Joana Isabel Silva Lopes Cavadas

Integrated Transit-Parking Planning

PhD Thesis in Doctoral Program in Transport Systems supervised by Professor
António Pais Antunes, Professor Nikolas Geroliminis and Professor Vikrant Vaze,
presented to the Department of Civil Engineering of the Faculty of Sciences and
Technology of the University of Coimbra

September 2017

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To my siblings, Sofia and Francisco.

Financial support

MITPortugal

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

Acknowledgements

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Abstract

Public transit systems are not only essential for urban mobility but are also advantageous from the fuel consumption, pollutant emissions and traffic congestion standpoints. In addition to this, transit also provides an alternative with acceptable levels of mobility to people who cannot own or drive a car. In fact, the main goal of having a transit system is to offer good quality service, where users travel easily at a low fare while reducing pollution and traffic congestion. This goal often results in serious financial problems for the transit operators, as their revenues are rarely enough to cover their expenses, requiring subsidies funded by local governments. In this context, we propose the integration of transit and parking systems as an option to decrease the subsidies of transit systems. This integration is developed considering two different views. A physical integration of the two systems; and an integration through prices (transit fares and parking fees), where two different standpoints are considered. One that assumes a regulated market, where the parking operator revenues will be used to fund the transit operator deficits; and another that assumes a fully deregulated market, where both transit operators and parking operators have a profit maximization goal.

The physical integration of the two systems was illustrated through an optimization-based study carried out for Coimbra (Portugal), with the goal of selecting the best locations for park-and-ride facilities so that car use inside the city is minimized. Park-and-ride facilities are parking lots located in the periphery of cities to intercept car trips coming from the suburbs, and divert them to transit. In this study, the transport mode choices were assumed to be dependent on the generalized travel costs of car, transit and park-and-ride according to a logit function. The main result was that the introduction of a park-and-ride network could reduce car use in Coimbra's city center by 19%.

The integration of transit and parking systems through prices in a regulated market was approached with an optimization model, where transit and parking are managed together to minimize the joint deficit of the respective operators, considering transit fares and parking fees as decision variables. The context of application of this model is a city divided into zones, where trips between each pair of zones can be made either by car or by bus, or not made if (generalized) travel costs are considered too high by the traveler. Modal choice in the city is described by a logit model of the generalized travel costs of both modes. In the case of car, these costs consist of vehicle depreciation, fuel, maintenance, travel time and parking fees, while time costs, discomfort costs and transit fares are the costs included in the transit

generalized travel cost. This model was applied to a case study in Coimbra, where both transit and parking systems become clearly profitable due to a substantial increase of prices. However, the relationship between demand and speed is not addressed in this model, as it is assumed that speed values remain unchanged even when modal choices change.

This shortcoming was handled by embedding on the optimization model a network level aggregate traffic model based on the macroscopic fundamental diagram (MFD), which determines the speeds and cruising-for-parking costs considering car travel demand. Due to the complexity of the optimization model, a solution method based on a traffic-equilibrium algorithm and a greedy algorithm was developed. Through the application of a case study inspired by the city of Coimbra, it was possible to verify that the joint operating deficits were decreased, leading to a profitable transit system.

An alternative SA algorithm was also developed in view of its future application to solve the previous model. If properly designed, algorithms of this type show good global optimum convergence properties. Otherwise, the quality of the best solution they return may be low or the computation time they require may be excessively long. The reason for this to happen may be because SA algorithms spend too much effort evaluating poor quality solutions. To avoid this, we hybridize a cross-entropy algorithm with a SA algorithm, in order to decrease the probability that a low-quality candidate solution is selected in each iteration. The results of a computational study developed for a facility location problem indicate that the hybrid algorithm clearly improves the classic SA algorithm.

The integration of transit and parking systems under a deregulated market was handled through a two-stage game-theoretic approach, assuming transit and parking operator as profit maximizers. The first stage decisions are parking capacity, transit frequencies and fleet size, whereas pricing decisions are made in the second stage, assuming the first-stage decisions known and fixed. The concept of subgame-perfect pure strategy Nash equilibrium was used to solve this game. By analyzing several hypothetical case studies (inspired by real-world situations), it was shown how the decisions of the operators are expected to interact.

In general, the proposed models and their applications contribute what we believe to be a significant addition to the literature. These integrated transit-parking planning models provide a better understanding of how park-and-ride networks and pricing schemes affect the city's mobility dynamics and modal choices, and insight into the impact of the decisions of transit and parking operators on their financial performance.

Resumo

Os sistemas de transportes públicos são não só essenciais à mobilidade urbana, mas também vantajosos em relação ao automóvel quanto ao consumo de combustível, emissão de poluentes e congestionamento do tráfego. Adicionalmente, os transportes públicos são uma opção que garante níveis aceitáveis de mobilidade a quem não conduz ou não tem automóvel. De facto, o principal objetivo de um sistema de transportes públicos é providenciar um serviço de qualidade através do qual os seus utilizadores possam viajar a custo relativamente baixo e, simultaneamente, contribuir para a diminuição da poluição e do congestionamento. A prossecução deste objetivo origina geralmente sérios problemas financeiros para os operadores de transportes públicos, uma vez que as receitas não são, em regra, suficientes para cobrir os custos do sistema, o que requer a subsidiação por entidades públicas. É neste contexto que analisamos a integração de sistemas de transportes públicos e de estacionamento como uma possibilidade para diminuir os subsídios dos transportes públicos. Esta integração dos dois sistemas é estudada de duas perspetivas distintas – integração física e integração através dos preços (dos bilhetes de transporte público e de tarifas de estacionamento) – e segundo dois pontos de vista diferentes: um que assume um mercado regulado, no qual as receitas do estacionamento são utilizadas para financiar os défices dos transportes públicos; e outro que assume um mercado totalmente desregulado, em que tanto o operador de transportes públicos como o operador do estacionamento têm como objetivo a maximização do lucro.

A integração física dos dois sistemas é analisada tendo por referência um estudo de otimização desenvolvido para Coimbra (Portugal), com o objetivo de selecionar localizações para estacionamentos park-and-ride que minimizem a utilização de automóveis no centro das cidades. Os estacionamentos park-and-ride localizam-se na periferia das cidades com o objetivo de intercetar as viagens de automóvel que vêm dos subúrbios. Neste estudo, assume-se que as escolhas modais dependem dos custos generalizados de viagem por automóvel, por transportes públicos ou pelos dois modos através de um parque de estacionamento periférico, de acordo com uma função logit. O principal resultado que obtivemos com a introdução de estacionamentos park-and-ride foi a redução do uso do automóvel no centro de Coimbra em 19%.

A integração de transportes públicos e estacionamento através de preços num mercado regulado foi analisada com base em um modelo de otimização no qual os transportes públicos e o estacionamento

são geridos em conjunto, a fim de diminuir o seu défice global, considerando os preços dos bilhetes e as tarifas de estacionamento como variáveis de decisão. O contexto para a aplicação deste modelo é uma cidade dividida em zonas, onde as viagens correspondentes a cada par origem-destino podem ser feitas ou de automóvel ou de transportes públicos, ou não ser realizadas caso o seu custo generalizado seja considerado muito elevado. A escolha do modo de transportes é descrita por um modelo logit dos custos generalizados dos vários modos. No caso do automóvel, estes custos contemplam a depreciação do veículo, o combustível, a manutenção, o tempo de viagem e a tarifa de estacionamento, enquanto o tempo de viagem, o desconforto e o preço do bilhete são contabilizados nos custos generalizados de uma viagem em transportes públicos. Este modelo foi aplicado ao estudo de caso de Coimbra, concluindo-se que ambos os sistemas se poderiam tornar bastante lucrativos como resultado de um aumento substancial de preços. Contudo, a relação entre volumes de tráfego e velocidades de circulação não foi tratada de forma apropriada neste modelo, pois que se considerou que aquelas velocidades permaneceriam constantes independentemente das escolhas modais. Esta lacuna foi ultrapassada através da inclusão, no modelo de otimização, de um modelo de tráfego agregado a nível de rede baseado no denominado diagrama fundamental, que determina as velocidades de circulação e os níveis de cruising-for-parking tendo em conta a procura de viagens de automóvel. Dada a complexidade do modelo, foi desenvolvido um método para o resolver baseado na combinação de um algoritmo de equilíbrio de tráfego com uma heurística de tipo greedy. A respetiva aplicação ao caso de Coimbra permitiu concluir que seria possível tornar o sistema de transportes públicos lucrativo.

Uma heurística alternativa baseada num algoritmo de simulated annealing (SA) foi também desenvolvida para futura resolução do modelo anteriormente apresentado. Os algoritmos SA apresentam boas propriedades de convergência para um ótimo global, mas podem tornar-se muito lentos se se quiser garantir soluções de boa qualidade. Essa lentidão decorre do facto do algoritmo passar muito tempo a analisar soluções de baixa qualidade. Para contornar este problema, hibridizámos um algoritmo de cross entropy com um algoritmo SA. Os resultados obtidos através de um estudo computacional desenvolvido para um problema de localização de equipamentos indicam que o algoritmo híbrido melhora claramente a performance do algoritmo SA clássico.

A integração de transportes públicos e estacionamento num mercado totalmente desregulado foi analisada através de uma abordagem por teoria dos jogos com dois estádios. O primeiro estádio tem como decisões a capacidade de estacionamento, a frequência dos transportes públicos e a dimensão da frota, ao passo que as decisões referentes aos preços são tomadas num segundo estádio, onde são assumidas como conhecidas e fixas as decisões tomadas no primeiro estádio. Cada estádio do jogo foi resolvido tendo em conta o conceito de equilíbrio de Nash. Em diversos estudos de caso hipotéticos (inspirados em situações reais) é examinada a forma como se dá a interação entre as decisões do operador de transporte público e do operador de estacionamento.

Em geral, acreditamos que os modelos propostos e as suas aplicações contribuem de forma significativa para a literatura. Os modelos em causa permitem apoiar as entidades responsáveis pelo planeamento de transportes públicos e do estacionamento, contribuindo para que as decisões que tomem sejam mais eficientes.

Chapter 1

Introduction

1.1 Context

The problems raised by the increase of automobile ownership and use – such as traffic congestion, pollutant emissions and land use occupancy – are not a new subject. In fact, the worldwide increase of mobility lead to losses of about 40 hours per year in traffic congestion in US and European countries (Inrix, 2016 [1]), cars are responsible for 44% of the total CO2 emissions and 70% of other greenhouse gas in urban areas (European Comission, 2009), and 40% to 60% of urban CBD are devoted to roads and parking in the US (Litman, 2014).

Public transport is probably the main alternative to deal with these transport problems (Schiller et al., 2010; Miller, 2014). Public transport systems (or "transit systems") – which include, for instance, buses, metro/subway and trams – promote equality of opportunity within the community, improve the quality of life (enabling people to enjoy access to a range of goods, services, people and places), in addition of being clearly advantageous from the environmental and economic standpoints (Kennedy, 2002; Vuchic, 2005). In fact, these systems should account for requirements with respect to network coverage or extensiveness, service frequency, in and out of vehicle time, delays, transit stop design and transfer time, comfort, regularity, safety and transit fares (Ceder et al., 2013; Wardman, 2004; Rodríguez-Núñez and García-Palomares, 2014; Guo and Wilson, 2011; van Lierop et al., 2017). These requirements expected to be met with transit systems come with a cost, usually higher than the service revenues collected from transit fares, bringing the need of subsidizing transit systems with public funds in order for them to continue operating (Parry and Small, 2009; Reynolds-Feighan et al., 2000). By subsidy it is understood a payment that does not require a direct exchange of goods or services of equal market value in return; it is used to accomplish a specific objective or has a specific effect (Black, 1995).

Besides the previously mentioned socio-economic reasons that support the need of subsidizing transit systems, two additional classic rationales are often advanced. The first one concerns the Mohring

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[1] http://scorecard.inrix.com/scorecard/default.asp (Retrieved on 26th August 2017)

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effect (Mohring, 1972) while the second refers to the un-priced negative externalities generated by tra-velling by car (such as congestion, pollution and noise).

The Mohring effect was a concept developed by Mohring (1972) that regards the relationship between users' waiting time, the increase of transit frequency and the consequent influence on marginal social costs, attempting at determining the optimum fare under first-best conditions [2] by including the waiting time costs into the user generalized costs. In other words, the Mohring effect accounts for the existence of economies of scale as a reasonable justification to sustain the existence of subsidies for the transit operators. This notion was further analyzed and improved by several authors with and without the in-clusion of first-best assumptions to justify the need (or not) of subsidization due to economies of scale (e.g. Vickrey, 1980; Small et al., 2007; Van Reeven, 2008; Basso and Jara-Díaz, 2010; 2012).

These studies can be considered as one of the two consolidated literature streams of transit plan-ning models, falling into microeconomic analysis and focused on establishing optimal transit pricing and subsidization rules for stylized cities. The other stream concerns operations research approaches, where transit management is planned mainly from the supply perspective, including features such as fre-quency, transit scheduling, routes and passengers' assignment, and usually with exogenous transit fares. Planning processes can be divided into three different stages: strategic, tactical and operational (Bar-nhart and Laporte, 2007). The first stage, or the strategic stage, refers to long-term decisions and seeks at maximizing service quality, including, for instance, the definition of routes as sequences of bus stops in order to meet the demand levels for each possible origin-destination route pair (i.e., *transit network design* and *passenger assignment*, respectively). The tactical stage concerns the decisions that are made in the light of the service that is provided to the user, including the management of bus frequencies for each line and time period (*transit networks frequencies*) and the schedules of each bus trip (*transit network timetabling*). Finally, the operational stage deals with the minimization of the operating costs of the system (e.g., *vehicle scheduling, duty scheduling, crew rostering and parking and dispatching*). For further details on the application of operations research in the study of transit systems, we suggest the re-views made by Barnhart and Laporte (2007), Desaulniers and Hickman (2007), Guihaire and Hao (2008), Kepaptsoglou and Karlaftis (2009) and Ibarra-Rojas et al. (2015). However, and as pointed out by Ceder (2007), Vuchic (2005) and Guihaire and Hao (2008), the interaction between supply, demand and pricing is usually excluded from this stream, and the integration of multi-modal possibilities in the transport system is not commonly addressed.

Putting ourselves in the transit operator shoes, it would be expectable that transit systems were de-signed and managed to be financial sustainable and no subsidies needed. However, such thought is not pursued by the transit operator due to their social responsibility of providing a quality service to the

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[2]A situation in which all prices match marginal costs is known as first best conditions. See Small et al. (2007) for a more detailed discussion on first-best and second-best issues.

population by increasing their mobility at low fares. This results on the general problem faced by transit operators: how can a quality service be offered while keeping reasonable asset and operating costs? Furthermore, would transit systems need to be subsidize if car externalities were properly charged to car users? In fact, transit has not been able to improve its market share all by itself, even when subsidies are provided to cover the deficits raised by operating its service. This statement has been observed in the real world, where an increase in the transit supply and/or in the service quality provided by major investments in several countries to improve the capability of their public transport to be more competitive, does not automatically led to an increase in transit demand and users' satisfaction (Friman, 2004).

One way of promoting the use of transit systems is through the implementation of park-and-ride facilities. Originated in the 1930s, these facilities are parking lots usually located in the periphery of cities as a mean to increase transit ridership (Noel, 1988). These facilities help the modal shift from car to transit, allowing to perform the trip by car in the least congested part of the trips and by transit in their most congested part, alleviating traffic congestion and other adverse external effects of cars. Park-and-ride facilities combine the two modes into a multi-modal transport system that takes advantages of the individual strengths of both systems while avoiding their weaknesses. The locations of these facilities should be carefully selected and they should also include characteristics such as the number of parking spaces (Bos, 2004), the safety for both travelers and cars (Shirgaokar and Deakin, 2005), and the quality and accessibility to the transit system (Burns, 1979). These characteristics of the location of park-and-ride facilities has been shown to be a key consideration to maximize the interception of 2% to 21% of car drivers in countries such as UK, France, Germany and Netherlands (Bos, 2004; Bos and van der Heijden, 2005).

Other measures considered in the literature have the purpose of persuading car users to switch to transit by reducing the attractiveness of car and by managing the negative externalities of car (dealing with one of the reasons that justify the subsidization of transit systems). As showed in the literature, car is usually considered to be more convenient, comfortable and provider of a greater individual freedom when compared to transit (Beirão and Cabral, 2007). The use of road pricing and parking fees are usually advocated as potential solutions to overcome the differences between transit and car with respect to the users' perception of the two modes.

Road pricing are direct charges collected for using roads, also referred to as congestion tolls, where the intention is to charge travelers for the externalities they create in terms of congestion. According to the literature, road pricing has been integrated with transit so that the maximization of the social welfare of both users and operators takes place by managing the transit fares and road pricing values, and by dealing with aspects linked to network design and/or levels of transit service (e.g. Tabuchi, 1993; Huang, 2000; 2002; Danielis and Marcucci, 2002; Mirabel and Reymond, 2011; Tirachini and Hensher, 2011; Basso et al., 2011). Examples of recent reviews of road pricing studies appear in Tsekeris and Voß

(2009) and de Palma and Lindsey (2011). However, road pricing schemes are rarely applied in the real world due to their unpopularity among drivers and to their difficult and costly implementation (Schade et al., 2000; Santos and Fraser, 2006; Jaensirisak et al., 2005; Ison and Rye, 2005; Noordegraaf et al., 2014).

