Regeneration and Grooming in Impairment-Aware Optical WDM Networks

Bruna Nogueira[1], Ricardo Mendes[1],
Lúcia Martins[12], Teresa Gomes[12], Rita Girão-Silva[12]
[1] Department of Electrical and Computer Engineering,
Faculty of Sciencies and Technology of University of Coimbra,
Pólo II, Pinhal de Marrocos 3030-290 Coimbra Portugal
[2] Institute for Systems and Computers Engineering at Coimbra,
Rua Sílvio Lima, Pólo II, 3030-290 Coimbra Portugal

João Santos[3] and Tibor Cinkler[4]
[3] Infinera Portugal,
Rua Irmãos Siemens, 1-1A
2720-093 Amadora, Portugal
[4] Department of Telecomm.and Media Informatics,
Budapest University of Technology and Economics,
Budapest, Hungary

This work addresses the Grooming, Routing and Wavelength Assignment with Regeneration (GRWAR) problem for meshed networks, in static traffic scenarios. It focuses primarily on the Grooming and Routing with Regeneration (GRR) problem. Physical impairments were considered in the model, imposing a limit in lightpath lengths without regeneration. This creates the opportunity of strategically placing transponders for both grooming and regeneration. The GRR problem is tackled lexicographically, which means that for a given network topology and a traffic matrix, the goal is to route and groom connection requests in a way that minimizes the number of transponders and, for that number of transponders, throughout minimum length routes. To solve this problem, an Integer Linear Programming (ILP) formulation was developed considering undirected networks and explicitly including the relationship between the different network layers. The results of the ILP model are used to validate a heuristic suited to be included in a Software Defined Network (SDN) platform in a multilayer core network.

The GRWAR problem is addressed after a solution for the GRR problem is obtained, using a first-fit approach to assign wavelengths to the lightpaths obtained by the ILP and the heuristic. The wavelength assignment problem is less critical, due to the large number of wavelengths per fiber.

The ILP and the heuristic were compared for small networks, with the heuristic providing good results in very short time, proving to be efficient and better suited for larger problems, as is confirmed through a set of proposed bounds.

***Index Terms*—Shortest Path, Traffic Grooming, Regeneration, Transponder, ILP, Heuristic, WDM**

I. Introduction

The problem of routing, assigning wavelengths and network resources to traffic demands (without traffic grooming) is well known as the Routing and Wavelength Assignment (RWA) problem [1]. The RWA problem applies to transparent networks and, in its purest form, does not account for wavelength conversion [2]. Demands must be routed end-to-end in a way such that if the physical paths assigned to them share an edge, the wavelengths assigned to those paths are different [2]. Typically, the metrics to minimize are the number of wavelengths, congestion, or a combination of the two [3]. When grooming is considered, the problem can be divided into a series of subproblems [4], which are solved in order to meet a network design or operational goal:

(1) Finding a virtual topology with a set of lightpaths.
(2) Routing of traffic demands in the virtual topology.
(3) Routing and wavelength assignment of lightpaths over the physical topology.

Most studies on the traffic grooming problem deal with all of the mentioned subproblems, addressing the Grooming, Routing and Wavelength Assignment (GRWA) problem [1], [5]. However, some works focus only on the virtual layer subproblems [6], addressing the Grooming and Routing (GR) problem, others study both the complete problem and some of the subproblems individually [5]. Note that the routing part of the GR problem can refer to routing only in the virtual layer or go deeper into the physical layer, disregarding only the wavelength assignment. As works on both impairment-aware and non-impairment-aware traffic grooming are herein referred, the problems RWA, GRWA and GR with regeneration constraints are referred to as Routing and Wavelength Assignment with Regeneration (RWAR), Grooming, Routing and Wavelength Assignment with Regeneration (GRWAR) and Grooming and Routing with Regeneration (GRR). The RWA problem is NP-complete [4], and since it is an integrating part of the GRWA/GRWAR problems, these problems are also NP-complete [4].

The strategic placement of transponders and regenerators is commonly addressed as a problem of translucent network design, where the goal is to obtain a translucent network that can achieve a network utilization equivalent to that of an opaque network while using a much smaller amount of strategically placed transponders/regenerators.

Traditionally, the problems of traffic grooming and regenerator placement are handled sequentially, although there are a few works that explore the combination of both [4], [7], [8]. These works address the problem of impairment-aware traffic grooming in Wavelength Division Multiplexing (WDM) networks, where the aim is to route traffic and lightpaths and to place regenerators and electronic grooming equipment in a way that minimizes the network cost. Although there are multiple sources of impairments [9], for a design problem like regenerator placement, a single impairment metric is sufficient, which could correspond to the worst of all impairments, or al-

Corresponding author: L. Martins (email: lucia@deec.uc.pt).

ternatively the metric of distance as it represents a determinant role in the signal quality [10].

The works that may be considered closer to our work are the ones that address the GRWAR problem. *Patel et al* [4] address the problem of traffic grooming and regenerator placement in a WDM optical network considering hop-constrained lightpaths due to physical impairments. An Integer Linear Programming (ILP) model for a directed network is formulated and an auxiliary graph based polynomial-time heuristic is proposed for non-blocking scenarios. A detailed Reconfigurable Optical Add-Drop Multiplexer (ROADM) node architecture and an associated cost model are proposed. Results show that jointly placing regenerators and electronic grooming equipment in the network reduces the network cost significantly. *Scheffel et al* [8] propose a “path over path” concept mapping traffic demands into grooming flows that are provisioned as transparent lightpaths. An ILP formulation to compute the optimal network design accounts for protected and unprotected traffic, assumes bidirectionality for every path, and solves grooming and routing in the physical layer. Wavelength assignment is performed in a subsequent optimization step by another ILP. The scalability of the problem size is enabled through the limitation of the solution space by an a priori selection of eligible grooming and physical paths. *Plunkte et al* [7] address the problem of the design of Dense Wavelength Division Multiplexing (DWDM) networks aiming at the minimization of its cost. Three different network types are considered: opaque, transparent and translucent networks. For each of them, a ILP formulation is proposed for routing the demands and considering a cost-effective data aggregation by muxponders. All formulations address both unprotected and protected (1+1) traffic.

More recent papers addressing the use of traffic grooming in WDM networks are [11–13]. In [11], variants of two metaheuristics for sparse grooming (i.e. grooming can only be performed in some nodes) are proposed. The authors consider dynamic traffic in the performed experiments, much as in [12], where a dynamic multicast traffic grooming problem in light trail optical WDM mesh networks is studied. In [13], a genetic algorithm heuristic for RWA is proposed, where grooming is performed and aiming at the minimization of the number of wavelengths and the cost of wavelength conversion and grooming.

More recent proposals considering traffic grooming are related with Elastic Optical Networks (EON) where subwavelength or superwavelength bandwidth channels can be assigned to each lightpath allowing a more efficient use of the available spectrum and avoiding the existence of excessive unused bandwidth in each lightpath [14], along with energy savings [15]. Note that the flexibility associated with the elasticity entails a higher complexity in the network design and control, but traffic grooming may help in reducing the impact of this increased complexity [16]. In EON, grooming can be done at the optical level, electronic level or both, giving rise in the latter case to the routing and spectrum assignment problem due to spectrum-continuity constraints. For a recent survey on different traffic grooming techniques performed at the optical level in EONs, see [17].

The flexibility of EON related with the granularity of the bandwidth assigned to each lightpath makes these networks suitable for dynamic traffic grooming [18]. A generic graph model similar to the one presented in [19] was used to represent the network at electronic and multiple spectrum layers. As in traditional WDM, a mixed strategy considering grooming protection in Internet Protocol (IP) over EON is also presented in [20] to improve network resilience. The benefits of traffic grooming in EON networks are also analysed in: [21], where impairments are modeled according to a nonlinear function and traffic grooming proves to be successful in reducing both the service blocking ratio and the number of used transceivers; [22], where a MILP problem is formulated and solved for small networks (and a heuristic for larger networks), aiming at minimizing the maximum slot index among all fibers; [23], where traffic grooming is considered in a problem aiming at the optimization of the use of ROADMs, taking into account the inter-node spectrum contention and the intra-node transponder contention; [24], where a problem of energy-efficient resource allocation is formulated and a two-stage algorithm is proposed, where in a first stage, the number of devices is minimized (traffic grooming is used in this stage) and on a second stage, the transponder parameters are selected, to minimize the total transponder power consumption.

