Optimal Emergency Vehicles Location: an approach considering the hierarchy and substitutability of resources

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Abstract

Decisions on where to locate emergency vehicles have a crucial impact on the quality of the emergency service that is provided to populations, with consequences in terms of mortality and quality of life. It is important to guarantee the access of the population to emergency care, not forgetting the need to guarantee the best possible use of all available resources. In this work a new integer linear programming model is presented that aims at optimizing the location of emergency vehicles, considering in an explicit way the substitutability possibilities among vehicles of different types, taken into account the type of care they can provide. Moreover, the assignment of variables to emergency episodes is also explicitly considered which allows the model to be more accurate when calculating the expected coverage obtained. Both deterministic and stochastic models are presented. In the stochastic model, uncertainty regarding emergency episodes is represented by using scenarios. The model is applied to a dataset built considering all the features which are present in real data.

Key Words:

Location, OR in health services, emergency vehicles, uncertainty

1. Introduction

The adequate location of health facilities is essential to guarantee a proper use of these services by the population, minimizing both their under and over utilization. The location of emergency vehicles is crucial to guarantee that pre-hospital care will be available where it is needed, within the recommended time window, and minimizing inequalities. Not locating in an optimal way health resources can lead to situations of increased costs, inconvenience to the user and, more importantly, an increase in the mortality and morbidity of the populations (Daskin & Dean, 2004).

The demand for health services, especially regarding emergency situations, is inherently uncertain, which increases the difficulty of the decision-making process. Emergency services are used daily by many people and play an essential role guaranteeing that the right care arrives at the right time and at the right place, reducing mortality and morbidity. Research on location of emergency vehicles can contribute to improvements in the provision of these services to the populations. Traditionally, emergency resource location problems have dealt with three types of decisions: where to locate, how many and what kind of resources to locate. In order to answer these questions, locations where emergency situations occur are usually assumed to be known, as well as the possible bases where resources can be located (Li, Zhao, Zhu & Wyatt, 2011). It is usually necessary to include constraints related to the maximum number of resources available, the maximum number of vehicles that can be located at each one of the bases, assurance of proper coverage and also constraints defining the available budget. The first models used to tackle emergency location problems were deterministic, and they did not explicitly consider the uncertainty associated with real situations, which constitutes a serious limitation of these models. One of the first known works that optimizes ambulance location can be attributed to Toregas, Swain, ReVelle & Bergman (1971), where the authors use a set covering model to establish the optimal location of emergency bases. The objective function represents the minimization of supply points, ensuring that each demand point is served by at least one supply point. Church & ReVelle (1974) present the first model of maximization of coverage (Maximal Coverage Problem). Coverage based models are still widely used. In coverage models, a maximum time or distance limit is established *a priori* and a demand point is only covered if it is closer to an emergency base than this defined limit (Ahmadi-Javid, Seyedi & Syam, 2015; Jánošíková, Jankovič & Márton, 2017). This definition of coverage has an underlying problem, since it does not tackle explicitly the possible unavailability of the resources placed within the time or distance limit of the demand point when the emergency episode occurs. One alternative is to consider the concept of double coverage. Double coverage considers the possibility of a population being covered by more than one resource, in case of one of them not being available (see, for instance, (Gendreau, Laporte & Semet, 1997)).

One of the advantages of using coverage-based models is the fact that these models are not very demanding in terms of the information they need. Furthermore, they can be used in combination with probabilistic models (Knight, Harper & Smith, 2012), as can be seen in the case of Shiah & Chen (2007). The authors include additional capacity restrictions on the number of available ambulances, approximating the model to the real situation.

In addition to coverage models, *p*-center models have also been used for vehicle emergency location, namely considering the minimization of the maximum distance between a demand point and an emergency base, possibly taking into account different capacity restrictions and limiting the number of ambulance bases available (Chanta, Mayorga & McLay, 2014; Yin & Mu, 2012).

An increased importance has been given to bridging the gap between the situation represented by the models and the reality of the pre-hospital emergency care context. One of the important developments is the explicit consideration of uncertainty. Emergency vehicles operate in highly complex environments, with multiple simultaneous emergency calls, and the occurrence of these calls will determine periods of unavailability for the allocated vehicles (Brotcorne, Laporte & Semet, 2003).

Stochastic models have the capability of dealing, in a more real setting, with questions associated with the availability of resources and adequate dimensioning of the emergency bases (Maleki, Majlesinasab & Sepehri, 2014), since they take explicitly into account situations where the resources are allocated to other emergency occurrences and cannot be allocated to new occurrences (Aboueljinane, Sahin & Jemai, 2013). Not being able to properly represent resource availability is not a new concern. Daskin (1983) presents a work where each resource is associated with a probability of being taken when it is needed. The model presented in Hogan & ReVelle (1986) follows a similar path. An extension to the work presented in Daskin (1983) considers two different servers and dependencies between the types of servers and servers of the same type (McLay, 2009). Each episode will receive exactly one vehicle, which can be different from the most adequate one if the latter is busy. Two types of vehicles are also considered in Chong, Henderson & Mark E. Lewis (2016). The authors develop two models (a Markov decision model and an integer programming model) under a budget constraint. Van Den Berg, Legemaate & van der Mei (2017) consider the location of different types of vehicles in the context of firefighter services, with a limited number of relocations being allowed. Ball & Lin (1993) present a model establishing a minimum confidence level in the service provided and considering the probability of resources not being available. Beraldi, Bruni & Conforti (2004) develop a stochastic model for the determination of how many vehicles to locate and where to locate them, using probabilistic constraints. Noyan (2010) introduces risk metrics to control the unfulfilled demand. Erkut, Ingolfsson & Erdoğan (2007) maximize survival, where the probability of survival depends not only on the time before the arrival of the means but also on the characteristics of the emergency medical team and of the emergency vehicle, among other things. Ingolfsson, Budge & Erkut (2008) present a model with uncertainty in resource availability and travel times. Knight et al. (2012) maximize survival, grouping patients into different categories according to the severity degree. The authors include queuing models to deal with resource congestion. Nickel et al. (2016) also present a stochastic model for ambulance location. Each scenario defines the number of ambulances requested by each demand point, being possible to generate all possible scenarios. However, the resulting problem would be impossible to solve due to its dimension. The authors choose to divide the model using different scenario subsamples, and running a heuristic procedure to build a global solution. Yoon & Albert (2017) present a model considering different priorities associated with the emergency episodes, including explicitly the existence of queues. Boujemaa, Jebali, Hammami, Ruiz & Bouchriha (2017) present a stochastic model, with different service levels.

Some authors have chosen to explicitly deal with the dynamic nature of these problems. This dynamic approach allows the consideration of ambulance reallocation decisions among different bases, aiming at improving the coverage. It is assigned to Gendreau, Laporte & Semet (2001) the presentation of the first model that includes the redistribution of emergency resources. Nogueira et al. (2016), for instance, presents a location model allowing the number of available ambulances in each base to be changed, in different periods of the day.

Akdoğan, Bayındır & Iyigun (2018) develop a stochastic model with the objective of minimizing the mean response time of emergency vehicles location, allowing the assignment of more than one vehicle to each episode. They define areas with different demands, each following a time homogeneous Poisson process. All the emergency vehicles are assumed to be similar.

In the real setting, there are different emergency care models that can be implemented. Until 1967, advanced life support (ALS) did not exist in prehospital care. All patients were transported to the hospital by professionals trained in basic life support (BLS) or by emergency technicians. The first time ALS care in the prehospital setting was mentioned was by Pantridge, a physician and cardiologist from Northern Ireland, with the proposal of organizing a mobile intensive care unit that could transport patients with acute myocardial infarction to the hospital, providing ALS, on site and during transportation (Liberman, Branas, Mulder, Lavoie & Sampalis, 2004). Since then, prehospital ALS units have been introduced, developed and expanded according to the possibilities and characteristics of each country (Liberman et al., 2004) .

In the organization of prehospital emergency care, two strategies can be used (Beuran M et al., 2012; Liberman et al., 2004). The "scoop and run / load and go" strategy, most commonly used in the United States, is based on the fast evacuation of the patient to the nearest emergency center, assisting the patient during this time, rather than trying to stabilize the situation at the place of occurrence of the episode. The care team is composed by paramedics or emergency technicians and their activities consist only of non-invasive interventions such as wound dressing, external bleeding control, spinal and fracture immobilization, oxygen administration, non-invasive cardiopulmonary function resuscitation and management of basic airway devices (oro / nasopharyngeal tubes and hand inflators). These techniques are easy to perform, require little additional on-site time, can be usually completed during travel, and have been widely accepted as necessary for the acute treatment of patients in the prehospital setting. Another strategy is known as the "stay and play / treat then transfer" strategy, used mainly in European countries. In this strategy, medical doctors and nurses go to the emergency occurrence, assisting and stabilizing the patient before moving the patient to a health care unit. Over the past 30 years, ALS in the prehospital setting has also been accepted as necessary for severe trauma victims and has been widely implemented, encompassing all the previously mentioned BLS techniques, as well as invasive procedures including intubation, intravenous access initiation, fluid replacement, drug administration. It is based on the reasoning that these interventions will not only prevent the deterioration of the patient's condition, like in BLS, but will also improve the patient's health status by stabilizing the patient before the arrival at the hospital. Some countries (Brazil, for instance) follow both strategies. In a performance comparative study of both models, in trauma situations, in the city of Catanduva, state of São Paulo, no significant differences in terms of health results were found (Gonsaga, Brugugnolli & Fraga, 2012). For Ebben et al. (2013) both strategies have their virtues and flaws, so it is risky to say which one is the most effective. In trauma situations, according to the same author, the approach to prehospital care should be decided considering the injury mechanism (blunt versus penetrating trauma), the distance from the trauma center (urban versus non-urban areas) and the level of resources available. The advantages of each system, according to the type of care needed, are still widely discussed today (Beuran M et al., 2012; De Souza Minayo & Deslandes, 2008; Dretzke, Sandercock, Bayliss & Burls, 2004; Ebben et al., 2013; Liu & Bai, 2018; O'Dwyer, Konder, Machado, Alves & Alves, 2013; Poppe et al., 2015).

In this work a new model for emergency vehicle location is presented. This model was developed in order to be able to represent the way in which emergency vehicles are organized in Portugal, where the strategy used is mostly "stay and play". However, the seriousness of the situation and the needed care that is foreseen in the hospital, have a greater preponderance in the decision of what strategy to adopt in each particular case.

