Asset classification under the IFRS 9 framework for the construction of a banking investment portfolio[*]

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Abstract

Under the IFRS 9 framework, we analyze the tradeoff of classifying a financial asset at amortized cost versus at fair value. Defining an impairment model and based on historical (2003-2019) data for the 10-year Portuguese Government bonds, we analyze the annual performance (income/comprehensive income) of different investment allocations. Setting as objectives the maximization of the income and the minimization of the semivariance of the comprehensive income, we suggest a bi-objective model in order to find efficient allocations. Given the non-smoothness of the semivariance function, we compute the solution of the suggested model by means of a multi-objective derivative-free algorithm. Assuming that the yields and funding rates follow a correlated mean-reverting process and that the bonds' rating dynamics are described by an ordinal response model, we show a possible approach to mitigate the estimation error ingrained in the proposed bi-objective stochastic model. Finally, we assess the out-of-sample performance of some of the suggested efficient allocations.

Keywords: *Asset Classification, Backtesting, IFRS 9, Derivative-Free Optimization, Sensitivity Analysis, Stochastic Simulation*

1   Introduction

In the portfolio optimization literature, it is possible to find several studies on backtesting and on the out-of-sample performance analysis of portfolios constructed under frameworks inspired by the seminal work of Markowitz (1952). When in the face of a portfolio optimization model, it is well known that special attention should be given to the inputs' estimation errors (see, e.g., Chopra and Ziemba 1993). Despite the sophisticated models that have arisen, over the years, the truth is that simple heuristics often are able to achieve superior out-of-sample performances (see, e.g., DeMiguel et al. 2009). This happens because a portfolio optimization model can be seen as a two-step model (Michaud, 1989): in one step the model's inputs need to be

estimated (with error); in the second step, the optimization is performed (the estimation error is optimized!). In turn, a heuristic, free of estimation error, leads to superior out-of-sample results (notwithstanding that in-sample it is a sub-optimal strategy). This does not mean that all the portfolio optimization models do not work in practice; it tells us that when building such portfolio optimization models somehow the estimation error sensitiveness must be addressed. As Gigerenzer (2015) states, in situations of risk (where we can generate future scenarios with a certain probability) optimization models can work just fine, and in situations of uncertainty (where we cannot anticipate anything about the future) heuristics that incorporate the intuition developed over the years, tend to produce better results.

Regarding optimization models for the bank balance sheet, the literature is sparser than for the case of portfolio optimization but still substantial. It had its beginning at least since the sixties with the paper of Kalman and Hammer (1967). In the eighties we highlight the papers of Koehn and Santomero (1980) and Kusy and Ziemba (1986). More recently we have, for example, the works of Diamond and Rajan (2000), Halaj (2013) and Schmaltz et al. (2014). Both the works of Halaj (2013) and Schmaltz et al. (2014) already address the impact of recent regulatory constraints imposed by Basel III. Nevertheless, these studies focus on the regulatory guidelines and overlook the impact of the more recent accounting rules, namely the financial asset classification system imposed by the International Financial Reporting Standard 9 (IFRS 9).

Accounting rules are one of the key variables in the optimization of a bank balance sheet. Motivated by the 2008 financial crisis, the International Accounting Standards Board (IASB) issued IFRS 9, on July 24, 2014, and replaced the International Accounting Standard 39 (IAS 39). IFRS 9 has become mandatory on January 1, 2018. Therefore, in 2018 banks had to adapt to the new accounting rules. From a bank's investment side, one of the practical pressing aspects issued by IFRS 9 lies on financial asset classification. According to the classification system established by IFRS 9, the financial manager faces the decision of classifying a financial asset either at amortized cost or at fair value: for assets at amortized cost, a decrease in market prices does not have an impact on the bank's capital, thus reducing risk; fair value assets, although impacted by variations in market prices, allow the possibility of realizing gains in case of an increase in market prices. This paper addresses the impact of this decision and investigates how it can be taken in an efficient way.

Motivated by the fact that banks tend to invest in sovereign debt (Gennaioli et al., 2018), we begin by designing a backtest on the 10-year Portuguese Government bonds for the period between 2003 and 2019. Due to the experienced downgrades during the 2011-2014 financial crisis, the history of the Portuguese bonds constitutes a very rich data set. The backtest consists in assessing the historical annual income and comprehensive income of different allocations to amortized cost and to fair value. In order to accomplish this, we need to compute the bonds' expected credit loss as imposed by IFRS 9. However, IFRS 9 does not give a definition of the impairment model, allowing each bank considerable discretion in its implementation. Thereby, in this paper, we suggest and implement a possible impairment model, based on the assumption that the marginal probability of default follows a Poisson process.

From the backtest's results, we stress that the higher the allocation of the investment to fair value, the higher the annual income achieved (higher investment return) and the greater the variability of the annual comprehensive income (greater investment risk). These results depend on two important determinants: the time horizon defined by the bank investment manager and the threshold from which she/he decides to realize gains at fair value.

According to the best of our knowledge, in the literature, there is no backtest on the classifi-

2

cation of financial assets at amortized cost versus fair value. Furthermore, no methodology that enables the bank investment manager to efficiently make this choice is known. In the particular case of fixed-income financial assets, the literature of portfolio theory has focused on the risk management side by suggesting alternative models to deal with different risk sources (see, e.g., Zenios 1995, Bertocchi et al. 2005, and Cassader et al. 2014). In a more recent study, Ortobelli et al. (2018) suggest a two-step optimization model, for the construction of a fixed-income portfolio, in order to cope with the minimization of the interest rate risk (immunization risk) and the maximization of return/risk metrics (namely, the Rachev and Sharpe ratios). As one can verify, none of these studies approach the portfolio selection problem from a holistic accounting/risk management perspective. All the studies mentioned above assume that assets are measured at fair value, meaning that the variations in market prices have a direct impact on capital so that a decrease in market values reduces the net position. However, for assets at amortized cost, variations in market prices do not have an impact on the net position of the bank, thus reducing capital risk. The approach presented in this paper enables the investment manager to decide the optimal mix between assets at amortized cost and at fair value, balancing the tradeoff between reducing risk (in the case of amortized cost) and the potential to realize gains (for assets at fair value).

To give a more substantial contribution to the literature, we construct an optimization model that allows the bank investment manager to directly find efficient allocations (amortized cost versus fair value) in a risk-return bi-dimensional space. We define as the model's objectives the maximization of investment income and the minimization of the semivariance of the comprehensive income. Both objectives have a direct impact on the bank's own funds and regulatory capital ratios. Given the non-differentiability of the semivariance objective function, we use a multi-objective derivative-free algorithm to find the efficient allocations.

Since in a stochastic optimization model we are optimizing objective functions with stochastic parameters, it is very important to explicitly cope with the estimation error. Motivated by this fact, in this paper we discuss and implement a possible approach to mitigate the estimation error present in the proposed stochastic bi-objective model. This approach is based on stochastic simulation and scenario generation. Finally, based on simulated paths (for a 10-year period), we assess the out-of-sample performance of efficient allocations, selected according to the proposed bi-objective model. An alternative approach is to compare performance against the hard-to-beat equally-weighted heuristic that corresponds to an equal allocation of the investment's amortized cost and fair value. For both considered objectives, the results suggest a robust performance of the efficient portfolios when compared to this benchmark.

The remainder of this paper proceeds as follows. In Section 2 we make a brief presentation on the IFRS 9 rules around the classification of financial assets. In Section 3 we perform a quantitative analysis: we begin by defining and implementing the impairment model; then, we design a backtest on the 10-year Portuguese Government bonds and analyze the respective results. In Section 4 the suggested bi-objective model is motivated, presented and discussed. We develop the stochastic simulation (scenario generation) and the out-of-sample performance assessment in Section 5. In the last section, Section 6, we present the main conclusions of the paper and discuss some possible directions for future work.

2 The IFRS 9 Framework

The classification system of IFRS 9 (IFRS Foundation©, 2017) is based on the business model in which assets are managed and their cash flow characteristics (KPMG, 2016). Under the IFRS 9 framework, financial assets are classified into one of the three categories: amortized cost (AC), fair value through other comprehensive income (FVOCI), and fair value through profit or loss (FVPL). The categories of held to maturity, loans and receivables, and available for sale, existing in the IAS 39 framework, are eliminated (KPMG, 2016).

According to the IFRS 9 (IFRS Foundation©, 2017) classification system, a financial asset is classified at AC if: the financial asset is held within a business model whose objective is to collect contractual cash flows, and the contractual terms of the financial asset give rise to cash flows that are solely payments of principal and interest. As examples of financial assets that can be classified at amortized cost, we have trade receivables, loan receivables, investments in government bonds that are not held for trading and investments in term deposits at standard interest rates. In turn, a financial asset is classified at FVOCI if the financial asset gives rise to cash flows that are solely payments of principal and interest, and it is held in a business model whose objective is achieved by collecting contractual cash flows and selling financial assets. As examples, we have investments in government or corporate bonds where the investment period is shorter than the maturity. If a financial asset does not fall into any of the two previous classifications, then it is classified as FVPL. As examples, we have held for trading financial assets.

An important practical implication of this classification system is that only loans, receivables, investments in debt instruments, and other similar financial assets can qualify to be measured at AC or at FVOCI (PWC, 2017a). From the perspective of a banking portfolio construction, in this paper we focus on the tradeoff between a financial asset being classified at AC or at FVOCI (hereafter FV). If a financial asset is classified at FV, it will allow realizing potential gains that may occur since the acquisition date to maturity, whereas if it is classified at AC this will not be possible. However, a financial asset classified at FV will expose the bank capital to market fluctuations. Therefore, according to the type of financial asset to include in the banking investment portfolio, the bank investment manager should carefully consider what business model to adopt.

Credit risk is the risk of default on a debt due to a borrower failing to fulfill its obligations. Within the IFRS 9 framework, the financial assets classified as AC or FV are impaired under a single impairment model (Volarević and Varović, 2018). IFRS 9 replaces the incurred loss model, used under the IAS 39, with a forward-looking expected credit loss (ECL). In the IAS 39 framework, value adjustments resulted from objective facts; in turn, the IFRS 9 impairment model focuses on possible future losses and therefore makes use of a greater amount of information (e.g., macroeconomic data). IFRS 9 allows each bank to define its own internal impairment model. Nevertheless, the ECL impairment model should be based on credit quality and must contemplate three stages.

Following Taylor (2017), stage 1 ("Performing"), corresponds to the case where there is no indication of a decline in credit quality since acquisition; for financial assets in this stage, the 12-month ECL must be recognized and the interest income is recognized on a gross basis (interest will be calculated on the gross carrying amount of the financial asset before adjusting for ECL). Stage 2 ("underperforming"), corresponds to the case where there is a significant increase in credit risk (SICR) since acquisition; when a financial asset is transferred to this stage, lifetime

ECL must be recognized; nevertheless, the interest income is still recognized on a gross basis. Finally, Stage 3 ("non-performing"), corresponds to the case where the financial asset is credit-impaired; the lifetime ECL continues to be recognized but the interest income is now recognized on a net basis (interest income will be calculated based on the gross carrying amount of the financial asset less the ECL).

3   Quantitative Analysis

Intending to help the bank investment manager in the design of an efficient strategy for asset classification, in this section we perform a quantitative analysis of the tradeoff between a financial asset being classified at AC or at FV.

The analysis relies on the data for the 10-year Portuguese Government bonds, for the period between 2003 and 2019. We have gathered monthly data for the yields[1]. As funding rates we have collected monthly data for the interest rate in term deposits (deposits with agreed maturity, up to one year) of individuals[2] (resident individuals in the monetary union and Portuguese emigrants outside the monetary union). Moreover, for the sample period, we have also gathered data of the Portuguese Government bonds credit ratings according to the Moody's Agency[3]. These monthly time series, corresponding each one to 204 observations, are reported in Figure 1.

Figure 1: Historical Data Gathered
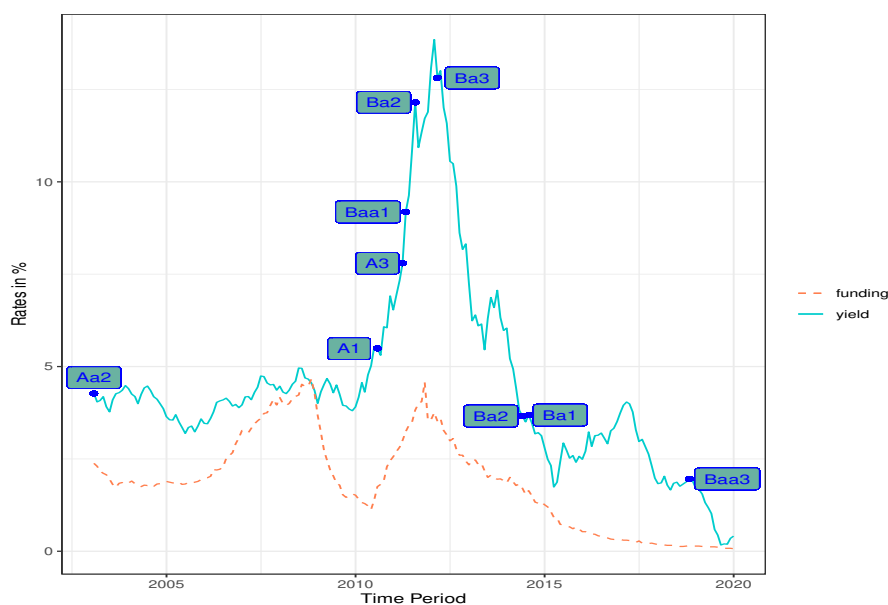
Displayed: monthly funding rates (interest rate on term deposits of individuals in Portugal), monthly yields of the 10-year Portuguese Government bonds and corresponding rating changes (according to the Moody's Agency notation) for the period 2003-2019, inclusive.