Different approaches may be used together with traffic grooming techniques. Impairment-aware traffic grooming and multipath routing techniques are jointly used in [25] to increase the spectral efficiency and to reduce resource consumption. In [26] the problem of protection in space division multiplexing (SDM) EONs, by generating primary paths and p-cycles, is addressed. By considering traffic grooming and spectrum overlap, a certain quality of transmission is assured, along with a decrease in the blocking of connections.

After this introductory section where some related work was presented, this paper is organized as follows: section II presents the motivation for this work (Subsection II-A), defines the GRR (Subsection II-B) problem, introduces the used notation (Subsection II-C) and presents the developed ILP and heuristic approaches to the problem (Subsections II-D and II-E). Section III explains how the problem of wavelength assignment was integrated with the approaches in Section II to tackle the GRWAR problem. Section IV presents the obtained results for both small and large networks, for which a set of bounds is proposed. Section V presents the main conclusions.

II. GRR Problem

A. Motivation for this Work

In the present work, a lexicographical ILP formulation for the GRR problem is presented for undirected networks with mesh topologies in a static traffic scenario. The objective is to guarantee the minimization of the number of transponders and after that the minimization of the overall lightpath lengths. Lexicographic optimization allows to consider more than one objective function, ordered by importance, being the first the most important one. Therefore for two objective functions, lexicographic optimization allows to obtain, in a set of alternative optimal solutions for the most important objective function, the one which is the best solution for the second objective function. A good survey on multi-objective optimization can be found in [27], in which lexicographical optimization is presented as a particular case where the different objective functions have an implicit preference.

The results obtained with this ILP model will be used for validation of the results of a heuristic approach, which is developed for the GRR problem. The solutions of the GRR problem are then input to a wavelength assignment algorithm, where regenerators are deployed if the need for wavelength conversion arises, addressing the GRWAR problem. Results show that, for small networks, the heuristic provides a number of transponders equal or close to the optimum, with a lower running time. For larger networks, the results of the heuristic are validated by means of bounds, as explained in Subsection IV-A.

This problem results from the evaluation of a novel strategy to manage, via an integrated Software Defined Network (SDN) platform, a multilayer core network constituted by a meshed optical network transporting traffic (of Multiprotocol Label Switching (MPLS) type, for instance). With the emergence of SDN-based control planes, the simultaneous evaluation of multiple layers is essential to produce resource-efficient routing solutions and is becoming increasingly important for operators' networks. Thus, the optimized computation of routes over DWDM, Optical Transport Network (OTN) and MPLS layers jointly becomes the focal point of this article.

As core networks transport aggregated traffic, it was considered that, for each traffic flow for an origin-destination pair (s, d), there is another traffic flow with the same characteristics in the opposite direction (from d to s), because the bandwidth differences between the uplink and downlink in the access vanish in the core network (considering asymmetrical traffic is outside the scope of this work). Therefore it was required that the network be undirected which implies that both traffic flows follow symmetrical paths, as it is usually the case in real core networks for ease of management. The same symmetry has been assumed for a long time in telephone networks.

It was considered that at the optical layer there is enough capacity to carry all the traffic demands, which is a realistic assumption in core DWDM networks. Nowadays, 100 Gbps per wavelength are widely used.

As far as we know, this work is the first attempt to formulate an ILP model for undirected networks with symmetric traffic considering grooming and regeneration combined. Almost all the approaches found in the literature related with grooming considered directed networks and when the undirected networks were considered, for instance in [28], there were not enough details in the models to maintain the relationship between the different network layers. This relationship is of paramount importance when an integrated network management is envisioned, as in this work.

Due to the innovative aspect of this work, we needed to formulate an ILP model and to develop new bounds to validate the heuristic, as it was impossible to compare our results with the results obtained by other approaches. Note that other ILP models in the literature usually simplify the problem by solving it for one direction only [7] or by considering a predefined set of paths [5] or both strategies [8].

B. Problem Statement

The aim of the GRR problem as stated here is to minimize the total number of transponders connected to MPLS routers while granting the fulfillment of all the requests in a given set of static traffic demands, respecting the regeneration impairment threshold Δ, through the establishment of lightpaths of minimum cost. The term lightpath will be used to refer to a translucent lightpath, a communication channel with a bandwidth equal to a full wavelength, at the optical layer, which may use different wavelengths throughout its route (thus relaxing the traditional wavelength continuity constraint), using wavelength converters, if necessary.

In the context of this work the optical signal regeneration problem is reduced to the simplest case where the considered impairment is only due to the distance of propagation, and thus the optical signal needs regeneration after traveling a distance equal to the impairment threshold, Δ, in kilometers. It is assumed that both optical signal regeneration and wavelength conversion capabilities are obtained through transponders. For signal regeneration purposes, two additional transponders must be used when the optical signal has traveled at most Δ. We consider this value to be constant for all the paths. For details on the calculation of this parameter and the influence of different aspects in that calculation, see [29], [30].

The minimization of the number of transponders must take combined advantage of both the grooming of low-speed connection requests into high-capacity lightpaths and the lightpaths regeneration needs.

The GRR problem leaves the wavelength assignment problem aside. Thus, the wavelength continuity constraint is not imposed, with the demands being routed through translucent lightpaths. At the optical level, it is considered that only the number of free wavelength channels in each fiber is known.

It is further considered, regarding the topology and technological requirements of networks this work applies to, that the nodes of the optical network consist of ROADM which are collocated with MPLS routers in the network. A pair of optical nodes is connected through a single (bidirectional) optical link with W wavelengths and one fiber for each direction. The distance between any two optical nodes is known. Regarding traffic grooming, it is assumed that a connection request cannot be divided into a set of diversely routed lower-speed connections, i.e., the traffic cannot be splitted.

C. Notation

The notation used in the problem formulation is displayed in Table I. Some parameters are defined for the physical network, while others are defined for the virtual topology. There are also some parameters that are used to establish the necessary relationships between the two layers. Some notation regarding the identification of the demands is also provided.

In terms of the parameters defined for the physical network, note that: (i) the cost of arc (m, n), c_{mn}, is an additive cost and in this work it will be the Euclidian distance between the

TABLE I: Notation list.

Physical layer	
$G(N, A)$	graph representing a network topology
N	set of nodes in the network
$k, m, n \in N$	end nodes of an optical link
A	set of arcs in the network
$(m, n) \in A$	directed arc connecting nodes m and n (with $m \neq n$)
c_{mn}	cost of arc (m, n)
p_{mn}	path originating at node m and terminating at node n
$N(p_{mn})$	set of nodes of the path p_{mn}
$A(p_{mn})$	set of arcs of the path p_{mn}
$c(p_{mn})$	cost of the path p_{mn}
$p_{mk} \diamondsuit p_{kn}$	concatenation of paths p_{mk} and p_{kn}
W	number of wavelengths of each arc
Virtual layer	
$G'(N, L)$	virtual topology
L	set of lightpaths
i, j, k	end nodes of a lightpath
l_{ij}	set of lightpaths originating at node i and terminating at node j
l_{ij}^{t}	t^{th} lightpath of l_{ij}
$p_{l_{ij}^{t}}$	optical lightpath associated with l_{ij}^{t}
$o_{l_{ij}^{t}}$	occupied capacity associated with l_{ij}^{t}
C	maximum bandwidth that may be carried by a lightpath
$l_{ik}^{t_1} \diamondsuit l_{kj}^{t_2}$	concatenation of lightpaths $l_{ik}^{t_1}$ and $l_{kj}^{t_2}$
V_{ij} or $\lvert l_{ij} \rvert$	number of lightpaths between node i and node j
$\lambda_{ij,t}^{sd,y,v}$	binary variable: 1 if $\Lambda_{y,sd}^{v}$ uses l_{ij}^{t} as an intermediate virtual link; 0 otherwise
Relation between the layers	
Paths in $G(N, A)$ are mapped as links in $G'(N, L)$	
$l(m, n)$	set of translucent lightpaths using the arc (m, n)
$P_{mn}^{ij,t}$	binary variable: 1 if l_{ij}^{t} is routed through arc (m, n); 0 otherwise
Demands	
s, d	source and destination nodes of an end-to-end connection request
Λ	set of traffic matrices
Λ_y	traffic matrix of a service class with bandwidth y
Λ_{sd}	set of traffic matrices between source node s and destination node d
$\Lambda_{y,sd}$	subset of the demands in Λ_{sd} with bandwidth y
$\Lambda_{y,sd}^{v}$	v^{th} demand in $\Lambda_{y,sd}$
Miscellany	
Δ	Regeneration impairment threshold