The existing emergency vehicle location models, even the ones that consider multiple types of vehicles (see, for instance, Chong, Henderson & Mark E. Lewis (2016) and McLay (2009)), are not appropriate to represent the current situation in Portugal. Actually, it is necessary not only to consider the existence of several different types of vehicles, providing different levels of assistance and considering coverage time limits that are different for different areas (namely urban and rural), but also the possibility of vehicle substitution. When a given type of vehicle is not available, it is sometimes possible to dispatch other vehicles of different types that are capable of providing the same level of assistance. The authors were not able to find published works that considered this substitutability capacity. An emergency episode is only considered covered if it receives all the vehicles that are appropriate to the situation, and there are possibly different sets of vehicle combinations that can provide the adequate assistance.

The developed model is different from the ones known from the literature in several different ways. It simultaneously considers:

1. Different types of vehicles capable of providing different levels of emergency healthcare;
2. The possibility of substitutability between emergency vehicles of different types;
3. The possibility of more than one vehicle, of similar or different types, being needed in a given emergency episode;
4. An incompatibility matrix that defines whether a given vehicle can or cannot be simultaneously assigned to two different emergency episodes, taking into account the time periods of occurrence of these episodes. This matrix allows the model to have a dynamic characteristic, without the need of having decision variables indexed to time, which is an added value regarding the dimension of the model.
5. The explicit consideration of the unavailability of vehicles due to previous assignments.

In the stochastic version of the model, uncertainty is explicitly taken into account considering a two-stage stochastic programming approach.

We can thus define the overall contribution of this work as being the development of a model that is able to incorporate more features present in the real setting, in the context of pre-hospital emergency care vehicles location, when compared with the existing models, leading to solutions that will better support the decision-making process. Features like vehicle substitution and vehicle unavailability due to previous occurrences bring more reality to the model and can have an important impact in the quality of the produced solution.

In the next section, the organization of the emergency vehicles in Portugal, that motivated the development of this work, is described. In section 3, the deterministic version of the model will be described and illustrated with small examples. In section 4, the stochastic version of the model is presented. Section 5 presents the application of the model to a problem inspired by real data. Section 6 concludes and suggests possible paths for future research.

2. Contextualization of the problem

In Portugal, the emergency response system is framed in what is known as the Integrated Medical Emergency System. This system uses both medical teams (including a medical doctor) and non-medical teams that belong to different organizations and have different assistance capacities. According to the National Medical Emergency Institute (INEM, 2019), the existing emergency resources can be of the following types (a summary of this information can be found in Table 1):

- Non-medical vehicle
  - ✓ Medical Emergency Motorcycles (MEM): vehicles manned by an emergency technician able to deliver BLS and External Automatic Defibrillation (EAD). They allow the initialization of stabilization measures of the victim and, according to the type of care required, they can prepare the victim to be transported by another vehicle.
  - ✓ Assistance Ambulances (AA): they are usually located at emergency medical centers (Volunteer Fire Departments or sections of the Portuguese Red Cross) and are manned by volunteers or professionals with specific training in prehospital emergency techniques. They can work with other means of emergency and have the capacity of transporting the victims to health units.
  - ✓ Medical Emergency Ambulances (MEA): they are manned by two emergency technicians. Their mission is to stabilize the victim autonomously or in complementarity with other means, and to transport the victim to the hospital. They have equipment for resuscitation and clinical stabilization, namely EAD. They have the capacity to transport the victim to a care unit. They are usually based at INEM bases.
  - ✓ Immediate Life Support (ILS) Ambulances (ILSA): they guarantee more differentiated health care than the previous means, such as resuscitation maneuvers. The crew consists of a nurse and a pre-hospital emergency technician. They can work in partnership with other means (with different levels of care differentiation) and have the capacity of defibrillation, cardiac monitoring, transmission of electrocardiographic data and they can transport the victim to a care unit. They are preferably based on basic urgency units.

- Medical vehicle

- ✓ Medical Emergency and Resuscitation Vehicles (MERV): they are intended for prehospital intervention. The crew is composed by a nurse and a medical doctor with competence and equipment for ALS. They aim to stabilize and monitor the transportation of victims to the hospital. They do not have the ability to transport victims, which forces them to work with another emergency vehicle whenever there is such a need.
- ✓ Emergency Medical Helicopters (EMH): they are manned by a team of pilots, a medical doctor and a nurse, and are assigned to primary response (the stabilization of the victim at the place of occurrence and transportation to the hospital as well as transportation between two health units). They allow for rapid intervention, even over long distances, and have all the equipment available in an ALS vehicle. Their operation availability is highly dependent on the weather conditions. They are located in heliports of hospital bases, in firemen quarters or aerodromes.

All these vehicles are managed in an integrated way by INEM, through its Urgent Patient Guidance Centers (CODU). These are telephone answering centers that, through defined algorithms and looking at the available resources, dispatch the necessary vehicles for each emergency occurrence. There are four CODUs in Portugal: one in the North (Porto), one in the Center (Coimbra), one in Lisbon and one in Algarve (Faro).

Type of vehicle	BLS	ALS	Physician	Nurse	Emergency Technician	Volunteer Firemen	Transportation	EAD
MEM	x				1		No	Yes
AA	x					2	Yes	No
MEA	x				2		Yes	Yes
ILSA	x	x		1	1		Yes	Yes
MERV	x	x	1	1			No	Yes
EMH	x	x	1	1			Yes	Yes

Table 1: Type of emergency vehicles that exist in the Portuguese emergency system and their main characteristics

According to the guidelines of INEM (2017), it is important to guarantee that BLS and ALS reach the emergency occurrence locations in the shortest possible time, since every minute without electrical defibrillation, in cardiac arrest victims with defibrillation rhythms, decreases the probability of survival by around 10% to 12%. If this waiting time is supported with BLS, this percentage decreases to 3% to 4%. Some countries established upper bounds for desirable assistance response times. In the United States of America, for instance, response time targets have been set as assisting 90% of life-threatening emergency episodes occurring in urban areas by a vehicle with transportation capability within 8:59 minutes. For rural and wilderness areas this target is changed by considering 15 and 30 minutes respectively. In the United Kingdom, 75% of the most critical emergency calls, in urban and rural areas, should be attended in 8 minutes and 95% of these episodes should be reached within 14 minutes in urban areas and 19 minutes in rural areas. Hong Kong has defined a time limit of 12 minutes that should be respected for 92% and 95% of all emergency episodes in rural and urban areas, respectively (Fitch , 2005). Although, as far as the authors know, there is no strict definition of maximum response times for Portugal, the time limits defined as reference by INEM (2019) are 15 minutes for urban areas and 30 minutes for rural areas. It should be highlighted that, although response time is an important aspect of prehospital care, it is not the only one influencing the success of the intervention. Data gathering and analysis of group of patients per type of diseases and evolution of health status under specific treatments, training of response teams and the cooperation of different entities are equally crucial factors for the success of the intervention (Salvucci, Kuehl, Clawson & Martin, 2004).

3. Deterministic Mathematical Model

In this section we will present the deterministic version of the model, making it easier to understand the underlying reasoning behind the model. In the next section, the stochastic version of the model will be presented. In the deterministic version, it is assumed that it is possible to know *a priori* which are the emergency episodes that will occur in the given planning time horizon. Each emergency episode will be characterized by several different features, including time of occurrence and number of vehicles of each type that need to be dispatched to guarantee a proper care. The model seeks to integrate a hierarchical view of the available resources, considering explicitly the possibility of vehicle substitution, if the needed vehicle is not available. For instance, the intermediate level support provided by ILSA can cover situations of BLS (AA and MEA), and the ALS vehicles can cover intermediate and basic level needs. The need of transporting the patient to a health unit has also to be considered, since there are vehicles that do not have this capability. For instance, although a MERV can supply the level of care of ILSA or MEA, it cannot replace them if the patient needs to be transported.

All the parameters used in the model are now presented. They are also presented in a table in Annex B.

T: time horizon

$i \in I$: set of possible bases for emergency vehicles

$j \in J$: set of locations where emergency episodes may occur

N_i: maximum number of vehicles that can be positioned in base i

$k \in K$: set of different types of vehicles, were each type k also determines the level of assistance of the respective vehicles ($l_k \in \{BLS, ILS, ALS\}, \forall k \in K$)

$v \in V$: set of existing vehicles. Each vehicle is characterized as being of a certain type (k_v).

$e \in E$: set of emergency episodes that occur during the defined time horizon. Each emergency episode is characterized by:

- $j_e \in J$: the location where the episode takes place;
- n_{ek}: number of vehicles of type k that will need to be dispatched, $\forall k \in K$;

— TS_{ek}: time period in which the assistance begins for vehicles of type k, $\forall k \in K$;[1]
— TSt_{ek}: time period in which the assistance ends for vehicles of type k, $\forall k \in K$.

Coverage matrix

For each location pair *i,j* and for each level of assistance *l*, the coverage matrix will give the information of whether *j* is in the coverage radius of base *i*. This coverage matrix represents the possibility of having different coverage time limits for different levels *l* of needed assistance.

$$a_{ijl} = \begin{cases} 1, \text{if the distance between } i \text{ and } j \text{ is within the maximum time limit} \\ \text{defined for level of assistance } l \\ (j \text{ is covered by vehicles in } i, \text{for assistance level } l), \forall i \in I, j \in J, \in \{ILS, ALS, BLS\} \\ 0, \text{otherwise} \end{cases}$$

Notice that it is possible to have different maximum times defined for different levels of assistance, so that a_{ijl} can take different values for different values of *l*.

Substitution matrix

The hierarchical relationship between vehicles that can be dispatched to a particular episode of emergency is made explicit in the substitution matrix. A vehicle of type *k'* can replace another vehicle of type *k* if and only if it provides at least the same level of care of vehicles of type *k* (also taking into account transportation capability).

$$c_{kk\prime} = \begin{cases} 1, \text{if a vehicle of type } k'\text{can substitute vehicle of type } k, \forall\ k, k' \in K \\ 0, \text{otherwise} \end{cases}$$

It should be stressed that, in general, $c_{kk\prime}$ can be different from $c_{k\prime k}$ (k' can replace k but the opposite cannot occur). Table 2 shows an example of a substitution matrix.

k	AA	MEM	MERV
AA	1	0	0
MEM	1	1	1
MERV	0	0	1

Table 2: Example of a substitution matrix that gathers the information of which vehicle types can or cannot substitute others

[1] These times are defined by type of vehicle to be possible to represent a situation where, at an early stage, only one type of vehicle is necessary but, later on, in the course of the episode, other types of vehicles are deemed necessary. In this situation, vehicles will not arrive all at the same time and they also do not end the assignment at the same time. It is assumed, however, that all the vehicles of the same type arrive at the same time and are free at the same time, which constitutes a limitation of the model.

