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A Study on SRLG-disjointness and Geodiverse Routing – a Trade-off Between Benefit and Practical Effort*†

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Abstract

The resilience to disasters is a very relevant problem in telecommunication networks. This work addresses the problem of 1+1 optical lightpath protection considering maximally SRLG-disjoint geodiverse paths, applied in the context of an optical network. The resilience to geographically correlated disasters is accomplished by

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guaranteeing geodiversity of the paths. This work focuses on estimating the increase of the path lengths and the increase in cost of the required transponders, compared to simple link disjointness (i.e. when no constraints on SRLG-disjointness or geodiversity are considered). Results in different networks allow to evaluate the effect of SRLG-disjointness to ensure some geodiversity.
Keywords: SRLG-disjoint, min-sum, geodiverse routing, optical networks, disaster resilience

1 Introduction

The resilience to disasters is a challenging problem in today's telecommunication networks [26]. Known risks of failure have to be taken into account in network planning and management, so that end-to-end connectivity may be guaranteed in the event of disasters, in particular in backbone networks [17, 20]. A set of links sharing a common risk of failure (for instance fibres sharing a cable or a duct) constitute a Shared Risk Link Group (SRLG). Two paths are SRLG-disjoint if they do not share a common risk of failure. As the problem of determining a pair of SRLG-disjoint paths is NP-complete [19], heuristics are used in large networks and/or when on-line routing is considered. In [32, 28, 16], heuristics seeking the minimization of some additive cost metric associated with the links of the paths, are proposed. Seeking to reduce the CPU needed by heuristics to calculate maximally SRLG-disjoint paths, the authors in [21] propose to use parallel processing, by offloading adequate parts of the algorithm to one or more graphics processing units (GPUs).

Usually a min-sum problem is formulated where the aim is the minimization of the sum of the cost of both paths in the pair, obtained considering SRLG-disjointness constraints. In [15] an exact algorithm for enumerating SRLG-disjoint path pairs, by non-decreasing order of their additive cost was proposed, and its computational efficiency was analysed. In [35] the "divide and conquer" strategy – first proposed as part of the heuristic called COnflicting Link Exclusion (COLE) [36] – is used for solving min-min SRLG disjoint routing problem.

In the context of multi-layer survivable routing, a formulation for multipath routing and resource assignment in a virtual optical network is presented in [37].

Broadcast communication using light-trees is considered in [38]. Here SRLGs are used to represent various failure scenarios, with a given probability. The authors seek the light-tree with minimum failure probability, and designate the problem reliable collective communication (RCC) and heuristics are put forward to solve RCC. A bicriteria model

to construct spanning trees over optical networks, with the objective to minimize jointly the number of different SRLGs and the total bandwidth usage cost of all the tree links is presented, and solved using an exact algorithm, in [12]. The authors based their approach in an algorithm for the minimal cost/minimal label spanning tree problem, by considering multiple labels per link [11].

Geographically correlated failures may be represented by SRLGs. The authors in [31] propose a model for representing the effect of regional failures with the shape of a disk with a certain radius, spanning a set of links. In that work, it is shown that the number of SRLGs covering the possible circular disk failures with a known radius is proportional to the number of nodes in the network, by a factor of 1.2. The closely related work [34] tackles the generation of SRLGs representing disk failures that affect a specific number of network nodes. In this case, the number of SRLGs is proportional to the multiplication of the number of nodes in the network with the number of affected nodes.

In cases when it is not possible to find fully SRLG-disjoint paths, one may try and find maximally SRLG-disjoint paths [28, 29, 18]. That is the problem to be considered in this work. Note that the devised path pair should be of min-sum cost and link or node-disjoint, for all traffic demands in a transparent optical network, i.e. guided over end-to-end optical lightpaths. Note that any additive cost may be considered. In this work, the cost is the fibre cable length. Hence, the cost of a path pair is the sum of the fibre length in kilometers of both paths.

The first objective of this work is to see the impact of the SRLGs in the total path length. Therefore, min-sum length path pairs are calculated for each demand with or without SRLG-disjointness constraints. This comparison allows to grasp the relevance of the SRLGs from a topological point of view.

In this paper we are not considering that SRLGs are associated to a duct or a cable with a set of links, but rather as a way to represent geographically correlated failures. That is also why: (i) we find SRLGs as explained in [18]; (ii) we are not calculating paths based on minimal total SRLG length, as we did in the previous study [22]. In that previous work, actual information from an existing network was taken into account and SRLGs actually represented a duct or a cable, unlike what is considered here.

The impact of geographically correlated failures may be taken into account in the selection of routing paths. In order to find geodiverse paths for a given node pair, different metrics may be considered. In [27], the authors propose a path diversification approach, where different paths are selected based on their diversity and also aiming at the improve-

ment of flow reliability.

A routing protocol for finding two paths separated by a certain distance in a physical network, GeoDivRP, is proposed in [7], where the cTGGD (compensated Total Geographical Graph Diversity) value is used to assess the geographical path diversity of a given topology. Following this work, two heuristics, iWPSP (iterative WayPoint Shortest Path) and MLW (Modified Link Weight), are proposed in [6, 9], allowing for a reduction in the complexity of the optimal routing algorithm. The GeoDivRP is further explored in [8] (by including information on critical regions) and in [10] (by proposing a multicommodity linear optimization formulation for the routing problem).

The work in [13] tackles the calculation of geodiverse paths, seeking to guarantee more resilience in the event of disasters. Two different routing algorithms are put forward in [2]. The first algorithm aims at selecting a pair of edge-disjoint paths which maximize the minimal spatial distance between them, which ensures maximum survivability against spatial-based simultaneous edge failures. The second algorithm aims at finding a pair of edge-disjoint paths with minimal path weight for the Active Path (AP) and with a Backup Path (BP) such that a certain distance between them is guaranteed.

The need to increase the network resilience to geographically correlated failures is tackled in this work. The approach proposed in [13] to calculate D-geodiverse path pairs, i.e. paths separated by D km, is considered. Therefore, the second objective is to study the impact of geodiverse routing (for various values of D) on the length of the paths (with SRLG-disjointness constraints, as explained earlier).

The final objective of this work is an assessment of the cost of the required transponders, to satisfy a set of demands using coherent interfaces at 100 Gb/s (or beyond) taking into account optical reach limitations [25]. As the fibre spectrum was modeled to be unconstrained, the lightpaths can always be guided over the calculated optimal path pair for each considered scenario. Note that the specific wavelength or spectrum assignment is not addressed, which keeps the problem simpler.

The results for maximally SRLG-disjoint paths while guaranteeing (or not) geodiversity in a network, are compared with those obtained when simple link disjointness (1+1 protection) is considered, i.e. without SRLG-disjointness and without any geodiversity constraints. The comparison is made in terms of the increase of the path lengths and the consequent increase in the cost of the required transponders. Note however that the increase is not in the same proportion, i.e. the increase in backup length by a certain factor leads to an increase in transponders costs, but by a smaller factor. Another observed result

is that SRLG-disjointness entails some geodiversity for most of the node pairs.

This paper is an extension of [22]. Experiments are performed on different networks of the sndlib set [24] for which different SRLG sets were devised as in [18], allowing to model geographically correlated failures. Given that geodiversity requires node-disjointness, we have formulated and solved a problem of maximally SRLG-disjoint paths among node-disjoint path pairs.

Specific strategies aiming at reducing the cost of transponders were considered in this paper. As the selection of transponders is performed for the longer path of the pair, we have formulated and solved variants of the proposed models, in which the aim is the minimization of the longer path of the pair considering a total path pair length close to the one obtained for the original models.

Due to the path lengths in the considered networks, it is expected that fully transparent paths may not be available for some demands. Therefore, we assume that regenerative transponders will have to be used along the paths. We will use the term regenerator to refer to a regenerative transponder specifically located in an intermediate node of the path, though it is still a transponder. The general term transponder may refer to any transponder, regardless of where it is located. A problem for devising the appropriate number and location of regenerators is formulated based on an approach in [3]. Once the length of the longest transparent segment is known in the paths of the path pair for each demand, the appropriate transponders may be selected. The transponders at the source and destination node for each demand will be identical to the regenerators along the path.

The selection of the transponders for each demand was also performed in a more accurate and flexible way, by solving a new Integer Linear Programming (ILP) problem. In the formulation of this new problem, we assume that the traffic may be split in any number of flows (not just in halves or quarters, as in [22]) and the used transponders may be selected so as to minimize the total cost.

After this Introduction section, the notation and the addressed problems are formally described in Section 2. After the details on the calculation of the transponders' reach and cost in Section 3, the formulation of problems related to the selection of regenerative transponders (regenerators) is presented in Section 4. Results are presented in Section 5 and the paper ends with some conclusions and proposals for future work (in Section 6).

