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UNIVERSIDADE D COIMBRA

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MULTIFRACTALITY IN BITCOIN

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Abstract

Bitcoin is the oldest and most attractive cryptocurrency. Since its launch in 2009, it has thrived, attracting the attention of investors, regulators and the public in general. Its price dynamics, characterized by extreme volatility, severe jumps and impressive appreciation as made bitcoin a different asset from those already existing in the financial market, attracted also the academia attention. The purpose of this dissertation is to examine fractality in the bitcoin market. This is not a new issue, however we intend to get a more clear and comprehensive overview, by applying the Multifractal Detrended Fluctuation Analysis (MF-DFA) and an Multifractal Regime Detecting Method (MRDM) in a high frequency setup. These techniques were applied to 1-min bitcoin returns of Bitstamp, from January 2013 to July 2020, and two other data sets constituted by hourly and daily returns of seven online exchanges (Bitfinex, Bitstamp, Cexio, Coinbase, Exmo, Gemini and Kraken), for the years of 2018-2020 and 2017-2020, respectively.

Our results point out that there is strong evidence that 1-min returns present multifractality, with smaller fluctuations being persistent behavior, while larger fluctuations being anti-persistent. This multifractality comes from the existence of significant long-range correlations, which cast some doubts on the informational efficiency of bitcoin at this frequency. But mainly multifractality comes from fat-tails distribution, which highlights the high level of risk that investors are subjected to in this market. Multifractality is mainly present during 2017, and in late 2018 and earlier 2019.

The comparison between exchanges show similar results at hourly frequency, namely that there is persistence in small fluctuations and anti-persistence in large fluctuations. Although multifractality mainly comes from fat-tail distribution, market efficiency may be low due to significant long-range correlations. There was no visible pattern relating multifractality and efficiency with the trading volume of those exchanges.

At a daily frequency one observe a direct relationship between multifractality and inefficiency with illiquidity (measured by trading volume). Surprisingly, at a daily frequency the main source of multifractality is not fat-tail distributions but instead the long-run correlations.

This complexity puts into perspective that although bitcoin has the potential for long-run attractiveness and increasing valuation, it is also subjected to a non-trivial short-run instability.

Resumo

A criptomoeda mais antiga e mais atrativa é a bitcoin. Desde o seu lançamento em 2009, tem prosperado, atraindo a atenção de investidores, reguladores e do público em geral. A sua dinâmica de preços, caracterizada por extrema volatilidade, saltos e apreciações impressionantes, fazem da bitcoin um ativo diferente dos já existentes no mercado financeiro, atraindo também a atenção da academia. O objetivo desta dissertação é examinar a fractalidade no mercado do bitcoin. Esta não é uma questão nova, mas pretendemos obter uma visão mais clara e abrangente, através da aplicação da *Multifractal Detrended Fluctuation Analysis* (MF-DFA) e de um *Multifractal Regime Detecting Method* (MRDM) numa configuração de alta frequência. Estas técnicas foram aplicadas aos retornos minuto-a-minuto da bitcoin da Bitstamp, de Janeiro de 2013 a Julho de 2020, e a dois outros conjuntos de dados constituídos por retornos horários e diários de sete casas de câmbio on-line (Bitfinex, Bitstamp, Cexio, Coinbase, Exmo, Gemini e Kraken), para os anos de 2018-2020 e 2017-2020, respetivamente.

Os nossos resultados apontam para a existência de fortes indícios de que os retornos 1-minuto apresentam multifractalidade, tendo as flutuações menores comportamentos persistentes, enquanto as flutuações maiores são anti-persistentes. Esta multifractalidade provém da existência de correlações de longo prazo significativas, que lançam algumas dúvidas sobre a eficiência informacional da bitcoin. Mas principalmente a multifractalidade provém da distribuição de caudas largas, o que realça o elevado nível de risco a que os investidores estão sujeitos neste mercado. A multifractalidade está principalmente presente durante 2017, e no final de 2018 e início de 2019.

A comparação entre as casas de câmbio mostra resultados semelhantes a uma frequência horária, nomeadamente que há persistência nas pequenas flutuações e anti-persistência nas grandes flutuações. Embora a multifractalidade provenha principalmente da distribuições com caudas largas, a eficiência do mercado pode ser baixa devido a correlações de longo prazo significativas. Não houve um padrão visível que relacionasse a multifractalidade e a eficiência com o volume de transação das casas de câmbio.

Numa frequência diária observa-se uma relação directa entre multifractalidade e ineficiência com iliquidez (medida pelo volume de comércio). Surpreendentemente, a uma frequência diária, a principal fonte de multifractalidade não são as distribuições com cauda largas, mas sim as correlações de longo prazo.

Esta complexidade coloca em perspetiva que, embora a bitcoin tenha o potencial de atração a longo prazo e de valorização crescente, está também sujeita a uma instabilidade não trivial a curto prazo.

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Chapter 1

Introduction

The cryptocurrency market, especially bitcoin, has gained special attention from investors, academics and regulators (Derbentsev et al., 2019). The history of bitcoin began in 2008, with a person or group of people with the pseudonym Satoshi Nakamoto publishing a white-paper on an electronic payment system without physical representation based on cryptography (Nakamoto, 2008). Just a few months latter, the bitcoin network was created based on an electronic public ledger called blockchain. Since the launch of bitcoin, many other cryptocurrencies were created[1], which, according to Bariviera et al. (2017), can be considered new synthetic tradable assets not supported by any financial intermediary or monetary authority.

Cryptocurrencies can be seen as open sources (anyone may enter in), decentralized (as no third trusted party is needed to validate the payments) networks. Transactions are available 24/7 (Cheng et al., 2019), and all transactions are public knowledge, although the system is pseudo-anonymous (public keys are known by all the network but real identity of users is not revealed). The blockchain is immutable, in the sense that once recorded, transactions cannot be altered or dismissed. Additionally, the blockchain has proven to be resilient to any attempt to hack the network. These features bring confidence and security to market participants; mainly by solving the double spending problem, i.e. by avoiding that a given digital entity is being used simultaneously in different payments (Sebastião and Godinho, 2021). Some cryptocurrencies have their supply artificially truncated.This is the case of bitcoin, whose supply is capped at 21 million units, hence due to its high attractiveness, the demand pressure is prone to result in long-term price appreciation, i.e. bitcoin is subjected to a long-run deflating process.

The finance literature on bitcoin and other cryptocurrencies has been increasing exponentially in the last five years. This dissertation belongs to this stream of the literature. More precisely, here we apply the fractal and multifractal concepts introduced by Benoit Mandelbrot (Mandelbrot and Hudson, 2010) to the bitcoin market. Multifractality may be related to observable properties of financial series, such as the existence of long memory and fat-tail distributions, which are not explainable by the traditional finance theory. In other words, when there is evidence of multifractality, this may imply that the market is informationally inefficient (Al-Yahyaee et al., 2018). In fact, if fractality exists,

[1] According to CoinMarketCap (https://coinmarketcap.com/1/, accessed on Oct. 20, 2020) there are more than 3,500 cryptocurrencies on the market, with bitcoin (BTC) being the currency with the largest market capitalization, more than 210 billion USD . Other important altcoins (i.e., alternative cryptocurrencies to bitcoin) are ethereum (ETH), litecoin (LTC), tether (USDT), ripple (XRP) and bitcoin cash (BCH).

then there are strong reasons to suspect that the Efficient Market Hypothesis (EMH), systematized by Fama (1970), does not hold, and most probably the price dynamics are better explained by the Fractal Market Hypothesis (FMH) of Peters (1994), according to which financial markets have different liquidity levels through time mainly because investors have different horizons.

The examination of multifractality in the bitcoin return series is pursued using the Multifractal Detrended Fluctuation Analysis (MF-DFA) and the Multifractal Regime Detecting Method (MRDM). The MF-DFA was proposed by Kantelhardt et al. (2002), and is an enhancement of the Detrended Fluctuation Analysis (DFA) of Peng et al. (1994), allowing the evaluation of multifractality of non-stationary time series and the characterization of series that do not present monofractal scaling behavior. The MRDM allows the characterization of the time-varying features of fractality itself, namely by identifying those periods of multifractality and monofractality dominance (Lee and Chang, 2015) .

Those methods where applied to 1-min bitcoin returns of Bitstamp, from January 2013 to July 2020, and two other data sets constituted by hourly and daily returns of seven online exchanges (Bitfinex, Bitstamp, Cexio, Coinbase, Exmo, Gemini and Kraken), for the years of 2018-2020 and 2017-2020, respectively.

To the best of our knowledge there are just a few papers addressing the multifractality of cryptocurrencies in a high frequency setup, and none of these papers compare the multifractality in different online exchanges. The idea to do this comparison is based on the hypothesis, already documented in the finance literature, that eventual misalignments between exchanges may produce distinct fractal dynamics. Also we remark that our MRDM differs from that of (Lee and Chang, 2015) because we use the estimates of the generalized Hurst exponents obtain from the MF-DFA.

This dissertation is organized as follows. Chapter 2 presents some basic concepts related to fractals and multifractals, explains the two sources of multifractality, and presents the Efficient Market and Fractal Market hypotheses. Chapter 3 reviews the most relevant empirical literature on fractality of cryptocurrencies in general and bitcoin in particular. In Chapter 4, the multifractal formalism and the MF-DFA are formally presented. Chapter 5 presents a description of the database used, descriptive statistics, and the main results obtained using the MF-DFA and MRDM. Finally, Chapter 6 concludes this dissertation.

Chapter 2

Fractal and Multifractal Theory

2.1 Introduction to fractal and multifractal theory

Financial time series are often generated by complex systems, which are formed by various parts that interact between themselves in a non-linear way, rendering these systems prone to phase transitions. These systems exhibit fluctuations that usually follow a scaling relation, characterized by a power function, over several orders of magnitude. It is here that fractal and multifractal concepts arise: through fractal or multifractal scaling exponents, the scaling laws allow to characterize such complex systems and, consequently, their data and states. More specifically, they help to predict the future behavior of the system (Kantelhardt, 2008). The ecosystem, the economy, the human brain, and in particular financial markets, are examples of such complex systems.

Fractal theory has become more and more recognized by academia due to the important results obtained in several areas of knowledge, such as biology and medicine (Ashkenazy et al., 2001; Peng et al., 1994), music (Su and Wu, 2006), meteorology (Baranowski et al., 2015), economics and finance (Barunik et al., 2012). Applications of fractal theory began in the 1960s, when Benoit Mandelbrot[1] use it to analyze cotton prices, concluding that their distribution followed a power law (Mandelbrot and Hudson, 2010). Mandelbrot's fractal theory began then to be widely used to analyze financial markets, casting some doubts on the conventional financial theory of Eugene Fama (Fama, 1970; Mandelbrot and Hudson, 2010). Over the years, fractal theory has proved useful in clarifying certain phenomena and features of financial series, which conventional financial theory have overlooked, such as that returns time series have fat-tail distributions, which relate to the occurrence of extreme events, show long memory, self-similarity, volatility clustering, and asymmetry (Gunay, 2014).

Basically, a fractal is a geometric shape where each part is identical to the whole. Objects that show this complex geometry cannot be characterized by an integer dimension, hence this led to the definition of fractal dimension – a new measure in fractal geometry. Within this definition there are two important notions: self-similarity and self-affinity. A self-similar fractal is one that scales in the same way in all directions, i.e., "is similar in shape to the element on the next scale higher up or lower down" (Mandelbrot and Hudson, 2010). This concept is associated with scale invariance because

[1]Mandelbrot was a Polish-born French and American mathematician considered the father of fractal geometry, since he coined the word "fractal" from the Latin "fractus" which means "broken".

regardless of the scale to be taken, the fractal is always similar. A self-affine fractal is one that scales more in one direction than another.

A multifractal is a fractal that scales in several different ways over distinct interwoven subsets of the series (Kantelhardt, 2008; Mandelbrot and Hudson, 2010). So, the main difference between a fractal and a multifractal is that the scaling behavior of the former is characterized by a single scaling exponent, while the behavior of the latter is characterized by multiple exponents. Hereafter, some concepts of fractal theory are formally presented.

A stochastic process $X(t)$ is self-similar if it satisfies:

$$X(qt) \stackrel{d}{=} q^H X(t) \tag{2.1}$$

where q is a positive constant (scaling factor), $t \geq 0$ is a time scale and H is the Hurst exponent[1] (Gunay, 2014; Jiang et al., 2020; Mandelbrot, 1972; Taqqu, 1978). Some authors also use the above scaling rule to describe a self-affine process, as they define it in a more general way (Mandelbrot et al., 1997). Equation (2.1) is a scaling rule where the Hurst exponent is constant and unique. If $0.5 < H < 1$, then the process is persistent and stationary. If $H = 0.5$, the process is not long-range dependent, i.e. there is no long memory. This is assumed to be the reference value, which usually refers to random walk processes. Hence, the concept of long memory can be measured by the Hurst coefficient and is considered to be a pervasive feature of financial time series. If $H > 1$, the process is non-stationary. If $H < 0.5$, the process is anti-persistent (presents a negative long memory), indicating that an increment (decrement) is more likely to be followed by a decrement (increment). If $H > 0.5$, the process is persistent (presents a positive long memory), indicating that an increment (decrement) of the series is likely to be followed by an increment (decrement) (Kristoufek and Vosvrda, 2013). One can observe long memory in the series through the dependence between observations and the presence of a power-law decaying autocorrelation function (Derbentsev et al., 2019; Kristoufek and Vosvrda, 2016; Mandelbrot and Hudson, 2010; Pascoal and Monteiro, 2014)

The Hurst exponent and the fractal dimension, denoted by D, are connected through the equality

$$D = 2 - H \tag{2.2}$$

for a self-affine process (Kristoufek and Vosvrda, 2013; Pascoal and Monteiro, 2014). The fractal dimension is a local characteristic of time series and is a measure of its roughness. For univariate series $1 < D \leq 2$. If $D = 1.5$ the process is uncorrelated, i.e, there is no persistence nor anti-persistence. Therefore, a low fractal dimension, $D \in [1, 1.5]$, means a persistent process, while a high fractal dimension $D \in [1.5, 2]$ means anti-persistent process (Kristoufek and Vosvrda, 2013).

Time series do not always present monofractal scaling behaviors, where a single scaling exponent may be used to characterize its behavior (Kantelhardt et al., 2002). In such cases, multiple scaling exponents are necessary to provide a complete description of the scaling behavior. So, for a multifractal process:

$$X(qt) \stackrel{d}{=} q^{h(q)} X(t) \tag{2.3}$$

[1] The first application of the Hurst exponent and long range dependence in economic time series was made by Mandelbrot, using Rescaled-Range Analysis, which became known as R/S analysis. Precisely the approach used by the renowned British hydrologist, H. E. Hurst.

where $t \geq 0$ is a time scale and $h(q)$ is the generalized Hurst exponent, which is not constant (Mandelbrot et al., 1997). Therefore, a way to distinguish multifractals from monofractals series is to look at the dependence of $h(q)$ on q. If $h(q)$ is independent of q, i.e. $h(q) = H$, then the series shows a monofractal behavior. If $h(q)$ is dependent on q, the series shows a multifractal behavior. The same interpretation of the classic Hurst exponent is applicable to the generalized Hurst exponent: Large fluctuations ($q > 0$) are persistent if $h(q) > 0.5$ and anti-persistent if $h(q) < 0.5$; small fluctuations ($q < 0$) are persistent if $h(q) > 0.5$ and anti-persistence if $h(q) < 0.5$ (Fang et al., 2018). The generalized Hurst exponent is a very important parameter in the Multifractal Detrended Fluctuation Analysis (MF-DFA), described in the Section 4.2.

