Inflation, Complexity and Endogenous Growth*

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Abstract

In this article, we argue that inflation increases complexity pertaining to knowledge production (or R&D). Then, we expand a recently developed complexity index based on entropy to include the effect of inflation. As a result of this new mechanism in an endogenous growth model, inflation is no longer *superneutral*. In the model, inflation can decrease economic growth in a nonlinear way, a sudden upward shock on inflation can severely hurt economic growth and an inflation cut can be responsible for a take-off. These effects are illustrated quantitatively.

Keywords: inflation, endogenous economic growth, complexity effects, entropy.

JEL Classification: O10, O30, O40, E22

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1 Introduction

This paper studies the effect of inflation on economic growth through the complexity-in-R&D channel. This mechanism is suggested by the relationship between inflation and complexity in R&D activities that is apparent in the data. This is a new mechanism in the literature, which complements other well-known mechanisms that are able to break the superneutrality of money, such as the money-in-utility channel, the liquidity/pecuniary-transaction-costs channel (e.g., Feenstra, 1986), and the channel of inflation uncertainty under irreversible investment (e.g., Pindyck, 1991).

Relying on the theoretical measure of complexity (complexity index) developed by Sequeira *et al.* (2018), and which connects complexity with the measure of technological varieties in the economy, we specify the theoretical relationship between complexity and inflation. Then, we incorporate this theoretical complexity index in an otherwise standard endogenous growth model of expanding technological varieties (i.e., the quantity of knowledge) and calibrate it in consistency with empirically suggested values of the complexity index.

The complexity effect developed in Sequeira *et al.* (2018) is *only* dependent on the level of knowledge, i.e., the higher the stock of knowledge accumulated by the economy, the more complexity hinders the production of new ideas. Here, in this article, we additionally argue that inflation is also a determinant of the complexity effect in knowledge production. In fact, in low-inflation economies, R&D activities are easily performed since prices of inputs are well-known and expectations of future input and prototype prices are easy to anticipate. However, in high-inflation countries, R&D activities have additional costs linked to tighter planning and studying different price scenarios both for the future potential products that arise from the knowledge production process and also for the respective inputs in the production phase of the projects. In face of those inflation-caused costs, some of the projects may be abandoned due to higher returns of shorter term alternatives.[1] This is the main theoretical contribution of this article: to highlight a new transmission channel of inflation through its effect on the complexity that affects R&D.

In our model, the impact of inflation on economic growth through the complexity channel is negative, but it also depends negatively on the level of inflation for the selected calibration of the complexity index. That is, the effect of inflation on economic growth is weaker as the inflation rate rises, because the (positive) impact of inflation on complexity is also weaker. Empirically, the negative long-run relationship between inflation and growth is emphasized by, e.g., Evers *et al.* (2009), Chu and Lai (2013), Chu *et al.* (2015, 2019). In particular, in

[1] Micro-evidence suggesting that firms behavior – e.g. cash holdings – is affected by uncertainty, which can be caused by inflationary processes avoidance, is found in Ramirez and Tadesse (2009).

describing the inflation-TFP-growth nexus, Evers *et al.* (2009) support that the causality channel runs from inflation to TFP growth. The nonlinearity of the relationship between inflation and growth is in line with the empirical evidence reported in, e.g., Burdekin *et al.* (2004) and Gillman *et al.* (2004).

Furthermore, given the selected calibration of the complexity index, the (negative) effect of inflation on economic growth depends negatively on the stock of knowledge, because the higher the latter the weaker the (positive) impact of inflation on complexity. The effect of inflation on growth also depends negatively on the size of the market measured by total labor force. These nonlinear effects of inflation on growth, which depend on the technological level as well as on the dimension of the market, are particularly important and novel to the literature. The estimation of the complexity index suggests that (at least) part of the modern innovations have a stabilizing role in the complexity of the economies, as the complexity index levels off despite the continuous increase in the measure of technological varieties. This can be interpreted as reflecting a relatively high level of complementarity of ideas in the modern knowledge system (Sequeira *et al.*, 2018). Then, the fact that the impact of inflation on complexity is higher when the measure of technological varieties is smaller suggests that the economies that benefit less from that complementarity (due to the small number of technological varieties) are more exposed to the detrimental effects of inflation on growth through the complexity channel. In particular, relatively high inflation in smaller and less technologically developed economies can effectively prevent these economies from moving to a sustained growth regime, or a sudden upward shock on inflation can severely hurt growth and move the economy to stagnation. In contrast, an inflation cut may be responsible for a take-off.

On the other hand, by considering that both in more technologically developed countries and in high inflation countries the (negative) effect of inflation on technological growth is attenuated, we show that the model is able to predict a U-shaped behavior of that effect across countries featuring different levels of inflation and at different development stages. This prediction follows from the observation that highly developed countries are also usually those with lower inflation and is in line with recent empirical findings by, e.g., López-Villavicencio and Mignon (2011).

The paper is organized as follows. Section 2 presents the knowledge production function, and proposes a theoretical measure of complexity (complexity index) that includes the effect of inflation. Section 3 presents the full model and the theoretical relationship between inflation and economic growth. Section 4 uses quantitative exercises to illustrate the results. Finally, Section 5 lays out the concluding remarks.

2 The knowledge production function, the complexity effect and inflation

2.1 The knowledge production function

Following Dinopoulos and Thompson (1999) and Barro and Sala-i-Martin (2004), among others, the knowledge production function features knowledge spillovers and also complexity costs associated with the scale of the market. It can be written as follows:

$$\Delta A_{t+1} = (A_{t+1} - A_t) = \delta \cdot A_t \cdot \frac{1}{L_t} \cdot L_t^{1-\chi(\cdot)} \cdot L_t^A \Leftrightarrow$$

$$\Leftrightarrow A_{t+1} = \delta \cdot A_t \cdot \frac{L_t^A}{L_t^{\chi(\cdot)}} + A_t \equiv f(A_t, L_t^A, L_t, \cdot), \tag{1}$$

where L_t^A is the aggregate amount of labor allocated to R&D activities, L_t is a scale variable (measure of market dimension) proportional to the size of total labor force in the economy, $\chi(\cdot)$ is a time-varying endogenous complexity index, which controls for the relationship between the scale of the economy and the (net) complexity costs on R&D and which depends on other variables in the model that will be discussed later; and δ is the productivity in R&D. The upper row of equation (1) decomposes the component of the knowledge production function that is external to R&D firms as follows: A_t denotes the knowledge spillovers (e.g., Romer, 1990), $1/L_t$ captures the market-scale complexity costs (as considered by e.g., Dinopoulos and Thompson, 1999), and $L_t^{1-\chi(\cdot)}$ captures the human spillovers. The latter arise because the productivity of the labor input in R&D firms benefits from the interaction with the overall human factor in the economy (as in, e.g., Lucas, 1988); but these benefits are curtailed as the complexity of the economy (controlled by $\chi(\cdot)$) also increases, as this implies an increasing diversity of human activities and thus of the 'technological distance' between them (e.g., Peretto and Smulders, 2002).[2] Thus, in this sense, $1/L_t^{\chi(\cdot)}$, in the lower row of (1), denotes the *net* market-scale complexity costs. In anticipation of the channel that will drive the non-neutrality of money in this model, we notice that inflation will increase the net market-complexity costs faced by agents that engage in R&D activities.

In the next subsection, we will present a new theoretical measure of complexity (complexity index) that includes the effect of inflation.

[2] This specification allows us to nest existing specifications in the literature as special cases: if $\chi = 0$, we recover the knowledge production function in Romer (1990) – no net complexity effects, full scale effects on growth; if $\chi = 1$, we get the function in, e.g., Dinopoulos and Thompson (1999) – full net complexity effects, no scale effects on growth. This is a crucially different mechanism from the duplication effect that is present in the semi-endogenous literature (e.g., Jones, 1995).