Parking is the final act of every car trip and cars are parked 80-97% of the time (RAC Foundation, 2004; Marsden, 2006; Shoup, 2005). It is a fundamental component of any vehicle trip and therefore its inclusion is essential in urban transportation models (Hensher and Button, 2007). Roth (1965) argues that parking must be treated as a good because the costs linked to the space used as parking should be paid according to the private value of the occupied land. Parking fees are a good option at fulfilling this role, together as being major features at reducing the individual transport use (Higgins, 1992), and consequently decreasing the negative externalities of car. Corroborating this result, the nonexistence of parking fees is the main factor of distortion in the choice of traveling mode in high density urban areas, where cheaper parking discourages travel by foot, bicycle and mass transit (Shoup, 2005). Several examples can be found in the literature to assess the influence of parking and their pricing in traffic networks under some of the following main subjects: policy oriented perspectives (see Shoup, 2005 for further descriptions and details), micro-economic analyses (e.g. Douglas Jr et al., 1975; Glazer and Niskanen, 1992; Arnott et al., 1991; Arnott and Rowse, 1999; 2009; Calthrop et al., 2000; Anderson and de Palma, 2004; Arnott and Inci, 2006) and cruising for parking (e.g. Shoup, 2006; Gallo et al., 2011; Geroliminis, 2015). Comprehensive reviews of the literature of economics of parking can be found in Marsden (2006), Fosgerau and De Palma (2013) and Inci (2015)

Although considered as a second best measure to solve traffic congestion (Verhoef et al., 1995; Calthrop et al., 2000), parking fees are a good substitute to road pricing schemes (Schade et al., 2000; Calthrop et al., 2000; Jaensirisak et al., 2005; Ison and Rye, 2005) with a straightforward implementation and a greater acceptance by drivers, who are willing to pay a higher parking fee instead of paying a road-pricing tolls (Albert and Mahalel, 2006; Dueker et al., 1998; Shoup, 2005; Albert and Mahalel, 2006; Marsden, 2006). As a result, parking fees can be considered as an important factor in reducing car use, changing parking type and location, and also affecting travel frequency, modal split, car occupancy, travel time and route (Higgins, 1992; Simićević et al., 2013; Kelly and Clinch, 2006; Glazer and Niskanen, 1992). Therefore, parking fees will be used as the car pricing policy in the integration of transit and parking features to minimize the deficits of the former.

1.2   Research Objectives

The general objective of this thesis is to provide models to assist local transport authorities (LTA) in the integration of transit and parking policies to manage the transit deficits, either indirectly by increasing the number of transit riders or directly by optimizing prices. These integrated transport planning

models[3] derive from two different strategies: physical integration and pricing integration.

The first objective concerns the physical integration of transit and parking through park-and-ride facilities. An optimization model capable of determining the best places to build park-and-ride facilities is developed, so that the minimization of the total traveled distance performed by car in the inner zone of the city occurs while complying to a budget constraint.

The second and third objectives of this thesis concern the integration of transit and parking through pricing policies schemes to circumvent transit financial problems. To the best of our knowledge, the integration of transit and parking through pricing features has not been addressed in the literature. Besides, the applicability of most of the models developed in the literature is hampered by the lack of spatial considerations about the urban area, especially the ones developed with micro-economic features, even though the conclusions previously summarized help at providing insights towards expectable results and users' behavior when changes on mode pricing occurs. Qualitative studies that assess the influence of parking in the transit usage, in terms of availability and/or pricing (Hess, 2001; Farag and Lyons, 2012), support our choice of integrating transit and parking through pricing schemes. The aim of providing models suitable to be applied in real-world case studies is guaranteed by embracing an operation research approach.

The second objective concerns the development of optimization models that determine the optima prices for a regulated market. In these models, it is ensured that an acceptable level of mobility is provided to the population. We also aim to assess how the optimization of parking fee values influence the modal shares and the expected joint revenues.

The third objective is related to the extreme scenario of deregulated market. The main goal is to develop a framework capable of analyzing what would occur if the transit operator and the parking operator were competing to improve their own revenues, without any concern of the effect of their actions towards the other operator's outcome. We also aim to include and study long-term and short-term decisions (such as supply and pricing, respectively), without any welfare-related constraints.

The objectives will be pursued by developing and applying optimization methods to real world cities. However, despite the efforts made in having access to real-world information to represent the problems as realistically as possible, there was the need in some cases to resort to hypothetical cities that were inspired in real-world data.

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[3]The expression 'integrated transport' was firstly used in the 1981 BBC Television political comedy Yes Minister, in an episode entitled 'The Bed of Nails' (Lynn and Jay, 1991). Sixteen years later, the UK government specified integration as an objective of its transport policy, with the election of a Labour Government that firmly shifted from realms of fiction to fact the phrase 'Integrated Transport Policy' (UK Round Table, 1997).

1.3   Outline

This thesis is divided into 7 chapters, where Chapters 1 and 7 are the introduction and the conclusion of the thesis, respectively. The main chapters, from Chapter 2 to Chapter 6, are all written in the format of scientific papers, allowing their reading to be successive or independent (although not all the chapters have been submitted yet, we aim to publish them as soon as possible). For this reason, there are some repetitions in concepts and information throughout the thesis that cannot be avoided. Although independent, the chapters of this thesis are not merely a collection of papers because there is a logical relation between then, as explained in the previous subsection.

Chapter 2 comprehends the physical integration of transit and parking, with recource to an optimization framework that aims at selecting the optimal places to install park-and-tide facilities. In this chapter, we present a study carried out for the city of Coimbra, aiming at determining the best possible locations for a given number of park-and-ride facilities under the objective of minimizing the kilometers traveled by car in the city center. This alternative to reduce the car usage in the city center is then compared to several alternative measures, such as decreasing bus fares, increasing parking fees and/or increasing bus commercial speeds.

Chapter 3 and 4 approaches the integration of transit and parking through pricing schemes in a regulated market. The methodologies used to address this issue are based on optimization models.

In Chapter 3, the integration of transit and parking is accomplished by optimizing the transit fares and the parking fees so that the maximization of the joint profit of the transit operator and the parking operator occurs, or at least the deficits of the transit operator can be minimized using the parking operator's profit to fund it. The demand assigned to each alternative (car and bus) reacts to changes on these prices, where the possibility of not making the trip if those prices are excessively costly in the users' viewpoints is also considered. This model is applied to the city of Coimbra, aiming at providing the best pricing schemes so that the main goal of minimizing the deficits of the transit operator is met. This approach also analyzes the importance of optimizing transit fares along with parking fees to help reducing the deficits of the transit system.

This model is then improved in Chapter 4 to take into account traffic conditions in the network. In this sense, the previous optimization framework is improved by embedding it with macroscopic fundamental diagrams techniques. These techniques account for the relationship between network space-mean vehicle density and flow in urban areas with small spatial heterogeneity by including the number of vehicles (accumulation) in the network, the average speed per traffic direction and the congestion conditions in the traffic network. This model was applied to a case study inspired by the municipality of Coimbra.

Due to the complexity and extremely high computation time of the optimization model developed

in Chapter 4, we developed a simulated annealing algorithm to solve and to provide good solutions (hopefully, optimum solutions) for the detailed model. However, the computation time that this heuristic required to solve the optimization framework was still too high. To overcome this problem, a cross-entropy procedure was introduced in the simulated-annealing algorithm to avoid spending too much effort in the evaluation of solutions of poor quality. This was still not enough to solve the complex model detailed in Chapter 4, but it seemed to be a good improvement to what has been developed in the literature in terms of hybrid algorithms, and also a good algorithm to solve facility location problems. Therefore, we decided to include this approach as the Chapter 5 of this thesis.

The third main objective of this thesis is addressed in Chapter 6. Here, a game-theoretic model of the interactions between transit and parking decisions is developed in order to provide insights of what would happen in the extreme scenario of deregulated markets. This model accounts for long-term and short-term decisions under competition between transit and parking operators, where these operators are profit maximizers with decisions depending on each other's decisions, attempting at optimizing their supply and their prices (long term and short-term decisions, respectively). We use the theoretical concept of Nash equilibrium to solve a two-stage game. We developed a case study setup inspired on real-world features and generated instances of cities to apply our framework and to draw conclusions about deregulated markets' options.

Finally, Chapter 7 summarizes the work presented in this thesis and the main conclusions withdrawn from it.

1.4 Dissemination

Most of the research upon which this thesis is based has been presented and discussed in several international and national conferences between 2014 and 2017. These are listed in Table 1.1.

Table 1.1: Conference Presentation.

Title	Authors	Conference (Date and Location)
Optimization-based park and ride facility planning	Joana Cavadas António Pais Antunes	Optimization 2014 (July 28-30, 2014 Guimarães, Portugal)
Optimization-based study on park-and-ride facility location for Coimbra (Portugal)	Joana Cavadas António Pais Antunes	12º Encontro do Grupo de Estudos em Transportes (12º GET) (January 5-6, 2015 Tomar, Portugal)
Setting supply and pricing policies for a transit network: An optimization approach	Joana Cavadas António Pais Antunes	CREATE Symposium on Future Mobility (July 8-9, 2015 Singapore)

Optimization approach to transit funding through parking fees	Joana Cavadas António Pais Antunes	18th Euro Working Group on Transportation (18th EWGT) (July 14-16, 2015 Delft, Netherlands)
An integration of an aggregated dynamic traffic model with advanced optimization techniques for strategic transit-parking planning	Joana Cavadas António Pais Antunes Nikolas Geroliminis	5th Symposium of the European Association for Research in Transportation (hEART 2016) (September 13-16, 2016 Delft, Netherlands)
Optimization Models for the Integration of Transit and Parking Policies	Joana Cavadas António Pais Antunes Nikolas Geroliminis	Mathematics of Complex Systems: from precision medicine to smart cities CoLaB Workshop (December 5-6, 2016 Coimbra, Portugal)
An integration of an aggregated dynamic traffic model with advanced optimization techniques for strategic transit-parking planning	Joana Cavadas António Pais Antunes Nikolas Geroliminis	14º Encontro do Grupo de Estudos em Transportes (14º GET) (February 20-21, 2017 Fátima, Portugal)
Computational study of a hybrid simulated-annealing cross-entropy algorithms for facility location problems	Joana Cavadas António Pais Antunes	2017 Learning and Intelligent Optimization Conference (June 19-21, 2017 Nizhny Novgorod, Russia)
Integration of an Aggregated Dynamic Traffic Model with Advanced Optimization Techniques for Strategic Transit-Parking Planning	Joana Cavadas António Pais Antunes Nikolas Geroliminis	First Triennial Conference - INFORMS Transportation and Logistics Society (TSL2017) (July 26-29 2017, Chicago, USA)
Game theoretic Approach to transit and parking planning under competition	Joana Cavadas Vikrant Vaze António Pais Antunes	Optimization 2017 (September 6-8, 2017 Lisbon, Portugal)

As previously mentioned, this thesis is organized as a collection of papers. Therefore, we refer in Table 1.2, to where we submitted them, as it is the case of the second Chapter, or to where we aim to submit. Chapter 2 not been altered in any aspect, with the exception of some layout-specific issues. Hence, some notation may differ from chapter to chapter of the thesis.

Table 1.2: Journal Articles.

Chapter	Title	Authors	Journal
2	Optimization-based study on the location of park-and-ride facilities	Joana Cavadas António Pais Antunes	Transportation Planning and Technology
3	An optimization approach to integrated transit-parking planning	Joana Cavadas António Pais Antunes	Transportation Research - part E

4	Integration of an aggregated dynamic traffic model with optimization techniques for strategic transit-parking planning	Joana Cavadas António Pais Antunes Nikolas Geroliminis	Transportation Research - part B
5	Computational study of a hybrid simulated-annealing cross-entropy algorithms for facility location problems	Joana Cavadas António Pais Antunes	Journal of Heuristics
6	Game-theoretic Approach to Transit and Parking Planning under Competition	Joana Cavadas Vikrant Vaze António Pais Antunes	Transportation Science

Chapter 2

Optimization-based study on the location of park-and-ride facilities

2.1 Introduction

Park-and-ride facilities are parking lots located in the periphery of cities aimed to intercept trips made by car with origin in the suburbs and transfer them to the transit system (Noel, 1988). Facilities of this type were first implemented in the United States in the 1950s and in the United Kingdom in the 1960s (Meek et al., 2008; Dijk and Montalvo, 2011), with the purpose of decreasing car use in urban areas, therefore contributing to mitigating the undesirable effects of the automobile with respect to traffic congestion and air pollution. On the negative side, park-and-ride is generally considered to favor car trips in suburban areas (Parkhurst, 1995; 2000), causing ridership losses in thin transit routes that may eventually be abandoned, with serious consequences for people in those areas that do not own a car, cannot afford to pay a taxi, and are left with virtually no option to travel to urban areas.

The positive and negative impacts of park-and-ride facilities critically depend on where they are located. The study we describe in this paper was carried out to shed light on the impacts of installing a park-and-ride network in the city of Coimbra, central Portugal (Figure 2.1). This city has a population of just over 60,000 inhabitants, but has always played a major role in the country because of the high-level services it offers notably in the administrative, education and health care sectors. Such services are spread across Coimbra's urban area generating a large number of trips particularly from and to its outskirts, where approximately 80,000 people live. Despite the dense public transit system of bus and a few trolley lines available in the municipality of Coimbra (city plus suburbs), these trips are predominantly made by car, provoking traffic congestion and air pollution problems especially in the historic city center. The fact that this part of the city was recently classified as UNESCO World Heritage is strongly contributing to make these problems more visible and urgent, and park-and-ride is being seen as a possible solution to attenuate them. At present, Coimbra does not have any true park-and-ride facilities. In

contrast, parking capacity is abundant inside the urban area, especially in the historic city center, either for free or at rather low fees, which obviously is an important factor for the supremacy of car in the municipality. The study was carried out in the framework of a long-standing collaboration in the transport domain between the University of Coimbra and the municipal council.
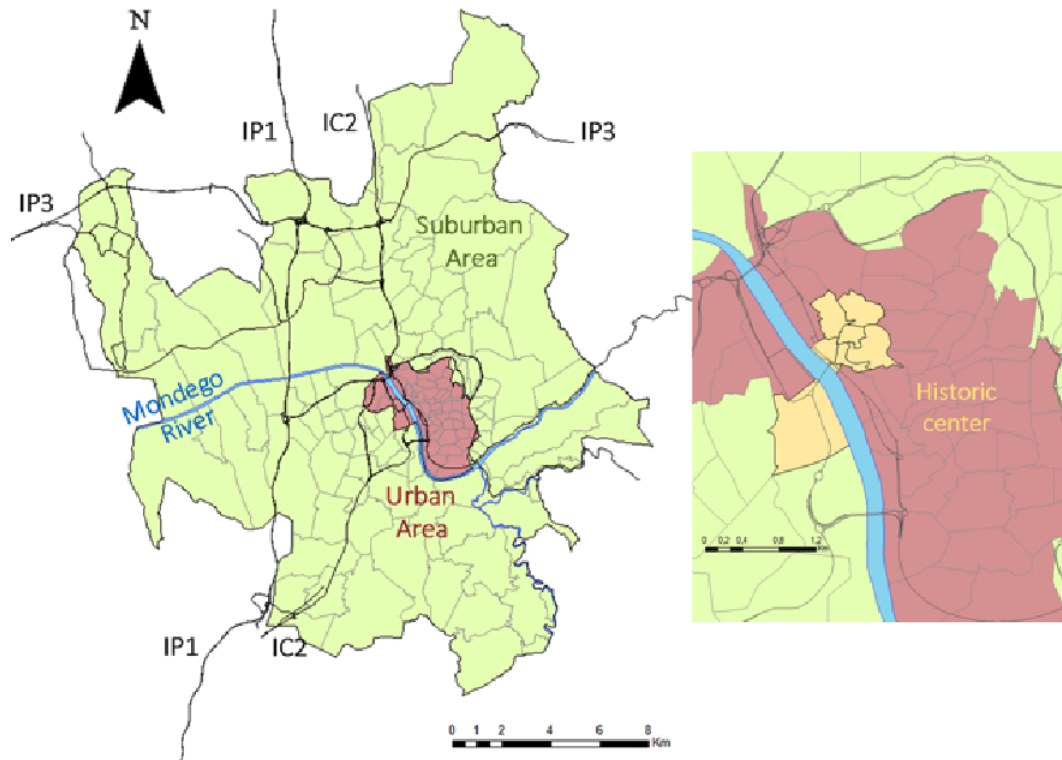
Figure 2.1: Urban and suburban areas (left) and historic city center (right) of the municipality of Coimbra.

The essential ingredient of the study was an optimization model aimed to determine the best possible locations for a given number of park-and-ride facilities in the periphery of a city, given a set of possible locations, under the objective of minimizing car use in its urban areas (and, concurrently, maximizing transit use). Based on the outcomes of the model, it is possible to assess thoroughly the impact of implementing the park-and-ride facility network (including the decrease in suburban transit use that many authors point out to be the most negative feature of park-and-ride). The model is, in particular, useful to compare the efficiency of park-and-ride solutions with respect to capturing car trips from the suburbs to the city and diverting them to transit against other measures that could be put forward with the same endeavor (e.g., increasing parking fees in urban areas).