The generalized Hurst exponent, $h(q)$, is also related to the classical scaling exponent, also called Rényi exponent, $\tau(q)$ defined by the standard partition function multifractal formalism, which is the simplest representation of the multifractal analysis, to be presented later:

$$\tau(q) = qh(q) - 1 \tag{2.4}$$

Unfortunately, this standard multifractal formalism does not produce satisfactory results for non-stationary series, hence new methods such as MF-DFA emerged (Kantelhardt, 2008). In addition, this scaling exponent appears in the definition of multifractal process with stationary increments (Calvet et al., 1997; Pascoal and Monteiro, 2014):

$$E[|X(t)|^q] = c(q)t^{\tau(q)+1} \tag{2.5}$$

If $\tau(q)$ is linear, then it means that the process is monofractal, because in that case $h(q) = H$ and therefore $\tau(q) = qH - 1$. Otherwise, if $\tau(q)$ is non-linear, and more precisely a concave function, then it is a multifractal process. According to Mandelbrot et al. (1997), a self-affine process that satisfies Equation (2.1) is multifractal since $X(t) \stackrel{d}{=} t^H X(1)$ and $E[|X(t)|^q] = E[|t^H X(1)|^q] = t^{Hq} E[|X(1)|^q]$. Therefore, according to Equation (2.5), $c(q) = E[|X(1)|^q]$ and $\tau(q) + 1 = qH$, with $\tau(q)$ linear.

Furthermore, one may also characterize the multifractal time series through the multifractal spectrum which relates two exponents: α (the Hölder exponent or singularity strength), defined as the exponent of the scale relationship of the fluctuation dimension on a time scale and $f(\alpha)$ (singularity spectrum), defined as the exponent in a scale relationship of the dimension of the subset of the series, i.e., the number of cells sharing the same α. This gives a metric on the asymmetry between small and large fluctuations. This spectrum is related to $\tau(q)$, via Legendre transform (Calvet et al., 1997) to be specified later. The width of the spectrum is given by:

$$\triangle\alpha = \alpha_{max} - \alpha_{min} \tag{2.6}$$

and determines the richness of the multifractality: the wider the spectrum, the richer the multifractality, meaning that there is a bigger difference between periods with small and large fluctuations (Gunay, 2014). Therefore, the multifractal spectrum is usually presented graphically by the plot of $f(\alpha)$ on α. The width and shape of the spectrum provide information about multifractality. In addition, if the series have a multifractal spectrum with a long left tail (right truncation), it means that the multifractal structure of the series is insensitive to small size local fluctuations. If the series have a multifractal spectrum with a long right tail (left truncation), it means that the multifractal structure of the series is insensitive to large size local fluctuations (Ihlen, 2012).

These relations between α, $f(\alpha)$ and $\tau(q)$ will be detailed below in Chapter 4. The degree of multifractality may also be directly measured by the range of generalized Hurst exponents $h(q)$, denoted by $\triangle h$:

$$\triangle h = h_{max}(q) - h_{min}(q) \tag{2.7}$$

In sum, there are several ways to analyze the multifractality in time series.

2.2 The sources of multifractality

According to Kantelhardt (2008) multifractality present in times series may result from two sources: (a) Fat-tails probability density functions, and (b) different long-range correlations of small and large fluctuations. Given that the effects of these two sources are mixed up in the original data, one needs to use two different data transformations in order to assess its relative importance: (i) Shuffled data via a shuffling technique and (ii) Surrogate data via a phase-randomization technique.

The shuffling process consists in randomly blend the observations of the original series, aiming at eliminating the existing correlations. Through this process, the importance of source (b) is removed, while the impact of source (a) remains in the transformed series (Gunay, 2014). If multifractality remains in the shuffled series, that means that the remaining multifractality is due to source (a) (Kantelhardt, 2008; Kantelhardt et al., 2002). One may perform this analysis using the generalized Hurst exponent applied to the shuffled data, $h_{shuf}(q)$. If multifractality comes only from source (b), the shuffled series will exhibit a simple random behavior, and $h_{shuf}(q) = 0.5$. On the other hand, if (a) is the only source present, $h(q)$ will not change because the multifractality remains, and $h_{shuf}(q) = h(q)$ (Bolgorian and Gharli, 2011; Kantelhardt et al., 2002). The phase-randomization technique is based on the randomization of the Fourier phases of original data aiming at eliminating the non-linearities stored in the phases. The resulting series are called surrogate series and the corresponding interpretation, similar to the previous one, may be applied to $h_{surr}(q)$ (Ashkenazy et al., 2001; Benbachir and El Alaoui, 2011).

Generally speaking, if the shuffled series remain multifractal, then fat-tails are at least in part the origin of multifractality, otherwise, if it turns monofractal, the long-range correlations are the source of multifractality. If the surrogate series remain multifractal, then the long range correlations are a source of multifractality, otherwise, if it turns monofractal, fat-tails are the source of multifractality. Additionally, the two sources of multifractality may be analyzed by comparing the different generalized Hurst exponents of the shuffled and surrogate series or by analyzing the multifractal spectrum $f(\alpha)$ vs α. If the difference between $h_{shuf}(q)$ for different q orders is very small (i.e., if the dependence of $h(q)$ on q for this series is low), this indicates that the multifractality in the original series is likely to derive also from the different long-range correlations of the small and large fluctuations. For the surrogate series, the same reasoning applies. If the difference between $h_{surr}(q)$ for different q orders is very small, this is an indication that the multifractality in the original series is likely to derive from the fat-tails. Also if the width of the spectrum in the shuffled series decreases, the multifractality has not disappeared, and therefore the long-range correlations are one of its sources in the original series. If the width of the spectrum in the surrogated series is small, then the series gives evidence of nearly monofractal behavior and the fat-tails are at the origin of multifractality in the original series (Bolgorian and Gharli, 2011; Gunay, 2014).

2.3 Efficiency on financial markets

The multifractal theory has several financial applications. Most notably, it may be used to test the possibility of investors to use past price information to obtain abnormal returns[1] and to assess the level of extreme risks they support. For instance, if multifractality arises from long-range correlations of small and large fluctuations then investors may profit on the information incorporated in past prices, if multifractality arises from fat-tails this means that investments are exposed to high risks, i.e. there is a higher probability of extreme events (Gunay, 2014). At this point, one should clarify two important frameworks: the Efficient Market Hypothesis (EMH) and the Fractal Market Hypothesis (FMH).

2.3.1 Efficient Market Hypothesis (EMH)

The basic ideas supporting EMH were introduced in 1900 by Louis Bachelier, a French mathematician. However, the EMH was only formally stated in the 1960s and 1970s by Eugene Fama, an American economist and Mandelbrot's PhD student. This hypothesis is by now part of the conventional financial theory and is the cornerstone of modern financial economics. The random walk model used by Bachelier is based on the laws of chance, independent increments and, hence, absence of memory. In this model, prices go up or down with equal probability and therefore are not predictable (Mandelbrot and Hudson, 2010). Accordingly, price movements follow a normal distribution. Fama (1970) built on the random walk and fair game concepts, arguing that the price system contains all the relevant information and therefore no one can obtain abnormal returns by using a particular information set. Depending on the information set considered, the market efficiency may be segmented into three levels or forms: (i) In the weak-form the information set contains all historical market information (such as the time series of prices, trading volumes, dividends, etc.). Hence, technical analysis has no value in the sense that looking at past market information does not allow trader to earn abnormal returns. (ii) In the semi-strong form the information set, besides historical market information, also includes all other public information (produced, for instance by firms, financial analysts, rating firms, etc.). Hence the price system fits rapidly to all new public information, and no one can profit on any piece of public information. (iii) In the strong-form efficiency the information set contains all relevant information, public and private, and no one, even those market participants that own private information may use it to obtain abnormal returns (Fama, 1970; Miloş et al., 2020; Pascoal and Monteiro, 2014).

Not only the basic assumptions of EMH that investors are rational and homogeneous, but also its corollaries that price changes are normally distributed and independent have been subjected to fierce criticisms (Mandelbrot and Hudson, 2010).

A given information set does not have the same impact on the investment decisions of all investors, simply because investors are not homogeneous (Peters, 1994). In reality investors have different

[1] Abnormal returns are returns higher than the overall market. Using the jargon of traders, abnormal returns means "beating the market".

investment horizons, treat the available information differently, do not take the same investment decisions and hence do not achieve the same returns. Even if one assumes that the information sets of all investors are the same, depending on the investment horizons, that information may imply large expected losses for some investors while for others it may result in an expected profit (Peters, 1994). In reality price changes do not follow a normal distribution. The distributions of price changes in financial markets, such as stock, bonds and currencies, are more picked and present fatter tails, and hence are more likely to follow a power law. Price changes are not independent of each other, i.e., they have memory. If prices go up (down) a lot today, there is a high probability that they will continue to go up (down) again tomorrow (Mandelbrot and Hudson, 2010; Peters, 1994).

2.3.2 Fractal Market Hypothesis (FMH)

Fractal Market Hypothesis (FMH) is an alternative to EMH and was defined in 1994 by Edgar E. Peters, an asset manager with contributions on financial economics (Peters, 1994). Briefly, FMH is based on the idea that investors have different horizons, and that is why trades occur and financial markets are liquid and stable. Liquidity means the easiness to trade, rapidly with low transaction costs. If a market is liquid then trades occur without causing major impacts on prices.

If investors treated information in the same way, they would make the same decisions and, consequently, there would not be anyone to trade against to. Markets would be completely illiquid. Because investors have different horizons, they will use information differently, even if they have the same set. Hence, the concept of "fair price" changes from investor to investor, and the market is formed by investors with complementary interests. Therefore, market liquidity exists because investors are not homogeneous, the diversity also comes from the fact that investors access to the information at different times and have different preferences and interpretations.

Liquidity in the market is necessary to guarantee the stable functioning of the market (Kristoufek, 2012), that is, a "stable market is a liquid market" (Peters, 1994). A stable market is a market where long-term investors intervene to stabilize the market in face of the forced trading of short-term investors. A market becomes unstable when long-term investors become short-term investors or cease to participate in the market. When long-term investors feel that the fundamental information is not reliable, panic installs into the market causing big price movements, contributing to fattening the tails of the distribution. According to Peters (1994), long-term investors become more nervous in periods of instability and crisis, when fundamental information looses its value, making short-term investors more active in the market and therefore dominant. Because prices are the result of a combination of technical information, important in the short term, and fundamental information, important in the long term, the unbalance between these two types of investors reduces the market liquidity and disrupts its stability. In sum, the causal relation runs from investor heterogeneity to market liquidity and from market liquidity to market stability.

This efficiency hypothesis was named "fractal" because of it somehow involves a self-similar structure - a characteristic of fractals - i.e., investors should share the same levels of risk, because long-term investors will counterbalance short-term investors to keep the market stable. This leads to different time horizons being intertwined. In particular, self-similarity is associated with complexity under magnification, i.e. if one considers shorter time periods, the smoothness that occurs in longer periods tend to decrease, and invariance in scale, i.e. whatever the scale to be taken, there is always similarity (Anderson and Noss, 2013). Kristoufek (2012) supports this hypothesis by concluding that the FMH gives good predictions of market dynamics in the turbulent times, specifically in the global financial crisis that began in 2007/2008. Anderson and Noss (2013) studied the Dow Jones industrial indexes and state that price series appear to exhibit self-similarity.

Chapter 3

Multifractality in Cryptocurrency Market

In financial markets, large fluctuations, caused by crises and crashes or speculative bubbles, occur more frequently than would be expected if price changes followed a normal distribution. Price time series often exhibit long tailed distributions which indicates the existence of multifractality (Mandelbrot, 1972). Also, according to the FMH of Peters (1994) market liquidity dynamics create significant long-range correlations, especially during periods caracterized by high turbulence. For instance, multifractality has been found in foreign exchanges (Gunay, 2014; Ioan et al., 2012) stock indexes (Benbachir and El Alaoui, 2011; Kumar and Deo, 2009; Miloş et al., 2020; Suárez-García and Gómez-Ullate, 2014; Wang et al., 2014), gold (Bolgorian and Gharli, 2011; Wang et al., 2011), bonds (Lim et al., 2007) and even in inflation rates (Fernandes et al., 2020). These results, mainly achieved using the MF-DFA , seem pervasive independently of the market under study being central or peripheral. However, there is no consensus on the source of multifractality. In those studies revealing persistence, the authors always point out that this is a clear evidence on the rejection of EMH (Barunik et al., 2012; Tiwari et al., 2019).

The cryptocurrency market is prone to frequent "fast markets" events characterized by extreme volatility and hence it is not surprising that an important stream of the financial literature on cryptocurrencies deals with the analysis of fractal structures and their implications in terms of informational efficiency. Obviously the majority of those studies deal only with bitcoin, as it is the most important cryptocurrency in terms of age, trading volume, price appreciation, market capitalization and public attention.

Urquhart (2016) was one of the first papers to address the efficiency of bitcoin. The author used several efficiency tests on the daily prices, among which the rescaled Hurst exponent (R/S Hurst). The main conclusion is that bitcoin is inefficient, especially in the first sub-sample until 2013. Kristoufek and Vosvrda (2016) and Kristoufek (2018) use the Efficiency Index of Kristoufek and Vosvrda (2013), based on long-range dependence, fractal dimension, and entropy, to assess the level of market efficiency. The first paper studies the efficiency of the gold prices in several fiat currencies and bitcoin, showing that gold prices expressed in bitcoin are among the less efficient. The second paper focus on the efficiency of bitcoin in US dollar and the Chinese yuan, showing that both price series are mostly inefficient, although efficiency increases in market cooling periods after bubble-type events. Fang et al. (2018) also study the bitcoin series in USD and yuans concluding that large fluctuations in the Yuan market are persistent, while in the USD market are anti-persistent. In the USD market the broad probability density functions are the source of multifractality. Therefore, the authors conclude that investors participating in the Chinese market are more speculative and less risk averse than those that participate in the US market. Bariviera (2017) and Bariviera et al. (2017) analyze the daily and intraday returns via the DFA. These authors concluded that the Hurst exponent was in the first years of bitcoin above 0.5, hence showing a persistent behavior, but since 2014 it declined to approximately 0.5, implying that the market is becoming more efficient. Tiwari et al. (2018) use a battery of robust long-range dependence estimators and conclude against the previous papers, arguing that the bitcoin market is mostly efficient, except for the periods of April to August 2013 and August to November 2016. On the other side, Jiang et al. (2018) categorically conclude that long memory is present in the bitcoin returns and that bitcoin is inefficient and it has not become more efficient over time. Lahmiri and Bekiros (2018) focused on price and returns of bitcoin to study chaos, randomness and multifractal properties. The sample was segmented into periods of stable prices and of exponentially increasing prices. Multifractality and long-range correlations were found to be present in returns and prices, with fat-tail distributions being the main cause of multifractality. In addition, multifractality was stronger in returns than in prices for negative q scales, while the opposite happened for positive q scales. Takaishi (2018) examined 1-min bitcoin returns highlighting that they present a fat tail distribution with a kurtosis much higher than three. The MF-DFA approach showed the existence of multifractality coming from both sources. Therefore, according to the author, bitcoin is similar to other assets in terms of stylized facts. Filho et al. (2018) apply the MF-DFA to prices quoted every 12h and find that multifractality arose from both fat tails and long-range correlations. The authors highlight that multifractality of bitcoin series is stronger when compared to other financial series. Al-Yahyaee et al. (2018) also reach this broad conclusion when comparing the bitcoin market with the gold, stock and foreign exchange markets via the MF-DFA. Zhang et al. (2019a) show that multifractality is evident in prices and volumes, and that fat-tails are the main reason of multifractality. However, prices and volumes are characterized by persistence and anti-persistence, respectively. Telli and Chen (2020) use the MF-DFA to investigate multifractality in the returns and volatility series. They found that returns and volatility are persistent, and the degree of multifractality in bitcoin is greater than in gold. Fat-tails and temporal correlations are both at the origin of the multifractality of the two series. Jiang et al. (2020) develop an empirical multifractal scaling hypothesis test to verify if there is a multifractal scaling in the presence of fat-tails and applied it to 1-min and daily bitcoin prices. They conclude that the scaling behavior of bitcoin is more akin to a multifractal model than to a heavy tail process, however the presence of multifractality in high frequency data is only revealed in smaller samples.