nodes; (ii) a path p_{mn} in G is defined as a sequence of arcs, $p_{mn} =< (m, k), \cdots, (\ell, n) >$, with $k, \ell, m, n \in N$; (iii) the cost of a path is given by the sum of the costs of the arcs that compose it, and is denoted by $c(p_{mn}) = \sum_{(\ell,k)\in A(p_{mn})} c_{\ell k}$; (iv) the concatenation of paths, $p_{mn} = p_{mk} \diamondsuit p_{kn}$, results in a longer path p_{mn} corresponding to the union of the operand sub-paths without the repetition of the last node of the left operand and first node of the right operand.

The virtual topology is formed by lightpaths connecting nodes. Regarding lightpaths: (i) a lightpath is defined by an optical path together with an associated occupied capacity $l_{ij}^{t} = (p_{l_{ij}^{t}}, o_{l_{ij}^{t}})$, with $0 \leq o_{l_{ij}^{t}} \leq C$; (ii) the concatenation of lightpaths is represented as $l_{ik}^{t_1} \diamondsuit l_{kj}^{t_2}$ and stands for an ordered sequence of lightpaths. In Figure 1, the concatenation of two lightpaths for the connection between nodes 1 and 8 is represented.

The low-speed connection requests are given in Λ, a set of traffic matrices of distinct service classes characterized by their bandwidths y.

D. ILP Formulation

The inputs to the problem are $G = (N, A)$, Λ, $c_{mn}, \forall (m, n) \in A$, W, C, and Δ. The variables of the problem are V_{ij}, $P_{mn}^{ij,t}$ and $\lambda_{ij,t}^{sd,y,v}$, defined in the appropriate domains.

Given the input values and variables, we intend to determine a virtual topology $G' = (N, L)$ where the nodes correspond

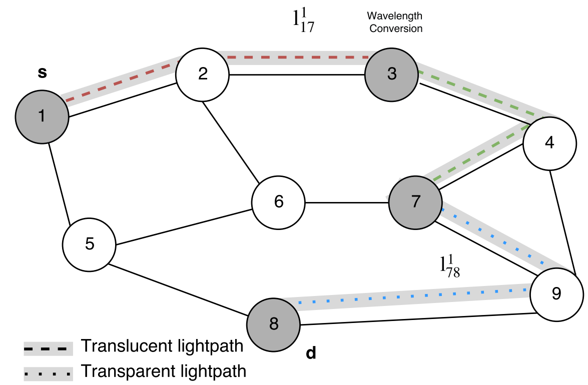

Fig. 1: The path from s to d is routed through the concatenation of l_{17}^{1} and l_{78}^{1}.

to the nodes in the physical topology and the arcs correspond to unidirectional lightpaths. Paths in $G = (N, A)$ are links in $G' = (N, L)$. This virtual topology has to be such that the total number of transponders is minimized (equivalent to the minimization of the number of lightpaths), as well as the length of the established lightpaths.

The ILP formulation of the traffic grooming problem was based on the approach presented in [1], considering several adaptations:

- The objective function is the minimization of the total number of lightpaths obtained to satisfy all the connection requests using lightpaths of minimum possible length, as opposed to the maximization of total network throughput.
- The costs of the arcs represent the Euclidian distance between the nodes they connect.
- The wavelength continuity constraint is discarded so as to incorporate the possibility of wavelength conversion in the lightpaths (translucent lightpaths).
- The number of transponders is not known and is directly related to the minimization objective.
- A distance proportional impairment of the optical signal is considered.

Furthermore, it is considered that each undirected link is represented by two directed arcs in opposite directions. For each traffic demand, the connection is bidirectional and symmetrical. In practice, unidirectional lightpaths will be used to carry traffic in both directions of a bidirectional connection request. To account for lightpath bidirectionality, for each lightpath l_{ij}^{t}, there will be an opposite lightpath l_{ji}^{t} routed through the opposite physical arcs. Analogously, to account for the bidirectionality of each connection request, for each request $\Lambda_{y,sd}^{v}$, a symmetrical request of the same order v, $\Lambda_{y,ds}^{v}$, routed through opposite lightpaths, will be considered. It is important to note that $\Lambda_{y,sd}^{v}$ and $\Lambda_{y,ds}^{v}$ must be routed along the very same intermediate nodes and physical arcs in opposite directions, as is guaranteed by constraints (21) and (22).

The lexicographical problem resolution is usually achieved in successive optimization steps, where the solution in one step is used as a constraint in the next steps. In this case, we have realized that one single step was sufficient, as the minimization of the number of transponders is guaranteed if the second term, which represents the length of the lightpaths is, in magnitude, much lower than unity, which is accomplished by an α factor in the second term of the objective function.

$$\min\left(\sum_{i,j} V_{ij} + \alpha \sum_{i,j,l,m,n} c_{mn} P_{mn}^{ij,l}\right) \tag{1}$$

s.t.:

$$0 \leq \sum_{i,j,t} P_{mn}^{ij,t} \leq W, \quad \forall (m,n) \in A \tag{2}$$

$$\sum_{m} P_{mk}^{ij,t} = \sum_{n} P_{kn}^{ij,t}, \quad \forall i,j,t,k,\ i \neq j,k; j \neq k \tag{3}$$

$$\sum_{n,t} P_{in}^{ij,t} = V_{ij}, \quad \forall i,j \tag{4}$$

$$\sum_{m,t} P_{mj}^{ij,t} = V_{ij}, \quad \forall i,j \tag{5}$$

$$\sum_{m,t} P_{mi}^{ij,t} = 0, \quad \forall i,j \tag{6}$$

$$\sum_{n,t} P_{jn}^{ij,t} = 0, \quad \forall i,j \tag{7}$$

$$P_{mn}^{ij,t} \leq \sum_{k} P_{ik}^{ij,t}, \quad \forall i,j,t,(m,n), m \neq i, k \neq i \tag{8}$$

$$P_{mn}^{ij,t} \leq \sum_{k} P_{kj}^{ij,t}, \quad \forall i,j,t,(m,n), n \neq j, k \neq j \tag{9}$$

$$\sum_{k} P_{mk}^{ij,t} \leq 1, \quad \forall m \neq k \tag{10}$$

$$\sum_{i,t} \lambda_{id,t}^{sd,y,v} = 1, \quad \forall \Lambda_{y,sd}^{v} \tag{11}$$

$$\sum_{j,t} \lambda_{sj,t}^{sd,y,v} = 1, \quad \forall \Lambda_{y,sd}^{v} \tag{12}$$

$$\sum_{i,t} \lambda_{is,t}^{sd,y,v} = 0, \quad \forall \Lambda_{y,sd}^{v} \tag{13}$$

$$\sum_{j,t} \lambda_{dj,t}^{sd,y,v} = 0, \quad \forall \Lambda_{y,sd}^{v} \tag{14}$$

$$\sum_{i,t} \lambda_{ik,t}^{sd,y,v} = \sum_{j,t} \lambda_{kj,t}^{sd,y,v}, \quad \forall \Lambda_{y,sd}^{v}, k \neq s, k \neq d \tag{15}$$