2.2 The theoretical complexity index with inflation

In this section, we expand the complexity index proposed by Sequeira *et al.* (2018), based on a generalized entropy index (Patil and Taillie, 1982; Tsallis, 1988), to include the effect of inflation highlighted above. Thus, the complexity index is as follows:

$$\chi(A_t, \mu_t) = max \left\{ 0, \begin{cases} b\frac{1-[A_t(1+\mu_t)^z]^{1-q}}{q-1} & , q \neq 1 \\ b\left(\ln(A_t) + z\ln(1+\mu_t)\right) & , q = 1 \end{cases} \right\}, \tag{2}$$

with b and z positive constants. Thus, b can be regarded as a scale-shifter parameter (it shifts units of A into units of the complexity index), whereas q is an elasticity parameter that maps relative changes in A into relative changes in the complexity index. Finally, $z > 0$ governs the effect of inflation on complexity in R&D.

The complexity index $\chi(A_t, \mu_t)$ arises as a positive and concave function of the technological level, A_t, as well as of the inflation rate, μ_t. We now extend some of the results obtained in Sequeira *et al.* (2018) that directly apply to our setup. The following results highlight that *(i)* there is a specific set of values of parameter q in equation (2) for which $\chi(A_t, \mu_t)$ converges to a constant, and *(ii)* there is a specific set of values of parameters q and b in equation (2) for which scale effects on growth vanish asymptotically.

Result 1. *With* $q > 1$, *then* $\lim_{A_t \to +\infty} \chi(A_t, \mu_t) = \lim_{\mu_t \to +\infty} \chi(A_t, \mu_t) = \frac{b}{q-1}$; *thus for* $b = q - 1$, $\chi(A_t, \mu_t)$ *converges to 1, as A and/or* μ *goes to infinity (with* $A_t > 0$*). With* $q \leq 1$, *then* $\chi(A_t, \mu_t)$ *goes to* $+\infty$, *as A and/or* μ *goes to infinity (with* $A_t > 0$*).*

Result 2. *With* $q > 1$, *then there is endogenous growth: (i) with positive scale effects if* $b < q - 1$; *(ii) with no scale effects if* $b = q - 1$; *(iii) with negative scale effects if* $b > q - 1$. *With* $q \leqslant 1$, *technological growth vanishes asymptotically. All in all, the degree of scale effects decreases with technological progress, since* $\chi(A_t, \mu_t)$ *increases in* A_t.

Results 1 and 2 state the asymptotic properties of the knowledge production function with respect to A and μ. According to these results, the operator that measures complexity in the knowledge production process implies either endogenous growth or stagnation.

Our first Lemma highlights that inflation increases complexity in R&D, that, depending on the magnitude of the q, the effect can be higher or lower for more technologically developed countries, and depending on the relationship between the parameters q and z, the effect can be higher or lower for higher levels of inflation (all proofs are presented in Appendix A).

Lemma 1. ***A.*** *Complexity in knowledge production increases with inflation.* ***B.*** *With* $q > 1$ *(*$q < 1$*), the higher the technological stock, the lower (the higher) the effect of inflation on* *complexity.* ***C.*** *With $z(q-1)+1>0$ ($z(q-1)+1<0$), the higher the inflation rate, the lower (the higher) the effect of inflation on complexity.*

In light of the results in Lemma 1, it is important to note that $q > 1$ implies that $z(q-1)+1>0$, for any $z > 0$. Together, these mean that more technological advanced countries may face smaller complexity effects arising from higher inflation and also that the higher the inflation rate the lower the complexity effects due to rising inflation. Note also that, from Lemma 1 and $z > 0$, we should expect that this parameter would crucially determine (quantitatively) the effect of inflation on complexity.

It is also worth noting that $q > 1$ implies that the complexity index levels off despite the continuous increase in the measure of technological varieties, A_t (see equation 2).[3] As noticed by Sequeira *et al.* (2018), this suggests that (at least) part of the modern innovations have a stabilizing role in the complexity of the economies, which, in turn, can be interpreted as reflecting a relatively high level of complementarity of ideas in the modern knowledge system. Then, the fact that the impact of inflation on complexity is higher when the measure of technological varieties is smaller suggests that the economies that benefit the least from that complementarity (due to a low A_t) are more exposed to the detrimental effects of inflation on growth through the complexity channel.

3 Full Endogenous Growth Model

The setup of the model is very close to that in Sequeira *et al.* (2018), which is a standard model of overlapping generations (OLG), now augmented by the existence of money. In this section, we first describe the setup of the endogenous growth model including the knowledge production function described above. Then, we characterize the steady-state and the transitional dynamics of this model, especially focusing on the effect of inflation on technological growth.

3.1 Model Setup

3.1.1 Households

The members of the young generation supply one unit of labor from which they earn nominal wages, $P_t w_t$, and smooth their consumption, dividing their income between the nominal consumption in the current period $P_t c_t^1$ and in the second period $P_{t+1} c_{t+1}^1$. P_t is the final-good price level in period t, which grows at the inflation rate μ_t, such that $P_{t+1} = (1+$

[3] $q > 1$ is indicated by both estimations in Sequeira *et al.* (2018) and our own estimations in Appendix B. Estimations in Appendix B provide indications that inflation precedes (in the Granger-causality sense) complexity in R&D, suggesting that it may help explain the increasing complexity phenomenon, but we are also able to empirically validate the extended functional form for the complexity index that includes inflation. This further supports us to include inflation acting on economic growth through the complexity-in-R&D channel in the full endogenous growth model, as carried out in the next section.

$\mu_{t+1})P_t$. The members of the old generation do not work and make a living from their savings. Young individuals born in period 1 maximize utility $u_t = log(c_t^1) + \beta log(c_{t+1}^1)$, where β is the discount factor, subject to the following constraints: $P_t c_t^1 = P_t w_t - P_t s_t$, where s_t are savings (in real value), and $P_{t+1}c_{t+1}^1 = (1 + i_{t+1})P_t s_t$, where i_{t+1} is the expected nominal interest rate. The monetary authority could, in period 0, issue money M_0 and transfer these notes to the members of the old generation who would then use them to buy goods from the young. The reason why in this setup money, M_t, is desired is that the young believe the notes to be valuable in the next period, thus, they would accept them in exchange for some of their goods in order to use them in the next period for buying from the new young generation etc.[4] This standard OLG setup provides a well-known solution for *per capita* real savings as a constant proportion of real wages: $s_t = \frac{\beta}{1+\beta} w_t$, which is maintained in the money model considered here. In fact, money demand is equal to (nominal) savings. Population has dimension L_t and grows at an exogenous rate n. An exogenous population growth rate allows one to consider a mechanism that enlarges the market while proving convenient in guaranteeing analytical tractability and in focusing the paper on the evolution of the technology side of the economy (e.g. Sequeira *et al.*, 2018).

3.1.2 Firms

A continuum of competitive firms produces a homogeneous final good using a Cobb-Douglas technology and employing physical capital, K_t, and labor, L_t^Y in each period t: $Y_t = A_t^\sigma K_t^\alpha L_t^{Y(1-\alpha)}$, where $0 < \alpha < 1$ is the share of physical capital in national income, $1 - \alpha$ is the share of labor in the national income (as usual in the Cobb-Douglas settings) and σ is a parameter that governs the returns to specialization. This allows us to proceed as Benassy (1996, 1998), Groot and Nahuis (1998), and Alvarez-Pelaez and Groth (2005) and disentangle the effect of returns to knowledge from the share of physical capital in the final-good production. The physical capital K_t is a CES aggregate of varieties of specialized capital goods, x_{jt}, which are the technological goods in the model: $K_t = A_t \left(\frac{1}{A_t}\sum_{j=1}^{A_t} x_{jt}^\alpha\right)^{\frac{1}{\alpha}}$. For simplicity and without any loss of generality, we assume that capital depreciates fully within one generation. Profit maximization yields the following first-order conditions: $w_t = (1-\alpha)\frac{Y_t}{L_t^Y}$ and $p_t = \alpha\frac{Y_t}{K_t}$, where p_t is the price (in real value) of the aggregate capital good. Using the equations for physical capital and its price, we obtain the demand for individual varieties: $x_{jt} = \frac{1}{A_t}\left(\frac{\alpha Y_t}{K_t^\alpha p_{jt}}\right)^{\frac{1}{1-\alpha}}$, where p_{jt} is the price (in real value) of each variety j at time t.