2 Addressed Problems

2.1 Notation

The physical network topology will be represented by $\mathcal{G} = (V, E)$ with $V = \{v_1, v_2, \ldots, v_{|V|}\}$ as the set of $|V|$ nodes and $E = \{e_1, e_2, \ldots, e_{|E|}\}$ as the set of $|E|$ network links or edges. These are undirected arcs represented by an unordered node pair $e_u = \{v_a, v_b\}$, with $v_a, v_b \in V$, $u = 1, 2, \ldots, |E|$.

An edge from E may be represented by two symmetrical (directed) arcs, i.e. edge $e_u = \{v_a, v_b\}$ may be represented by $a_k = (v_a, v_b)$ and $a_{k'} = (v_b, v_a)$. The set of the directed arcs corresponding to the links in E will be denoted by A.

Let ϱ be the total number of risks affecting more than one edge in the network. The set of existing failure risks will be denoted by Y, with $Y = \{y_1, y_2, \ldots, y_\varrho\}$. The set A_r is the set of arcs that defines the r-th SRLG, i.e. this set includes the set of arcs that fail if the event associated with risk y_r occurs. Note that $\cup_{r \in R} A_r$ may be only a subset of A as an arc $a \in A$ may not belong to any $A_r, r = 1, \ldots, \varrho$.

Let R be the set of indexes of all SRLGs in the network. For ease of notation, SRLG A_r will be simply identified by r, with $r = 1, \ldots, \varrho$. Given the arc a_k, then ϕ_k is the set of indexes of SRLGs in R containing arc a_k, i.e. $r \in \phi_k$ if and only if $a_k \in A_r$.

A path is a continuous sequence of different nodes from one source node, v_s or simply s, to a destination (or target) node v_t or simply t, (with $s, t \in V$), and is represented by $p = \langle s \equiv v_1, v_2, \ldots, v_n \equiv t \rangle$, where $(v_i, v_{i+1}) \in A, i \in \{1, \ldots, n-1\}$ and n is the number of nodes in the path. Let A_p be the set of arcs of path p and E_p the set of the corresponding edges. Let $\mathcal{P}_{st}$ represent the set of all paths from s to t in the network. A path pair is identified by (p_1, p_2). The set of path pairs, from s to t, which are edge-disjoint is designated by $\mathcal{P}^2_{st}$.

Let φ^r_p be the set of arcs in path p belonging to SRLG r, i.e. if $a_k \in \varphi^r_p$, then $a_k \in A_r$ and $a_k \in A_p$. The indexes of the union of all SRLGs of path p is denoted by $R_p = \{r \in R : A_r \cap A_p \neq \emptyset\}$. The indexes of the SRLGs shared by path pair (p, q) is represented by $R_{p,q} = R_p \cap R_q$.

The length of arc $a_k \in A$ is given by ℓ_k, and the length of path p will be denoted by $\bar{\ell}_p$, with $\bar{\ell}_p = \sum_{a_k \in A_p} \ell_k$.

We now define some additional notation, that is useful for the formulation of the ILP problems. The following indexes will be used: $j, k = 1, \ldots, |A|$ identify arcs a_j or a_k; $i = 1, \ldots, |V|$ identifies a node v_i; s and t identify the source node and the destination node,

respectively (as already mentioned); $m = 1, 2$ identifies whether a path is the active one ($m = 1$) or the backup one ($m = 2$); $r = 1, \ldots, \varrho$ identifies the risks and the corresponding SRLGs (as already mentioned). The indexes of the arcs leaving node v_i are identified by $\mathcal{E}(i^+)$ and the indexes of the arcs entering node v_i are identified by $\mathcal{E}(i^-)$. Note that any arc a_k leaving node v_i will have a symmetrical arc $a_{k'}$ entering the same node.

A set of parameters is calculated and known beforehand:

- $h_{r,k}$ indicates whether arc a_k belongs to SRLG r:

$$h_{r,k} = \begin{cases} 1 & \text{if } r \in \phi_k \\ 0 & \text{otherwise} \end{cases}$$

  As previously stated, $r \in \phi_k$ is equivalent to $a_k \in A_r$.

- the length of the arcs ℓ_k.

The binary decision variables used in the formulation are:

- $x_{k,m}$ indicates whether arc a_k belongs to path p_m:

$$x_{k,m} = \begin{cases} 1 & \text{if } a_k \in p_m \\ 0 & \text{otherwise} \end{cases}$$

- $z_{r,m}$ indicates whether SRLG r affects any arc of path p_m:

$$z_{r,m} = \begin{cases} 1 & \text{if } r \in R_{p_m} \\ 0 & \text{otherwise} \end{cases}$$

- g_r indicates whether SRLG r includes arcs belonging simultaneously to both paths p_1 and p_2:

$$g_r = \begin{cases} 1 & \text{if } \exists a_j \in \varphi^r_{p_1} \text{ and } \exists a_k \in \varphi^r_{p_2} \\ 0 & \text{otherwise} \end{cases}$$

For all the formulated problems, the arcs forming the solution path pair (p_1^*, p_2^*) are given in the decision variables $x_{k,1}$ and $x_{k,2}$.

2.2 Optimization problems without SRLG-disjointness

We formulate problem $\mathscr{P}_0$, which is a basic problem of finding the link-disjoint path pair with min-sum length, which is easily solved by the Suurballe's [30] or Bhandari's algo-

rithm [4]. This is a very simple form of protection, usually referred to as 1+1 protection.

We present the formulation here for a demand originating in node s and terminating in node t, as the formulations of the other problems include some of the constraints of $\mathscr{P}_0$.

$$\min \sum_{k=1}^{|A|} \ell_k \left(x_{k,1} + x_{k,2}\right)$$

subject to:

$$\sum_{k \in \mathcal{E}(i^+)} x_{k,m} - \sum_{k \in \mathcal{E}(i^-)} x_{k,m} = \begin{cases} 1 & \text{if } v_i = s \\ -1 & \text{if } v_i = t \\ 0 & \text{otherwise} \end{cases} \quad \forall i = 1, \ldots, |V|; m = 1, 2 \tag{1}$$

$$x_{k,1} + x_{k,2} \leq 1 \quad \forall k = 1, \ldots, |A| \tag{2}$$

$$x_{k,1} + x_{k',2} \leq 1 \quad \forall k = 1, \ldots, |A| \tag{3}$$

$$x_{k,m} + x_{k',m} \leq 1 \quad \forall k = 1, \ldots, |A|; m = 1, 2 \tag{4}$$

$$\sum_{k \in \mathcal{E}(i^+)} x_{k,m} + \sum_{k \in \mathcal{E}(i^-)} x_{k,m} \leq 2 \quad \forall i = 1, \ldots, |V|; m = 1, 2 \tag{5}$$

$$\text{binary} \quad x_{k,m} \quad \forall k = 1, \ldots, |A|; m = 1, 2 \tag{6}$$

Constraints (1) are the usual flow conservation constraints. An arc (or an arc and its symmetrical) cannot be used in both paths simultaneously, which is guaranteed by constraints (2)-(3). Constraints (4)-(5) prevent the formation of cycles (or loops). An arc and its symmetrical cannot be used in the same path; any node in either path cannot have more than two of its arcs in that path, which guarantees that certain configurations of cycles do not appear.

In the formulation of this problem, constraint (3) may be omitted, as we are dealing with positive costs in the objective function, which is of min-sum type. For a similar reason, constraints (4) and (5) may also be omitted, as the objective function of $\mathscr{P}_0$ naturally leads to the avoidance of any superfluous links in the solution.

Let θ_{st} be the solution of problem $\mathscr{P}_0$, i.e. the minimal length of an edge-disjoint path pair for a demand between s and t. Problem $\mathscr{P}_0$ is instrumental in the sense that it must be solved so that the value of θ_{st} for every (s, t) demand is known.

2.2.1 Variant of $\mathscr{P}_0$

We now formulate $\mathscr{P}_0'$, which is a variant of the previous problem. This variant aims at finding solutions (path pairs) with length close to the minimal total length obtained when $\mathscr{P}_0$ was solved, but with a smaller length for the longest path of the pair.

The selection of transponders for a demand takes into consideration the longest path of the path pair obtained for that demand. Therefore, it may be important to reduce the length of the longest path of the pair (for each (s,t) demand). In this framework, we want to find the path pair that satisfies the total length θ_{st} (with some pre-defined tolerance Δ_θ) and for which the longest path has minimum length. This is equivalent to minimizing the difference between the length of the AP and of the BP: $\left|\sum_{k=1}^{|A|}(x_{k,1}\ell_k) - \sum_{k=1}^{|A|}(x_{k,2}\ell_k)\right|$. This absolute value expression must be linearized.