Several papers address the issue of fractal structures of bitcoin and other cryptocurrencies, mainly showing that these structures may be quite different and that they have a time-varying nature. For instance, Caporale et al. (2018) find some evidence of persistence in bitcoin, litcoin, ripple and dash, which implies market inefficiency, although this inefficiency tends to be less expressive in more recent periods. Mensi et al. (2019) use the asymmetric multifractal detrended fluctuation analysis to study high frequency data from bitcoin and ethereum. The main conclusions are that multifractality is present in both markets, efficiency has a time-varying feature but bitcoin is more inefficient than ethereum. Costa et al. (2019) analyze bitcoin, ethereum, ripple and litecoin through the DFA and found that bitcoin appears to be the most efficient, although the persistent behavior is predominant at the end of 2017. Derbentsev et al. (2019) examine the multifractal properties of the most capitalized cryptocurrencies through R/S analysis and DFA. Persistent is observable for almost the full period, although during periods of turbulence and crisis in the market, cryptocurrencies series present anti-persistence. Cheng et al. (2019) use the DFA and MF-DFA to investigate the price movements of four cryptocurrencies: bitcoin, ethereum, Ripple and EOS, concluding that bitcoin and ethereum markets revealed a strong momentum effect (long-range memory), while ripple and EOS showed price reversals (anti-persistence behavior). Stosic et al. (2019) apply the MF-DFA to the price and volume series of fifty cryptocurrencies. The authors conclude that large fluctuations dominated the multifractal behavior in price changes (left-skewed spectrum), while the opposite happened with volume changes. Besides, the price changes do not present any kind of significant correlations, while the volume changes present anti-persistence. They also highlight that fat tails are the origin of multifractality for both price and volume changes. Bariviera (2020) study the informational efficiency of the eighty-four largest cryptocurrencies through the generalized Hurst exponent, obtained from a multi-scale methodology. Multifractal dynamics are mostly present in this market, but, for the largest cryptocurrencies, a monofractal process seems to fit. The shuffling process resulted in a reduction of multifractality, but the author continue to believe on the existence of the two sources of multifractality. Mnif et al. (2020) applies MF-DFA to the five biggest cryptocurrencies to study their efficiency through the magnitude of a Long-memory index related to the generalized Hurst exponent, and the herding behavior at the time of COVID-19. Most of cryptocurrencies were multifractal before the COVID-19 and bitcoin proved to be the most efficient one, but less efficient than ethereum after the pandemic. A clear exception to the main stream of the literature is Zhang et al. (2019b). This paper analyzes the long range memory of bitcoin, ethereum, ripple and litecoin using hourly prices. The authors argue that the Hurst exponent is approximately 0.5, indicating relative efficiency, so the price movements can be characterized by a random walk. Nevertheless, there is evidence of long memory in certain periods.

In a nutshell, the main conclusions that one may highlight from all the previous mentioned papers is that the bitcoin market presents multifractality, coming from not only fat-tails but also from long-range correlations. Multifractality seems to be stronger in bitcoin than in other more traditional financial markets. The existence of significant long-range correlations casts some doubts on the informational efficiency of the market, although there is some weak evidence that efficiency is increasing as the market matures. The fractal structure of bitcoin and other cryptocurrencies may be quite different and have a time-varying structure.

Chapter 4

Methodology and Interpretations

4.1 The Multifractal Formalism

The multifractal formalism is one way among others to reveal the scaling behavior and characterize the time series. Therefore, this formalism is introduced here in order to obtain the instrument known as the full spectrum, through the partition function. The following is based on the articles of Halsey et al. (1986) and Salat et al. (2017), where the moment method and box-counting approaches to calculate the spectrum $f(\alpha)$ are detailed.

The probability related to the i-th box, p_i, that is, the probability that the phenomenon occurs in the i-th box, is defined by

$$p_i := \frac{N_i}{N} = \int_{i^{th}box} d\mu(x) \tag{4.1}$$

where μ represents the probability measure, N the number of boxes, or the times that a phenomenon occurs in domain D, and N_i is the number of times that a phenomenon occurs in the i-th box. This verifies $p_i \sim r^{\alpha_i}$, where r is the unit of the mesh grid which covers D, i.e., the size of the i-th piece which divides D. Remark that we have $\mu_i(r) \sim r^{\alpha_i}$, where α_i is the Hölder dimension, such that $\alpha_i = \log \mu_i(r)/\log r$ (Zhao et al., 2015). In addition, it is necessary to define $N(\alpha_i)$ as the number of times that α occurs in each interval $[\alpha_i, \alpha_i + d\alpha_i]$, which verifies

$$N(\alpha_i) \sim \rho(\alpha_i)\, d\alpha_i r^{-f(\alpha_i)} \tag{4.2}$$

where ρ is the density function used, which takes into account the D dimension, and $f(\alpha)$ represents the dimensions of the set where the singularities of strength α can be found. Now, let us consider a partition function $Z(q)$ such that $Z(q) := \sum_i p_i^q \sim r^{\tau(q)}$. Notice that each q is associated to an α with a predominant contribution to the total value of the measure:

$$Z(q) := \sum_i p_i^q \sim \sum_i r^{\alpha_i q} \sim \int_\alpha N(\alpha) r^{\alpha q} \sim \int_\alpha \rho(\alpha) r^{-f(\alpha)} r^{\alpha q}\, d\alpha \sim \int_\alpha \rho(\alpha) r^{\alpha q - f(\alpha)}\, d\alpha \tag{4.3}$$

So, considering r small enough, $Z(q)$ is almost entirely characterized in terms of α, such that $\tau(q)$ given by $\alpha q - f(\alpha)$ is minimal. Therefore, the full spectrum giving f as a function of α can be obtain after computing τ.

4.2 The Multifractal Detrended Fluctuation Analysis (MF-DFA)

The Multifractal Detrended Fluctuation Analysis (MF-DFA) (Kantelhardt et al., 2002), especially suitable for time series, is based on five steps. Let's consider a series x_i of length N with a compact support, i.e, $x_i = 0$ only for an insignificant portion of values. The first three steps are identical with the classical DFA (Peng et al., 1994).

First, from series x_i and being $\bar{x}$ its mean, the profile is computed as

$$Y(j) = \sum_{i=1}^{j} (x_i - \bar{x}), \qquad j = 1, \dots, N \tag{4.4}$$

The average doesn't have to be necessary taken into account since it would be eliminated by the later detrending.

The second step consists in splitting $Y(j)$ into $N_s = int(N/s)$ non-overlapping segments of equal length s, where s is a given time scale, *int* represents the rounding down to the nearest integer, and N is often not a multiple of the scale s. The same procedure has to be considered starting from the end of the sample in order not to ignore the residual part of the series not yet considered. Hence, $2N_s$ segments should be obtained.

Next, after calculating the local trends for every segments by a least-square fit of the profile to a polynomial function, the variances are given by

$$F^2(s, \upsilon) \equiv \frac{1}{s} \sum_{j=1}^{s} \{Y[(\upsilon - 1)s + j] - y_\upsilon(j)\}^2, \; for \; \upsilon = 1, \dots, N_s \tag{4.5}$$

$$F^2(s, \upsilon) \equiv \frac{1}{s} \sum_{j=1}^{s} \{Y[N - (\upsilon - N_s)s + j] - y_\upsilon(j)\}^2, \; for \; \upsilon = N_s + 1, \dots, 2N_s \tag{4.6}$$

where $y_\upsilon(j)$ is the polynomial trend fitted to segment υ. In the fitting procedure, the polynomials can be linear, quadratic, cubic or of higher degrees. This procedure is done for different polynomial orders, m. Hence, an MF-DFA of order m, trends of order m in $Y(j)$, or order $m - 1$ in the original series, are eliminated. Therefore, to estimate the type of the polynomial trend in the time series, a comparison of the quality of trend fitting for different orders of DFA is necessary (Kantelhardt et al., 2002).

The fourth step consists in obtaining the q order fluctuation function and by averaging over all segments:

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\upsilon=1}^{2N_s} [F^2(\upsilon, s)]^{\frac{q}{2}} \right\}^{\frac{1}{q}}, \; for \; q \neq 0 \tag{4.7}$$

It is apparent that $F_q(s)$ increases with s. Also, $F_q(s)$ depends on the order m, which is necessary not superior to $s - 2$. Steps 2 to 4 must be repeated for several time scales s in order to know the dependence of $F_q(s)$ on the time scale s for different values of q. Finally, the scaling behavior of fluctuation functions is determined by analyzing log-log plots of $F_q(s)$ versus s for each value of q. The following relationship is obtained:

$$F_q(s) \sim s^{h(q)} \tag{4.8}$$

i.e., $F_q(s)$ increases as a power-law, for large values of s, if the series x_k are long-range power-law correlated. Here $h(q)$ is called the generalized Hurst exponent. The q-th order generalized Hurst exponent is the slope of $\log(F_q(s)) \sim \log(s)$. For $q > 0$, $h(q)$ specifies the scaling behavior of the segments with larger fluctuations, while $q < 0$, $h(q)$ specifies the scaling behavior of the segments with small fluctuations. Thus, the small and large fluctuations will present a different behavior. Moreover, for $q = 2$ and stationary series, $h(2)$ is the classic Hurst exponent, H (Kantelhardt, 2008). The fluctuation function of order 0, $F_0(s)$, is not obtainable from (4.7), so the following procedure is defined:

$$F_0(s) \equiv \exp\left\{\frac{1}{4N_s}\sum_{\upsilon=1}^{2N_s}\ln[F^2(\upsilon,s)]\right\} \sim s^{h(0)} \tag{4.9}$$

4.3 The Multifractal Formalism and MF-DFA

In order to show the relationship between $h(q)$ and the Rényi exponent $\tau(q)$, characterizing the behavior of the partition function, the standard multifractal formalism and the standard box counting formalism are related to the MF-DFA, in the context of stationarity (Kantelhardt et al., 2002). First, to make such a comparison, stationary, the normalized sequence x_i of length N with compact support is considered. Hence, in the third step of the MF-DFA, Equation (4.5) takes the form:

$$F^2(s,\upsilon) \equiv [Y(\upsilon s) - Y((\upsilon-1)s)]^2,\ for\ \upsilon = 1,\ldots,N_s. \tag{4.10}$$

Remark that it is not necessary to apply the detrending procedure, as no trend has to be eliminated. Proceeding with the MF-DFA, the next step allows, substituting 4.10 into 4.7, to obtain

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}|Y(\upsilon s) - Y((\upsilon-1)s)|^q\right\}^{\frac{1}{q}} \sim s^{h(q)}\ i.e.\ \frac{1}{2N_s}\sum_{\upsilon=1}^{2N_s}|Y(\upsilon s) - Y((\upsilon-1)s)|^q \sim s^{qh(q)} \tag{4.11}$$

Since $N_s = N/s$, assuming that N is divisible by s, the previous expression can be written as:

$$\sum_{\upsilon=1}^{N_s}|Y(\upsilon s) - Y((\upsilon-1)s)|^q \sim s^{qh(q)-1} \tag{4.12}$$

The standard box counting formalism is introduced in this context by defining the partition function

$$Z_q(s) \equiv \sum_{\upsilon=1}^{N/s}|p_s(\upsilon)|^q \sim s^{\tau(q)} \quad with \quad p_s(\upsilon) \equiv \sum_{i=(\upsilon-1)s+1}^{\upsilon s} x_i = Y(\upsilon s) - Y((\upsilon-1)s) \tag{4.13}$$

where $p_s(\upsilon)$ is the box probability, that is identical to the sum of the numbers x_i within each segment υ of size s. Therefore, the expression (4.12) is equivalent to expression (4.13), so it can be verified that $Z_q(s) \sim s^{qh(q)-1}$ and therefore:

$$\tau(q) = qh(q) - 1. \tag{4.14}$$

Also, $\tau(q)$ is related to the generalized multifractal dimension $D(q)$, which is sometimes used in certain studies instead of $\tau(q)$, given by:

$$D(q) \equiv \frac{\tau(q)}{q-1} = \frac{qh(q)-1}{q-1} \tag{4.15}$$

In a monofractal series with compact support, $D(q)$ depends on q, unlike $h(q)$, which is independent of q. Hence

$$\alpha(q) = \tau'(q) = \frac{d\tau(q)}{dq} = \frac{d[qh(q)-1]}{dq} = h(q) + q\frac{dh(q)}{dq} \tag{4.16}$$

and equating equations (4.14) and $\tau(q) = \alpha q - f(\alpha)$, where $f(\alpha)$ is the spectrum of singularity and α is the Hölder exponent, referred to in Section 2.1:

$$\begin{aligned} & qh(q) - 1 = \alpha q - f(\alpha) \\ \Leftrightarrow\ & f(\alpha) = q(\alpha - h(q)) + 1 \end{aligned} \tag{4.17}$$

Thus, through $h(q)$, the multifractal spectrum can be specified, which also characterizes the multifractal series.

4.4 The presence of multifractality in the MF-DFA

In order to detect the existence of multifractality in financial time series, it is necessary to check certain characteristics in the graphs obtained through the MF-DFA. Starting with the plot $\log F_q(s)$ vs s of the series, where $F_q(s)$ is the q-order fluctuation function, an indication of a monofractal behavior can be the parallel relationship that the fluctuation function presents in different scales for different orders of q.

The slopes of the regression lines give the generalized Hurst exponents. In the multifractal case, the slopes are dependent on q and the differences between the fluctuation functions for negative and positive qs are more pronounced in small segment sizes. If it is a monofractal case, these differences are the same for different q orders, thus $h(q)$ is a constant H, for all q orders (Gunay, 2014).

Regarding the generalized Hurst exponents $h(q)$, more specifically the plot $h(q)$ vs q, the distance of the values of the generalized Hurst exponent from each other for different orders, is a sign of multifractality. Therefore, an approximately equal behavior of $h(q)$ for different q orders are evidence of near monofractal behavior, while wide variations of $h(q)$ according to the variation of q are evidence of strong multifractality in the series. Usually, in the case of multifractality, the function described by the graph $h(q)$ vs q is non-linear and decreasing on q (Benbachir and El Alaoui, 2011; Bolgorian and Gharli, 2011; Gunay, 2014; Miloş et al., 2020; Wang et al., 2014).

The plot $\tau(q)$ on q concerns the classical multifractal scaling exponent for the partition function on different orders of q. The monofractal behavior is associated with the linear form of $\tau(q) = qH - 1$, since $h(q) = H$ is constant, while a nonlinear form of $\tau(q)$ represents the existence of multifractality. Moreover, the further apart from linearity the curve is, the stronger the multifractality is (Wang et al., 2014).

Finally, concerning the multifractal spectrum, that is, the function $f(\alpha)$, a monofractal behavior is reflected in the shape of the dense spectrum, in which its curve is reduced to a single point $\alpha = H$ with $f(\alpha) = 1$. On the contrary, a multifractal behavior translates into a spectrum curve with a single-humped shape, presenting a concave downward parabola shape. Also, for a small degree of multifractality, the width of the spectrum will have a small range and for strong multifractal series, the width will have a large range (Telli and Chen, 2020). The relations between the plots $h(q)$, $\tau(q)$ and $f(\alpha)$ results from the analysis exposed in the previous sections.

Chapter 5

Empirical Results

5.1 Data and preliminary analysis

The first data set analyzed in this dissertation consists on the time series of 1-min bitcoin prices from January 01, 2013 to July 26, 2020. The database, retrieved from the Bitcoincharts website API[1], contains the trading information on bitcoin from one of the main bitcoin online exchanges – Bitstamp. The data file has the timestamp in Unix time, the trading price in USD, and the trading volume in bitcoin units. A significant part of these volumes are not integers, because bitcoin may be traded in multiples of 10^{-10} of a bitcoin, i.e. a satoshi. These raw data was then sampled considering the last price before each minute-by-minute observation point. The final data is formed by 3,981,601 observations.

The second data set contains the bitcoin hourly prices in USD of seven online exchanges: Gemini, Coinbase, Kraken and Bitfinex from USA and Bitstamp, Cexio and Exmo from Europe, from January 1, 2018 to October 12, 2020. This data set, with a total of 24,372 observations, was retrieved from the Cryptodatadownload website[2].

Finally, a third data set with daily bitcoin prices in USD timestamped at 00:00:00 for the period from January 1, 2017 to December 2, 2020 (1,432 observations) for the set of the seven online exchanges, was retrieved from Bitcoinity.org website[3].

The exchanges, ranked by decreasing liquidity, measured by the trading volume, are characterized below in Table 5.1. The table shows the continent of the location of their headquarters, the launching year, average hourly trading volume (01/01/2018 - 10/12/2020) and average daily trading volume (01/01/2017 - 12/02/2020).