In the specialized capital goods sector (in which there is monopoly power), each firm maximizes profits $\pi_{jt} = (p_{jt} - r_t)\, x_{jt}$, where r_t is the real interest rate at time t, from which, after substituting x_{jt} from the demand for varieties, we obtain the usual profit

[4] Money has *exchange* value, following Samuelson (1958).

maximizing (real) price as $p_{jt} = p_t = r_t/\alpha$. Using the profits equation from the specialized capital goods sector and the profits maximizing price, we obtain the following expression for profits: $\pi_{jt} = \pi_t = (1-\alpha)\alpha Y_t/A_t$. Since all varieties are produced in the same quantities, $x_{jt} = x_t$ and, thus, $K_t = A_t x_t$.

The number of varieties, A_t, is increased according to the motion law in equation (1). The free-entry condition into the R&D sector, which only employs labor, is $w_t L_t^A = \pi_t \Delta A_{t+1}$,[5] which equates the costs and the profits of inventing ΔA_{t+1} new units. Using equation (1), this yields $w_t \frac{L_t^{\chi(A_t)}}{\delta A_t} = \pi_t$. We equate both equations for profits. Then, we use the equations for the wages and profits and the labor market clearing condition, $L_t = L_t^A + L_t^Y$, to obtain the shares of labor employed in the R&D sector and in the final-goods sector:

$$l_t^Y = \frac{L_t^Y}{L_t} = \min\left\{1, \frac{1}{\alpha\delta L_t^{1-\chi(A_t,\mu_t)}}\right\};\ l_t^A = \frac{L_t^A}{L_t} = \max\left\{0, 1 - \frac{1}{\alpha\delta L_t^{1-\chi(A_t,\mu_t)}}\right\}. \tag{3}$$

3.1.3 Monetary Authority

The monetary authority setup follows e.g., Chu and Cozzi (2014) and Gil and Iglésias (2020). As underlying in subsection 3.1.1, the nominal money supply is denoted by M_t. As mentioned earlier, the growth rate of P_t is the inflation rate, μ_t. The monetary authority adopts an inflation targeting framework, where the monetary policy instrument is the nominal interest rate, i_t. By Fisher equation, $i_t = r_t + \mu_t$ – note that the real interest rate does not depend on i_t nor on μ_t –, the inflation rate is endogenously determined, as $\mu_t = i_t - r_t$,. The monetary authority adjusts the money supply, given the inflation rate, in order to obtain the desired nominal interest rate. As usual in the literature, we consider that, to balance its budget, the monetary authority returns the seigniorage revenues to households as a lump-sum transfer.

3.2 Equilibrium dynamics: transitional dynamics and steady state

Using the first-order conditions for the consumer problem, the production function, the capital market clearing condition, $K_{t+1} = L_t.s_t$, and the *per capita* versions of the relevant variables, such that $y_t = \frac{Y_t}{L_t}$ is *per capita* income, $k_t = \frac{K_t}{L_t}$ is physical capital *per capita*, and $c_t = \frac{C_t}{L_t}$ is consumption *per capita*, the model can be summarized by the following equations:

[5] In line with, e.g., Strulik *et al.* (2013) and Sequeira *et al.* (2018), we make the simplifying assumption that a patent holds for one period (i.e. one generation) and that afterwards the monopoly right to produce a good is sold at price π_{t+1} to someone chosen at random from the next generation. Through this simplification we get rid of intertemporal (dynastic) problems of patent holding and patent pricing while keeping the basic incentive to create new knowledge intact.

$$s_t = \frac{\beta}{1+\beta} w_t, \tag{4}$$

$$\Delta A_{t+1} = (A_{t+1} - A_t) = \delta \cdot A_t \cdot \frac{l_t^A}{L_t^{\chi(A_t,\mu_t)-1}}, \tag{5}$$

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{L_t}{L_{t+1}} s_t, \tag{6}$$

$$w_t = (1-\alpha) y_t / l_t^Y, \tag{7}$$

$$y_t = (A_t)^\sigma (l_t^Y)^{1-\alpha} k_t^\alpha = c_t + (1+\mu_t) k_{t+1}, \tag{8}$$

$$L_{t+1} = (1+n_t) L_t. \tag{9}$$

Inserting (4) into (6), then substituting w_t from expression (7) and finally using (3) and the first part of (8), we obtain the difference equation for physical capital *per capita*:

$$k_{t+1} = a \frac{A_t^\sigma L_t^{\alpha(1-\chi(A_t,\mu_t))} k_t^\alpha}{(1+n)}, \tag{10}$$

where $a \equiv \beta(\alpha\delta)^\alpha (1-\alpha)/(1+\beta)$.

Using equations (1) and (3), we derive another difference equation that, together with equation (10), describes recursively the dynamics of this model:

$$A_{t+1} = f(A_t, \mu_t, L_t), \tag{11}$$

where

$$f(A_t, \mu_t, L_t) = \begin{cases} A_t & \text{if } L_t^{1-\chi(A_t,\mu_t)} \leq 1/(\alpha\delta) \\ \left[\delta\left(L_t^{1-\chi(A_t,\mu_t)} - \frac{1}{\alpha\delta}\right) + 1\right] A_t & \text{if } L_t^{1-\chi(A_t,\mu_t)} > 1/(\alpha\delta) \end{cases}$$

When the complexity index reaches unity, $\chi(\cdot) = 1$, then equations (10) and (11) become free of scale effects. When $\chi(\cdot) < 1$, positive scale effects are present, but decreasing as $\chi(\cdot)$ increases. In case $\chi(\cdot) > 1$, negative scale effects arise.

In particular, A_t follows a piecewise dynamics triggered by the (exogenous) dynamics of population, L_t, and inflation, μ_t, as described in Lemma 2.[6]

Lemma 2. ***A.*** *If* $L_t^{1-\chi(A_t,\mu_t)} > 1/(\alpha\delta)$*, an increase in inflation,* μ_t*, implies a decrease in knowledge growth and no inflation always implies higher growth.* ***B.*** *For a sufficiently high* L_t *and low* $\chi(\cdot)$*, an increase in* μ_t *may imply that the economy regresses to a stagnation stage. For a sufficiently high* L_t *and low* $\chi(\cdot)$*, a decrease in* μ_t *may imply that the economy enters in the growth stage.*

[6] This piecewise structure is implicit in the innovation-driven endogenous growth models, such as those by Romer (1990) and Dinopoulos and Thompson (2000), but is taken explicitly here.

As a corollary of Lemma 1 and the empirical estimations in Table B.3, it is also possible to highlight some results concerning the impact of inflation on growth and their dependency on the level of technological development and inflation that a country faces. Lemma 3 summarizes these results.

