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Analysis and comparison of thermodynamic behavior for Stirling and Ericsson cycles

Submitted in Partial Fulfillment of the Requirements for the Degree of Master in Mechanical Engineering in the specialty of Energy and Environment

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Análise e comparação do comportamento termodinâmico de ciclos Stirling e Ericsson

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"To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination and marks real advance in science."
Albert Einstein

"De sonhar ninguém se cansa, porque sonhar é esquecer, e esquecer não pesa e é um sono sem sonhos em que estamos despertos."
Fernando Pessoa

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Abstract

Stirling and Ericsson engines are external heat engines that offer the ability to use many different heat sources to provide reliable and sustainable power. In this thesis, we compare the Stirling and Ericsson cycles in order to determine in which situations one engine produces more net work output than the other. The net work output equations are derived and are analyzed for three different scenarios: (i) equal mass and temperature limits, (ii) equal mass and pressure or volume, and (iii) equal temperature and pressure or volume limits. The comparison is performed by calculating when both cycles produce the same net work output and then analyzing which one produces more net work output based on how the parameters are varied. In general, the results demonstrate that Stirling engines produce more net work output at higher pressures and lower volumes, and Ericsson engines produce more net work output at lower pressures and higher volumes. For certain scenarios threshold values are calculated to illustrate precisely when one cycle produces more net work output than the other. This thesis can be used to inform the design of the engines and particularly to determine when either a Stirling or Ericsson should be selected for a particular application.

Keywords: Stirling, Ericsson, Engine, Themodynamics

Resumo

Os motores Stirling e Ericsson são motores de calor externos que oferecem a capacidade de usar muitas fontes de calor diferentes para fornecer energia confiável e sustentável. Nesta tese, comparámos os ciclos Stirling e Ericsson para determinar em que situações um motor produz mais trabalho do que o outro. As equações de trabalho são derivadas e são analisadas para três cenários diferentes: (i) massa e limites de temperatura iguais, (ii) massa e pressão ou volume iguais, e (iii) limites de temperatura e pressão ou volume iguais. A comparação é realizada calculando quando ambos os ciclos produzem o mesmo trabalho e, em seguida, analisando em que situações um produz mais trabalho baseado em como os parâmetros são variados. Em geral, os resultados demonstram que os motores Stirling produzem mais trabalho com pressões mais altas e menores volumes, e os motores Ericsson produzem mais trabalho com pressões mais baixas e volumes maiores. Para certos cenários, os valores de limiar são calculados para ilustrar precisamente quando um ciclo produz mais trabalho do que o outro. Esta tese pode ser usada para informar o design dos motores e particularmente para determinar quando um Stirling ou Ericsson devem ser selecionados para uma aplicação específica.

Palavras Chave: Stirling, Ericsson, Motor, Termodinâmica

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Nomenclature

W	Work	kJ
P	Pressure	kPa
V	Volume	m^3
T	Temperature	K
m	Mass of the working fluid	kg
R	Ideal gas constant	$kJ\,kg^{-1}\,K^{-1}$

Subscripts

max	maximum
min	minimum
H	High
L	Low

1 Introduction

Due to an increasing worldwide demand for clean energy, there is a corresponding need for alternative technologies to provide reliable and sustainable energy to meet this demand. Small-scale external heat engines, such as Stirling and Ericsson engines, offer the ability to use many different heat sources to provide reliable and sustainable power, such as solar thermal and biomass. Stirling and Ericsson cycles are both considered to have the Carnot efficiency ($\eta = 1 - T_L/T_H$) as their maximum theoretical efficiencies, but they accomplish this using different thermodynamic cycles, with constant volume and constant pressure regeneration, respectively, and thus they are inherently different.

a) **b)**

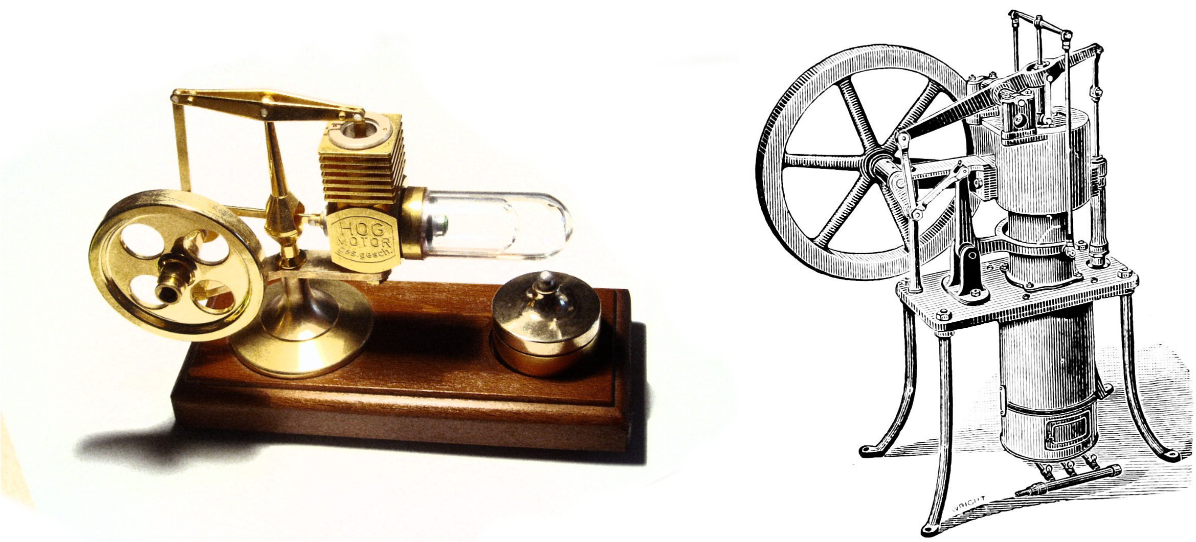

Figure 1.1: a) Stirling engine b) Ericsson engine.

1.1. Stirling Engine

Throughout the years, several works studied and developed Stirling and Ericsson engines. Stirling engines have been tested and analysed for three main configurations: the α-type , with two power pistons in separate cylinders, one hot and one cold; the

β-type, with a single power piston within the same cylinder and same shaft as a displacer piston; and the γ-type, where the power piston is located in a separate cylinder alongside the displacer piston cylinder.

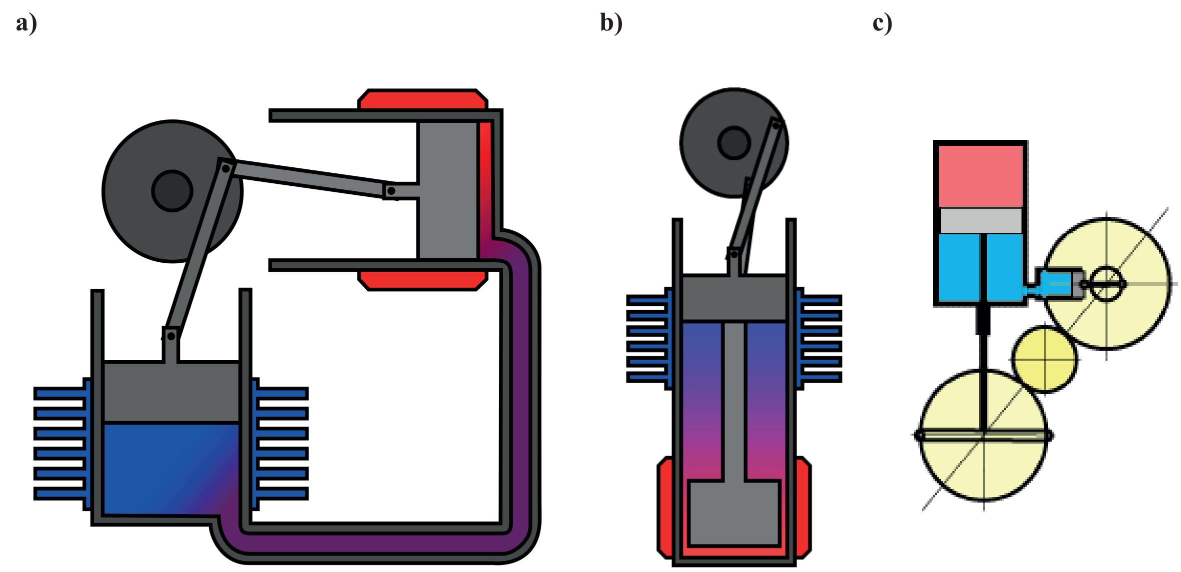

Figure 1.2: a) α-type b) β-type c) γ-type Stirling engines.