[1]http://api.bitcoincharts.com/v1/csv/.
[2]https://www.cryptodatadownload.com/data/.
[3]http://data.bitcoinity.org/markets/price/5y/USD?c=e&t=l.

Table 5.1 Online exchanges

	Headquarter	Year	Hourly volume (BTC)	Daily volume (USD)
Bitfinex	America	2012	18,572,760	228,991,353,026
Coinbase	America	2012	13,108,974	158,496,869,065
Bitstamp	Europe	2011	9,334,135	103,626,571,751
Kraken	America	2011	6,320,323	62,047,382,036
Gemini	America	2014	3,095,720	40,254,511,611
Cexio	Europe	2013	479,672	7,101,667,828
Exmo	Europe	2014	414,553	4,103,315,927

The time series of bitcoin prices were then used to compute the logarithmic returns, such that $r_t = \log(P_t/P_{t-1})$, where P_t is the transaction price at time t.

Figure 5.1 plots the series of bitcoin 1-min prices and their resulting logarithmic returns, with Unix time converted into calendar time.

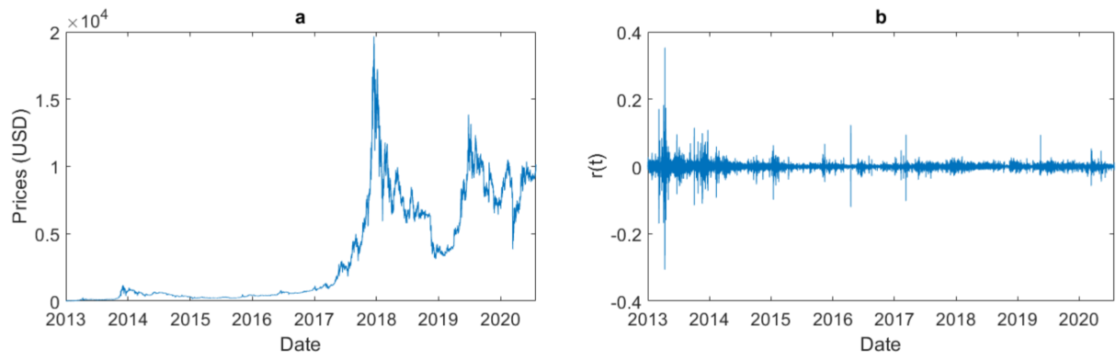


Fig. 5.1 Bitcoin 1-min prices (a) and bitcoin 1-min returns (b)

A striking feature that can visualize from Figure 5.1a is that bitcoin prices have increased their scale considerably. During 2013, prices were relatively low and seem stable. At the end of that year the market saw its first period of exponential price increases, i.e. the first bubble-like event. During the following three years prices remain at similar levels without significant long run increases or decreases. 2017 was characterized by exponential increasing prices – the second and more important bubble in our sample. At the beginning of 2017, bitcoin was quote almost at 1,000USD and at the end of that year it achieved the absolute maximum in this sample of almost 20,000USD. In 2018, the bubble burst and prices showed a sharp decline. In November that year one may even observe a "negative" jump halving the price from 6,000USD to 3,000USD. In the first semester of 2019 there was the third long duration bubble, while in the second semester of that year there was a reverse tendency. Finally, in the year 2020 there is a sharp decrease in February/March, which is associated with the impact of the Covid-19 pandemic crisis. Our sample ends in July 2020, when bitcoin was quoted around 10,000USD. Nevertheless, one should notice that bitcoin is now living the most important bubble in all of financial history of mankind. At the time of writing (March 13, 2021) bitcoin price reached more than 57,000USD. According to Jiang et al. (2020), the 2013 and 2017 peaks come from boosts in the popularity of bitcoin. The sharp decline in the beginning of 2018 was associated with the launch of Bitcoin futures by the the Chicago Board Options Exchange (CBOE) and the Chicago Mercantile Exchange (CME) (Sebastião and Godinho, 2020). Figure 5.1b presents the history of 1-min returns. At this frequency, one may observe that returns are more volatile in the beginning of the sample, so the stability of prices in that period were more "an illusion" due to the change in its scale during the overall sample.

Table 5.2 presents the descriptive statistics of bitcoin 1-min returns. One can verify that returns present a positive mean very close to zero, positive skewness, indicating more probability mass on the right side of the distribution, a very high kurtosis in comparison to the reference value 3 of the normal distribution, showing that the distribution is highly leptokurtic. This extremely high kurtosis, which is pervasive at various periodicities, has itself being addressed in the literature on multifractality (Bariviera, 2017; Filho et al., 2018). To complement the strong evidence that the return series are non-normal, Table 5.2 also shows the Jarque-Bera test, which peremptorily rejects the null hypothesis of normality. The first order autocorrelation is significantly negative, corroborating the result present in Takaishi (2018), for the 1-min returns of the bitcoin price index in the period 2014-2016.

Table 5.2 Descriptive statistics of bitcoin 1-min returns

	Returns
Number of observations	3,981,600
Mean (10^{-6})	1.6637
Standard Deviation	0.0021
Median	0.0000
Maximum	0.3529
Minimum	-0.3065
Skewness	0.5099
Kurtosis	959.59
Jarque-Bera normality test (10^{11})	1.5181
First order autocorrelation	-0.1931

In order to obtain a more detailed analysis, the full sample was partitioned into three sub-samples (Figure 5.2): the first sub-sample is from 01/01/2013 to 08/30/2015, the second sub-sample is from 08/31/2015 to 01/27/2019, and finally the third sub-sample is from 01/28/2019 to 07/26/2020. As highlighted before, now one can observe that in fact prices vary considerably in all the three sub-samples, showing increasing and decreasing trends of several months or even years. Table 5.3 presents the descriptive statistics of bitcoin returns in those three sub-samples. The mean in each sub-sample is very close to zero and the maximum and minimum of the full sample are both located in the first sub-sample. Only the second sub-sample shows a negative skewness, which gives the indication of a long left tail, while the others sub-samples show positive skewness. Kurtosis is very high in all sub-samples, the highest being found in the first sub-sample and the lowest in the second sub-sample. Therefore, the series are highly leptokurtic independently of the period considered.

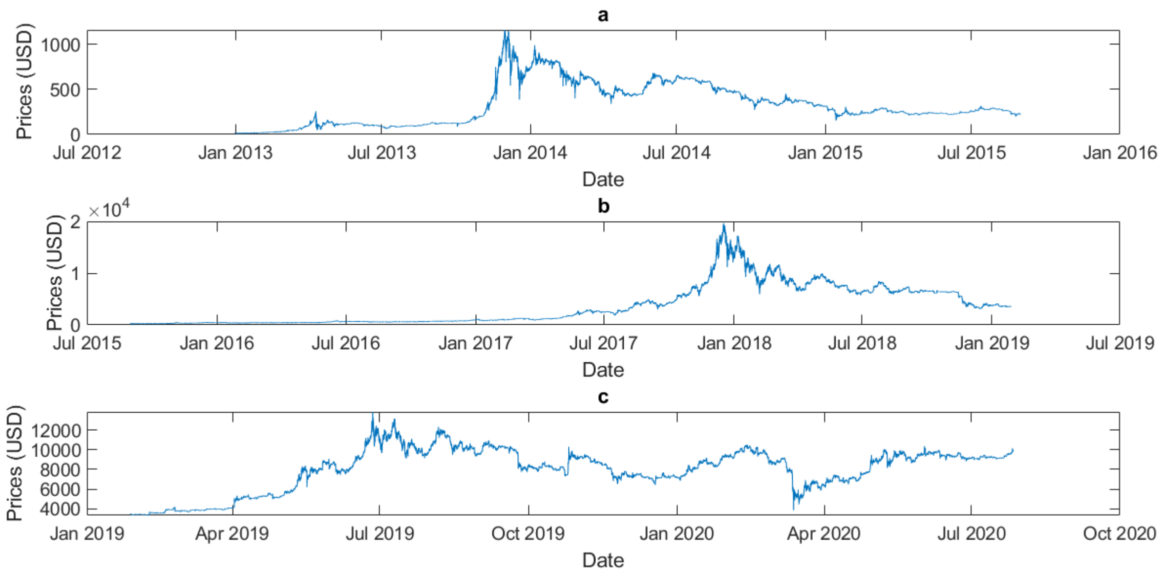


Fig. 5.2 Bitcoin prices in: (a) 1st sub-sample (b) 2nd sub-sample (c) 3rd sub-sample

Table 5.3 Descriptive Statistics of bitcoin returns on the sub-samples

	1st subsample	2nd subsample	3rd subsample
Observations	1,401,120	1,794,240	786,240
Mean (10^{-6})	2.0366	0.1525	1.3157
Standard Deviation	0.0030	0.0015	0.0012
Median	0.0000	0.0000	0.0000
Maximum	0.3529	0.1227	0.0938
Minimum	-0.3065	-0.1205	-0.0572
Skewness	0.5394	-0.2829	0.1609
Kurtosis	652.38	103.70	150.24
Jarque-Bera normality test (10^8)	246.18	7.5814	7.1020

In order to have a more complete picture on what has happen in the bitcoin market, namely in terms of trading intensity, we also looked at the historical data of the daily volume of the overall bitcoin market. These data was collected from the CoinMarketCap website[1] from April 29, 2013 to July 30, 2020 (2,650 observations). This site provides aggregated data on eligible online exchanges. The data file is formed by the daily opening and closing, the lowest and highest prices (which are commonly referred to as OLHC prices), volume, and market capitalization, all in USD. The daily volume in bitcoin units is obtained by dividing the volume in USD by the average of the OLHC prices. Figure 5.3 plots of the daily volume of bitcoin.

[1] https://coinmarketcap.com/currencies/bitcoin/historical-data/.

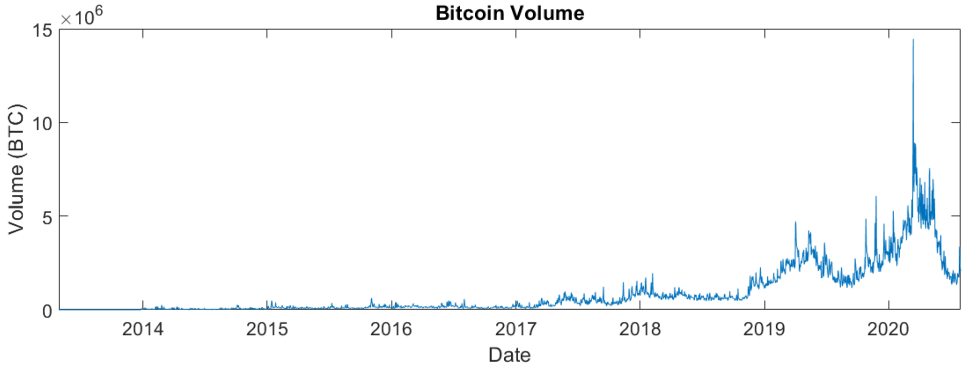


Fig. 5.3 Daily volume of bitcoin

Daily trading volume shows a stable linear increase during the years of 2013-2016. In 2017 the daily trading volume increased visibly and then remained stable in 2018. In 2019 it increased in the first semester and decreased in part of the second semester, which was directly related with the price tendencies. Interestingly, in 2020 volume and price trends are inversely related, that is higher volume was associated with selling pressure and decaying prices, with the peak in trading volume occurring during the Covid-19 pandemic crisis. Table 5.4 presents the descriptive statistics of bitcoin volume. On average, during the overall period, the eligible exchanges traded around 848 bitcoins per day. The daily volume shows positive skewness and some excess kurtosis. As suggested in the finance literature, the Jarque-Bera test is applied to the natural logarithm of volumes, but even with this transformation the log-volume is clearly non-normal.

The full sample was then divided into four sub-samples. 1st sub-sample from 04/29/2013 to 12/31/2016, 2nd sub-sample from 01/01/2017 to 12/31/2018, 3rd sub-sample from 01/01/2019 to 12/31/2019, and finally, 4th sub-sample from 01/01/2020 to 07/30/2020. Figure 5.4 shows the volume charts and Table 5.5 presents the descriptive statistics of bitcoin volumes of these sub-samples. All the statistics increase through the four sub-samples, except the skewness and kurtosis, which are lower in the 2nd and 3rd sub-samples.

Table 5.4 Descriptive Statistics of bitcoin volume

	Volume
Observations	2,650
Mean	848,75
Standard Deviation	1,320,441
Median	208,575
Maximum	14,450,842
Minimum	0.0000
Skewness	2.6429
Kurtosis	13.009
Jarque-Bera normality test	97.018

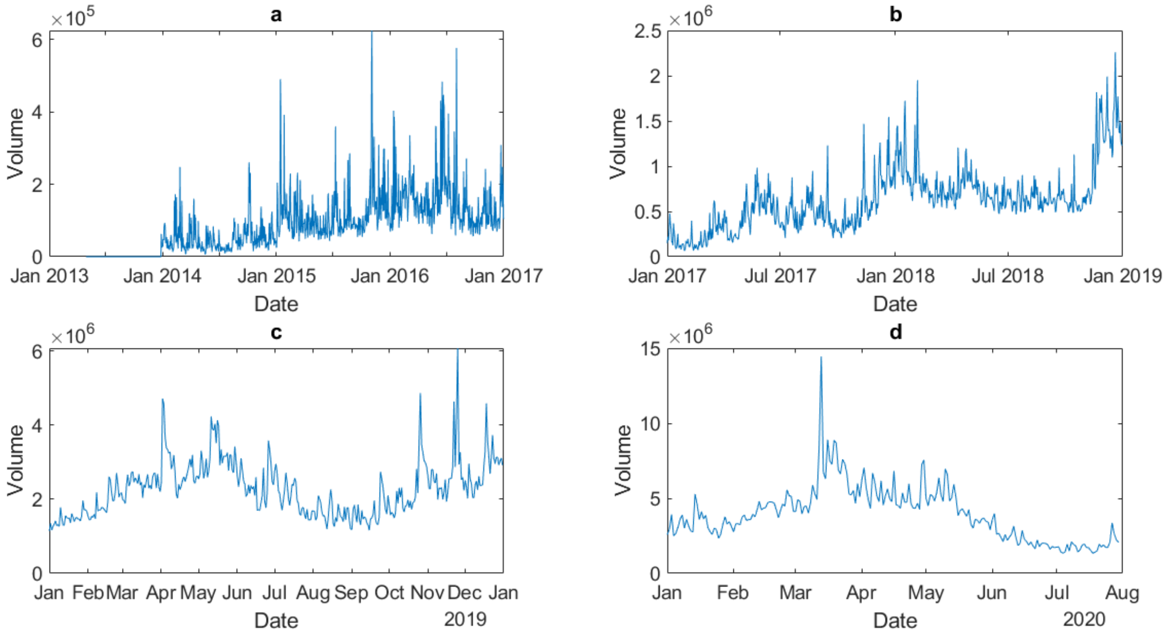


Fig. 5.4 Bitcoin volume in: (a) 1st sub-sample (b) 2nd sub-sample (c) 3rd sub-sample (d) 4th sub-sample

Table 5.5 Descriptive Statistics of bitcoin volume in four sub-samples

	1st sub-sample	2nd sub-sample	3rd sub-sample	4th sub-sample
Observations	1,343	730	365	212
Mean	87,428	628,819	2,280,344	3,964,183
Standard Deviation	80,710	340,641	733,539	1,834,469
Median	78,155	601,178	2,254,127	3,858,410
Maximum	624,124	2,260,484	6,071,189	14,450,842
Minimum	0.0000	66,068	1,141,826	1,337,546
Skewness	1.7116	1.1555	1.0690	1.2654
Kurtosis	8.2939	5.1663	5.1690	7.1543
Jarque-Bera normality test	43.640	113.70	3.965	3.9130

5.2 Multifractality in the bitcoin 1-min returns

This section applies the MF-DFA to the bitcoin 1-min logarithmic returns, using the code provided by Ihlen (2012). To apply the method we define the following parameters: (1) the scaling parameter ranges from $s_{min} = 64$ to $s_{max} = 262,144$ in powers of two, the parameter q ranges from -5 to 5 with a step of 0.1 and the polynomial order is $m = 1$. According to Ihlen (2012), one should choose $10 < s < N/10$, where N is the length of the time series and the value of the minimum scale should be higher than the polynomial order used for detrending. Therefore, the minimum and maximum scales were chosen to be powers of two between those boundaries, namely the last value is the highest power of two lower than $N/10$.