We are interested in analyzing the impact of annual decisions (namely, looking at the annual

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[1]This data is publicly available on https://bpstat.bportugal.pt/. We have collected all the available data in which the first observation corresponds to January 2003.

[2]Monthly data, for the funding rates, is publicly available on https://bpstat.bportugal.pt.

[3]This data is publicly available on https://pt.countryeconomy.com/governo/ratings/portugal.

income and the annual comprehensive income); thus, we have annualized all the collected data. Note that, for the funding rates and yields, the monthly data correlation coefficient is equal to 0.6539 and the corresponding annualized data coefficient equals 0.6601. Thereby, the use of annual data does not lead to any loss of generality. Regarding the rating, according to the Moody's Agency notation we have sixteen (considering modifiers) different rating levels: B3, B2, B1 $\mapsto$ highly speculative grade; Ba3, Ba2, Ba1 $\mapsto$ non-investment (speculative) grade; Baa3, Baa2, Baa1 $\mapsto$ medium-low grade; A3, A2, A1 $\mapsto$ medium-high grade; Aa3, Aa2, Aa1 $\mapsto$ high grade; Aaa $\mapsto$ prime. By assigning a numerical value to each possible rating level (1 to 16), this allows us to compute the annual average (rounded value) rating for the sample under analysis[4]. The three annualized series (yields, funding rates, and rating levels) are reported in Figure 2.

Even though our analysis could be applied to any other financial assets, we decided to focus on the 10-year Portuguese Government bonds. The evolution of the 10-year Portuguese Government bonds has been exhibiting very unique dynamics as we can see in Figure 1. In particular, the data set shows a substantial number of downgrades which allows us to assess the impacts of the numerous changes in credit rating. Moreover, banks make substantial investments in sovereign debt (Gennaioli et al., 2018), given the low capital requirements and high liquidity. Thereby, we have chosen to carry out the analysis on the 10-year Portuguese Government bonds which, according to our best knowledge, corresponds to the first time that such an analysis is made. Due to the fact that these bonds have experienced significant downgrades during the 2011-2014 financial crisis, the computation of the ECL, through an impairment model definition, constitutes a challenging task.

Figure 2: Annualized Data
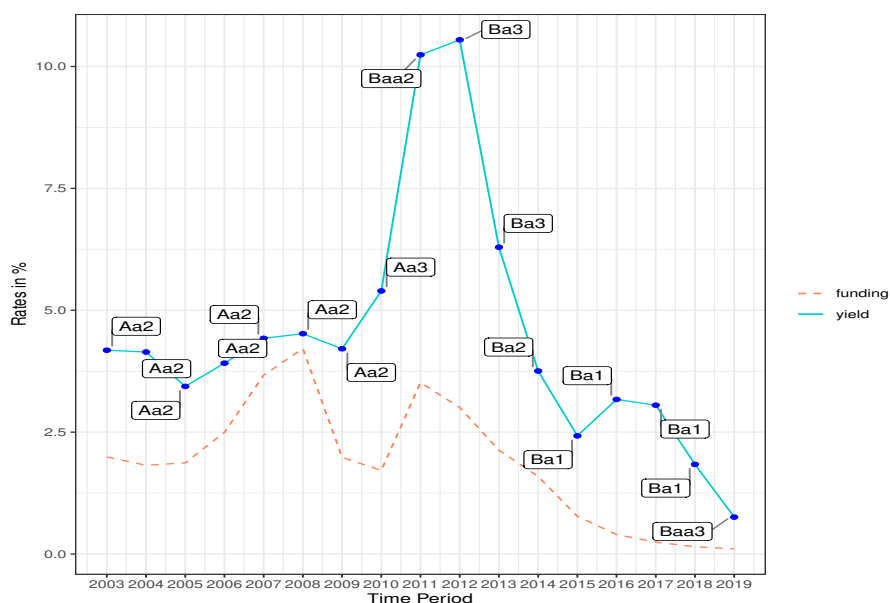
Displayed: annual average funding rates (interest rate on term deposits of individuals in Portugal), annual average yields of the 10-year Portuguese Government bonds and corresponding annual average rating levels (according to the Moody's Agency notation) for the period 2003-2019, inclusive.

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[4]In the monthly data collected we observe eight different rating levels (see Figure 1): Ba3, Ba2, Ba1, Baa3, Baa1, A3, A1, and Aa2.

3.1 Impairment Model Definition and Implementation

One of the great "freedoms" given to the banks by IFRS 9, in the implementation of the impairment model, is related to the computation of the ECL. Since IFRS 9 does not stipulate how to compute the ECL, it is not straightforward to implement it. Banks are expected to develop their internal models making use of past and future information.

In order to quantify the bond credit risk, we begin by defining the lifetime ECL as

$$ECL^{(lifetime)} = \sum_{t=1}^{T} ECL_t, \tag{1}$$

where ECL_t represents the expected credit loss at time t and T is the bond maturity. Following KPMG (2017), ECL_t depends on four factors: (1) the discount factor; (2) the survival probability up to time t; (3) the probability of default between time t and time $t+1$; (4) the loss given default (LGD). Thereby we can model the lifetime ECL as

$$ECL^{(lifetime)} = \sum_{t=1}^{T} \exp\left(-rt\right) \times \exp\left(-\lambda t\right) \times \lambda \times LGD, \tag{2}$$

where r represents the discount rate and λ defines the intensity of the Poisson process that represents the marginal probability of default. Equation (2) can be approximated by the following integral

$$LGD \int_0^T \lambda \exp\left(-(r+\lambda)t\right)dt = LGD \times \frac{\lambda}{\lambda+r} \times \left[1 - \exp\left[-(\lambda+r)T\right]\right]. \tag{3}$$

We can observe from Equation (3), that the loss increases with LGD and maturity. Moreover, it has an asymptote in $\lambda/(\lambda+r)$, i.e., $\lambda/(\lambda+r)$ is an upper bound to the loss.

Regarding the stage allocation, IFRS 9 does not define what a significant increase in credit risk (SICR) is. A SICR determines if an asset should change from stage 1 to stage 2. A bank must identify the factors that indicate a SICR based on the specific nature of the financial asset (PWC, 2017b). Commonly there are three elements taken into account in this identification (see, PWC 2017b, for further details): a quantitative element, a qualitative element, and the 30 days past due backstop indicator in IFRS 9 paragraph 5.5.11 (IFRS Foundation©, 2017). Since great "freedom" is given to banks in defining what constitutes a SICR, several banks are undecided about which indicators to use (Ernst & Young, 2017). However, a great number of banks have declared to make use of the low credit risk simplification as their main indicator for debt securities (Ernst & Young, 2017). In the context of the IFRS 9 framework, a financial asset can be classified as having low credit risk if it has a rating of "investment grade" or above (Taylor, 2018). According to IFRS 9 paragraph 5.5.10 (IFRS Foundation©, 2017), if a financial asset is classified at a reporting date as having low credit risk, then it is assumed that a SICR has not occurred since initial recognition. This can be seen as an absolute threshold (PWC, 2017b).

In our analysis, we assume the following criteria: stage 1 - investment-grade financial assets or financial assets that have not had a downgrade greater or equal than 3 rating categories since origination; Stage 2 - speculative grade financial assets or financial assets that have had a downgrade of at least 3 rating categories since origination.

According to the described methodology, in order to compute a bond's ECL, three inputs are needed: the intensity of the Poisson process (λ), the discount rate (r), and the LGD. The

intensity of the Poisson process can be calibrated using probabilities of default. Nevertheless, we highlight that probabilities of default inferred from market instruments, such as credit default swaps, must not be used in the computation of the ECL. Just as insurers do not use insurance premiums to infer probabilities associated with events (e.g., mortality tables), credit default swaps should not be used to infer probabilities of default, given the uncertainties relating to market risk premia, counterparty risks, and market frictions (for further details see, e.g., Jarrow 2012). Thus, to calibrate the Poisson process intensity, we use the sovereign issuer-weighted cumulative default rates, made public by Moody's Investors Service (2019). For general purposes, we consider, without modifiers, six different rating levels (in ascending order: B, Ba, Baa, A, Aa, Aaa). Therefore, each sovereign issuer-weighted cumulative default rate can be seen as a $p_{k,n}$ element, with $k = 1, \ldots, 6$; $n = 1, \ldots, 10$ (where k represents each rating level and n the number of years considered in Moody's Investors Service 2019). We have calibrated each intensity $\lambda^{(k)}$ (with $k = 1, \ldots, 6$) through the minimum squares method. Accordingly, each intensity, $\lambda^{(k)}$, corresponds to the solution of the following optimization problem

$$\min_{\lambda^{(k)} \in \mathcal{R}} \sum_{k=1}^{K} \sum_{n=1}^{N} [1 - \exp(-\lambda^{(k)} \times n) - p_{k,n}]^2, \tag{4}$$

where $K = 6$, $N = 10$, and $1 - \exp(-\lambda^{(k)} \times n)$ represents the model's n-year probability of default. Problem (4) has a nonlinear but smooth objective function; thereby it can be solved straightforwardly by a nonlinear optimization algorithm. We have used the Broyden-Fletcher-Goldfarb-Shanno algorithm (this algorithm was originally and simultaneously published by Broyden 1970, Fletcher 1970, Goldfarb 1970 and Shanno 1970), also known as the BFGS algorithm. The results are reported in Table 1.

Table 1: Calibrated Intensities

Rating Level	Intensity
Aaa	0.000390%
Aa	0.112%
A	0.349%
Baa	0.273%
Ba	1.106%
B	2.477%

This table reports the Poisson process intensities (see Equation (3)) for each rating level. These intensities correspond to the solution of Problem (4), using as input parameters the sovereign issuer-weighted cumulative default rates (Moody's Investors Service, 2019).

Moody's Investors Service (2019) also reports the historical average value-weighted sovereign recovery rate (RR), $RR = 0.41$. Thereby the corresponding historical average LGD is equal to 0.59 ($LGD = RR - 1$). We use this historical value for the computation of the ECL.

As discount rate, r (see Equation (3)), we use the 12-month Euribor rate[5] for the period under analysis, 2003-2019.

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[5]Source: https://www.euribor-rates.eu/.

Once defined the three inputs needed to implement the methodology described in Section 3.1, the lifetime ECL in year i of a bond acquired in year j, $ECL_{i,j}^{(\text{lifetime})}$, is given by

$$
\begin{aligned}
ECL_{i,j}^{(\text{lifetime})} &= LGD \times \left(\frac{\lambda_i^{(k)}}{\lambda_i^{(k)}+r_i} \right) \times \left[1 - \exp\left[-(\lambda_i^{(k)} + r_i)[10 - (i-j)]\right] \right], \\
&\forall j \in J = \{1, \ldots, s-1\}, \\
&\forall i \in I = \{l \in \{(j+1), \ldots, (j+10)\} \wedge l \leq s\},
\end{aligned}
\tag{5}
$$

where s is the number of sample periods ($s = 17$, corresponding to the number of years for the period 2003-2019), r_i is the discount rate for year i, and $\lambda_i^{(k)}$ represents the intensity of the Poisson process (see Table 1) associated to the rating level k (1,2,3,4,5 or 6) observed in year i.