From this table, it is possible to conclude that a vehicle of type MEM can be substituted by any other type of vehicle, but the opposite is not allowed. MERV vehicles, for instance, are not substitutes to AA because they do not offer transportation capability.

There are situations where a vehicle can be substituted by two additional vehicles. This can happen, for instance, with ILSA vehicles, that can be substituted by a MERV and AA (MERV provides a higher level of care but it does not have the transportation capability). These situations will be properly represented by constraints in the model, as explained later. For now, let us define the following notation:

$$r_k = \begin{cases} 1, \text{if vehicle of type } k \text{ can be substituted by more than one vehicle}, \forall k \in K \\ 0, \text{otherwise} \end{cases}$$

$g_k = \{(k', k''): \text{vehicle of type } k \text{ can be substituted by a pair of vehiclesof types } k' \text{and } k'', k', k'' \in K\}, \forall k \in K$

Assume that $(k, k) \in g_k, \forall k \in K$.

Incompatibility matrix

If two episodes have overlapping time periods of occurrence, then the same vehicle cannot be dispatched to both of them. For this reason, there is a matrix of incompatibilities that defines whether or not two episodes are overlapping and thus are incompatible. Two episodes will be considered incompatible if they occur, in whole or in part, at the same time.

$$b_{ee'k} = \begin{cases} 1, \text{if vehicles of type } k \text{ cannot be assigned to both episodes } e \text{ and } e', \forall e, e' \in E, k \in K \\ 0, \text{otherwise} \end{cases}$$

Notice that the time periods in which a vehicle is assigned to an episode can also include the time spent to prepare the vehicle after the episode, so that it is ready to be assigned to another episode.

Availability matrix

It cannot usually be assumed that all the emergency vehicles are available at the beginning of the defined planning time horizon. In fact, some emergency vehicles can be unavailable because they are still being used in an episode that started before the current planning time period and that has not yet ended. Moreover, a vehicle can also be unavailable because it is in maintenance, for instance. These situations can be represented using this binary matrix.

$$d_{ev} = \begin{cases} 1, \text{if vehicle } v \text{ can be assigned to episode } e, \forall e \in E, v \in V \\ 0, \text{otherwise} \end{cases}$$

The decision variables to be defined in the model determine how to locate the existing emergency vehicles among the available potential bases, taking into account the needs of the emergency episodes that occur during the defined time horizon.

Decision variables

$$y_i = \begin{cases} 1, \text{if location } i \text{ has vehicles located there}, \forall i \in I \\ 0, \text{otherwise} \end{cases}$$

$$h_{vi} = \begin{cases} 1, \text{if vehicle } v \text{ is located at } i, \forall v \in V, i \in I \\ 0, \text{otherwise} \end{cases}$$

$$z_e = \begin{cases} 1, \text{if episode } e \text{ receives all the needed vehicles within the maximum time limit}, \forall e \in E \\ 0, \text{otherwise} \end{cases}$$

$$x_{vek} = \begin{cases} 1, \text{if vehicle } v \text{ is assigned to episode } e \text{ to fulfill level } k \text{ needs} \\ \text{(even if it is of a different level)}, \forall v \in V, e \in E, k \in K \\ 0, \text{otherwise} \end{cases}$$

In this model, the objective function is the maximization of the total number of episodes covered.

$$Max\ Z = \sum_{e \in E} z_e \tag{1}$$

With this objective function, many optimal alternative solutions exist, especially if there are more resources than the ones strictly needed. To eliminate many of these alternative optimal solutions, the objective function can be changed as follows:

$$Max\ Z = \sum_{e \in E} z_e - \varepsilon\left(\sum_{v \in V} \sum_{i \in I} h_{vi} + \sum_{i \in I} y_i + \sum_{v \in V} \sum_{e \in E} \sum_{k \in K} x_{vek}\right) \tag{2}$$

Parameter ε represents a very small positive number.

The constraints can be defined as follows:

- An episode *e* is *covered* if and only if it receives all the necessary vehicles of all the needed types within the defined time limits. These restrictions consider the possibility of one vehicle being substituted by another one but not by more than one vehicle ($r_k = 0$). If it is possible to substitute vehicle *k* by more than one vehicle, then constraints (4) to (10) apply.

$$n_{ek} z_e \leq \sum_{v \in V} c_{kk_v} x_{vek}, \forall e \in E, k \in K: r_k = 0 \tag{3}$$

- When a vehicle of a given type can be substituted by more than one vehicle of other types, then it is more complicated to guarantee that the correct number of vehicles assigned to each episode is in accordance with what the episode needs. This is done by using auxiliary variables. Define auxiliary integer variables $O_{ekk'}, \forall e \in E, k, k' \in K, k': \exists(k', k'') \in g_k$. These variables will count the number of vehicles of type k' that are assigned to *e*, such that k' can substitute *k* in conjunction with other k'' ($(k', k'') \in g_k$). Furthermore, let $q_{ekk'k''}, \forall e \in E, k, k', k'' \in$

$K: r_k = 1, (k', k'') \in g_k, k' < k''$, represent the number of vehicles belonging to the pair $(k', k'') \in g_k$ that are simultaneously dispatched to episode *e*. These variables will guarantee that no vehicle is counted more than once. Then, if vehicle k can be substituted by more than one vehicle (namely the pair $(k', k'') \in g_k$) the following constraints hold:

- Variable O_{ekk} represents the number of vehicles that are dispatched to episode *e* without being substituted by more than one vehicle.

$$O_{ekk} \leq \sum_{v \in V} c_{kk_v} x_{vek}, \forall e \in E, k \in K: r_k = 1 \qquad (4)$$

- Episode *e* is covered if and only if it receives the necessary vehicles, either exactly the ones it should receive from the needed vehicle types or from direct substitutability (O_{ekk}) or pairs of others that are equivalent in terms of care ($\sum_{(k',k'') \in g_k, k' < k''} q_{ekk'k''}$).

$$n_{ek} z_e \leq O_{ekk} + \sum_{(k',k'') \in g_k, k' < k''} q_{ekk'k''}, \forall e \in E, k \in K: r_k = 1 \qquad (5)$$

- The number of vehicles of type k' that contribute to the substitution of vehicles of type *k* are limited by the number of vehicles of this type that are dispatched to episode *e*, considering the vehicles of this type that were already needed. These constraints guarantee that no vehicle is counted more than once for the same episode. This could happen if the episode needs different vehicles of different types and a given vehicle *v* of type *k'* can simultaneously be the sole substitute for one type of needed vehicle, but also belongs to a pair (k', k'') for another type of vehicle, for instance.

$$O_{ekk'} \leq \sum_{v \in V} c_{k'k_v} x_{vek'} - n_{ek'}, \forall e \in E, k, k' \in K: r_k = 1, \exists (k', k'') \in g_k \qquad (6)$$

- The total number of pairs of vehicles $(k', k'') \in g_k$ that are used to substitute vehicles of type *k* is limited by the lower number of vehicles of each type. If k' belongs to more than one pair $(k', k'') \in g_k$, then the same vehicle cannot contribute to more than one substituting pair.
  Imagine that 2 vehicles of type k' and 1 vehicle of type k'' are being dispatched. This means that only one pair (k', k'') can be counted.

$$\sum_{(k',k'') \in g_k: k' < k''} q_{ekk'k''} \leq O_{ekk'}, , \forall e \in E, k, k' \in K, k' < k'': \exists (k', k'') \in g_k \qquad (7)$$
$$\sum_{(k',k'') \in g_k: k' < k''} q_{ekk'k''} \leq O_{ekk''}, , \forall e \in E, k, k'' \in K, k' < k'': \exists (k', k'') \in g_k \qquad (8)$$

— In each episode *e*, an emergency vehicle can only contribute to one given level of assistance.

$$\sum_{k \in K} x_{vek} \leq 1, \forall e \in E, v \in V \qquad (9)$$

— The emergency vehicle *v* can only attend episode *e* from base *i* if it is located there, and if *e* occurs within the coverage radius of *i* for that level of care, and it is available.

$$x_{vek} \leq d_{ev} \sum_{i \in I} a_{i j_e l_k} h_{vi}, \forall v \in V, e \in E, k \in K \qquad (10)$$

— An emergency vehicle can only be assigned to two episodes if their occurrence time periods do not overlap.

$$\sum_{k' \in K} x_{vek'} + \sum_{k' \in K} x_{ve'k'} \leq 2 - b_{ee'k}, \forall v \in V, k \in K, e, e' \in E: e < e' \quad (11)$$

— There is a maximum number of vehicles that can be located at each base, and vehicles can only be assigned to a base that has been prepared.

$$\sum_{v \in V} h_{vi} \leq N_i y_i, \forall i \in I \quad (12)$$

— Each emergency vehicle can only be located at one base.

$$\sum_{i \in I} h_{vi} \leq 1, \forall v \in V \quad (13)$$

— If a given episode is not fully covered, then it will not have any resource assigned. This constraint will eliminate many alternative optimal solutions that would exist (M represents a very large and positive number).

$$\sum_{v \in V} \sum_{k \in K} x_{vek} \leq M z_e, \forall e \in E \quad (14)$$

$$y_i \in \{0,1\}, \forall i \in I$$
$$h_{vi} \in \{0,1\}, \forall v \in V, i \in I$$
$$z_e \in \{0,1\}, \forall e \in E$$
$$x_{vek} \in \{0,1\}, \forall v \in V, e \in E, k \in K$$
$$q_{ekk'k''} \geq 0 \text{ and integer}, \forall e \in E, k, k', k'' \in K: (k', k'') \in g_k$$
$$O_{ekk'} \geq 0 \text{ and integer}, \forall e \in E, k, k' \in K: \exists (k', k'') \in g_k$$

Constraints (4) to (8) deserve a more detailed explanation. They account for the situation in which one vehicle of type k can be substituted by simultaneously dispatching one vehicle of type k' and another one of type k''. In this situation, the coverage of episode e regarding assistance from vehicles of type k can be achieved by vehicles of type k (or equivalent vehicles according to the substitutability matrix) or by pairs of other types of vehicles.
These restrictions could also be adapted to situations in which a vehicle can be substituted by more than two vehicles (which does not happen in the Portuguese case).