Within the relatively meager literature on park-and-ride facility planning, two streams were of interest to our work. One of them focuses on location principles and guidelines for these facilities. Amongst the sometimes contradictory and confusing empirical indications that come out of this literature, there is consensus on two points: first, park-and-ride facilities should be located close to major highways and placed next to the sections where congestion starts to be felt, as this enhances their visibility and attractiveness (Burns, 1979; O'Flaherty, 1997; Spillar, 1997); second, park-and-ride facilities are unlikely

to entice drivers if they need to deviate significantly from their usual routes (Lam et al., 2001; Bos et al., 2004). These and other desirable features of park-and-ride facilities are also highlighted in a recent paper by Cornejo et al. (2014), where a comprehensive approach to individually evaluate potential park-and-ride facilities is presented. The other literature stream relates to park-and-ride facility location models and tools. There are several early contributions to this stream (e.g. Schneider et al., 1976; Schoon, 1980; Sargious and Janarthanan, 1983), but the most interesting ones are relatively recent. One of them is due to Faghri et al. (2002) and consists in a hybrid knowledge-based expert system GIS tool to assist planners in determining the location for park-and-ride facilities. Another one is the optimization model appeared in Wang et al. (2004). This model, later improved by Liu et al. (2009), applies to a stylized, linear mono-centric city, aims at determining the best location and parking fee for one park-and-ride facility, and assumes that mode choice is deterministic (the least-cost mode is used). The types of trips considered are car, rail and park-and-ride. Two alternative objectives are dealt with: profit maximization for a company that jointly operates the rail service and the park-and-ride facility; and social cost minimization. Models sought for real cities were proposed in Horner and Groves (2007) and Farhan and Murray (2008). The former paper introduces a linear flow-capturing model to determine the locations for a given number of park-and-ride facilities that minimize the distance traveled by car in a monocentric city (this setting was extended to a polycentric city by Khakbaz et al. (2013)). The Farhan and Murray (2008) location-allocation model considers three objectives, namely maximizing the trips intercepted by park-and-ride facilities, minimizing travel time between facilities and major highways, and maximizing the utilization of existing park-and-ride facilities. A feature common to all these models is that they do not address (properly) mode/route competition issues. The only model we know of where such issues are suitably taken into account is due to Aros-Vera et al. (2013). This model maximizes the car trips intercepted by a given number of park-and-ride facilities assuming that the choice between car and park-and-ride is made according to a logit function (Ben-Akiva and Lerman, 1985; Ortúzar and Willumsen, 2011), and its application is exemplified for a hypothetical city. The inclusion of the logit function in the Aros-Vera et al. (2013) model returns a mixed-integer nonlinear model that was linearized by the authors with an approach similar to the one developed by Haase (2009). In Haase and Müller (2014) it is shown that this linearization approach is efficient when compared to possible alternatives. The work developed by Chen et al. (2016) also considers a logit function to represent mode choice, with the options being 'driving' or 'not driving'. The combined nonlinear mode split and traffic assignment model proposed by these authors to optimally locate and size rail-based park-and-ride facilities aims to maximize the number of public transport riders in a city and its suburbs. The model is solved through a genetic algorithm and applied to the location of rail-based park-and-ride facilities in the city of Melbourne.

The optimization model we developed for the Coimbra study integrates an objective that we believe is more relevant than the ones accounted in the literature referred to above. Our goal is the minimiza-

tion of the person-kilometer (pkm) distance traveled by car in urban areas, assuming that travelers have three different alternatives to make their trip (car, bus and a combination of both modes through a park-and-ride facility). Our model captures the behavior of drivers that use park-and-ride facilities, which typically do not deviate significantly from their least-cost route. These are significant improvements to the existing models. Other innovative features of our work are the sensitivity analysis of model results (to changes in important parameters, namely sensitiveness of travelers to cost differences and propensity of drivers to route deviations) and the computational study on model solving. Finally, we compare the implications of implementing the park-and-ride facilities with those of implementing other measures aimed to achieve the same goal (e.g., increasing parking fees in central areas or decreasing bus fares).

The remainder of this article is structured as follows. In the next section, we provide a detailed description of the problem dealt with in the Coimbra study. This is followed by the presentation of the optimization model we propose to represent park-and-ride location problems and the explanation of how it behaves for a hypothetical application example. Afterward, we describe the data we used in the application of the optimization model to Coimbra and the results we obtained through it. Finally, in the last section, we summarize the main conclusions of the study and indicate some directions for further work on this topic.

2.2   Problem Description

As stated in the introductory section, the problem dealt in this chapter consisted in determining locations for installing park-and-ride facilities in the municipality of Coimbra, with the purpose of diverting trips originated in the suburbs from car to bus in the urban area.

The essential information for our study was taken from a recent mobility survey (2009), for which the municipality of Coimbra was divided into 157 trip generation zones, 51 located in the urban area and 106 in the suburban area (Figure 2.1). According to this survey, an average of 250,400 trips was made daily in the municipality, of which 108,500 (43%) connected the city with the outskirts in both directions. The number of city inbound and outbound trips were approximately the same, but their distribution across the day was naturally different – as depicted in Figure 2.2, the former prevailed until 9 AM and by 2 PM (people returning home after spending the morning in the city), and the latter in the rest of the day with a peak at 5 PM. The main origin-destination trip flows are shown in Figure 2.3. The survey also revealed that car was clearly the dominant mode for trips between the city and the outskirts with a modal share of 71%, followed by bus with a modal share of 28% (the trips captured by other modes, namely train, motorcycle and bicycle, were negligible). Approximately 25% of these trips are unlikely to change mode, notably because some travelers need a car during their stay in the city, or because they do not own or cannot drive a car.
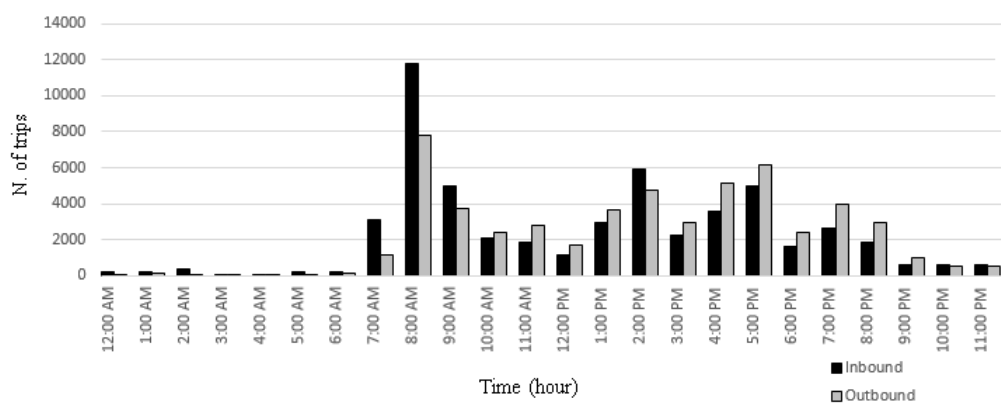
Figure 2.2: Distribution of inbound and outbound trips across the day in the municipality of Coimbra.
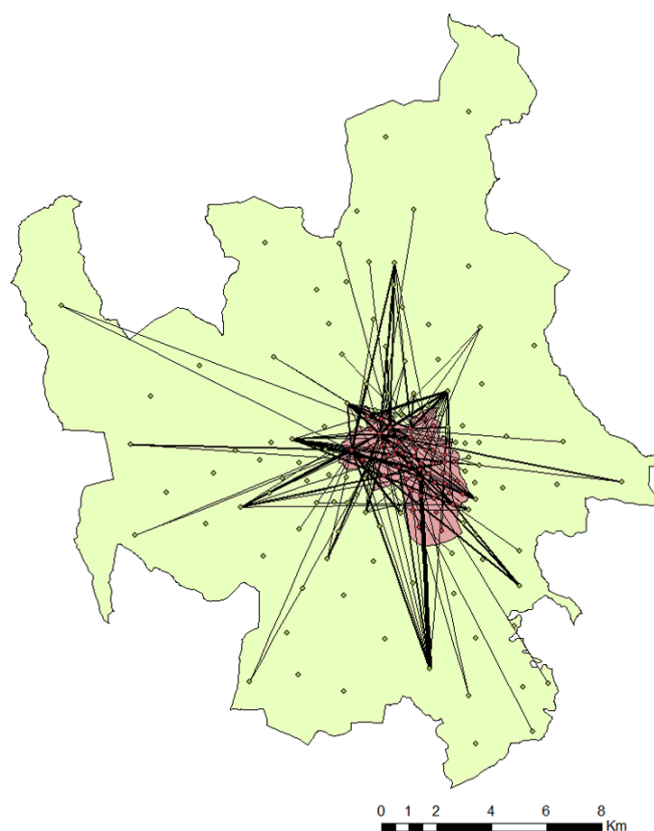
Figure 2.3: Main origin-destination trip flows between urban and suburban zones in the municipality of Coimbra.

Taking into account the fact that Coimbra's population, as well as its geographic distribution, is expected to remain practically unchanged at least in the near future (the land available in the municipality to accommodate significant urban developments is scarce, which contributes to explain why its population grew on average only 0.1% annually over the last 30 years), the problem we had to address consisted therefore in determining how many of the 28,600 car trips that are currently made between the suburbs and the city (in this direction) by travelers who may change mode could be diverted to park-and-ride facilities, thus becoming bus trips in the urban area, by installing such facilities in the best possible places.

In order to select possible locations for the park-and-ride facilities, we conducted a detailed map

analysis of land availability next to city entrances complemented with local visits, and ended up by pinpointing the seven sites marked on Figure 2.4. The sites we have selected meet the guidelines for the location of individual park-and-ride facilities recommended in the literature (see e.g. Spillar, 1997; Cornejo et al., 2014). In fact, they are positioned along Coimbra's ring road and, in general, intercept a large number of trips between the city and the outskirts (notably those corresponding to the main origin-destination pairs identified in Figure 2.3. Moreover, all the sites are located in highly visible places immediately before the first sections where congestion typically occurs. Another advantage of the sites selected is the fact that they are already served by the existing bus network, thus not requiring an update of this network at least in terms of route design.

With our study, in addition to knowing where to install park-and-ride facilities in the periphery of Coimbra, we wanted to compare the impacts of such measure with those of other measures that could be put forward with the same objective, namely decreasing bus fares, increasing parking fees and/or increasing bus operating speed.

Figure 2.4: Possible sites for the location of park-and-ride facilities in the municipality of Coimbra.

2.3 Optimization Model

The problem described in the previous section could be handled through an enumeration approach. Indeed, since we considered only 7 possible locations for the park-and-ride facilities, there are only 127 possible configurations for the park-and-ride facility network (from just one facility in one of the locations to facilities in all 7 locations). However, we decided to handle it through an optimization model. The main reason was because, though our present focus was Coimbra, we are looking forward to applications to larger cities, like Lisbon and Oporto, where the number of possible facility locations is much larger. Moreover, in the case of these and many other cities, the demographic evolution is much more

uncertain than that of Coimbra. In these conditions, the problem involved in the determination of the best configuration for a park-and-ride facility network would certainly be very difficult, if not impossible, to tackle efficiently without resorting to (robust) optimization models.

Below, we present the formulation of the optimization model we developed to represent Coimbra's park-and-ride location problem, illustrate the results it can provide through a small application example, and discuss methods for its resolution.

2.3.1 Model Formulation

The optimization model designed to represent Coimbra's park-and-ride facility location problem comprises the following features:

- The city and its suburbs are divided in two sets of trip generation zones, $\mathbf{Z}^C$ and $\mathbf{Z}^S$, respectively, connected by a road network.

- The total number of trips between any pair of zones $i \in \mathbf{Z}^S$ and $j \in \mathbf{Z}^C$ is known and equal to T_{jk}.

- The trips can be made by car (mode a), by bus (b) or by a combination of car and bus through a set of N routes that include a mode change in a park-and-ride facility.

- The travel distance corresponding to the least-cost path between any pair of zones i and j for trips made through mode/route $k \in \mathcal{M}$ is equal to D_{ijk}. Part of this distance, D^S_{ijk}, is made in the suburbs (outside the city), and the other part, D^C_{ijk}, is made inside the city.

- The trips between any pair of zones i and j only include a given park-and-ride facility if the corresponding route involves a shortest travel distance that does not exceed in a given percentage, $\delta\%$, the shortest travel distance by car, D_{ija} (δ therefore represents the propensity of drivers to route deviations). The set of possible modes/routes for zone pair ij is therefore $\mathcal{M}_{ij}(\delta) = \{a, b\} \cup N_{ij}(\delta)$, where $N_{ij}(\delta) = \{r \in N : D_{ijr} \leq (1 + \delta)D_{ija}\}$.

- The proportion of trips made through mode/route k between any pair of zones i and j, x_{ijk}, is assumed to be given by a logit function in terms of the least (generalized) travel costs for the modes/routes in competition, C_{ijk}, and of a parameter expressing the sensitiveness of travelers to cost differences, β.

- The objective is to minimize the travel distance made by car inside the city, D^C_a, for a given maximum number of park-and-ride facilities to install in the municipality, $P \leq N$ (P is a proxy for the costs involved in the construction and operation of the facilities, and also in the expansion of bus services that is expected to accompany the implementation of the park-and-ride facility network).

- The key decisions to be made are the locations for the facilities to include in the park-and-ride facility network. They are represented by binary variables y_n, $n \in N$, which are equal to one if a facility is installed in location n, and otherwise are equal to zero. These decisions determine the proportions of trips made by the various modes/routes, x_{ijk}, thus the distance traveled by car inside the city.

Using the notation introduced above, the optimization model can be formulated as follows:

$$\text{Max} \qquad D_a^c = \sum_{i \in \mathbf{Z}^S} \sum_{j \in \mathbf{Z}^C} D_{ija}^c \cdot T_{ij} \cdot x_{ija} \tag{2.1}$$

subject to

$$x_{ijk} = \frac{e^{-\beta c_{ijk}}}{e^{-\beta c_{ija}} + e^{-\beta c_{ijb}} + \sum_{n \in N_{ij}(\delta)} y_n \cdot e^{-\beta c_{ijn}}}, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C, k \in a, b \tag{2.2}$$

$$x_{ijk} = \frac{y_k \cdot e^{-\beta c_{ijk}}}{e^{-\beta c_{ija}} + e^{-\beta c_{ijb}} + \sum_{n \in N_{ij}(\delta)} y_n \cdot e^{-\beta c_{ijn}}}, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C, k \in N_{ij}(\delta) \tag{2.3}$$

$$\sum_{n \in N} y \leq P \tag{2.4}$$

$$x_{ijk} > 0, \qquad \forall i \in \mathbf{Z}, j \in \mathbf{Z}^C, k \in \mathcal{M}_{ij}(\delta) \tag{2.5}$$

$$y_n \in \{0, 1\}, \qquad \forall n \in N \tag{2.6}$$

The objective-function (2.1) of this optimization model minimizes the distance traveled by car inside the city, which is obtained by multiplying the number of trips between each pair of zones (one suburban and the other urban) by the proportion of trips made by car and by the travel distance for the trips through the least-cost path, and then adding the results for all zone pairs. The proportion of trips made by car is calculated in parallel with the proportion of trips made by the other modes/routes through constraints (2.2) and (2.3), the former corresponding to the car and bus modes and the latter to the combination of both modes with a mode change at a park-and-ride facility. They describe modal split according to a logit function, but this function does not appear in the usual form because the impedance terms corresponding to the park-and-ride routes ($e^{-\beta C_{ijk}}$) are multiplied by the location variables (y_n) to ensure that only the routes involving installed facilities are taken into account in the computation of modal shares. Constraint (2.4) specifies the maximum number of park-and-ride facilities to be installed (at most P variables y_n may be equal to one). Finally, constraints (2.5) and (2.6) specify the domain of the decision variables.

The optimization model presented above is integer and nonlinear, because decision variables y_n are binary and appear in the denominator of constraints (2.2) and (2.3). This could be a problem because nonlinear combinatorial optimization models are generally rather difficult to solve. However, using the linearization approach proposed by Haase (2009), in the case of this optimization model it is possible to replace the nonlinear constraints by the equivalent linear constraints (2.7) to (2.10), thus converting the

initial nonlinear formulation into the following linear formulation:

$$\text{Max} \qquad D_a^c = \sum_{i \in \mathbf{Z}^S} \sum_{j \in \mathbf{Z}^C} D_{ija}^c \cdot T_{ij} \cdot x_{ija} \tag{2.1}$$

subject to

$$x_{ijk} \leq y_k, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C, k \in \mathcal{M}_{ij} \tag{2.7}$$

$$\sum_{k \in \mathcal{M}_{ij}(\delta)} x_{ijk} = 1, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C \tag{2.8}$$

$$x_{ijk} \leq \frac{e^{-\beta \cdot C_{ijk}}}{e^{-\beta \cdot c_{ijl}}} + (1 - y_l), \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C, k \in \mathcal{M}_{ij}(\delta), l \in N_{ij}(\delta), k \neq l \tag{2.9}$$

$$x_{ijk} \leq \frac{e^{-\beta \cdot C_{ijk}}}{e^{-\beta \cdot c_{ijl}}}, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C k \in \mathcal{M}_{ij}(\delta), l \in N_{ij}(\delta), k \neq l \tag{2.10}$$

$$\sum_{n \in N} y_n \leq P \tag{2.4}$$

$$x_{ijk} > 0, \qquad \forall i \in \mathbf{Z}^S, j \in \mathbf{Z}^C, k \in \mathcal{M}_{ij}(\delta) \tag{2.5}$$

$$y_n \in \{0, 1\}, \qquad \forall n \in N \tag{2.6}$$

Both this model and the equivalent nonlinear one can be classified as (uncapacitated) p-hub location models of the multiple-allocation and non-strict hubbing type (Alumur and Kara, 2008; Daskin, 2013). Indeed, there are a given number of hubs (park-and ride facilities) to locate and, unlike for the more widespread p-median models (Daskin, 2013), demands (trips) are associated with origin-destination pairs and not with nodes. They are classified as multiple-allocation and non-strict hubbing because trips with a given origin may be made through more than one hub or without going through a hub. However, the models are different from typical p-hub location models because trips are distributed across routes depending on the travel costs associated with the routes (according to a logit function), instead of being made through the least-cost route.