Problem $\mathscr{P}_0'$ may be formulated as:

$$\min \mathcal{M}$$

subject to: constraints (1)-(6) and

$$\sum_{k=1}^{|A|} \ell_k (x_{k,1} + x_{k,2}) \leq \mathcal{L}_{st}\Delta_{\mathcal{L}} \tag{7}$$

$$\sum_{k=1}^{|A|} (x_{k,1}\ell_k) - \sum_{k=1}^{|A|} (x_{k,2}\ell_k) \leq \mathcal{M} \tag{8}$$

$$-\left(\sum_{k=1}^{|A|} (x_{k,1}\ell_k) - \sum_{k=1}^{|A|} (x_{k,2}\ell_k)\right) \leq \mathcal{M} \tag{9}$$

where $\mathcal{L}_{st} = \theta_{st}$ and $\Delta_{\mathcal{L}} = \Delta_\theta$. Let $\theta_{st}^{\mathcal{M}}$ be the solution of this problem.

2.3 Optimization problems with node-disjointness, SRLG-disjointness and no geodiversity constraints

The calculation of the minimum number of SRLGs in common, that is the number of SRLGs shared by a maximally SRLG-disjoint path is an ancillary problem for obtaining the path pair of minimal total length among all maximally SRLG-disjoint path pairs (which are also node-disjoint). Let this problem be identified as $\mathscr{P}_{1N}$. Afterwards, we can formulate problem $\mathscr{P}_{2N}$, which tackles the calculation of the minimal total length among the

maximally SRLG-disjoint path pairs (which are also node-disjoint). Finally, we formulate $\mathscr{P}'_{2N}$, which is a variant of problem $\mathscr{P}_{2N}$. This variant aims at finding solutions (path pairs) with length close to the minimal total length obtained when $\mathscr{P}_{2N}$ was solved, but with a smaller length for the longest path of the pair.

Problems $\mathscr{P}_{1N}$ and $\mathscr{P}_{2N}$ are instrumental as they provide information needed for the subsequent formulation of problem $\mathscr{P}'_{2N}$.

2.3.1 Minimizing the number of SRLGs in common

To keep the text self-contained, we present a formulation for node-disjoint path pairs, for a demand originating in node s and terminating in node t, similar to the one present in [19] for SRLG-disjoint paths:

$$\min \sum_{r=1}^{\varrho} g_r$$

subject to: constraints (1), (4)-(5) and

$$x_{(s,t),1} + x_{(s,t),2} \leq 1 \tag{10}$$

$$\sum_{k \in \mathcal{E}(i^+)} (x_{k,1} + x_{k,2}) \leq 1 \quad \forall v_i \in V \setminus \{s,t\} \tag{11}$$

$$\sum_{k=1}^{|A|} h_{r,k} x_{k,m} \leq |A| z_{r,m} \quad \forall r = 1, \ldots, \varrho; m = 1, 2 \tag{12}$$

$$z_{r,1} + z_{r,2} - g_r \leq 1 \quad \forall r = 1, \ldots, \varrho \tag{13}$$

$$\text{binary} \quad x_{k,m}, z_{r,m}, g_r \quad \forall k = 1, \ldots, |A|; m = 1, 2; r = 1, \ldots, \varrho \tag{14}$$

In the formulation of this problem, constraints (2) and (3) may be omitted, because we added the node-disjointness constraint (11). Due to the removal of (2), constraint (10) had to be added, to guarantee that the direct arc between s and t (if it exists) is not used in both paths simultaneously.

Constraint (12) allows to know if an SRLG affects an arc of a path. On the one hand, if at least an arc a_k belonging to SRLG r (i.e. $h_{r,k} = 1$) is used in path p_m (i.e. $x_{k,m} = 1$) then the constraint is satisfied only if $z_{r,m} = 1$; on the other hand, if none of the arcs in path p_m is affected by risk r, then the left-hand side of the inequation is 0, which means that $z_{r,m}$ may be 0 or 1. However, the constraint (13) and the fact that we are minimizing the number of variables g_r which are 1, will lead to $z_{r,m} = 0$. In fact, a variable g_r will be 1 only if $z_{r,1}$ and $z_{r,2}$ are simultaneously 1 (i.e. both paths have arcs affected by the risk

r), as guaranteed by constraint (13) and the considered objective function.

Let μ_{st} be the value obtained as a result of the resolution of $\mathscr{P}_{1N}$ for an (s,t) demand.

2.3.2 Minimizing the path-length of a node-disjoint path pair with minimal number of SRLGs in common

The formulation for node-disjoint path pairs, for a demand originating in node s and terminating in node t (similar to the link-disjoint formulation in [18]) is presented next:

$$\min \sum_{k=1}^{|A|} \ell_k \left(x_{k,1} + x_{k,2}\right)$$

subject to: constraints (1), (10)-(14) and

$$\mu_{st} = \sum_{r=1}^{\varrho} g_r \tag{15}$$

Let Λ_{st} be the solution of problem $\mathscr{P}_{2N}$, i.e. the minimal length of a node-disjoint path pair for a demand between s and t, with minimal number of common SRLGs.

2.3.3 Variant of $\mathscr{P}_{2N}$

Similarly to what was considered earlier, we want to find the maximally SRLG-disjoint path pair that satisfies the total length Λ_{st} (with some pre-defined tolerance Δ_Λ) and for which the longest path has minimum length.

Problem $\mathscr{P}'_{2N}$ may be formulated as:

$$\min \mathcal{M}$$

subject to: constraints (1), (10)-(14), (15) and (7)-(9), where $\mathcal{L}_{st} = \Lambda_{st}$ and $\Delta_{\mathcal{L}} = \Delta_\Lambda$. Let $\Lambda_{st}^{\mathcal{M}}$ be the solution of this problem.

2.4 Optimization problems with SRLG-disjointness and geodiversity constraints

2.4.1 D-geodiverse path pair

In [13] the geographical distance between two paths is shown to depend only on the distance of the edges of the paths. Let $\delta(e_u, e_w)$ designate the (minimal) distance between two edges e_u and e_w. The same notation will be used if arcs are considered instead of edges: $\delta(a_j, a_k)$ designates the distance between the edges corresponding to the arcs a_j and a_k. The geodiversity value of path pair $(p, q) \in \mathcal{P}^2_{st}$ is represented by $D_{p,q}$ and given by:

$$D_{p,q} = \min_{e_u \in E_p, e_w \in E_q} \delta(e_u, e_w) \tag{16}$$

When e_u, e_w have a common end node, the distance $\delta(e_u, e_w) = 0$, except if the common node is the source or the destination of the path pair, in which case $\delta(e_u, e_w)$ is calculated as in [13]. More specifically, the distance between two edges $e_u = (s, v_1)$ and $e_w = (s, v_2)$ is the minimum of two distances: the distance between node v_1 and edge e_w and the distance between node v_2 and edge e_u – and similarly for a pair of edges with node t in common.

For each node pair (s, t), $D^{Max}_{st} = \max_{(p,q) \in \mathcal{P}^2_{s,t}} D_{p,q}$ represents the maximal geographical distance that can be achieved for any path pair between s and t. This problem requires the calculation of the maximum geodiversity that can be achieved between every node pair of interest for a demand originating in node s and terminating in node t. A formulation for solving the ancillary problem of calculation of D^{Max}_{st} can be found in [13].

In this context, as SRLG-disjointness is also a requirement, the calculation of the maximal geographical distance that can be achieved takes into account the value of μ_{st}, obtained when problem $\mathscr{P}_{1N}$ was solved. Let this maximal distance be identified by $D^{Max}_{st|\mu_{st}} = \max_{(p,q) \in \mathcal{P}^2_{s,t}: |R_{p,q}| = \mu_{st}} D_{p,q}$. Therefore, the path pairs considered in the calculation of $D^{Max}_{st|\mu_{st}}$ are those such that the minimal number of shared SRLGs is μ_{st}, as in the following formulation:

$$\max \xi$$

subject to: constraints (1), (10)-(14), (15) and

$$x_{j,1} + x_{k,2} - \vartheta_{jk} \leq 1 \quad \forall j, k = 1, \ldots, |A| \tag{17}$$

$$(\delta(a_j, a_k) - M)\, \vartheta_{jk} - \xi \geq -M \quad \forall j, k = 1, \ldots, |A| \tag{18}$$

$$\text{binary} \quad \vartheta_{jk} \quad \forall j, k = 1, \ldots, |A| \tag{19}$$

where M is a sufficiently large number.