Lemma 3. *Under the conditions of Lemma 2, with* $q > 1$ *and* $z > 0$ *(implying that* $z(q+1)+1 > 0$*), the higher the inflation rate and/or the higher the technological stock, the lower the (absolute) effect of inflation on technological progress. Under the conditions of Lemma 2, and with* $\chi(\cdot) < 1$*, the higher the population, the higher the (absolute) effect of inflation on technological progress.*

This Lemma 3 implies that in more developed countries as well as in high inflation countries the negative effect of inflation on technological growth tends to *be attenuated.* As highly developed countries are also usually those with lower inflation, this opens the door to different growth-inflation relationships, depending on the specific features of the countries and their position on the transitional path. Moreover, in larger countries, the (negative) effect of inflation on technological growth tends to be larger, meaning that, *ceteris paribus*, larger countries may be highly hurt by inflation. This is consistent with the empirical evidence that suggests that the growth-inflation relationship differs a lot across countries (in our case, across countries in different development stages) – see, e.g., Burdekin *et al.* (2004) and López-Villavicencio and Mignon (2011). Note also that we focus on the empirically plausible case of $q > 1$, which is well documented in Sequeira *et al.* (2018) and in Appendix B, Table B.3, for the US case. We cannot exclude, however, that for some economies $q < 1$ occurs. If this would be the case, note that the effect of inflation on technological growth would also be negative, but for high inflation and highly technologically advanced countries the (negative) relationship would be stronger.

As in Sequeira *et al.* (2018), with increasing population, only $\chi(A_t, \mu_t) = 1$ guarantees a feasible steady state with positive growth; consequently, $g^*_{A_t} = \frac{\Delta A_{t+1}}{A_t} = \left(\delta - \frac{1}{\alpha}\right)$ and $g^*_{k_t} = \frac{\Delta k_{t+1}}{k_t} = \left(\frac{\sigma}{1-\alpha}\right)\left(\delta - \frac{1}{\alpha}\right)$, and there is a feasible steady state with endogenous growth if and only if $\delta\alpha > 1$. That is, as $A_t \to \infty$ (or $\mu_t \to \infty$), this model evolves to a steady state characterized by endogenous economic growth, depending only on the primitive parameters of the model, if $\chi(A_t, \mu_t)$ converges to a constant equal to unity (i.e., scale effects vanish) under increasing population.

It is worth noting that, although in such a steady-state equilibrium inflation has no influence on growth, it is not the case along the transitional dynamics. That is, in our model, money is not superneutral as long as the economy is on its transitional growth path for finite values of A_t (and μ_t).

In the next section, we calibrate the model and evaluate quantitatively the behavior of the economy governed by this model.

4 Calibration and quantitative effects of inflation

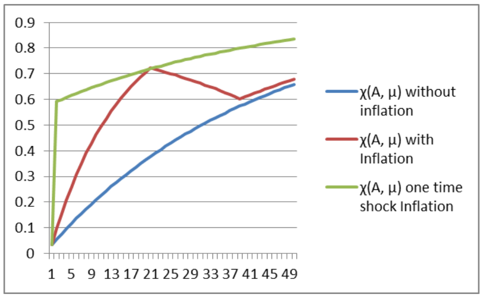
4.1 Calibration

The value for the share of physical capital is often regarded as constant, around 0.36, as a stylized fact (see e.g., Elsby *et al.*, 2013). Thus we use $\alpha = 0.36$. We draw the value for returns to knowledge, $\sigma = 0.2$, from Coe *et al.* (2009: Table 4) for the group of G7 countries with the larger updated sample considered in that article. This value is consistent with the average empirical values for the output elasticity to R&D yielded by country studies reported in Hall *et al.* (2009: Table 5), which oscillate between of 0.18 (considering only domestic R&D) and 0.235 (considering both domestic and international R&D), and which are based on estimates for the group of OECD and G-7 countries. We set the value of δ (productivity in the R&D sector) such that the model replicates an annual average growth of GDP per worker in the United States of 1.87% (average between 1950 and 2000). The values that shape the entropy function for the complexity index – equation (2) – come from the empirical exercise shown in Appendix B and are depicted in Table B.3 (using data for the US).

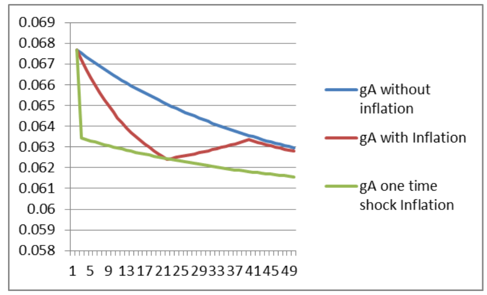
4.2 The effect of inflation shocks on transitional dynamics

We show four different quantitative illustrations of the effects of inflation on both the complexity index and productivity (or knowledge) growth. We always illustrate the effect of inflation comparing it to a baseline without inflation in the complexity function. The first two illustrations simply use two different assumptions for inflation. First, we assume an hump-shaped evolution of inflation resembling a *stylized* evolution of inflation in the US during the twentieth century (see e.g., Reinhart and Rogoff, 2014): it begins at nearly 0%, gradually increases towards 10% (converging to this value after 20 periods) and then begins to slowly decrease towards a value of 1% (converging to this value after 40 periods from the beginning). Second, we assume a one-time shock on inflation from 0% to 10% (in the first period), lasting forever. Those exercises are depicted in Figure 1. Third, we illustrate the effect of inflation in creating technology cycles, increasing the volatility of both the complexity index and the technological growth rate (see Figure 2). Finally, we illustrate a situation when a sudden drop in inflation can induce the take-off of an economy from stagnation to growth (see Figure 3).

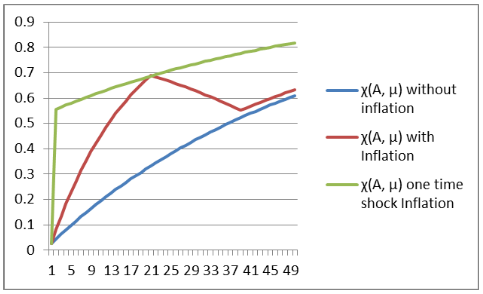
Figure 1 highlights result **A** in Lemma 2, showing that an increasing path of inflation increases complexity (and decreases technological growth) when compared to the no infla-



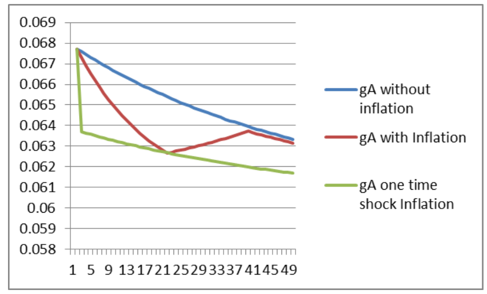
(a) Complexity Index $q = 1.360$; $b = 0.348$; $z = 25.92$.



(b) Knowledge Growth g_A; $q = 1.360$; $b = 0.348$; $z = 25.92$.



(c) Complexity Index $q = 1.285$; $b = 0.291$; $z = 27.26$.



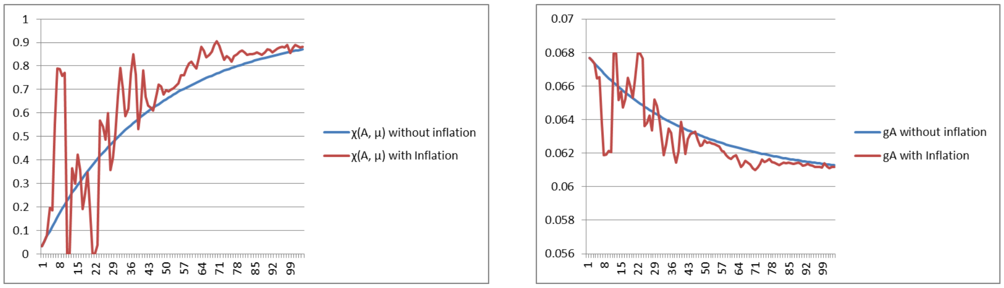
(d) Knowledge Growth g_A; $q = 1.285$; $b = 0.291$; $z = 27.26$.