In turn, the one-year (12-month) ECL in year i of a bond acquired in year j, $ECL_{i,j}^{(\text{one-year})}$, can be computed by

$$
\begin{aligned}
ECL_{i,j}^{(\text{one-year})} &= LGD \times \left(\frac{\lambda_i^{(k)}}{\lambda_i^{(k)}+r_i} \right) \times \left[1 - \exp\left[-(\lambda_i^{(k)} + r_i)\right] \right], \\
&\forall j \in J, \ \forall i \in I.
\end{aligned}
\tag{6}
$$

3.2   Backtest

In this section, we compute and analyze the annual income and comprehensive income corresponding to different investment allocations on the 10-year Portuguese Government bonds. The price of a bond depends on the coupon rate c, yield y and time to maturity T. Therefore, the bond price, $P(c, y, T)$, can be computed as

$$
P(c, y, T) = \sum_{t=1}^{T} \frac{c}{(1+y)^t} + \frac{1}{(1+y)^T}.
\tag{7}
$$

Given the sum of the first n terms of a geometric progression, Equation (7) can be expressed as

$$
\begin{aligned}
P(c, y, T) &= c \sum_{t=1}^{T} \frac{1}{(1+y)^t} + \frac{1}{(1+y)^T} \\
&= c \left(\frac{1}{1+y} \right) \left[\frac{1 - \left(\frac{1}{1+y}\right)^T}{1 - \left(\frac{1}{1+y}\right)} \right] + \frac{1}{(1+y)^T} \\
&= \frac{c}{y} \left[1 - \frac{1}{(1+y)^T} \right] + \frac{1}{(1+y)^T} \\
&= \frac{c}{y} - \frac{c}{y} \frac{1}{(1+y)^T} + \frac{1}{(1+y)^T} \\
&= \frac{c}{y} + \frac{1}{(1+y)^T} \left(1 - \frac{c}{y} \right).
\end{aligned}
\tag{8}
$$

9

A bank investment portfolio must obey double-entry accounting requirements (see, e.g., Horngren et al. 1998; Weygandt et al. 2012, for further details). Accordingly, at AC we measure the originated income (difference between the margin impact and the impairment flow), in year i by the bonds' investment made in year j, $x_{i,j}^{(ac)}$, as

$$
\begin{aligned}
x_{i,j}^{(AC)} &= MI_{i,j}^{(AC)} - IF_{i,j}^{(AC)} = (y_j - f_i) \times w^{(AC)} - (IS_{i,j}^{(AC)} - IS_{i-1,j}^{(AC)}), \\
&\forall j \in J, \ \forall i \in I,
\end{aligned}
\tag{9}
$$

where y_j represents the 10-year Portuguese Government bond yield in year i, f_i is the funding rate in year i, $w^{(AC)}$ is the proportion of the bonds' investment allocated to AC and $IS_{i,j}^{(AC)}$ is the impairment's stock in year i, of the proportion of the bonds' investment (in year j) allocated to AC. In case of the existence of a SICR (see Section 3.1), $IS_{i,j}^{(AC)}$ is computed as

$$
\begin{aligned}
IS_{i,j}^{(AC)} &= ECL_{i,j}^{(\text{lifetime})} \times w^{(AC)}, \\
&\forall j \in J, \ \forall i \in I.
\end{aligned}
\tag{10}
$$

If a SICR did not occur, then $IS_{i,j}^{(AC)}$ is computed as

$$
\begin{aligned}
IS_{i,j}^{(AC)} &= ECL_{i,j}^{(\text{one-year})} \times w^{(AC)}, \\
&\forall j \in J, \ \forall i \in I.
\end{aligned}
\tag{11}
$$

In turn, at FV, we measure the originated income (difference between the margin impact and the impairment flow plus the gains if realized), in year i by the bonds' investment made in year j, $x_{i,j}^{(FV)}$, as

$$
\begin{aligned}
x_{i,j}^{(FV)} &= MI_{i,j}^{(FV)} - IF_{i,j}^{(FV)} + \mathbb{1}_A(x) \times [P(y_j, y_i, (j+10) - i) - 1] \times w^{(FV)} \\
&= (y_j - f_i) \times w^{(FV)} - (IS_{i,j}^{(FV)} - IS_{i-1,j}^{(FV)}) + \\
&\quad + \mathbb{1}_A(x) \times [P(y_j, y_i, (j+10) - i) - 1] \times w^{(FV)}, \\
&\forall j \in J, \ \forall i \in I.
\end{aligned}
\tag{12}
$$

where $P(\cdot, \cdot, \cdot)$ is defined in Equation (7), $IS_{i,j}^{(FV)}$ is the impairment's stock in year i, of the proportion of the bonds' investment (in year j) allocated to FV (it is computed in an analogous way as described in Equation (10) and Equation (11)), $w^{(FV)}$ is the proportion of the bonds' investment allocated to FV, and $\mathbb{1}(\cdot)$ represents the indicator function, $A = \{y_j - y_i >= \tau\}$. τ defines a threshold from which the bank investment manager decides to realize the gains or not. The careful selection of the normative level for this decision variable determines what proportion of the investment should be allocated to FV. Note that the decision to realize gains, in a given period, may lead to a sacrifice of future gains through the margin impact.

The comprehensive income, at AC, in year i by the bonds' investment made in year j, $z_{i,j}^{(AC)}$, is equal to the income, $x_{i,j}^{(AC)}$. At FV, the comprehensive income (income plus other comprehensive income), in year i by the bonds' investment made in year j, $z_{i,j}^{(FV)}$, is computed as

$$
\begin{aligned}
z_{i,j}^{(FV)} &= I_{i,j}^{(FV)} + OCI_{i,j}^{(FV)} = MI_{i,j}^{(FV)} + MV_{i,j}^{(FV)} \\
&= (y_j - f_i) \times w^{(FV)} + \\
&\quad + [P(y_j, y_i, (j+10) - i) - 1] \times w^{(FV)} \\
&\quad - [P(y_j, y_{i-1}, (j+10) - (i-1)) - 1] \times w^{(FV)}, \\
&\quad \forall j \in J, \ \forall i \in I,
\end{aligned}
\tag{13}
$$

where $OCI_{i,j}^{(FV)}$ and $MV_{i,j}^{(FV)}$ represent, respectively, the other comprehensive income and the market fluctuations, in year i by the bonds' investment made in year j.

In what follows we consider that the bank investment manager invests a fixed amount, M, each year on the 10-year Portuguese Government bonds[6]. The annual income and the annual comprehensive income are the results of different invested amounts. For example, the income in 2005 results from the investments made in 2003 and 2004. In turn, the income of 2006 results from the investments made in 2003, 2004 and 2005; and so on. The results are presented in percentage of the annual fixed amount invested (M). The bank investment manager faces each year, the tradeoff between what proportion of the investment must be allocated to AC and what proportion must be allocated to FV. The proportion of the investment allocated to AC must obey the rule that a bond acquired in year j will be in the bank investment portfolio until maturity, i.e., until the year $j + 10$. In turn, the proportion of the investment allocated to FV should follow the rule that a bond acquired in year j will be sold in year i with $j \leq i \leq j + 9$ if $y_j - y_i \geq \tau$, otherwise it will be in the bank investment portfolio until maturity, i.e., the year $j + 10$. We begin by realistically assuming that $\tau = 0.05$ (500 basis points). All the gains are reinvested in the next period at FV. We consider four different settings:

- Setting #1 - the case where the bank investment manager allocates the investment totally to AC ($w^{(AC)} = 1$ in Equation (9)) or to FV ($w^{(FV)} = 1$ in Equation (12) and Equation (13));

- Setting #2 - the case where the bank investment manager allocates 50% of the investment to AC ($w^{(AC)} = 0.5$ in Equation (9)) and 50% to FV ($w^{(FV)} = 0.5$ in Equation (12) and Equation (13));

- Setting #3 - the case where the bank investment manager allocates 25% of the investment to AC ($w^{(AC)} = 0.25$ in Equation (9)) and 75% to FV ($w^{(FV)} = 0.75$ in Equation (12) and Equation (13));

- Setting #4 - the case where the bank investment manager allocates 75% of the investment to AC ($w^{(AC)} = 0.75$ in Equation (9)) and 25% to FV ($w^{(FV)} = 0.25$ in Equation (12) and Equation (13)).

3.2.1 Historical Annual Income and Comprehensive Income

In Figure 3 and Figure 4 we report, respectively, the annual income and the annual comprehensive income, for each of the four considered settings. There is one year that stands out

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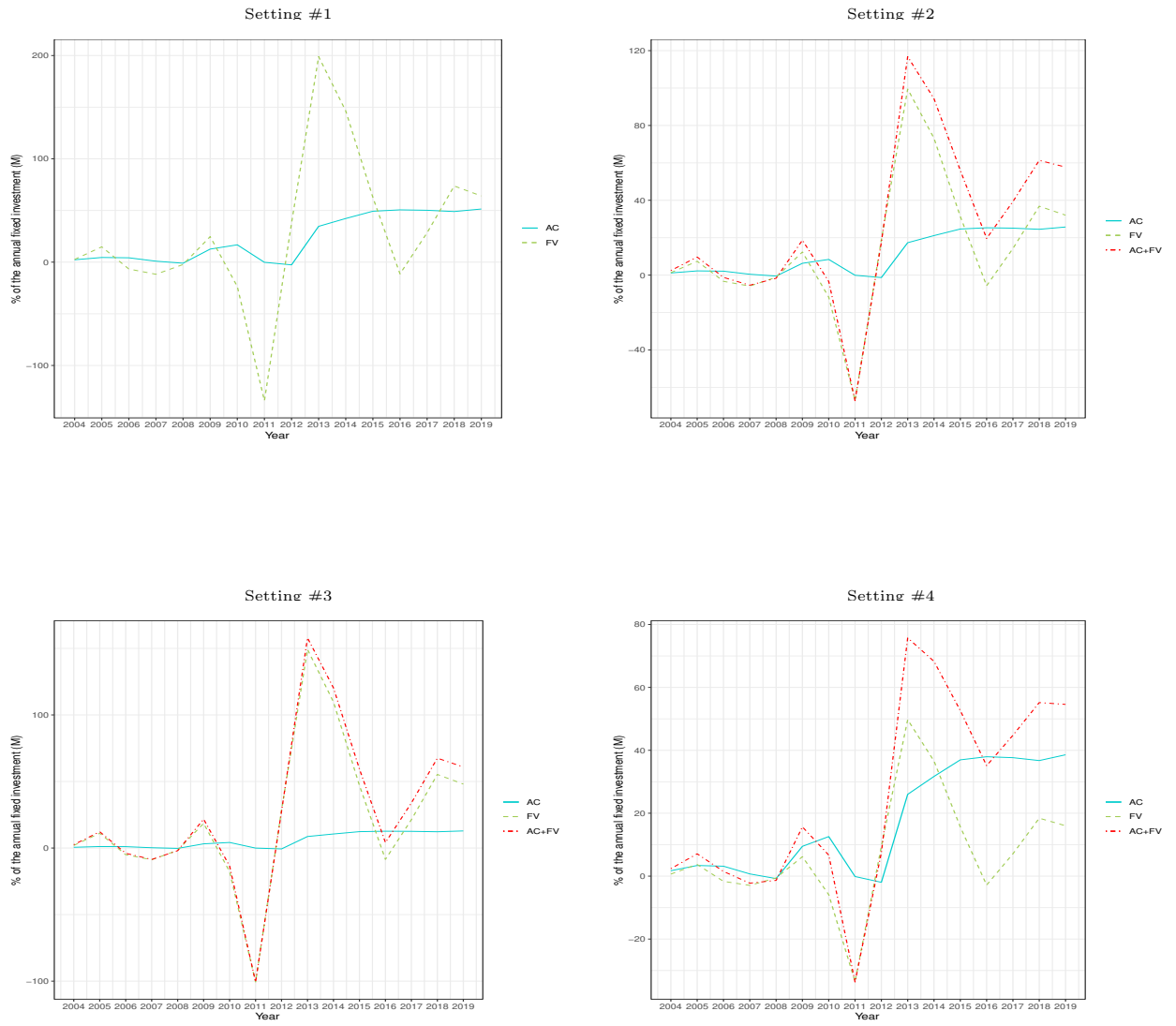[6]In our model, we assume that each investment decision is made at the end of the corresponding year, the first one made at the end of 2003 and the last one made at the end of 2018.

Figure 3: Annual Income
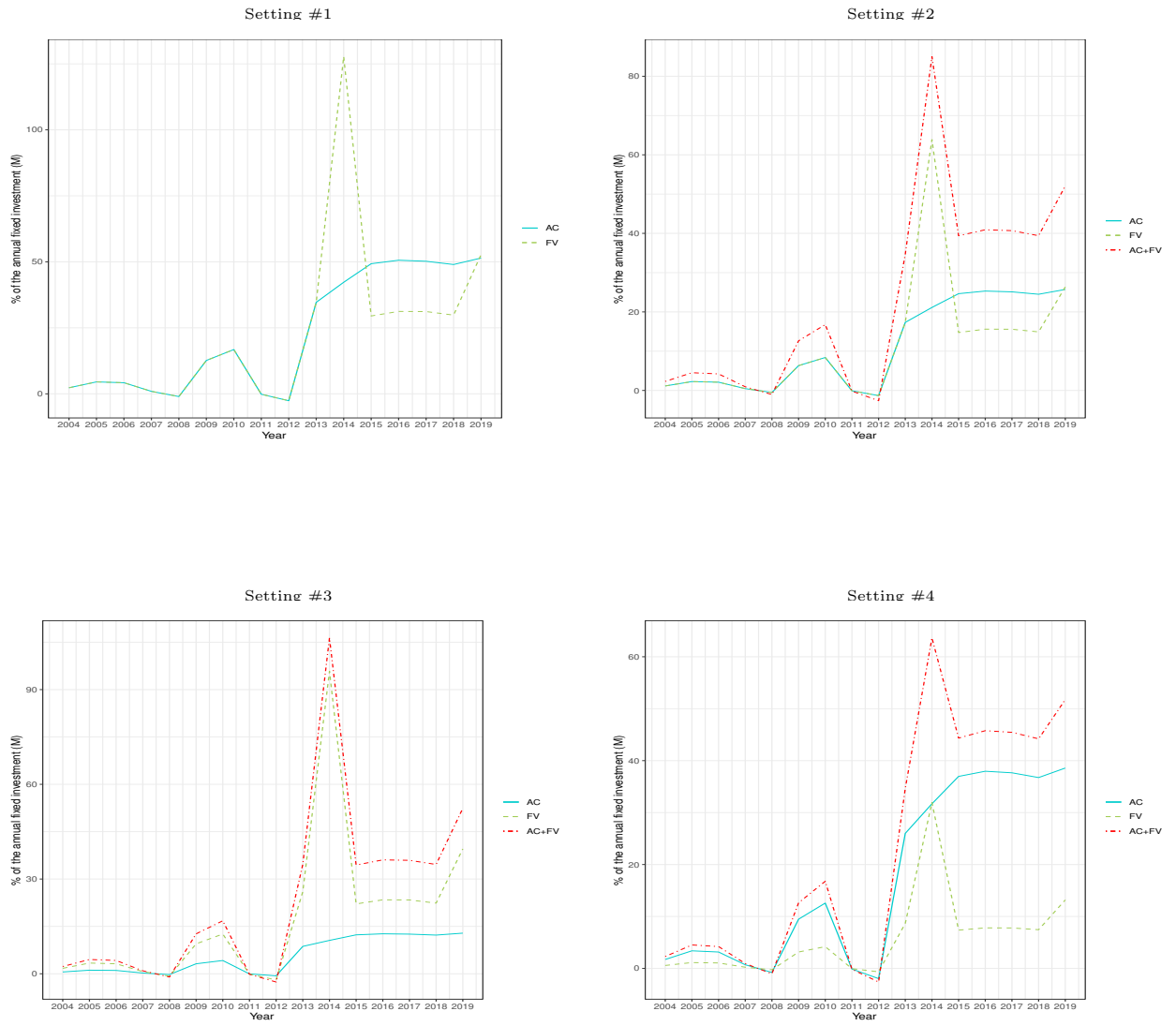
Displayed: annual income for each of the four considered settings.