The binary variable ϑ_{jk} is 0 if arcs j and k cannot be both in the path pair. Indeed if $\vartheta_{jk} = 0$, then equation (17) becomes $x_{j,1} + x_{k,2} \leq 1$ and therefore, if arc j is in the first path then arc k cannot be in the second path (and vice-versa). This will happen if these two arcs are not geodiverse. In this case, equation (18) becomes $\xi \leq M$ and is always satisfied. If they are geodiverse, then $\vartheta_{jk} = 1$ and both arcs can be used in the path pair and equation (18) becomes $\xi \leq \delta(a_j, a_k)$.

The solution of the previous problem for demand (s, t) is the value $D^{Max}_{st|\mu_{st}}$, which can be calculated in advance for all (s, t) pairs. This formulation implies that geodiversity is considered to be less relevant than the number of common SRLGs, i.e. the maximal distance is the best that can be achieved while still respecting the value of the minimal number of SRLGs in common obtained for node-disjoint path pairs.

One can define, for a desired D-geodiversity:

$$D_{st} = \min\left(D, D^{Max}_{st|\mu_{st}}\right) \tag{20}$$

We assume the networks are bi-connected, hence at least a node-disjoint pair of paths exists for every node pair in the network. By using value D_{st} when calculating a D-geodiverse path pair from s to t, we ensure a solution will always exist for any $D > 0$ in a bi-connected network, because we relax D to each node pair maximal possible value D_{st}.

For the calculation of the geodiversity we must obtain the geographical distance of each link to every other link and from each link to every node. To accomplish this, we take into account the coordinates of the nodes in a xy plane, rather than geographical information about the nodes or the edges (due to the difficulty in obtaining this type of information). The edges are considered to follow the shortest lines between the corresponding end nodes, which results in an *optimistic* calculation of the paths distances. We could have scaled up the obtained distance by some factor, or alternatively scale down the desired geodiversity. Therefore, the solution for a problem with a geodiversity of D km will be *optimistic* in the sense that the real geodiversity is probably lower.

For the formulation of optimization problems involving geodiversity constraints, we consider that D_{st} (as calculated in equation (20)) is known, along with the binary values $d_{k,j}$, which indicate whether arcs a_k and a_j are geodiverse (for a connection from s to t):

$$d_{k,j} = \begin{cases} 1 & \text{if } \delta(a_k, a_j) < D_{st} \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

2.4.2 Minimizing the total path-length of a path pair, under (maximal) SRLG-disjointness and geodiversity constraints

Problem $\mathscr{P}_{2G}$ is similar to problem $\mathscr{P}_{2N}$. It also relies on the result obtained for problem $\mathscr{P}_{1N}$ for each demand and some of the constraints are the same. The difference is that now a constraint on geodiversity is also considered.

$$\min \sum_{k=1}^{|A|} \ell_k \left(x_{k,1} + x_{k,2}\right)$$

subject to: constraints (1), (4)-(5), (10)-(14), (15) and

$$\left(x_{k,1} + x_{j,2}\right) d_{k,j} \leq 1 \quad \forall k, j = 1, \ldots, |A| \tag{22}$$

Constraint (22) guarantees the geodiversity. On the one hand, if $d_{k,j} = 0$ then the constraint is always satisfied, which means that the arcs a_k and a_j may be freely used in the paths, as they are geodiverse; on the other hand, if $d_{k,j} = 1$ then the constraint is satisfied only if the arcs a_k and a_j are not used in both paths, as they are not geodiverse.

A lexicographic optimization approach is used, as this problem is solved after having found a solution for problem $\mathscr{P}_{1N}$. Let Γ_{st} be the solution of problem $\mathscr{P}_{2G}$. This problem is another ancillary problem, which finds the value of Γ_{st} for each (s, t) demand, required for the formulation of $\mathscr{P}'_{2G}$.

2.4.3 Variant of $\mathscr{P}_{2G}$

Similarly to what was considered earlier, we want to find the maximally SRLG-disjoint path pair with geodiversity constraints that satisfies the total length Γ_{st} (with some pre-defined tolerance Δ_Γ) and for which the longest path has minimum length.

Problem $\mathscr{P}'_{2G}$ may be formulated as:

$$\min \mathcal{M}$$

subject to: constraints (1), (4)-(5), (10)-(14), (15), (22) and (7)-(9), where $\mathcal{L}_{st} = \Gamma_{st}$ and $\Delta_\mathcal{L} = \Delta_\Gamma$. Let $\Gamma^{\mathcal{M}}_{st}$ be the solution of this problem.

3 Calculation of transponders' reach and cost

After solving problems$\mathscr{P}_0$, $\mathscr{P}_{2N}$ and $\mathscr{P}_{2G}$ (and their respective variants), the length of the optimal path pairs for each demand is known. Given this information along with a traffic matrix with demands in Gb/s, the aim is to calculate the cost of each solution in terms of required Reconfigurable Optical Add-Drop Multiplexers (ROADMs). Note that the length of the paths determines the possible/feasible transponders that can be used, due to their optical reach. According to the optical reach, the feasible transponders may have different rates and capacities, that will have an impact on the cost of the transponders.

3.1 Transponder reach

According to the model proposed in [25] for the calculation of the maximum reach of a lightpath, we have to consider a certain threshold of the Bit Error Rate (BER), which is related to a minimal value of the Optical Signal-to-Noise Ratio (OSNR). In the calculation of the OSNR, the noise contribution from Amplified Spontaneous Emission (ASE) and Non-Linear Interference (NLI) due to the Kerr effect in the fibre are considered. The generalized Gaussian noise model may be used:

$$\text{OSNR} = \frac{P_{Tx,ch}}{P_{\text{ASE}} + P_{\text{NLI}}} \tag{23}$$

where $P_{Tx,ch}$ is the input power per channel (ch) of the transmitter (Tx).

The ASE noise power is given by

$$P_{\text{ASE}} = \left[N_s(G-1)Fh\nu\right] B_n \tag{24}$$

where B_n is the bandwidth at which the noise is measured, N_s is the number of amplifier spans, G is the performance gain of the Erbium Doped Fibre Amplifier (EDFA), F is the EDFA's noise figure, h is the Planck effect quantum, and ν is the frequency.

The noise power due to NLI is given by

$$P_{\text{NLI}} = \left(\frac{2}{3}\right)^3 N_s B_n \psi \tag{25}$$

$$\psi = \gamma^2 L_{eff} P_{Tx,ch}^3 \frac{\log_2\left(\pi^2 |\beta_2| L_{eff} N_{ch}^2 R_s^2\right)}{\pi |\beta_2| R_s^3} \tag{26}$$

where γ is the fibre nonlinearity coefficient, L_{eff} is the effective fibre length, N_{ch} is the

number of channels, R_s is the baud rate and β_2 is the fibre dispersion.

The expressions for P_{ASE} (equation (24)) and P_{NLI} (equation (25)) are replaced in equation (23). The resulting equation may be solved in order to get N_s. Therefore, the maximum number of spans for the considered minimal OSNR is given by

$$N_{s_{\max}} = \frac{P_{Tx,ch}}{\text{OSNR}_{\min} \cdot B_n \left((G-1)Fh\nu + \left(\frac{2}{3}\right)^3 \psi \right)} \tag{27}$$

Finally, the optical reach of a lightpath is given by $N_{s_{\max}} L_s$, where L_s is the length of the span.

3.2 Transponder cost

We consider normalized transponder costs calculated in reference to a state-of-the-art 100 Gb/s coherent transponder implemented by a single carrier with Dual-Polarization Quadrature Phase-Shift Keying (DP-QPSK) and roughly 34 Gbaud electronic speed.

The costs of more advanced transponders are assumed to mainly depend on three factors (see Table 1):

- the number of optical carriers (lasers). We assume that it is more cost-efficient to choose a transponder with two or more carriers than choosing two or more transponders with a single carrier.

- the maximum baudrate. We assume that doubling the maximal baudrate entails an increase in the cost of roughly one-third.

- the adaptability of the transponder capacity. A fixed transponder which supports a single modulation format and thus a single capacity only is cheaper than a modulation-flexible transponder supporting multiple modulation formats and related capacities. For example, DP-QPSK-only modulation is cheaper than the adaptable support of DP-QPSK, DP-8QAM (Quadrature Amplitude Modulation) and DP-16QAM modulations. Therefore, we assume that the capacity can be flexibly doubled. Let $C_{\max}$ and $C_{\min}$ represent the maximum and minimum capacity of the transponders induced by modulation adaptivity. The corresponding formula in Table 1 is based on [14, eqs. (33)-(34)], adapted so that doubling the capacity entails a 5% extra cost.