Figure 1: Evolution of the "model" series for the complexity index and technological knowledge growth without and with inflation ($L_0 = 1.1$, $A_0 = 1.1$). Blue lines represent the no-inflation case. Red lines describe the case of an inflation rate beginning at 0% and gradually increasing to 10% and, then gradually decreasing to 1%. Green lines describe a one-time shock from 0% to 10%. The alternative calibrations of q, b, and z are inspired, for the purpose of illustration, in the values depicted in Table B.3, Appendix B.

tion case and also it increases complexity (and decreases technological growth) as inflation increases. Furthermore, as this exercise introduces a hump-shaped path for inflation (see the red lines in Figure 1), it also highlights that while increasing inflation might play a role in the productivity slowdown, the following decreasing inflation path may give some support to the recovery of productivity. This *stylized* figure matches the empirical evidence according to which small inflation changes may induce small changes in economic growth - while a small increase in inflation may have a small detrimental effect on growth, also small inflation downturns may have small boosting effects on growth. The one-time inflation shock additionally highlights how much can a sudden inflation shock hurt economic growth visible as an immediate drop of nearly 0.4 percent points in the economic growth rate for a one-off increase of 10 percentage points in inflation (see the green lines in Figure 1). These two numerical exercises are also consistent with the small empirical effects of inflation on economic growth for reasonably small changes in inflation, but significant effects of sudden rises in the inflation rate.

Figure 2 shows the effect of considering a high volatile inflation in the complexity index and in the growth rate of technology when comparing to the case without inflation.[7] These results seems to indicate that the consideration of inflation in such a model may introduce

[7]We use the same US data series for the inflation rate that is plotted in Appendix B, Figure B.1.

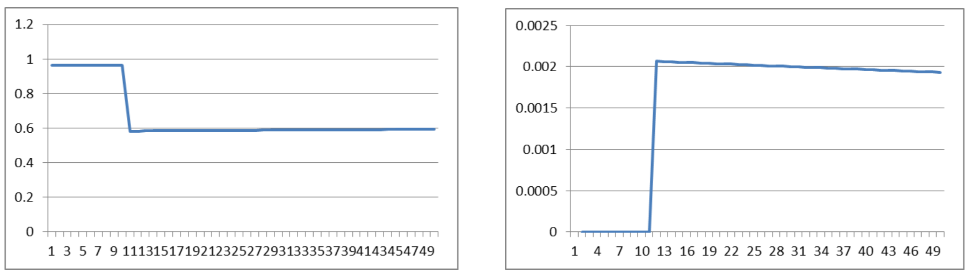


(a) Complexity Index $q = 1.360$; $b = 0.348$; $z = 25.92$.

(b) Knowledge Growth g_A; $q = 1.360$; $b = 0.348$; $z = 25.92$.

Figure 2: Evolution of the "model" series for the complexity index and technological knowledge growth without and with inflation ($L_0 = 1.1$, $A_0 = 1.1$) – the case of period-to-period changes in the inflation rate (using annual data series). Blue lines represent the no-inflation case. Red lines describe the case with period-to-period changes in the inflation rate.

short-term movements in technological growth resembling business cycles, allowing us to analyse both the short and the long-run effects of changing inflation.



(a) Complexity Index $q = 1.360$; $b = 0.348$; $z = 25.92$.

(b) Knowledge Growth g_A; $q = 1.360$; $b = 0.348$; $z = 25.92$.

Figure 3: Evolution of the "model" series for the complexity index and technological knowledge growth with inflation ($L_0 = 1.1$, $A_0 = 1.1$, $\delta = 2.72$), representing a take-off after a one-time negative inflation shock.

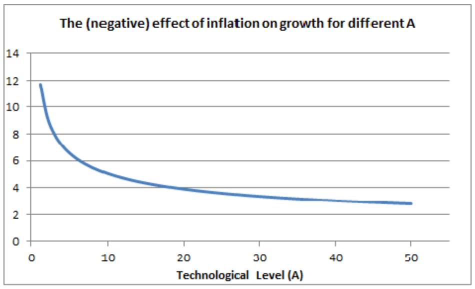
Figure 3 highlights that an (negative) inflation shock can induce a take-off from a stagnation equilibrium, as was theoretically pointed out in part **B** of Lemma 2. A drop in inflation may determine a decrease in complexity costs so that it may become profitable to invest in R&D. However, after the one-off decrease in inflation, the increasing complexity induced by the ensuing technological growth implies that technological stock is increasing, but at decreasing rates.

All these results highlight the main features of the model. First, the *non-neutrality* result emerges from the transitional dynamics. Second, high volatile inflation – when compared to state variables such as technology – implies that the transitional dynamics resembles growth cycles.[8] Finally, the channel through which inflation influences growth is naturally non-linear, implying different effects during the process of development, a result that will be fully studied in the next section.

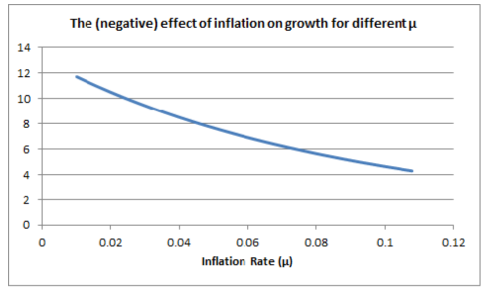
[8]For literature on technological growth cycles see, e.g., Stiglitz (1993) and Wald (2005).

4.3 Quantifying the non-linear effect of inflation on growth

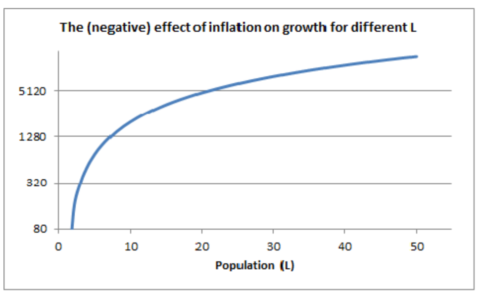
In this section, we quantify the effect of inflation on technological growth, using the results in Lemmas 2 and 3 above. In particular, we are interested in quantifying the effects of inflation on (technological) growth for different levels of population, L_t, technological development, A_t, and inflation, μ_t. This way, we wish to assess how the model addresses the existing empirical evidence on these effects.



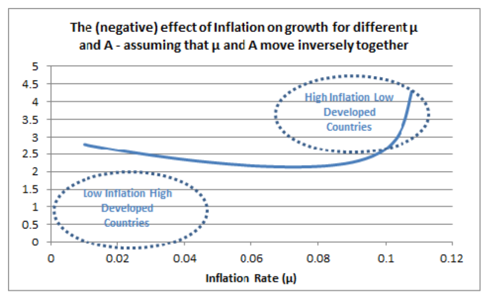
(a) calibrated parameters and variables: $q = 1.360$; $b = 0.348$; $z = 25.92$; $\delta = 5$; $\alpha = 0.36$; $L = 1.1$, $\mu = 1\%$.



(b) calibrated parameters and variables: $q = 1.360$; $b = 0.348$; $z = 25.92$; $\delta = 5$; $\alpha = 0.36$; $L = 1.1$, $A = 1.1$.



(c) calibrated parameters and variables: $q = 1.360$; $b = 0.348$; $z = 25.92$; $\delta = 5$; $\alpha = 0.36$; $A = 1.1$, $\mu = 1\%$.



(d) calibrated parameters and variables: $q = 1.360$; $b = 0.348$; $z = 25.92$; $\delta = 5$; $\alpha = 0.36$; $L = 1.1$.

Figure 4: The (negative) effect of inflation for different levels of technological development, A, inflation, μ, and population, L. Vertical axis always measure $-\frac{\partial g_A}{\partial \mu}$.