Figure 4: Annual Comprehensive Income

Displayed: annual comprehensive income for each of the four considered settings.

immediately in all these figures, the 2011 year. In May 2011, the Portuguese Government signed the €78 billion bailout program (European Comission, 2011) with the European Commission (Eurogroup), the European Central Bank, and the International Monetary Fund. In this year the 10-year Portuguese Government bonds' yield climbed to 10.24% (see Figure 2) and as the result we observe a considerable drop in the income (see Figure 3) and comprehensive income (see Figure 4).

At the end of 2011, in setting #1, regardless of the allocation type (AC or FV), there is a drop of 0.16% of M (€0.16 for each €100 of the fixed amount, M, invested each year from 2003 until 2011) in the income and a drop of exactly the same magnitude in the comprehensive income if the allocation is done at AC; if the allocation is done at FV a more significant drop, equal to 134.14% of M (€134.14 for each €100 of the fixed amount, M, invested each year from 2003 until 2011), is observed in the comprehensive income.

Moreover, between 2004 and 2013, the yearly originated income (see Figure 3) is always the same independently of the allocation type. In 2014 we observe a superior income originated at FV in comparison with the income originated at AC. This results from realized gains, since the bonds acquired in 2010 and 2011 at the yields of 10.24% and 10.55%, respectively, are sold in 2014 because the defined threshold is exceeded, i.e., the bond yield of 2014 is equal to 3.75%. In 2015, despite gains realized in 2004 being reinvested at FV, we observe a decline (compared to the case where the investment is totally allocated to AC) in the income. This decline is explained by the margin drop due to the liquidation carried out in the previous year of 2014.

Still in setting #1, in terms of annual volatility, we observe that, both in terms of income and comprehensive income, if the investment is totally allocated to FV we have more volatility than if the investment is totally allocated to AC. In fact, looking to the case of income (see Figure 3), if the investment is allocated totally to FV we observe an annual income standard deviation equal to 32.39% and a standard deviation equal to 22.67% in the case that the investment is made totally in AC. In the case of comprehensive income (see Figure 4), if the investment is totally made to FV the standard deviation of the annual comprehensive income is equal to 74.35%. In turn, if the investment is allocated totally to AC this number falls to 22.67% (more than three times lower). This was somehow expected since an asset classified at FV is subject to market variations.

Regarding settings #2, #3, and #4, at the end of 2011 the drop in the income (see Figure 3) has the same magnitude as the one observed in setting #1 but has different origins. In setting #2, a drop of 0.08% of M results from the allocation to AC and the other 0.08% of M from the allocation to FV. In setting #3, we observe a drop of 0.04% of M from the allocation to AC and a drop of 0.12% of M from the allocation to FV. In setting #4, a drop of 0.12% of M results from the allocation to AC and a drop of 0.04% of M results from the allocation to FV. In terms of comprehensive income, the observed drop, at the end of 2011, has different magnitudes. In setting #2, it drops 67.15% of M: 0.08% of M from the allocation to AC and 67.07% of M from the allocation to FV. In setting #3, it drops 100.65% of M: 0.04% of M from the allocation to AC and 100.61% of M from the allocation to FV. Finally, in setting #4, the comprehensive income drops 33.66% of M: 0.12% of M from the allocation to AC and 33.54% of M from the allocation to FV.

We note that the biggest drop is observed in the case that the bank investment manager decides to allocate the investment totally to FV: in setting #1, where the allocation is all to FV, a drop of 134.14% of M is observed. The lowest drop is observed when the bank investment manager decides to allocate the investment totally to AC: in setting #1, where we have a full

allocation to AC, we observe a drop of 0.16% of M.

Summing up, the annual income and the annual comprehensive income (see Figure 3 and Figure 4, respectively) in settings #2, #3 and #4 exhibit a similar behavior as observed in setting #1 (differing only in the percentage that is allocated to AC and to FV). The higher the percentage allocated to FV the more volatile will be the annual income and the annual comprehensive income of the investment.

In order to understand the most advantageous allocation, from the point of view of the bank investment manager, in addition to observing the annual variations of the income and the comprehensive income, it is important to compute (for the defined time horizon) the annual average value of the income and the comprehensive income. These results are presented in Figure 5 and Figure 6, respectively.

On the one hand, the maximum annual average income (23.40% of M) and the maximum annual average comprehensive income (29.02% of M) are obtained in the case where the bank investment manager decides to allocate the investment totally to FV (see setting #1, allocation at FV, in Figure 5 and Figure 6, respectively). On the other hand, the minimum annual average income (22.81% of M) and the minimum annual average comprehensive income (22.81% of M) are observed in the case where the bank investment manager allocates the investment totally to AC (see setting #1, allocation to AC, in Figure 5 and Figure 6, respectively).

These numbers contrast with the settings where the investment manager decides to implement mixed allocations (settings #2, #3, and #4, see Figure 5 and Figure 6): the annual average income in settings #2, #3 and #4 is equal to 23.11% of M, 23.25% of M and 22.96% of M, respectively; in turn, the annual average comprehensive income in settings #2, #3, and #4 is equal to 25.92% of M, 27.47% of M, 24.37% of M, respectively. According to these data, we infer that the higher the allocation to FV the higher the annual average income and the annual average comprehensive income.

So, given the assumptions under which this backtest was performed, the best choice for the bank investment manager would be to allocate the investment totally to FV.

Nevertheless, these results are dependent on two very important assumptions: the investment time horizon (in the backtest case seventeen years - from 2003 to 2019) and the value assigned to the threshold (τ) that defines when gains at FV should be realized (in the backtest $\tau = 0.05$). Let us analyze the importance of these assumptions more deeply, focusing on setting #1 (this without losing generality), where the bank investment manager decides to allocate the investment totally to AC or totally to FV.

3.2.2 The Importance of the Investment Horizon

Regarding the investment time horizon, in practice, we know that the bank investment manager has incentives to choose shorter time horizons. Instead of the seventeen-year investment period, let us consider three different time horizons: 3, 5, and 10 years. Considering a rolling-window approach (for further details see, e.g., DeMiguel et al. 2009), we compute the annual average income and the annual average comprehensive income, for the cases where the investment is allocated totally to AC or totally to FV. The results are reported in tables 2 to 4, for 3-year, 5-year and 10-year time horizons, respectively. Contrasting these results with the seventeen year's investment period (see setting #1 in Figure 5 and Figure 6), we observe that the bank investment manager has a greater gain in deciding to allocate the investment totally to FV instead of to allocate the investment totally to AC. In the case of a 3-year time horizon (see Table 2) and

Figure 5: Average Results - Income

Setting #1   Setting #2, #3, and #4
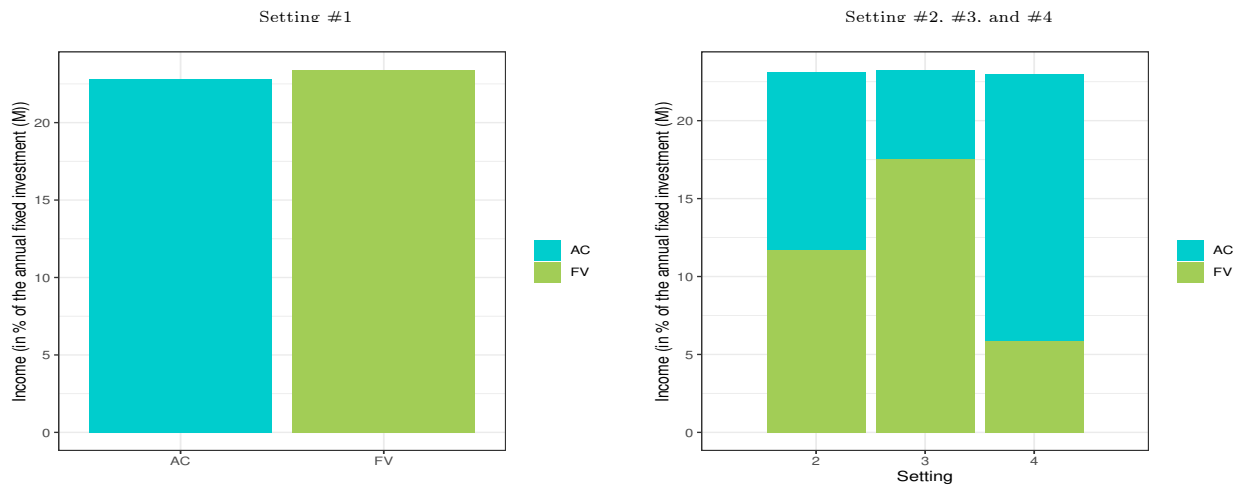
Displayed: average results, for the period 2003-2019, of the income for each of the four considered settings.

Figure 6: Average Results - Comprehensive Income

Setting #1   Setting #2, #3, and #4
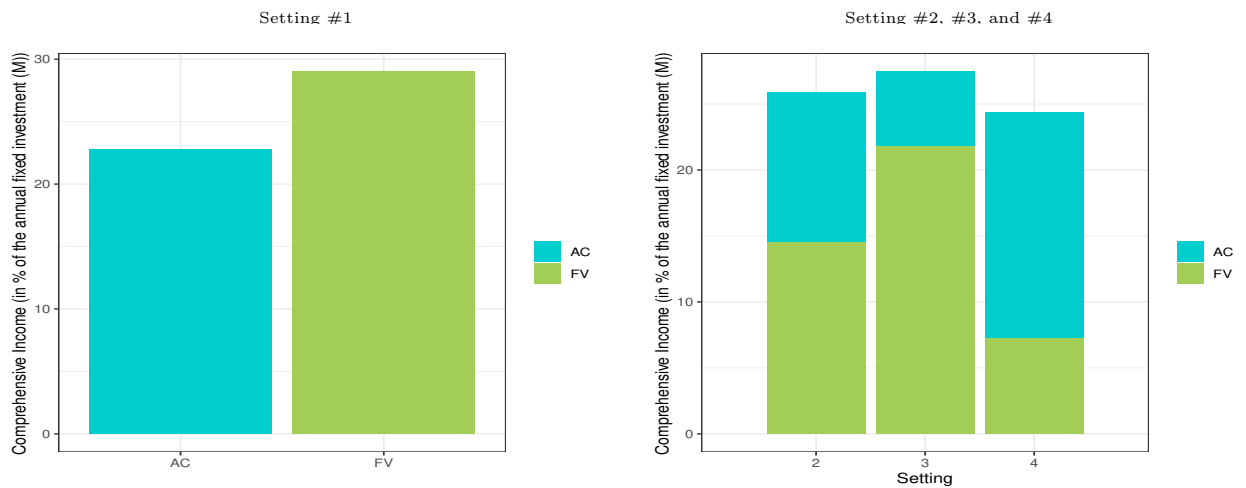
Displayed: average results, for the period 2003-2019, of the comprehensive income for the four considered settings.

considering the entire sample period (2003-2019), the annual average income/comprehensive income for the investment totally allocated to AC is equal to 7.69% / 7.69%, and for the investment totally allocated to FV is equal to 10.57% / 14.19%. For the case of a 5-year time horizon (see Table 3), the investment totally allocated to AC originates an annual average income/comprehensive income equal to 24.21% / 24.21%, and the investment totally allocated to FV originates an annual income/comprehensive income equal to 35.37% / 31.02%, considering the entire sample period (2003-2019). Finally, in the case of a 10-year time horizon (see Table 4) and considering the entire sample period (2003-2019), the annual average income/comprehensive income for the investment totally allocated to AC is equal to 67.88% / 67.88%, and for the investment totally allocated to FV is equal to 83.66% / 106.08%.

Thereby, when the bank investment manager considers, as investment time horizon, the entire sample period, 2003-2019 (see setting #1 in Figure 5 and Figure 6), she/he has an upside (from allocating the investment totally to FV in contrast with totally to AC) of 0.58% in the annual average income, and 6.21% in the annual average comprehensive income. In turn, this upside is greater in the cases that the investment time horizon is shorter. In the 3-year case (see Table 2), this upside is equal to 2.89% in the annual average income, and 6.50% in the annual average comprehensive income. For the 5-year case (see Table 3), we observe an upside of 11.17% in the annual average income, and 6.82% in annual average comprehensive income. In the 10-year case (see Table 4), the upside is equal to 15.79% in the annual average income, and 38.20% in the annual average comprehensive income. We highlight that these results are dependent on the yields' trajectory. In hindsight, we know that, in the data set under analysis, there was a great downward trajectory for the bond yields from 2012 to 2015 (see Figure 2), which allows for the realization of gains at FV.