Figure 5.5 presents the plot of $\log_2 F_q(s)$ vs. $\log_2 s$ for 101 different q orders and the plot of $\log_2 F_q(s)$ vs. s for the values $q = -5$, $q = 0$ and $q = 5$.

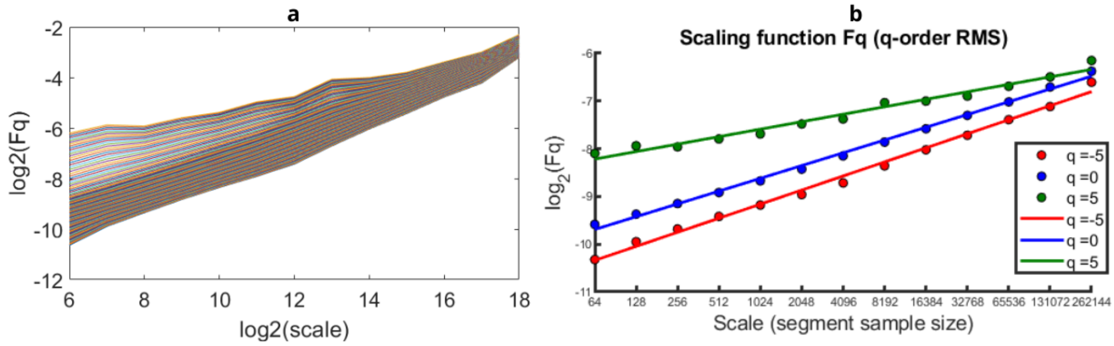


Fig. 5.5 (a) The plot of $\log_2 F_q(s)$ versus $\log_2 s$ for 101 different q orders and (b) the plot of $\log_2 F_q(s)$ versus s for three q orders.

Noticeably, the presence of multifractality may be detected through the non-parallel relationship, i.e., the slope changes for different orders q. In addition, a significant difference of the fluctuation function can be verified for different orders in small size segments (left section of each graphs). Therefore, this is a first indication that the series of 1-min returns presents multifractality. Figure 5.6 shows the plot of $h(q)$ and $\tau(q)$ on q.

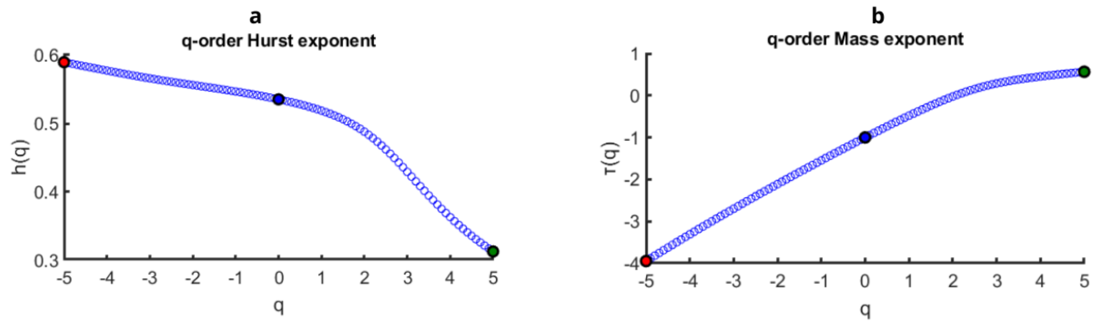


Fig. 5.6 (a)The plot of $h(q)$ on q and (b)the plot of $\tau(q)$ on q

Figure 5.6a shows that the variation of $h(q)$ with respect to q is remarkable, presenting a non-linear decreasing shape. The generalized Hurst exponent, $h(q)$, is the slope of the regression lines in Figure 5.5. Most fluctuations are persistent ($h(q) > 0.5$), but this behavior is more evident in small fluctuations, i.e, for $q < 0$. For $q > 1.6$, the fluctuations are anti-persistent (Zhang et al., 2019a). Therefore, for small fluctuations, the bitcoin market presents long memory. According to Figure 5.6b, the Rényi exponent $\tau(q)$ presents also a visible increase for values of q up to 2.5, and a non-linear behavior on q, which results from $\tau(q) = qh(q) - 1$, and $h(q)$ not constant. These results strengthens the claim that the 1-min returns are multifractal. Finally, the multifractal spectrum $f(\alpha)$, which characterizes the multifractal behavior, is shown in Figure 5.7. The curve presents a single-humped shape, representing multifractality.

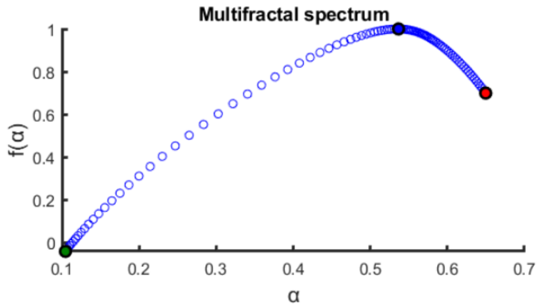


Fig. 5.7 The multifractal spectrum $f(\alpha)$ versus α

Several insights can be obtained from the multifractal spectrum. First, the degree of multifractality measured by Equation (2.7) is $\triangle h = 0.5895 - 0.3131 = 0.2764$. This is a lower value than the one obtained by Takaishi (2018) using 1-min data from January 1, 2014 to December 31, 2016. Second, the width of the spectrum given by Equation (2.6) is $\triangle\alpha = 0.6497 - 0.1046 = 0.5451$. Third, one of the important values in the spectrum is α_0, which gives the maximum of $f(\alpha)$, that is, $f(\alpha_0) = 1$, and hence measures the persistent behavior. In this case, $\alpha_0 = 0.5366$, which is a value relatively close to 0.5 (Telli and Chen, 2020). Fourth, the asymmetry parameter $A = (\alpha_{max} - \alpha_0)/(\alpha_0 - \alpha_{min})$ gives information about the multifractal spectrum. If $A < 1$, the spectrum is left-skewed and the scaling behavior of large fluctuations dominates the multifractal behavior. If $A > 1$, the spectrum is right-skewed and small fluctuations dominate the multifractal behavior (Telli and Chen, 2020). For these time series $A = 0.2618$, which indicates that the spectrum is left-skewed.

In order to study the EMH of the 1-min returns, we use the Market Efficiency Measure (MEM) (Al-Yahyaee et al., 2018; Wang et al., 2009) defined as follows:

$$MEM = \frac{1}{2}(\mid h(q_{min}) - 0.5 \mid + \mid h(q_{max}) - 0.5 \mid) \tag{5.1}$$

According to the EMH, $h(q)$ should take the value of 0.5 for all q orders. Therefore, a market is efficient if MEM is equal to 0. The further away from 0 is the value of MEM, the lower the degree of market efficiency. For these time series $MEM = 0.1382$, which clearly indicates inefficient.

We proceed by studying the sources of multifractality. Figure 5.8 shows the results of the MF-DFA applied to the shuffled series. The lines $\log_2 F_q$ vs. s are not parallel for all the different q orders, although for $q = 0$ and $q = -5$ there seems to be an almost parallel relationship. This may be interpreted as a weaker evidence of multifractality than in the original data. In the graph of $h_{shuf}(q)$ on q, there is a higher variation of $h_{shuf}(q)$ for $q < 0$, while for $q > 0$ $h_{shuf}(q)$ is approximately constant. Only for a few orders of q, $h_{shuf}(q)$ presents a value visibly below 0.5. Therefore, the dependence of $h_{shuf}(q)$ on q is higher in the original series than in the shuffled series. Also, the Rényi exponent presents a smaller non-linearity compared to the original series. These results suggest that the multifractality comes from the long-range correlations but also from fat-tails distribution, since there is still evidence of multifractality in the shuffled series.

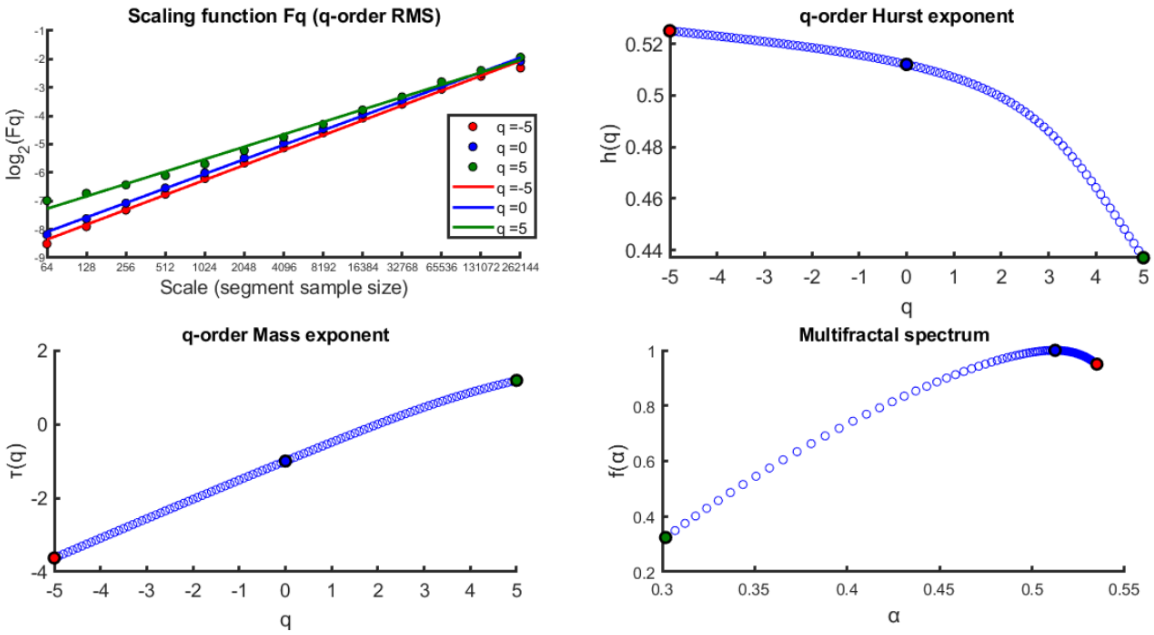


Fig. 5.8 The MF-DFA for the shuffled series of the bitcoin 1-min returns

Table 5.6 compares the the degree of multifractality, $\triangle h$, the width of the spectrum, $\triangle\alpha$, and the asymmetry parameter, A, of the original and shuffled series. All these values are lower for the transformed series indicating that multifractality is weaker in the shuffled series and thus long-range correlations are also a source of multifractality. This result is in accordance with Telli and Chen (2020), Stosic et al. (2019), Takaishi (2018) and Filho et al. (2018).

Table 5.6 Original series vs shuffled series: Degree of multifractality, width of the multifractal spectrum and asymmetry parameter

Original Series		Shuffled Series	
$\triangle h$	0.2764	$\triangle h_{shuf}$	0.0880
$\triangle\alpha$	0.5451	$\triangle\alpha_{shuf}$	0.2332
A	0.2618	A_{shuf}	0.1073

In order to obtain the surrogate series we apply the Fourier Transform (FT), which is the basis of the phase randomization, using the method also used in Theiler et al. (1992) and Schreiber and Schmitz (2000)[1]. Figure 5.9 shows the results of the MF-DFA for these series.

[1]The Matlab code was obtained in Temu (2020). Surrogate Data (https://www.mathworks.com/matlabcentral/fileexchange/4612-surrogate-data), MATLAB Central File Exchange. Retrieved on October 23, 2020.

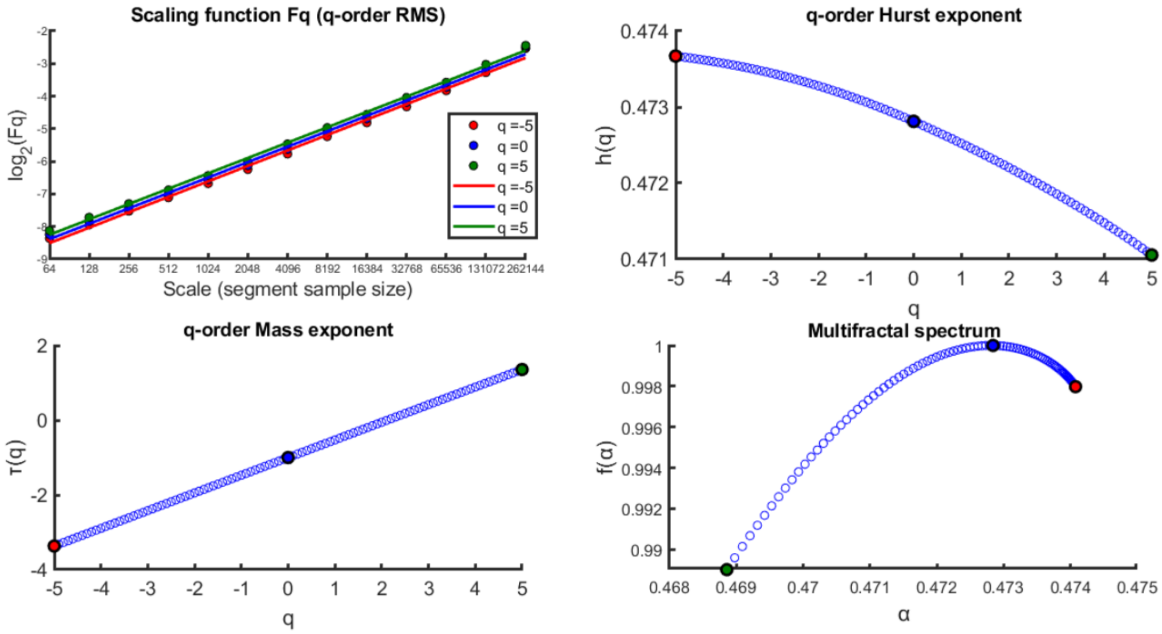


Fig. 5.9 The MF-DFA for the surrogate series of bitcoin 1-min returns

As for the plot of $\log_2 F_q$, the slopes for different orders are apparently parallel, which indicates that the surrogate series appear to have an almost monofractal behavior. There is a small variation of $h(q)$ with q, since for $-5 \leq q \leq 5$, $h(q)$ varies approximately around 0.47. Therefore, the dependence of $h(q)$ on q is much higher in the original series than in the surrogate series. The Rényi exponent $\tau(q)$ as a function of q is almost linear compared to the original series. The existing multifractality in the surrogate series is therefore very weak, which seems to indicate that the importance of long-range is somehow reduced, when compared with the importance of fat-tails as a source of multifractality.

Table 5.7 presents a similar comparison as in 5.6 but now for the surrogate series. The degree of multifractality, $\triangle h_{surr}$, and the width of the multifractal spectrum, $\triangle \alpha_{surr}$ are very small, and hence the multifractality is much weaker in the surrogate series. Therefore, the weak multifractality remaining in the surrogate series is due to the long-range correlations.

In sum, both sources of multifractality are present in the 1-min returns, but clearly fat-tails is the most important one. This is in accordance with the literature, for instance Telli and Chen (2020), Stosic et al. (2019), Takaishi (2018) and Filho et al. (2018), conclude that both sources of multifractality are present in the bitcoin returns, while Zhang et al. (2019a) and Lahmiri and Bekiros (2018) go further indicating that the fat-tails distribution is the main source of multifractality.

Table 5.7 Original series vs surrogate series: Degree of multifractality, width of the multifractal spectrum and asymmetry parameter

Original Series		Surrogate Series	
$\triangle h$	0.2764	$\triangle h_{surr}$	0.0026
$\triangle \alpha$	0.5451	$\triangle \alpha_{surr}$	0.0052
A	0.2618	A_{surr}	0.3333

In order to have a more precise idea on the contribution of each source we use the procedure of Baranowski et al. (2015). The contribution to multifractality of long-range correlations is quantified as:

$$| h_{corr}(q) | = | h(q) - h_{shuf}(q) | \tag{5.2}$$

while the contribution to multifractality of fat-tails distribution is quantified by:

$$| h_{dist}(q) | = | h(q) - h_{surr}(q) | \tag{5.3}$$

Results are shown in Figure 5.10.

