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Characterizing the spatial variability of groundwater quality using the entropy theory: II. Case study from Gaza Strip

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Abstract:

This paper, the second in the series, uses the entropy theory to describe the spatial variability of groundwater quality data sets. The application of the entropy theory is illustrated using the chloride observations obtained from a network of groundwater quality monitoring wells in the Gaza Strip, Palestine. The application involves calculating information measures, such as transinformation, the information transfer index and the correlation coefficient. These measures are calculated using a discrete approach, in which contingency tables are used. An exponential decay fitting approach was applied to the discrete models. The analysis shows that transinformation, as a function of distance, can be represented by the exponential decay curve. It also indicates that, for the data used in this study, the transinformation model is superior to the correlation model for characterizing the spatial variability. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS correlation; entropy; information; spatial variability; Gaza Strip; Palestine

INTRODUCTION

Entropy theory (information theory) came to be viewed as a statistical concept at the beginning of the twentieth century. About 50 years later, it found its way into engineering and mathematics, notably through the work of Shannon in communication engineering. Shannon (1948) used entropy as a measure of uncertainty in the mind of someone receiving a message that contains noise. Later, in 1957, Jaynes made use of Shannon's entropy metric to formulate the maximum entropy principle that formed a basis for estimation and inference problems (Golan *et al*., 1997). In 1972 Amorocho and Esplidora were the first to apply the entropy concept to hydrological modelling (Singh, 1997). Since then, there has been a great variety of entropy applications in hydrology and water resources management (e.g. Rajagopal *et al*., 1987; Singh and Rajagopal, 1987; Singh, 1998; Harmancioglu *et al*., 1999). Entropy theory can be used in modelling and decision-making in environmental and water resources, especially in developing countries (Singh, 2000).

Entropy theory also has been applied to assess and evaluate monitoring networks with respect to: water quality (Harmancioglu *et al*., 1994, Ozkul *et al*., 2000), rainfall (Krastanovic and Singh, 1992) and groundwater (Bueso *et al*., 1999; Mogheir and Singh, 2002). Most of these applications involve applying entropy theory to the evaluation, assessment and design of monitoring networks, and they used an analytical approach with a presumed knowledge of the probability distributions of the random variables involved. In the first paper of this series, Mogheir *et al*. (2004) adopted discrete and analytical approaches using a synthetic

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data set, where the data were spatially correlated and fitted the normal distribution function. Under these conditions, it was found that there was a reasonable agreement between discrete and analytical approaches for developing the transinformation model (T model), and it was shown that the T model also could be used instead of the correlation model (C model) to characterize the spatial variability.

In this paper, a different set of data is used. The set of data includes groundwater quality from the Gaza Strip monitoring network (chloride data). For these data, the spatial correlation is low and the normal distribution function does not fit. The objective of this paper is to:

1. use a discrete approach (contingency table) for calculating information measures, such as transinformation (T), information transfer index (ITI) and correlation coefficients.
2. apply an exponential decay fitting approach to the discrete T model and C model;
3. use the T model and C model to describe the spatial variability of the Gaza Strip data set.

GAZA STRIP GROUNDWATER QUALITY DATA

The set of data used in the analysis is part of groundwater quality data from the Gaza Strip, Palestine. The data were selected from the groundwater quality data monitored in the middle part of the Gaza Strip. This part of the Gaza Strip is the area with the most serious problems of seawater intrusion. More than 150 wells are used to monitor the groundwater quality in this area. In this study, 26 monitoring wells that monitor chloride were selected. Each well has 52 chloride data measured between 1972 and 1997. Chloride is measured twice per year: in winter and summer. The winter cycle is considered to be taken in April and May whereas the summer cycle is in October and November. The locations of these 26 wells in the middle part of the Gaza Strip are shown in Figure 1. The chloride time-series of the 26 wells are presented in Table I. In the table, $\bar{x}$ is the mean and S_x is the standard deviation of the chloride data. The spatial variation of the mean of the chloride time-series in each well is presented in Figure 2. The contour lines were drawn using the kriging technique, which is an option in the Surfer-7 mapping program (Golden Software, 1999). Additionally, the chloride time-series of some of these wells are plotted in Figure 3. The groundwater data in the Gaza Strip (quality and water level) were summarized and presented by the Palestinian Water Authority (PWA, 2000). These data were also used in the modelling of the Gaza Strip aquifer by Metcalf and Eddy (2000).

METHOD

The method used in this study follows that presented in Mogheir *et al*. (2004). A contingency table is used for the discrete approach. The discrete models' results were smoothed using the moving average method. For convenience, the base *e* and the unit *nats* were used for computing numerical results.

This study differs from Mogheir *et al*. (2004) mainly in the analytical approach. As the Gaza Strip data, which were used in this study, do not follow the Gaussian distribution function, and their spatial correlations are low, an exponential decay curve is fitted to the discrete models and to the smoothed discrete models (exponential decay fitting approach).