3.2.3 The Importance of the Threshold Reinvestment Level

In Table 5 we report the annual average (for the seventeen years' sample) income and annual average comprehensive income, for different values of τ (the threshold that determines if the bank investment manager realizes gains and reinvests them, at FV, in the following period). Of course, the τ value does not have any influence on the annual income and the annual comprehensive income when the investment is totally allocated to AC. Thus, the annual average income and the annual average comprehensive income are always equal to 22.81% (see Figure 5 and Figure 6, respectively) independently of the chosen τ value. From the results presented in Table 5, we observe that for τ equal to $0.01, 0.02, 0.03, 0.04$ and 0.06, the annual average income is always lower in the case that the investment is totally allocated to FV (in contrast with the case where the investment is allocated totally to AC).

Analyzing the comprehensive income variations, only with τ equal to 100 basis points, the annual average comprehensive income at FV is lower than the annual average comprehensive income at AC. Note that with values of τ equal or greater than 700 basis points, the bank investment manager reaches the higher values of annual average comprehensive income (higher than the initial case considered of $\tau = 0.05$, see Table 5). With τ equal to 900 basis points, the bank manager obtains the best values of annual average income (25.79% of M) and annual average comprehensive income (31.95% of M). Somehow, these are expected results. The lower the τ value, the lower the possible gains a bond classified at FV can originate, which, in turn, is an incentive to allocate the bond to AC instead of allocating it to FV. An analogous reasoning can be applied to the case of higher τ values but with an upper bound (note that with τ equal

Table 2: Annual Average Income and Comprehensive Income for a 3-Year Investment Time Horizon

Investment Time Horizon	Allocation Type	Income	Comprehensive Income
2003-2005	AC	3.41%	3.41%
	FV	3.41%	8.72%
2004-2006	AC	2.37%	2.37%
	FV	2.367%	1.427%
2005-2007	AC	0.41%	0.41%
	FV	0.41%	-4.64%
2006-2008	AC	0.01%	0.01%
	FV	0.01%	-2.26%
2007-2009	AC	2.53%	2.53%
	FV	2.53%	4.45%
2008-2010	AC	3.85%	3.85%
	FV	3.85%	-3.01%
2009-2011	AC	2.38%	2.38%
	FV	2.38%	-27.22%
2010-2012	AC	5.11%	5.11%
	FV	5.11%	-8.58%
2011-2013	AC	11.23%	11.23%
	FV	11.23%	38.27%
2012-2014	AC	10.39%	10.39%
	FV	33.48%	43.68%
2013-2015	AC	5.95%	5.95%
	FV	5.95%	25.84%
2014-2016	AC	3.53%	3.53%
	FV	3.53%	3.32%
2015-2017	AC	2.92%	2.92%
	FV	2.92%	1.82%
2016-2018	AC	3.78%	3.78%
	FV	3.78%	14.33%
2017-2019	AC	3.63%	3.63%
	FV	3.63%	17.35%

This table reports the annual average income and the annual average comprehensive income (in % of the annual fixed investment, M), for both cases where the bank investment manager decides to allocate the bonds' investment totally to AC or totally to FV (setting #1). The results are reported to a 3-year investment time horizon.

Table 3: Annual Average Income and Comprehensive Income for a 5-Year Investment Time Horizon

Investment Time Horizon	Allocation Type	Income	Comprehensive Income
2003-2007	AC	2.99%	2.99%
2003-2007	FV	2.99%	-0.23%
2004-2008	AC	1.04%	1.04%
2004-2008	FV	1.04%	-2.14%
2005-2009	AC	2.05%	2.05%
2005-2009	FV	2.05%	1.59%
2006-2010	AC	4.25%	4.25%
2006-2010	FV	4.25%	-2.39%
2007-2011	AC	4.26%	4.26%
2007-2011	FV	4.26%	-23.65%
2008-2012	AC	5.30%	5.30%
2008-2012	FV	5.30%	-15.26%
2009-2013	AC	8.79%	8.79%
2009-2013	FV	8.79%	18.86%
2010-2014	AC	13.70%	13.70%
2010-2014	FV	35.07%	42.66%
2011-2015	AC	17.80%	17.80%
2011-2015	FV	34.36%	44.63%
2012-2016	AC	14.79%	14.79%
2012-2016	FV	21.52%	25.98%
2013-2017	AC	9.14%	9.14%
2013-2017	FV	9.14%	14.39%
2014-2018	AC	6.54%	6.54%
2014-2018	FV	6.54%	15.79%
2015-2019	AC	6.19%	6.19%
2015-2019	FV	6.19%	19.66%

This table reports the annual average income and the annual average comprehensive income (in % of the annual fixed investment, M), for both cases where the bank investment manager decides to allocate the bonds' investment totally to AC or totally to FV (setting #1). The results are reported to a 5-year investment time horizon.

Table 4: Annual Average Income and Comprehensive Income for a 10-Year Investment Time Horizon

Investment Time Horizon	Allocation Type	Income	Comprehensive Income
2003-2012	AC	4.17%	4.17%
	FV	4.17%	-10.94%
2004-2013	AC	6.08%	6.08%
	FV	6.08%	10.32%
2005-2014	AC	8.97%	8.97%
	FV	18.46%	25.39%
2006-2015	AC	13.35%	13.35%
	FV	20.60%	31.66%
2007-2016	AC	17.29%	17.29%
	FV	22.72%	28.83%
2008-2017	AC	20.37%	20.37%
	FV	23.67%	29.58%
2009-2018	AC	22.85%	22.85%
	FV	24.01%	34.80%
2010-2019	AC	27.59%	27.59%
	FV	29.02%	38.95%

This table reports the annual average income and the annual average comprehensive income (in % of the annual fixed investment, M), for both cases where the bank investment manager decides to allocate the bonds' investment totally to AC or totally to FV (setting #1). The results are reported to a 10-year investment time horizon.

Table 5: Annual Average Income and Comprehensive Income for Different τ Values

Threshold	Income	Comprehensive Income
$\tau = 0.01$	18.63%	19.45%
$\tau = 0.02$	21.52%	23.46%
$\tau = 0.03$	19.34%	25.06%
$\tau = 0.04$	21.62%	26.96%
$\tau = 0.05$	23.40%	29.02%
$\tau = 0.06$	22.04%	29.02%
$\tau = 0.07$	23.93%	31.33%
$\tau = 0.08$	24.50%	31.33%
$\tau = 0.09$	25.79%	31.95%
$\tau = 0.10$	22.81%	31.95%

This table reports the annual average income and the annual average comprehensive income (in % of the annual fixed investment, M), for the case where the bank investment manager decides to allocate the bonds' investment totally to FV (setting #1), for 10 different thresholds (τ) that determine when gains should or should not be realized.

to 1000 basis points the annual average income obtained with the allocation totally made to FV is the same as if the allocation is totally made to AC).

The analysis of the tradeoff between classifying a bond at AC or at FV constitutes, as we can see from the backtest's results, a crucial decision for the bank investment manager. It is therefore important to have a quantitative model that allows the bank investment manager to systematically identify efficient allocations (AC or FV) according to a risk-return target. The income originated by the investment presents itself as a good return target. As a risk target, we suggest considering the downside of the comprehensive income, which effectively impacts the bank's own funds and regulatory capital ratios. In the next section, we propose to build and analyze a model, over the space of all feasible allocations to AC or FV, in which, on the one hand, the bank investment manager intends to maximize the income and, on the other hand, she/he intends to minimize the downside of the comprehensive income.

4 A Bi-Objective Model and the Pareto Front

In this section, we propose a bi-objective model designed over the feasible space of all possible allocations (to AC or to FV), in which the bank investment manager maximizes the annual income and minimizes the annual downside of the comprehensive income. Such a framework allows the bank investment manager to directly identify efficient allocations according to two defined objectives. As the risk target, we suggest using a downside risk measure, since downside risk measures realistically only account for the negative part of the deviations from a certain predefined level (for a detailed survey on downside risk measures, see Nawrocki 1999 and Estrada 2006).

Roy (1952) was one of the earliest contributors (with the Roy's safety first principle) for further development of downside risk measures. Posteriorly, Bawa (1975) and Fishburn (1977) introduced the lower partial moments. Fishburn (1977) showed that the lower partial moments' framework allows the representation of several Von Neumann-Morgenstern utility functions (Von Neumann and Morgenstern, 1953), instead of considering that the preferences of the investor follow a particular utility function (such as a quadratic utility function). Therefore, this groundbreaking framework enables accommodating different levels of the investor's risk aversion (Nawrocki, 1999).

The framework for lower partial moments promoted further research (see, e.g., Balzer 1994 and Sortino and Price 1994) in the semivariance (also known as the lower partial measure of degree 2; see Bawa 1975 and Fishburn 1977). These studies showed the practical importance of the semivariance as a downside risk measure. In fact, already Markowitz (1959), in the late 1950s, stated that the semivariance is a much more plausible risk measure than the variance. However, for computational reasons, the semivariance is often neglected (Estrada, 2008). It is possible to overcome the endogeneity problem of the semicovariance matrix (see, e.g., Ben Salah et al. 2018) using the heuristic method described in Estrada (2008).

In this paper, as a measure of the downside of the comprehensive income, we make use of the semivariance (the lower partial measure of degree 2). Before we show how the semivariance of the annual comprehensive income can be computed, let us introduce additional notation. Let $c(w^{(AC)}, w^{(FV)}) = [c_1(w^{(AC)}, w^{(FV)}), \ldots, c_{s-1}(w^{(AC)}, w^{(FV)})]^\top$ represent the $(s-1) \times 1$ vector of the annual comprehensive incomes. Thereby each annual comprehensive income is given by

$$c_{i-1}(w^{(AC)}, w^{(FV)}) = \sum_{j \in \{1, \ldots, i-1\}} z_{i,j}^{(AC)} + z_{i,j}^{(FV)}, \; \forall i = 2, \ldots, s. \tag{14}$$

Accordingly, we can compute the semivariance of the annual comprehensive income, $SMV(c(w^{(AC)}, w^{(FV)}))$, as

$$SMV(c(w^{(AC)}, w^{(FV)})) \;\; = \;\; \frac{1}{s-1} \sum_{i \in \{1, \ldots, s-1\}} \min\left(c_i(w^{(AC)}, w^{(FV)}) - B, 0\right)^2, \tag{15}$$

where B represents a benchmark target. In this paper we establish B as the sample mean of the annual comprehensive income distribution. Therefore,

$$B = \frac{1}{s-1} \sum_{i \in \{1, \ldots, s-1\}} c_i(w^{(AC)}, w^{(FV)}). \tag{16}$$

Defining the semivariance of the annual comprehensive income as in Equation (15) leads to an endogenous definition, i.e., a change in the bonds' investment allocation ($w^{(AC)}$ and $w^{(FV)}$) changes the years when the annual comprehensive income underperforms its mean (the mean is also affected), which in turn changes the elements of the annual comprehensive income semivariance.

Defining the annual average income, $AVI(w^{(AC)}, w^{(FV)})$, as

$$AVI(w^{(AC)}, w^{(FV)}) = \frac{1}{s-1} \sum_{j \in J} \sum_{i \in I} x_{i,j}^{(AC)} + x_{i,j}^{(FV)}, \tag{17}$$

the bank investment manager can decide the efficient allocation (to AC or to FV) by finding the solution of the following bi-objective problem

$$
\begin{aligned}
&\max_{(w^{(AC)}, w^{(FV)}) \in \mathbb{R}^2} \quad AVI(w^{(AC)}, w^{(FV)}) \\
&\min_{(w^{(AC)}, w^{(FV)}) \in \mathbb{R}^2} \quad SMV(c(w^{(AC)}, w^{(FV)})) \\
&\text{subject to} \quad w^{(AC)} + w^{(FV)} = 1.
\end{aligned}
\tag{18}
$$

The solution of Problem (18) is given in the form of a Pareto front. $(w_*^{(AC)}, w_*^{(FV)}) \in \mathbb{R}^2$ belongs to the Pareto frontier if it corresponds to a nondominated point, i.e., there is no other feasible point $(w^{(AC)}, w^{(FV)}) \in \mathbb{R}^2$ such that

$$
AVI(w^{(AC)}, w^{(FV)}) > AVI(w_*^{(AC)}, w_*^{(FV)})
$$

$$
\bigwedge
\tag{19}
$$

$$
SMV(c(w^{(AC)}, w^{(FV)})) < SMV(c(w_*^{(AC)}, w_*^{(FV)})).
$$

We have no access to the derivatives of both the objective functions of Problem (18), since both functions are endogenous. Therefore, we solve Problem (18) using a derivative-free algorithm. Among the derivative-free algorithms available for multi-objective optimization, direct multisearch[7] performs the best in a robust numerical exercise (test set of more than 100 problems) reported in Custódio et al. (2011). Moreover, in terms of the purity metric (Bandyopadhyay et al., 2004), on the test set considered by Custódio et al. (2011), direct multisearch exhibited a better efficiency and robustness than the popular non-dominated sorting genetic algorithm - NSGA-II (Deb et al., 2002). Thus, we have chosen this algorithm to find the Pareto front solution of Problem (18).