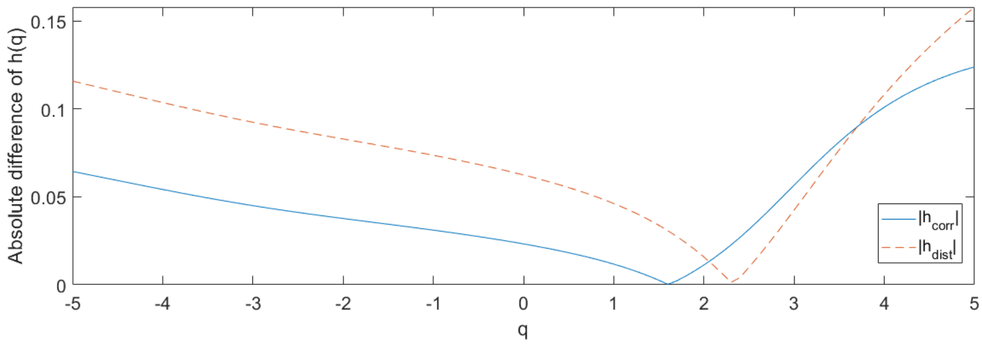


Fig. 5.10 Absolute differences between the generalized Hurst exponents of original and shuffled series, $| h_{corr}(q) |$, and between the the generalized Hurst exponents of original and surrogate series, $| h_{dist}(q) |$, as a function of q

Non-zero values of $| h_{dist}(q) |$ and $| h_{corr}(q) |$ prove that both sources of multifractality are present, however, for almost all values of q, $| h_{dist}(q) |$ is greater than $| h_{corr}(q) |$, meaning that the multifractality of the 1-min returns results mainly from the fat-tails distribution.

5.3 Multifractal Regime Detecting Method (MRDM)

This section uses the Multifractal Regime Detecting Method (MRDM), introduced by Lee and Chang (2015), to verify the time ranges when multifractality predominates. Basically MRDM consists on a loop of a multifractality test applied within overlapping fixed width windows.

The multifractality test is the following: Firstly, different maximum scales in powers of two are considered, s_{max}. Then, the MF-DFA is applied for each one of those maximum scales, and the values of the generalized Hurst exponents for $q = 1$ and $q = 2$ are estimated. By proceeding this way, we end up with two vectors, $h(1)$ and $h(2)$, with the same number of elements as the elements considered for s_{max}. Finally, the Wilcoxon rank sum test is used on the null of multifractality such that: $H_0 : h(1) = h(2)$. As in Lee and Chang (2015) we also used a significance level of 0.05.

It should be remarked that the procedure used here differs from Lee and Chang (2015) since $h(1)$ and $h(2)$ are estimated using the MF-DFA, while those authors estimate these generalized Hurst exponents according to the procedure proposed by Di Matteo et al. (2003).

In order to check the multifractality in the 1-min returns, the test is applied using the scales $s = 2^6, ..., s_{max}$ with $s_{max} = 2^7, ..., 2^{18}$, $s = 2^6, ..., s_{max}$ with $s_{max} = 2^7, ..., 2^{13}$ and $s = 2^{13}, ..., s_{max}$ with $s_{max} = 2^{14}, ..., 2^{18}$. The results are presented in Table 5.8. According to the Wilcoxon rank sum test, the null hypothesis is rejected, with a significance level of 0.05, only for the higher scales, implying that smaller cycles are better characterized by monofractality.

Table 5.8 Wilcoxon rank sum test for $q = 1$ and $q = 2$

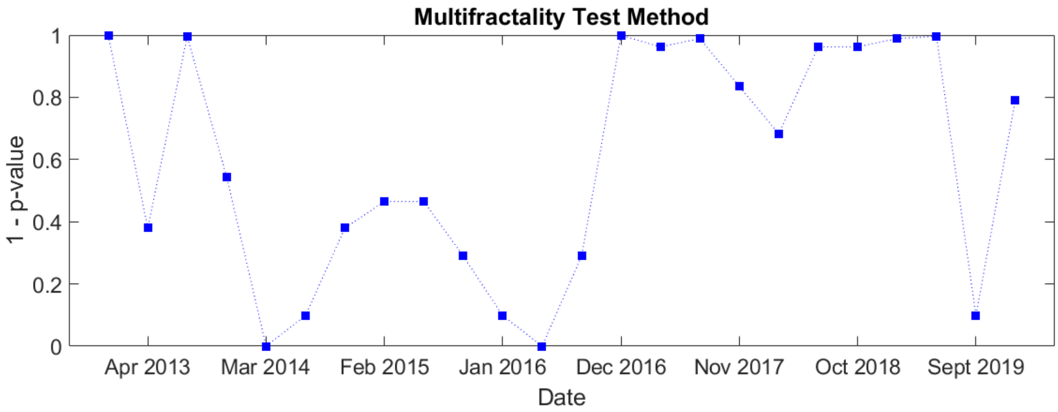
s_{max}	$p-value$	Result
2^7 to 2^{18}	0.0885	Monofractal
2^7 to 2^{13}	0.1649	Monofractal
2^{14} to 2^{18}	0.0317	Multifractal

It should be noted that this test only compares $h(q)$ with $q = 1$ and $q = 2$, i.e. very close values. Table 5.9 shows the test results using $q = 1$ and $q = 3$. Now, multifractality is clearly evident in all the maximum scales considered. So, the farther apart are the values of q, the bigger will be the differences between the $h(q)$ vectors, and hence more compelling evidence on multifractality will be drawn from the test.

Table 5.9 Wilcoxon rank sum test for $q = 1$ and $q = 3$

s_{max}	$p-value$	Result
2^7 to 2^{18}	0.0000	Multifractal
2^7 to 2^{13}	0.0006	Multifractal
2^{14} to 2^{18}	0.0079	Multifractal

Before the application of MRDM, we performed another preliminary analysis by conducting the multifractality test on non-overlapping periods with a duration of 4 months, i.e. the test with $q = 1$ and $q = 2$ is performed on 24 samples with a length 161,280 minutes. Therefore, each period of 4 months does not refer to calendar months but to the standard value of minutes per week and weeks per month. The results are present in Figure 5.11. The points represent the value of one minus the p-value of the test, and hence higher values indicate higher probabilities of multifractality being present in those 4-month periods.



Note: The dates on the x-axis refer to the beginning of the interval of each 161280 minutes period.

Fig. 5.11 Multifractality test applied for every 4 months of the 1-min returns

Figure 5.11 highlights that multifractality is most probably present in the beginning and end of 2013, early 2017 to November 2017, and July 2018 to September 2019. So, multifractality predominates mainly in the three last years of the sample (2017 to 2019). Comparing these latter findings to the 1-min bitcoin price chart (Figure 5.1), it can be seen that the periods where multifractality predominates are years with fast and sharp bitcoin price growths. Monofractal behavior is predominant in calmer or moderately growing periods.

After this precursory analysis we present the MRDM procedure. Let's consider a sample window with fixed width $n\triangle$, which is rolled forward by $\triangle$ units each time. In each window the aforementioned multifractality test is performed and the result of the test (1 if the test reject the null, and 0 otherwise), is recorded. The result for window i may be denoted by $1_{M(i)}$, where 1 is the indicator function. Now, let us consider that the period of interest is $[t, t+\triangle]$, which is included in n moving window samples, as shown in Figure 5.12.

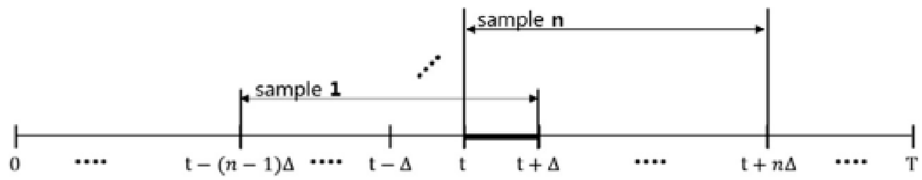


Fig. 5.12 The time range $[t, t+\triangle]$ included in n sample windows. Source: Lee and Chang (2015)

Hence, there are n windows in the the period $[t-(n-i)\triangle, t+i\triangle]$ that contain period $[t, t+\triangle]$. Multifractality is checked in this time range through the proportion index:

$$I(t) = \frac{1}{n} \sum_{i=1}^{n} 1_{M(i)} \tag{5.4}$$

Therefore, $I(t)$ represents the proportion of the n windows containing $[t, t+\triangle]$ that reject H_0. According to Lee and Chang (2015), the existence of multifractality in $[t, t+\triangle]$ is given if the proportion index is above a certain threshold value, I_M, that they assume equal to 0.7. The parameters chosen for the application of the method were: a sample window of length $n\triangle = 161,280$ minutes, which is equivalent to 4 months, each window moves forward $\triangle = 10,080$ minutes each time, which is equivalent to 1 week, and the threshold value $I_M = 0.7$. Therefore, for each week $n = 16$. The results of MRDM applied to our sample are shown in Figure 5.13.

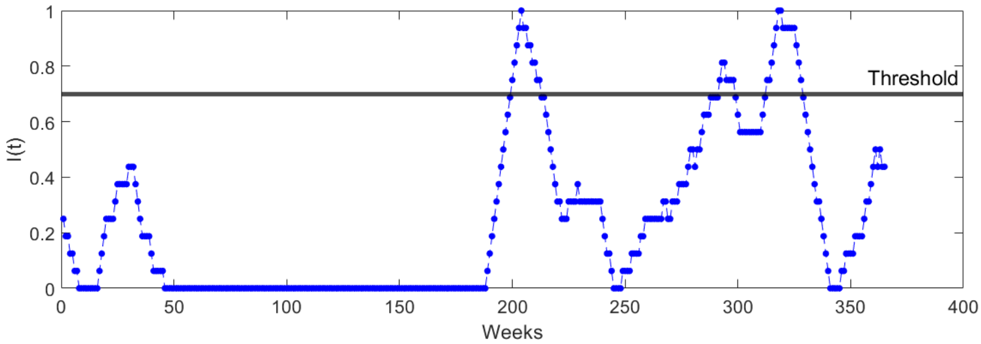


Fig. 5.13 MRDM applied to the 1-min bitcoin returns

The proportion index is computed over 365 weeks, so each value in the graph represents one week. The first week is from April 15, 2013 at 00:00 to April 21, 2013 at 23:59, leaving 15 weeks behind, in order to have the associated period $[t-(n-i)\triangle, t+\triangle]$. The second week goes from April 22, 2013 at 00:00 to April 28, 2013 at 23:59 and so on. The last week is from April 06, 2020 at 00:00 to April 12, 2020 at 23:59 in order to have the associated period $[t, t+(n+1)\triangle]$.

Figure 5.13 presents the results of the MDRM. In the first half of the sample there are several consecutive weeks where $I(t) = 0$, which clearly indicates monofractality. This behavior happens from week 46 to week 188 (February 2014 to November 2016). For the majority of the weeks there are always some windows where monofractality is rejected, however considering the threshold of $I_M = 0.7$, the existence of multifractality is only identifiable in weeks 200 to 212, 292 to 298 and 313 to 328, which roughly correspond to the years 2017, late 2018 and early 2019. These results are in agreement with what was found in the previous multifractality test for non-overlapping 4-month periods.

5.4 Multifractality of bitcoin in different online exchanges

This section examines the multifractal behavior of bitcoin in different online exchanges in the period after the burst of the second bubble. We use the second data set of hourly returns from January 1, 2018 to October 12, 2020 and the third data set with daily returns from January 1, 2017 to December 2, 2020.

The idea to study multifractality in different exchanges is based on the results present in the finance literature, according to which bitcoin prices in multiple online exchanges are not completely arbitraged away, even those formed in exchanges with the higher market shares (Makarov and Schoar, 2020; Pieters and Vivanco, 2017). Hence transmission of information and volatility spillovers between exchanges can last for several hours or even days and price discovery may occur at different paces

(Matkovskyy, 2019; Pagnottoni and Dimpfl, 2019; Sebastião et al., 2018). So, arguably, price formation processes may be different, which may imply that fractal structures on those exchanges are distinct.

5.4.1 Hourly data

The MF-DFA parameters are the same as those used in Section 5.2 for 1-minute returns, except for the scaling parameter that ranges from $s_{min} = 16$ to $s_{max} = 2,048$ in powers of two. The main results of the original, shuffled and surrogate series are shown in Table 5.10. Figure A.1, Figure A.2, and Figure A.3 in Appendix present graphically the application of the method to the three series, respectively. All exchanges present similar results, meaning that multifractality does not change much among different exchanges. The differences in the fluctuation function for different orders of q are significant in small size segments, contrary to what is seen for large size segments. The generalized Hurst exponent is decreasing with q. For small fluctuations, $h(q) > 0.5$ (persistent behavior), while for large fluctuations, $h(q) < 0.5$ (anti-persistent behavior). Also, $\tau(q)$ is non-linear, which is a signal of multifractality of the series. Finally, the multifractal spectrum, which is left-skewed $(A < 1)$, presents a curve with a single-humped shape for all exchanges.

Table 5.10 Multifractality in different exchanges using hourly data

	$\triangle h$	$\triangle \alpha$	A	MEM	$\triangle h_{shuf}$	$\triangle \alpha_{shuf}$	$\triangle h_{surr}$	$\triangle \alpha_{surr}$
Bitfinex	0.2209	0.4195	0.5606	0.1104	0.1013	0.2148	0.0090	0.0212
Coinbase	0.2583	0.4949	0.7007	0.1291	0.0973	0.2081	0.0242	0.0481
Bitstamp	0.2551	0.4907	0.6478	0.1275	0.1149	0.2422	0.0158	0.0328
Kraken	0.2577	0.4971	0.7405	0.1288	0.1430	0.2909	0.0173	0.0380
Gemini	0.2556	0.4908	0.6779	0.1278	0.1358	0.2753	0.0133	0.0280
Cexio	0.2453	0.4755	0.6431	0.1226	0.1172	0.2379	0.0160	0.0340
Exmo	0.2326	0.4565	0.5016	0.1163	0.1194	0.2491	0.0169	0.0324

According to Table 5.10, it can be seen that there are four cryptocurrencies exchange with very similar values, Coinbase, Bitstamp, Kraken and Gemini, presenting a stronger multifractal behavior, with Coinbase showing higher values, while Bitfinex, Cexio and Exmo have slightly lower multifractality. In terms of efficiency, measured by MEM, Bitfinex and Exmo stand out as the more efficient exchanges. Hence there is no discernible pattern relating multifractality and efficiency with the liquidity of the exchanges (measured by trading volume) or even regarding their continent of origin.

For the shuffled series, the fluctuation function for different orders presents differences ($h(q)$ decreases as q increases), $\tau(q)$ has a non-linear behavior, and the multifractal spectrum, which is left-skewed $(A < 1)$, presents a curve with a single-humped shape. Regarding the surrogate series, the evidences of multifractality are weaker when comparing with the original and shuffled data. The main results summarized in Table 5.10, indicate that multifractality is weaker in the shuffled and surrogate series, because both the degree of multifractality and the width of the spectrum of the shuffled series ($\triangle h_{shuf}$ and $\triangle \alpha_{shuf}$) and the surrogate series ($\triangle h_{surr}$ and $\triangle \alpha_{surr}$) decreased in relation to the values of the original series. Therefore, it can be stated that, at this frequency, the multifractality of these exchanges comes from both sources.

