mathcal{L}}\cdot i)\cdot y_{\mathcal{L}}(j,k-1,t)$ and, from the functional form of the probability of success in imitation-R&D (2.12), the R&D expenditures are $y_{\mathcal{L}}(j,k-1,t) = q^{\frac{\alpha-1}{\alpha}}\cdot y_{\mathcal{L}}(j,k,t)$.

[39] I.e., $\int_0^J r(t)\cdot V_L(j,k,t)\cdot dj = \int_0^J (q-1)\cdot(1+\Omega_L\cdot i)\cdot x_L(j,k,t)\cdot dj - \int_0^J q^{\frac{\alpha-1}{\alpha}}\cdot y_L(j,k,t)\cdot dj$ and $\int_J^1 r(t)\cdot V_H(j,k,t)\cdot dj = \int_J^1 (q-1)\cdot(1+\Omega_H\cdot i)\cdot x_H(j,k,t)\cdot dj - \int_J^1 q^{\frac{\alpha-1}{\alpha}}\cdot y_H(j,k,t)\cdot dj$.

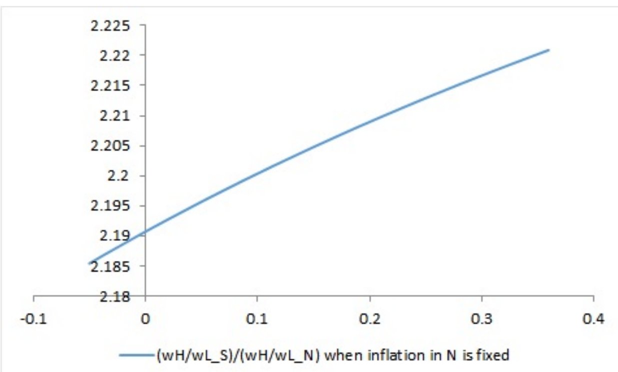
Appendix OD Inflation in the South rises – Figure 3



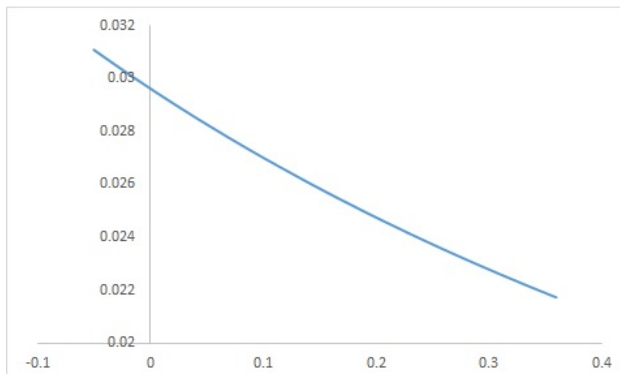
(a) Threshold final good for different inflation rates



(b) Skill premium for different inflation rates



(c) Relative South-North skill premium for different inflation rates



(d) Economic growth rate for different inflation rates

Figure 3: Changes in the endogenous threshold final good, panel (a), skill premium, panel (b), relative South-North skill premium, panel (c), and economic growth rate, panel (d), considering different inflation rates in the South