Table 1: Factors that influence the cost of transponders

Number of carriers	Carrier	1	2	4	
	Factor	1	1.6	2.56	
Maximal baudrate	Gbaud	34	42	52	65
	Factor	1	1.1	1.2	1.3
Adaptability of capacity	$\frac{C_{\max}}{C_{\min}} 0.525^{\log_2\left(\frac{C_{\max}}{C_{\min}}\right)}$				

4 Use of regenerative transponders

As mentioned earlier, the length of the paths determines the possible/feasible transponders that can be used, due to their optical reach. In particular, when a path pair is considered for a certain (s,t) demand, the longest of the two paths is determinant in selecting the most appropriate transponder to be used. Therefore, it is advantageous to have the longest path of the pair as short as possible, which is the reasoning behind the formulation of problems $\mathscr{P}'_0$, $\mathscr{P}'_{2N}$ and $\mathscr{P}'_{2G}$.

In the networks that will be considered, paths with length higher than the maximum reach of the available transponders are expected. For those long paths it is not possible to have a fully transparent path and the path will in fact be composed of fully transparent segments, for which appropriate regenerative transponders, herein simply termed as regenerators, will have to be used. Therefore we are considering regenerators at specific intermediate nodes, followed by fully transparent segments, where an impairment threshold is satisfied. The regenerators will be selected from the available set of transponders, already mentioned in Section 3.

In this work, we consider a regenerator placement problem, as tackled in [3], in particular the SRSRRP (Single Request Survivable Routing and Regenerator Placement) problem. The resolution of this problem aims at finding $\mathcal{R}_{st}$, which is the minimum number of intermediate nodes where regenerators will be necessary for a demand (s,t). Each demand is considered separately, i.e. the demands are not considered in a network as a whole. Therefore, dedicated regeneration is performed, in the sense that transponders are not shared among different demands.

Note that in each intermediate node, back-to-back regenerative transponders will be used. Also one transponder is necessary in the source node and another in the destination node, making the total number of used transponders $2\,(\mathcal{R}_{st}+1)$.

Other information that can be obtained from the solution of this problem is the regen-

erator placement (i.e. the intermediate nodes where regenerators are necessary) and the path pair for each demand that satisfies the constraints of each specific problem.

For the sake of generality, we assume that the two paths of the pair can share regenerators on nodes which are common to both the AP and the BP, which is designated in [3] as a dedicated-shared variant of the SRSRRP problem. We assume that the BP may be used only in the event of failure of the AP. Naturally when we consider node-disjoint path pairs, there will be no sharing of intermediate regenerators, but the transponders at the source and destination nodes may be shared.

Note that in the following calculations no information on the traffic for the demand is considered.

4.1 Notation

The impairment value η_k is known for each arc $a_k \in A$. The impairment threshold Δ_η for each transparent segment in a path is also known. There cannot be any segment in a path with impairment higher than that threshold Δ_η, where the impairment of the segment is the sum of the impairment values of the arcs in that segment.

The binary decision variables used in the formulation are:

- $x^i_{k,m}$ is 1 if arc a_k belongs to path p_m and node v_i is its last regenerator node (or the source node) before encountering the arc a_k; it is 0 otherwise;
- $\sigma^{a,b}_m$ (with $a \neq b$) is 1 if path p_m uses a regenerator at node v_a directly followed by a regenerator at node v_b (where v_a may be the source node, which always has a transponder); it is 0 otherwise;
- ς^i is 1 if a regenerator (that may be shared) is necessary at node v_i; it is 0 otherwise.

4.2 Formulation of problem $\mathscr{P}^\eta_0$

The problem $\mathscr{P}^\eta_0$ of finding a link-disjoint path pair for a specific (s, t) demand with regenerators along those paths, such that the number of regenerators for both paths is minimal, is tackled in [3]. In particular, we are considering the variant identified as dedicated-shared by the authors, with an additional constraint on the total length of the path pair and constraints on the difference between the length of the AP and the BP.

This problem may be formulated as:

$$\min \sum_{i=1}^{|V|} \varsigma^i$$

subject to:

$$\sum_{k \in \mathcal{E}(s^+)} x^s_{k,m} = 1 \quad \forall m = 1, 2 \tag{28}$$

$$\sum_{k \in \mathcal{E}(b^-)} x^a_{k,m} - \sum_{k \in \mathcal{E}(b^+)} x^a_{k,m} = \sigma^{a,b}_m \quad \forall v_b \in V \setminus \{s; t\}; v_a \in V : a \neq b; m = 1, 2 \tag{29}$$

$$\sum_{k \in \mathcal{E}(b^+)} x^b_{k,m} - \sum_{\substack{a=1 \\ a \neq b}}^{|V|} \sigma^{a,b}_m = 0 \quad \forall v_b \in V \setminus \{s; t\}; m = 1, 2 \tag{30}$$

$$\sum_{\substack{a=1 \\ a \neq b}}^{|V|} \left(\sigma^{a,b}_1 + \sigma^{a,b}_2 \right) \leq 2\varsigma^b \quad \forall v_b = 1, \ldots, |V| \tag{31}$$

$$\sum_{\substack{i=1 \\ v_i \neq t}}^{|V|} \sum_{k \in \mathcal{E}(t^-)} x^i_{k,m} = 1 \quad \forall m = 1, 2 \tag{32}$$

$$\sum_{i=1}^{|V|} \left(x^i_{k,1} + x^i_{k,2} \right) \leq 1 \quad \forall k = 1, \ldots, |A| \tag{33}$$

$$\sum_{i=1}^{|V|} \left(x^i_{k,1} + x^i_{k',2} \right) \leq 1 \quad \forall k = 1, \ldots, |A| \tag{34}$$

$$\sum_{k \in \mathcal{E}(s^-)} \sum_{i=1}^{|V|} \left(x^i_{k,1} + x^i_{k,2} \right) = 0 \tag{35}$$

$$\sum_{k \in \mathcal{E}(s^+)} \sum_{\substack{i=1 \\ v_i \neq s}}^{|V|} \left(x^i_{k,1} + x^i_{k,2} \right) = 0 \tag{36}$$

$$\sum_{k \in \mathcal{E}(b^-)} \sum_{\substack{a=1 \\ a \neq b}}^{|V|} x^a_{k,m} \leq 1 \quad \forall v_b \in V \setminus \{s\}; m = 1, 2 \tag{37}$$

$$\sum_{i=1}^{|V|} \left(x^i_{k,m} + x^i_{k',m} \right) \leq 1 \quad \forall k = 1, \ldots, |A|; m = 1, 2 \tag{38}$$

$$\sum_{k=1}^{|A|} \eta_k x_{k,m}^i \leq \Delta_\eta \quad \forall i = 1, \ldots, |V|; m = 1, 2 \tag{39}$$

$$\sum_{k=1}^{|A|} \left(\ell_k \sum_{i=1}^{|V|} \left(x_{k,1}^i + x_{k,2}^i \right) \right) \leq \mathcal{L}_{st} \Delta_{\mathcal{L}} \tag{40}$$

$$\sum_{k=1}^{|A|} \left(\ell_k \sum_{i=1}^{|V|} \left(x_{k,1}^i - x_{k,2}^i \right) \right) \leq \mathcal{M}_{st} \Delta_{\mathcal{M}} \tag{41}$$

$$-\left(\sum_{k=1}^{|A|} \left(\ell_k \sum_{i=1}^{|V|} \left(x_{k,1}^i - x_{k,2}^i \right) \right) \right) \leq \mathcal{M}_{st} \Delta_{\mathcal{M}} \tag{42}$$

$$\text{binary} \quad x_{k,m}^i, \sigma_m^{a,b}, \varsigma^i \quad \forall k = 1, \ldots, |A|; i, a, b = 1, \ldots, |V| \text{ (with } a \neq b); m = 1, 2 \tag{43}$$

where $\mathcal{L}_{st} = \theta_{st}$ (obtained when $\mathscr{P}_0$ was solved) and $\Delta_{\mathcal{L}} = \Delta_\theta$ (a pre-defined tolerance, which may be different from the one considered in the resolution of $\mathscr{P}_0'$ – we considered it to be the same). The value of $\mathcal{M}_{st}$ is $\theta_{st}^{\mathcal{M}}$ (obtained when $\mathscr{P}_0'$ was solved) and $\Delta_{\mathcal{M}}$ is a tolerance value.

Constraints (28)-(32) are the usual flow conservation constraints. Constraint (28) is formulated for the source node, where a transponder should be placed (although it is not accounted for in the objective function, due to the format of constraint (31)) and it guarantees that an arc leaving s will be considered for each of the paths.