In this deterministic model it is assumed that all the episodes that will occur are known (set E), along with the characteristics of each one. Considering the objective function of maximization of the total number of covered episodes, and with the constraints presented, the model would tend to benefit the less demanding episodes, for which it is easier to achieve z_e =1. Imagine that there is an episode e_1 that needs several vehicles of different types. This episode occurs before two other episodes (e_2 and e_3), that occur sequentially and with no overlapping time periods. Episode e_1 has occurrence time periods that overlap with the occurrence times of e_2 and e_3, so that vehicles cannot be simultaneously assigned to e_1 and e_2 or to e_1 and e_3 . It would be better not to assign resources to e_1, and to guarantee instead the coverage of e_2 and e_3, since this would be better in terms of the objective function. However, in a real setting, this would never happen since when e_1 occurs there is no possibility of knowing that e_2 and e_3 will occur, because they occur later in time. If such a situation is allowed to happen, the value of the coverage calculated in the objective function would be biased, since it would be obtained thanks to a "knowledge of the future": the allocation of vehicles was not made according to the episodes that occurred until the present moment, but also taking into account what would happen in the future. The following solution was developed. Assume that all the episodes $e \in E$ are ordered by time of occurrence (that is, if *e*<*e*' then *e* begins before *e*'). A new restriction is added that, in conjunction with (14), guarantees that if *e* and *e*' have periods of intersection related to assistance by vehicles of type *k* ($b_{ee'k} = 1$), and if vehicle *v* that was available for *e* was not assigned to this episode that occurs first ($x_{vek} = 0$) and $n_{ek} \geq 1$, if episode *e* did not receive all the vehicles it needed ($z_e = 0$), then vehicle *v* cannot be assigned to episode *e*' (if it was available it should be used to ensure coverage of the episode that happens first).

$$\sum_{k' \in K} x_{ve'k'} \leq \left(2 - \sum_{i \in I} a_{ij_{e'}l_k} h_{vi} - \sum_{i \in I} a_{ij_e l_k} h_{vi}\right) + 1 - (b_{ee'k} - x_{vek} - z_e) + \frac{\sum_{v' \in V} x_{v'ek}}{n_{ek}} + \\ + M\left(1 - c_{kk_v}\right) + \sum_{k' \in K} \sum_{e'': e'' < e} b_{e''ek'} x_{ve''k'}, \\ \forall v \in V, e, e' \in E: e < e', k \in K: n_{ek} \geq 1 \qquad (15)$$

This constraint will only be of interest if vehicle *v* is within the coverage radius of both episodes *e* and *e'*. If this is not the case, then $\left(2 - \sum_{i \in I} a_{ij_{e'}l_k} h_{vi} - \sum_{i \in I} a_{ij_e l_k} h_{vi}\right)$ will be greater or equal to 1, and the constraint is redundant.

If $b_{ee'k} = 1$, $x_{vek} = 0$, $z_e = 0$ and the total number of vehicles of type *k* assigned to *e* is less than n_{ek} $\left(\frac{\sum_{v' \in V} x_{v'ek}}{n_{ek}} < 1\right)$ (*e* is not being covered because it has not received sufficient vehicles of type *k*), we are left with $\sum_{k' \in K} x_{ve'k'} \leq 1 - (1 - 0 - 0) +$ a number less than 1. This causes $\sum_{k' \in K} x_{ve'k'}$ to be less than or equal to a number less than 1, implying that $\sum_{k' \in K} x_{ve'k'}$ has to take the value zero (that is, vehicle *v* cannot go to the future episode *e*'). If $c_{kk_v} = 0$, then vehicle *v* does not contribute to type *k*, and the constraint is redundant. If $\sum_{k' \in K} \sum_{e'': e'' < e} b_{e''ek'} x_{ve''k'} > 0$, this vehicle is already being used in another episode *e*'' occurring before *e* and which has intersections with it (this latter addend causes this restriction to have effect only if this vehicle is not already being used in an episode e'', and therefore it is not in fact available to be assigned to e). In any other situation, we have $x_{ve'k}$ less than or equal to a number equal to or greater than 1, so the variable can already take the value 1.

A small example will now be described to illustrate the model characteristics and the importance of constraints (15).

Example 1

Imagine that there are two possible locations for placing emergency services, each capable of receiving at most two vehicles. There are five locations where emergency episodes can occur, and each one will have exactly one emergency episode. For the sake of simplicity, let us assume that episode 1 occurs in location 1, episode 2 in location 2 and so on. Figure 1 illustrates the time periods in which these episodes occur.

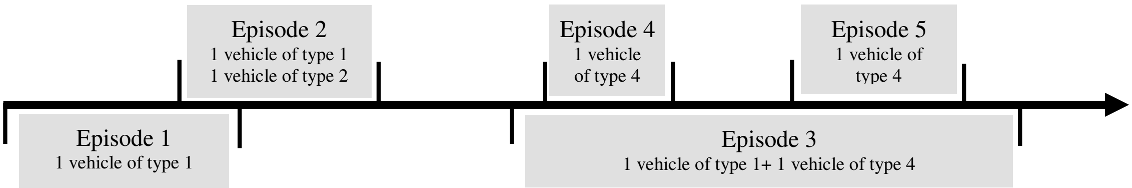

Figure 1: Timeline for the occurrence of episodes in example 1

Let us also assume that there are four vehicles available, each one of a different type k. Table 3 presents the number of resources needed by each one of the emergency episodes. For now, let us assume that there is no possibility of substitution between vehicles, all vehicles are available at the beginning of the planning horizon, and locations 1 to 5 are all inside the coverage radius of both base locations 1 and 2.

	Vehicle type 1	**Vehicle type 2**	**Vehicle type 3**	**Vehicle type 4**
Episode 1	1	0	0	0
Episode 2	1	1	0	0
Episode 3	1	0	0	1
Episode 4	0	0	0	1
Episode 5	0	0	0	1

Table 3: Resources needed by each one of the emergency episodes (example 1)

The timeline presented in Figure 1 will determine the incompatibility matrix, and the corresponding values $b_{ee'k}$. Table 4 presents these values.

***k*=1**	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Episode 1	0	1	0	0	0
Episode 2	1	0	0	0	0
Episode 3	0	0	0	0	0
Episode 4	0	0	0	0	0
Episode 5	0	0	0	0	0

***k*=2**	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Episode 1	0	0	0	0	0
Episode 2	0	0	0	0	0

Episode 3	0	0	0	0	0
Episode 4	0	0	0	0	0
Episode 5	0	0	0	0	0

***k*=3**	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Episode 1	0	0	0	0	0
Episode 2	0	0	0	0	0
Episode 3	0	0	0	0	0
Episode 4	0	0	0	0	0
Episode 5	0	0	0	0	0

***k*=4**	Episode 1	Episode 2	Episode 3	Episode 4	Episode 5
Episode 1	0	0	0	0	0
Episode 2	0	0	0	0	0
Episode 3	0	0	0	1	1
Episode 4	0	0	1	0	0
Episode 5	0	0	1	0	0

Table 4: Incompatibility matrix between different emergency episodes (example 1)

The optimal solution to this problem is as follows: episode 1 and 3 are the only ones covered. Base 2 is opened, and the two needed resources are located there. Vehicle 1 is assigned to both episodes 1 and 3. Episode 3 also needs vehicle 4. Episode 2 cannot be covered because one needed resource is already assigned to episode 1 and it is not available, and no other vehicle can be a substitute. Episodes 4 and 5 can also not be covered because the needed resource is assigned to episode 3.

Now imagine that the problem is solved without constraints (15). In this case, the model, that maximizes the total number of covered episodes, will take advantage of its "knowledge of the future": it will be better to guarantee the coverage of episodes 4 and 5 than to guarantee the coverage of episode 3. So, instead of dispatching the needed and available resources to episode 3 when this episode happens, these available vehicles will be held so that they can be later assigned to the other two episodes. In this case, three episodes will be covered. This is a biased value for coverage, since this would never occur in a real setting.

Example 2

Let us now assume that vehicle of type 3 can substitute vehicles of type 4. In this case, three resources will be used, and episodes 1, 3, 4 and 5 can be covered. As each base can have, at most, 2 vehicles, both bases will be operational. Base 1 will have vehicles 3 and 4, base 2 will have vehicle 1. It will be possible to assign vehicle 3 as being of type 4 to episodes 3 and 4 (x_{344} and x_{354} are equal to 1). There are optimal alternative solutions, both in terms of the location of the vehicles (since all locations are inside the coverage radius of each potential base), and regarding the assignment of vehicle 3 (it could also be assigned to episode 3 and vehicle 4 to episodes 4 and 5).

If we now change the coverage matrix, stating that location 1 is only covered by using vehicles of base 1, then the needed vehicle to assure coverage of this episode would be mandatorily located at base 1.

Example 3

Let us now work with a slightly different example, where the occurrence of episodes is depicted in figure 2. Table 5 shows the vehicles that need to be assigned to each episode.

	Vehicle type 1	Vehicle type 2	Vehicle type 3	Vehicle type 4
Episode 1	0	0	0	1
Episode 2	0	1	0	0
Episode 3	1	0	0	1
Episode 4	0	0	0	1
Episode 5	0	0	0	1

Table 5: Resources needed by each one of the emergency episodes (example 3)

Constraint (15) deals with the situation where, despite one episode not being fully covered (episode 3), other incompatible episodes can be served (episodes 4 and 5) because the episode is not covered due to the fact that the needed resource is already being used in a prior event that has not yet finished (episode 1).

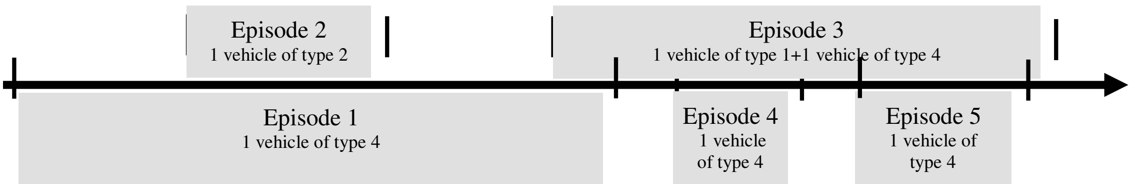

Figure 2: Timeline for the occurrence of episodes in example 3

The optimal solution in this example is x_{121}, x_{222}, x_{414}, x_{444} and x_{454} equal to 1, guaranteeing the coverage of all the episodes but episode 3.