$$\sum_{v,i,t} \lambda_{id,t}^{sd,y,v} = |\Lambda_{y,sd}|, \quad \forall \Lambda_{y,sd} \tag{16}$$

$$\sum_{v,j,t} \lambda_{sj,t}^{sd,y,v} = |\Lambda_{y,sd}|, \quad \forall \Lambda_{y,sd} \tag{17}$$

$$\sum_{s,d,y,v} y \cdot \lambda_{ij,t}^{sd,y,v} \leq C, \quad \forall i,j,t \tag{18}$$

$$\sum_{(m,n)} c_{mn} P_{mn}^{ij,t} \leq \Delta, \quad \forall i,j,t \tag{19}$$

$$\lambda_{ij,t}^{sd,y,v} \leq \sum_{(m,n)} P_{mn}^{ij,t}, \quad \forall \Lambda_{y,sd}^{v}, i,j,t \tag{20}$$

$$P_{mn}^{ij,t} = P_{nm}^{ji,t}, \quad \forall (m,n), \forall i,j,t \tag{21}$$

$$\lambda_{ij,t}^{sd,y,v} = \lambda_{ji,t}^{ds,y,v}, \quad \forall \Lambda_{y,sd}^{v}, \forall i,j,t \tag{22}$$

$$V_{ij} \in \mathbb{N}_0, \quad P_{mn}^{ij,t} \in \{0,1\}, \quad \lambda_{ij,t}^{sd,y,v} \in \{0,1\} \tag{23}$$

- Equation (2) ensures that the number of lightpaths routed through an arc is constrained by the number of wavelengths W it supports.
- Equations (3)-(7) are flow continuity constraints. Equations (6)-(10) guarantee that no cycles are formed.
- Equations (11) and (12) guarantee that a low-speed request $\Lambda_{y,sd}^{v}$ employs one and only one lightpath terminating at the demand's destination node d or originating at the demand's origin node s.
- Equations (13) and (14) ensure that a given request $\Lambda_{y,sd}^{v}$ does not use any lightpath terminating at the source node s nor originating at the destination node d.
- Equation (15) ensures that the connection request $\Lambda_{y,sd}^{v}$ routed through an intermediate lightpath terminating at node k is continued by the employment of a new lightpath originating at node k.
- Equations (16) and (17) force the fulfillment of all connection requests.
- Equation (18) ensures that the aggregate traffic in a lightpath does not exceed the wavelength capacity C.
- Equation (19) ensures that each lightpath is routed in the optical layer through a path with a cost (i.e., distance) of at most Δ km.
- Equation (20) guarantees that a demand only uses a given lightpath l_{ij}^{t} if this lightpath exists, that is, if it is associated with a physical path.
- Equation (21) ensures that if a lightpath l_{ij}^{t} is routed through an arc (m,n), there is an opposite lightpath l_{ji}^{t} routed through the directed arc (n,m).
- Equation (22) ensures that if a connection request $\Lambda_{y,sd}^{v}$ uses lightpath l_{ij}^{t} as an intermediate virtual link, there will be a symmetrical request $\Lambda_{y,ds}^{v}$ using the symmetrical lightpath l_{ji}^{t}.

When sub-optimal results are obtained (or $\alpha = 0$ in the previous formulation), isolated loops in lightpaths that contain the correct path may be formed, as the flow conservation constraint (3) makes it possible. A post-processing script was developed in order to remove such loops from the solutions provided by the optimizer. When the lexicographical method is employed in the optimization, this processing is no longer necessary as the cost of the lightpaths is minimal only in the absence of loops.

E. Heuristic

An intermediate step of the proposed heuristic consists of the creation of a logical auxiliary graph that represents the upper layer of this two-layer problem. Because the algorithm involves both layers, some extra notation was included in order to make the separation between both layers clearer as already mentioned. The logical reachability graph is denoted by $G' = (N, L)$ where N is the set of nodes (the same set of nodes of G) and L is the set of logical arcs. A logical arc between i and j in G' has a direct correspondence to a path between i and j in the underlying physical network G. Such arc may be referred to as $la_{ij}^{u,t}$ so as to distinguish logical arcs that connect the same pair of nodes from one another. The value of u refers to the u^{th} logical arc between i and j, and the value of t relates to a particular existing lightpath. As it will be detailed in the description of the heuristic, a logical arc may result from an existing lightpath or not. If $t \neq 0$,

t implies that $la_{ij}^{u,t}$ had origin in lightpath l_{ij}^{t}. Otherwise, it did not result from an existing lightpath. Each logical arc is characterized by a pair $la_{ij}^{u,t} = (p_{la_{ij}^{u,t}}, c_{la_{ij}^{u,t}})$, where $p_{la_{ij}^{u,t}}$ denotes the specific underlying physical path of the arc in G, and $c_{la_{ij}^{u,t}}$ represents the cost of the arc in G'. A path in G' is denoted similarly to a path in G but with an appended apostrophe $'$. p'_{ij} represents a logical path between i and j and is given by a sequence of logical arcs. The path in G that corresponds to a specific p'_{ij} in G' can be obtained through the concatenation of the physical paths underlying each logical arc, that is, $p_{ij} = p_{la_{ik}^{u_1,t_1}} \diamondsuit \cdots \diamondsuit p_{la_{mj}^{u_2,t_2}}$. For simplicity, let $p_{ij} \dashleftarrow p'_{ij}$ represent this operation. Note that underlying paths of different logical arcs may share one or more physical arcs and, as such, the physical path that results from this operation may contain loops.

The heuristic presented next is based on the use of shortest path routing of requests in sorted order (largest aggregated requests first, and shortest path length as tie breaker for secondary ordering) over a logical graph where already instantiated lightpaths are represented as direct links between the connected nodes. Two secondary orderings of demands were considered – shortest and longest shortest path lengths first. For each set of ordered demands, the routing and grooming is solved for a considered decreasing number of wavelengths per fiber until it is not possible to fulfill all the requests. The solution of the heuristic is the best among the set of solutions obtained through the combination of the demand orders and the number of wavelengths per fiber.

The heuristic is described at a higher level in Algorithm 1, but further details concerning the implementation of certain steps are provided in Algorithms 2 and 3. The output of the heuristic is the set of lightpaths established to carry the requests in Λ, referred to as L_{LP}. The number of transponders will be twice the number of lightpaths in this list.

Algorithm 1 is composed of two main sections: an outer combinatory section that conditions an inner core section. The core section (steps 7–15) is responsible for the whole process of grooming and routing of a given set of demands, providing as an output the set of lightpaths established to carry the demands in that set. This section is explained in detail later on in this text. The combinatory facet, which includes the remaining steps, results from two main observations during the development of the heuristic: (1) the order in which the demands present themselves for routing can have significant influence on the overall routing results; (2) the variation of the number of wavelengths per arc, because of different capacity conditions, creates different opportunities for demand routing. For those reasons, different orders D of the demands in the traffic matrix and different numbers of wavelengths per arc w are used to run the core section. For each order D, the number of wavelengths per fiber w is decremented one by one from the maximum value (and actual physical number of wavelengths per link) W. Let $\{D, w\}$ define a particular combination of a reordered traffic matrix and a particular number of wavelengths per arc. For each $\{D, w\}$, the core algorithm is run. The decrease in the number of wavelengths per link goes on until the network does not have enough capacity and the heuristic fails to route all the demands. Each time a minimum number of lightpaths provided by the core algorithm is registered, the list of lightpaths and routing information for each demand is stored. The output variable L_{LP} corresponds to the minimal set of lightpaths needed for routing all the demands.