Scollo *et al.* (2013) redesigned the pistons and seals of a α-type engine improving the performance and Campos *et al.* (2012) used a mathematical model to do a thermodynamic optimization of the cycle achieving a 225% variation of calculated efficiency.

Cinar *et al.* (2005) and Sripakagorn & Srikam (2011) developed and tested the performance of a β-type Stirling engine, where the power of both engines reached satisfactory values. Cheng *et al.* (2013) also developed and tested a β-type Stirling engine and compared the experimental data to numerical predictions to verify a numerical model. The numerical predictions on the shaft power were higher than the experimental data by 12% $\sim$ 20%. Paul & Engeda (2015) modeled a β-type Stirling engine and were able to predict engine power and brake specific fuel consumption over a wide range of engine speeds and mean pressures.

Cinar & Karabulut (2005) designed, manufactured and tested a γ-type Stirling engine where they obtained the power output for different working fluids. Parlak *et al.* (2009) used a numerical model to obtain a thermodynamic analysis of a γ-type Stirling engine. This was used to maximize the power output and thermal efficiency of the engine. Bert *et al.* (2014) simulated a generic Stirling engine and experimentaly validated the results with a γ-type Stirling engine. The validated model was associated with a optimization algorithm to develop a Stirling engine design optimization tool for the kinematics of the engine.

There have been thorough reviews of the Stirling engine by Kongtragool & Wongwises (2003), Thombare & Verma (2008) and Wang *et al.* (2016).

1.2. Ericsson Engine

The research of the Ericsson engine is not as extensive as the Stirling engine, however, Sisman & Saygin (1999) studied the efficiency of the Ericsson cycle for different working fluids. An open cycle Ericsson engine (the working fluid is constantly being renovated) equipped with valves was modeled and analysed by Bonnet *et al.* (2005) for micro-generation purposes. A similiar engine was modeled by Touré & Stouffs (2014), where the relationships between the geometrical characteristics of the engine, its operating parameters, its power and efficiencies were established. Creyx *et al.* (2016) established a dynamic model and compared the effects of the air intake pressure and temperature conditions and the effects of the timing of intake and exhaust valve closing between the dynamic model and a steady-state thermodynamic model. It showed that the intake air pressure maximizing the Ericsson engine's indicated mean pressure was situated between 6 and 8 bar and that the highest temperatures improved the engine's performances.

The above research has been instrumental in the development and improvement of these engines but there has yet to be a comparison between Stirling and Ericsson engines.

1.3. Comparing Stirling and Ericsson engines

Previous work included analyses that apply to both Stirling and Ericsson engines, but both engines aren't compared, such as the work by Kaushik & Kumar (2001), where they used finite time thermodynamics to optimise the power output and the thermal efficiency of both engines. Tyagi *et al.* (2002) maximized the power output minus power loss and found that it is the increasing function of the internal irreversibility parameter, the reservoir temperature, heat capacitance ratios and the effectiveness on the cold and regenerative side heat exchangers. Badescu (1992) studied the influence of design and climatological parameters on a solar receiver for a Stirling or Ericsson engine.

Creyx *et al.* (2013) optimized an engine operating with an open Joule cycle (similiar to the Ericsson cycle but the compression and expansion processes are adiabatic instead of isothermal) or Ericsson cycle adapted for biomass upgrading in order to find another alternative to Stirling engines already used for the same application. Wojewoda & Kazimierski (2010) investigated an externally heated valve Joule engine and compared its efficiency and power output to a Stirling engine, having both similiar values. Hachem *et al.* (2015) made a comparison based on an exergetic analyses of a γ-type Stirling Engine and an Open Joule Cycle Engine, where the Stirling engine presented higher global performances due to the presence of a regenerator.

To date, there is a lack of understanding of when it is more advantageous to use the Stirling or Ericsson cycles.

1.4. Objectives

In this thesis, we compare the Stirling and Ericsson cycles and determine when one is more advantageous than the other. Since the purpose of a heat engine is to convert heat energy to mechanical work, we base the comparison on determining the situations where one cycle has a higher net work output than the other. The working fluid is assumed to be an ideal gas with $PV{=}mRT$, thus there are four main parameters determining the net work output of both cycles:

- Pressure
- Volume
- Mass of the working fluid
- Temperature

We determine the influence of each parameter by calculating the work produced by each cycle over a range of values while a subset of the other parameters are fixed.

By calculating the values at which the work is equal for both cycles, we are able to determine over which range of parameters each cycle will produce more or less work relative to the other. With this analysis, we are able to understand when one cycle is more advantageous than the other, which can be used in the design of engines and determine when to select one over the other for a particular application.

2 Theoretical Model

The theoretical Stirling and Ericsson cycles have isothermal heat addition and heat rejection processes. The main differences between them are the regeneration processes with two constant volume regeneration processes for the Stirling cycle and two constant pressure regeneration processes for the Ericsson cycle.

The comparison of both cycles is made by analyzing in which cases one cycle produces more net work than the other. The expressions for the net work are derived for each cycle by integrating the expression for the work:

$$dW = PdV \tag{2.1}$$

2.1. Stirling cycle

The net work of the Stirling cycle is calculated with the following integration, using the reference volumes listed in the P-v diagram in Fig. 2.1:

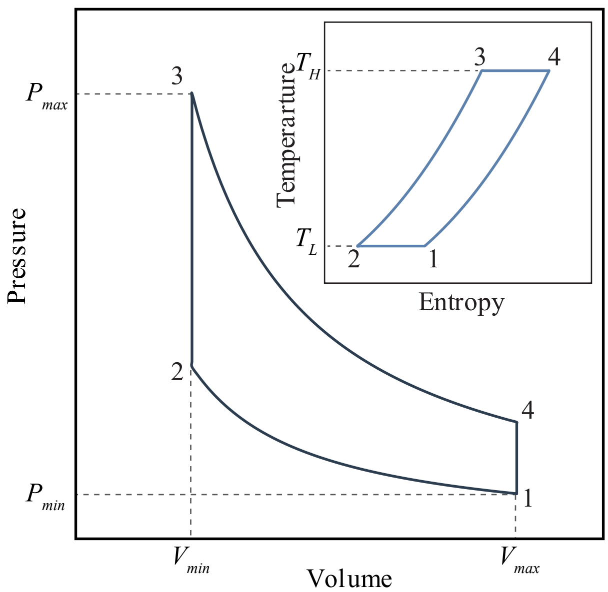

Figure 2.1: P-v diagram of Stirling Cycle, T-s diagram (inset).

$$W_S = \int_{V_1}^{V_2} PdV + \int_{V_3}^{V_4} PdV \tag{2.2}$$

From Fig. 2.1 we denote the maximum and minimum volumes for the processes as:

$$V_1 = V_4 = V_{max} \tag{2.3}$$

$$V_2 = V_3 = V_{min} \tag{2.4}$$

Replacing these volumes in Eq. 2.2 and rewriting the pressure according to the ideal gas relation, the net work of the Stirling cycle can be written as:

$$W_S = \int_{V_{max}}^{V_{min}} \frac{mRT_L}{V} dV + \int_{V_{min}}^{V_{max}} \frac{mRT_H}{V} dV \tag{2.5}$$