4. Stochastic Model

The previous described model is missing a very important characteristic: the ability of explicitly considering the underlying uncertainty that is associated with the existence of emergency episodes. In order to bring our model closer to the real context of emergency situations, a stochastic version is now described, explicitly acknowledging that it is not possible to know with certainty which episodes will be occurring during the defined time horizon.

The model developed is a two-stage stochastic model, with uncertainty included in the model using a set of different scenarios. The set of scenarios will be represented by $s \in S$. Each scenario will have a given probability of occurrence $p_s, \forall s \in S$. The decision variables belong to one of two sets: either they belong to the first stage set, meaning that they have to be decided before the actual realization of the uncertainty, or they belong to the second stage set, and will be defined after the random events are known (also named recourse variables). Monte Carlo simulation is used to build the scenarios, and the sample average approximation method is used (Birge & Francois, 2011).

The classical linear two-stage programming problem can be formulated as follows (a minimization problem is shown without any loss of generality):

$$\min f(\zeta) = c^T\zeta + E_\Theta[R(\zeta,\Theta)]$$
$$\text{Subject to } A\zeta = b; \zeta \in \Psi$$

$E_\Theta[R(\zeta,\Theta)]$ denotes the mathematical expectation with respect to Θ. $R(\zeta,\Theta)$ is the optimal value of the second stage problem for a given scenario s ($\Theta(s)$) which can be formulated as:

$$\min\ r(\Theta)^T\omega$$
$$\text{Subject to } \Gamma(\Theta)\zeta + \Upsilon(\Theta)\omega = \phi(\Theta); \omega \in \Omega$$

Variables ζ are the first stage decision variables, and ω are the second stage decision variables. $\Theta(\mathrm{r},\Gamma,\Upsilon,\phi)$ corresponds to the data of the second stage problem, which is considered to be random (it is scenario dependent). In the present case, ζ corresponds to variables y_i and h_{vi} (the location variables that will not be scenario dependent). Second stage variables ω correspond to variables z_{es} and x_{veks} (similar to variables z_e and x_{vek} in the deterministic version, but that are now indexed to the different scenarios that are going to be included in the model). $E_\Theta[R(\zeta,\Theta)]$ is replaced by a Monte Carlo estimate such that each scenario will be a given realization of Θ ($\Theta(s)$).

$$z_{es} = \begin{cases} 1, \text{if episode } e \text{ receives all the needed vehicles within} \\ \text{the maximum time limit, under scenario } s\ , \forall e \in E_s, s \in S \\ 0, \text{otherwise} \end{cases}$$

$$x_{veks} = \begin{cases} , \text{if vehicle } v \text{ is assigned to episode } e \text{ to fulfill level } k \text{ needs under scenario } s \\ \text{(even if it is of a different level)}, \forall v \in V, s \in S, k \in K, e \in E_s \\ 0, \text{otherwise} \end{cases}$$

The parameters defined previously remain valid for this model, so we will only present those which must change to accommodate the existence of scenarios. Each scenario will have its own set of emergency episodes, $e \in E_s$. $\Theta(\mathrm{r},\Gamma,\Upsilon,\phi)$ in the classical two-stage programming problem formulation represents the data associated with the emergency episodes, that have a random nature.
The incompatibility matrix will now also depend on the different scenarios.

$$b_{ee'ks} = \begin{cases} 1, \text{if vehicles of type } k \text{ cannot be assigned to both episodes } e \text{ and } e' \\ \text{under scenario } s, \forall e, e' \in E_s, k \in K, s \in S \\ 0, \text{otherwise} \end{cases}$$

It is also possible that the *a priori* availability of vehicles is different for different scenarios:

$$d_{evs} = \begin{cases} 1, \text{if vehicle } v \text{ can be assigned to episode } e, \text{under scenario } s, \forall e \in E_s, v \in V, s \in S \\ 0, \text{otherwise} \end{cases}$$

The number of vehicles of type k that will need to be dispatched to emergency episode e belonging to scenario s will now be represented by n_{eks}.

The objective function will now be the maximization of the expected total number of covered emergency episodes, considering the probability of occurrence of each scenario ($p_s, \forall s \in S$):

$$Max\ Z = \sum_{s\in S}\sum_{e\in E_s} p_s z_{es} - \varepsilon(\sum_{v\in V}\sum_{i\in I} h_{vi} + \sum_{i\in I} y_i + \sum_{v\in V}\sum_{e\in E}\sum_{k\in K}\sum_{s\in S} p_s x_{veks}) \tag{16}$$

Parameter ε represents a very small positive number.

Considering the classical two-stage programming model formulation, $\min c^T\zeta$ corresponds to $\min \varepsilon(\sum_{v\in V}\sum_{i\in I} h_{vi} + \sum_{i\in I} y_i)$ whilst $\min r(\Theta)^T\omega$ represents $\min -z_{es} + \varepsilon(\sum_{v\in V}\sum_{e\in E}\sum_{k\in K} x_{veks})$. Constraints $A\zeta = b$ in the classical two-stage programming model formulation are represented by constraints (12-13), defined for the deterministic version and that should also hold in the stochastic model. Constraints $\Gamma(\Theta)\zeta + \Upsilon(\Theta)\omega = \phi(\Theta)$ are represented by (16-27), that are defined as (these constraints follow directly from (3) to (11)):

— An episode belonging to a given scenario s is "covered" if and only if it receives all the necessary vehicles of all the needed types in that scenario.

$$n_{eks} z_{es} \le \sum_{v\in V} c_{kk_v} x_{veks}, \forall s \in S, e \in E_s, k \in K: r_k = 0 \tag{17}$$

— Define auxiliary integer variables $O_{ekk's}, \forall s \in S, e \in E_s, k \in K, k': \exists(k',k'') \in g_k$ which represent the number of vehicles of type k' that are dispatched to episode e substituting vehicles of type k, under scenario s. Furthermore, let $q_{ekk'k''s}, \forall s \in S, e \in E_s, k \in K: r_k = 1, (k',k'') \in g_k, k' < k''$, represent the number of vehicles belonging to the pair , $(k',k'') \in g_k$ that are simultaneously dispatched to episode e under scenario s. Then, if vehicle k can be substituted by more than one vehicle (namely the pair $(k',k'') \in g_k$) the following constraints hold:

$$n_{eks} z_{es} \le O_{ekks} + \sum_{(k',k'')\in g_k, k'<k''} q_{ekk'k''s}, \forall s \in S, e \in E_s, k \in K: r_k = 1 \tag{18}$$

$$O_{ekks} \le \sum_{v\in V} c_{kk_v} x_{veks}, \forall s \in S, e \in E_s, k \in K: r_k = 1 \tag{19}$$

$$O_{ekk's} \le \sum_{v\in V} c_{k'k_v} x_{vek's} - n_{ek's}, \forall s \in S, e \in E_s, k, k' \in K: \exists(k',k'') \in g_k \tag{20}$$

$$\sum_{(k',k'')\in g_k: k'<k''} q_{ekk'k''s} \le O_{ekk's}, \forall s \in S, e \in E_s, k, k' \in K: \exists(k',k'') \in g_k \tag{21}$$

$$\sum_{(k',k'')\in g_k: k'<k''} q_{ekk'k''s} \le O_{ekk''s}, \forall s \in S, e \in E_s, k, k'' \in K: \exists(k',k'') \in g_k \tag{22}$$

— An emergency vehicle can only contribute to one given level of assistance, in each episode belonging to a given scenario.

$$\sum_{k\in K} x_{veks} \le 1, \forall s \in S, e \in E_s, v \in V \tag{23}$$

— The emergency vehicle *v* can only attend episode e, belonging to scenario *s*, from *i* if it is located there, and if *e* occurs within the coverage radius of *i* for that level of care, and it is available in that scenario.

$$x_{veks} \le d_{evs} \sum_{i\in I} a_{ij_e l_k} h_{vi}, \forall v \in V, s \in S, e \in E_s, k \in K \tag{24}$$

— An emergency vehicle can only be assigned to two episodes if their occurrence time periods do not intersect, considering the scenario where both episodes belong.

$$\sum_{k'\in K} x_{vek's} + \sum_{k'\in K} x_{ve'k's} \le 2 - b_{ee'ks}, \forall v \in V, k \in K, s \in S, e, e' \in E_s\colon e < e' \tag{25}$$

Constraints (14) and (15) have also to be changed as follows:

— If a given episode is not fully covered, then it will not have any resource assigned.

$$\sum_{v\in V} \sum_{k\in K} x_{veks} \le M z_{es}, \forall s \in S, e \in E_s \tag{26}$$

— It is not possible to anticipate the future in each scenario, regarding the assignment decisions.

$$\begin{aligned}\sum_{k'\in K} x_{ve'k's} \le {} & \left(2 - \sum_{i\in I} a_{ij_{e'} l_k} h_{vi} - \sum_{i\in I} a_{ij_e l_k} h_{vi}\right) + 1 - (b_{ee'ks} - x_{veks} - z_{es}) + \frac{\sum_{v'\in V} x_{v'eks}}{n_{eks}} \\ & + M\left(1 - c_{kk_v}\right) + \sum_{k'\in K} \sum_{e'':e''<e} b_{e''ek's} x_{ve''k's}, \\ & \forall v \in V, s \in S, e, e' \in E_s\colon e < e', k \in K\colon n_{eks} \ge 1\end{aligned} \tag{27}$$

5. Case study based on real data

One of the objectives of the development of the model was to see whether it would be possible to reach a solution, in terms of vehicle locations, that would improve the emergency episodes coverage when compared with the current real setting (the current location of emergency vehicles). To apply the model, it is necessary to generate a set of randomly generated scenarios, but it is also important that these scenarios reflect what is expected to happen in reality. So, the analysis of the available data was the starting point for scenario generation.

The available data includes information about all the emergency occurrences that required the assignment of ALS vehicles during one year, and data published by INEM (2019) for the last 3 years, that allowed the calculation of the total number of occurrences (BLS plus ALS) for the same period. It was possible to collect information about the time and location of individual ALS calls but only aggregated information about the time and location of individual BLS calls. All scenarios were built considering this baseline information. These scenarios were then fed into the model which was solved using Cplex. We will first describe how the scenarios were built, and then we will analyze the obtained optimal solution. The solution that corresponds to the current location of the existing vehicles was also tested using the generated scenarios, and it is compared with the optimal solution obtained. As this comparison could be unfair for the current location of vehicles, both solutions are also compared in a new dataset composed of new scenarios not included in the instance that originated the optimal solution. In order to see whether the stochastic version of the model brings benefits when compared to the deterministic version, an optimal solution using this deterministic version was also calculated.