Although we believe that the assumptions upon which this optimization model is based are quite reasonable, at least four of them may be contested. First, the model only considers the car and bus modes. The reason for this is because car and bus are the only relevant modes in Coimbra, but extending the model to encompass other modes is straightforward. Second, the model assumes car and bus trips to be made through least-cost paths (at current travel speeds). This is more questionable, but it would be possible to split trips across several routes as a function of their costs (according e.g. to a logit function) at the expenses of an increase in model size. Third, the model considers that travel speeds do not change as a function of traffic. We acknowledge this would be a weakness of the model if the traffic changes provoked by the implementation of the park-and-ride facility network were substantial. However, it is implausible that changes related only to trips between the city and its outskirts have a substantial overall impact on travel conditions in an urban area (though they may have in some road segments). Fourth and final, the model disregards the uncertainty inherent to the evolution of travel demand. The reason

for this is because, as mentioned before, Coimbra changed very little in the past and is unlikely to change much in the next years to come. But we recognize that in other contexts this may be a major issue and, as stated in the concluding section, we plan to address it in the future.

2.3.2 Application Example

In order to explain the behavior of the optimization model described above and illustrate the results it can provide, we present and discuss in the sequel the results of its application to the hypothetical radio-centric city depicted in Figure 2.5. It is a city with a population of about 100,000 distributed across 9 zones (neighborhoods) connected by four radial roads and by a ring road that separates the inner zones (5 to 9) from the outer zones (1 to 4). The average number of daily trips from the outer zones to the inner zones is shown in Table 2.1. The values of parameters β (sensitiveness of travelers to cost differences) and δ (propensity of drivers to route deviations) are assumed to be 1.0 and 0.25, respectively. These values and the generalized travel costs in this city are the same as the ones we used for Coimbra (see Section 2.4). At present, 49.8×10^3 pkm are traveled daily by car in the inner city. The objective is to decrease this traffic as much as possible by installing two park-and-ride facilities next to the four intersections of the radial roads with the ring road (sites A to D).
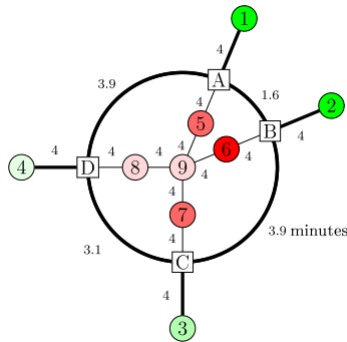
Figure 2.5: Configuration of the hypothetical city.

Table 2.1: Average number of daily trips between the outer zones and the inner zones of the hypothetical city.

Outer Zone	Trips				
	Inner Zone				
	5	6	7	8	9
1	2795.5	2795.5	993.9	751.7	1490.9
2	3106.1	4472.7	1503.4	883.5	1987.9
3	1104.4	1503.4	1118.2	537.3	993.9
4	626.4	662.6	402.9	372.7	497.0

We applied the model to the hypothetical city, and the result was that the best locations for the park-and-ride facilities would be sites A and B. Choosing these locations would allow reducing car use in the

inner city by 15.5%, from 49.8 to 42.1×10^3 pkm (Table 2.2). Park-and-ride would capture 16% of the trips. Only trips starting in both zones 1 and 2 would use both of these facilities, since they are the closest ones to that departure zones. For instance, as displayed in Figure 2.6, 11% of the trips from zone 1 to zone 9 would use facility A, which is on the least-cost path between both zones, and 7% would use facility B, which is not so conveniently located but still attracts some drivers (their travel distance would exceed the length of least-cost path by less than 25%). On the contrary, for trips from zone 3 to zone 9, the facilities would be too far off the least-cost path and would capture no trips.

Table 2.2: Optimum solution features for the park-and-ride network of the hypothetical city (β=1.0 and δ=0.25).

Park-and-ride network	-	{A, B}
Distance traveled by car in the inner city (10^3pkm)	49.8	42.1
Modal share (%) Car	74	62
Bus	26	22
Park-and-ride	-	16
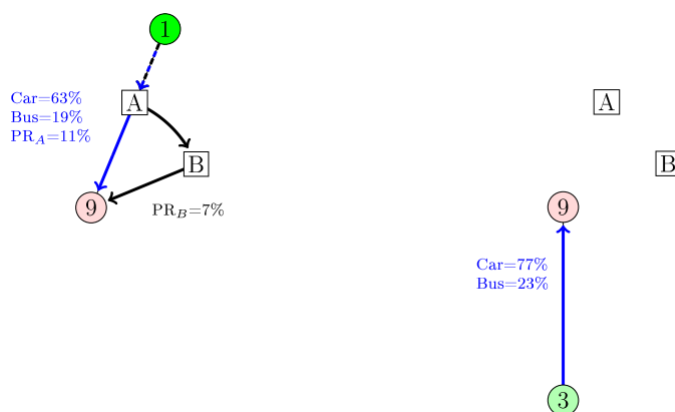
Figure 2.6: Optimum routes and modal shares for trips from zones 1 and 3 to zone 9 of the hypothetical city (β=1.0 and δ=0.25).

The behavior of the model can be further appraised by looking at the best alternative to the optimum solution, which consists in placing the park-and-ride facilities in sites B and C (instead of A and B). As shown in Table 2.3, this second-best solution would be clearly worse than the first-best one, as it would lead to a decrease of car use in the inner city by just 12.7%. This means that only about 80% of the gains made possible by the introduction of the parking-and-ride facility network would be realized. The use of park-and-ride facilities would change considerably. For instance, 11% of the trips from zone 3 to zone 9 would be captured by the facility located at C, which is on the least-cost path between both zones (Figure 2.7). However, the fact that there would be no facility at A would make park-and-ride less attractive to drivers in zones 1 and 2. In particular, only 8% of the trips from zone 1 to one 9 would resort to park-and-ride (against 18 % if the first-best solution were implemented).

Table 2.3: Second-best solution features for the park-and-ride network of the hypothetical city (β=1.0 and δ=0.25).

Park-and-ride network		{B, C}	Optimum
			{A, B}
Distance traveled by car in the inner city (10^3 pkm)		43.5	42.1
Modal share (%)	Car	64	62
	Bus	23	22
	Park-and-ride	13	16
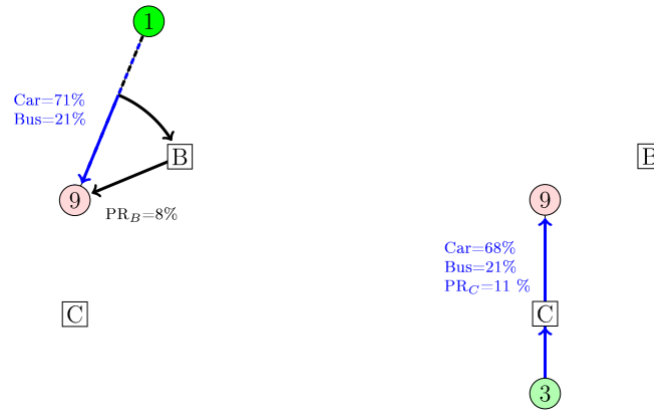
Figure 2.7: Second-best routes and modal shares for trips from zones 1 and 3 to zone 9 of the hypothetical city (β=1.0 and δ=0.25).

Additional insights on the behavior of the model can be gained by analyzing how the optimum solution changes in response to variations in the values of parameters δ (propensity of drivers to route deviations) and β (sensitivity of travelers to travel costs).

With respect to the first parameter, we considered two extreme cases (in addition to δ= 0.25): δ= 0, meaning that drivers will only use park-and-ride facilities if they are right on the least-cost path to their destinations; and δ = inf, meaning that drivers will use park-and-ride facilities no matter how far they are off their least-cost path (but increasingly less as travel costs grow). Our conclusion was that, in both cases, the optimum locations for the park-and-ride facilities would remain the same (that is, sites A and B), but the modal share of park-and-ride, which was 16% for δ= 0.25, would be 15% and 18% respectively for δ = 0 and δ = inf (Table 2.4). Using the example of trips from zone 1 to zone 9, the relatively low modal share in the former case can be explained by the fact that there would be no trips captured by the park-and-ride facility located at B, because it is not on the least-cost path between both neighborhoods (Figure 2.8). The high modal share in the latter case can be illustrated by the trips from zone 3 to zone 9, which would not exist if δ = 0.25 and are now in part (8%) made through the park-and-ride facilities despite the fact that, for using these facilities, drivers would have to deviate considerably from their least-cost path.

Table 2.4: Optimum solution features for the park-and-ride network of the hypothetical city under different values of δ (β=1.0).

δ		0	0.25	∞
Park-and-ride network		{A, B}	{A, B}	{A, B}
Distance traveled by car in the inner city (10^3 pkm)		42.8	42.1	40.9
Modal share (%)	Car	63	62	60
	Bus	22	22	22
	Park-and-ride	15	16	18
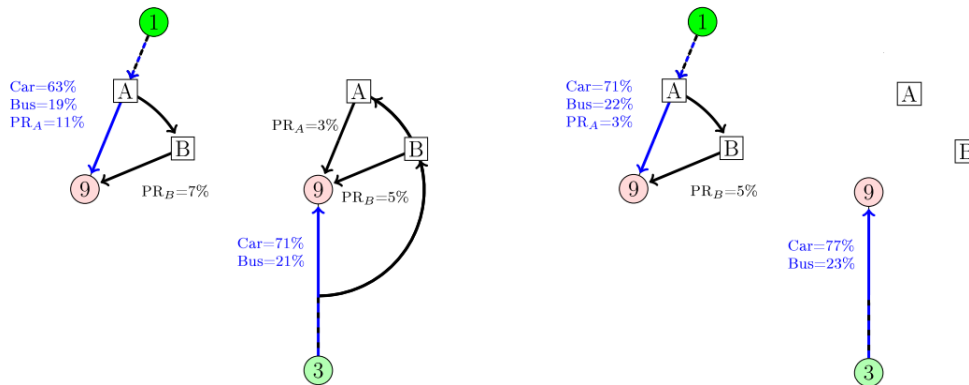
Figure 2.8: Optimum routes and modal shares for trips from zones 1 and 3 to zone 9 of the hypothetical city (β=1.0): δ=0 (left) and $\delta = \infty$ (right).

Regarding the other parameter under consideration, we considered the cases of β=0.5 and β= 2.0 (in addition to β=1.0). Again in these cases the optimum locations for the park-and-ride facilities would be A and B (it is not surprising that these locations do not change because, as seen before, they would be clearly better than any alternatives). In contrast, as shown in Table 2.5, car use in the inner city would vary considerably as the value of β increases, from 31.7×10^3 pkm when β=0.5 to 57.0×10^3 pkm (79.8% more) when β=2.0. This happens because higher values of β signify a stronger preference for the least-cost mode, which is car in this example. For any pair of zones, the travel modes would not change with changes in the value of β, but their shares would change. For instance, and as displayed in Figure 2.9, the number of trips from zone 1 to zone 9 made through the park-and-ride facilities would raise from 2% to 18% with the increase of β from 0.5 to 2.0, but, similarly to what happens when β=1.0, no trips from zone 3 to zone 9 would be made through such facilities.

Table 2.5: Optimum solution features for the park-and-ride network of the hypothetical city under different values of δ (β=1.0).

β		1	0.5	2
Park-and-ride network		{A, B}	{A, B}	{A, B}
Distance traveled by car in the inner city (10^3 pkm)		42.1	31.7	57
Modal share (%)	Car	62	47	84
	Bus	22	28	11
	Park-and-ride	16	25	5
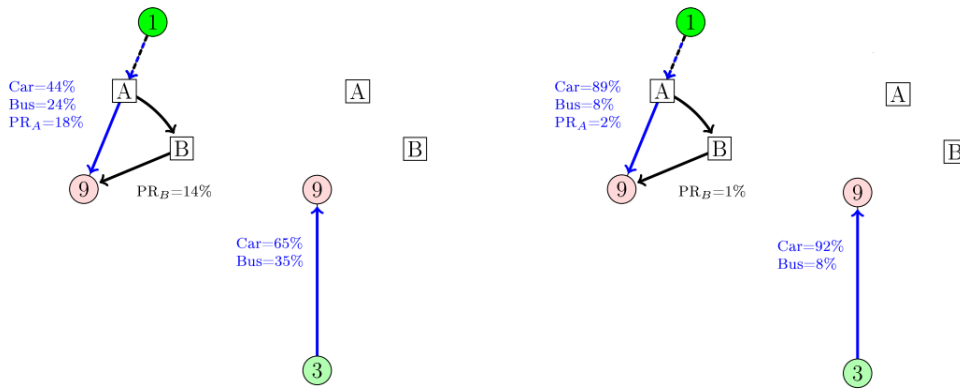
Figure 2.9: Optimum routes and modal shares for trips from zones 1 and 3 to zone 9 of the hypothetical city (δ=0.25): β=0.5 (left) and β=2.0 (right).

2.3.3 Model Solving

The fact that it is possible to cast the park-and-ride facility location problem into a linear combinatorial formulation has profound implications with respect to model solving. Indeed, in general, nonlinear combinatorial optimization models remain generally quite challenging to cope with (Burer and Letchford, 2012; Hemmecke et al., 2010). This was the situation also for linear models until the 1990s. However, thanks to the vast progresses accomplished since then, large instances of our model should be possible to solve today to exact optimality within very reasonable computational effort using branch-and-cut approaches and respective software implementations.

We used one of the top-quality software packages available – XPRESS Version 7.7 (FICO, 2014) – to solve the optimization model, starting with the resolution of the application example presented in the previous subsection. Without surprise, in this case solutions were obtained instantaneously. Then, we solved a collection of randomly generated instances involving cities with the same radiocentric shape as in the application example but with up to 160 zones and 20 possible sites for installing up to 20 park-and-ride facilities. The results we obtained can be summarized by stating that instances involving 8, 12, 16 and 20 possible sites (the key feature with respect to computational effort) took at most 7 minutes, 55 minutes, 7 hours and 30 hours to solve. The time consumed in the resolution of the 16-site and especially the 20-site instances was certainly rather long, but infrastructure planning problems like the one we are dealing with have a long-term nature and normally do not require quick answers. Moreover, only very big cities will have more than 20 entrances where to install park-and-ride facilities. Therefore, it can be said that, in general, it will not be necessary to resort to specialized or heuristics methods to solve our model, but this cannot be excluded in applications to the largest metropolitan areas.

2.4 Model Data

In order to apply the optimization model presented in the previous section to the park-and-ride facility location problem faced by the city of Coimbra, we first had to prepare the data needed to run it: origin-destination trips (designated by T in the model); generalized travel costs (C); travel distances (D); and mode/route choice parameters (β and δ). In this section, we provide detailed information on how these data were obtained.

Origin/destination trips

The number of origin-destination trips between the urban and suburban zones of Coimbra was taken directly from the mobility survey carried out in the municipality in 2009. Since the population of the municipality, as well as its spatial distribution, almost remained unchanged in recent years and probably will not change much at least in the near future, we assumed that the park-and-ride facility network of Coimbra could be designed considering that number of trips.

Generalized travel costs

The generalized travel costs for trips made by car or bus comprise components that depend on the mode. In the case of car, the components are vehicle costs, parking costs and time costs, and in the case of bus they are fare costs, time costs and discomfort costs (expressing the relative lack of comfort that characterizes bus trips when compared to car trips).

For the municipality of Coimbra, the following reference values were considered:

a) Car mode

- Vehicle costs - the average costs of owning and using a car are 0.30 €/km considering vehicle depreciation, fuel, maintenance, insurance and taxes.

- Parking costs - the average fee for a parking space in the 9 zones where parking is paid is 1.0 € (this average reflects the fact that the parking fee in these zones is 0.5 €/hour, that the average parking time is 4 hours, and that approximately 50% of car users have access to free parking).

- Time costs - the average value of travel time is 8.0 €/hour (80% of the hourly disposable income), which is equivalent to 0.20 €/km given the average speed for car trips of 40 km/h.

b) Bus mode

- Fare costs - the average flat rate for any trip is 1.0 € (this average reflects the substantial discounts offered to frequent users, seniors, disabled persons and students).

- Time costs - the average value of travel time was taken to be the same as for car users (8.0 €/hour), which is equivalent to 0.40 €/km considering the average operating speed of buses of 20 km/h.