After integrating, we have:

$$W_S = mRT_L[ln\, V]_{V_{max}}^{V_{min}} + mRT_H[ln\, V]_{V_{min}}^{V_{max}} \tag{2.6}$$

and rearranging yields:

$$W_S = mR(T_H - T_L)\, ln\left(\frac{V_{max}}{V_{min}}\right) \tag{2.7}$$

2.2. Ericsson cycle

Similarly, integrating each process in the Ericsson cycle, with the volumes shown on the P-v diagram in Fig. 2.2, the net work is:

$$W_E = \int_{V_1}^{V_2} PdV + \int_{V_2}^{V_3} P_{max} dV + \int_{V_3}^{V_4} PdV + \int_{V_4}^{V_1} P_{min} dV \tag{2.8}$$

Based on the volumes shown in Fig. 2.2 and the ideal gas relation, we can rewrite the volumes according to the maximum and minimum volumes and pressures as follows:

$$V_1 = \frac{mRT_L}{P_{min}} = V_{min}\left(\frac{P_{max}}{P_{min}}\right) \tag{2.9}$$

$$V_2 = V_{min} \tag{2.10}$$

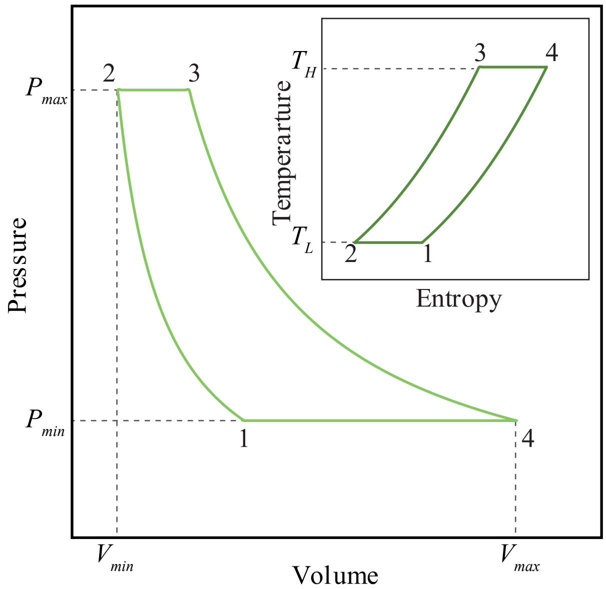

Figure 2.2: P-v diagram of Ericsson Cycle, T-s diagram (inset).

$$V_3 = \frac{mRT_H}{P_{max}} = V_{max}\left(\frac{P_{min}}{P_{max}}\right) \tag{2.11}$$

$$V_4 = V_{max} \tag{2.12}$$

Substituting these expressions into Eq. 2.8:

$$\begin{aligned} W_E = &\int_{V_{min}\left(\frac{P_{max}}{P_{min}}\right)}^{V_{min}} \frac{mRT_L}{V}\,dV + \int_{V_{min}}^{V_{max}\left(\frac{P_{min}}{P_{max}}\right)} P_{max}\,dV \\ &+ \int_{V_{max}\left(\frac{P_{min}}{P_{max}}\right)}^{V_{max}} \frac{mRT_H}{V}\,dV + \int_{V_{max}}^{V_{min}\left(\frac{P_{max}}{P_{min}}\right)} P_{min}\,dV \end{aligned} \tag{2.13}$$

After integrating, we have:

$$\begin{aligned} W_E = &mRT_L[ln\,V]_{V_{min}\left(\frac{P_{max}}{P_{min}}\right)}^{V_{min}} + P_{max}[V]_{V_{min}}^{V_{max}\left(\frac{P_{min}}{P_{max}}\right)} \\ &+ mRT_H[ln\,V]_{V_{max}\left(\frac{P_{min}}{P_{max}}\right)}^{V_{max}} + P_{min}[V]_{V_{max}}^{V_{min}\left(\frac{P_{max}}{P_{min}}\right)} \end{aligned} \tag{2.14}$$

and rearranging yields:

$$W_E = mR(T_H - T_L)\,ln\left(\frac{P_{max}}{P_{min}}\right) \tag{2.15}$$

Eqs. 2.7 and 2.15 give the net work for the Stirling and Ericsson cycles respectively and will be used as the basis for the comparison between the two cycles.

3 Results & Discussion

Examination of Eqs. 2.7 and 2.15 shows that the only difference in the net work between the Stirling and Ericsson cycles is the presence of the volume ratio in the Stirling work equation and pressure ratio in the Ericsson work equation. This indicates the net work output of the two cycles is equal when the volume ratio of the Stirling cycle is equal to the pressure ratio of the Ericsson. This fact alone does not reveal much about how the cycles compare with each other because the two cycles will often be compared when certain operating parameters are the same in each of the cycles. This leads us to analyze how the parameters influence the net work output of the cycles, to provide a more meaningful comparison.

The ideal gas relation limits how the thermodynamic parameters can vary throughout each of the cycles relative to one another. In order to understand how all of the parameters influence the net work output both cycles are compared for three different scenarios: (i) equal mass and temperature limits, where pressure and volume ratios are varied, (ii) equal mass and pressure or volume ratios, where the temperature limits are varied, and (iii) equal temperature and pressure or volume limits, where the mass is varied. The three analyses provide a broad basis for comparison to assess which of the cycles are more favorable under which operating conditions.

3.1. Comparison with equal mass and temperature limits

For the first comparison, the two cycles are considered to have an equivalent mass of working fluid and the same high and low temperature limits. The fixed mass and temperature limits enable an analysis of how the pressure or volume ratio influences the net work output. The influence of the pressure ratio is analyzed first followed by the volume ratio.

To analyze the influence of the pressure ratio on the net work output, the ideal gas relation is substituted into Eq. 2.7:

$$W_S = mR(T_H - T_L)\, ln\left(\left(\frac{P_{max}}{P_{min}}\right)_S \frac{T_L}{T_H}\right) \tag{3.1}$$

In order to calculate when the two cycles have equivalent net work output, Eqs. 2.15 and 3.1 can be equated and simplified by exploiting the equal mass and temperature limits, as follows:

$$\left(\frac{P_{max}}{P_{min}}\right)_S \frac{T_L}{T_H} = \left(\frac{P_{max}}{P_{min}}\right)_E \tag{3.2}$$

Since the temperature ratio in Eq. 3.2 is always less than one, this result shows that in order to generate the equivalent net work output when mass and temperature limits are fixed, the Stirling cycle requires a higher pressure ratio than the Ericsson cycle. The pressure ratio of the Stirling cycle must be higher in proportion to the temperature ratio, as shown in Fig. 3.1a. The solid lines in Fig. 3.1a represent the equivalent net work output, while above the lines the Ericsson cycle produces more work and below the lines the Stirling cycle produces more work. Fig. 3.1a shows that for the same pressure ratio, the Ericsson cycle produces more net work output when the mass of the working fluid and the temperature limits are fixed.