5.1. Dataset

The scenarios for the stochastic model were developed using *R* software environment. The geographical area of interest was the Coimbra district, located at the central part of Portugal. At the present moment there are 35 vehicles available that are distributed among 34 existing bases. The location of the existing bases is presented in Figure 3. The number of vehicles of each type presently located in each base is given in Table 6. In the developed model, three different types of vehicles are defined: BLS, including MEA, AA and MEM; ILS, including ILSA; ALS, including MERV. There is also one EMH available. This vehicle is not included in the model, because it is only used in very specific and limited situations.

Figure 3: Location of existing bases for emergency vehicles in the area under study

	k						k				
Bases	**1 AA**	**2 MEM**	**3 MEA**	**4 ILSA**	**5 MERV**	**Bases**	**1 AA**	**2 MEM**	**3 MEA**	**4 ILSA**	**5 MERV**
1	1	0	0	0	0	**18**	0	0	0	0	0
2	1	0	0	0	0	**19**	1	0	0	0	0
3	1	0	0	0	0	**20**	1	0	0	0	0
4	0	0	0	0	1	**21**	1	0	0	0	0
5	0	0	0	0	1	**22**	1	0	0	0	0

6	0	0	0	0	0	**23**	1	0	0	0	0
7	0	0	0	0	0	**24**	1	0	0	0	0
8	0	1	3	0	0	**25**	1	0	0	0	0
9	1	0	0	0	0	**26**	1	0	0	0	0
10	1	0	0	0	0	**27**	1	0	0	0	0
11	1	0	0	0	0	**28**	1	0	0	0	0
12	0	0	0	0	0	**29**	1	0	0	0	0
13	1	0	0	0	0	**30**	1	0	0	0	0
14	0	0	0	1	1	**31**	1	0	0	0	0
15	1	0	0	0	0	**32**	1	0	0	0	0
16	0	0	0	0	0	**33**	1	0	0	0	0
17	0	0	0	0	0	**34**	1	0	1	1	1

Table 6: Number and location of existing vehicles (current situation)

Table 7 presents the substitutability matrix for the available types of vehicles. A vehicle of type 4 can also be substituted by sending simultaneously a vehicle of type 5 and a vehicle of type 1 or 3. This means that $r_4 = 1$ and $g_4 = \{(4,4), (1,5), (3,5)\}$. This situation has been represented by using constraints (18)-(22) accordingly.

k	**1**	**2**	**3**	**4**	**5**
1	1	0	1	1	0
2	1	1	1	1	0
3	1	0	1	1	0
4	0	0	0	1	0
5	0	0	0	0	1

Table 7: Substitutability matrix between different types of vehicles

As this region has different areas, with different patterns of emergency situations, 16 different areas were defined, as depicted in Figure 4. Each area is defined as a polygon, through the use of GPS coordinates. Some polygons overlap. The biggest polygon represents the entire region. Smaller polygons represent areas that experience larger volumes of emergency calls, such as city centers and roads with a pattern of high number of accidents.

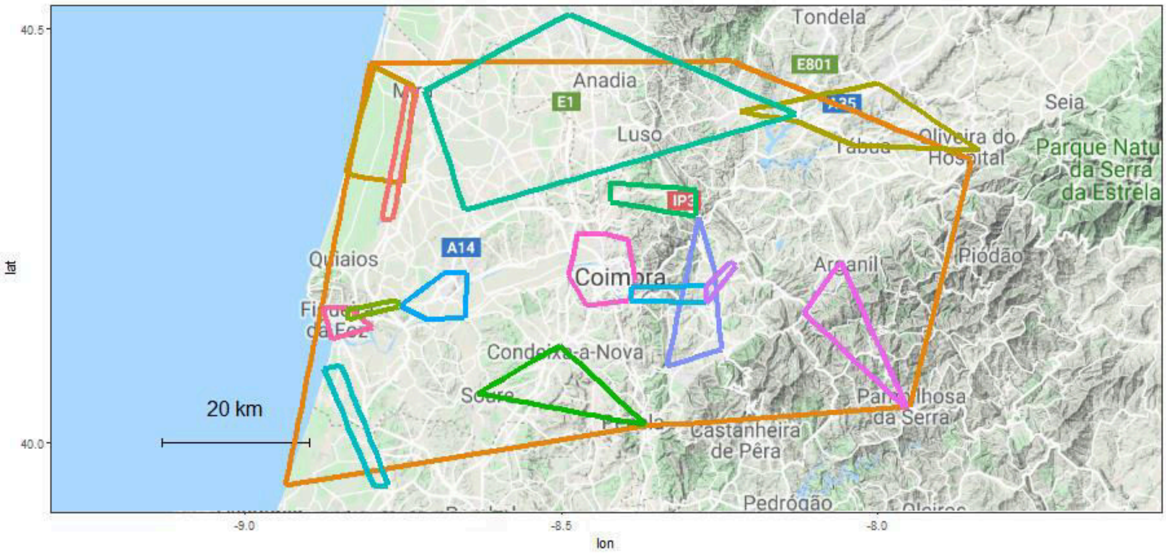

Figure 4: Definition of different areas with different patterns regarding the occurrence of emergency episodes, patterns that were taken into account when generating the scenarios.

Each scenario will correspond to a 24-hour day period. The number of episodes in each scenario is randomly generated using a Poisson distribution with λ adjusted by considering the best fit to the known data (the parameters were estimated by considering the maximum likelihood criterion, different distributions were tested and the Poisson distribution was chosen according to the Akaike information criterion). In the present case, two different types of emergency episodes were defined: medical episode or trauma episode. In the first case λ is equal to 56.57, and in the second case is equal to 9.35. The type of emergency episode will influence the random generation of the number and type of vehicles that should be assigned to the episode. If the emergency is a trauma episode, for instance, then it is mandatory to have at least one MERV vehicle and one AA or MEA. The location of each episode is first randomly assigned to one of the existing polygons, according to the existing probabilities of occurrence for each area. The episode is then randomly located inside the polygon. The time limits considered for building the coverage matrix were the reference time limits from INEM (2019): 15 minutes for urban areas and 30 minutes for rural areas. All the areas were defined as rural except for the polygons associated with the cities of Coimbra and Figueira da Foz. Actual driving times by road were calculated by using the Google Maps API, within *R*. Emergency vehicles usually drive faster than common vehicles, but no data was available that could support some kind of conversion between "normal" driving times and "emergency vehicles driving times", so the option was to use these values. Furthermore, the area under study does not usually experience heavy traffic. The start time of each emergency episode was also randomly generated according to the occurrence patterns throughout the day (it is more likely that emergency episodes occur from 9.00 to 21.00 than from 2.00 to 4.00, for instance). Figure 5 presents the probability of a given episode occurring in each hour of the day. The total duration (in minutes) of each episode was randomly generated, assuming these times follow a Gamma distribution (this was the distribution that presented the best fit to the available real data, with shape α=4.2123, rate β=0.0692 (mean equal to 30.03 and standard deviation equal to 29.67). According to the existing data, vehicles were unavailable only 3% of the time (due to vehicle maintenance operations or other non-call activities), and this was the probability used to build the availability matrix.

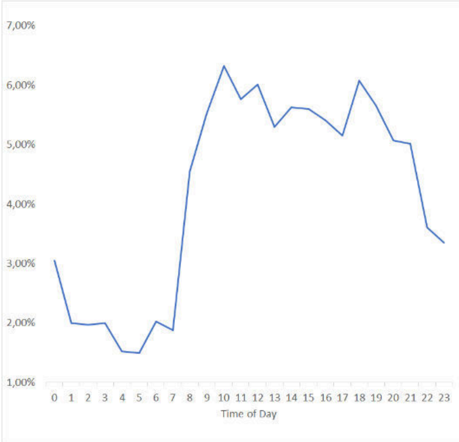

Figure 5: During the day, the probability of occurrence of an emergency episode is not always the same. This figure shows the pattern with which emergency episodes occur during one day (24 hour period).

When using a sample average approximation, it is necessary to define the number of scenarios to generate. There is no way of determining the optimal number of scenarios. Increasing the sample size will allow the calculated solution to converge to the optimal solution of the two-stage stochastic model (Birge & Francois, 2011). In practice, it is necessary to reach a compromise between the number of scenarios considered and the dimension of the problem, and the inherent difficulties in calculating a solution. In this work we have considered a total of 30 scenarios. This number was chosen because it was possible to observe that the differences in the objective function obtained with different sets of 25 scenarios each were small (the model was solved five times, each time with 25 scenarios independently generated, and the expected coverage rate was always within the interval [89.0%, 89.6%]). It was possible to solve the model with 30 scenarios in a reasonable computational time. With more scenarios, not only did the computational time increase considerably, but the general solver had also memory problems.

The created dataset encompasses a total of 30 different scenarios, and 1978 total emergency episodes. Figure 6 shows the location of the emergency episodes generated for these 30 scenarios.

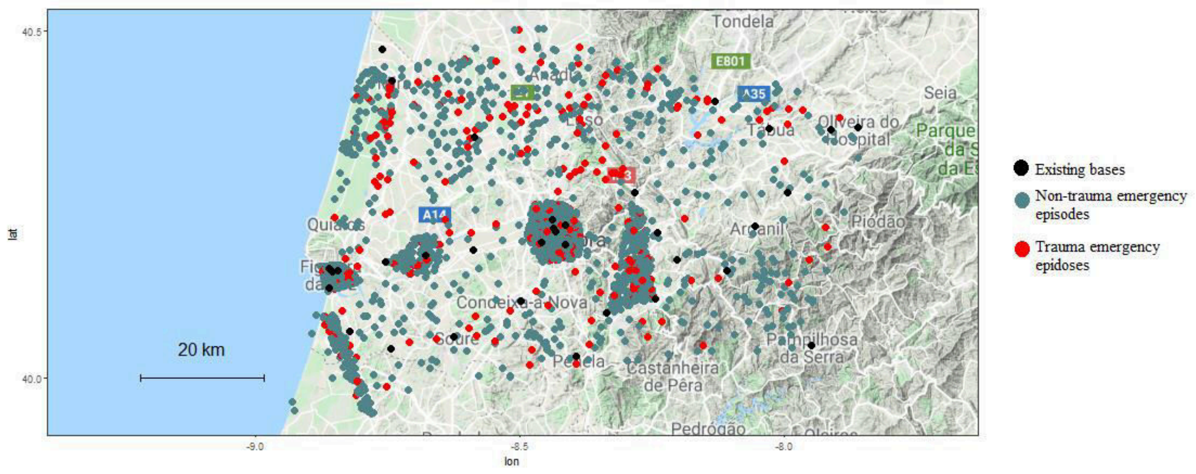

Figure 6: Location of the emergency episodes for 30 scenarios

The pseudo-code used to build the scenarios as well as all the generated data are available as supplementary material.