- Discomfort costs - the value of these costs was estimated to be 0.30 €/km (this value was estimated together with β; see explanations below under the heading mode/route choice parameters).

Therefore, the current car (C_a) and bus (C_b) generalized travel costs in the municipality of Coimbra can be expressed as a function of distance (D^a and D^b) in the following manner:

$$C_a = 0.5 \times D^a + 1, \qquad (C_a \text{ in } €, D^a \text{ in km}) \tag{2.11}$$

$$C_b = 0.7 \times D^b + 1, \qquad (C_b \text{ in } €, D^b \text{ in km}) \tag{2.12}$$

If the park-and-ride facility network is implemented in the same cost conditions and it is assumed that the mode change has a discomfort cost of 1.0 € (approximately equal to the current cost for a bus transfer), the generalized travel cost for a park-and-ride route (C_r) could be expressed as a function of the distances covered by car (D_a) and bus (D_b) in this manner:

$$C_r = 0.5 \times D^a + 0.7 \times D^b + 2, \qquad (C_r \text{ in } €, D^a, D^b \text{ in km}) \tag{2.13}$$

Travel distances

The travel distances between the different urban and suburban zones of Coimbra were calculated within the Geographic Information System we have developed in ArcGIS 10.2 to organize and process data for Coimbra's transport network. Specifically, we determined distances through the least-cost route for three types of trips: by car, by bus, and by both modes through each park-and-ride facility. The calculations were performed using the Shortest Path command available in the Network Analyst toolbar of ArcGIS 10.2.

Mode/route choice parameters

The mode/route choice parameters were determined in two different ways. We will focus first on β and then on δ.

The value for parameter β was obtained through the calibration of a logit function using the data for origin-destination trips by mode and generalized travel costs available for Coimbra. The calibration of a logit function is not straightforward when it is necessary to take into account three or more transport modes; see e.g. Ben-Akiva and Lerman (1985) and Train (2009). However, as explained in Ortúzar and Willumsen (2011, Subsection 6.5.4 Calibration of Binary Logit Models), it can be performed by simple

regression analysis after a suitable transformation of variables if, as is the case in Coimbra, only two modes are at stake. Moreover, an additional parameter can be included in the logit function to capture costs other than the ones that can be measured objectively. These costs are typically associated with (relative) discomfort costs. We applied this type of analysis and obtained, after rounding, β=1.0 and γ (bus discomfort costs) = 0.30€/km (the value we used earlier in this section to calculate bus generalized travel costs). The fitness between observed values and modeled values (through the logit function) was not particularly high (R^2=0.65), but both β and γ were very significantly different from zero (t-test>>2). Therefore we decided to rely on these parameter values for our study.

The value for parameter δ was defined based on a user survey carried out for Coimbra's ECOVIA park-and-ride service (Seco et al., 1999). This service was introduced in Coimbra in the mid 1990s and discontinued ten years later mainly because the areas where the three parking facilities were located changed considerably (the main park was placed near the stadium, in a part of the city that was fully renovated for the 2004 European Football Championship) and/or were too close to the city center. The user survey made it clear that very rare drivers had extended the length of their trip by more than 25% to take advantage of the park-and-ride service. Based on this information, we decided to consider δ = 0.25 in our study.

2.5   Study Results

The goal of the study described in this chapter was to identify locations for a set of park-and-ride facilities to install in the municipality of Coimbra such that car use in its urban area could be reduced as much as possible.

For analyzing the impact of this measure, our base scenario was that parking fees and bus fares would remain the same as today, and that the (average) operating speed of buses would not change as well. Then, to deepen the analysis, we considered seven alternative scenarios, involving changes in parking fees, bus fares and/or bus operating speeds, as follows:

   I. Moderate increase (10%) of parking fees in the paid parking zones.

  II. Moderate decrease (10%) of bus fares.

 III. Substantial increase (25%) of parking fees in the paid parking zones.

  IV. Substantial decrease (25%) of bus fares.

   V. Substantial increase (25%) of parking fees in the paid parking zones coupled with the introduction of parking fees (0.3 €/hour) in the remaining urban area.

  VI. Substantial increase (25%) of bus operating speed.

VII. Combination of measures IV, V and VI.

Below, we first present and discuss the results obtained for the base scenario, and then compare them with the results obtained for the alternative scenarios.

2.5.1 Base Scenario

One of the main results we have obtained for the base scenario of our study was that, on its own, the implementation of a park-and-ride facility network would allow reducing car use in the city of Coimbra by as much as 29.0%, from 81.1 to 57.6×10^3 pkm (Figure 2.10 and Table 2.6). Another relevant result was that park-and-ride could attract up to a modal share of 26% in the municipality.

The previous outcomes could only be achieved if facilities were built in all the 7 sites selected for possible park-and-ride facility location. However, even if only 3 facilities were built, it would already be possible to decrease car use in the city by 19.1% (from 81.1 to 65.6×10^3 pkm) and achieve a park-and-ride share of 17%. As shown in Figure 2.11, these facilities should be located at sites A, B and D, and would intercept 2162, 1965 and 1713 daily trips, respectively (based on this number of trips and taken essentially into account their distribution over the day it would not be difficult to set suitable sizes for the park-and-ride facilities). Installing a fourth facility would reduce car use in the city by an extra 4.0% and increase the park-and-ride share by the same percentage, but the contribution of the additional facilities would be relatively small (6.1% and 5% in total, respectively).

It is worth noting that, as the number of park-and-ride facilities increases, the configuration of the network changes smoothly, always by the addition of a facility to the ones that were already included in the network. Indeed, if only one facility were to be built, than it should be located at site A; if the network were to comprise two facilities, one should be in that same site (A) and the other in site B; if it were to comprise three facilities, two should be in those same sites (A and B) and the other in site D; and so forth.

It is also worth noting that the optimal locations of park-and-ride facilities almost do not change when the propensity of drivers to route deviations, δ, varies 0.15 around the reference value of 0.25 (that is, when $\delta \in [0.10, 0.40]$). Indeed, the only change we observed was for P=3 and $\delta \leq 1.5$. In this case, the optimal locations are A, D and G (instead of A, B and D in all other cases).
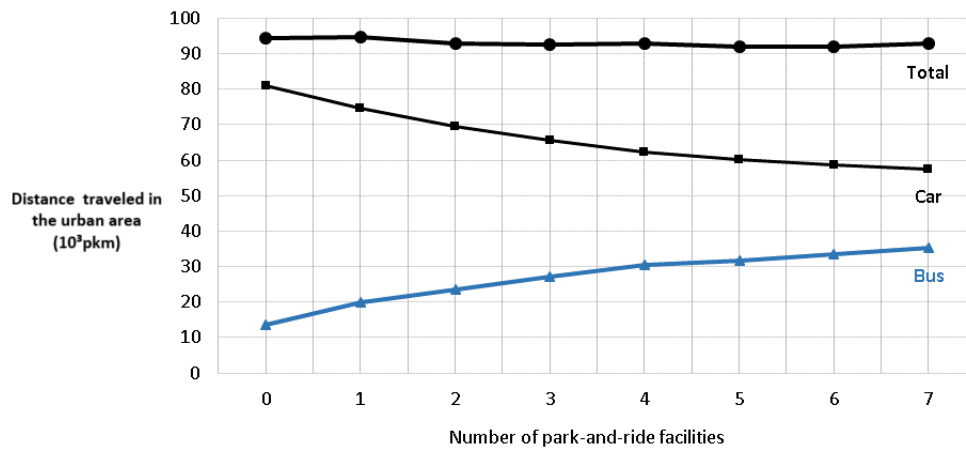
Figure 2.10: Distance traveled in the urban area of the municipality of Coimbra as a function of the number of park-and-ride facilities.

Table 2.6: Optimum solution features for the park-and-ride network of the hypothetical city under different values of δ (β=1.0).

Number of facilities		0	1	2	3	4	5	6	7
Optimum park-and-ride network		-	{A}	{A, D}	{A, B, D}	{A, B, D, G}	{A, B, C, D, G}	{A, B, C, D, E, G}	{A, B, C, D, E, F, G}
Distance traveled in	Car	81.1	74.8	69.5	65.6	62.4	60.1	58.6	57.6
the urban area	Bus	13.5	19.9	23.5	27	30.6	31.7	33.6	35.1
(10^3pkm)	Total	94.6	94.7	93	92.6	93	91.8	92.2	92.7
Distance traveled	Car	131.3	134.4	138.1	141.3	142.7	145.1	145.5	145.5
in the suburban area	Bus	16.3	14.7	14	13.1	12.3	11.9	11.6	11.5
(10^3pkm)	Total	147.6	149.1	152.1	154.4	155	157	157.1	157
Distance traveled	Car	212.4	209.2	207.6	206.9	205.1	205.2	204.1	203.1
in the municipality	Bus	29.8	34.6	37.5	40.1	42.9	43.6	45.2	46.6
(10^3pkm)	Total	242.2	243.8	245.1	247	248	248.8	249.3	249.7
	Car	84	78	73	69	66	64	63	62
Modal share (%)	Bus	16	15	14	14	13	13	12	12
	Park-and-ride	-	7	13	17	21	23	25	26

Figure 2.11: Optimum locations for three park-and-ride facilities in the municipality of Coimbra (figures in brackets represent the number of trips intercepted by each facility).

In line with one of the main arguments against park-and-ride, the more facilities would be included in the park-and-ride facility network the larger would be the total distance traveled in the municipality. However, it should be mentioned here that, even if 7 facilities were built, this distance would just increase from 242.2 to 249.7×10^3 pkm (3.1%). This growth of travel distance would be the outcome of a decrease in the use of car (from 212.4 to 203.1×10^3 pkm if 7 park-and-ride facilities were built) combined with an increase in the use of bus (from 29.8 to 46.6×10^3 pkm).

With respect to the previous results, it is important to underline that it is not enough to look at the costs of building and operating the facilities (for which the number of facilities is a proxy) when planning the development of a park-and-ride facility network. Probably more relevant are the costs of providing additional bus services – the distance traveled by bus in the municipality of Coimbra would increase by 34.6% (from 29.8 to 40.1×10^3 pkm) even if only 3 park-and-ride facilities were installed, and this would certainly entail more costs for Coimbra's bus company (SMTUC) and an aggravation of their already serious deficits.

As could be expected, the increase of bus use in the municipality would only occur in the urban area. For instance, the distance traveled by this mode in the city would double if a 3-facility park-and-ride network were implemented, from the current 13.5×10^3 pkm to 27.0×10^3 pkm, and would further increase to 35.1×10^3 pkm if 7 facilities were built. In contrast, for this number of facilities, bus use in the suburbs would decrease from 16.3 to 11.5×10^3 pkm (29.4%). Even if only 3 facilities were built, the decrease would still be of 19.6%, thus significant enough to put routes that are already rather thin today under additional pressure, eventually leading to their abandonment. Subsequently, car could become the only alternative to travel to the city from some suburbs. This could be another detrimental effect of implementing a park-and-ride facility network in Coimbra.

To conclude the presentation and discussion of the results we obtained for the base scenario, we comment here on the shape of the catchment areas of park-and-ride facilities (the catchment area of a given park-and-ride facility is defined as the set of trip generation zones that resort to that particular facility more than to any other park-and-ride facilities). As illustrated in Figure 2.12 for Coimbra's optimum park-and-ride facility network with P=3, such catchment areas can be extremely irregular (as noted e.g. by Horner and Grubesic 2001). Among several other factors, the reason for this irregularity has to do with the competition between park-and-ride facilities and with the different patterns of origin-destination trips for different trip generation zones. Therefore, it does not seem reasonable to assume, like some authors do, that the catchment area of a park-and-ride facility has a given pre-specified shape (parabolic or other).

2.5.2 Other Scenarios

In order to analyze whether measures involving changes on parking fees, bus fares and/or bus operating speeds would be a good alternative to the implementation of a park-and-ride facility network in the municipality of Coimbra with respect to diverting trips between the suburbs and the city from car to bus in the urban area, we compared the results obtained for scenarios I to VII with P=0 (the current situation, i.e., no park-and-ride facility network) with the results obtained for the base scenario with P=3. We performed the comparison considering a network of 3 facilities because, as seen before, the corresponding solutions would represent a well-balanced compromise between car use reduction in the city and the effort to be made by the municipal council to build and operate the facilities and increase the level of bus services.
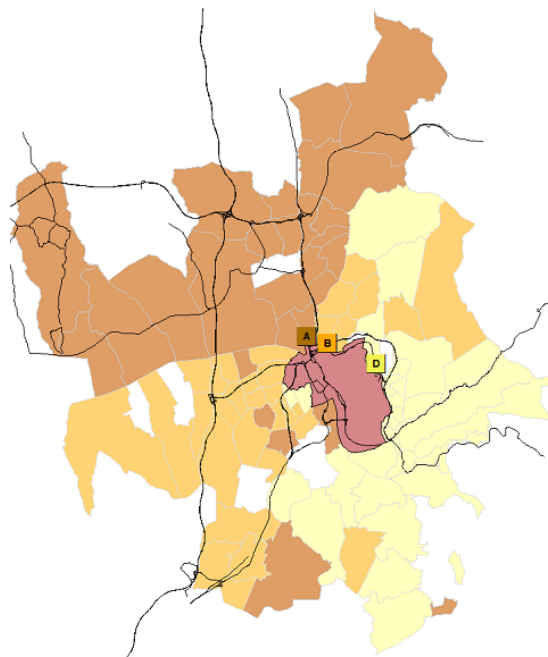
Figure 2.12: Shape of the catchment areas for the three optimum park-and-ride facilities in the municipality of Coimbra.

The results we obtained show that the weaker measures (corresponding to scenarios I and II) would fall very short of matching the impacts of implementing the park-and-ride facility network with respect to the distance traveled by car in the urban area, and even the stronger measures taken separately (scenarios III to VI) would not be enough for that purpose. As a matter of fact, and as shown in Figure 2.13, even the most effective of the latter measures, increasing bus commercial speed by 25%, would only lead to a 9% decrease reduction of car use in the city, against the 19% achieved through a 3-facility park-and-ride network. For equaling the impact of this measure (and indeed surpassing it by 5%), it would be necessary to combine the stronger measures together, which would certainly be very difficult both technically and politically (increasing bus commercial speed by 25% would inevitably require major changes in traffic organization, generalizing paid parking in the city would certainly motivate harsh protests).
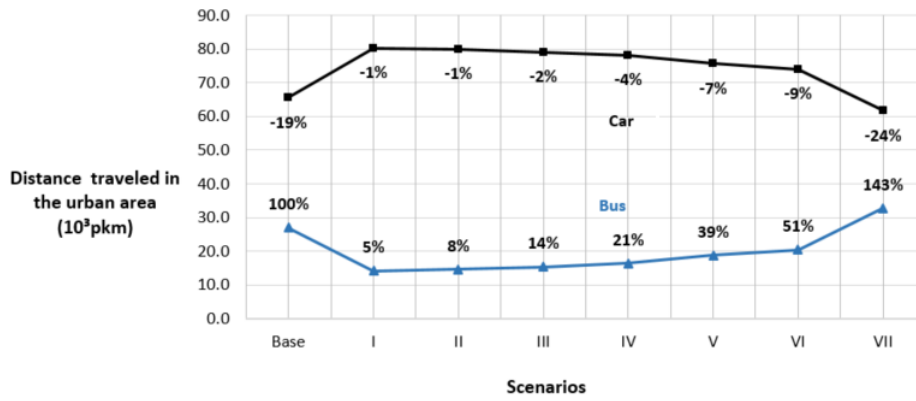
Figure 2.13: Distance traveled in the urban area of the municipality of Coimbra by car and bus for the base scenario with 3 park-and-ride facilities and the 7 alternative scenarios.

In addition to assessing the impacts of the measures under consideration when applied as an alternative to the development of a 3-facility park-and-ride network, we also evaluated their effects if they were carried out as a complement to the network's implementation. In line with the previous results, and as shown in Figure 2.14, only for scenario VII the implications of the additional measures would be very significant, as they would lead to a decrease of the distance traveled by car in the urban area for trips originated in the outskirts by 47% (against 19% for the base scenario, i.e., if no complementary measures were applied). Any other scenarios would be much less effective, though the ones corresponding to the moderate measures (increasing parking prices and decreasing bus fares by 10%) could be interesting because they would enhance the impact of the park-and-ride facility network (by 2 or 3%, respectively) and should be relatively easily accepted by residents.
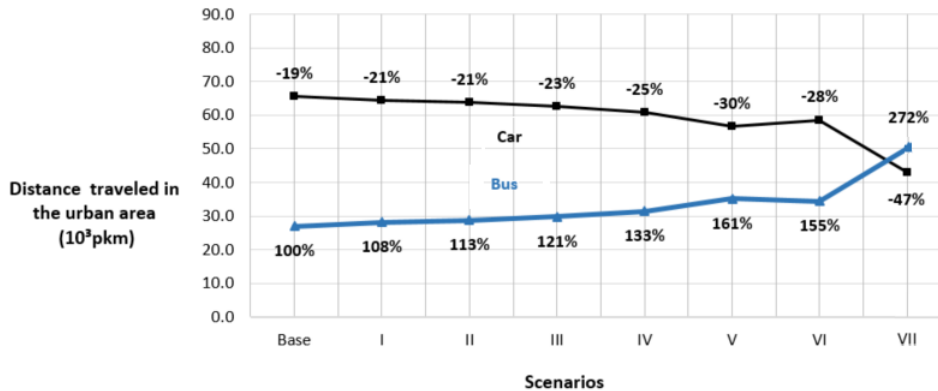
Figure 2.14: Distance traveled in the urban area of the municipality of Coimbra by car and bus for the base scenario with 3 park-and-ride facilities and the 7 complementary scenarios.