Algorithm 1 GRR

Input: $G(N, A)$, Λ, Δ,W, C
Output: L_{LP}
1: $D_{sp} \leftarrow lexicoSort_{y,sp}(\Lambda)$
2: $D_{lp} \leftarrow lexicoSort_{y,lp}(\Lambda)$
3: $minLP \leftarrow \infty$
4: **for all** $D \in \{D_{sp}, D_{lp}\}$ **do**
5: $w \leftarrow W$
6: **while** $w > 0$ **do**
7: $FFDemands \leftarrow 0$
8: $L'_{LP} \leftarrow \emptyset$
9: **for all** $\Lambda_{y,sd}^{v} \in \Lambda$ in the order given by D **do**
10: $G'(N, L) \leftarrow genLogGraph(G(N, A), \Lambda_{y,sd}^{v}, \Delta, L'_{LP}, W, C)$
11: $p'_{sd} \leftarrow shortestPath(G'(N, L), s, d, y)$
12: **if** $p'_{sd} \neq \emptyset$ **then**
13: $\{L'_{LP}, FFDemands\} \leftarrow grooming(G'(N, L), \Lambda_{y,sd}^{v}, L'_{LP}, p'_{sd}, FFDemands, \Delta)$
14: **end if**
15: **end for**
16: **if** $FFDemands = |\Lambda|$ and $|L'_{LP}| < minLP$ **then**
17: $minLP \leftarrow |L'_{LP}|$
18: $L_{LP} \leftarrow L'_{LP}$
19: **else if** $FFDemands \neq |\Lambda|$ **then**
20: break
21: **end if**
22: $w \leftarrow w - 1$
23: **end while**
24: **end for**

Note that D represents two different orders of Λ, D_{sp} and D_{lp}. Both result, in a first step, from the ordering of Λ by decreasing order of aggregated bandwidth. That is, the first demands to be serviced are those between the node pair (s, d) presenting the highest value for the sum of bandwidth requests. For requests of different pairs (s, d) for which the aggregated traffic has the same value, the tiebreaker is the length of the shortest path from s to d, computed with no capacity considerations. D_{sp} gives priority to the demands which yield the shortest paths and D_{lp} gives priority to the demands which yield the longest paths. This two step ordering is a lexicographical ordering, hence the names $lexicoSort_{y,x}$ where y represents the bandwidth objective and x the path length objective with $x \in \{sp, lp\}$.

Having explained the outer combinatory section, herein follows the explanation of how the core section operates internally until the set of lightpaths for a given set of demands is obtained. It begins by setting the number of routed demands $FFDemands$ to zero and the set of lightpaths L'_{LP} that carry the demands in D to an empty set. Then, each demand $\Lambda_{y,sd}^{v} \in D$ is orderly taken, and three essential operations are performed:

Generation of an auxiliary graph $G'(N, L)$: The generation of the auxiliary logical graph G' is detailed in Algorithm 2. The aim is to build a reachability graph, which is a quite common procedure in impairment-aware problems. An auxiliary logical arc $la_{ij}^{u,t}$ will exist:

(1) If L_{LP} contains a lightpath l_{ij}^t between i and j with sufficient spare capacity for the demand;

(2) Else, if the shortest path between i and j in G, p_{ij}, exists such that $c(p_{ij}) \leq \Delta$ and that every $(m, n) \in A(p_{ij})$ has capacity for a new lightpath.

A lightpath is created when a logical arc resulting from case (2) is a part of the chosen end-to-end path for a demand, which is explained later on when looking into Algorithm 3. Thus, whenever a logical arc $la_{ij}^{u,t}$ is created, in both cases, it is associated with an underlying physical path $p_{la_{ij}^{u,t}}$ that corresponds to the current shortest path between i and j in G with enough capacity for the demand. However, there is a difference in the costs assigned to both types of logical arcs. In case (1), to force the reutilization of already existing lightpaths through the grooming of several requests, the cost assigned to the logical arc is always inferior to the one assigned in case (2). In the latter case, if such an arc is chosen to be a part of the path p'_{sd}, it will imply the creation of a new lightpath and thus the deployment of two more transponders. This way, using in the final path a pre-existing lightpath or a set of pre-existing lightpaths will contribute, in almost all situations, to a lower overall cost of p'_{sd}. In case (1), the cost assigned to a logical arc $la_{ij}^{u,t}$ deriving from a lightpath l_{ij}^t is a function of the physical arcs the lightpath/logical arc spans. More specifically, it is set to the number of arcs of the lightpath, $|A(p_{l_{ij}^t})|$, divided by the total number of arcs in the network, $|A|$. In the other case, the cost of the logical arc is set to unity. Note that the cost of a path resulting exclusively from the reutilization of preexisting lightpaths would only cost more than the creation of a new lightpath if it used more than the number of arcs in the network. This could happen only in very long paths with several cycles.

Computation of the shortest path between s and d in G', p'_{sd}: After obtaining G', the shortest path from s to d in G', p'_{sd}, is computed using Dijkstra's algorithm for the shortest path. This path is a sequence of logical arcs and, since logical arcs are associated with an underlying physical path, p'_{sd} can be easily expanded to obtain the corresponding physical path p_{sd} in G, as it was already explained. If the network faces high load, even if just locally, there may be lack of capacity to reach the destination, in which case no path p'_{sd} will be found.

Routing and grooming of $\Lambda_{y,sd}^t$ through p'_{sd}: Assuming a path p'_{sd} is found, the next step is to route the demand, taking advantage of traffic grooming for sharing existing lightpaths if possible. $(\Lambda_{y,sd}^v)_{LP}$ denotes the ordered sequence of lightpaths selected to carry the demand $\Lambda_{y,sd}^v$. The path p'_{sd} is a sequence of logical arcs which may exist as lightpaths or not. Different logical arcs may share common physical resources and, as such, it is possible that the concatenation of two logical arcs translates into the occurrence of loops at the physical layer.

Algorithm 2 genLogGraph

Input: $G(N, A)$, $\Lambda_{y,sd}^v$, Δ , L_{LP},W, C
Output: $G'(N, L)$

$G'(N, L) \leftarrow (N, \emptyset)$
for all node pair $(i, j) \in N^2$, $i \neq j$ **do**
    $u \leftarrow 1$
    $LP_{ij} \leftarrow \{l_{ij}^t \in L_{LP} : C - o_{l_{ij}^t} \geq y\}$
    if $LP_{ij} \neq \emptyset$ **then**
        for all $l_{ij}^t \in LP_{ij}$ **do**
            $p_{la_{ij}^{u,t}} \leftarrow p_{l_{ij}^t}$
            $c_{la_{ij}^{u,t}} \leftarrow \frac{|A(p_{l_{ij}^t})|}{|A|}$
            $L \leftarrow L \cup \{la_{ij}^{u,t}\}$
            $u \leftarrow u + 1$
        end for
    else
        $A' \leftarrow \{(m, n) \in A : |l(mn)| < W\}$
        $p_{ij} \leftarrow shortestPath(G(N, A'), i, j)$
        if $p_{ij} \neq \emptyset \wedge c(p_{ij}) \leq \Delta$ **then**
            $p_{la_{ij}^{1,0}} \leftarrow p_{ij}$
            $c_{la_{ij}^{1,0}} \leftarrow 1$
            $L \leftarrow L \cup \{la_{ij}^{1,0}\}$
        end if
    end if
end for

When a lightpath is established, it is advantageous that it is reused. For that reason, when p'_{sd} contains one or more lightpaths, we choose not to interfere with them for loop removal. However, in parts of p'_{sd} that consist of a logical arc or of a concatenation of logical arcs that have no allocated resources, loops at the physical level will be removed. After loop removal, new logical arcs are computed to create new lightpaths. This helps greatly at reducing loops but does not eliminate completely the possibility of occurrence, because of the existence of a lightpath in p'_{sd} with allocated demands. In a more technical explanation, that is what is done in Algorithm 3.

Path p'_{sd} is analyzed, one logical arc $la_{ij}^{u,t}$ at a time, from s to d. If a lightpath l_{ij}^t is not found (i.e., $t = 0$ in step 6), the underlying physical path $p_{l_{ij}^t}$ is concatenated to an auxiliary path variable p_{xy} (steps 15-16). This concatenation proceeds for every logical arc until a logical arc for which a lightpath exists is found. Then, two actions must be performed:

(i) The lightpath is reused to route the demand and the capacity of network elements must be updated (steps 13-14).

(ii) Remove loops in p_{xy} and create corresponding lightpaths (steps 8-11).