Similarly to the pressure ratio analysis, to analyze the influence of the volume ratio on the net work output, the ideal gas relation is substituted into Eq. 2.15:

$$W_E = mR(T_H - T_L)\, ln\left(\left(\frac{V_{max}}{V_{min}}\right)_E \frac{T_L}{T_H}\right) \tag{3.3}$$

In order to calculate when the two cycles have the equivalent net work output, Eqs. 2.7 and 3.3 can be equated and simplified by exploiting the equal mass and temperature limits, as follows:

$$\left(\frac{V_{max}}{V_{min}}\right)_S = \left(\frac{V_{max}}{V_{min}}\right)_E \frac{T_L}{T_H} \tag{3.4}$$

There is a reverse similarity in the results for the pressure and volume ratios. Since the temperature ratio in Eq. 3.4 is always less than one, this result shows that in order to generate equivalent net work output when mass and temperature limits are fixed, the Ericsson cycle requires a higher volume ratio than the Stirling cycle. The volume ratio of the Stirling cycle must be higher in proportion to the temperature ratio, as

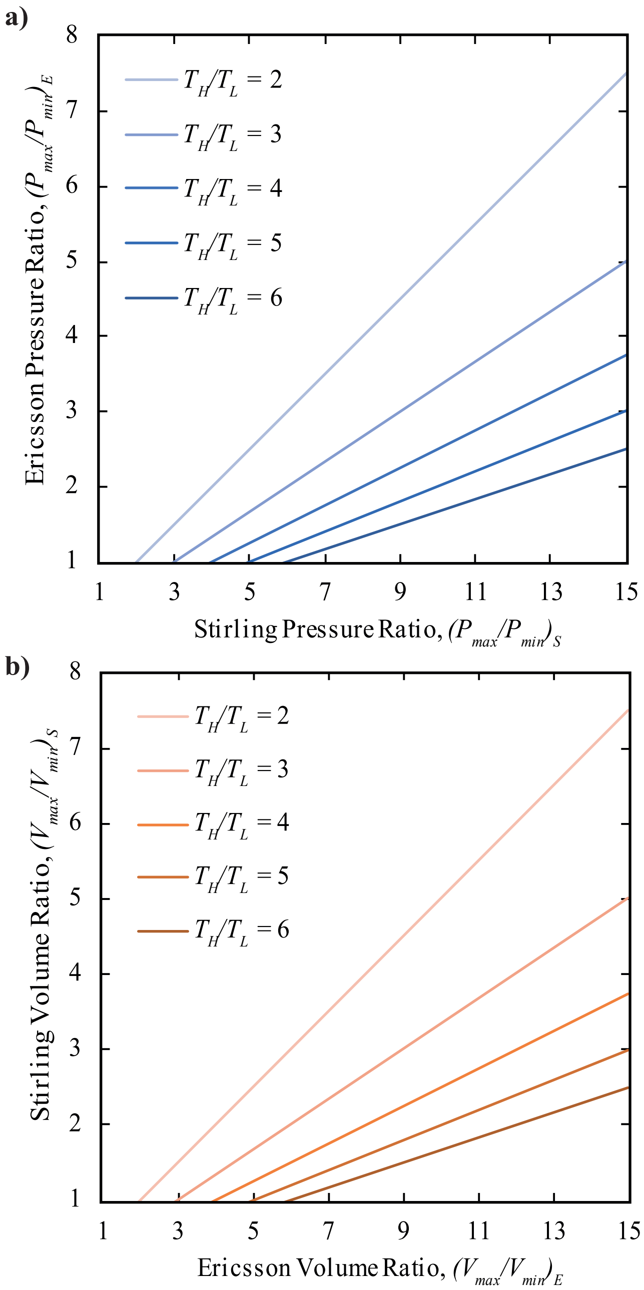

Figure 3.1: a) Pressure ratio relation, b) Volume ratio relation between Stirling and Ericsson cycles for equal net work.

shown in Fig. 3.1b, which is the opposite of the pressure ratio results. Fig. 3.1b shows that for the same volume ratio, the Stirling cycle produces more net work output when the mass of the working fluid and the temperature limits are fixed.

The results of this analysis can be applied to a general rule about the selection of Stirling or Ericsson engines in practice, when the mass and temperature limits are fixed. If the pressure ratio is more limiting for the engine design than the engine size, an Ericsson engine is recommended. If the reverse is true, and the engine size is a more critical constraint than the higher pressure ratio, a Stirling engine is recommended.

In order to demonstrate how the results apply to the design of Stirling and Ericsson engines more directly, two specific cases are examined: (i) one where the maximum pressure and volume are fixed, and (ii) one where the minimum pressure and volume are fixed. The specific operating parameters are listed in Table. 3.1, corresponding to a small stationary engine.

Operating Parameters	**Case (i)**	**Case (ii)**
Working fluid	air	air
m (mg)	30	30
T_H (K)	773.15	773.15
T_L (K)	298.15	298.15
P_{max} (kPa)	1013.3	—
V_{max} (m^3)	6.28×10^{-5}	—
P_{min} (kPa)	—	101.33
V_{min} (m^3)	—	6.28×10^{-6}

Table 3.1: Set parameters for small scale engine (refers to Fig. 3.2).

For fixed maximum pressures and volumes, Eqs. 2.7 and 2.15 simplify to yield the same net work output for both cycles, as follows:

$$W_S = W_E = mR(T_H - T_L)\, ln\left(\frac{P_{max}V_{max}}{mRT_H}\right) \tag{3.5}$$

The cycles can then be plotted on a P-v diagram, as shown in Fig. 3.2a, to enable a comparison of the resulting minimum pressure and volume values. In both cases, the Stirling's volume ratio is equal to the Ericsson's pressure ratio to yield the same net work output. For this scenario, the Stirling cycle requires a lower minimum pressure value and higher minimum volume value, and the Ericsson cycle requires a lower minimum volume value and higher minimum pressure. This illustrates the need for the Stirling cycle to have a higher pressure ratio and the Ericsson cycle to have a higher volume ratio to generate the same amount of work.

Similarly, for fixed minimum pressures and volumes, Eqs. 2.7 and 2.15 simplify to

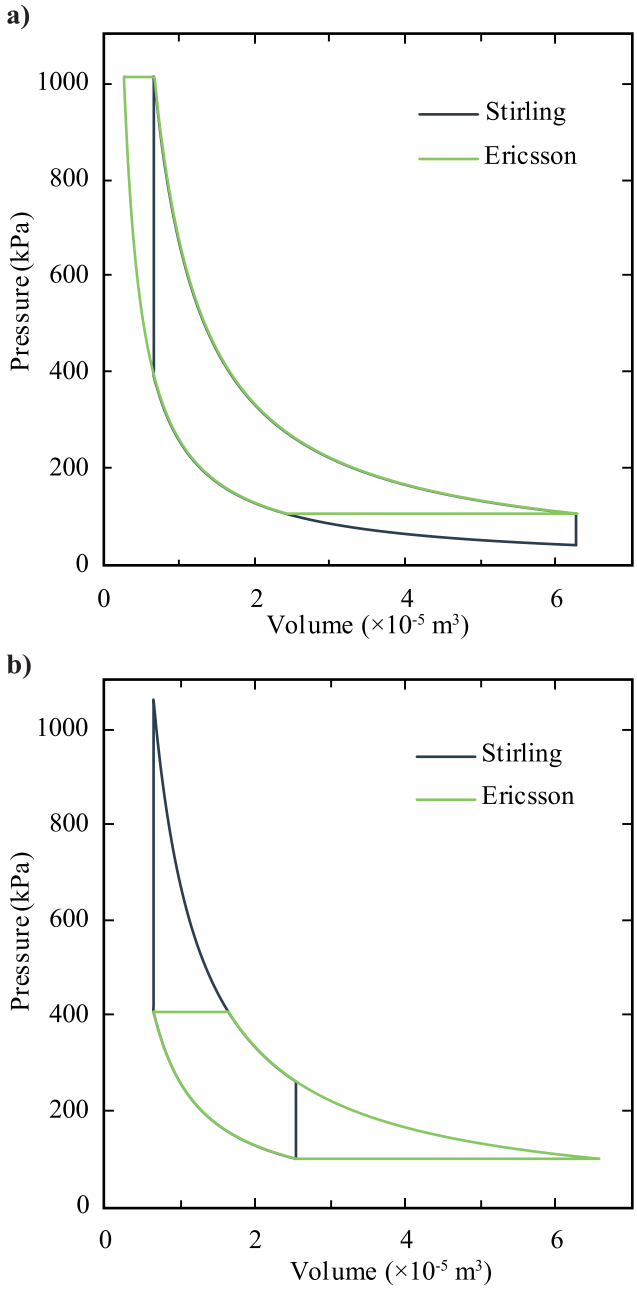

Figure 3.2: P-v diagrams of Stirling and Ericsson cycles for fixed mass and temperature limits and a) Fixed maximum pressure and volume, b) Fixed minimum pressure and volume.

yield the same net work output for both cycles, as follows:

$$W_S = W_E = mR(T_H - T_L)\, ln\left(\frac{mRT_L}{P_{min}V_{min}}\right) \tag{3.6}$$

The resulting P-v diagrams are plotted in Fig. 3.2b, and, as in the previous case, the Stirling's volume ratio is equal to the Ericsson's pressure ratio to yield the same net work output. For this scenario, the Stirling cycle requires a lower maximum volume value and higher maximum pressure, and the Ericsson cycle requires a higher maximum volume value and lower maximum pressure value. This confirms the need for the Stirling cycle to have a higher pressure ratio and the Ericsson cycle to have a higher volume ratio to generate the same amount of work. The P-v diagram in Fig. 3.2b gives a visual demonstration of the requirement for the Stirling to have a higher pressure and the Ericsson to have a higher volume when the mass, temperature limits, and minimum pressure and volume values are fixed.