Harmancioglu *et al*. (1999) investigated the fitting of a semi-exponential curve to the discrete T model. The analysis of the synthetics data (Mogheir *et al*., 2004) and the shape of the discrete T model, smoothed by the moving average method, of the chloride data set signified that the exponential decay curve could be the best representation of the discrete T model, and could be presented as (e.g. Motulsky, 1999)

$$T(d) = G\ \mathrm{e}^{(-Kd)} + Q \tag{1}$$

where the exponential decay curve starts with $T_0 = G + Q$ at distance $(d) = 0$; and the curve decays to reach Q value with a constant rate K. The units of G and Q are expressed in the same way as the T unit (nats),

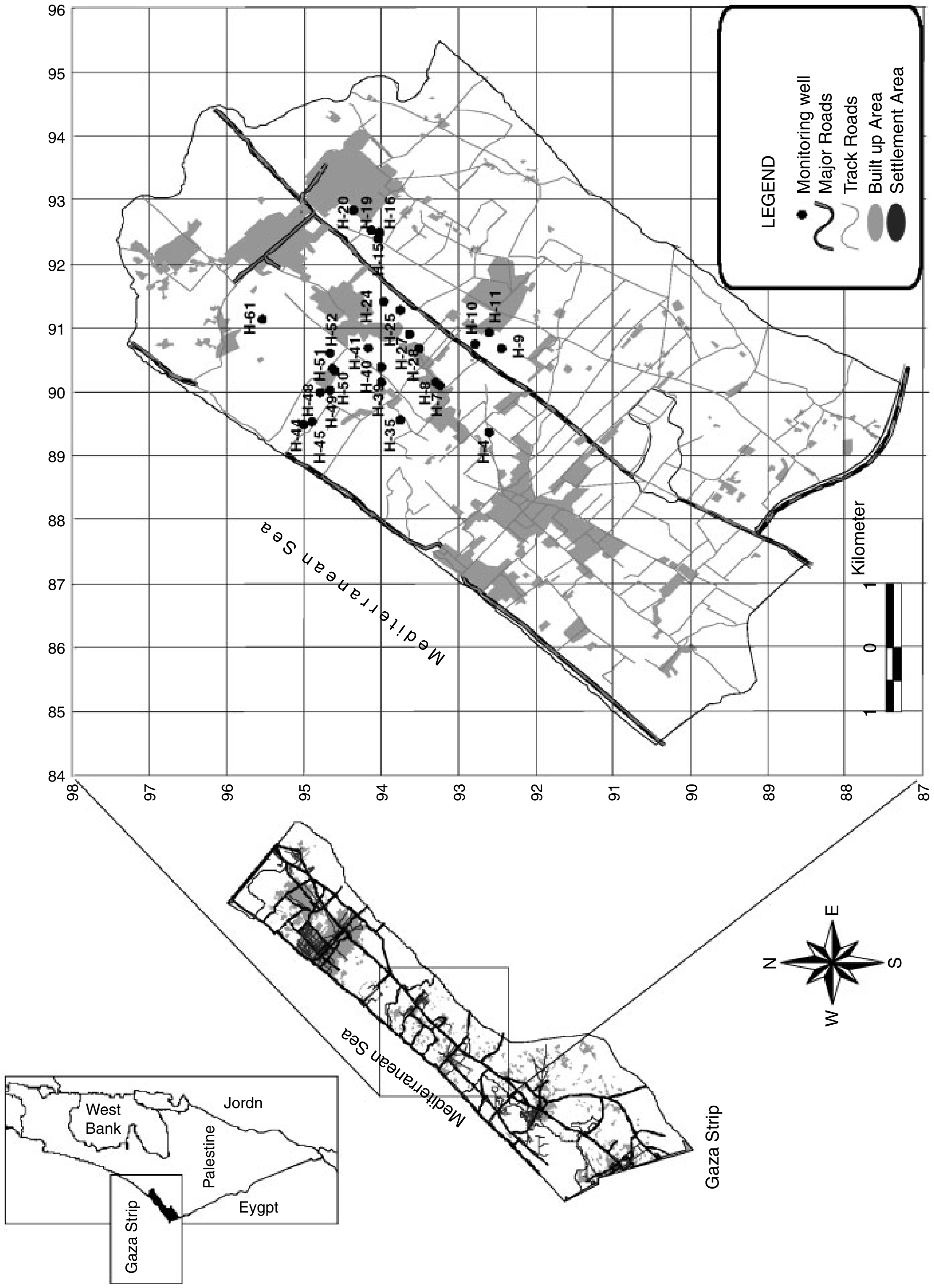

Figure 1. Location of selected groundwater quality monitoring wells in the middle part of the Gaza Strip, Palestine

Table I. Chloride (mg/L) time-series of the 26 monitoring wells as presented in Figures 1 and 2