In Figure 4, the (absolute value of the negative) effect of inflation on technological growth – the negative of equation (12) in Appendix A – is plotted against technological development (Figure 4a), inflation (Figure 4b), and population (Figure 4c). In these figures, the nonlinear (convex) relationship between the effect of inflation on growth and both the level of technological development and the level of inflation is visible. In fact, as shown in Lemmas 2 and 3, the higher the inflation rate and the level of technological development, the less intense the effects of inflation on growth. For example, for an initial inflation rate of 2% per year, an increase in inflation of 0.1 percent points yields, *ceteris paribus*, a drop in the growth rate of 1.051 percent points – see Figure (4b). However, for an initial inflation rate of 10%, an increase in inflation of 0.1 percent points yields, *ceteris paribus*, a drop in growth of 0.464 percent points – see again Figure (4b).[9] Following a similar reasoning,

[9]This is consistent with the nonlinear negative influence of inflation on growth found e.g., in Burdekin *et al.* (2004), Gillman *et al.* (2004) and Gil and Iglésias (2020). Note that this corresponds to an empirical effect that takes the technological level (or the stock of knowledge) as constant.

for a technological level of 20, an increase in inflation of 0.1 percent points yields, *ceteris paribus*, a drop in the growth rate of 3.91 percent points – see Figure (4a). However, for a technological level of 50, an increase in inflation of 0.1 percent points yields a drop in growth of 2.79 percent points – see again Figure (4a). On the contrary, the higher the population level, the more inflation shocks hurt technological growth.

These results may justify why inflation has been reported to have so different effects in different countries by the empirical literature (see, e.g., López-Villavicencio and Mignon, 2011). In fact, we may notice that more developed countries tend to have lower and more stable inflation rates than less developed countries.[10] Figure (4d) highlights the growth effect of inflation for different combinations of inflation rates and technological development, assuming that the inflation rate and technological development are negatively correlated. The exercise yields a nonlinear (negative) effect of inflation on growth such that, for high-inflation and low-developed countries, the effect should be high (reflecting the dominant effect of a low A_t) and, for countries with intermediate levels of development and of inflation, the effect should be the lowest. Interestingly, the effect of inflation strengthens again for the most developed countries with low inflation. This last figure is an incision on the overall relationship between the level of technology, A_t, the level of inflation rate, μ_t, and the effect of inflation on growth that is shown in Figure 5. The different grey-shadowed branches in this figure highlight the nonlinear relationship between inflation and growth once the technological development level is taken into account.

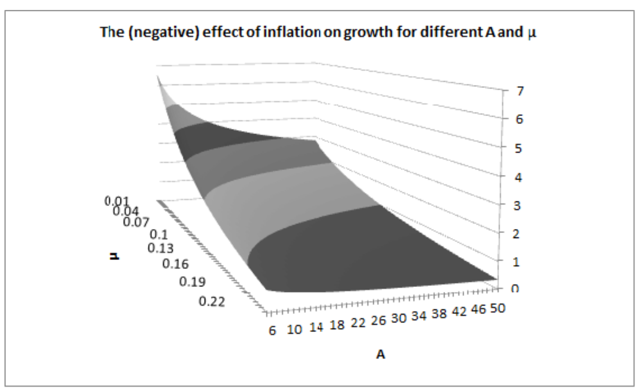


Figure 5: The (negative) effect of inflation for different levels of technological development, A, and inflation, μ.
Note: calibrated parameters and variables: $q = 1.360$; $b = 0.348$; $z = 25.92$; $\delta = 5$; $\alpha = 0.36$; $L = 1.1$.

Despite its simplicity, the model is capable of replicating a number of empirical results about the relationship between inflation and economic growth that have been highlighted in the recent literature. For example, the model features a nonlinear negative effect of inflation

[10] Here, we identify less developed countries as those with a lower level of the stock of ideas or technological stock, A_t. This means that a less developed country has a lower A_t than more developed countries but is equal to them in any other parameter. To access the robustness of our results to this simplifying assumption, we present results with an alternative set of parameter values in Appendix C.

on technological growth consistent with the theoretical results in Gil and Iglésias (2020) and the empirical evidence in Burdekin *et al.* (2004) and Gillman *et al.* (2004). However, as shown above, it also encompasses country heterogeneity in what regards the behavior of the (negative) effect of inflation on growth emphasized by, e.g., López-Villavicencio and Mignon (2011) in cross-country data. In the model, this is obtained by combining the dependence of that effect on the initial level of the inflation rate and on the technological development of the countries.

5 Conclusions

Inflation has been pointed out as a negative determinant of economic growth although possibly with nonlinear effects. Inflation is deemed a source of noise at the macroeconomic level that distorts incentives to invest in different assets. Bearing this in mind, in this paper, we uncover a new channel through which inflation can deter economic growth: complexity in R&D activities. The rationale is that more inflation implies more planning, prospection and coordination costs for the R&D firms.

The effect of inflation on growth that arises from this new channel has three main features that are empirically sound: (1) it is negative; (2) sudden inflation shocks may severely hurt economic growth; (3) high inflation volatility also implies high economic growth volatility. This third feature highlights the link between inflation, business cycles and economic growth. Furthermore, we show that under certain circumstances, a sudden inflation drop can cause a (late) takeoff from stagnation to growth.

Finally, the model also addresses important nonlinearities in the relationship between inflation and economic growth, a highly debated issue in the literature. For a relatively high level of complementarity of ideas in the modern knowledge system, we obtain that in more developed countries as well as in high inflation countries the negative effect of inflation on technological growth tends to be *smaller*. As high (low) developed countries are also usually those with lower (higher) inflations rates, this opens the possibility for different growth-inflation relationships depending on the specific features of the countries and their position on the transitional path. This is consistent with the empirical evidence according to which the growth-inflation relationship differs a lot across countries and specifically across countries with different initial inflation rates. Finally, the fact that the model points out that the effect of inflation on growth may depend on the technological development of the countries suggests that this variable should be taken into account when empirically assessing the effect of inflation on economic growth in growth regressions.

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A Proofs of Lemmas

Lemma 1

Proof. **A.** $\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t} = (A_t(1+\mu_t)^z)^{-q}\, bzA_t(1+\mu_t)^{z-1} = \frac{bz}{(1+\mu_t)^{1+(q-1)z}(A_t)^{q-1}} > 0$, for $z > 0$. **B.** Derive the expression of the derivative above in order to A_t and observe that $\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t \partial A_t} < 0$ for $q > 1$ and $\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t \partial A_t} > 0$ for $q < 1$. **C.** Derive the expression of the derivative above in order to μ_t, $\frac{\partial^2 \chi(A_t,\mu_t)}{\partial \mu_t^2}$ and evaluate it for $z(q-1)+1 > 0$ and $z(q-1)+1 < 0$. □

Lemma 2

Proof. Take the derivative of g_A on the inflation rate μ_t in equation (11) as:

$$\frac{\partial g_A}{\partial \mu_t} = -\delta \cdot \ ln\left(L_t\right) \ \cdot \left(L_t^{1-\chi(A_t,\mu_t)}\right) \cdot \left[\frac{z}{(1+\mu_t)^{1+(q-1)z}\left(A_t\right)^{q-1}}\right] < 0. \tag{12}$$

This proves A. The threshold equations for the two branches of (11) yield the result in B. □

Lemma 3

Proof. On the effect of the level of inflation on the relationship between inflation and growth:

$$\frac{\partial^2 g_A}{\partial \mu_t^2} = -\delta \cdot \ ln\left(L_t\right) \ \cdot \left(L_t^{1-\chi(A_t,\mu_t)}\right)\left[-\left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}\right)^2 ln\left(L_t\right) + \frac{\partial^2 \chi(A_t,\mu_t)}{\partial \mu_t^2}\right].$$

Note that $\frac{\partial^2 \chi(A_t,\mu_t)}{\partial \mu_t^2} = -\frac{(1+(q-1)z)}{1+\mu_t}\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}$. Substituting this result in the expression above yields:

$$\frac{\partial^2 g_A}{\partial \mu_t^2} = \delta \cdot \ ln\left(L_t\right) \ \cdot \left(L_t^{1-\chi(A_t,\mu_t)}\right) \cdot \left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}\right) \cdot \left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t} ln\left(L_t\right) + \frac{(1+(q-1)z)}{1+\mu_t}\right).$$

From Lemma 1, $\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t} > 0$. Thus, for $z(q-1)+1 > 0$, which always hold for $z > 0$ and $q > 1$, $\frac{\partial^2 g_A}{\partial \mu_t^2} > 0$, meaning that, for higher inflation rates, the effect of inflation on technological growth is less intense.