3.2. Comparison with equivalent mass and pressure or volume ratio

For the second comparison, the two cycles are considered to have the same mass of the working fluid, and there are two cases: (i) one with the same pressure ratio, and (ii) one with the same volume ratio. Fixing the mass of the working fluid and the pressure or volume ratio allows for variation of the temperature limits. This case is useful for practical consideration because the low temperature limit is often governed by the atmospheric temperature and the high temperature is often governed by which heat source is selected.

3.2.1. Equivalent pressure ratios

In order to calculate when the two cycles have equivalent net work output for equal pressure ratio, Eqs. 2.15 and 3.1 can be equated and simplified by exploiting the equal mass of the working fluid and pressure ratio, as follows:

$$\frac{P_{max}}{P_{min}} = \left(\frac{T_L}{T_H}\right)_S^{\frac{(T_H - T_L)_S}{(T_H - T_L)_E - (T_H - T_L)_S}} \tag{3.7}$$

Unlike the previous cases, in this case the high and low temperatures appear individually, and not exclusively as a temperature ratio, so there is a need to fix the high or low temperatures in the analysis. Since the temperature of the heat sink is commonly more restrictive than the temperature of the heat source, the comparison is more relevant when the low temperature is fixed, and the engines are compared based on the temperature range of the heat sources (high temperature) that can be selected.

Eq. 3.7 can be rearranged by fixing an equivalent low temperature, as follows:

$$T_{H,E} = T_{H,S} + \frac{ln\left(\frac{T_L}{T_{H,S}}\right)}{ln\left(\frac{P_{max}}{P_{min}}\right)}(T_{H,S} - T_L) \tag{3.8}$$

This relation is plotted in Fig. 3.3b for a low temperature value of 25°C, with the solid lines showing the equivalent net work output, which illustrates that a Stirling cycle requires a higher high temperature value than the Ericsson to yield the same net work output when the mass and pressure ratio are equal. Below the solid line represents when the Stirling cycle produces more net work output, and correspondingly, above each solid line represents when the Ericsson cycle produces more net work output. The P-v diagrams for this case are shown in Fig. 3.3b using the specific parameters listed in Table. 3.2. It can be seen in Fig. 3.3b that for this scenario the Stirling cycle requires a lower volume ratio to produce the same work as the Ericsson cycle, and as a result the high temperature requirement for the Stirling cycle is higher. Conversely, the Ericsson cycle requires a higher volume ratio, which results in a lower high temperature requirement.

Operating Parameters	**Case (i)**	**Case (ii)**
Working fluid	air	air
m (mg)	30	30
T_L (K)	298.15	298.15
$T_{H,S}$ (K)	773.15	—
$T_{H,E}$ (K)	—	773.15
P_{max} (kPa)	1013.3	—
P_{min} (kPa)	101.33	—
V_{max} (m^3)	—	3.14×10^{-5}
V_{min} (m^3)	—	3.14×10^{-6}

Table 3.2: Parameters used for specific comparison (refers to Figs. 3.3b and 3.4b).

3.2.2. Equivalent volume ratios

Similarly to the pressure ratio analysis, in order to calculate when the two cycles have equivalent net work output for equal volume ratio, Eqs. 2.7 and 3.3 can be equated

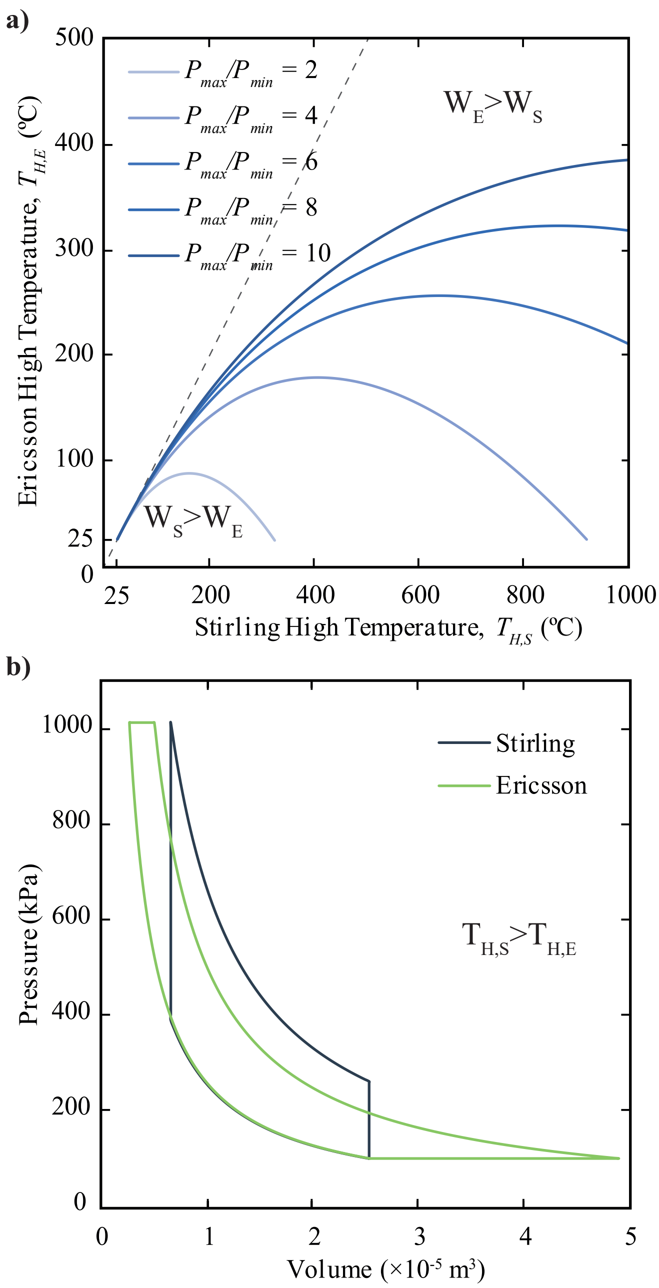

Figure 3.3: a) High temperature relation between Stirling and Ericsson cycles for fixed pressure ratios and TL=25 °C for equal net work output, b) P-v diagrams of Stirling and Ericsson cycles for fixed mass, pressure limits and low temperature for equal net work output.

and simplified by exploiting the equal mass of the working fluid and volume ratio, as follows:

$$\frac{V_{max}}{V_{min}} = \left(\frac{T_L}{T_H}\right)_E^{\frac{(T_H - T_L)_E}{(T_H - T_L)_S - (T_H - T_L)_E}} \tag{3.9}$$

Eq. 3.9 can be rearranged by fixing an equivalent low temperature, as follows:

$$T_{H,S} = T_{H,E} + \frac{ln\left(\frac{T_L}{T_{H,E}}\right)}{ln\left(\frac{V_{max}}{V_{min}}\right)}(T_{H,E} - T_L) \tag{3.10}$$