Date	Monitoring well																									
	H-10	H-11	H-15	H-16	H-19	H-20	H-24	H-25	H-27	H-28	H-35	H-39	H-4	H-40	H-41	H-45	H-44	H-48	H-49	H-50	H-51	H-52	H-61	H-7	H-8	H-9
01-05-72	294	378	805	581	595	1064	392	455	539	364	798	644	343	742	700	525	574	725	427	441	399	816	707	875	427	637
28-10-72	322	385	840	539	616	1008	406	560	532	515	790	679	329	742	658	469	644	686	574	504	483	693	700	882	413	609
26-04-73	462	378	826	623	700	1001	420	476	539	462	798	721	322	756	672	518	707	756	427	623	511	714	728	910	483	588
23-10-73	371	385	840	616	770	1022	399	483	532	525	784	805	315	749	714	476	686	847	574	665	546	742	812	924	497	609
21-04-74	322	378	910	623	700	1015	420	483	546	525	798	693	336	756	672	462	707	756	427	609	532	726	756	889	483	588
18-10-74	322	406	840	623	770	1015	406	560	518	525	791	750	315	749	714	448	686	847	518	595	518	742	812	882	497	609
16-04-75	462	406	910	595	700	1001	420	483	546	455	819	693	511	756	672	462	714	756	427	609	511	770	756	910	504	588
13-10-75	371	399	945	644	770	1008	413	560	532	532	812	679	427	721	714	511	714	854	574	616	546	735	928	924	518	595
10-04-76	385	420	882	644	763	1043	434	476	553	539	756	721	399	756	742	518	735	847	560	623	532	805	728	861	511	588
07-10-76	399	432	882	644	721	1016	455	476	560	567	861	805	420	770	735	476	725	931	574	665	581	791	812	945	553	602
05-04-77	350	434	924	686	791	1029	406	504	553	539	791	717	420	721	686	462	678	854	504	609	567	749	700	952	584	574
02-10-77	399	392	791	616	707	903	448	441	511	497	798	756	357	707	679	455	602	896	574	616	574	756	686	840	497	574
31-03-78	371	406	805	525	791	917	399	417	504	525	812	735	371	714	742	476	735	924	581	644	588	742	700	842	504	644
27-09-78	385	392	805	609	700	903	441	441	532	497	812	756	264	735	735	455	725	980	574	616	574	791	924	840	497	616
26-03-79	371	406	826	630	791	917	399	417	504	525	812	735	371	749	686	476	678	1008	581	644	588	749	700	854	504	644
22-09-79	399	385	840	609	749	949	441	455	518	546	812	728	378	735	665	434	602	1015	588	693	665	756	924	875	525	616
20-03-80	364	406	868	816	693	924	441	417	532	553	749	721	392	763	700	441	623	1008	581	707	679	931	735	861	532	679
16-09-80	406	406	868	651	742	949	441	483	602	560	798	749	385	756	700	399	630	1043	756	742	665	854	777	968	546	497
15-03-81	364	420	882	658	693	924	469	490	532	553	749	756	392	763	665	420	630	1008	581	707	700	966	735	861	518	490
11-09-81	399	420	890	651	756	1036	518	483	602	560	861	861	434	840	784	420	658	1022	574	742	693	854	777	854	525	497
10-03-82	462	504	959	777	833	924	518	490	623	637	861	840	448	861	791	420	693	1169	581	707	705	1071	910	952	602	490
06-09-82	476	450	1001	756	861	1071	532	483	644	651	861	861	441	840	784	399	665	1162	588	826	784	1015	1093	959	630	553
05-03-83	455	504	1001	756	840	924	518	490	651	658	819	840	448	1008	840	441	707	1190	581	924	847	1134	840	945	630	546
01-09-83	497	511	1029	812	861	1057	525	483	658	658	791	882	455	889	784	420	707	1155	756	902	819	1085	910	945	644	595
28-02-84	490	504	1064	791	875	1057	546	602	672	651	889	854	497	861	826	434	735	1071	791	896	840	1169	910	980	651	623