5.2. Calculating and analyzing the optimal solution

Two different instances were solved. In the first instance we let the model determine the optimal location of all the emergency vehicles, maximizing the expected total coverage. In the second instance, we fixed the vehicles in their current bases and looked for the solution that maximizes the expected coverage.

The instances were solved by Cplex, version 12.7, using Intel Xeon Silver 4116, 2.1 gigahertz, 12-core processor, 128 gigabyte RAM. Cplex is capable of eliminating many of the existing constraints and also fixing several binary variables in the pre-solve stage, resulting in a model with 122982 constraints and 71207 variables. The first instance took 6.7 hours solve to optimality. Assuming that this is a strategic decision, this computational time can be considered reasonable. The second instance (with the location variables fixed) took 50.03 seconds to solve to optimality.

The results of the optimal location are shown in Table 8. These locations can be compared with the current ones presented in Table 6.

			k						***k***		
Bases	**1 AA**	**2 MEM**	**3 MEA**	**4 ILSA**	**5 MERV**	**Bases**	**1 AA**	**2 MEM**	**3 MEA**	**4 ILSA**	**5 MERV**
1	1	0	0	0	0	**18**	0	0	0	0	0
2	0	0	0	1	0	**19**	1	0	0	0	0
3	1	0	0	0	0	**20**	1	0	0	0	0
4	1	0	0	0	1	**21**	1	0	0	0	0
5	0	0	1	0	0	**22**	1	0	0	0	0
6	1	0	0	0	0	**23**	1	0	0	0	0
7	0	0	0	0	0	**24**	0	0	0	0	0
8	1	0	0	0	1	**25**	1	1	0	0	0
9	0	0	0	0	0	**26**	0	0	1	0	0
10	1	0	0	0	0	**27**	1	0	0	0	0
11	0	0	0	0	0	**28**	0	0	0	0	0

12	0	0	0	1	0	**29**	0	0	1	0	0
13	0	0	1	0	0	**30**	1	0	0	0	0
14	2	0	0	0	1	**31**	1	0	0	0	0
15	1	0	0	0	0	**32**	1	0	0	0	0
16	1	0	0	0	0	**33**	1	0	0	0	0
17	1	0	0	0	0	**34**	2	0	0	0	1

Table 8: Optimal location of emergency vehicles (solution produced by using the proposed model)

The differences between the current solution and the optimal one are highlighted in grey. The optimal locations for emergency vehicles achieve better coverage results than the existing one, as shown in Table 9.

Scenarios	Total number of episodes	Current solution		Optimal solution	
		Covered episodes	Coverage rate %	Covered episodes	Coverage rate %
1	64	53	82.81	55	85.94
2	53	49	92.45	52	98.11
3	70	62	88.57	67	95.71
4	62	51	82.26	52	83.87
5	70	63	90.00	66	94.29
6	59	47	79.66	48	81.36
7	77	67	87.01	71	92.21
8	71	63	88.73	65	91.55
9	69	62	89.86	65	94.20
10	55	45	81.82	49	89.09
11	72	59	81.94	63	87.50
12	68	58	85.29	62	91.18
13	51	43	84.31	47	92.16
14	66	51	77.27	52	78.79
15	64	58	90.63	59	92.19
16	61	51	83.61	54	88.52
17	56	39	69.64	47	83.93
18	72	59	81.94	63	87.50
19	68	54	79.41	61	89.71
20	66	54	81.82	62	93.94
21	67	60	89.55	63	94.03
22	80	72	90.00	73	91.25
23	65	56	86.15	59	90.77
24	77	63	81.82	67	87.01
25	55	48	87.27	48	87.27
26	75	60	80.00	67	89.33
27	84	69	82.14	75	89.29
28	58	47	81.03	49	84.48
29	67	56	83.58	61	91.04
30	56	47	83.93	50	89.29

Table 9: Comparison of the achieved coverage between the current and the optimal solution. For each one of the generated scenarios, the total number of scenarios as well as the covered ones are shown in the current and the optimal solution.

The observed changes can be summarized as follows:

— Some vehicles were relocated to areas with higher population density and, therefore, with a greater number of occurrences, such as the reinforcement of base 4 with a vehicle with transport capacity;

- Replacement of vehicles in areas that have, in the current solution, lower coverage, but that present high accident rates. An example is a MERV, that was previously assigned to base 5, and that is now moved to base 8, which is closer to two major roadways where severe trauma episodes are more likely to happen;
- Reallocation of vehicles to areas where the emergency episodes pattern justifies the existence of these means. An example of this replacement is, for instance, one AA vehicle that is positioned in bases 16 and 17, the placement in base 25 of another BLS vehicle, or the placing of an emergency vehicle on bases 6 and 12;
- Adjustment of the locations of BLS and ALS vehicles, relocating vehicles that have the capacity of transporting patients to bases where than can be more useful. As an example, base 14 is reinforced with one such vehicle.

The optimal solution found is equal to or better than the current one in terms of coverage for all the scenarios. The expected coverage of the optimal solution is equal to 89.5%, whilst it is equal to 84.2% when the current location of vehicles is considered. Moreover, the worst result in all scenarios is equal to 78.8% of coverage, whilst this value is equal to 69.6% in the current solution. There was an overall gain of approximately 5% in the total number of episodes covered (plus 106 episodes covered), using exactly the same number of available vehicles.

Comparing the current and optimal locations using the dataset used to build the solved instance can give a biased result, benefiting the optimal solution (it is the optimal solution for exactly that set of scenarios). To eliminate any bias resulting from using the dataset that originated the solved instance, we independently generated 15 new scenarios, using the same described procedure. Then, the locations of the emergency vehicles were fixed for both the current locations and the optimal locations calculated before. The results are shown in Table 10.

		Current solution		**Optimal solution**	
Scenarios	**Total number of episodes**	**Covered episodes**	**Coverage rate %**	**Covered episodes**	**Coverage rate %**
1	62	55	88.71	57	91.94
2	62	54	87.10	56	90.32
3	82	65	79.27	71	86.59
4	72	58	80.56	63	87.50
5	67	58	86.57	61	91.04
6	66	57	86.36	57	86.36
7	47	41	87.23	40	85.11
8	54	46	85.19	50	92.59
9	54	44	81.48	47	87.04
10	66	54	81.82	56	84.85
11	73	62	84.93	62	84.93
12	60	47	78.33	49	81.67
13	58	51	87.93	51	87.93
14	62	55	88.71	57	91.94
15	66	54	81.82	59	89.39

Table 10: Comparison of the achieved coverage between the current and the optimal solution for a new dataset with 15 scenarios.

The optimal solution found presented also the best results in this new and unseen dataset. The expected coverage of the current location is 84.4%, and of the calculated one is 87.95%. The worst coverage achieved by the optimal solution is better than the worst coverage found using the current distribution of emergency vehicles (81.67% against 78.33%). The optimal solution is capable of covering 35 episodes (3.7%) more than the current solution.

Figure 7 depicts all the episodes from all scenarios that are not being covered in the optimal solution. Around 19% of these episodes are trauma medical emergencies. From the observation of this image, it is possible to conclude what would be the available bases that would need to have more vehicles, if possible.

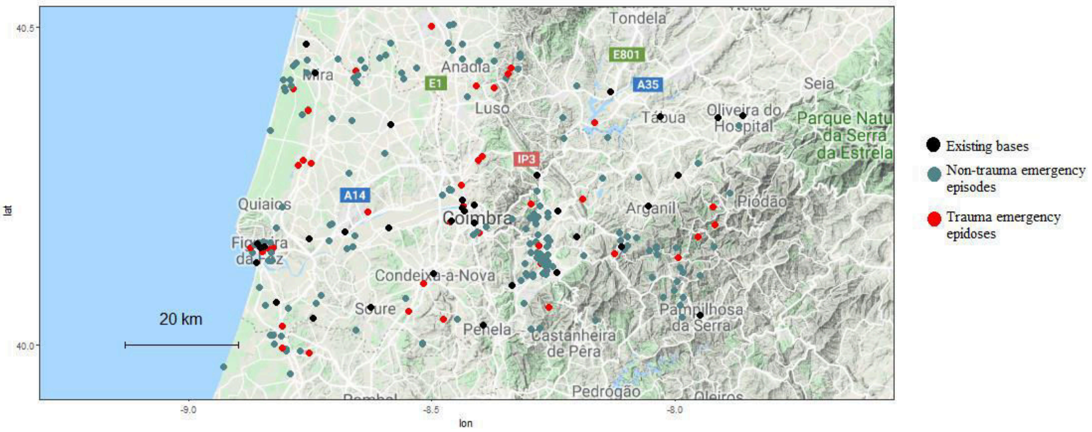

Figure 7: Location of the emergency episodes that are not covered considering the location of vehicles proposed by the model

It is important to point out that, in a real setting, all episodes receive assistance. The emergency episodes that are not being covered in this model would be episodes where assistance would arrive later than it should, would receive vehicles providing a lower level of assistance, or vehicles originating from other bases not belonging to the defined area of interest would be called. The use of vehicles stationed in the outskirts of the region of interest is not being considered in this model, and it can be relevant especially when emergency episodes occur near the geographic limits of the defined area, or when there are several victims that require the activation of vehicles from different bases. It should also be pointed out that constraints (26) will not allow any vehicle to be dispatched to episodes that will not be covered. The model can thus consider as available some vehicles which would not be available in a real setting. This is equivalent to assuming that episodes occurring when all vehicles within the coverage radius are busy leave the system and will not stay in queue. A similar assumption has also been followed by other authors (Chong et al., 2016), considering the situation of all vehicles being busy. Our assumption is however stronger, since we are considering the situation where all vehicles within the coverage radius of the episode are busy.