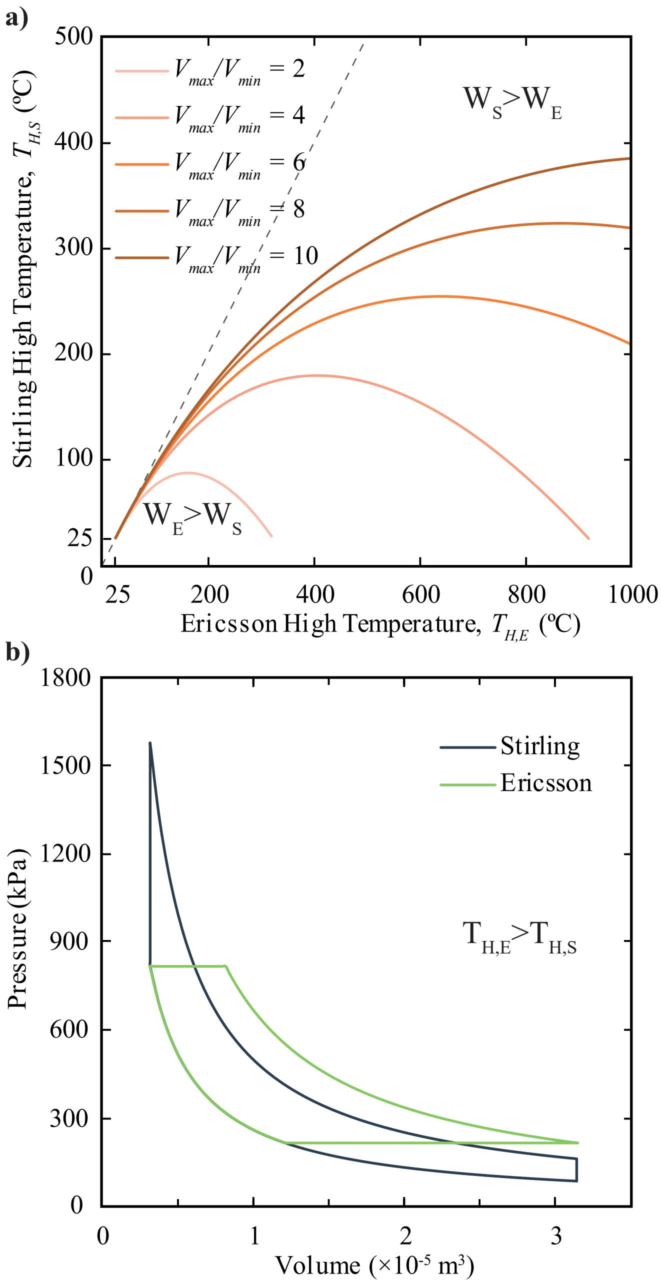

Figure 3.4: a) High temperature relation between Stirling and Ericsson cycles for fixed volume ratios and TL=25 °C for equal net work output, b) P-v diagrams of Stirling and Ericsson cycles for fixed mass, volume limits and low temperature for equal net work output.

This relation is plotted in Fig. 3.4b for a low temperature value of 25°C, with the solid lines showing the equivalent net work output, which illustrates that a Stirling cycle requires a lower high temperature value than the Ericsson to yield the same net work output when the mass and volume ratio are equal. Above the solid line represents when the Stirling cycle produces more net work output, and correspondingly, below each solid line represents when the Ericsson cycle produces more net work output. The P-v diagrams for this case are shown in Fig. 3.4b using the specific parameters listed in Table. 3.2. It can be seen in Fig. 3.4b that for this scenario the Ericsson cycle requires a lower pressure ratio to produce the same work as the Stirling cycle, and as a result the high temperature requirement for the Ericsson cycle is higher. Conversely, the Stirling cycle requires a higher pressure ratio, which results in a lower high temperature requirement.

3.3. Comparison with equivalent temperature limits and pressure or volume ratios

For the third comparison, the two cycles are considered to have the same high and low temperature limits, and there are two cases: (i) where the pressure ratio is equivalent and the maximum volume is fixed, and (ii) where the volume ratio is equivalent and the maximum pressure is fixed. Fixing the temperature limits along with either the pressure or volume ratios allows for variation of the mass of the working fluid. This situation is arguably more relevant for practical application since the mass of the working fluid is rarely a primary design consideration and the other engine characteristics, such as operating pressures, size (volume), and the temperature of the available heat source and heat sink are commonly more restrictive.

3.3.1. Equivalent pressure ratio and fixed maximum volume

To compare the cycles with equivalent pressure ratios, Eqs. 2.15 and 3.1 can be simplified by exploiting the equal temperature limits and pressure ratio and since the maximum volume is also fixed, the masses of the working fluid of the Stirling and Ericsson cycles are calculated using the maximum volume and minimum pressure, corresponding to point 1 in Fig. 2.1 and point 4 in Fig. 2.2, respectively, which yields:

$$W_S = V_{max}P_{min}\left(\frac{T_H}{T_L} - 1\right) ln\left(\frac{P_{max}T_L}{P_{min}T_H}\right) \tag{3.11}$$

$$W_E = V_{max}P_{min}\left(1 - \frac{T_L}{T_H}\right) ln\left(\frac{P_{max}}{P_{min}}\right) \tag{3.12}$$

Non-Dimensional net work can be achieved by dividing the network by the maximum volume and minimum pressure, as follows:

$$\frac{W_S}{V_{max}P_{min}} = \left(\frac{T_H}{T_L} - 1\right) ln\left(\frac{P_{max}T_L}{P_{min}T_H}\right) \tag{3.13}$$

$$\frac{W_E}{V_{max}P_{min}} = \left(1 - \frac{T_L}{T_H}\right) ln\left(\frac{P_{max}}{P_{min}}\right) \tag{3.14}$$

Eqs. 3.13 and 3.14 can be equalized to demonstrate the relationship between the temperature and pressure ratios when the cycles have equal net work output:

$$\frac{P_{max}}{P_{min}} = \left(\frac{T_H}{T_L}\right)^{\frac{\frac{T_H}{T_L}}{\frac{T_H}{T_L}-1}} \tag{3.15}$$

Eq. 3.15 is plotted in Fig. 3.5a with the solid line showing where the net work output of both cycles is equal. At the higher pressure ratios above the line, the Stirling cycle produces more net work, and at the lower pressure ratios below the line, the Ericsson cycle produces more net work. To show how this impacts a specific engine design, the inset shows the P-v diagram corresponding to the parameters listed in Table. 3.3. The P-v diagram illustrates how the Ericsson cycle requires a lower minimum volume than the Stirling cycle to produce the same amount of net work output. This result ultimately indicates that for a given temperature ratio, with a size restriction (i.e. fixed maximum volume), there is a threshold pressure ratio that determines which cycle produces more net work. For example, as shown in Fig. 3.5b, where Eqs. 3.13 and 3.14 are plotted, if the temperature ratio is fixed at 2.6, the threshold pressure ratio is 4.7, and below this value an Ericsson cycle produces more net work output, while above it a Stirling cycle produces more net work output. Correspondingly, as shown in the inset of Fig. 3.5b, if the pressure ratio is fixed at 4.7, the threshold temperature ratio is 2.6, and below this value a Stirling cycle produces more net work output, while above it an Ericsson cycle produces more net work output. Since engine