Algorithm 3 performs these two actions in reverse order, so that the final path of the demand is obtained in the right order of lightpaths. In step (ii), if p_{xy} is not empty, it will contain a portion of the physical path p_{sd} that corresponds to a portion of p'_{sd} where no lightpaths were found. This portion will be processed for loop removal. After that, new logical arcs for which the concatenation is free of loops are obtained.

This process is explained with more detail in Algorithm 4, in Appendix A.

Algorithm 3 grooming

Input: $G'(N,L)$, $\Lambda^v_{y,sd}$, L_{LP}, p'_{sd}, $FFDemands, \Delta$
Output: L_{LP}, $FFDemands$

```
1: p_sd ⇠ p'_sd
2: (Λ^v_{y,sd})_LP ← ∅
3: newLPs ← ∅
4: p_xy ← ∅
5: for all p_{la^{u,t}_{ij}} ∈ p'_sd do
6:     if t ≠ 0 then
7:         if p_xy ≠ ∅ then
8:             remove possible existing loops in p_xy
9:             {L_LP, newLPs} ← createNewLightpaths(p_xy, L_LP, y, Δ)
10:            (Λ^v_{y,sd})_LP ← (Λ^v_{y,sd})_LP ◈ newLPs
11:            p_xy ← ∅
12:        end if
13:        (Λ^v_{y,sd})_LP ← (Λ^v_{y,sd})_LP ◈ l^t_ij
14:        o_{l^t_ij} ← o_{l^t_ij} + y
15:    else if |l(mn)| < W,  ∀(m,n) ∈ A(p_{la^{u,t}_{ij}}) then
16:        p_xy ← p_xy ◇ p_{la^{u,t}_{ij}}
17:    else
18:        {L_LP} ← deallocateDemandResources((Λ^v_{y,sd})_LP, L_LP)
19:        return
20:    end if
21: end for
22: if p_xy ≠ ∅ then
23:     remove possible existing loops in p_xy
24:     {L_LP, newLPs} ← createNewLightpaths(p_xy, L_LP, y, Δ)
25:     (Λ^v_{y,sd})_LP ← (Λ^v_{y,sd})_LP ◈ newLPs
26: end if
27: FFDemands ← FFDemands + 1
```

This process of creating new lightpaths from p_{xy} has to be repeated after the outmost for loop, in case no lightpaths are encountered in p'_{sd} or if p'_{sd} does not terminate with a lightpath (steps 22-26).

Note that the logical path p'_{sd} is calculated with a shortest path algorithm having in mind that the underlying physical arcs have enough capacity for the demand. The problem is that due to the already mentioned possibility of loop occurrence at the physical layer, if a demand uses the same arc twice, then it would need twice the capacity that was searched for. If the case happens where a certain part of the path only has capacity to support a single new lightpath, and it has to be used more than once (by more than one logical arc), then the routing of the demand through p'_{sd} is not feasible. If so, the lightpaths, capacity and transponders allocated for the demand that is being routed have to be deallocated (step 18). This process is explained in Algorithm 5, in Appendix A.

III. GRWAR Problem

In order to extend the GRR approach to the GRWAR problem, the lightpaths obtained with both the ILP and the heuristic are input to a wavelength assignment algorithm which assigns wavelengths to each lightpath. When wavelength continuity constraint cannot be met throughout the entire underlying physical path of a lightpath, due to lack of available wavelengths, the algorithm places regenerators for wavelength conversion.

In [31], the problem of impairment aware routing and wavelength assignment with regeneration placement (RWARP) is tackled. Capacity constraints are considered, while focusing on minimizing the number of regenerators. An ILP formulation and an efficient heuristic for the RWARP problem are proposed.

The wavelength assignment algorithm is described in [31] and can be summarized as follows. Given an impairment threshold Δ, a virtual topology is created for each lightpath such that each arc in the virtual topology corresponds to a subpath in G having at least a free common wavelength, i.e. the same free wavelength in all arcs of the subpath, and a cost lower than Δ, where the path cost is defined as the sum of the cost of the arcs. The final path is then chosen as the concatenation of virtual arcs of (one of) the shortest path in the virtual topology, as this corresponds to the path with minimal regeneration requirements. The free wavelength of the virtual arc is then assigned to each of the subpaths of the final path.

IV. Simulation and Results

The networks to be tested were originally obtained from [32], where both topology and end-to-end requests are provided, and were later modified. To reduce the demands to two classes of service, each source-destination pair is associated with all the demands concerning that pair of nodes. Half of the entire set, consisting of the source-destination pairs which require the least aggregate bandwidth in the original file, have their individual bandwidth requests set to 10 Gbps. From the remaining half, the 35% with the lowest aggregate bandwidths have their bandwidth requests set to 40 Gbps and the 15% with the highest aggregate bandwidth are associated with both a demand of 10 Gbps and a demand of 40 Gbps. The variation of the number of demands was obtained by replicating the above mentioned demands a certain number of times. Note that the values 10 and 40 Gbps were chosen in order to increase the opportunity of grooming, given the available capacities in the test networks. The data rates considered are low, but still aligned to what is generally employed on commercial deployments.

As the number of necessary lightpaths is only obtained after the optimization process, in this formulation, an upper bound to the number of lightpaths between two nodes i and j ($|l_{ij}|$) is estimated. In practice, this number is limited by W times the number of possible end-to-end disjoint optical paths between i and j. For higher values of W, this translates into a large unnecessary limit and waste of memory for the creation of variables and constraints. So the boundary $\mathcal{L} = |\Lambda|$ has been defined, which means $t \in \{1, \cdots, \mathcal{L}\}$. As a result, in most cases, not all $\mathcal{L}$ lightpaths are necessary, and for some values of t, l^t_{ij} has no real significance, because it has no optical path p_{ij} associated with it.

The ILP was bound to run on 4 cores and for at most 24 hours, after which only an upper bound is obtained. The tests were performed on a computer with an Intel(R) Xeon(R) CPU X5660 2.80 GHz and 48 GB RAM. The Java API of IBM ILOG CPLEX Optimization Studio V12.6.1 [33] was used for the ILP. As the optimizer takes more than the time bound to run in most networks, the following results were obtained from subnetworks (see Appendix B) of the *polska* network, for different numbers of nodes, links and demands. The original network contains 12 nodes, 18 links and 66 demands (one for each pair of nodes).

Table II shows the results obtained for the GRWAR problem for both ILP and heuristic approaches. For each network, the table provides: the average length of all the established lightpaths; the percentual network occupation, given by the ratio between the number of wavelengths used by each lightpath (considering that each lightpath uses one wavelength in each link it traverses) and the total number of wavelengths available in the network ($W\cdot|A|$); the maximum number of used wavelengths in a link; the number of required transponders, and the elapsed time until the solution is met. In case the optimizer reaches the time limit, the value "time-out" appears on the elapsed time field, and the solution obtained by the ILP may or may not be optimal, thus representing an upper bound to the optimal number of transponders. The last three columns of the table present results on bounds for the number of transponders, which will be explained in the next section.

In the performed experiments, the value $C = 100$ Gbps was considered. Additionally, to guarantee that the second term in equation (1) is much lower than unity, α was set to $1/\sum_{(m,n)\in A} c_{mn}\cdot|A|\cdot|\Lambda|$. The values assigned to Δ are larger than the distance of the longest links in the network under consideration, ensuring that they are not disregarded due to the impossibility of performing regeneration in the middle of a link.

Regarding the number of transponders, results show that, for small networks, the heuristic reasonably approximates the optimal solution. The results of the heuristic match those obtained in the ILP in one test, are outperformed by the optimizer in five and are able to provide a better solution than the optimizer in two of the tests, in which cases the results of the optimizer are sub-optimal but all the demands are routed. Note that two transponders are needed per lightpath, which means that any non-optimal solution will differ from the optimal by a multiple of 2. Excluding the cases where the heuristic outperformed the optimizer because of time-out, the average relative error is of 11% and the maximum relative error is of 25%. In the solutions obtained by both the heuristic and the optimizer, there was not a need for placing regenerators due to wavelength continuity constraints.