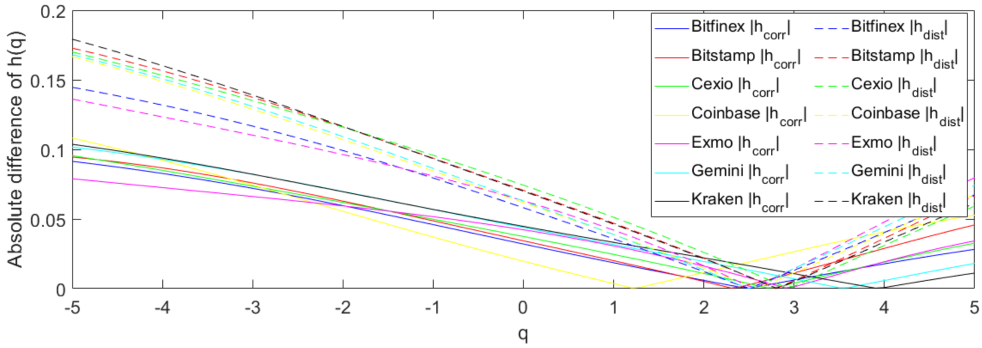


Fig. 5.14 Absolute difference of the generalized Hurst exponents for original and shuffled series $\mid h_{corr}(q) \mid$ and original and surrogate series $\mid h_{dist}(q) \mid$ as a function of q, for the 7 exchanges using hourly data

Figure 5.14 presents the results of $\mid h_{corr}(q) \mid$ and $\mid h_{dist}(q) \mid$ computed according to Equation 5.2 and Equation 5.3, respectively. The plots indicate that the effect of fat-tails distribution is more important than long-range correlations for all exchanges.

5.4.2 Daily data

This sub-section applies the MF-DFA to the daily bitcoin returns of the different cryptocurrencies exchanges. The scaling parameter ranges now from $s_{min} = 16$ to $s_{max} = 128$ in powers of two. The main results of the original, shuffled and surrogate series are shown in Table 5.11. Figure A.4, Figure A.5, and Figure A.6 in Appendix, present graphically the application of the method to the three series, respectively.

Table 5.11 Multifractality in different exchanges using daily data

	$\triangle h$	$\triangle\alpha$	A	MEM	$\triangle h_{shuf}$	$\triangle\alpha_{shuf}$	$\triangle h_{surr}$	$\triangle\alpha_{surr}$
Bitfinex	0.2736	0.4845	3.0714	0.1783	0.0595	0.1083	0.0904	0.1728
Coinbase	0.2876	0.5010	5.4562	0.1981	0.0725	0.1396	0.0653	0.1332
Bitstamp	0.2980	0.5211	3.0490	0.1929	0.0404	0.0945	0.1002	0.1911
Kraken	0.2981	0.5180	3.0532	0.1946	0.0568	0.1078	0.0459	0.1016
Gemini	0.2914	0.5066	3.3188	0.1968	0.0606	0.1304	0.0494	0.0945
Cexio	0.3411	0.5772	4.0061	0.2465	0.0916	0.1783	0.1024	0.1930
Exmo	0.3298	0.5604	4.3524	0.2457	0.0486	0.1050	0.1002	0.1903

Again, exchanges exhibit multifractal behavior. Cexio and Exmo, two European exchanges and the ones with the lower trading volume, have higher values of $\triangle h$ and $\triangle\alpha$, suggesting a stronger multifractal behavior, and a higher value of MEM implying that are the least efficient exchanges. On the contrary, Bitfinex, the most liquid exchange, is the one that presents a weaker multifractal behavior and higher efficiency. In fact, one may observe at a daily frequency a direct relationship between multifractality and inefficiency with iliquidity (measured by trading volume).

The shuffled series continue to present a multifractal behavior. In the surrogate series, there is also evidence of multifractality, but weaker, since $\log F_q$ seem parallel in different orders and the variation of $\triangle h$ as a function of q is very small in some cases. Hence, multifractality in the exchanges under scrutiny also comes from both sources at a daily frequency.

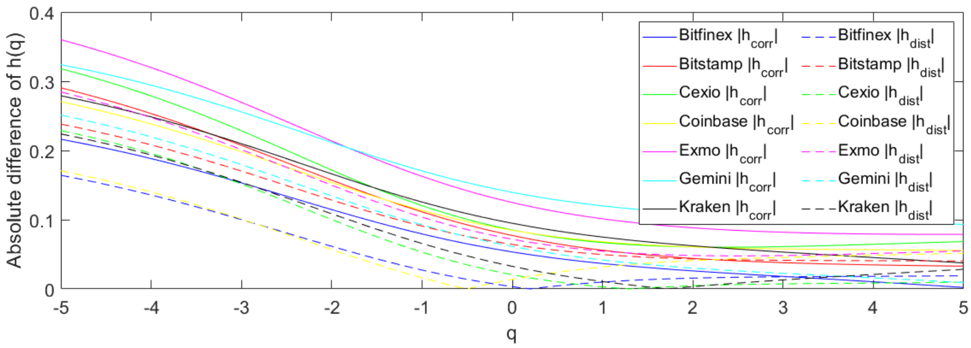


Fig. 5.15 Absolute difference of the generalized Hurst exponents for original and shuffled series $\mid h_{corr}(q) \mid$ and original and surrogate series $\mid h_{dist}(q) \mid$ as a function of q, for the 7 exchanges using daily data

The absolute values of $h_{corr}(q)$ and $h_{dist}(q)$ are shown in Figure 5.15 for daily data. Contrary to what we have seen so far, at a daily frequency, $\mid h_{corr}(q) \mid$ is higher than $\mid h_{dist}(q) \mid$ in all exchanges for almost all values of q. This means that the multifractality of the daily data results mostly from long-range correlations.

Chapter 6

Conclusion

Bitcoin is a decentralized digital asset based on a public, immutable and resilient ledger, called blockchain. Since its launch in 2009, bitcoin has thrived, attracting the attention of individual and institutional investors, regulators and policy makers, and the public in general. Recently, due to its impressive price appreciation, trading volume and market capitalization, the economics and finance literature has been piling up at an significant rate. Most of this literature focus on gathering evidence on stylized facts of bitcoin price dynamics, which appear to be quite different from other more traditional and regulated financial markets. One way to characterize financial price dynamics is by studying its fractal properties.

This dissertation aims at analyzing fractality of the bitcoin market, and also to provide some insights on its informational efficiency. This study is conducted using the Multifract Detrended Fluctuation Analysis (MF-DFA) of Kantelhardt et al. (2002) and the Multifractal Regime Detecting Method (MRDM) of Lee and Chang (2015). The implementation of MRDM has an originality twist because we use the estimations of the generalized Hurst exponentes obtained from the MF-DFA.

Those methodologies are applied to 1-min bitcoin returns (in USD) of Bitstamp (one of the most mature, liquid and reputable cryptocurrency online exchanges) from January 2013 to July 2020. Two other data sets constituted by hourly and daily returns of seven online exchanges (Bitfinex, Bitstamp, Cexio, Coinbase, Exmo, Gemini and Kraken) for the years of 2018-2020 and 2017-2020, respectively, are also used to check if eventual misalignments between these exchanges are visible on different fractal dynamics.

Our results point out that bitcoin 1-min returns are non-normal. In fact, these returns are prone to extreme volatility events, are subjected to jumps (the minimum and maximum returns in the sample are -30.65% and 35.29%) and are highly leptokurtic. There is strong evidence that these returns present multifractality, more specifically smaller fluctuations have a persistent behavior, while larger fluctuations have an anti-persistent behavior. Overall, multifractality comes from the existence of significant long-range correlations, which cast some doubts on the informational efficiency of bitcoin prices at Bitstamp. But mainly multifractality comes from fat-tails distribution, which highlights the high level of risk that investors are subjected to in this market.

The results also highlight that fractal dynamics are themselves time-varying. The MRDM shows that, roughly speaking, multifractality is mainly present in the last half of the series. More precisely, during 2017, and in late 2018 and earlier 2019. Monofractality in the period from February 2014 to November 2016 is undoubtedly supported by MRDM. The analogy of Mandelbrot and Hudson (2010) on the "wind tunnel" may shed some light on a possible explanation of these changes between monofractal and multifractal behaviors. According to Mandelbrot and Hudson (2010), the complex behavior of the wind inside a tunnel arises when the wind speed (i.e. the tunnel pressure) increases, or when the tunnel opening narrows. Therefore, one may argue that the complexity in the bitcoin market is caused by the pressure of increasing demand (that is, an increase in the "tunnel pressure") or by a more inelastic supply (that is, a "narrow tunnel opening"). Noticeably, the periods of multifractality are characterized by rapid and sharp growth in bitcoin prices resulting from demand increases, enhanced by higher "public attention" and bitcoin attractiveness.

A second set of results were obtained for hourly and daily bitcoin returns from different exchanges. These exchanges present similar results at hourly frequency: multifractality, with persistence in small fluctuations and anti-persistence in large fluctuations, mainly coming from fat-tail distributions, but still market efficiency may be low due to long-range correlations. There was no visible pattern relating multifractality and efficiency with the trading volume of those exchanges. At a daily frequency one observe a direct relationship between multifractality and inefficiency with illiquidity (measured by trading volume), and now, a striking result is that the main source of multifractality is not fat-tail distributions but instead the long-run correlations.

In sum, an everlasting drawback to long-run attractiveness and increasing valuation of bitcoin is its short-run instability, as if these two features were the two faces of the same coin.

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Appendix A

MF-DFA and multifractal sources on different online exchanges

A.1 Using hourly data (01/01/2018 - 10/12/2020)

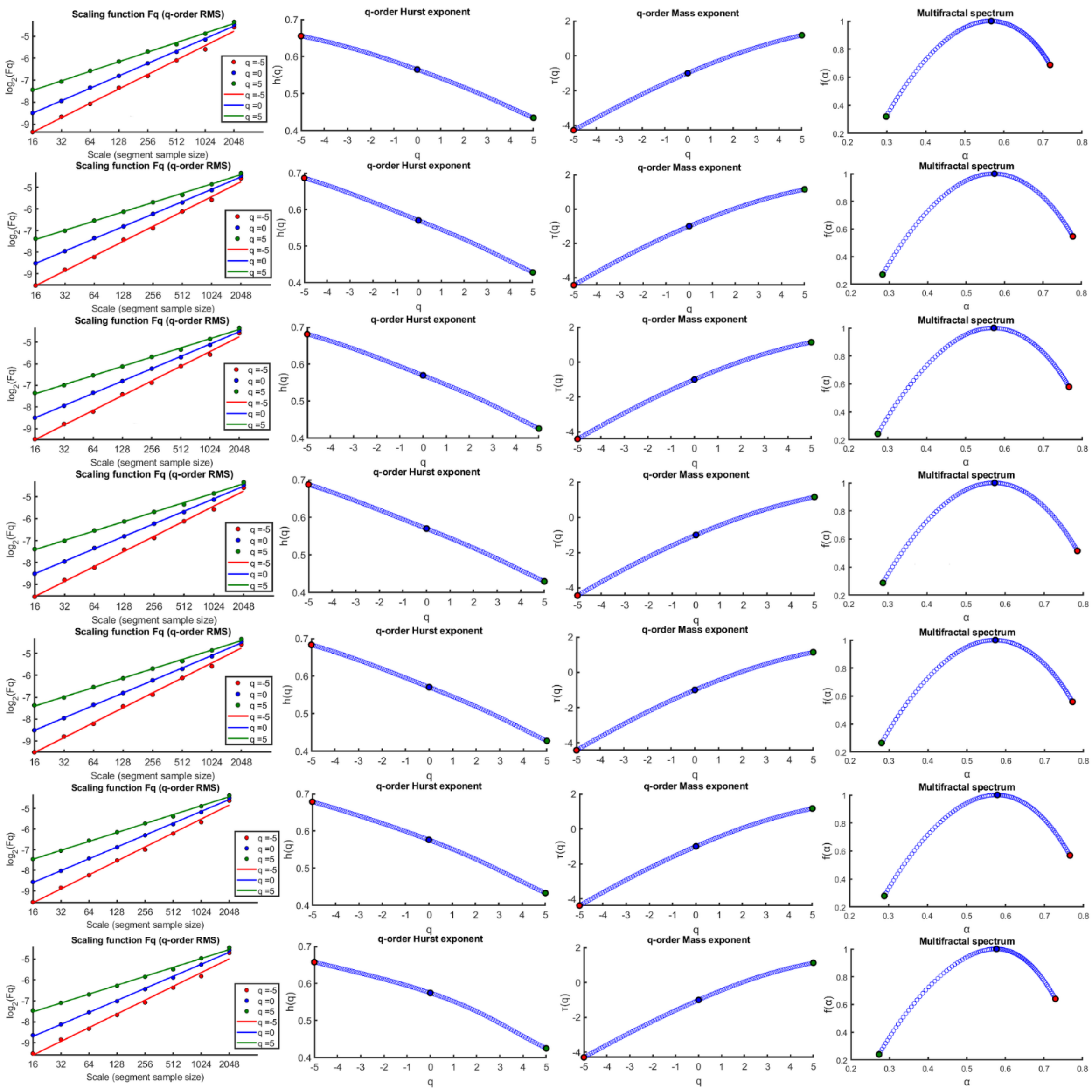

Fig. A.1 The MF-DFA method for hourly data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line)

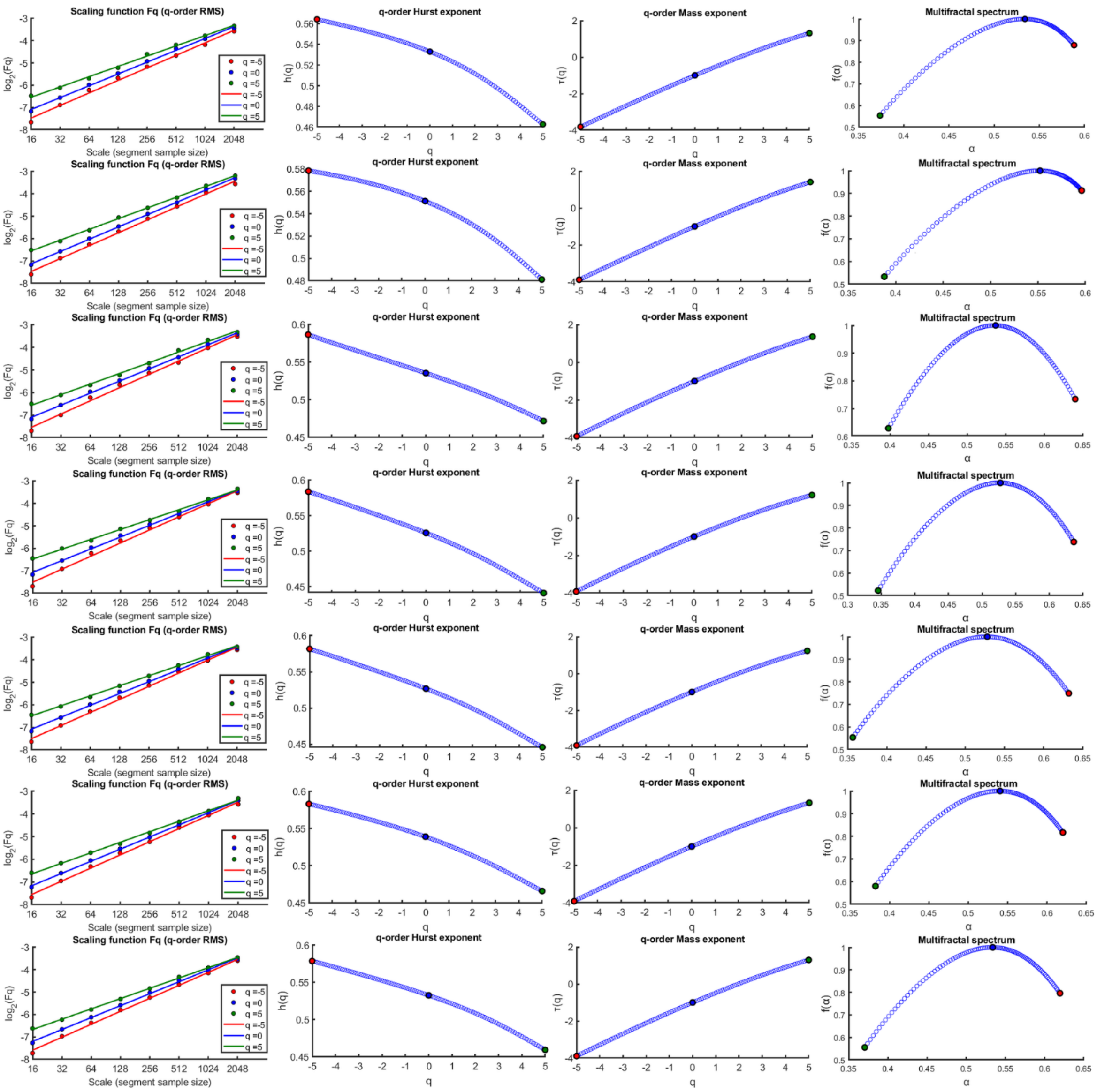

Fig. A.2 The MF-DFA method for the shuffled series of hourly data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line)