To understand whether the stochastic model brings an added value when compared with its deterministic version, the deterministic model has also been used and the corresponding solution compared with the one obtained in the stochastic version. The input data is composed of the expected number of episodes for one day. The respective locations correspond to the centroids obtained by creating clusters using all the episodes generated in the 30 scenarios dataset (using the *k*-means clustering algorithm). Applying the obtained optimal solution for the 30 scenarios dataset, the average coverage obtained is 84% (lower than what is obtained with the stochastic solution). Looking at the worst scenario, the coverage drops to 73.4%. This solution was also applied to the out-of-sample dataset with 15 scenarios. The coverage obtained was, once again, lower than with the stochastic solution: 84.5% on average (31 less occurrences covered) and 78.7 for the worst-case scenario.

Restrictions (27) have been added to assure that the model does not make vehicle assignments taking into account future emergency events that should not be known in advance. We have also tested the importance of these constraints, namely to see if, without them, the obtained coverage would be much different. Indeed, the average coverage increases, in the 30 scenario database, to 94.5% (an increase of 5.6% when compared with the 89.5% average coverage) and, in the worst case scenario, it increases to 81.8% (a 3.8% increase when compared with the 78.8% worst case coverage). If the constraints are not added to the model, the objective function will, indeed, be biased. Furthermore, it was possible to conclude that, without these constraints, the computational time increases significantly: more than 48 hours were needed to calculate the optimal solution.

We have also tested the influence of the number of available vehicles in the coverage obtained (Table 11). Assuming that the number of available vehicles is the double (keeping the fleet composition pattern unchanged), it would be possible to increase the coverage to 95.5% in the 30 scenarios dataset, being the coverage in the worst-case scenario equal to 89.9%. If we assume that the number of available vehicles has tripled, then not all of them would be needed to achieve 99.3% of coverage: in total 65 vehicles of type 1, 1 vehicle of type 2, 10 vehicles of type 3, 6 vehicles of type 4, 15 vehicles of type 5. This information is summarized in Table 11. The option to double or triple the number of available vehicles instead of assuming that it would be possible to acquire new vehicles of each of the existing types has to do with the fact that we do not have information about the costs associated with each of these vehicle types. The composition of the existing fleet has implicit information about the vehicle relative costs, so it makes more sense to consider similar fleet compositions.

Number of available vehicles	Average coverage	Worst-case coverage	Percentage of vehicles used
Current situation	89.5%	78.8%	100%
Doubling the number of vehicles	95.5%	89.9%	100%
Tripling the number of vehicles	99.3%	95.1%	95% of all available vehicles: • 90% of all vehicles of type 1 • 33% of all vehicles of type 2 • 83% of all vehicles of type 3 • 100% of all vehicles of type 4 • 100% of all vehicles of type 5

Table 11: Sensitivity analysis with respect to the number of available vehicles for the 30 scenarios dataset, keeping the same vehicle type mix

6. Conclusions

The mathematical model developed is, as far as the authors know, different from the models proposed in the literature for locating emergency vehicles. The built-in matrices (covering, availability, incompatibility and substitution) give the model a dynamic character, removing limiting assumptions related to the uncertainty of demand and supply. The fact that the model explicitly includes the assignment of vehicles to emergency episodes is an added value that allows for a better representation of the real setting and improves the calculation of the expected coverage. The stochastic version of the model allows for the inclusion of different scenarios, so that it is possible to find solutions that present greater robustness, taking into account the inherent uncertainty of these contexts. The scenarios were generated in a way that truly represent the real situation.

The solutions obtained show that the model is behaving as expected considering the input data, and it is coherent with the needs that result from the demand of each of the defined areas within the region of interest.

One of the issues that may occur if the model is applied to a wider geographical area has to do with the model size. The size of the model will grow significantly if more episodes in each scenario are considered, or if more scenarios are included. The general solver was capable of solving the 30 scenarios instance, having almost 2000 episodes in total, in reasonable computational times. However, for larger instances, it would be interesting to develop a metaheuristic dedicated to this problem and assess whether it could calculate good quality solutions in reasonable computational times. It will also be interesting to explore the application of algorithms based on Bender’s decomposition, for instance.

It is also important to explicitly state some limitations of the proposed approach, some of which can motivate future work. The generated scenarios are not differentiating different days of the week. A deeper real data analysis can bring some insights on whether this should be an important factor to take into account, being possible to have different vehicle locations during weekend and weekdays. A similar thing can be said regarding summer months, where there is usually a flow of people from the inland areas to the coast and the arrival of emigrants that, in some locations, increase significantly the number of inhabitants. This could justify different vehicle locations throughout the year. The model is also assuming that the location of vehicles does not change during the day or the week. Considering that there are different patterns of emergency occurrence during the day, it could be better to develop a solution where vehicles could indeed change their positions. However, such approach would also have some disadvantages. It is important to notice that these changes usually generate a strong resistance from the human resources.

The developed model considers the possibility of not sending vehicles to emergency episodes. This lost call assumption could also be dropped in future works, assuming instead that all episodes receive some kind of assistance (even if it is not the most appropriate of it does not arrive within the defined coverage window).

It would also be interesting to study the impact of having different maximum assistance times in the definition of the coverage matrices. The more demanding these times are, the less coverage can be guaranteed with the same set of available vehicles. The model developed would also allow us to study the existing compromises between these maximum times and the resources necessary to maintain acceptable levels of coverage.

Although our model maximizes the expected number of episodes that receive all the needed resources within the adequate time window, other objective functions could be used. Considering the stochastic formulation, it could be interesting to maximize the total minimum coverage (the coverage of the scenario that is in the worst situation), or maximize the equity between different areas (namely urban and regional areas, guaranteeing that the difference in coverage is minimized). As there is a clear compromise between the number of available vehicles and the achieved coverage, it is also possible to evolve to multiobjective models. In the presented model, an episode is either covered or not. It could be possible to consider a partial coverage of an episode, namely if it receives vehicles that are not the ones it should receive.

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Annex A

Abbreviation	Meaning
AA	Assistance Ambulances
ALS	Advanced Life Support
BLS	Basic Life Support
CODU	Urgent Patient Guidance Centers
EAD	External Automatic Defibrillation
EMH	Emergency Medical Helicopters
ILS	Immediate Life Support
ILSA	Immediate Life Support Ambulances
INEM	National Medical Emergency Institute
MEA	Medical Emergency Ambulances
MEM	Medical Emergency Motorcycles
MERV	Medical Emergency and Resuscitation Vehicles

Annex B

Indexes and Sets

Index, set	Meaning
T	Time horizon.
$i\,, I$	Set of possible bases for emergency vehicles $i, i \in I$.
k, K	Set of different types of vehicles $k, k \in K$; each *k* will also determine the level of assistance of those vehicles l_k.
l_k	Level of assistance of vehicles of type k; $l_k \in \{BLS, ILS, ALS\}, \forall k \in K$.
v, V	Set of existing vehicles v; $v \in V$.
k_v	Type of vehicle *v*; $v \in V$; $k_v \in K$.
s, S	Set of existing scenarios s; $s \in S$.
$e, E\ /\ E_s$	Set of emergency episodes e that occur during the considered time horizon; $e \in E$. These emergency episodes will be scenario dependent in the stochastic model, so in this case $e \in E_s$.
j, J	Set of locations j where emergency episodes may occur; $j \in J$.

Data

j_e	The location *j* where the episode *e* takes place, $j_e \in J, e \in E/E_s, s \in S$.
n_{ek}/n_{eks}	Number of vehicles of type *k* that will need to be dispatched to episode e (this can be scenario dependent in the stochastic model), $k \in K, e \in E/E_s, s \in S$.
TS_{ek}/TS_{eks}	Time period in which the assistance for vehicles of type *k* for episode e is required. This value can be scenario dependent. $k \in K, e \in E/E_s, s \in S$.
TSt_{ek}/TSt_{eks}	Time period in which the assistance ends for vehicles of type *k* in episode e; this is the moment when the vehicle is declared operational by the crew after assisting episode *e*. It can be scenario dependent. $k \in K, e \in E/E_s, s \in S$.
N_i	Maximum number of vehicles that can be positioned in base *I; $i \in I$.*
a_{ijl}	Binary value defining whether location *j* is in the coverage radius of *i* for level of assistance *l*. $i \in I, j \in J, l \in \{BLS, ILS, ALS\}$.
$c_{kk'}$	Binary value that defines whether vehicle of type *k'* can substitute *k*. $k, k' \in K$.
c_{kk_v}	The same as the previous one, but considering whether vehicle *v* (of type k_v) can substitute vehicles of type *k*. $v \in V, k, k_v \in K$.
r_k	Binary value indicating whether vehicles of type *k* can be substituted by more than one vehicle. If r_k is equal to one, then set g_k will define the pairs of types of vehicles that can substitute type *k*. $k \in K$.
$b_{ee'k}/b_{ee'ks}$	Binary value that will determine whether episodes *e* and *e´* have any intersection in their occurrence times. If this is true, then the same vehicle can't be assigned to both episodes. These values can be scenario dependent. $e \in E/E_s, k \in K, s \in S$.
p_s	Probability of scenario $s \in S$.

Variables

x_{vek}/x_{veks}	Binary decision variable that will decide whether vehicle v will be assigned to episode *e* providing level of assistance *k*. This last index is necessary to account for vehicle substitutability. These variables can be scenario dependent. $v \in V, e \in E/E_s, k \in K, s \in S$.

y_i	Binary decision variable that will determine whether base i will have vehicles based there, $i \in I$.
h_{vi}	Binary decision variable that will determine whether vehicle v is located i, $v \in V, i \in I$.
z_e/z_{es}	Binary decision variable that will determine whether episode e receives all the vehicles it needs respecting the coverage radius. It can be scenario dependent. $e \in E/E_s, s \in S$.
$O_{ekk'}O_{ekk's}$	Auxiliary variable that represents the number of vehicles of type k' that are dispatched to episode e substituting vehicles of type k. It can be scenario dependent. $e \in E/E_s, k, k' \in K$.
$q_{ekk'k''}$ $/q_{ekk'k''s}$	Auxiliary variable representing the maximum number of vehicles belonging to the pair $(k', k'') \in g_k$ that are simultaneously dispatched to episode e. It can be scenario dependent. $e \in E/E_s, k, k', k'' \in K$