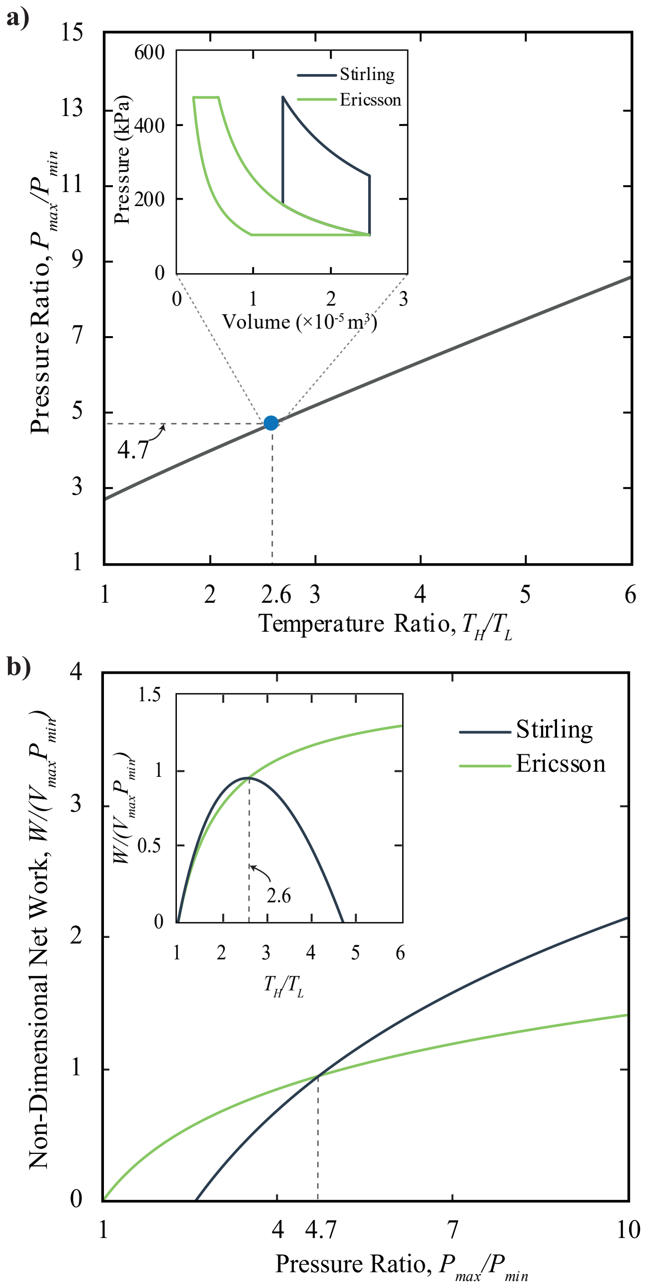

Figure 3.5: a) Relation between pressure and temperature ratio for equal net work output, P-v diagram of Stirling and Ericsson cycles for fixed temperature and pressure limits and maximum volume for equal net work output (inset), b) Non-Dimensional net work in function of pressure ratio and temperature ratio (inset).

design is commonly limited by a size restriction, the operating pressure limits, and the temperature of the available heat source and sink, this result is a useful tool for determining which of the two cycles should be used for specific scenarios.

Operating Parameters	Case (i)	Case (ii)
Working fluid	air	air
T_H (K)	773.15	773.15
T_L (K)	298.15	298.15
P_{min} (kPa)	101.33	—
V_{max} (m^3)	2.51×10^{-5}	—
V_{min} (m^3)	—	4.71×10^{-6}
P_{max} (kPa)	—	1013.3

Table 3.3: Parameters used for specific comparison (refers to Figs. 3.5 and 3.6).

3.3.2. Equivalent volume ratio and fixed maximum pressure

To compare the cycles with equivalent pressure ratios, Eqs. 2.7 and 3.3 can be simplified by exploiting the equal temperature limits and volume ratio and since the maximum pressure is also fixed, the masses of the working fluid of the Stirling and Ericsson cycles are calculated using the maximum pressure and minimum volume, corresponding to point 3 in Fig. 2.1 and point 2 in Fig. 2.2, respectively, which yields:

$$W_S = P_{max}V_{min}\left(1 - \frac{T_H}{T_L}\right) ln\left(\frac{V_{max}}{V_{min}}\right) \tag{3.16}$$

$$W_E = P_{max}V_{min}\left(\frac{T_H}{T_L} - 1\right) ln\left(\frac{V_{max}T_L}{V_{min}T_H}\right) \tag{3.17}$$

Non-Dimensional net work can be achieved by dividing the network by the maximum pressure and minimum volume, as follows:

$$\frac{W_S}{P_{max}V_{min}} = \left(1 - \frac{T_H}{T_L}\right) ln\left(\frac{V_{max}}{V_{min}}\right) \tag{3.18}$$

$$\frac{W_E}{P_{max}V_{min}} = \left(\frac{T_H}{T_L} - 1\right) ln\left(\frac{V_{max}T_L}{V_{min}T_H}\right) \tag{3.19}$$

Eqs. 3.18 and 3.19 can be equalized to demonstrate the relationship between the temperature and volume ratios when the cycles have equal net work output:

$$\frac{V_{max}}{V_{min}} = \left(\frac{T_H}{T_L}\right)^{\frac{\frac{T_H}{T_L}}{\frac{T_H}{T_L}-1}} \tag{3.20}$$

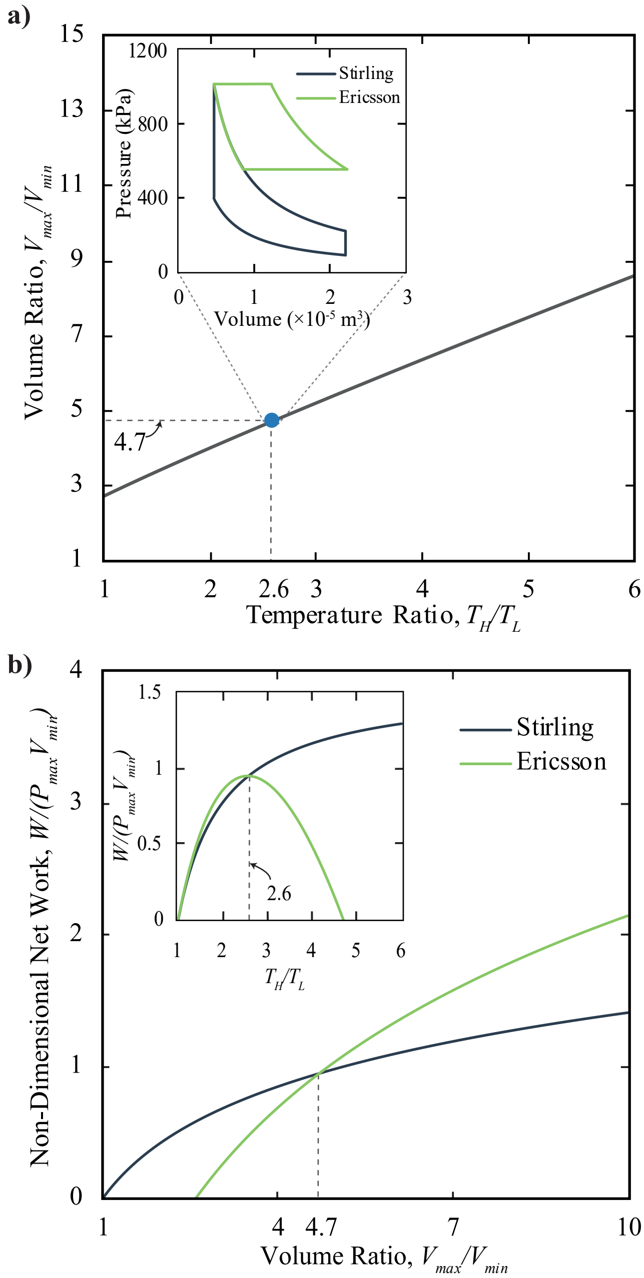

Figure 3.6: a) Relation between volume and temperature ratio for equal net work output, P-v diagram of Stirling and Ericsson cycles for fixed temperature and volume limits and maximum pressure for equal net work output (inset), b) Non-Dimensional net work in function of volume ratio and temperature ratio (inset).