In terms of running times, the heuristic clearly outperforms the ILP, running in fractions of seconds to units of seconds, while the ILP runs in hundreds of seconds to tenths of hours, reaching the 24 hour bound in five of the eight tests.

Other metrics of interest should be examined: concerning the average length of lightpaths, the ILP manages to obtain better solutions as well. Since the objective function used in the optimization process minimizes both the number of transponders and the length of lightpaths, this result is expected.

The overall network capacity usage is consistently lower for the solutions of the optimizer, only surpassing the capacity used by the heuristic for sub-optimal results in which the heuristic achieves a lower number of transponders. Concerning the maximum number of wavelengths required in a link to carry all the demands, results show that the ILP approach is less demanding, in the majority of the cases, although the heuristic is able to match or even outperform the ILP in a few, in which the ILP did not find the solution within the 24h time limit. Overall, the heuristic provides a very good trade-off between results, which are slightly worse, and running times, which are 3-4 orders of magnitude lower.

As a final remark, note that the link usage is very low because it is assumed in this problem that the capacity in the network is always greater than the one needed for carrying the traffic demands.

A. Larger Networks

In order to assess the quality of the heuristic for realistically sized networks, some bounds were considered for the number of transponders needed to carry the traffic of a given traffic matrix, in a given network. For the computation of such bounds, it is assumed that there is traffic between all pairs of nodes in a given network, and the impairment is not taken into account.

Lower Bound (LB): this parameter is a lower bound to the number of transponders in a network. It corresponds to twice the number of lightpaths needed to carry the total traffic entering or leaving each node, without considering how the traffic is routed or groomed inside the network. This lower bound is, in almost all cases, impossible to achieve because it is obtained without taking into account the topology of the network. Let us consider $LB(i)$ as the total number of transponders needed to route the total amount of traffic entering and leaving node i. $LB(i) = \left\lceil \frac{\sum_{s,d,y} y\left(|\Lambda_{y,id}| + |\Lambda_{y,si}|\right)}{C} \right\rceil$ Then, the total number of transponders is given by $LB = 2\left\lceil \frac{\sum_i LB(i)}{2} \right\rceil$.
However, as traffic splitting was not allowed in this work, the lower bounds presented are not computed exactly as in the above expressions, and will be equal to or slightly superior to the values obtained through them.
Lower Bound Approximation (LBA): in order to have a more realistic value for the previous bound, now taking into account the topology of the network, a LBA is proposed. This is in fact not a bound but rather an approximation, because there can be optimal solutions with lower or higher values for the number of transponders than the LBA values. The LBA is calculated considering that all the demands are routed through the shortest path and so the total traffic computed previously for $LB(i)$ is now distributed over the adjacent arcs of i.
If $T(i)_n$ is the fraction of traffic that leaves node i to all the destination nodes for which the shortest path

TABLE II: Results obtained for the GRWAR problem.

Input data						ILP (Lexi) / Heuristic					Bounds for Transp.		
Network	$\|N\|$	$\|A\|$	$\|\Lambda\|$	Δ	W	Avg LP Cost [km]	Capacity Usage (%)	W$_{max}$	Transp. + Reg.	Time [s]	LB	LBA	UB
polska	6	6	17	1000	48	322.65 / 491.04	3.12 / 4.51	2 / 4	14 + 0 / 14 + 0	496.30 / 0.681	12	14	18
			34			415.97 / 466.95	5.90 / 6.94	4 / 5	20 + 0 / 22 + 0	2537.67/ 0.984	18	20	28
			51			373.90 / 430.45	7.99 / 10.07	5 / 5	30 + 0 / 32 + 0	time-out / 1.49	26	28	42
			68			433.37 / 459.77	10.76 / 11.81	7 / 8	36 + 0 / 38 + 0	time-out / 1.72	34	34	50
	7	8	24			305.74 / 432.89	3.12 / 5.21	2 / 4	18 + 0 / 22 + 0	25620.21 / 1.71	14	20	26
			48			368.50 / 380.18	6.51 / 5.99	5 / 4	30 + 0 / 28 + 0	time-out / 1.66	24	28	40
	8	10	32			324.90 / 397.71	3.54 / 5.41	3 / 5	24 + 0 / 30 + 0	time-out / 1.81	18	24	38
abilene	12	15	75	3000	48	1434.84 / 1558.69	9.44 / 8.75	12 / 10	72 + 0 / 66 + 0	time-out / 7.66	38	50	102

uses arc (i,n) and $LBA(i)_n$ is the total number of transponders needed to route that amount of traffic, then $\sum_n T(i)_n = \sum_{s,d,y} y\left(|\Lambda_{y,id}| + |\Lambda_{y,si}|\right)$ and, similarly to the previous bound, $LBA = 2\left\lceil \frac{\sum_{i,n} LBA(i)_n}{2} \right\rceil$ with $LBA(i)_n = \left\lceil \frac{T(i)_n}{C} \right\rceil$. As before, it was not considered how the traffic is routed or groomed inside the network, and traffic splitting for each demand is not allowed.

Upper Bound (UB): the upper bound for the number of transponders is calculated considering: (i) the total end-to-end demands that fulfill whole lightpaths (NT_{fll}); (ii) for the other demands, all are routed through the shortest path, and the number of lightpaths is obtained by examining the amount of bandwidth required in each arc and by considering that no lightpath spans more than one physical link. Note that this corresponds to a feasible solution where the flexibility for grooming in the network is the maximum possible. $UB_{(m,n)} = \left\lceil \frac{\sum_{s,d,y:(m,n)\in \mathrm{shpath}(s,d)} y|\Lambda_{y,sd}|}{C} \right\rceil$ and $UB = NT_{fll} + 2\left\lceil \frac{\sum_{(m,n)\in A} UB_{(m,n)}}{2} \right\rceil$

The last three columns of Table II show the bounds obtained for each network case. Results show that the LBA bound represents a better approximation for the given networks, with all instances presenting a deviation of up to 11% except for the last result, with a deviation of 24%. The average deviation is of 6%. The LB value is also reasonably close to the results, presenting an average deviation of 18%, with all instances presenting a deviation of up to 25% except for the last result, with a deviation of 42%. The upper bound is the more distant to the obtained results, being on average 43% higher than the results obtained with the heuristic.

Once the bounds are reasonably close to the optimizer results for small sub-networks, they were subsequently used as a reference to validate the quality of the proposed heuristic when applied to larger networks.

Table III shows the number of transponders obtained with the heuristic for different networks, as well as the corresponding values for the three computed bounds. Again, the networks are obtained from [32], considering only one demand per pair of nodes, and it was assumed that each arc provides 48 wavelength channels, as was done before. In order to understand the influence of the impairment awareness in the number of transponders of a solution, the number of transponders obtained for each network without considering an impairment is given in field T. The number of transponders obtained considering the impairment Δ is given in field T_Δ. Similarly to Table II, the average cost of lightpaths and the time of execution are also presented. Additionally, the average cost of lightpaths in terms of hops (number of arcs) is also given, as well as the maximum and average cost in terms of hops of the set of shortest paths between every pair of nodes. The number of end-to-end shortest paths whose cost is superior to Δ is also presented.

It is quite evident that the consideration of impairments can have a great influence on the number of transponders obtained by the heuristic in a given solution. This influence is more visible for cases in which the distances involved in end-to-end paths (here estimated by the shortest paths) are considerably greater than the impairment threshold, which is presented in the field representing the number of shortest paths greater than Δ ($nSP > \Delta$). These cases include the networks of *india* and *nobel_eu*. Since the bounds are computed without consideration of an impairment, they should only be compared quantitatively to the number of transponders T obtained by the heuristic in the same conditions. The bounds LB, LBA and UB present average deviations from the result of the heuristic of 29%, 24% and 78%, respectively. Note that there is a network (*dfn_bwin*) for which the value of LBA matches the value of UB. This is explained by the network's topology, which is fully meshed (Avg SP Hops = 1.0). Therefore, each arc is the shortest path between each pair of nodes, and the computation of the bound LBA is equal to the one of the UB. These are the only two instances in which the LBA is more distant than the LB bound from the heuristic solution. If this network is not considered, the average deviation of the LBA bound is 18%.