2.6 Conclusion

Park-and-ride solutions have long been seen as capable of giving a significant contribution to mitigate the traffic congestion and pollution problems faced by cities, and the results of the study we described in this chapter clearly confirmed this idea. Indeed, the introduction of a park-and-ride network with

only three facilities in Coimbra could reduce car use in its urban areas by 19.1% (and increase bus use in the municipality by 34.7%). For achieving the same outcome with respect to diverting trips from the suburbs to the city by decreasing bus fares, increasing parking fees and increasing bus commercial speeds, the combination of measures required would be rather difficult to implement, both from the technical and the political point of view. We also confirmed that the development of the park-and-ride facility network would lead to a decrease of bus use in suburban areas (by 19.6%), which could put pressure on the abandonment of some bus routes there and, if no action were taken, have harmful consequences for the mobility of the residents of those areas.

One of the study features that deserves to be underlined is the fact that it was based on an optimization model that takes into account mode/route competition effects (through a logit function). Despite the fact that park-and-ride solutions are quite widespread and that their effectiveness critically depends on the locations of the park-and-ride facilities, only a few optimization models have been proposed in the literature for the purpose at hand. The model we used improves on this model notably because its objective is to minimize car use in urban areas, whereas the previous model was built around a covering objective (intercepting as much car trips as possible, no matter the distance they would travel until reaching their destinations).

As is shown in this chapter, our model, in its current form, can already provide useful insights on where to install park-and-ride facilities in the periphery of a city and on their implications regarding the use of car and bus (and other modes, if relevant). However, we recognize that the model still has a number of limitations. For instance, as already mentioned in the section where the model was presented, it does not take into account the changes in traffic patterns caused by the implementation of the park-and-ride facility network, as well as any uncertainty issues. A limitation of the model that we did not allude to before has to do with capacity constraints. In many cases these constraints will not be so important because park-and-ride facilities are located in the periphery of cities and, as it happens in Coimbra, it will not be too difficult to find there vacant land where to install them, but in other cases conditions may be different. In order to overcome these limitations, the changes that need to be performed in the model would undoubtedly make it more complex and much more difficult to solve than it is at present.

In the future, we expect to work on the improvements required to overcome the limitations of the model. But we plan to do this only in the medium term. Indeed, in the short run and continuing to focus on Coimbra, our priority will be to analyze how the park-and-ride facility network should develop if a light rail system were built in the municipality. Such system should already be in place, but its implementation had to be postponed because of the serious debt crisis that is affecting Portugal since 2010. Now that the worst austerity days seem to be over, there are some chances that the light rail project is resumed, and, given its expected impact, any other initiatives relating to transport in Coimbra need to take it into account.

Chapter 3

An optimization approach to integrated transit-parking planning

3.1 Introduction

Urban transit plays a crucial role with respect to the pursuit of sustainable development goals in several dimensions. As a matter of fact, by helping to reduce fuel consumption, pollutant emissions and traffic congestion in relation to car, transit is clearly advantageous from the environmental and economic standpoints (Kennedy, 2002; Schiller et al., 2010; Miller, 2014). In addition to this, it performs a key social function by providing people who cannot own or drive a car with acceptable levels of mobility (Lucas et al., 2001; Preston and Rajé, 2007; Lucas, 2012).

These settings explain why, notably in the European Union and in the United States, local and sometimes central governments are strongly involved in the provision of urban transit either producing this type of service or subsidizing private companies to do so. In cities like Brussels, Rome and Stockholm, recovery ratios (i.e. the percentage of operating costs funded by transit revenues) are only about 35%, and in Cleveland and Detroit they are below 25% (Figure 3.1). This leads to the aggravation of public deficits that are already very harmful in countries affected by debt crises, like most Southern European countries. For this reason, transit in these countries is being put under great pressure. In less developed countries, transit is generally assured by private companies that either are not subsidized or, when they are, the subsidies they receive are rather low. However, this often leads to strong and occasionally violent public protests against high transit fares such as the ones happened in Brazil in recent years, which will eventually lead (and are already leading) to a stronger involvement of the public sector in the supply of urban transit.
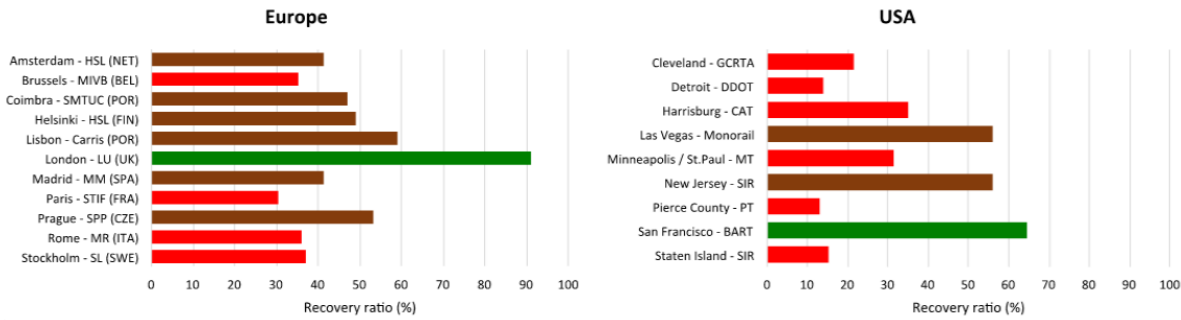
Figure 3.1: Recovery ratio of public transport for some European countries and USA states – between 2007 and 2016.

Despite the support they get from the public sector, the truth is that there is a long-term trend for transit modes to have low market share both in European Union countries (EU-28) and in the United States (USA). In fact, the use of transit systems remained stable in the past 17 years on both EU-28 and USA, with average transit modal shares around 18.2% and 7.8%, respectively. A slight increase of around 1% in the transit modal share has occurred in the USA between 2000 and 2015, against a decrease of 0.2% in the EU-28 during the same period. These conclusions are drawn taking into account the data displayed in Figure 3.2, where the transit modal shares were assessed based on the total number of passenger-kilometers made in both urban and non-urban trips.
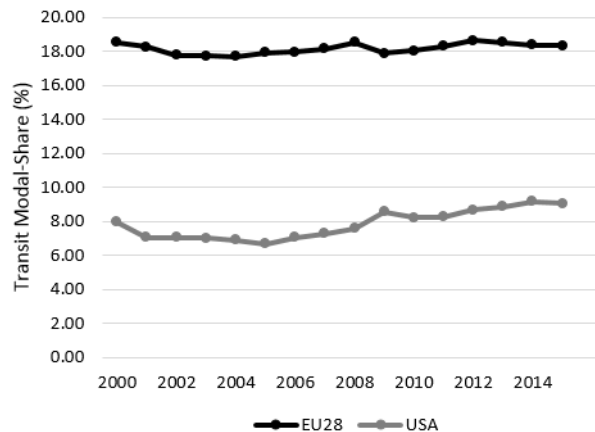
Figure 3.2: Transit market share for the European Union 27 and USA between 1995 and 2009. [Source: European Commission and US Bureau of Transportation Statistics].

As one could expect considering the role of transit in the promotion of a more sustainable urban mobility, the scientific community has devoted a great deal of attention to the subject, and in particular to transit planning models. This attention has essentially consubstantiated in two literature streams.

The first stream of transit planning models is based on microeconomic analysis approaches, and focuses on the establishment of optimal transit pricing, supply and subsidization rules in stylized cities (e.g. monocentric cities). Examples of contributions to this stream include Mohring (1972), Vickrey (1980), Small et al. (2007), Van Reeven (2008) and Basso and Jara-Díaz (2010, 2012).

The other stream relies on operations research approaches. It focuses on real cities, but as the con-

tents of authoritative textbooks like Ceder (2007) and Vuchic (2005) make clear, transit fares, and therefore transit demand, are taken as given (that is, the interaction between supply and demand is not taken into account). Recent reviews of this literature stream appear in Desaulniers and Hickman (2007), Guihaire and Hao (2008), Kepaptsoglou and Karlaftis (2009) and Ibarra-Rojas et al. (2015).

According to the analysis conducted by the International Association of Public Transport (of Public Transport, 2015), managing urban transit either by pricing and/or supply measures might not be enough to decrease transit operating deficits or to increase transit's modal share. As a matter of fact, and in line with Figure 3.2, issues related to private vehicles should also be considered so that problems such as congestion and fuel consumption can be addressed. According to the literature, road congestion pricing and parking fees are advocated as potential solutions to these problems (Vickrey, 1963; Eliasson et al., 2009; Beirão and Cabral, 2007; Dueker et al., 1998; Shoup, 2005).

Road pricing schemes are vastly explored in the transport economics literature upon the claim of being an efficient policy to relieve congestion. A revision of this literature is presented in Tsekeris and Voß (2009), where the authors highlight the scarce number of studies that relate road pricing with other network management measures. In what concerns the integration of road pricing with transit fares, its main focus is in micro-economic approaches with the goal of minimizing the total social costs of a competitive transit/highway system (Tabuchi, 1993; Huang, 2000; 2002; Danielis and Marcucci, 2002; Mirabel and Reymond, 2011), without any consideration of the cost of the pricing scheme or its practical implementation. Indeed, real-world applications of this type of measure are rare (Santos and Fraser, 2006; Noordegraaf et al., 2014; Börjesson et al., 2014), mainly due to their unpopularity and/or difficulty of implementation (Schlag and Schade, 2000; Jaensirisak et al., 2005; Ison and Rye, 2005; Eliasson, 2014; Börjesson et al., 2014).

Parking fees are an attractive alternative to road pricing, not only because they are more easily accepted by users but also because they are more straightforward to implement (Dueker et al., 1998; Shoup, 2005; Albert and Mahalel, 2006; Marsden, 2006; Beirão and Cabral, 2007), even though they are a second-best measure for solving traffic congestion problems (Verhoef et al., 1995; Calthrop et al., 2000). Similar to what is expected from road pricing schemes, parking fees are also capable of influencing transit ridership and driving behavior with respect to, for instance, chosen route or departure time (Balcombe et al., 2004; Simićević et al., 2013). However, to the best of our knowledge, few studies explore the relationship between parking and transit systems (Inci, 2015), and the integration of transit and parking through pricing schemes has not been addressed in the literature.

The model we propose in this chapter intends to fill the gap in the literature concerning the integration of transit and parking systems through pricing schemes using an operations research approach. Our goal is to develop an approach for assisting city councils (local governments) in the development of integrated transit-parking policies with the objective of optimizing the financial performance of these

policies while ensuring given minimum levels of (motorized) mobility in a city. Each one of the two transit and parking systems is managed by an operator that answer to or is controlled by the city council. The key decision variables are transit fares and parking fees. Our model can be classified as strategic, since it does not look into the detailed configuration of transit networks – the idea is that it will serve to identify and pre-assess transit-parking policy solutions for subsequent in-depth analyses possibly through simulation approaches.

This chapter is organized as follows. In the next section, we present the ingredients and formulation of our planning model, which is analyzed and discussed for a small-scale hypothetical city in Section 3.3. The computational effort involved in solving the model is examined in Section 3.4. This is followed by a real-world application to Coimbra, a municipality in central Portugal with a population of approximately 150,000. In the final section of this chapter, we provide a summary of conclusions and indicate directions for further research.

3.2   Optimization Model

The integrated transit-parking planning model introduced in this chapter is applied to a city divided into $\mathbf{Z}$ trip-generation zones. The set of trip(-generation) zones is $\mathbf{Z} = \{1, \ldots, Z\}$.

Trips can be done by transit (B) or by car (A). For the sake of making explanations easier, we limit the presentation of the model to two modes, but its extension to a larger number is rather straightforward. The total number of trips between two zones, Q_{ij}, corresponds to having free transit and parking systems. The number of trips made by transit and car are q_{ijB} and q_{ijA}, $i, j \in \mathbf{Z}$, respectively. If transit and/or parking are paid, some trips will not be made. We call them lost trips (O), and designate their number by q_{ijO}. It is assumed that, for each trip zone pair, the number of trips by each mode, as well as the number of lost trips, can be described by logit functions of the generalized travel costs for the two modes, C_{ijB} and C_{ijA}, as given by equations (3.1)-(3.3), where θ and β are statistical calibration parameters that capture the sensitivity of travelers to travel costs.

$$q_{ijB} = \frac{e^{-\theta \cdot C_{ijB}}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i, j \in \mathbf{Z} \tag{3.1}$$

$$q_{ijA} = \frac{e^{-\theta \cdot C_{ijA}}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i, j \in \mathbf{Z} \tag{3.2}$$

$$q_{ijO} = \frac{e^{-\beta}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i, j \in \mathbf{Z} \tag{3.3}$$

Transit generalized travel costs (C_{ijB}) comprise time costs, discomfort costs (the latter express the relative lack of comfort of transit trips when compared to car trips) and transit fares. The time and discomfort costs are assumed to be proportional to the transit travel distance, D_{ijB}. Transit fares are assumed to vary across trip zone pairs according to transit zones defined *a priori*. Each trip zone pair, ij,

belongs to a transit zone $r \in \mathbf{R}$, where $\mathbf{R}$ represents the set of transit zones. The transit fare for this trip zone is denoted as p_{rB}. Therefore, the transit generalized travel costs can be expressed by equation (3.4), where C_B represents the time and discomfort costs per unit of distance and L_{ijr} is a binary parameter equal to one if trip zone pair ij belongs to transit zone r, and equal to zero otherwise.

$$C_{ijB} = C_B \cdot D_{ijB} + \sum_{r \in R} p_{rB} \cdot L_{ijr}, \quad i, j \in \mathbf{Z} \tag{3.4}$$

Car generalized travel costs (C_{ijA}) comprise vehicle costs (depreciation, fuel, maintenance, insurance, taxes), time costs and parking fees in the destination zones. The total vehicle and time costs are obtained by multiplying the vehicle and time costs per unit of distance, C_A, by the car travel distance, D_{ijA}. The parking fee in zone j is denoted by p_{jA}. Because these costs are defined per car trip and the demand is measured in person-trips, we use the average car occupancy rate, τ, to convert car trips into person-trips. The car generalized travel costs are given by equation (3.5).

$$C_{ijA} = \frac{C_A \cdot D_{ijA} + p_{jA}}{\tau}, \quad i, j \in \mathbf{Z} \tag{3.5}$$

Two different possibilities are considered to set the parking fees (p_{jA}): one that assumes parking fees as known, and another that takes parking fees as decision variables.

In the case of unknown parking fees, it is considered that their value is given by the model as the result of optimizing a fixed number of different parking fee levels. In this case, the model assigns each trip zone to a parking fee level. The set of trip zones assigned with the same parking fee level is called a parking zone.

Let $\mathbf{U}$ be the set of parking fee levels and $\hat{p}_{uA}$ be the decision variable that describes the parking fee assigned to each parking fee level u of set $\mathbf{U}$. To link the parking fee levels to the parking fee applied to a given trip zone, the binary decision variable w_{ju} is considered. This binary decision variable takes 1 if the parking fee level u, given by $\hat{p}_{uA}$, is charged to each car that parks at j, and zero otherwise. Each trip zone will have one and only one parking fee level. Each trip zone has a known and fixed parking capacity S_j, and it is assumed that the assignment of a parking fee level to a trip zone depends on the number of car users willing to park at that same trip zone. Let x_j be a binary decision variable that takes 1 if the number of car users willing to park at j exceeds by $\psi\%$ the parking capacity S_j of trip zone j, and zero otherwise, where ψ is the parking demand coverage ratio.

The parking fee charged in each trip zone j, p_{jA}, is expressed by equation (3.6), which relates the parking fee levels $\hat{p}_{uA}$ and the binary decision variable w_{ju}. We ensure that only a single parking fee level is assigned to a trip zone through constraints (3.7). The relationships between parking fee levels and parking capacities are given by equations (3.8) and (3.9), where M is a sufficiently large bound. If equation (3.8) holds, the number of car users willing to park at zone j does not exceed $\psi\%$ of the parking

capacity and constraint (3.9) sets the binary variable x_j to take the value 0 and parking is free in the trip zone j. In the case of being equation (3.9) the one that is active, the number of car users willing to park at zone j exceeds $\psi\%$ of this trip zone parking capacity, constraint (3.8) ensures that x_j takes the value 1 and this trip zone should be assigned to a parking fee level p_{jA}.

$$p_{jA} = \sum_{u \in \mathbf{U}} \bar{p}_{uA} \cdot w_{ju}, \tag{3.6}$$

$$\sum_{u \in \mathbf{U}} w_{ju} = x_j, \qquad j \in \mathbf{Z} \tag{3.7}$$

$$\sum_{i \in \mathbf{Z}} q_{ijA} \cdot Q_{ij} \leq \tau \cdot \psi \cdot S_j + M \cdot y_j, \qquad j \in \mathbf{Z} \tag{3.8}$$

$$\sum_{i \in \mathbf{Z}} q_{ijA} \cdot Q_{ij} \geq \tau \cdot \psi \cdot S_j - M \cdot (1 - y_j), \qquad j \in \mathbf{Z} \tag{3.9}$$

As mentioned in the introduction, the management of each transit system and parking system is made by two different operators, named transit operator and parking operator, respectively. These two entities answer to or are controlled by the city council.