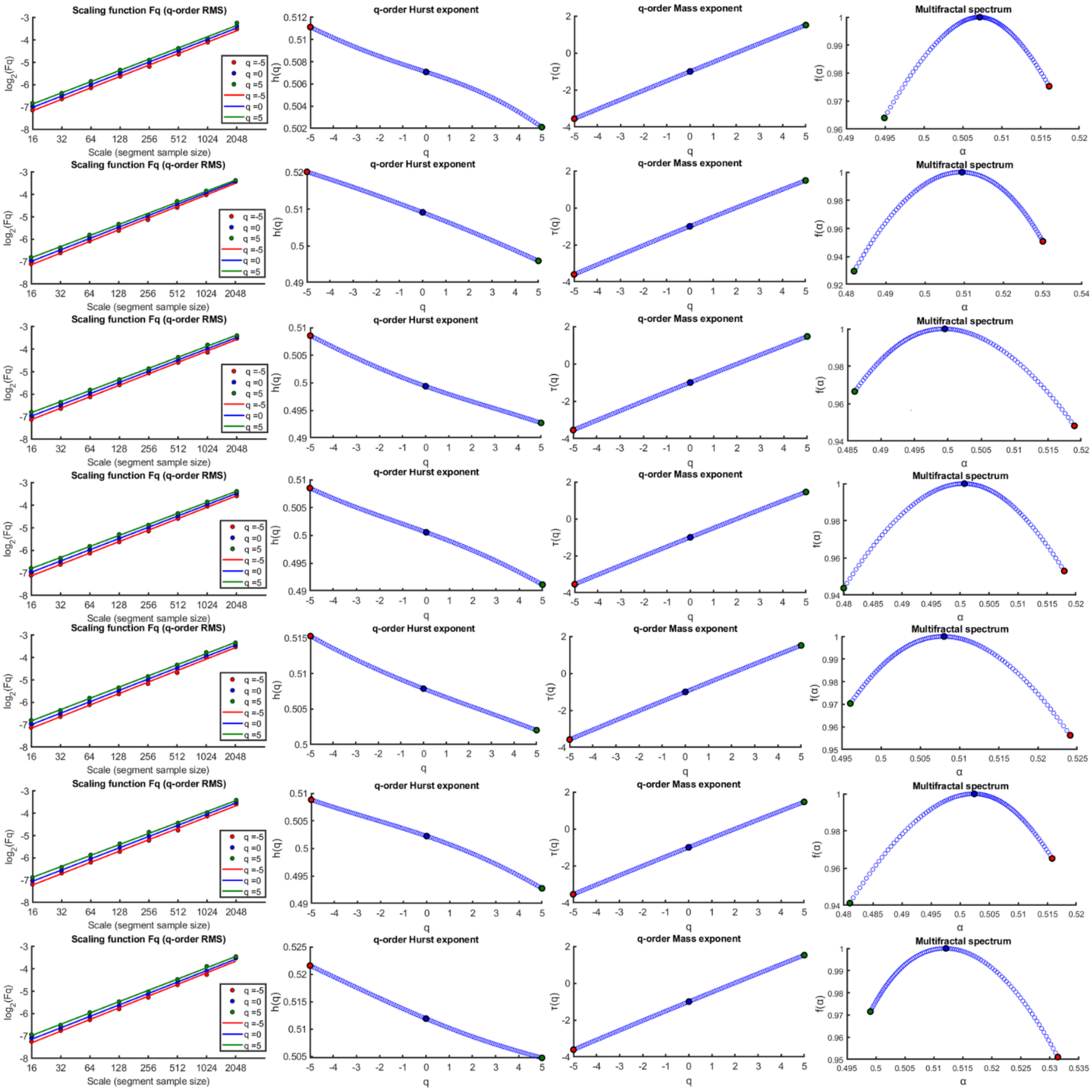

Fig. A.3 The MF-DFA method for the surrogated series of hourly data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line).

A.2 Using daily data (01/01/2017 - 12/02/2020)

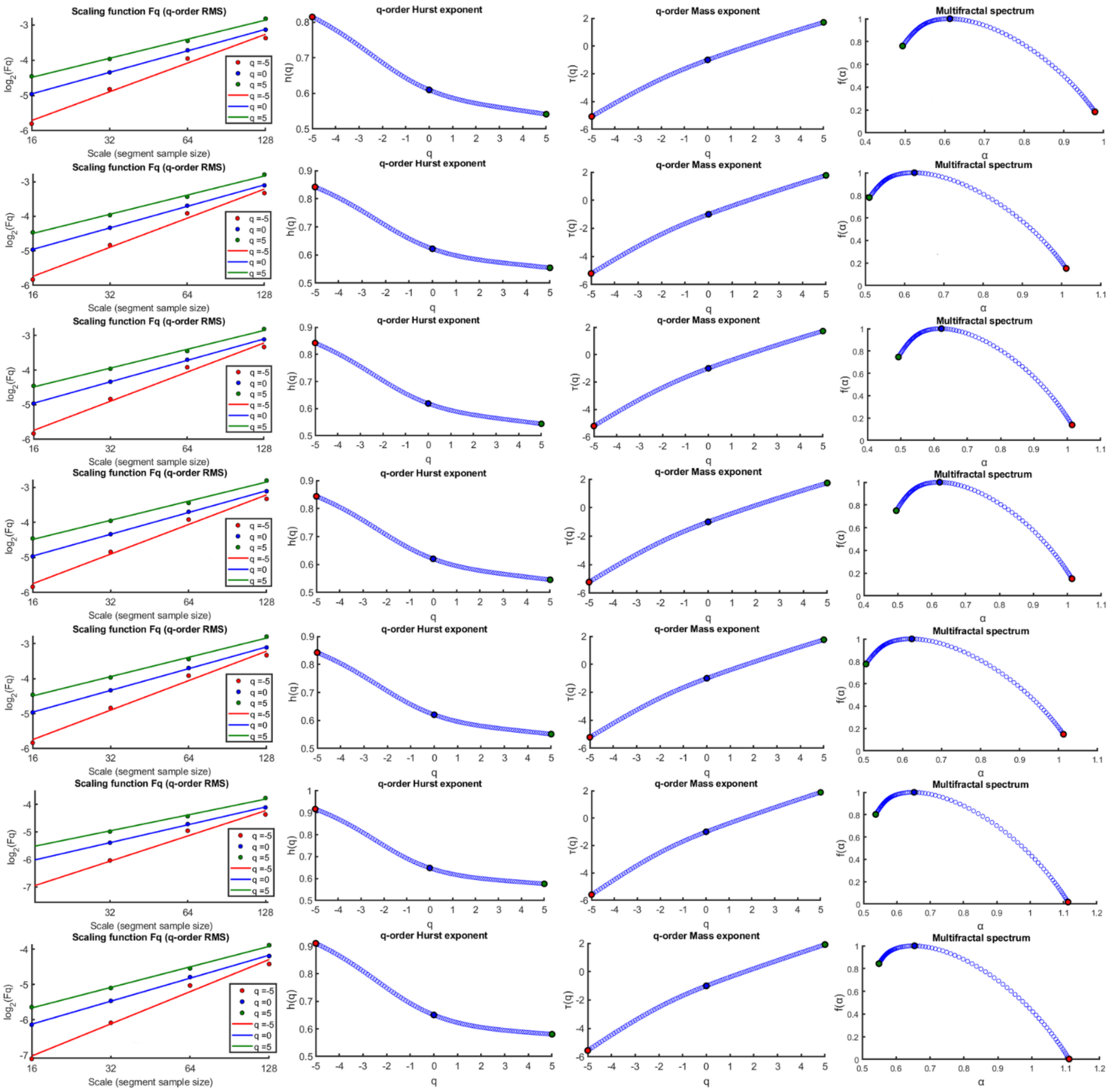

Fig. A.4 The MF-DFA method for daily data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line)

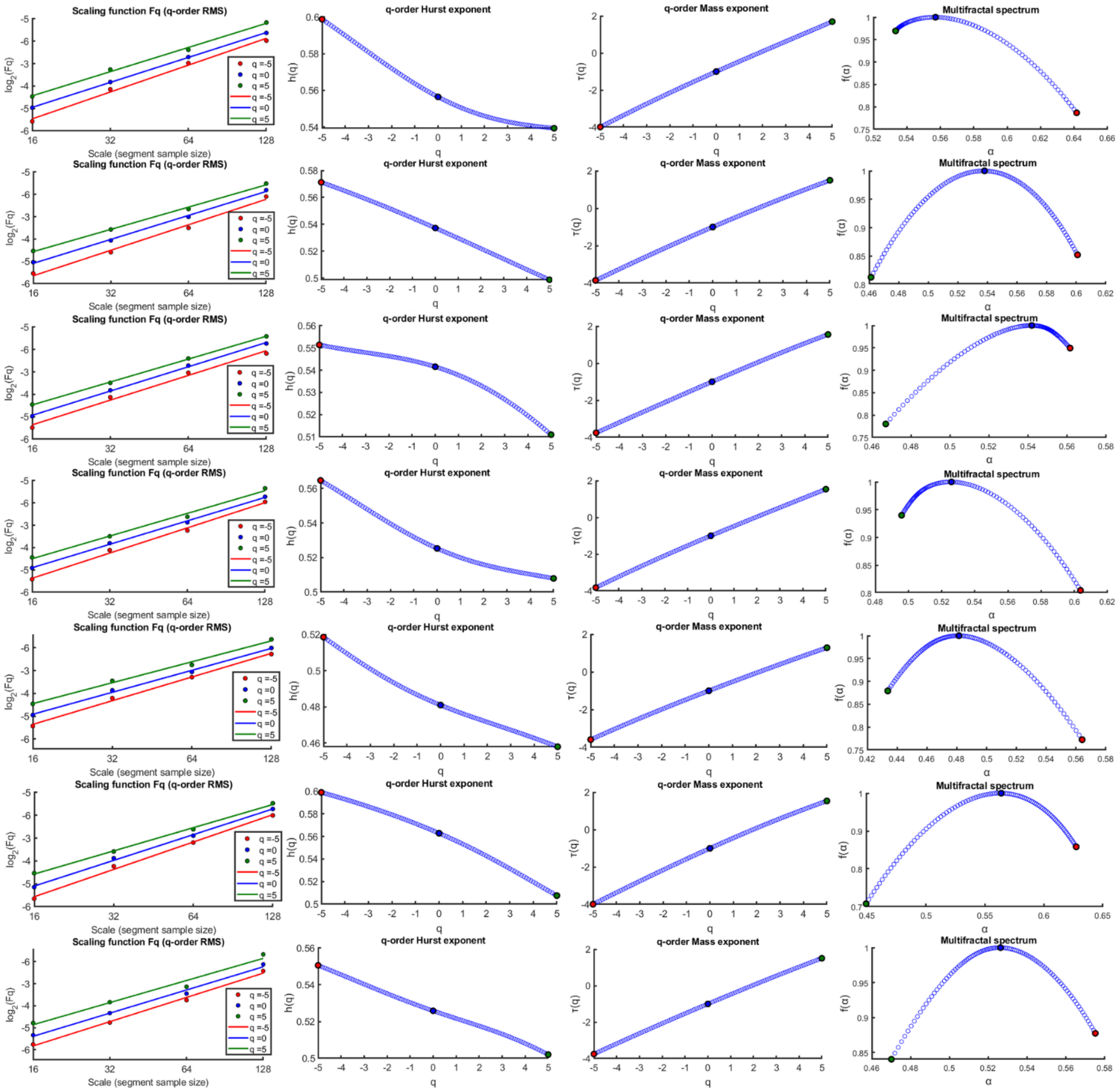

Fig. A.5 The MF-DFA method for the shuffled series of daily data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line)

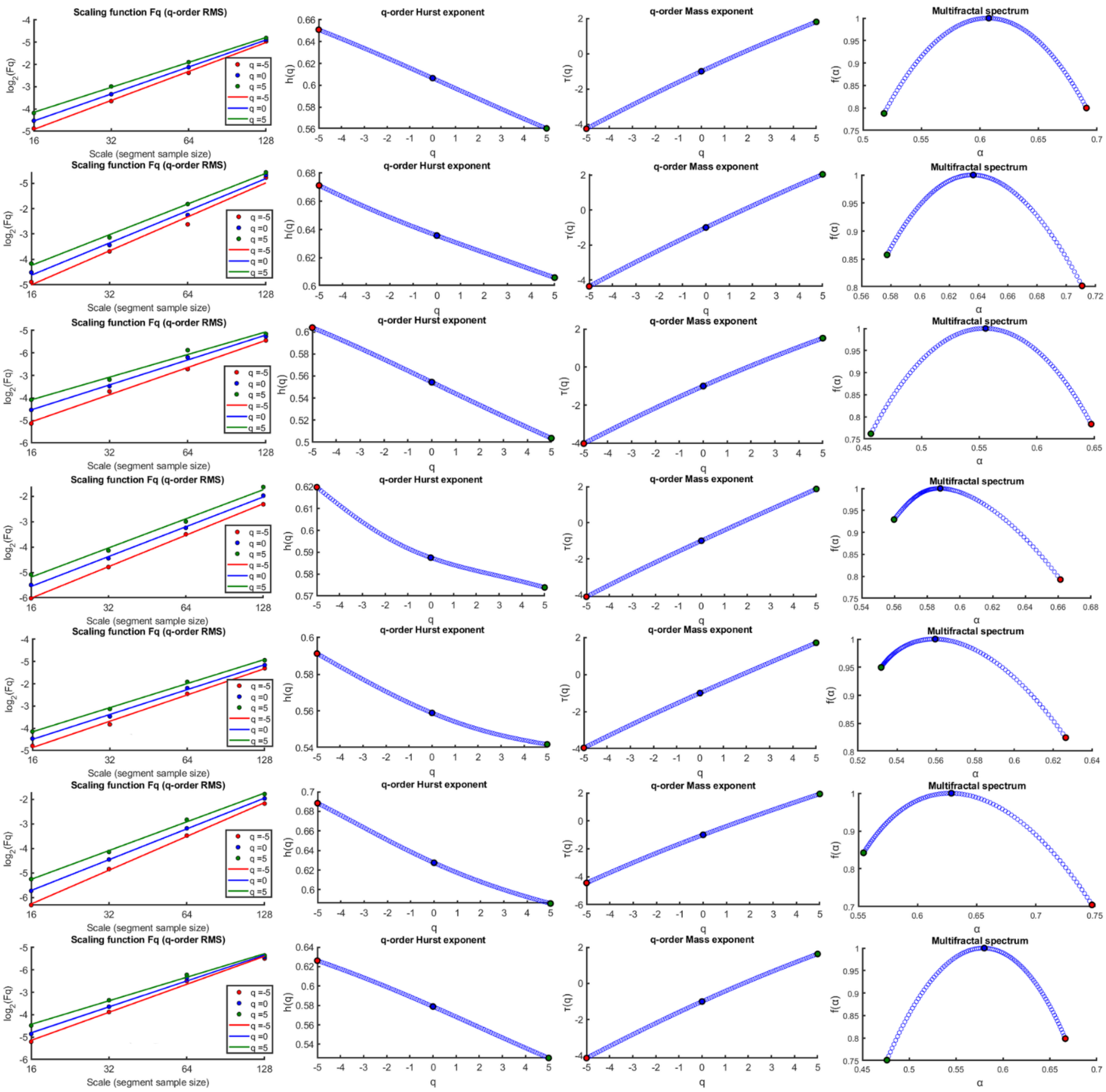

Fig. A.6 The MF-DFA method for the surrogated series of daily data of Bitfinex (1st line), Coinbase (2nd line), Bitstamp (3rd line), Kraken (4th line), Gemini (5th line), Cexio (6th line), and Exmo (7th line).