Since the LB and the LBA do not take into account the whole topology of the network, as the hop-count diameter of the network and the degree of the nodes increase, the greater are the relative errors of these bounds, as can be seen for networks *india* and *nobel_eu* (see Avg SP Hops and Max SP Hops). On the other hand, in fully meshed networks, as is the case of *dfn-bwin*, LBA is equal to UB, because LBA is computed based on the shortest paths, and the heuristic manages to find better solutions.

The heuristic also proves to have a good performance in the presence of impairment, as can be seen comparing the number of shortest paths longer than the impairment ($nSP > \Delta$) to the added number of transponders when impairment is considered ($T_\Delta - T$).

In conclusion, the results of the heuristic are reasonably

TABLE III: Comparison between the heuristic results and the proposed bounds.

Input Data					**Heuristic**								**Bounds for Transp.**		
Network	$\lvert N\rvert$	$\lvert A\rvert$	$\lvert\Lambda\rvert$	Δ	T	T_Δ	Avg LP Cost [km]	Avg LP Hops	Avg SP Hops	Max SP Hops	Time [s] GR	nSP $> \Delta$	LB	LBA	UB
abilene	12	15	75	3000	66	66	1558.69	1.91	2.64	6	7.699	21	38	50	102
			150		92	110	1719.94	2.05			10.61		92	82	164
polska	12	18	75	1000	66	66	490.63	2.18	2.2	5	9.78	1	40	54	94
			150		94	94	485.576	2.19			14.21		74	84	152
dfn-bwin	10	45	51	1000	46	46	406.26	1	1	1	5.06	0	28	90	90
			102		64	64	407.42	1			8.28		52	90	90
nobel_eu	28	41	434	1500	368	476	1063.96	2.42	3.61	8	281.47	197	222	248	852
			868		526	822	1088.5	2.42			206.98		430	450	1324
india	35	80	684	3000	604	694	2038.94	2.37	3.23	9	1132.28	294	330	394	1130
			1368		844	1144	2130.14	2.47			1402.19		644	700	1788

close to the LBA and LB bounds, confirming that it provides good results in very short times.

V. Conclusion

This work addresses the problem of grooming and regeneration in WDM networks, with the objective of minimizing the number of transponders and regenerators, as well as the length of the established lightpaths for the minimum number of transponders.

Results show that the heuristic provides an impressive trade-off between results and running times, with an average deviation of 11% from the optimizer results (not considering the cases where the optimizer returns sub-optimal solutions).

Regarding the ILP formulation, it was not possible to obtain results for realistically sized networks. The possibility of relaxation of some constraints could be explored in order to reduce its complexity. The use of a restricted set of precomputed physical and logical paths in the ILP can also be explored to reduce the solution space and execution times.

To assess the performance of the heuristic for larger networks, two bounds and a bound approximation were proposed, showing that the heuristic presents good results in very short time.

One of the possible directions of this work is its integration in survivability contexts. In the future, it is intended to address survivable impairment-aware traffic grooming, for instance using MPLS recovery techniques.

Appendix A
Additional Algorithms

In Algorithm 4, each new logical arc is obtained by scanning the new loop free auxiliary path p_{xy} and creating a list of unregenerated segments, that is, a list of subpaths of p_{xy}, p_{rq}, such that their length is maximum without surpassing Δ (starting at node x) (step 2). For each unregenerated segment, a new lightpath will be created, transponders will be placed at its end nodes, and the capacity of network elements will be updated (steps 3-11).

Algorithm 4 createNewLightpaths

Input: p_{xy}, L_{LP}, y, Δ
Output: L_{LP}, $newLPs$
1: $newLPs \leftarrow \emptyset$
2: $P_\Delta \leftarrow \{p_{rq} \in p_{xy} : \ c(p_{rq}) \leq \Delta \wedge c(p_{rq}) + c_{qk} > \Delta\}$ ▷ Let k be the successor of q in p_{xy}
3: **for all** $p_{rq} \in P_\Delta$ **do**
4: $\quad l_{rw}^{|l_{rq}|+1} \leftarrow (p_{rq}, y)$
5: $\quad L_{LP} \leftarrow L_{LP} \ \cup \ \{l_{rq}^{|l_{rq}|+1}\}$
6: $\quad newLPs \leftarrow newLPs \ \textcircled{$\diamond$} \ l_{rq}^{|l_{rq}|+1}$
7: $\quad$ **for all** $(m, n) \in A(p_{rq})$ **do**
8: $\qquad |l(mn)| \leftarrow |l(mn)| + 1$
9: $\qquad$ Place Transponders for $l_{rq}^{|l_{rq}|+1}$ in nodes r and q
10: $\quad$ **end for**
11: **end for**

Algorithm 5 deallocateDemandResources

Input: $(\Lambda_{y,sd}^v)_{LP}$, L_{LP}
Output: L_{LP}
1: **for all** $l_{ij}^t \in (\Lambda_{y,sd}^v)_{LP}$ **do**
2: $\quad o_{l_{ij}^t} \leftarrow o_{l_{ij}^t} - y$
3: $\quad$ **if** $o_{l_{ij}^t} = 0$ **then** ▷ lightpath purposely created for this demand
4: $\qquad L_{LP} \leftarrow L_{LP} \ \setminus \ l_{ij}^t$
5: $\qquad$ Remove Transponders for l_{ij}^t from nodes i and j
6: $\quad$ **end if**
7: **end for**
8: $(\Lambda_{y,sd}^v)_{LP} \leftarrow \emptyset$

Appendix B
Networks used for tests

Relevant information regarding the sub-networks of the network *polska* used in this paper is given in Table IV. Each table concerns a specific sub-network, specifying the amount of bandwidth required between each pair of nodes, as well as the existence of a link connecting that same pair ("y" if it exists, "n" otherwise).

Acknowledgment

Lúcia Martins, Teresa Gomes and Rita Girão-Silva acknowledge financial support by Fundação para a Ciência e a Tecnologia (FCT) under project grant UID/MULTI/00308/2019 and FEDER Funds and National Funds through FCT under project CENTRO-01-0145-FEDER-029312. Bruna Nogueira and Ricardo Mendes acknowledge Infinera Portugal for a MSc scholarship during which this work was developed.

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TABLE IV: Bandwidth requests and topology of sub-networks *polska_6_6_17*, *polska_7_8_24* and *polska_8_10_32*.

(a) polska_6_6_17

S	D	BW	Link
0	1	10	n
0	2	40	n
0	3	10	n
0	4	40 + 10	y
0	5	40 + 10	n
1	2	10	n
1	3	10	y
1	4	40	y
1	5	40	n
2	3	10	n
2	4	10	y
2	5	40	y
3	4	10	n
3	5	10	y
4	5	40	n

(b) polska_7_8_24

S	D	BW	Link
0	1	10	n
0	2	10	n
0	3	10	n
0	4	10	n
0	5	40	y
0	6	40 + 10	n
1	2	40	n
1	3	10	n
1	4	10	n
1	5	40	y
1	6	10	y
2	3	40 + 10	y
2	4	40	n
2	5	40	n
2	6	40 + 10	y
3	4	10	n
3	5	10	n
3	6	40	y
4	5	10	y
4	6	10	n
5	6	40	y

(c) polska_8_10_32

S	D	BW	Link
0	1	10	n
0	2	40	y
0	3	40	n
0	4	10	n
0	5	10	n
0	6	40 + 10	y
0	7	40 + 10	n
1	2	10	y
1	3	10	n
1	4	10	y
1	5	40	n
1	6	40	y
1	7	40	n
2	3	40	n
2	4	10	n
2	5	40 + 10	y
2	6	10	n
2	7	40	n
3	4	40	n
3	5	10	n
3	6	10	y
3	7	40	y
4	5	40 + 10	y
4	6	10	n
4	7	10	y
5	6	10	n
5	7	10	n
6	7	40	n

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