On the effect of the level of technological development on the relationship between inflation and growth:

$$\frac{\partial^2 g_A}{\partial \mu_t \partial A_t} = -\delta \cdot ln\left(L_t\right) \cdot \left(L_t^{1-\chi(A_t,\mu_t)}\right)\left[-\left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}\right)\left(\frac{\partial \chi(A_t,\mu_t)}{\partial A_t}\right) ln\left(L_t\right) + \frac{\partial^2 \chi(A_t,\mu_t)}{\partial \mu_t \partial A_t}\right].$$

Note that $\frac{\partial^2 \chi(A_t,\mu_t)}{\partial \mu_t \partial A_t} = -\frac{(q-1)}{A_t}\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}$. Substituting this result in the expression above yields:

$$\frac{\partial^2 g_A}{\partial \mu_t \partial A_t} = \delta \cdot \ ln\left(L_t\right) \ \cdot \left(L_t^{1-\chi(A_t,\mu_t)}\right) \cdot \left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}\right) \cdot \left[\frac{\partial \chi(A_t,\mu_t)}{\partial A_t} ln\left(L_t\right) + \frac{(q-1)}{A_t}\right].$$

From Lemma 1, $\frac{\partial \chi(A_t,\mu_t)}{\partial A_t} > 0$. Thus, for $q > 1$, $\frac{\partial^2 g_A}{\partial \mu_t \partial A_t} > 0$, meaning that for higher technological development, the effect of inflation on technological growth is less intense.

On the effect of the population on the relationship between inflation and growth:

$$\frac{\partial^2 g_A}{\partial \mu_t \partial L_t} = -\delta \cdot \ ln\left(L_t\right) \ \cdot \left(L_t^{-\chi(A_t,\mu_t)}\right) \cdot \left(\frac{\partial \chi(A_t,\mu_t)}{\partial \mu_t}\right) \cdot \left[\left(1-\chi(A_t,\mu_t)\right) ln\left(L_t\right) + 1\right],$$

yielding $\frac{\partial^2 g_A}{\partial \mu_t \partial L_t} < 0$ for $(1-\chi(A_t,\mu_t))ln(L_t)+1 > 0$. $L_t > 1$ and $\chi(A_t,\mu_t) < 1$ are sufficient conditions for this to be verified. □

B Empirical Illustration for the USA

B.1 The empirical complexity effect and inflation

By applying logs to equation (1) and solving for χ, we get the recursive equation:

$$\chi = \frac{\ln\delta + \ln A_t + \ln L_t^A - \ln(\Delta A_{t+1})}{\ln L_t}.$$

To obtain empirical values for χ over time, we consider the calibrated values of δ and the U.S. time series data for A, L^A and L. The parameter δ is adjusted such that we obtain a steady-state growth rate in the model of 1.87%. This is the average annual growth rate of GDP per worker in the U.S. between 1950 and 2000, from the PWT 8.1. We use the data on total labor force for L between 1950 and 2000, from the Penn World Tables (PWT) 8.1. For the number of workers employed in R&D, L^A, we use the Number of Full-Time-Equivalent (FTE) R&D scientists and engineers in R&D-performing companies from the National Science Foundation. We use the U.S. patent stock from 1870 to 2000 (from the U.S. Patent Office) as a proxy of A_t.[11] In order to present results for a larger time span than the directly available data would allow us to, we extrapolate backwards the series until 1914. In order to extrapolate the series for L, we use the annual averaged growth rates from the decennial growth rates provided by the series in Baier *et al.* (2006) for the labor force. And in order to extrapolate backwards the series for L^A (employment in R&D), we use the contemporaneous relationship between L^A and R&D expenditures as a share of output, for the period between 1954 and 2000 (available from Ang and Madsen, 2015). We obtain the series for χ that is depicted in Figure 1.

Now, we wish to test for the (possible) causal statistical relationship between the series for the inflation rate and the series for χ. To that end, we use the inflation rate from 1914 to 2014 from the Federal Reserve Board of the U.S. We also depict this series in Figure 1. Before running the standard Granger-causality tests, we must, however, test the series for stationarity.

As becomes clear from the analysis of the unit-root tests in Table B.1, we reject the no-stationary null hypothesis for both series.[12] This enables us to perform a Granger-causality

[11] Following Ang and Madsen (2015), the initial patent stock is obtained by using the Solow model steady state value of $A_0/(\delta + g)$, where A_0 is initial patent granted, δ is the rate of depreciation (assumed to be 15%), and g is the growth rate in patent issued over the period for which patent applications data are first available to 2000. We use a series for patent issued belonging to classifications 1 to 5 (chemical, computers and communications, drugs and medical, electrical and electronics, mechanical) in the NBER Classification (Marco *et al.*, 2015). The objective was to include patents directly linked with innovations in high-advanced intermediate inputs (excluding some patents in low technology inputs - such as agriculture - and in final consumption goods - such as amusement devises). In further tests, we also use total issued patents, for a matter of comparison. However, tests with the total issued and applications of patents did not significantly change our results. For growth rates based on patent series g_A the null of having an unit root is clearly rejected by an Augmented Dickey-Fuller test (constant and trend included). Results of the tests are available upon request.

[12] In particular, since the trend is statistically significant in both cases, the unit-root tests suggest that

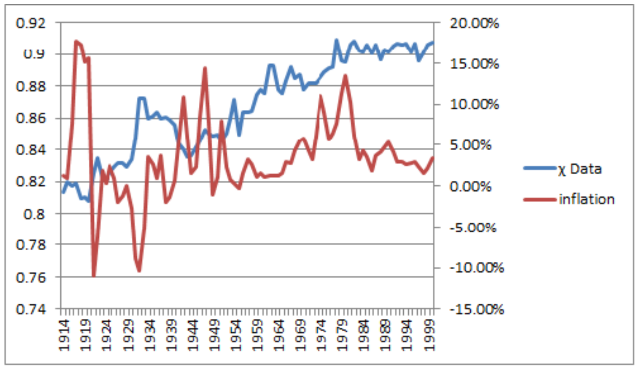


Figure B.1: Empirical series for χ and for the inflation rate, μ, between 1914 and 2000

Table B.1: Unit-Root Tests

Variables	Lags	ADF	$p-value_{ADF}$	Bandwith	Phillips-Perron	$p-value_{PP}$
χ	0	-3.51**	0.0451	1	-3.53**	0.0421
μ	2	-3.89**	0.0165	5	-4.136***	0.0082

Notes: Time period for both variables: 1914-2000. Tests include trend and constant. ADF - Augmented Dickey-Fuller. Automatic lag choice for the ADF test using the AIC (Akaike Info Criteria). Bandwith choice for the Phillips-Perron test using Newey-West automatic and Bartlett kernel. *** indicates significance at the 1% level; ** indicates significance at the 5% level; * indicates significance at the 10% level.

test to these series.

Table B.2: Granger-Causality Tests

Null Hypothesis	Lags	F-statistic	p-value
χ does not Granger-cause μ	2	0.357	0.7012
μ does not Granger-cause χ	2	3.768	0.0273**

Notes: Time period for both variables: 1914-2000. Tests include trend and constant. *** indicates significance at the 1% level; ** indicates significance at the 5% level; * indicates significance at the 10% level.

Table B.2 shows that the empirical evidence clearly points to the existence of Granger-causality directed from inflation to complexity in R&D. In what follows, we show that this theoretical foundation for the complexity effect can reasonably match the empirical series, including inflation, given the available data for the US.