26-08-84	511	525	1050	826	903	1071	546	588	686	658	868	868	469	875	819	392	714	1183	854	875	833	1120	945	966	658	567
22-02-85	637	511	1064	840	910	1071	567	602	707	651	826	854	483	840	819	392	721	1190	875	917	840	1239	924	959	665	525
21-08-85	511	525	1085	840	903	1099	546	588	742	665	917	868	483	931	826	392	910	1225	854	959	903	1141	945	994	644	567
17-02-86	504	518	1106	868	889	1078	567	490	735	672	847	833	490	903	854	406	686	1218	910	924	1001	1204	945	1001	651	525
16-08-86	525	504	1218	882	903	1099	588	483	749	700	882	868	497	882	826	392	714	1204	966	959	903	1141	1001	1015	658	567
12-02-87	532	511	1092	931	917	1092	574	602	686	679	798	952	483	1113	854	406	784	1211	910	1015	980	1330	987	1029	721	553
11-08-87	525	504	1127	910	903	1099	588	588	707	700	882	868	483	882	826	392	714	1204	966	959	903	1141	945	1015	658	567
07-02-88	567	518	1092	931	917	1015	581	602	742	679	931	854	490	924	861	406	819	1169	847	1001	980	1330	945	1029	721	546
05-08-88	546	518	1148	910	917	1106	595	588	791	679	994	819	497	917	826	392	777	1106	966	959	952	1239	1001	994	707	556
01-02-89	616	525	1106	903	917	1113	574	602	735	693	1029	840	483	924	882	420	812	1162	931	1001	980	1330	987	1001	784	590
31-07-89	553	483	1155	938	917	1085	588	588	770	686	1043	819	504	931	826	406	812	1106	938	1027	1001	1225	1001	966	707	569
27-01-90	623	532	1015	952	945	1134	581	602	763	693	1029	868	504	917	882	378	889	1211	917	1001	1036	1309	966	970	700	567
26-07-90	623	525	1232	959	938	1134	469	569	798	721	1029	854	490	924	826	455	812	1204	931	1064	1015	1323	1001	956	707	580
22-01-91	637	532	1169	980	945	1134	637	602	812	714	1029	840	504	924	924	448	833	1169	924	1099	1036	1449	966	952	770	574
21-07-91	644	518	1169	980	924	1141	651	490	840	840	1071	845	490	924	917	497	840	1106	945	1057	1015	1323	987	931	784	567
17-01-92	798	532	1169	1085	945	1106	644	455	840	756	1029	819	504	959	959	434	931	1162	686	966	1050	1519	861	917	770	619
15-07-92	686	525	1176	1085	931	1085	714	490	819	735	1113	826	490	959	980	546	959	1106	749	959	1001	1323	987	910	784	609
11-01-93	812	546	1176	1027	910	1085	714	455	819	721	1099	819	504	959	980	476	959	1162	686	966	1050	1449	861	920	805	705
10-07-93	812	539	1148	1029	920	1085	749	742	833	721	1099	819	497	959	1050	511	959	1043	847	966	910	1323	1106	796	805	602
06-01-94	875	588	1148	1057	952	1211	749	742	833	791	1099	819	497	959	1050	476	959	1162	686	966	910	1519	1012	952	784	616
05-07-94	875	588	1190	1029	940	1085	735	728	819	791	1085	819	497	994	1050	511	945	1106	847	959	910	1323	1020	931	784	546
01-01-95	868	588	1190	1386	1001	1085	707	707	798	756	1022	819	532	959	1078	539	1050	1162	777	966	896	1505	1090	917	777	560
30-06-95	882	602	1169	1050	959	1190	714	686	777	749	1050	763	525	994	1050	490	994	1043	847	862	945	1323	1111	910	791	581
27-12-95	861	588	1167	1085	977	1085	714	714	826	749	889	714	504	1071	1099	609	1071	1015	791	910	1015	1351	1155	1043	777	649
24-06-96	882	588	1183	1108	1036	1190	710	644	840	728	1085	767	497	1043	1131	567	1029	1008	854	910	938	1333	1190	910	829	569
21-12-96	946	672	1185	1107	1050	1085	767	631	826	744	1022	739	503	1116	1129	604	1061	988	837	910	816	1358	1180	1019	921	588
19-06-97	956	690	1190	1144	1060	1134	801	617	890	756	1050	751·8	497	1130	1096	687	1090	1136	872	882	901	1306	1217	1085	821·8	608
$\bar{x}$	552	486	1022	833	850	1049	549	544	677	635	904	794	445	872	836	462	784	1043	720	826	791	1089	911	934	643	583
S_x	192	79	143	197	112	77	120	89	124	105	117	66	68	116	141	63	139	150	170	170	193	267	144	61	126	43