Constraints (29)-(30) are formulated for the intermediate nodes and they establish the relations between the x variables (that identify the arcs of the path) and the σ variables (that identify the sequence of nodes where regenerators will be necessary). As for constraint (31), it establishes the relation between the sequence of nodes where regenerators will be necessary and the consequent placement of regenerators, represented by the ς variables.

Constraint (32) is formulated for the destination node, to guarantee that the paths will end in this node.

Constraints (33)-(34) are link-disjointness constraints and they guarantee that: an arc (or an arc and its symmetrical) cannot be used simultaneously in both paths.

Constraints (35)-(38) guarantee that no cycles (or loops) occur. Constraints (35)-(36) are formulated for the source node and they guarantee that: no arcs of the path enter the source node (35); there are no transponders in the path before the source node (36).

Constraint (37) is formulated for the intermediate nodes and the destination node and it guarantees that arcs entering a node v_b may have transponders in other nodes before that node v_b is encountered in a path.

Constraint (38) guarantees that an arc and its symmetrical cannot be used simultaneously in the same path.

Constraint (39) is the impairment constraint, and it guarantees that any segment between two consecutive transponders will have an impairment value under the maximal possible impairment threshold. As explained earlier, the impairment of a transparent segment is the sum of impairments of the arcs in that segment.

Constraint (40) is a constraint on the total length of the path pair, similarly to constraint (7) previously used in the formulation of problem $\mathscr{P}_0'$.

Constraints (41) and (42) are similar to constraints (8) and (9) and they guarantee that the length of the longest path of the pair is minimal.

4.3 Formulation of problem $\mathscr{P}_{2N}^{\eta}$

The problem $\mathscr{P}_{2N}^{\eta}$ of finding a node-disjoint and maximally SRLG-disjoint path pair for a specific (s,t) demand with regenerators along those paths, such that the number of regenerators for both paths is minimal, may be formulated as:

$$\min \sum_{i=1}^{|V|} \varsigma^i$$

subject to: constraints (28)-(32), (35)-(43), (13), (15) and

$$x^s_{(s,t),1} + x^s_{(s,t),2} \leq 1 \tag{44}$$

$$\sum_{k \in \mathcal{E}(b^+)} \sum_{a=1}^{|V|} \left(x^a_{k,1} + x^a_{k,2} \right) \leq 1 \quad \forall v_b \in V \setminus \{s,t\} \tag{45}$$

$$\sum_{k=1}^{|A|} h_{r,k} \left(\sum_{i=1}^{|V|} x^i_{k,m} \right) \leq |A| z_{r,m}, \quad \forall r = 1, \ldots, \varrho; m = 1,2 \tag{46}$$

$$\text{binary} \quad z_{r,m}, g_r \quad \forall m = 1,2; r = 1, \ldots, \varrho \tag{47}$$

where $\mathcal{L}_{st} = \Lambda_{st}$ and $\Delta_{\mathcal{L}} = \Delta_{\Lambda}$. Also $\mathcal{M}_{st} = \Lambda^{\mathcal{M}}_{st}$ and $\Delta_{\mathcal{M}}$ is a tolerance value.

In this formulation, we have considered node-disjointness constraints (44)-(45) similar

to equations (10)-(11). Naturally in this problem the regenerators cannot be shared.

Constraints (13) and (46), which is similar to equation (12), are SRLG-disjointness constraints. The minimal number of SRLGs in common is guaranteed by constraint (15).

4.4 Formulation of problem $\mathscr{P}_{2G}^{\eta}$

The problem $\mathscr{P}_{2G}^{\eta}$ of finding a geodiverse (for distance D_{st}) and maximally SRLG-disjoint path pair for a specific (s,t) demand with regenerators along those paths, such that the number of regenerators for both paths is minimal, may be formulated as:

$$\min \sum_{i=1}^{|V|} \varsigma^i$$

subject to: constraints (28)-(32), (35)-(47), (13), (15) and

$$\sum_{i=1}^{|V|} \left(x_{k,1}^i + x_{j,2}^i\right) d_{k,j} \leq 1 \quad \forall k,j = 1,\ldots,|A| \tag{48}$$

where $\mathcal{L}_{st} = \Gamma_{st}$ and $\Delta_{\mathcal{L}} = \Delta_{\Gamma}$. Also $\mathcal{M}_{st} = \Gamma_{st}^{\mathcal{M}}$ and $\Delta_{\mathcal{M}}$ is a tolerance value. The parameters $d_{k,j}$ were defined in equation (21), according to the distance D_{st} in equation (20).

In this formulation, we have considered the geodiversity constraint (48) similar to equation (22). Naturally in this problem the regenerators cannot be shared.

4.5 Selection of transponders

In paper [22], the selection of the appropriate transponders was made in such a way that if a single transponder could not support the necessary traffic volume, the traffic was evenly split (in halves or in quarters) and all the necessary transponders (2 or 4) were equivalent. A more accurate way of performing these calculations may lead to a better selection of transponders (i.e. with a lower total cost) to be used in a fully transparent optical network or for a transparent segment.

Assuming the following data is known for each demand:

- $\mathscr{B}_{st}$: length of the longest transparent segment of the path pair for the (s,t) demand. Note that this value may be equal to the longest path of the path pair (usually it is the BP) for the (s,t) demand if no regenerative transponders are needed in any intermediate node for this demand;

- $\mathscr{D}_{st}$: traffic for the (s,t) demand.

and for each transponder of type $\tau \in \mathcal{T}$:

- reach_τ: optical reach [km], calculated as described in subsection 3.1;
- capacity_τ: capacity [GB/s];
- cost_τ: cost [a.u.], calculated as described in subsection 3.2.

The binary decision variables

$$y_{\tau\omega} = \begin{cases} 1 & \text{if a total of } \omega \text{ transponders of type } \tau \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

are defined for each transponder type $\tau \in \mathcal{T}$. We assume the maximum number of transponders of a certain type to be used for each demand is given by Ω, so ω may take one of the values $1, \ldots, \Omega$. The number ω of transponders of a certain type τ to be used is related to the number of sub-flows in which a flow will be split. In fact, if a single type τ of transponder is used for the (s,t) demand, then $\omega = \left\lceil \frac{\mathscr{D}_{st}}{\text{capacity}_\tau} \right\rceil$ assuming the transponder of type τ satisfies $\text{reach}_\tau \geq \mathscr{B}_{st}$.

The following problem is formulated and solved for each demand:

$$\min \sum_{\tau \in \mathcal{T}} \left(\text{cost}_\tau \sum_{\omega=1}^{\Omega} \omega y_{\tau\omega} \right)$$

subject to:

$$\text{reach}_\tau \geq \mathscr{B}_{st} y_{\tau\omega} \quad \forall \tau \in \mathcal{T}; \omega = 1, \ldots, \Omega \tag{49}$$

$$\sum_{\tau \in \mathcal{T}} \left(\text{capacity}_\tau \sum_{\omega=1}^{\Omega} \omega y_\tau \right) \geq \mathscr{D}_{st} \tag{50}$$

$$\sum_{\omega=1}^{\Omega} y_{\tau\omega} \leq 1 \quad \forall \tau \in \mathcal{T} \tag{51}$$

$$\text{binary} \quad y_{\tau\omega} \quad \forall \tau \in \mathcal{T}; \omega = 1, \ldots, \Omega \tag{52}$$

If $\text{reach}_\tau \leq \mathscr{B}_{st}$, then the transponder of type τ cannot be used for this demand. Note that in this case, for equation (49) to be satisfied, the value of $y_{\tau\omega}$ must be 0 for every ω, and therefore this transponder type will not be selected. If $\text{reach}_\tau \geq \mathscr{B}_{st}$, then the

transponder of type τ may or may not be used for this demand, in the quantity given by ω. Equation (49) is always satisfied in this case, regardless of the value of $y_{\tau\omega}$. In order to solve this problem, we need all the results $\mathscr{B}_{st}$ for each of the problems ($\mathscr{P}_0^\eta$, $\mathscr{P}_{2N}^\eta$ or $\mathscr{P}_{2G}^\eta$) for any network.

4.6 Resolution approach

The resolution approach to find the set of transponders with minimal cost to be used for each demand, is a lexicographic one.

1. We start by solving one of the problems $\mathscr{P}_0^\eta$, $\mathscr{P}_{2N}^\eta$ or $\mathscr{P}_{2G}^\eta$, with the impairment threshold $\Delta_\eta = \max_{\tau\in\mathcal{T}} \text{reach}_\tau$. We will assume that the impairment value of an arc is the length of the arc, i.e. $\eta_k = \ell_k$. Let $\mathcal{R}_{st}$ be the solution obtained, i.e. the minimal number of intermediate nodes where regenerators will be necessary, for demand (s,t). Each transparent segment in any of the paths in the pair has a maximal length of Δ_η, which is guaranteed by constraint (39).