Using the entire available time window (2003-2019) to estimate the inputs' parameters, the solution of Problem (18) is presented in Figure 7. Now, the bank investment manager has a way to find efficient allocations (to AC or to FV) according to her/his own risk-return preferences. Thereby, e.g., $w_{10} = [w_{10}^{(AC)}, w_{10}^{(FV)}]^\top$, where $w_{10}^{(AC)} = 0.0441$ and $w_{10}^{(FV)} = 0.9559$, is the allocation corresponding to the top[8] 10% of the SMV (see Figure 7). $w_{50} = [w_{50}^{(AC)}, w_{50}^{(FV)}]^\top$, where $w_{50}^{(AC)} = 0.2667$ and $w_{50}^{(FV)} = 0.7333$, is the allocation corresponding to the top 50% of the SMV (see Figure 7). Finally, $w_{90} = [w_{90}^{(AC)}, w_{90}^{(FV)}]^\top$, where $w_{90}^{(AC)} = 0.7062$ and $w_{90}^{(FV)} = 0.2938$, is the allocation corresponding to the top 90% of the SMV (see Figure 7). We observe that the proposed framework incorporates the intuition that the greater the allocation to FV, the greater the expected risk. In turn, the expected return also increases with the allocation to FV.

Corresponding this framework to a stochastic optimization setting, it is important to take into account the results' sensitivity to the estimation error present in both the objectives. Particularly, we know that the semivariance, being an asymmetric risk measure, is many times more sensitive to estimation error, than symmetric risk measures that make use of more information (Satchell, 2010). One possible way to mitigate this estimation error is through a

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[7] A derivative-free solver based on direct multisearch, `dms`, is publicly available under request at http://www.mat.uc.pt/dms/.

[8] Let *cdf* be the normalized cumulative distribution function of $SMV(w^{(AC)}, w^{(FV)})$. Then, the top x% of risk is defined by the allocation $(w^{(AC)}, w^{(FV)})$ such that $cdf(w^{(AC)}, w^{(FV)}) >= x\%$.

Figure 7: Pareto Front
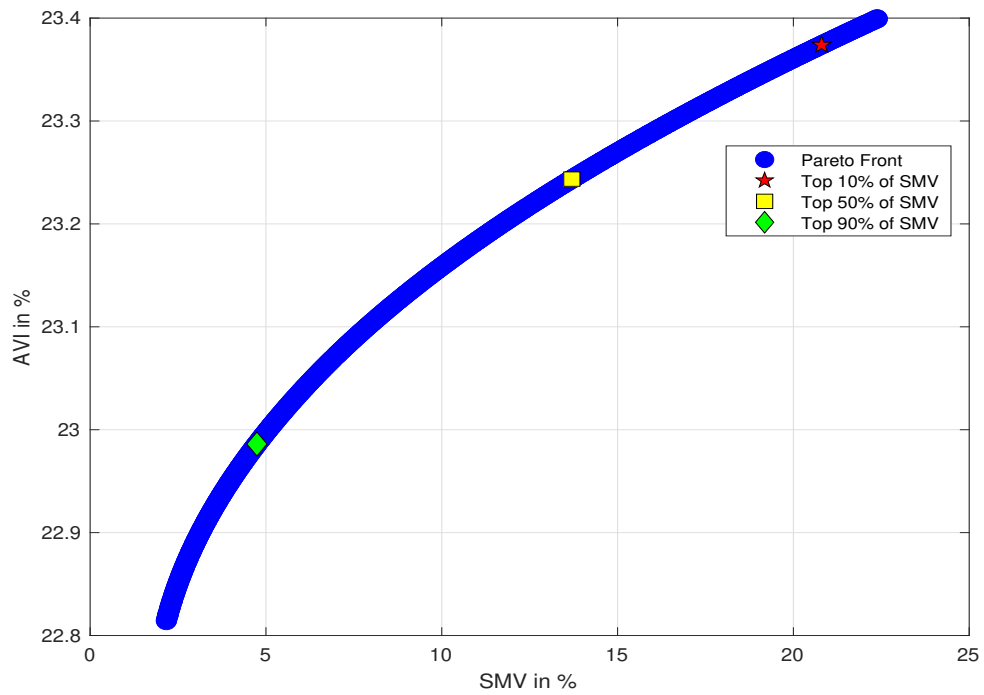
*Displayed: Pareto front, solution of Problem (18), using all the sample period (2003-2019) for the input parameters'
estimation. The threshold, τ, used for the computation of the annual income at FV (see Equation (12)), was equal to 500
basis points. In this figure are also plotted the allocations that correspond to the top 10%, 50% and 90% of risk.*

resampling methodology as proposed by Michaud and Michaud (2007). The idea is straightforward. The bank investment manager generates different scenarios for which a solution of the proposed model is found. The average of the obtained solutions corresponds to the optimal efficient solution. But how to simulate the different scenarios for the complex problem that we have in our hands? In the next section, we address this issue by presenting and exemplifying a possible approach.

5 Stochastic Simulation and Out-of-Sample Performance

Simulating future scenarios is an arduous task that mainly requires a significant amount of modeling. In this section, we will consider just a possible way to simulate future scenarios in our framework. We assume only one general premise: that the future resembles the past. Of course, this premise is quite debatable, but our aim here is just to have a systematic way of simulating scenarios instead of predicting the future. We are interested in simulating different possible "futures" that are generated according to the historical available information. On the one hand we need to simulate the yields and funding rates (incorporating the observed historical correlation between these two rates), and on the other hand we need to simulate the rating dynamics.

Regarding the simulation of the yields and funding rates, in this study, we make use of a correlated mean-reverting process. Since our aim is to simulate (not to predict) different possible future states that somehow are a mirror of the past information, we will not allow negative rates and thereby we decided to make use of a squared Bessel stochastic process (Chou and Lin, 2006). The squared Bessel stochastic process is the basis of the Cox-Ingersoll-Ross model (Cox et al., 1985) and it is well known in the financial literature as the CIR process. Given an interest rate, r_t, a CIR process can be defined by the following stochastic differential equation

$$dr_t = \alpha(\mu - r_t)dt + \sigma\sqrt{r_t}dW_t, \tag{20}$$

where α is the mean reverting speed, μ is the mean reverting level and $\{W_t, \ t \geq 0\}$ represents a standard Brownian motion. The interest rate, r_t, thus follows a diffusion process with drift $\alpha(\mu - r_t)$ (it is linear and ensures the mean reverting property) and diffusion $r_t\sigma^2$ (it is proportional to the interest rate and ensures the positiveness of r_t). If $2\alpha\mu \geq \sigma^2$ (with $\alpha, \mu, \sigma \geq 0$), the CIR process defined by Equation (20) has a marginal density that is gamma distributed (Cox et al., 1985). Following Cox et al. (1985) and Kladivko (2007), given r_t (at time t) the density of $r_{t+\Delta t}$ (at time $t + \Delta t$) is given by

$$p(r_{t+\Delta t}|r_t; \alpha, \mu, \sigma, \Delta t) = ce^{-u-v}\left(\frac{v}{u}\right)^{(q/2)} I_q(2\sqrt{uv}), \tag{21}$$

with $c = \frac{2\alpha}{\alpha^2(1-e^{-\alpha\Delta t})}$, $u = cr_t e^{-\alpha\Delta t}$, $v = cr_{t+\Delta t}$, $q = \frac{2\alpha\mu}{\sigma^2} - 1$, and $I_q(2\sqrt{uv})$ is the modified Bessel function of the first kind and order q (for further details see, e.g., Arfken and Weber 2012). The transition density given by Equation (21) was for the first time derived by Feller (1951). Given this closed-form expression for the transition density it is straightforward to estimate the parameters α, μ and σ by maximum likelihood. In fact, according to Equation (21) and assuming that there are N observations ($r_{t_n}, \ n = 1, \ldots, N$), being equally spaced with time step Δt, the likelihood function of a CIR process can be expressed as

$$L(\alpha, \mu, \sigma) = \prod_{n=1}^{N-1} p(r_{t_{n+1}} | r_{t_n}; \alpha, \mu, \sigma, \Delta t). \tag{22}$$

Then the log-likelihood function is given by

$$\log[L(\alpha, \mu, \sigma)] = \sum_{n=1}^{N-1} \log[p(r_{t_{n+1}} | r_{t_n}; \alpha, \mu, \sigma, \Delta t)]. \tag{23}$$

According to Equation (21), Equation (23) can be rewritten as

$$
\begin{aligned}
\log[L(\alpha, \mu, \sigma)] \;=\;& (N-1)\log(c) + \\
&+ \sum_{n=1}^{N-1} \left[-u_{t_n} - v_{t_{n+1}} + \frac{1}{2} q \log\left(\frac{v_{t_{n+1}}}{u_{t_n}} \right) + \log[I_q(2\sqrt{u_{t_n} v_{t_{n+1}}})] \right],
\end{aligned}
\tag{24}
$$

where $u_{t_n} = c r_{t_n} e^{-\alpha \Delta t}$ and $v_{t_{n+1}} = c r_{t_{n+1}}$. Given that the logarithm is a monotonically increasing function, maximizing the log-likelihood function (Equation (24)) is equivalent to maximizing the likelihood function (Equation (22)). Thereby, the maximum likelihood estimators, $\hat{\alpha}$, $\hat{\mu}$, and $\hat{\sigma}$, correspond to the solution of the following maximization problem

$$(\hat{\alpha}, \hat{\mu}, \hat{\sigma}) = \arg \max_{(\alpha, \mu, \sigma)} \log[L(\alpha, \mu, \sigma)]. \tag{25}$$

The numerical solution of Problem (25) can be obtained by the Nelder-Mead method (Nelder and Mead, 1965). To ensure convergence to the global optimum, typically the starting values are the estimates given by ordinary least squares (Kladivko, 2007). A MATLAB® implementation of this procedure can be found in Kladivko (2007).

Assuming that the yields and the funding rates follow

$$dy_t = \alpha^y(\mu^y - y_t)dt + \sigma^y \sqrt{y_t} dW_t^y, \tag{26}$$

and

$$df_t = \alpha^f(\mu^f - f_t)dt + \sigma^y \sqrt{f_t} dW_t^f, \tag{27}$$

respectively, we have calibrated through maximum likelihood (using as initial estimates for the optimizing procedure those obtained through ordinary least squares) Equation (26) and Equation (27). Using all the available historical information, monthly data from 2003 to 2019, the monthly parameters estimated were: $\alpha^y = 1.614$, $\alpha^f = 1.588$, $\mu_y = 0.03023$, $\mu_f = 0.01023$, $\sigma_y = 0.2735$, $\sigma_f = 0.2033$ and $E[dW_t^y dW_t^f] = \rho dt = 0.6539 dt$.

Now we can use Equation (26) and Equation (27) to simulate different future states for the yields and funding rates. As we want to take into account the historical correlation between the yields and the funding rates, instead of simulating, separately, each one of the equations (Equation (26) and Equation (27)), we consider a correlated vector-valued CIR process of the form

$$dR_t = \alpha \left[\mu - R_t\right] dt + \Sigma \sqrt{R_t} dW_t \wedge E[dW_t^y dW_t^f] = \rho dt \Longleftrightarrow$$

$$\begin{bmatrix} dy_t \\ df_t \end{bmatrix} = \begin{bmatrix} \alpha^y & 0 \\ 0 & \alpha^f \end{bmatrix} \left(\begin{bmatrix} \mu^y \\ \mu^f \end{bmatrix} - \begin{bmatrix} y_t \\ f_t \end{bmatrix} \right) dt + \tag{28}$$

$$+ \begin{bmatrix} \sigma_y \sqrt{y_t} & 0 \\ \rho \sigma_f \sqrt{f_t} & \sigma_f \sqrt{f_t} \sqrt{1 - \rho^2} \end{bmatrix} \begin{bmatrix} dV_t^y \\ dV_t^f \end{bmatrix},$$

where V_t^y and V_t^f are independent standard Brownian motions. Therefore, future yields and funding rates can be simulated, according to Equation (28), following discretization in time by a forward Euler approach (see Deelstra and Delbaen 1998, for further details). Through discretization, negative values can occur. One possible way to handle the negative values is by means of the Higham-Mao method (Higham and Mao, 2005). Assuming that $t = 0$ is the time period corresponding to the last sample observation (December 2019), Figure 8 presents one simulated path (for 10 years) of the yields and funding rates.