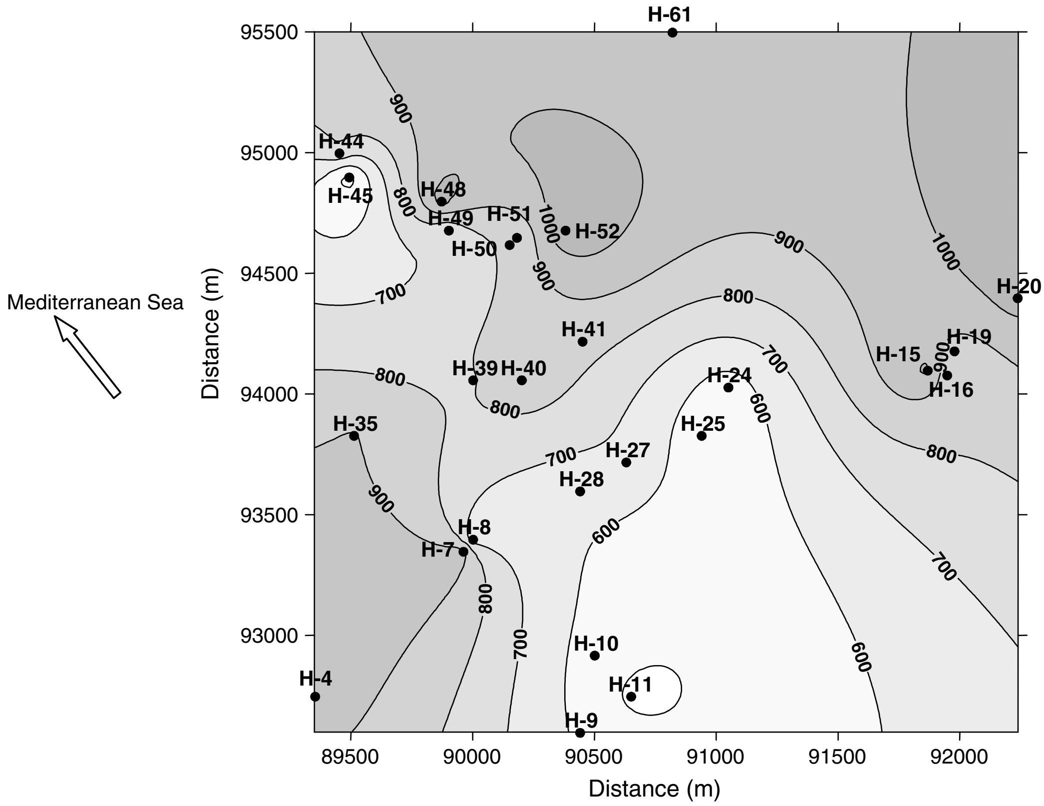

Figure 2. Chloride contour map for the middle part of the Gaza Strip. The average of chloride data (mg/L) was used in drawing the contour map

whereas K is expressed in the inverse unit used by d (1/m). Note that Equation (1) was also used to represent the analytical lognormal T, ITI and correlation models.

The fitting of the exponential decay curve to the discrete models was performed using the least-square fitting procedure with the GRAPHPAD PRISM statistical software (Motulsky, 1999). The coefficient of determination was used to quantify the goodness of fit between the exponential decay curve and discrete models. The coefficient of determination (R^2) was computed as (e.g. Motulsky, 1999)

$$R^2 = 1{\cdot}0 - \frac{SS_{\text{reg}}}{SS_{\text{tot}}} \tag{2}$$

where SS_{reg} is the sum of the squares of the residuals between the discrete model and the best-fit exponential decay curve, and SS_{tot} is the sum of the squares of the residuals between the discrete model and the horizontal line through the mean.

As in Mogheir *et al*. (2004), the T model and C model were compared to characterize the spatial variability of the Gaza Strip data set.

COMPARISON OF DISCRETE AND EXPONETIAL DECAY FITTING APPROACHES

Correlation model (C model)

The discrete C model is obtained by computing the correlation values using the discrete approach and the distance between wells. The discrete C-Model data is smoothed by using the moving average method