2. Next we solve a problem aiming at the minimization of the maximal length of any transparent segment in any of the paths of the pair, subject to a constraint on $\mathcal{R}_{st}$, which is the minimal number of intermediate nodes where regenerators will be necessary. This will allow us to consider transponders with a reach lower than $\max_{\tau\in\mathcal{T}} \text{reach}_\tau$.

   In this problem, the value of Δ_η is not known beforehand and is actually the parameter that we wish to minimize. A problem formulated as

   $$\min \Delta_\eta$$

   subject to: constraints of the considered problems $\mathscr{P}_0^\eta$, $\mathscr{P}_{2N}^\eta$ or $\mathscr{P}_{2G}^\eta$ and

   $$\sum_{i=1}^{|V|} \varsigma^i \leq \mathcal{R}_{st} \tag{53}$$

   is solved.

   Let $\mathcal{S}_{st}$ be the solution obtained, i.e. the length of the longest transparent segment in any of the paths of the pair for demand (s,t). Note that if $\mathcal{R}_{st} = 0$, then $\mathcal{S}_{st}$ is actually the length of the longest path in the pair.

Table 2: Network characteristics: $|V|$, $|E|$, average node degree (AND) [24], link density (LD) [24], network diameter (ND), average clustering coefficient (ACC), assortativity

Network	$\|V\|$	$\|E\|$	AND	LD	ND	ACC	Assort.
polska	12	18	3.00	27.27%	4	0.1472	-0.0435
nobel-eu	28	41	2.93	10.85%	8	0	0.0534
cost266	37	57	3.08	8.56%	8	0	-0.0151

Note that the solutions for the path pairs obtained after this resolution approach should be very close to the ones obtained when $\mathscr{P}'_0$, $\mathscr{P}'_{2N}$ or $\mathscr{P}'_{2G}$ were solved, because of constraints (40)-(42).

3. Finally, we select the actual transponders to be used. This is a simplification, as we will assume that all the segments in the paths will have the same set of transponders, regardless of the actual length of each individual segment.

   To achieve this, we consider the problem defined in Subsection 4.5 for the selection of transponders of minimal cost, with $\mathscr{B}_{st} = \mathcal{S}_{st}$. Note that in this problem information on the traffic $\mathcal{D}_{st}$ for the demand is a parameter of the problem. Let the solution be $\mathcal{C}_{st}$.

4. The final cost value will be $2\left(\mathcal{R}_{st} + 1\right) \cdot \mathcal{C}_{st}$.

5 Results

In this paper, we consider three different networks, all from the sndlib set [24], for which some topology characteristics are displayed in Table 2. The polska network has a dimension similar to the DT-backbone network provided by Deutsche Telekom (DT) and considered in [22]. The other two networks have a higher dimension and they present paths longer than the ones in the DT-backbone network or the polska network, as they span a much wider geographical area. These longer paths ensure that there is the need for regeneration.

The topological information included in the table provides some insight into the networks: (i) the average node degree, which is similar to all the networks; (ii) the link density, which is the highest for the polska network (it is the densest of the considered networks); (iii) the network diameter, i.e. the highest of the minimal hop count between all pairs of nodes in the network, is the same for the nobel-eu and the cost266 networks; (iv) for these two networks, the average clustering coefficient [33] is 0, which goes to show that they are

not very dense; (v) the assortativity [23] of the nobel-eu network is positive, indicating a correlation between nodes of similar degree.

In the sndlib files, information on the longitude and latitude of the nodes for the networks is available. Given this information, the coordinates of the nodes in a xy plane were obtained by a sinusoidal projection [5]. The coordinates of the nodes in the xy plane are used to calculate the length of the edges which are simply the Euclidean distance between the end-nodes of the edges. They are also used to calculate the geodiversity in terms of the geographical distance of each link to every other link and from each link to every node, as explained earlier.

For the polska and nobel-eu networks, we considered ten different instances with different SRLG sets, which were generated as described in [18], which should allow for a realistic SRLG distribution. For the cost266 network five different instances were considered.

The number of generated SRLGs is half the number of edges in each network and for each generated SRLG, the target number of edges is 2 to 4 (randomly (uniformly) selected value). For each edge, the target number of SRLGs is 0 to 4 (uniformly selected value). The generation of each SRLG starts with a random (uniform) selection of a node. Afterwards, the edges with an end node at a distance less than a pre-defined value χ from the selected node are candidates for that SRLG. Each candidate edge which hasn't exceeded its assigned SRLG target yet, may be included in the SRLG with a probability of 10%.

In this generation of SRLGs, the value of χ is defined for each network, based on the average and minimum distance between all the node pairs in that network (Euclidean distance between the xy coordinates of the nodes). Therefore, the edges in each SRLG are located in a χ-radius neighborhood, i.e. they are in close geographic proximity and should share common risk faults.

The information on the traffic matrices for each network is based on the information provided in the sndlib files and on the estimated population served by each network. The approach we used is arguable as the total traffic volume in a backbone network also depends on the number of backbone nodes, but we still think it may be used for the sake of experimentation. Note that information on the traffic matrices in a network is not available from network operators, so some kind of estimation always has to be considered.

Considering the total traffic in the matrix provided by DT for the DT-backbone network in [22] and the total population of Germany, we get an estimate of traffic per user (denoted by $\wp$). Given estimates of the populations served by each of the networks, a multiplication of those values of population per $\wp$ gives the total traffic we should have

Table 3: Average value of the lengths (in km) of the edge-disjoint paths of min-sum length for the nobel-eu network

Problem	Average length of AP [km]	Average length of BP [km]
$\mathscr{P}_0$	1382.01	2064.31
$\mathscr{P}'_0$	1426.22	2020.11

for each network. Taking the traffic matrix provided in sndlib for each network as a basis, we applied a multiplication factor to each demand, so that the total traffic has the desired value. Afterwards, a final correction factor of 2 to 5 is applied to account for higher traffic values. Note that in a backbone network, the traffic values are related not only to the population, but also to the number of nodes in the backbone.

All the ILP problems were solved using CPLEX 12.8 [1].

Regarding the lengths of the paths for the AP, the BP and the path pair, we will focus on the results (in terms of the paths) of problems: (i) $\mathscr{P}_0^\eta$ – edge-disjoint path pair of min-sum length, with minimal length for the longest path of the pair; (ii) $\mathscr{P}_{2N}^\eta$ – node-disjoint path pair with minimal number of SRLGs in common and min-sum length, with minimal length for the longest path of the pair; (iii) $\mathscr{P}_{2G}^\eta$ – path pair with minimal number of SRLGs in common and min-sum length, with minimal length for the longest path of the pair and geodiversity constraints, with a distance D – problem $\mathscr{P}_{2G}^\eta(D)$. We considered different values of D including a value sufficiently high so that for all the demands the value of D_{st} is $D_{st|\mu_{st}}^{Max}$.

As mentioned previously, the problems described in Section 2 are instrumental, to find out the values of θ_{st}, μ_{st}, Λ_{st}, $\mathcal{M}_{st}^\Lambda$, $\Gamma_{st}(D)$ and $\mathcal{M}_{st}^\Gamma(D)$. The tolerance values Δ are always 1.001.

For illustration of the benefits of the use of the path pair in which the longest path of the pair has minimal length, results for the average value of the lengths of the paths is displayed in Table 3 for the nobel-eu network, when $\mathscr{P}_0$ and $\mathscr{P}'_0$ were run. As expected, the length of the BP is slightly lower for $\mathscr{P}'_0$, which is accompanied by a slight increase in the length of the AP. The variation is not very high, but it could be enough to allow for the use of cheaper transponders. This is the reason why the following results focus on the path pairs for which the longest path has minimal length.

In Figure 1 results for the relative variation of the lengths of the paths with respect to the corresponding length of the path obtained with $\mathscr{P}_0^\eta$ are displayed. Different instances with different SRLG sets were considered. The average values are in the bars and the

standard deviation values are in the lines.

The results for node- and maximally SRLG-disjointness (min#SRLG) and the results for maximally SRLG-disjointness and geodiversity constraints with smaller values of D are similar. This shows that node-disjointness along with maximally SRLG-disjointness already provides some geodiverse results.

Networks polska, nobel-eu and cost266 have different sizes and different distances to be considered. It is noticeable that in the smaller network, the results for geodiversity are very different for D varying from 50 km to 150 km. The results for Dmax are similar to the results for `D=150km`. As for the larger networks, the results for lower D do not show a great variation, which again reinforces the idea that for large networks, node-disjointness and maximally SRLG-disjointness guarantee some geodiversity. The average variation of path length for Dmax is clearly higher than for lower D. The standard variation values are relatively large, possibly due to the impact of selected SRLGs.