Figure 8: Simulated Path of the Yields and Funding Rates
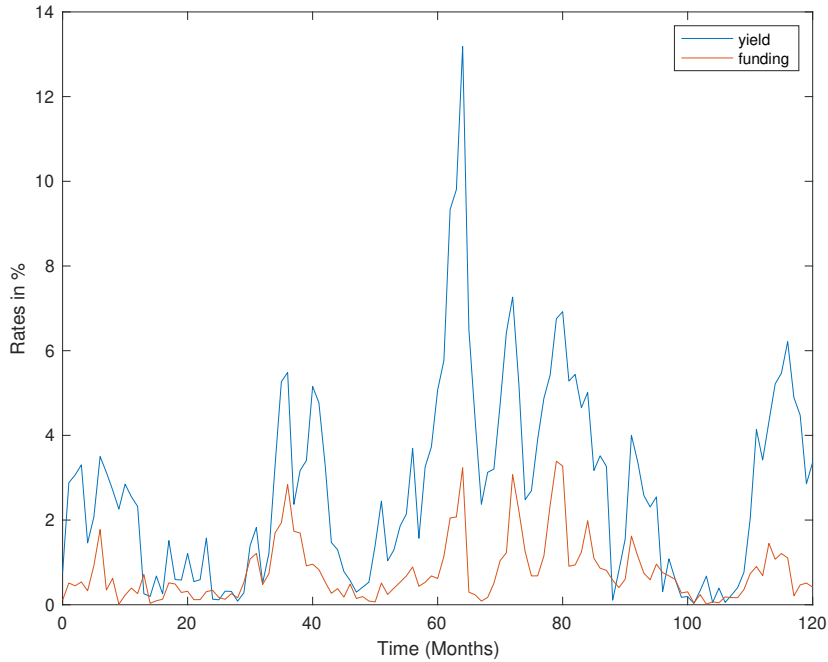
Displayed: one simulated path of the yields and funding rates. We assume that the yields and funding rates follow the dynamics described by Equation (28). The first time period, $t = 0$, corresponds to the last historical sample observation (December 2019).

In order to simulate the ratings dynamics, we constructed an ordered response model (Mckelvey and Zavoina, 1975). Ordered response models (also known as ordinal cumulative link models) to modeling the ratings dynamics have been widely employed and analyzed in the credit rating literature (see, e.g., Afonso 2003, Bissondoyal-Bheenick et al. 2006, Afonso et al.

2009 and Erdem and Yusuf 2014). As our focus is on simulation and not on prediction, in this paper we are not interested in studying or analyzing the determinants of sovereign bond ratings (as done, e.g., in Cosset and Roy 1991 and Cantor and Packer 1996). Our focus is also not the search for the best prediction methodology (as done, e.g., in Ozturk et al. 2016). Our aim is to construct a simple (but not simplistic) model that allows us to simulate the 10-year Portuguese Government bonds ratings using as a risk factor only the 10-year Portuguese Government bonds' yield.

In the gathered historical sample (monthly data from January 2003 to December 2019) we observe eight different rating levels (see Figure 9): Ba3, Ba2, Ba1, Baa3, Baa1, A3, A1, and Aa2.

Figure 9: Historical Rating Levels
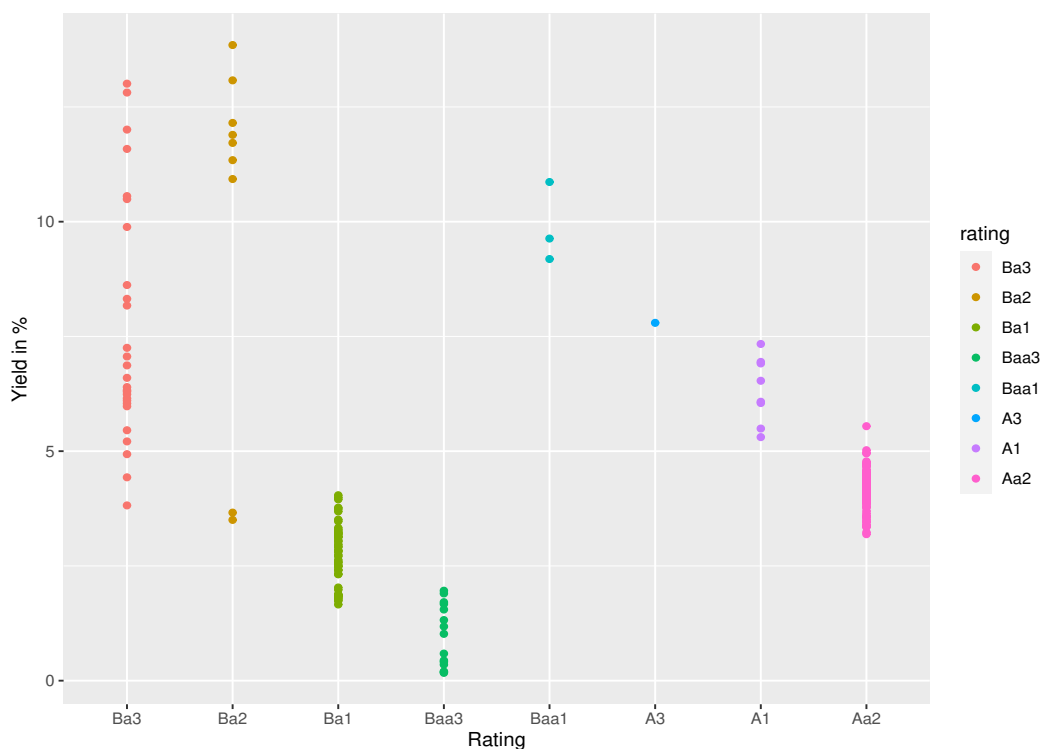
Displayed: distribution of the rating levels as function of the 10-year Portuguese Government bonds' yield, observed during the sample period (January 2003 to December 2019).

These rating levels can be represented in ordinal form as

$$j = \begin{cases} 8, & \textit{if the rating is} \quad \text{Aa2}, \\ 7, & \textit{if the rating is} \quad \text{A1}, \\ 6, & \textit{if the rating is} \quad \text{A3}, \\ 5, & \textit{if the rating is} \quad \text{Baa1}, \\ 4, & \textit{if the rating is} \quad \text{Baa3}, \\ 3, & \textit{if the rating is} \quad \text{Ba1}, \\ 2, & \textit{if the rating is} \quad \text{Ba2}, \\ 1, & \textit{if the rating is} \quad \text{Ba3}. \end{cases} \tag{29}$$

Now we can define an ordinal response model by letting r_t^* be a latent variable such that

$$r_t^* = \gamma y_t + \varepsilon_t, \quad \forall t \in \{1, \ldots, s\}, \tag{30}$$

where y_t represents the bond yield and ε_t is the stochastic disturbance term with *cdf* given by $F(\cdot)$. Accordingly, the rating level at time t, r_t, is defined by

$$r_t = j \quad \text{if} \quad c_{j-1} \leq r_t^* \leq c_j \quad \text{for} \quad j = 1, \ldots, 8, \tag{31}$$

where $-\infty \equiv c_0$, c_1, $\ldots$, c_7, $c_8 \equiv +\infty$ are the strictly ordered cutoffs determined by the model. The cumulative probabilities are given by

$$Prob(r_t \leq j) = Prob(r_t^* \leq c_j) = Prob\left(\varepsilon_t \leq c_j - \gamma y_t\right) = F(c_j - \gamma y_t). \tag{32}$$

Now assuming that ε_t follows a standard logistic distribution, we have

$$Prob(r_t \leq j) = \frac{e^{(c_j - \gamma y_t)}}{1 + e^{(c_j - \gamma y_t)}} = \frac{1}{1 + e^{-(c_j - \gamma y_t)}}. \tag{33}$$

Thereby, by choosing the logistic function as the link function, it follows that

$$\text{logit}(Prob(r_t \leq j)) = c_j - \gamma y_t, \tag{34}$$

where $\text{logit}(\cdot)$ is no more than the odds' logarithm, i.e.,

$$\text{logit}(p) = \log\left(\frac{p}{1 - p}\right). \tag{35}$$

The described model can be estimated by maximum likelihood. Typically, ordinal response models can be estimated using a regularized Newton-Raphson algorithm with step-halving (line search) and analytical expressions for the gradient and Hessian of the negative log-likelihood function (Christensen, 2018). A detailed implementation in `R` through the `ordinal` package can be found in Christensen (2018). We present the estimation results of the ordinal regression in Table 6.

The results presented in Table 6 can be interpreted as follows. Given a 1% increase in the bond yield, the likelihood of 8 versus 1-7 (or analogously, 7 versus 1-6, or 6 versus 1-5, or 5 versus 1-4, or 4 versus 1-3 or 3 versus 1-2 or 2 versus 1) decreases approximately 20%, this in the log odds scale. Thus, we have a methodology to simulate the funding rates, the yields, and the corresponding rating levels. Proceeding accordingly, we started by simulating 100 paths (according to the vector-valued CIR process defined by Equation (28)). As such, we have computed the rating levels according to the ordinal response model defined by Equations (30)

Table 6: Estimation Results of the Ordinal Regression

Coefficient	Estimate	
c_1	-2.888^{***}	(0.3366)
c_2	-2.487^{***}	(0.3068)
c_3	-1.103^{***}	(0.2432)
c_4	-0.8158^{***}	(0.2429)
c_5	-0.7603^{***}	(0.2434)
c_6	-0.7414^{***}	(0.2435)
c_7	-0.5867^{***}	(0.2441)
γ	-20.04^{***}	(4.717)
Log-Likelihood	-301.22	
Observations	204	

*This table reports the estimation parameters of the ordinal response model defined by Equation (30), Equation (31), Equation (32) and Equation (33). The corresponding standard error is presented in parenthesis. *** means that the result is statistically significant at the 1% level.*

to (33), inclusive. In Figure 10 we report the distribution of the simulated rating levels as a function of the simulated 10-year Portuguese Government bonds' yield.

Since our interest lies in analyzing the impact of annual decisions (on annual income and annual comprehensive income), and similarly to what we did in the previous sections, we have annualized the simulated 100 paths' data. The annual simulated bond yield and funding rate correspond to an annual average. Following the correspondence defined by Equation (29), the annual simulated rating level also corresponds to an annual average (rounded value). An annualized simulated path (yield, funding, and rating level) is presented in Figure 11.

Following the framework proposed in Section 4, we exemplify now how the bank investment manager can implement a methodology that intends to mitigate the estimation error, based on the work of Michaud and Michaud (2007), for determining optimal efficient allocations (AC or FV) to the 10-year Government bonds.

Firstly, for each one of the 100 simulated paths (yield, funding and rating level), the efficient Pareto front (solution of Problem (18)) is computed. Secondly, on each obtained Pareto front an efficient allocation (AC and FV) is chosen according to the defined risk-return objectives. This choice can be made based on any decision rule adopted by the bank investment manager. In this paper we decided to choose three different efficient allocations (also defined in the previous section): the allocations corresponding to the top 10%, 50% and 90% of risk, $w_{10}^{*(AC)}$, $w_{50}^{*(AC)}$ and $w_{90}^{*(AC)}$, respectively[9]. Thus, for each of the 100 generated paths, we have three efficient allocations: $w_{i,10}^{(AC)}$, $w_{i,50}^{(AC)}$ and $w_{i,90}^{(AC)}$, with $i = 1, \ldots, 100$. Finally, the corresponding optimal efficient allocations are given by the average allocations. That is, the optimal efficient allocations

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[9]Note that an efficient allocation (AC and FV) is completely determined solely by the proportion of the investment that is allocated at AC.

Figure 10: Simulated Rating Levels
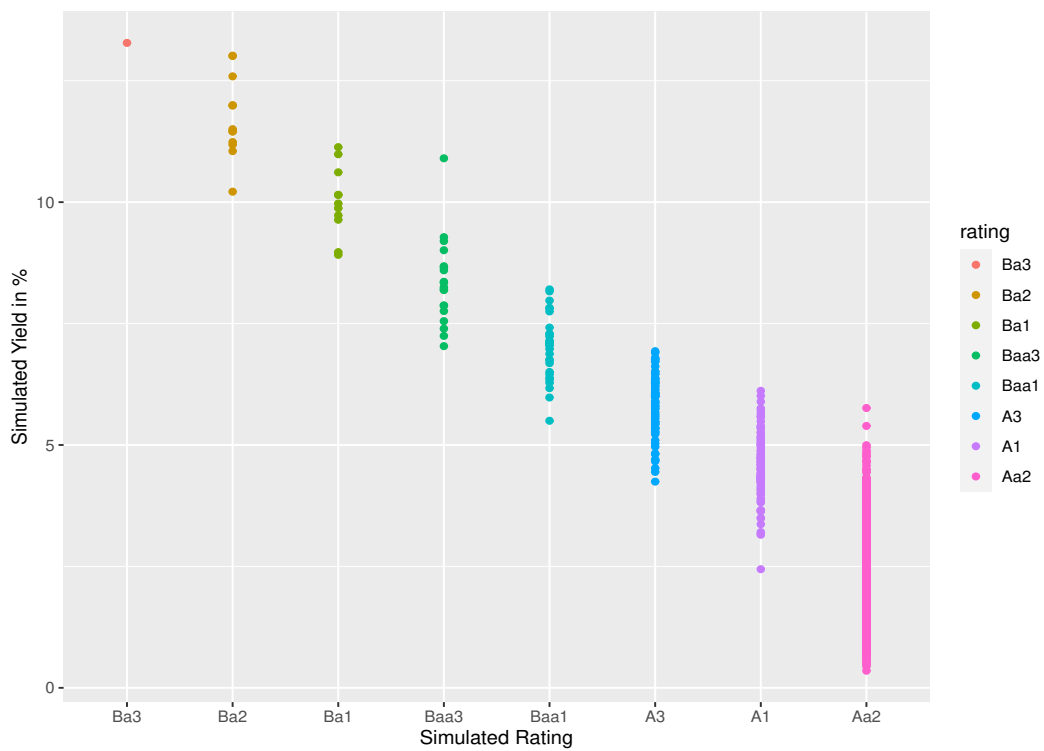
Displayed: distribution of the simulated rating levels as a function of the simulated 10-year Portuguese Government bonds' yield. These results are reported to a simulation of 100 yields paths.