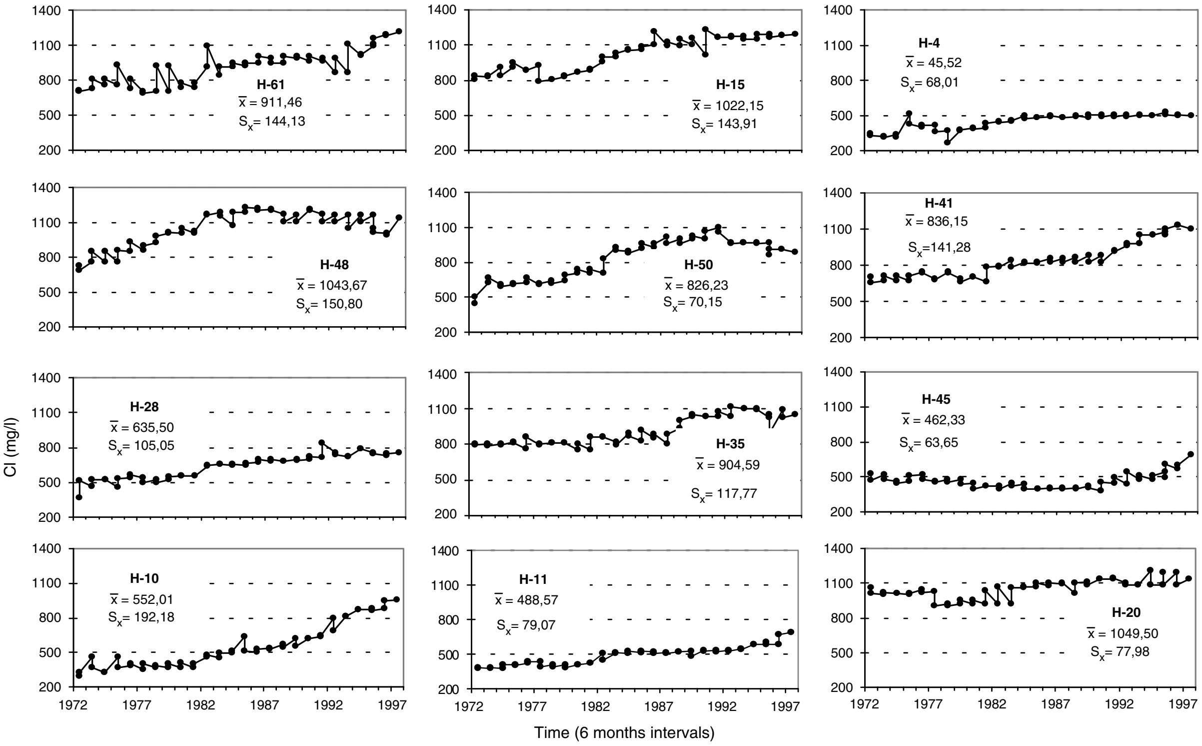

Figure 3. Chloride time-series of 12 monitoring wells (H-10, H-11, H-20, H-28, H-35, H-45, H-48, H-50, H-41, H-61, H-15 and H-4) for the period winter 1972 to summer 1997, used in the analyses. In the graph $\bar{x}$ is the mean of the chloride time-series and S_x is the standard deviation

(DCM_{MA}). The exponential decay fitting approach is applied to the discrete C model and DCM_{MA}. A summary of the best-fit equations of the exponential decay curve to the discrete T, lognormal T, ITI, correlation models and R^2 values for each model is presented in Table II.

The discrete C model (DCM), the C model smoothed by the moving average method (DCM_{MA}) and the exponential decay of the discrete C model (DCM_{ED}) are plotted in Figure 4. This figure and Table II show that DCM_{ED} does not fit the discrete C model well, as $R^2 = 0{\cdot}07$, which is very low. The coefficient R^2 is increased by applying the exponential decay fitting approach to the DCM_{MA} ($R^2 = 0{\cdot}22$). Nevertheless, for both the DCM_{MA} and discrete C models the coefficient R^2 is quite small. Therefore, the exponential decay curve, which was selected to present the discrete C model, does not infer the spatial variability of the chloride data adequately.

Table II. Fitting discrete models with the exponential decay curve applied to the Gaza Strip data

Model type	Fitting equation	R^2
Discrete C model	$r(d) = 0{\cdot}43\ e^{(-0{\cdot}0033\ d)} + 0{\cdot}53$	0·07
Discrete T model	$T(d) = 0{\cdot}29\ e^{(-0{\cdot}0087\ d)} + 0{\cdot}90$	0·33
Lognormal discrete T model	$T(d) = 0{\cdot}90\ e^{(-0{\cdot}0102\ d)} + 0{\cdot}59$	0·43
Discrete ITI model	$\mathrm{ITI}(d) = 0{\cdot}39\ e^{(-0{\cdot}0359\ d)} + 0{\cdot}61$	0·57

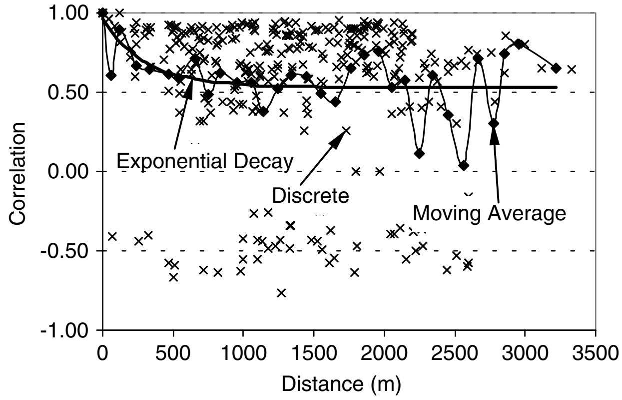

Figure 4. Correlation models for the groundwater quality monitoring network in the middle part of Gaza Strip (chloride)

Transinformation model (T model)

The discrete T model was obtained in the same way, by computing the T values using the discrete approach and the distance between wells. The discrete T model data were smoothed using the moving average method (DTM_{MA}). The exponential decay fitting approach is applied to the discrete T model and DTM_{MA}. For the discrete T model, the R^2 coefficient is 0·33, which is smaller than that for $DTM_{MA}(R^2 = 0{\cdot}71)$. This indicates that the exponential decay curve fits the DTM_{MA} much better than does the discrete T model. The discrete T model, the DTM_{MA} and exponential decay of the discrete T model (DTM_{ED}) are plotted in Figure 5.

T-model using logarithmic chloride data

As the normal distribution did not fit the chloride data well, the lognormal distribution was assumed. The chloride logarithmic data from the Gaza Strip monitoring wells are used to compare the discrete and exponential decay fitting approaches in obtaining the T values. The logarithmically transformed chloride data are used to check the fitting of the normal function by constructing the histogram and plotting the probability diagram. The chi-square test was used to assess the adjustment of the lognormal distribution to the empirical data.