To that end, we compare the empirical series obtained previously for complexity with the theoretical one that comes from the insertion of the empirical series for A_t and μ_t in the complexity function (2). In particular, using the empirical series for χ_t, A_t and μ_t, we estimate b, q and z in equation (2) by GMM (Generalized Methods of Moments) such that we obtain the best possible fit between the theoretical and the empirical series.

As an empirical proxy for A_t, we use either the number of the patents for the most technological sectors (NBER classifications 1 to 5 – see also footnote 2) or the total number of patents. Table 3 shows the estimation results. In both cases, the results indicate that the estimates of q are statistically significant and that $q > 1$, therefore excluding the case

both series are time-trend-stationary.

of complexity growing forever and thus technological growth vanishing asymptotically (see Results 1 and 2, above). Moreover, the estimates show that b is particularly close to but different from $q-1$.[13] When we use the data on the patents for the most technological sectors, the estimates imply $b < q-1$ ($b/(q-1) = 0.97$), indicating that small (positive) scale effects remain in the long-run (recall Result 2). When we use the data on total patents, we get $b > q-1$ ($b/(q-1) = 1.02$), indicating that small (negative) scale effects remain in the long-run.[14]

The estimates in Table B.3 also show that $z > 0$, which confirms that the complexity index is a positive function of the inflation rate – and thus the assumption made in equation (2). For our purposes, it is important to note that statistical non-significance of the estimate of z is rejected at the 5% or 10% levels, which is an indication that the functional form of the complexity index with inflation is empirically validated.

Table B.3: Estimation of the Complexity Index with Inflation

$\hat{b}$	$\hat{q}$	$\hat{z}$	$S.E._b$	$S.E._q$	$S.E._z$
0.348***	1.360***	25.92*	0.039	0.052	13.19
0.291***	1.285***	27.26**	0.044	0.063	13.70

Notes: US yearly data for 1914-2000. Line 1 uses patents classified in sectors 1 to 5 (most technological patents) as a proxy for A_t and line 2 uses total number of patents. GMM non-linear estimation. Instruments: R&D expenditures (as a share of output) and time. Standard-errors (S.E.) were computed using estimation of weighting matrix HAC (Bartlett kernel, Newey-West fixed bandwidth). *** indicates significance at the 1% level; ** indicates significance at the 5% level; * indicates significance at the 10% level.

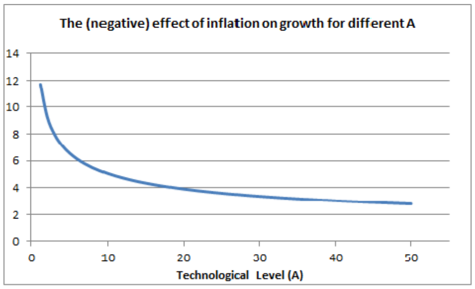
C Alternative Calibrated Parameters Set for Developing Countries

This alternative calibration exercise is set to fit the reality of developing countries such as those in South Asia. This exercise intends to be a robustness check of our main results to different values for the calibrated parameters. The value for the share of physical capital is set to $\alpha = 0.5$, according to Trapp (2014). We draw the value for returns to knowledge, $\sigma = 0.163$, from Ang and Madsen (2013) for the group of South Asian countries. We set the value of δ (productivity in the R&D sector) such that the model replicates an annual average growth of GDP per worker in the South Asian Countries of 2.32% (average between 1950 and 2000) – according to Bolt and van Zanden (2020). The values that shape the entropy function for the complexity index – equation (2) – are adapted by using the comparison

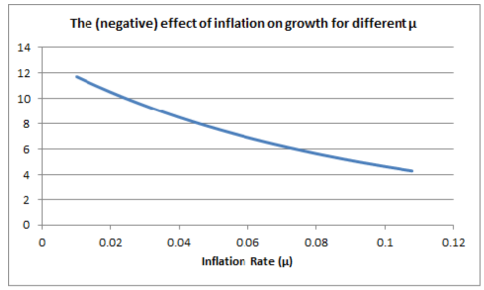
[13] Standard tests allow us to reject the null hypothesis that $q = 1$ and that $b = q-1$ at the 1% level of significance.

[14] All results regarding the estimates of q and b are consistent with the ones presented in Sequeira *et al.* (2018: Table 1), although we consider a smaller time span due to the data available for inflation rates. In particular, our results, as theirs, encompass both small negative and small positive scale effects in the long-run.

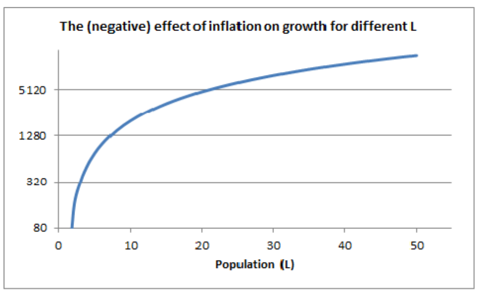
of developed (European) and developing (Asian) countries provided in Ang and Madsen (2015). The inflation rate is now 4% (instead of the 1% inflation rate considered before), fitting the most current inflation rate values in South Asia countries. Corresponding figures to Figures 4 and 5 in the main text are presented below. The overall conclusions in the main text are thus maintained.



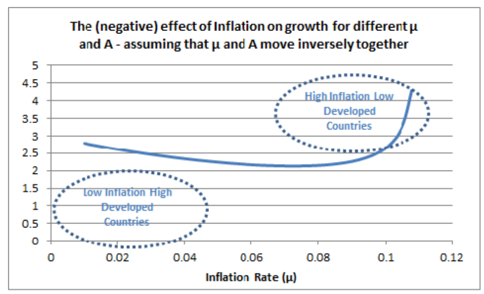
(a) calibrated parameters and variables: $q = 1.5$; $b = 0.51$; $z = 25.92$; $\delta = 3.24$; $\alpha = 0.5$; $L = 1.1$, $\mu = 4\%$.



(b) calibrated parameters and variables: $q = 1.5$; $b = 0.51$; $z = 25.92$; $\delta = 3.24$; $\alpha = 0.5$; $L = 1.1$, $A = 1.1$.



(c) calibrated parameters and variables: $q = 1.50$; $b = 0.51$; $z = 25.92$; $\delta = 3.24$; $\alpha = 0.5$; $A = 1.1$, $\mu = 4\%$.



(d) calibrated parameters and variables: $q = 1.50$; $b = 0.51$; $z = 25.92$; $\delta = 3.24$; $\alpha = 0.5$; $L = 1.1$.

Figure C.1: The (negative) effect of inflation for different levels of technological development, A, inflation, μ, and population, L. Vertical axis always measure $-\frac{\partial g_A}{\partial \mu}$.

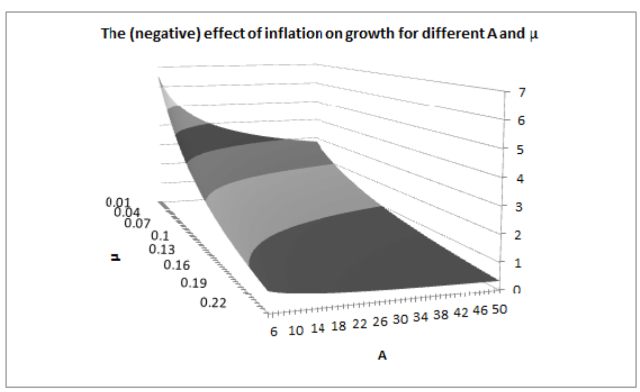


Figure C.2: The (negative) effect of inflation for different levels of technological development, A, and inflation, μ.

Note: calibrated parameters and variables: $q = 1.50$; $b = 0.51$; $z = 25.92$; $\delta = 3.24$; $\alpha = 0.5$; $L = 1.1$.