Figure 11: Annualized Simulated Path
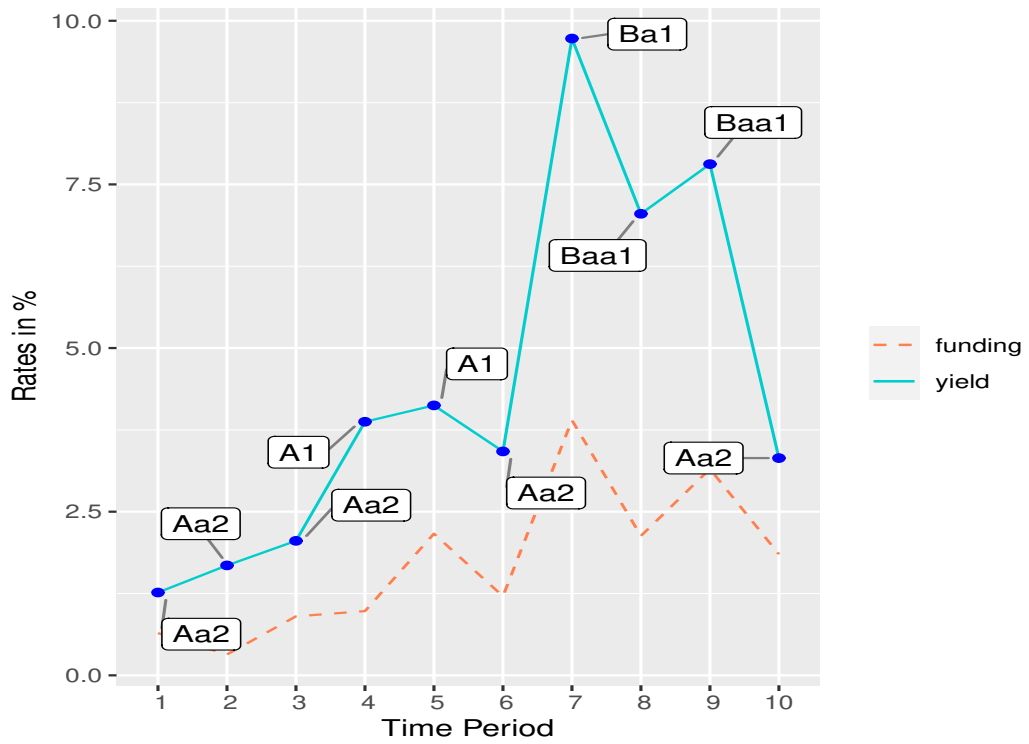
Displayed: one of the 100 simulated paths. The yields and funding rates are simulated according to the vector-valued CIR process defined by Equation (28) and the rating levels according to the ordinal response model defined by Equation (30), Equation (31), Equation (32), and Equation (33).

corresponding to the top 10%, top 50%, and top 90% of risk are given, respectively, by $w_{10}^{*(AC)} = \frac{1}{100}\sum_{i=1}^{100} w_{i,10}^{(AC)}$, $w_{50}^{*(AC)} = \frac{1}{100}\sum_{i=1}^{100} w_{i,50}^{(AC)}$, $w_{90}^{*(AC)} = \frac{1}{100}\sum_{i=1}^{100} w_{i,90}^{(AC)}$. The main distributional values of these efficient allocations are reported in tables 7 to 9.

Table 7: Efficient Allocation — Top 10% of Risk

	min value	*max* value	*average* value ($w_{10}^{*(AC)}$)
$w_{i,10}^{(AC)}$	0.0348	1	0.6293
AVI	-0.005%	39.43%	10.54%
SMV	0.04%	84.75%	12.34%

This table reports the main distributional values of the efficient allocation corresponding to the top 10% of risk, along the computed 100 Pareto fronts (one for each simulated path). We report the minimum, maximum and average value of the allocation to AC and the risk-return metrics (SMV-AVI).

Table 8: Efficient Allocation — Top 50% of Risk

	min value	*max* value	*average* value ($w_{50}^{*(AC)}$)
$w_{i,50}^{(AC)}$	0.2086	1	0.6999
AVI	-0.005%	37.17%	10.23%
SMV	0.04%	57.18%	8.33%

This table reports the main distributional values of the efficient allocation corresponding to the top 50% of risk, along the computed 100 Pareto fronts (one for each simulated path). We report the minimum, maximum and average value of the allocation to AC and the risk-return metrics (SMV-AVI).

Table 9: Efficient Allocation — Top 90% of Risk

	min value	*max* value	*average* value ($w_{90}^{*(AC)}$)
$w_{i,90}^{(AC)}$	0.5450	1	0.8425
AVI	-0.005%	32.70%	9.62%
SMV	0.04%	19.38%	2.84%

This table reports the main distributional values of the efficient allocation corresponding to the top 90% of risk, along the computed 100 Pareto fronts (one for each simulated path). We report the minimum, maximum and average value of the allocation to AC and the risk-return metrics (SMV-AVI).

In Table 7 we report the results for the efficient allocation corresponding to the top 10% of risk, i.e., the efficient allocation, corresponding to the top 10% of the SMV, chosen on each of the 100 Pareto fronts (one for each simulated path). We observe that the minimum allocation

to AC is equal to 0.0348 (which means that the maximum allocation to FV is equal to 0.9652) and the maximum allocation to AC is equal to 1 (the minimum allocation to FV is equal to 0). We also report the minimum values of -0.005% / 0.04% in the AVI/SMV space. In turn, the maximum values in the AVI/SMV space are given by 39.43% / 84.75%. The average allocation that we were looking for is equal to 0.6293 to AC (0.3707 to FV), which corresponds to a 10.54% / 12.34% value in the AVI/SMV space.

Analogously, in Table 8 and Table 9 we find the results for the efficient allocations corresponding to the top 50% and top 90% of risk, respectively. The average efficient allocation corresponding to the top 50% of risk is equal to 0.6999 to AC (0.3001 to FV), which attains a 10.23% / 8.33% value in the AVI/SMV bi-dimensional space. In turn, the average efficient allocation corresponding to the top 90% of risk is equal to 0.8425 to AC (0.1575 to FV), which attains a 9.62% / 2.84% value in the AVI/SMV bi-dimensional space.

In the remainder of this section, we provide some insight about the out-of-sample performance of the efficient allocations according to the bi-objective model (see Problem (18)). Thus, we use each of the 100 simulated paths as an out-of-sample path. Given the in-sample (2003-2019 is the in-sample time window) efficient allocations computed in the previous section ($w_{10} = [0.0441, 0.9559]^\top$, $w_{50} = [0.2667, 0.7333]^\top$ and $w_{90}[0.7062, 0.2938]^\top$), we assess the performance of these efficient allocations in each of the out-of-sample paths. It is interesting to contrast these results with other three different allocations: the allocation where the bank investment manager decides to allocate the investment totally to AC ($w_{AC} = [1, 0]^\top$), totally to FV ($w_{FV} = [0, 1]^\top$) and 50%/50% of the investment distributed equally between AC and FV ($w_{EW} = [0.5, 0.5]^\top$). The results are presented in Table 10.

Table 10: Out-of-Sample Performance

Allocation Type	AVI			SMV		
	min	*max*	*average*	*min*	*max*	*average*
w_{10}	-0.005%	39.33%	10.52%	1.38%	83.34%	22.35%
w_{50}	-0.005%	36.58%	10.15%	0.90%	50.07%	13.54%
w_{90}	-0.005%	31.15%	9.43%	0.24%	9.26%	2.74%
w_{AC}	-0.005%	29.26%	8.95%	0.04%	1.74%	0.31%
w_{FV}	-0.005%	39.88%	10.59%	1.50%	90.93%	24.36%
w_{EW}	-0.005%	33.70%	9.77%	0.54%	24.29%	6.72%

This table reports the performance, on the 100 simulated paths (corresponding to a 10-year out-of-sample period), of six different allocations: three in-sample efficient allocations (w_{10}, w_{50} and w_{90}) computed on the Pareto front solution of Problem (18), the allocation where the bank investment manager decides to allocate the investment 100% to AC (w_{AC}), 100% to FV (w_{FV}) and 50%/50% of the investment distributed equally between AC and FV (w_{EW}).

Based on the results presented in Table 10, the efficient allocations computed according to the suggested model exhibit a robust out-of-sample performance. In fact, the considered in-sample efficient allocations are very consistent in both objectives (AVI and SMV). Note that w_{10}, w_{50} and w_{90} represent the *ex-ante* allocations corresponding to the top 10%, 50% and

90% of the risk, respectively (see Figure 7) and thereby w_{10} leads to the *ex-ante* highest values of AVI and SMV, followed by the w_{50} allocation and finally the w_{90} allocation. This pattern remains untouchable *ex-post* (see Table 10). Moreover, the w_{10} and w_{50} allocations outperform the w_{AC} and w_{EW} allocations in terms of *ex-post* average annual income (for the 10-year out-of-sample period). This result is even more noteworthy, taking into account two aspects. First, it is achieved without using any explicit methodology to deal with the parameter estimation error (as the one presented in the beginning of this section). Second, it is well known that an equally-weighted allocation (w_{EW}) is a hard-to-beat (in terms of out-of-sample performance) benchmark (for additional detail, reference is made to DeMiguel et al. 2009). Lastly, even if we were to consider an *ex-post* return/risk type measure (in this case AVI/SMV) the w_{90} allocation outperforms the equally-weighted allocation.

From the results presented in Table 10 it also stands out that the best *ex-post* performance in terms of average annual income (10.59%) is attained by the case where the bank investment manager decides to allocate the investment totally to FV (w_{FV}). Nevertheless, this allocation also corresponds to the highest *ex-post* risk (the annual semivariance of the comprehensive income equals 24.36%).

6   Conclusions and Future Research

IFRS 9 intends to harmonize accounting with risk management. From the perspective of a banking investment portfolio construction, this is not an easy task due to the requirements of using both historical and forward-looking information. The bank investment manager faces the critical decision on the allocation of securities to fair value and amortized cost. We have addressed this important issue.

Since IFRS 9 does not give an explicit definition for the impairment model, in this paper we have suggested a banking impairment model in order to compute a bonds' expected credit loss. We assume that the marginal probability of default is given by a Poisson process. From the suggested impairment model results an expected credit loss that increases with the loss given default, as with the bond's maturity. In the implementation of the impairment model, we consider the existence of a significant increase in credit risk if a bond has a rating level classified as "speculative grade" or if a bond has had a downgrade of at least 3 rating categories since origination.

A backtest on the 10-year Portuguese Government bonds was performed for the period between 2003 and 2019. The backtest was constructed using only three inputs: yields, funding rates, and the corresponding rating levels. The results suggest that the higher the proportion of the investment allocated to fair value, the higher the variability of the annual income and the annual comprehensive income. Moreover, for the full sample period (2003-2019) if the bank investment manager decides to allocate the investment totally to fair value, she/he obtains a higher value of annual average income and annual average comprehensive income than if she/he decides to allocate the investment totally to amortized cost. Nevertheless, we have shown that these results are sensitive to the choice of the threshold from which the bank investment decides to realize gains at fair value. We have also shown that these results rely on the investment time horizon which in turn can affect the bank investment manager decisions.

Based on risk-return objectives, a bi-objective optimization model was proposed to allow the bank investment manager to directly find efficient allocations, i.e., which proportion of the

investment should be allocated to amortized cost and which proportion should be allocated to fair value. As a return measure, we have defined the average annual income. In turn, as a risk measure, we have defined the semivariance of the annual comprehensive income. Our proposed model generates a Pareto front. The solution of the defined non-smooth bi-objective problem, using the data related to the 10-year Portuguese Government bonds for the period from January 2003 to December 2019, was computed by means of a derivative-free algorithm based on direct multisearch. Given the significant downgrades as a result of the 2011-2014 financial crisis, the Portuguese Bond yields constitute a very unique data set.

We have suggested and implemented a methodology to mitigate the bi-objective optimization model estimation error. Based on the resampling methodology proposed in Michaud and Michaud (2007), we suggested an idealization of different possible future paths that can be used to compute different Pareto fronts. The optimal solution corresponds to the average efficient allocations found in each of the Pareto fronts. As inputs of the bi-objective model, we have the yield, funding, and rating levels, so we needed to simulate these variables. The yields and funding rates were simulated by a correlated vector-valued Cox-Ingersoll-Ross process. In turn, the rating levels were simulated using an ordinal response model. We have exemplified this approach for a 10-year period. Finally, we have also used the simulated paths to perform an out-of-sample analysis.

The results suggest that the efficient allocations given by the proposed approach are robust and very competitive in terms of both defined optimization objectives (the average annual income and the semivariance of the annual comprehensive income). Overall, the performed quantitative analysis seems to suggest that, for the long run, the higher the proportion of the investment allocated to fair value the higher the average annual income achieved but also the higher the semivariance of the annual comprehensive income. In this paper, we have looked to the bank investment portfolio in terms of recent accounting rules, namely under the IFRS 9 framework. We could observe the several complexities that arise when we try to develop quantitative analyses that incorporate accounting rules. However, we are convinced that the bank investment manager needs to have this type of analysis at her/his disposal, to be able to make more informed decisions. Therefore, as future work, we want to extend this analysis to other asset classes and to the incorporation of recent regulatory rules inherent to the Basel III framework.

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