After fitting the lognormal function of the chloride data from the Gaza monitoring wells, the lognormal discrete T model (lognormal DTM) is obtained by computing the T values of the logarithm of the chloride data, using the discrete approach and the distance between wells. The lognormal discrete T model is smoothed

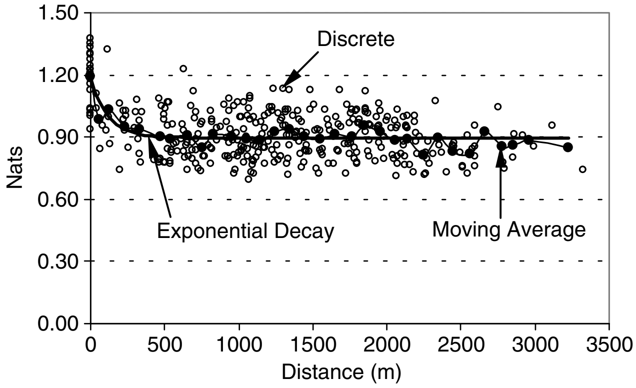

Figure 5. Transinformation models for the groundwater quality monitoring wells in the middle part of Gaza Strip (chloride)

using the moving average method (lognormal DTM_{MA}). The exponential decay fitting approach is applied to the lognormal discrete T model and to the lognormal DTM_{MA}. Figure 6 illustrates the lognormal discrete T model, the lognormal DTM_{MA} and the exponential decay of the lognormal discrete T model (lognormal DTM_{ED}). From Figure 6, it can be seen that, for the lognormal DTM_{MA}, R^2 is 0·68 which is greater than the R^2 value obtained by the lognormal discrete T model ($R^2 = 0{\cdot}43$). As for all the T models, the exponential decay curve fits better to the DTM_{MA} than that to the discrete T model. The lognormal DTM_{ED} is compared with the DTM_{ED}. As shown in Figure 6, the minimum value of the transinformation in the lognormal DTM_{ED} is 0·3 nats less than that found in the DTM_{ED}. Additionally, the initial value of the transinformation in the lognormal DTM_{ED} is 0·3 nats greater than that in the DTM_{ED}. This indicates that the T model is sensitive to the type of distribution of the data, whether its normal or lognormal.

CHARACTERIZATION OF SPATIAL VARIABILITY

When comparing the correlation model (C model) and the transinformation model (T model), to characterize the spatial variability of the chloride data, Figure 4 shows that the discrete C model is highly scattered and the exponential decay curve does not fit to the discrete C model well. This is also found where $R^2 = 0{\cdot}07$ and 0·22 for the discrete C model and DCM_{MA}, respectively. On the other hand, Figure 5 shows that the exponential decay curve fits to the DTM_{MA} better than it does to the discrete T model, as $R^2 = 0{\cdot}33$ and 0·71 for the discrete T model and DTM_{MA}, respectively. Furthermore, the R^2 values are greater if the logarithmically transformed chloride data are used.

As the ITI and correlation models have the same range from 0 to 1, they are compared in Figure 7, which demonstrates that there is less scatter in the discrete ITI model, which is smoothed by the moving average method ($DITIM_{MA}$), than there is in the DCM_{MA}. The R^2 value for $DITIM_{MA}$ is 0·79, which is greater than that for the DCM_{MA}($R^2 = 0{\cdot}22$). These values suggest that the exponential decay curve is representing the ITI model much better than it represents the C model. As a result, it can be inferred from Figures 5–7 that the T model and ITI model represent the dependency between wells better than the discrete C model.

In the above analysis, the dependency is described by an exponential decay model, which is relevant to the T model because the T value is maximized at a distance equal to zero. The maximum T value equals the average of the marginal entropies of the 26 wells. There is a sharp drop in the T value when the distance is around 500 m. With a further increase in the distance, T becomes essentially constant. Therefore, what is significant for the spatial assessment and redesign of monitoring wells is selecting the distance at which T has a minimum steady value. The prescribed 500 m value may be adopted as the recommended distance between wells. This distance can be utilized in the assessment stage under the following conditions.

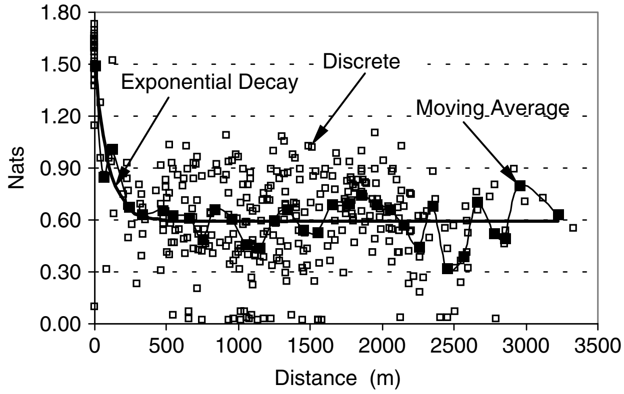

Figure 6. Lognormal T models applied to the chloride data. The lognormal probability distribution was used in the analyses