The objective of this integrated approach is to minimize the joint operating deficit of both transit and parking operators. We opt for this objective because it corresponds to a critical concern of policy-makers. By focusing on a deficit minimization objective (while ensuring a minimum mobility goal), we believe to be closer to the way policy-makers look at transit and parking systems planning.

The transit operator deficits are the difference between the costs of operating the transit system and the revenues collected by the transit fares (i.e., $\sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot \sum_{r \in \mathbf{R}} p_{Br} \cdot L_{ijr}$). Those costs comprise a variable component, obtained by multiplying the number of trips and the distance for each trip by a unit cost M_B^V, and a fix component, M_B^F. Hence, the transit operating deficits, R_B, can be calculated as described by equation (3.10).

$$R_B = M_B^V \cdot \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot D_{ijB} + M_B^F - \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot \sum_{r \in \mathbf{R}} p_{Br} \cdot L_{ijr} \tag{3.10}$$

The parking operator deficits are the difference between the costs of operating the parking system and the revenues it makes from the parking fees (i.e., $\sum_{i,j \in \mathbf{Z}} q_{ijA} \cdot \frac{p_{jA}}{\tau}$). The operating costs include a variable component, obtained by multiplying the number of cars parked in each zone by a unit cost M_A^V, and a fix component, M_A^F. These operating costs only exist in trip zones where parking is paid (i.e., zones for which $x_j = 1$). Therefore, the parking operator deficits, R_A, are given by equation (3.11).

$$R_A = M_A^V \cdot \sum_{i,j \in \mathbf{Z}} \frac{q_{ijA}}{\tau} \cdot x_j + M_A^F - \sum_{i,j \in \mathbf{Z}} q_{ijA} \cdot \frac{p_{jA}}{\tau} \tag{3.11}$$

The decisions to be made for achieving the joint deficit minimization objective have been introduced before in this section, but we repeat them here:

- q_{ijA} - number of car trips made between trip zones i and j

- q_{ijB} - number of transit trips made between trip zones i and j

- q_{ijO} - number of lost trips between trip zones i and j

- p_{rB} - transit fare for transit zone r

- p_{jA}- parking fee for trip zone j

- $x_j = 1$ if parking is paid in trip zone j ($x_j = 0$ otherwise).

Under these settings, the optimization model to solve is as follows:

$$\text{Min} \quad \begin{aligned} & M_B^V \cdot \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot D_{ijB} + M_B^F + M_A^V \cdot \sum_{i,j \in \mathbf{Z}} \frac{q_{ijA}}{\tau} \cdot y_j + M_A^F - \\ & - \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot \sum_{r \in \mathbf{R}} p_{rB} \cdot L_{ijr} - \sum_{i,j \in \mathbf{Z}} q_{ijA} \cdot \frac{p_{jA}}{\tau} \end{aligned} \tag{3.12}$$

$$q_{ijB} = \frac{e^{-\theta \cdot C_{ijB}}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i,j \in \mathbf{Z} \tag{3.1}$$

$$q_{ijA} = \frac{e^{-\theta \cdot C_{ijA}}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i,j \in \mathbf{Z} \tag{3.2}$$

$$q_{ijO} = \frac{e^{-\beta}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\beta}} \cdot Q_{ij}, \quad i,j \in \mathbf{Z} \tag{3.3}$$

$$p_{jA} = \sum_{u \in \mathbf{U}} \bar{p}_{uA} \cdot w_{ju}, \tag{3.6}$$

$$\sum_{u \in \mathbf{U}} w_{ju} = x_j, \quad j \in \mathbf{Z} \tag{3.7}$$

$$\sum_{i \in \mathbf{Z}} q_{ijA} \cdot Q_{ij} \leq \tau \cdot \psi \cdot S_j + M \cdot y_j, \quad j \in \mathbf{Z} \tag{3.8}$$

$$\sum_{i \in \mathbf{Z}} q_{ijA} \cdot Q_{ij} \geq \tau \cdot \psi \cdot S_j - M \cdot (1 - y_j), \quad j \in \mathbf{Z} \tag{3.9}$$

$$\sum_{i,j \in \mathbf{Z}} q_{ijO} \leq Q_{\min} \tag{3.13}$$

$$\sum_{i \in \mathbf{Z}} q_{ijA} \leq \tau \cdot (S_j + z_j), \quad j \in \mathbf{Z} \tag{3.14}$$

$$p_{rB} \in \mathbb{R}, \quad r \in \mathbf{R} \tag{3.15}$$

$$\bar{p}_{uA} \in \mathbb{R}, \quad u \in \mathbf{U} \tag{3.16}$$

$$w_{ju}, x_j \in \{0,1\}, \quad j \in \mathbf{Z}, u \in \mathbf{U} \tag{3.17}$$

The objective function (3.12) of this model expresses the minimization of the joint operating deficits. Constraints (3.1)-(3.3) define the logit function setting the demand for each mode according to its costs. Constraints (3.6)-(3.7) account for the links between parking fee levels and the parking fee charged in each trip zone, whereas the relationships between parking fee levels and trip zone parking capacities are given by constraints (3.8)-(3.9). The minimum mobility goal is defined by setting a maximum value for

the trips that are not made due to their costs, i.e., by setting the upper bound Q_{min} for lost trips (3.13). Constrains (3.14) guarantee that the parking capacity is not exceeded. Constraints (3.15)-(3.17) set the decision variables domain.

The optimization model presented above is, in our opinion, a valuable tool for assisting local administrations in the design of integrated transit-parking plans. Despite this, we would like to acknowledge here one of its possible limitations. In our model, travel times and distances for transit and car (and therefore part of the respective generalized travel costs) are implicitly assumed to remain unchanged with variations in the number of trips for each mode. This assumption is obviously unacceptable in a city severely affected by traffic congestion, where modal share variations will certainly affect the dynamics of traffic (e.g., speed and route choices). Indeed, in these conditions, if 50% of the car trips are diverted to transit, a substantial decrease in car travel times will certainly occur. Hence, our model should be applied only in cities that are not severely affected by traffic congestion (except, possibly, in relatively short periods of the day) or only if changes in transit and parking prices are small and expected to have little impact on travel times and distances. This limitation of our model will be handled by us in the future, as stated in the concluding section of this chapter.

3.3 Application Example

We now present an example of the application of the proposed optimization model to a small hypothetical city, in order to illustrate the behavior of the model and the benefits of integrating transit and parking systems. The features of this city are detailed in a first subsection, which is followed by two subsections that account for two different cases. In the first case, we assume that transit and parking systems are managed separately, i.e., the parking fees charged by the parking operator are kept constant, whereas the transit fares are optimized. The second case analyzes the integration of the two systems, where both transit fares and parking fees are optimized so that the joint operating deficits of the transit operator and the parking operator are minimized.

3.3.1 Hypothetical City Features

The hypothetical city we will use to provide insights into the model's behavior was generated partly at random based on features of real-world cities. The total population of this city is 75,300, a value that was randomly generated taking into account the population range for small-sized cities as listed in a recent OECD report (OECD, 2016). The city is divided in six trip-generation zones classified as inner zones (A and B), the ones where most productive activities are concentrated, and outer zones (C to F), the ones where residential uses are dominant (Figure 3.3). The number of people living and working in each zone is presented in Table 3.1.
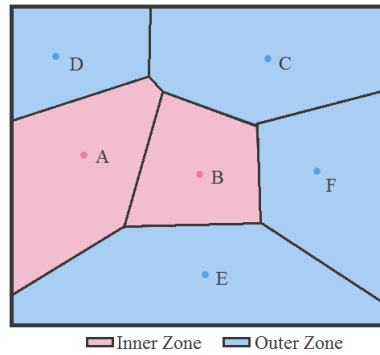
Figure 3.3: Configuration of the hypothetical city.

The potential number of trips between two zones, shown in Table 3.2, was calculated assuming that, on average, people make at most 1.5 (inter-zonal) trips per day, and that trips originating in a given zone are distributed by the other zones proportionally to the employment in these zones and inversely proportionally to the travel costs between them. These trips can be made by car or transit. The total parking capacity in the inner city is 8,800 places in zone A and 9,900 in zone B. In every zone of the outer city, parking capacity largely exceeds demand.

Table 3.1: Population and employment of each zone.

Trip Zones	A	B	C	D	E	F
Population (10^3)	3.7	6.5	20.3	18.6	14.7	11.5
Employment (10^3)	15.9	14.7	1.2	3.2	2.4	0.25

Table 3.2: Number of potential trips (demand) between each OD pair.

	Trips (10^3)					
Origin	Destination					
	A	B	C	D	E	F
A	0	2.78	0.72	1.61	1.02	0.42
B	3.93	0	1.41	1.28	2.13	1.19
C	4.01	5.59	0	2.3	1.58	1.83
D	6.42	3.63	1.65	0	1.14	0.62
E	3.89	5.73	1.07	1.08	0	1.34
F	2.68	5.36	2.09	0.99	2.24	0

Transit and parking services are provided by two different operators, which are controlled by the city council. At present, the same fare of 0.6€ is paid in every transit trip made in the city, and the same parking fee of 1.2€ (per parking) is applied in the two inner city zones. Parking is free in the outer city zones. The unit travel costs included in expressions (3.4) and (3.5) are 0.7€/km and 0.5€/km for transit and car respectively.

Under these conditions, the total number of (inter-zonal) daily trips made in the city is 54.9×10^3,

with a modal share of 58% for car and 42% for transit (Table 3.3). The lost trips in the city amount to 16.8×10^3, per day (23% of the potential trips). The parking occupation rate is 91% in the inner zones, and the parking fee paid there is 1 and below 60% in the outer zones (i.e., outer zones).

The parking demand coverage ratio was assumed to be 60% ($\psi = 0.6$).

For the number of trips and transit fares referred to above, the operating deficit of the transit operator is $13.5 \times 10^3 \text{€}$/day, with daily operating costs of a total of $27.2 \times 10^3 \text{€}$/day and revenues of around 50% of this value ($13.7 \times 10^3 \text{€}$/day). The parking operator makes a revenue of $7.4 \times 10^3 \text{€}$/day, and its costs represent 57% of the revenues achieved through the parking fees. These values are shown in Table 3.3.

In the same table, we present the social cost (SC) corresponding to the transit and parking systems of the city, which is a simplified way of expressing the costs of these systems to the society. This cost is obtained by summing the generalized travel costs of users with the operating deficits of the two operators without including the costs/revenues paid/collected by transit fares and parking fees, as detailed in equation (3.18). In the current situation, this calculation yields to a social cost of $41.24 \times 10^6 \text{€}$/year.

Table 3.3: Current situation.

Transit fare (€)		0.6
Parking fee (€)		1.2
Operating deficit (10^6 €/year)	Transit	3.67
	Parking	-2.01
	Total	1.66
Social cost (10^6 €/year)		41.24
Number of trips (10^3/day)	Transit	22.9 (42%)
	Parking	32 (58%)
	Total	54.9
Parking occupation (%)	Zone 1	91
	Zone 2	91

$$
\begin{aligned}
NSC = \quad & M_B^V \cdot \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot D_{ijB} + M_B^F + M_A^V \cdot \sum_{i,j \in \mathbf{Z}} \frac{q_{ijA}}{\tau} \cdot x_j + M_A^F + \\
+ \quad & \sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot \left(C_B \cdot D_{ijB} \right) + \sum_{i,j \in \mathbf{Z}} q_{ijA} \cdot \left(\frac{C_A \cdot D_{ijA}}{\tau} \right) + \sum_{i,j \in \mathbf{Z}} q_{ijO} \cdot \beta
\end{aligned}
\tag{3.18}
$$

3.3.2 Separate transit-parking policy

In this subsection, we analyze which transit fares should be charged by the transit operator to minimize the joint operating deficits while keeping fixed the parking fees to their current value (i.e., 1.2€ per

parking place in zones 1 and 2, and free parking in the remaining zones).

This analysis was developed by optimizing the transit fares so that the transit operator minimizes its operating deficits, considering three (policy) scenarios. The maximum mobility loss represents the proportion of the current number of lost trips that is incremented to this amount, which defines the value of Q_{min} considered in equation (3.13):

Scenario S_1: a single transit fare for every transit trip, with infinite maximum mobility loss;

Scenario S_2: a single transit fare for every transit trip, with 5% of maximum mobility loss;

Scenario S_3: two different transit fares, one for the set of trips with origin and destination in the city center (Region 1) and another for the remaining trips, assuming that the value assigned to the former is smaller than the value of the later. The maximum mobility lost is set to 5%.

A transit fare of 0.98€ is the result of applying the model to Scenario S_1, corresponding to a transit modal-share of 35% and a transit operator operating deficit of 6.9×10^3€ (Table 3.4). When comparing these results with the current situation previously detailed, an increase on the transit fare is observed, justifying the decrease on the transit modal share. The joint operating deficits change from its current value of 3.67×10^6€/day to 1.91×10^6€/day (Table 3.3 and 3.4, respectively). Similar conclusions can be drawn for the social cost of this scenario, improving from its current value of 41.24×10^6€/year to 40.64×10^6€/year (Table 3.3 and Table 3.4, respectively).

In Scenario S_2, where the number of lost trips is bounded to a maximum increase of 5% of its current value (i.e., increasing 5% of the current lost trips of 16.8×10^3, which is equal to 17.6×10^3), the optimum transit fare becomes 0.83€ and the transit modal-share increases 3% when comparing this Scenario S_2 to Scenario S_1. The operating deficits increase from 1.91×10^6€/year on Scenario S_1 to 2.52×10^6€/year on Scenario S_2 (Table 3.4). This trend is observed when comparing the social costs, where a change from 40.64×10^6€/year to 40.87×10^6€/year occurs.

Table 3.4: Optimum solution features for Scenarios S_1 and S_2.

Scenario		S_1	S_2
Maximum mobility loss (%)		∞	5
Transit fare (€)		0.98	0.83
Parking fee (€)		1.2	1.2
Operating deficit (10^6 € /year)	Transit	1.91	2.52
	Parking	-2.37	-2.23
	Total	-0.46	0.29

Social costs (10^6 €/year)		40.64	40.87
Number of trips (10^3/day)	Transit	19 (35%)	20.5 (38%)
	Parking	34.6 (65%)	33.6 (62%)
	Total	53.6	54.1
Parking occupation (%)	Zone 1	99	96
	Zone 2	100	96

Adding the possibility of having two different transit fares according to the transit zones $r_1, r_2 \in \mathbf{R}$, the results shown in Table 3.5 are obtained for Scenario S_3. It must be highlighted the similarity between these results and the ones obtained for Scenario S_2. In fact, the transit fare of Scenario S_2 was optimized to take the value 0.83€, which is close to the transit fares obtained in Scenario S_3 (0.79€ and 0.84€ for transit zones 1 and 2, respectively). The possibility of having two different transit fares slightly increases the transit operator's operating deficits when compared to the Scenario S_2, but the differences on the modal-shares of the two scenarios can be neglected due to being minimal.

Table 3.5: Optimum solution features for Scenario S_3.

Scenario		S_3
Maximum mobility loss (%)		5
Transit fare (€)	Transit zone 1	0.79
	Transit zone 2	0.84
Parking fee (€)		1.2
Operating deficit (10^6 € /year)	Transit	2.5
	Parking	-2.23
	Total	0.27
Social costs (10^6 €/year)		40.85
Number of trips (10^3/day)	Transit	20.5 (38%)
	Parking	33.6 (62%)
	Total	54.1
Parking occupation (%)	Zone 1	96
	Zone 2	96
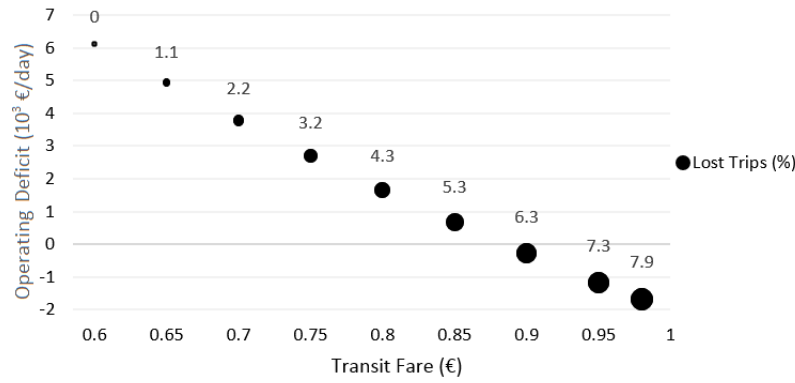
Figure 3.4: Operating deficit features for different transit fares without bounding the number of trips that lost.

As expected, if the transit fare is increased, the deficit of the transit system decreases as well as the trips that are made by transit or by car. For instance, to achieve a balanced transit structure, where no profit or deficit occurs, the transit fare must be increased from the current 0.6€ to 0.89€. This increase leads to an increment of 6% in the number of trips that are not made for being unaffordable in the passengers' perspective.