Eq. 3.20 is plotted in Fig. 3.6a with the solid line showing where the net work output of both cycles is equal. The results here show similar patterns to the previous case with the maximum volume restriction. At the higher volume ratios above the line, the Ericsson cycle produces more net work, and at the lower volume ratios below the line, the Stirling cycle produces more net work. To show how this impacts a specific engine design, the inset shows the P-v diagram corresponding to the parameters listed in Table. 3.3. The P-v diagram illustrates how the Stirling cycle requires a lower minimum pressure than the Ericsson cycle to produce the same amount of net work output. This result ultimately indicates that for a given temperature ratio, with a maximum pressure restriction, the volume ratio will determine which cycle produces more net work. For example, as shown in Fig. 3.6b, where Eqs. 3.18 and 3.19 are plotted, if the temperature ratio is fixed at 2.6, the threshold volume ratio is 4.7, and above this value a Stirling cycle produces more net work output, while below it an Ericsson cycle produces more net work output. Correspondingly, as shown in the inset of Fig. 3.6b, if the volume ratio is fixed at 4.7, the threshold temperature ratio is 2.6, and above this value an Ericsson cycle produces more net work output, while below it a Stirling cycle produces more net work output. Similar to the previous case, since engine design is commonly limited by the attainable minimum and maximum volumes, the maximum operating pressure, and the temperature of the available heat source and sink, this result is a useful tool for determining which of the two cycles should be used for specific scenarios. This case is more applicable when the minimum volume is more critical and the previous case is more applicable when the minimum pressure is more critical.

4 Conclusions

In this thesis, Stirling and Ericsson cycles have been compared to determine which one produces more net work output for three different scenarios: (i) equal mass and temperature limits, where pressure and volume ratios are varied, (ii) equal mass and pressure or volume ratios, where the temperature limits are varied, and (iii) equal temperature and pressure or volume ratios, where the mass is varied.

For the first scenario, when both cycles are compared with equal working fluid mass and temperature limits, the Ericsson cycle produces more net work output when the pressure ratio is restricted, and the Stirling cycle produces more net work output when the volume ratio is restricted. Overall this indicates that when the mass of the working fluid and the temperature limits are equal, an Ericsson engine produces more net work output if the pressure ratio is more limiting, and a Stirling engine produces more net work output if the engine size (volume) is a more critical design constraint.

For the first case of the second scenario, when both cycles are compared for a fixed low temperature value and with equal mass and pressure ratios, the Stirling cycle requires a higher high temperature value than the Ericsson to yield the same net work output. For the second case of the second scenario, when both cycles are compared for a fixed low temperature value and with equal mass and volume ratios, the Ericsson cycle requires a higher high temperature value than the Stirling to yield the same net work output.

For the first case of the third scenario, when both cycles are compared for a specific temperature ratio and maximum volume, the Stirling cycle produces more net work output above a threshold pressure ratio value, and when both cycles are compared for a specific pressure ratio and maximum volume, the Ericsson cycle produces more net work output above a threshold temperature ratio. For the second case of the third scenario, when both cycles are compared for a specific temperature ratio and maximum pressure, the Stirling cycle produces more net work output above a threshold volume ratio value, and when both cycles are compared for a specific volume ratio and maximum pressure, the Ericsson cycle produces more net work output above a threshold temperature ratio. The equations were derived to obtain these threshold values from the operating parameters.

The results in this thesis reveal the various situations when one cycle produces more net work output than the other, which can be used to inform the design of the engines and determine when either a Stirling or Ericsson engine should be selected for a particular application.

Bibliography

Badescu, V. 1992. Optimum operation of a solar converter in combination with a Stirling or Ericsson heat engine. *Energy*, **17**(6), 601–607.

Bert, Juliette, Chrenko, Daniela, Sophy, Tonino, Le Moyne, Luis, & Sirot, Frédéric. 2014. Simulation, experimental validation and kinematic optimization of a Stirling engine using air and helium. *Energy*, **78**, 701–712.

Bonnet, S, Alaphilippe, M, & Stouffs, P. 2005. Energy, exergy and cost analysis of a micro-cogeneration system based on an Ericsson engine. *International Journal of Thermal Sciences*, **44**(12), 1161–1168.

Campos, MC, Vargas, JVC, & Ordonez, JC. 2012. Thermodynamic optimization of a Stirling engine. *Energy*, **44**(1), 902–910.

Cheng, Chin-Hsiang, Yang, Hang-Suin, & Keong, Lam. 2013. Theoretical and experimental study of a 300-W beta-type Stirling engine. *Energy*, **59**, 590–599.

Cinar, Can, & Karabulut, Halit. 2005. Manufacturing and testing of a gamma type Stirling engine. *Renewable energy*, **30**(1), 57–66.

Cinar, Can, Yucesu, Serdar, Topgul, Tolga, & Okur, Melih. 2005. Beta-type Stirling engine operating at atmospheric pressure. *Applied Energy*, **81**(4), 351–357.

Creyx, M, Delacourt, E, Morin, C, Desmet, B, & Peultier, P. 2013. Energetic optimization of the performances of a hot air engine for micro-CHP systems working with a Joule or an Ericsson cycle. *Energy*, **49**, 229–239.

Creyx, M, Delacourt, E, Morin, C, & Desmet, B. 2016. Dynamic modelling of the expansion cylinder of an open Joule cycle Ericsson engine: A bond graph approach. *Energy*, **102**, 31–43.

Hachem, Houda, Creyx, Marie, Gheith, Ramla, Delacourt, Eric, Morin, Céline, Aloui, Fethi, & Nasrallah, Sassi Ben. 2015. Comparison Based on Exergetic Analyses of Two Hot Air Engines: A Gamma Type Stirling Engine and an Open Joule Cycle Ericsson Engine. *Entropy*, **17**(11), 7331–7348.

Kaushik, SC, & Kumar, S. 2001. Finite time thermodynamic evaluation of irreversible Ericsson and Stirling heat engines. *Energy Conversion and Management*, **42**(3), 295–312.

Kongtragool, Bancha, & Wongwises, Somchai. 2003. A review of solar-powered Stirling engines and low temperature differential Stirling engines. *Renewable and Sustainable energy reviews*, **7**(2), 131–154.

Parlak, Nezaket, Wagner, Andreas, Elsner, Michael, & Soyhan, Hakan S. 2009. Thermodynamic analysis of a gamma type Stirling engine in non-ideal adiabatic conditions. *Renewable Energy*, **34**(1), 266–273.

Paul, Christopher J, & Engeda, Abraham. 2015. Modeling a complete Stirling engine. *Energy*, **80**, 85–97.

Scollo, LS, Valdez, PE, Santamarina, SR, Chini, MR, & Baron, JH. 2013. Twin cylinder alpha stirling engine combined model and prototype redesign. *International Journal of Hydrogen Energy*, **38**(4), 1988–1996.

Sisman, Altug, & Saygin, Hasan. 1999. On the power cycles working with ideal quantum gases: I. The Ericsson cycle. *Journal of Physics D: Applied Physics*, **32**(6), 664.

Sripakagorn, Angkee, & Srikam, Chana. 2011. Design and performance of a moderate temperature difference Stirling engine. *Renewable Energy*, **36**(6), 1728–1733.

Thombare, DG, & Verma, SK. 2008. Technological development in the Stirling cycle engines. *Renewable and Sustainable Energy Reviews*, **12**(1), 1–38.

Touré, Abdou, & Stouffs, Pascal. 2014. Modeling of the Ericsson engine. *Energy*, **76**, 445–452.

Tyagi, SK, Kaushik, SC, & Salhotra, R. 2002. Ecological optimization and performance study of irreversible Stirling and Ericsson heat engines. *Journal of Physics D: Applied Physics*, **35**(20), 2668.

Wang, Kai, Sanders, Seth R, Dubey, Swapnil, Choo, Fook Hoong, & Duan, Fei. 2016. Stirling cycle engines for recovering low and moderate temperature heat: A review. *Renewable and Sustainable Energy Reviews*, **62**, 89–108.

Wojewoda, Jerzy, & Kazimierski, Zbyszko. 2010. Numerical model and investigations of the externally heated valve Joule engine. *Energy*, **35**(5), 2099–2108.