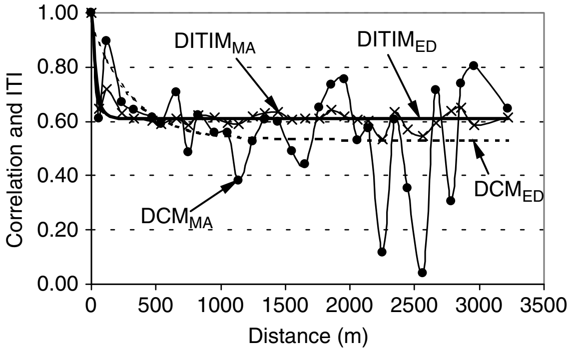

Figure 7. Comparison between C model and ITI model using the discrete and exponential fitting approaches, for the chloride data. In the figure: DCM_{MA} = smoothed discrete C model by the moving average method; $DITIM_{MA}$ = smoothed discrete ITI model by the moving average method; DCM_{ED} = exponential decay of the discrete C model; $DITIM_{ED}$ = exponential decay of the discrete ITI model

1. If the distances between wells are less than the recommended distance, then there is available transinformation (redundant information) between wells.
2. If the distances between the existing wells are greater than the recommended distance, then the transformation between wells is less than the minimum transinformation value (not enough information).
3. The adequate information that can be available between wells is found only where the distances between wells equal the recommended distance and the transinformation is minimum.

These arguments afford efficient criteria to assess and redesign the existing wells according to that recommended distance and minimum redundant information between wells. Consequently, the number of wells can be extended or reduced.

It is also useful for redesigning groundwater quality monitoring networks, and developing an analytical equation to relate T and distance. This equation can form an exponential decay curve, as in the synthetic data (Mogheir *et al.*, 2004) and the chloride data example, or any other type of curve. The monitoring network redesign procedure also might need to look at the variations of the value of T and the shape of the T model by changing the number of wells and the size of the time-series used for constructing the T model.

CONCLUSIONS

This article has presented a comparison between the discrete and exponential decay fitting approaches, using a groundwater quality data set from the Gaza Strip (chloride data). The following conclusions can be drawn.

1. The exponential decay fitting approach shows that the exponential decay curve does not fit to the discrete correlation model well.
2. The exponential decay curve fits to the discrete T model, the lognormal discrete T model and the discrete ITI model much better than does to the discrete correlation model.
3. The characteristics of the exponential decay of the lognormal discrete T model, such as the minimum T and initial T, differ from those of the exponential decay of the discrete T model.
4. The discrete T and ITI models are superior to the discrete correlation model for characterizing the spatial variability by means of an exponential decay model.

The exponential decay T model can be used to evaluate a groundwater monitoring network. Furthermore, the T model can be used to redesign the monitoring network by either increasing or decreasing the number of wells. The assessment and redesigning of a groundwater quality monitoring network, using the sensitivity of the T model to the number of monitoring wells and the size of time-series, are part of an ongoing study by the first author.

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APPENDIX

List of symbols and abbreviations

Symbols.

G	the value of transinformation where distance equals 0 deducted from Q (NATS)
K	the transinformation decay rate (1/m)
$\mathrm{ITI}(d)$	information transfer index as a function of distance (NATS)
Q	the end value of transinformation at which the distance is maximum (NATS)
R^2	coefficient of determination
$r(d)$	correlation as a function of distance.
SS_{reg}	sum of the squares of the residuals between the discrete model and the best fit curve (analytical model)
SS_{tot}	sum of squares of the residuals between the discrete model and the horizontal line through the mean
S_{x}	sample standard deviation of variable x
$T(d)$	transinformation as a function of distance (NATS)
$\overline{x}$	sample mean of variable x

Abbreviations.

C model	correlation model
DCM	discrete correlation model
$\mathrm{DCM_{ED}}$	exponential decay of the discrete C model
$\mathrm{DCM_{MA}}$	smoothed discrete correlation model by the moving average method
DITIM	discrete ITI model
$\mathrm{DITIM_{ED}}$	exponential decay of the discrete ITI model
$\mathrm{DITIM_{MA}}$	smoothed discrete ITI model by moving average method.
DTM	discrete transinformation model
$\mathrm{DTM_{ED}}$	exponential decay of transinformation model
$\mathrm{DTM_{MA}}$	smoothed discrete transinformation model by the moving average method
ITI model	information transfer index model
Lognormal $\mathrm{DTM_{ED}}$	exponential decay of lognormal discrete transinformation model
Lognormal DTM	lognormal discrete transinformation model
Lognormal $\mathrm{DTM_{MA}}$	smoothed lognormal discrete transinformation model by the moving average method
T model	transinformation model

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