For the cost analysis, the parameters used to calculate the transponder optical reach (see equation (27)) were: $N_{ch} = 96$; $\gamma = 1.37$ [(W.km)$^{-1}$]; $L_{eff} = (1 - e^{-2\alpha L_s})/(2\alpha)$; $\alpha = 0.22$ [dB/km]; $L_s = 80$ [km]; $\beta_2 = -\mathcal{D}\lambda^2/(2\pi c)$ [ps^2/km]; $\mathcal{D} = 17$ [ps/(km·nm)]; $B_n = 12.48$ [GHz]; $G = 10^{0.1\alpha L_s}$; $F = 6.5$ [dB]; $P_{Tx,ch} = -2$ [dBm]; and c the light velocity in free space. Additionally it was considered an OSNR penalty of 4 [dB] which accounts for ageing, implementation imperfections, and additional imperfections inside the fibre infrastructure.

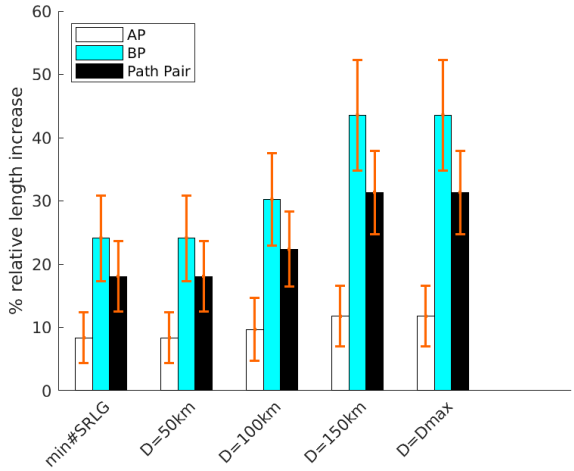
We considered different types of transponders with different properties (cost, adaptable capacity and reach). The reach and normalized cost of each transponder were calculated as explained in Section 3.

The cheapest transponders capable of transporting the traffic of a demand in both paths of the pair were always selected. As explained in Subsection 4.5, if a single transponder cannot support the necessary end-to-end traffic volume, the traffic must be split and more transponders will be required in a fully transparent optical network or for a transparent segment.

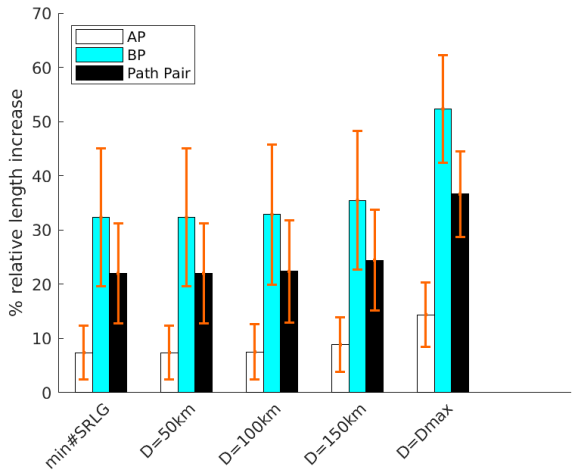
As explained in Section 4, if a fully transparent path is not possible, then the maximum length transparent segment in any of the paths in the pair is considered in the calculation of the necessary transponders.

Note that in these problems, no fibre capacity constraints were taken into account.

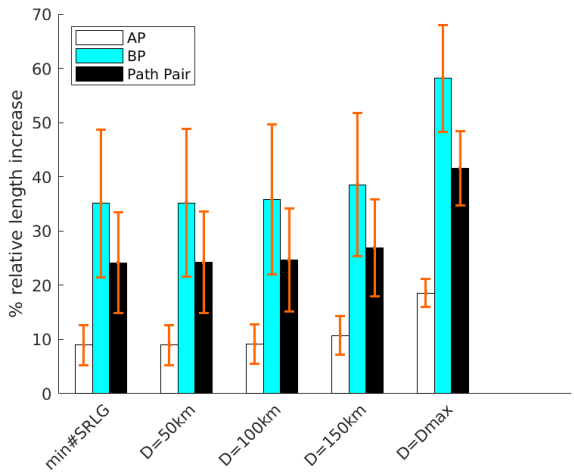
In Figure 2, results for the relative variation of the costs of the transponders to be used in the paths, with respect to the corresponding cost of transponders to be used in the path



(a) Results for the polska network



(b) Results for the nobel-eu network



(c) Results for the cost266 network

Figure 1: Variation of the lengths of the paths (AP, BP and path pair) obtained for the different problems $\mathscr{P}^{\eta}_{2N}$ (min#SRLG) and $\mathscr{P}^{\eta}_{2G}$ for different distances D, in relation to the results of the basic solution (obtained with problem $\mathscr{P}^{\eta}_{0}$)

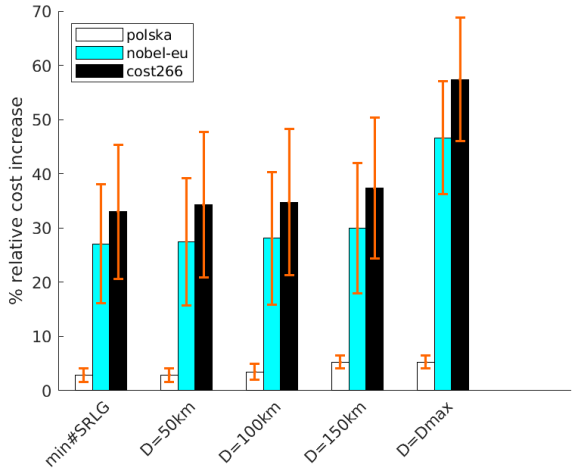


Figure 2: Variation of the cost of the transponders for the different problems $\mathscr{P}^{\eta}_{2N}$ (min#SRLG) and $\mathscr{P}^{\eta}_{2G}$ for different distances D, in relation to the results of the cost of the basic solution (obtained with problem $\mathscr{P}^{\eta}_{0}$)

pair obtained with $\mathscr{P}^{\eta}_{0}$ are displayed. Different instances with different SRLG sets were considered. The average values are in the bars and the standard deviation values are in the lines.

The pattern of variation of the transponders costs is similar to the pattern for the path lengths, as expected as one of the main parameters influencing the cost of the transponders is the reach of the transponder. For longer paths, transponders with longer reachs are necessary.

Notice however, that the variation is not in the same proportion. For instance, for the nobel-eu network, the relative variation of length of the BPs (usually the longest path of the pair) increases from 35.5% to 52.3% when D varies from 150 km to Dmax, whereas the relative variation of cost of the transponders in the same situation increases from 30.0% to 46.6%.

6 Conclusion

This work focuses on the impact of SRLG-disjointness in the length of a path pair for each demand in a network. Problems for devising (i) edge-disjoint path pairs, (ii) node-disjoint and maximally SRLG-disjoint path pairs, and (iii) maximally SRLG-disjoint path pairs with geodiversity were formulated, in order to find the path pair with min-sum length, such that the longer path of the pair has minimal length, for each of the cases.

Considering the results for the performed experiments, we realize that SRLG-disjointness entails a certain geodiversity, as the results for maximally SRLG-disjointness with and without geodiversity constraints are similar for the smaller values of D. This was expected, as the SRLG sets were generated taking into account some geographical information. This result shows that associating links in SRLGs that encompass some geographical information and finding node- and maximally SRLG-disjoint path pairs should guarantee some degree of geodiversity, which is obviously desired in disaster-prone areas. Therefore, some benefits of geodiversity may be accomplished by simply guaranteeing maximally SRLG-disjointness (with node disjointness as well).

The cost of the transponders to be used for such paths was also analysed. If a transparent path is not possible, regenerators will be necessary in intermediate nodes. A problem for calculation of the number and location of regenerators for each path pair is put forward and solved. Another outcome of this problem is the path pair itself. The actual selection of the transponders to use is accomplished by solving another ILP problem, aiming at the minimal cost of the set of transponders that satisfy the reach of the longest transparent segment in any of the path pairs. For the selection of transponders, information on the traffic for every demand is taken into account.

As the required geodiversity distance increases, the length of the paths also increases, which means that the cost of transponders will be higher. Note however that the cost increase is not proportional to the path length increase.

Regarding future work, we plan to consider the selection of transponders in a more refined way, so that the length of each segment in the path is taken into account, and not only the length of the longest transparent segment. We also plan to formulate a problem that encompasses the different aspects in transponder selection, rather than the lexicographical approach that was proposed here.

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