These results make possible to verify the existence of an extreme point solution as a result of parking capacity constraints, because the increase of transit fares increases the transit generalized costs, which increases the number of car trips. This increase origins parking capacity problems, where parking demand is higher than the parking supply, leading to infeasible solutions. This is observed by the infeasibility of the solutions achieved if the transit fare is higher than 0.98€ (the maximum value achieved in Figure 3.4), even when the constraint regarding the maximum value of lost trips is dropped.

3.3.3 Integrated transit-parking planning

We will now focus on how the integration of parking and transit systems in the hypothetical city impacts the financial performance of both systems. This was developed by optimizing the transit fares and the parking fees, attempting at reducing the subsidization level of the transit operator by incorporating the parking operator revenues as transit funds, while keeping fixed the levels of supply provided by the two operators (i.e., transit supply and parking capacity). This analysis considers the following 3 scenarios for the pricing configurations, with an upper bound for the trips that are lost (i.e., Q_{min}) equal to a maximum increase of 5% of the current number of lost trips in equation (3.13):

Scenario I_1: single transit fare and a single parking fee;

Scenario I_2: single transit fare and two parking fee levels;

Scenario I_3: two different transit fares - one for the set of trips with origin and destination in the city center (Region 1) and another for the remaining trips, assuming that the value assigned to the former is smaller than the value of the later - and two parking fee levels;

We start by analyzing Scenario I_1, where it is assumed a single transit fare and a single parking fee level to be optimized so that the number of lost trips remains below 5% of its current value, as previously considered. The results achieved by the model are displayed in Table 3.6.

Table 3.6: Optimum solution features for Scenario I_1.

Scenario		I_1
Maximum mobility loss (%)		5
Transit fare (€)		0.88
Parking fee (€)		1.14
Operating deficit (10^6 €/year)	Transit	2.33
	Parking	-2.11
	Total	0.22
Social cost (10^6 €/year)		40.73
Number of trips (10^3/day)	Transit	19 (35%)
	Parking	34.6 (65%)
	Total	53.6
Parking occupation (%)	Zone 1	99
	Zone 2	100

A decrease in the parking fee to 1.14€ is observed for this Scenario I_1, when compared to the current price of 1.2€, together with an increase on the transit fare from the current 0.6€ to 0.88€. These changes affect the modal shares, with transit losing 5% of its current ridership to car (Table 3.3 and Table 3.6, respectively). Although the total operating deficit remains positive (0.22×10^6€/year), there is a decrease of 36% on the transit operating deficits (from the current 3.67×10^6€/year to 2.33×10^6€/year, see Table 3.3 and Table 3.6, respectively). On the parking side, a decrease on the operating deficits is reached, changing from the current -2.01×10^6€/year to -2.11×10^6€/year (Table 3.3 and Table 3.6, respectively).

Comparing the results for Scenario I_1 (Table 3.6) with the results achieved for Scenario S_2, where only the transit fare is optimized (Table 3.4), an increase in the transit fare is observed as well as an expectable loss of passengers of 1 %, and a decrease of the parking fee. This new set of prices decreases the generalize cost of car and increases the generalized cots of transit, which justifies the changes on the modal-shares. As expected, changes on the joint operating deficits are also observed, decreasing from 0.29×10^6€/year to 0.22×10^6€/year. This is a direct consequence of decreasing by 7.5% the transit operating deficits (from 2.52×10^6€/year to 2.33×10^6€/year), as opposite to increasing the operating deficits of the parking operator from -2.23×10^6€/year to -2.11×10^6€ /year (Table 3.5 and 3.6, respectively). This increase is mainly justified by the decrease of parking fees (from 1.20€ to 1.14€), since the number of car trips increasing from 33.6 thousand to 34.3 thousand is not enough to accommodate the difference on the parking operator's gains achieved by collecting parking fees. Note that the parking capacity limits are

almost met in the two inner zones in this first Scenario (Table 3.6).

We proceed by analyzing scenarios I_2 and I_3, with results shown in Table 3.7. By increasing the number of parking fees to two instead of one (Scenarios I_2 and I_1, respectively), a slight impact on the single transit fare's optimum value occurs and it changes from 0.88€ to 0.87€, while the parking fee changes from 1.14€ to 1.11€ and 1.20€ (zones 1 and 2, respectively). A similar trend is observed in Scenario I_3, where the transit fares are 0.86€ and 0.89€ for transit zones 1 and 2 (see Figure 3.6), respectively, and parking fees for zones 1 and 2 are equal to 1.12€ and 1.15€, respectively. These two scenarios generate modal-shares and joint operating deficits similar to the ones generated by scenario I_1. The transit share of 35% in Scenario I_1 is increased to 37% and 36% for Scenarios I_2 and I_3, respectively, whereas the joint operating deficits change from 0.22×10^6€/year to 0.21×10^6€/year for Scenarios I_2 and I_3.

Table 3.7: Optimum solution fea1tures for Scenarios I_2 and I_3.

Scenario		I_2	I_3
Maximum mobility loss (%)		5	5
Transit fare (€)	Transit zone 1	0.87	0.86
	Transit zone 2		0.89
Parking fee (€)	Parking zone 1	1.11	1.12
	Parking zone 2	1.2	1.15
Operating deficit (10^6 €/year)	Transit	2.36	2.31
	Parking	-2.14	-2.10
	Total	0.22	0.21
Social costs (10^6 €/year)		40.76	40.71
Number of trips (10^3/day)	Transit	20 (37%)	19.7 (36%)
	Parking	34.1 (63%)	34.3 (64%)
	Total	54.1	54
Parking occupation (%)	Zone 1	100	100
	Zone 2	97	99

These analyses were developed with a maximum value for the mobility that is loss by the population, which is defined by setting an upper bound equal to adding 5% on the number of lost trips in the current scenario. We now aim to identify the trade-off between the collected operating deficits and the number of lost trips. This analysis is developed by fixing the transit fare values between 0.6€ and 2.8€ and optimizing the parking fee levels without considering any restrictions towards the number of lost trips. These results are displayed in Figure 3.5, where the lost trip proportions represent the proportion of trips that are lost towards their current value of 16.8×10^3/day (Table 3.3). E.g., when the transit fare is set to 0.7€, the optimization model returns that the optimum parking fee value is 2.31€ and a loss of 19.07×10^3/day that corresponds to an increase of 13.5% of the lost trips, which is the value shown in Figure 3.5.
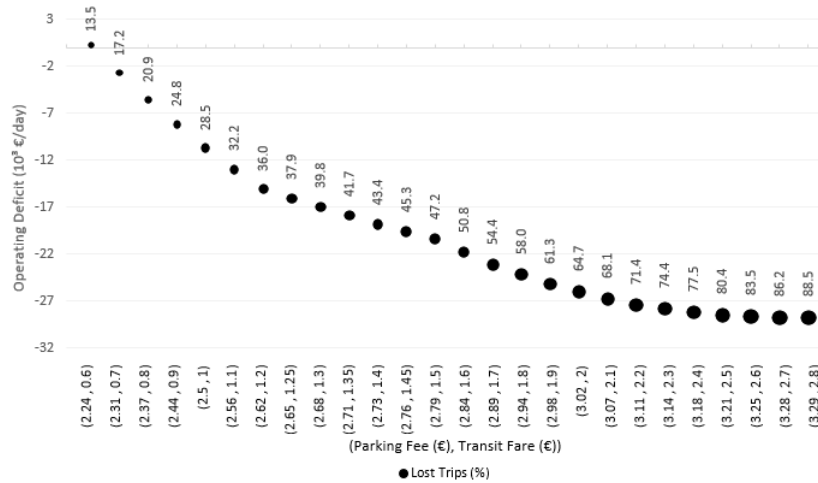
Figure 3.5: Optimal solutions features for a single parking fee and lost trips while varying the values of the single transit fares.

As previously pointed out, there is a strong relationship between having a self-financing system and the social responsibility of offering an affordable service. This is shown in Figure 3.5, where by setting the transit fare to 0.6€ and the parking fee to 2.24€ the system collects near zero joint operating deficits. In this case, an increase of 13.5% of the current lost trips is achieved, which corresponds to about 26.5% of the total trips (in the current pricing situation, this loss is about 23%). This increase on the number of lost trips is more reasonable than the increase required to achieve a transit system that is able to cover its own expenses (Figure 3.4).

The relationship between the total number of trips that are made and the values of both transit fares and parking fees, which is also shown in Figure 3.5, highlight the main reason why this lost trip "mode" was added into the demand logit model. In fact, if prices are too high, some potential trips will not be made because the trip's utility does not worth its total generalized cost.

It is worth noting that, even in extreme cases with increases of prices of more about 150% towards the current pricing (e.g., a transit fare of 1.5€ and a parking fee of 2.79€), the share of lost trips is 33.5% against the current 23.4%. This smooth change is related to the generalized travel costs included in the logit function. Transit fares and parking fees are only a fraction of those costs, therefore increasing them does not necessary mean that the generalized travel costs will increase in the same proportion. By way of example, let us consider a car trip with a generalized cost of 5€, where 1.2€ correspond to parking fees and 3.8€ to the remaining costs. If an increase of 170% occurs in the parking fee (to around 3.3€), the total generalized cost of the trip will increase to 7.1€, which corresponds to a 40% increase over the original generalized cost.

3.4   Model Solving

The developed optimization model results in a combinatorial-nonlinear formulation. These characteristics are generally quite challenging to cope with (Burer and Letchford, 2012; Hemmecke et al., 2010), and there a few software systems able to handle these types of formulations. According to the literature, BARON (Tawarmalani and Sahinidis, 2005) is one of the most suitable such systems (Burer and Letchford, 2012; Bussieck and Vigerske, 2014). This software implements a global optimization approach based on polyhedral branch-and-cut algorithms (Kesavan and Barton, 2000).

To get some insights on the computation effort of BARON software, we developed a computational study that related the size of problem instances (i.e., the number of zones) with the computation time that BARON software took at finding the optimum combination of prices. This study was developed considering 10 randomly-generated instances for each considered size of problem instances (between 6 and 30 zones), and the pricing schemes for two scenarios:

1) a single transit fare and a single parking fee level;

2) a single transit fare and two parking fee levels.

The average computation times of the solving the model for the randomly generated problem instance according to their size are shown in Figures 3.6 and 3.6 for scenarios #1) and #2), respectively.
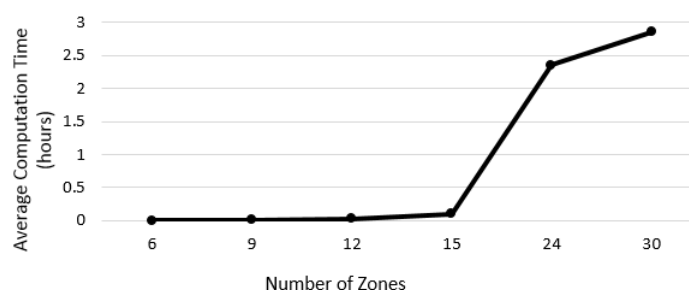
Figure 3.6: Computational effort for Scenario # 1).
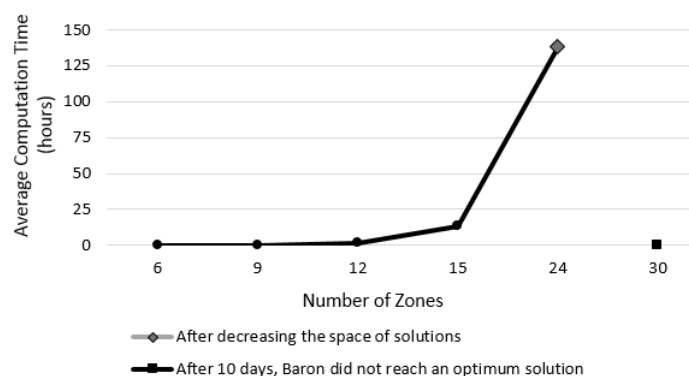
Figure 3.7: Computational effort for Scenario # 2).

As expected, if the size of the problem increases, the time required to find the optimal set of prices also increases due to increase of the model's complexity. This conclusion is observed regardless the pricing configuration. However, if two parking fee levels are considered, the effort is strongly higher than if a single parking fee level is considered, even in instances with smaller sizes. E.g., for instances with 15 zones, the average computation time with a single parking fee level, i.e. scenario #1, is close to zero hours (Figure 3.6) while this value increase to around 13 hours if two parking fee levels are considered, i.e. scenario #2 (Figure 3.7). This substantial growth is not only justified by adding a parking fee level but also by requiring the assignment of each parking fee level to each one of the 15 zones while ensuring that the parking capacity restrictions are met. When the size of the problem instances is increased to 24 zones, a substantial rise is observed on the computation time of scenario #1 when compared to smaller problem instances. This is why the size of the space of solutions for problem instances with size greater or equal 24 zones was decreased when optimizing the prices for scenario #2. However, this change was not enough to deal with problem instances with 30 zones in a reasonable amount of computation time (Figure 3.7).

Considering these results, we argue the need of developing a heuristic to find the optimal solutions for large problem instances, with and without single transit fares and single parking fee levels. This consideration will be address in our future work.

3.5   Case Study: Coimbra

In this section, we explore and analyze the application of the developed optimization model to the municipality of Coimbra. We start by detailing the settings of the municipality taking into account the data of a mobility survey conducted in Coimbra during 2009. This is followed by analyzing the possibility of the transit system to exclude the need of subsidization by adjusting the transit fares and parking fees, where the revenues of the parking operator cover the deficits of the transit operator.

3.5.1   Settings

The municipality of Coimbra has 156,000 inhabitants with a total area of 316.8 km^2, which was divided into 25 zones (Figure 3.8). This zonation was based on the geographic boundaries of the 18 parishes that compound the municipality of Coimbra, and their division considering the similarities of the social characteristics of the population observed through the data gathered by the mobility survey. For simplicity, it was assumed that all trips had their starting and their ending in a centroid designed in each zone, while the effort required to reach this geographic place from any other geographic place in the zone is negligible. The centroid of each zone was designed as the geographic location closest to all departures and arrivals trip's locations in each zone. The 25 zones are also grouped into 2 main regions, as

shown in Figure 3.9, where Region 1 represents the center of the municipality and Region 2 represents the periphery.
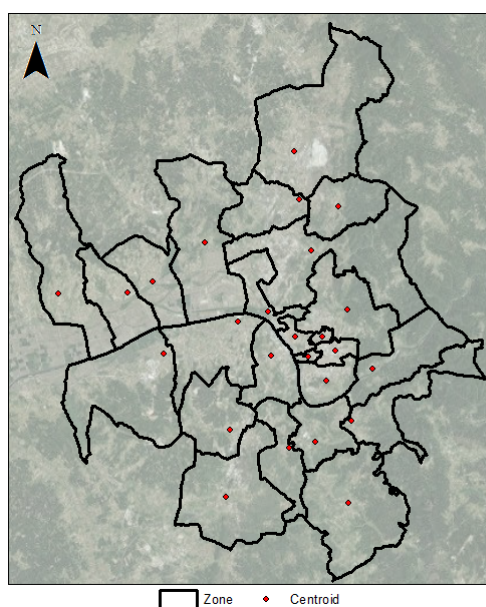
Figure 3.8: Division of the municipality of Coimbra in 25 zones.

The transit operator that manages the transit system running in Coimbra charges an average of 0.6€ per transit trip, according to the accounting reports made by that same operator (SMTUC, 2013). The parking fees charged to parking users are estimated based on the data collected by the mobility survey, leading to 4 parking fee levels ($\{0.01, 0.02, 0.14, 0.17\}$) distributed across the 25 zones of the municipality as displayed in Figure 3.9. These values are the result of dividing the total number of cars parked in a zone by the total parking fee revenue collected in that same zone.
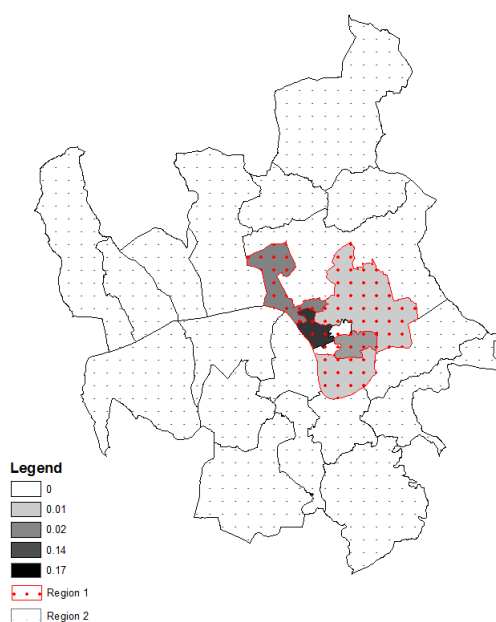
Figure 3.9: Current average parking fees per zone.

According to the survey conducted in Coimbra, an average of 153,200 daily trips are done by car and by transit in the municipality between all the OD pairs and excluding the trips with the same origin and destination. These trips are split between transit trips (25%) and car trips (75%), with a car occupancy rate equal to $\tau = 1.2$ passenger per vehicle. The total number of trips that would be done if both transit fares and parking fees were zero was estimated with an extrapolation of the surveys' data. This procedure yielded a total of 157,400 potential trips during a regular day if both parking fees and transit fares were set to zero.

The unit travel costs per travelled km of transit was estimated to be 0.7€/km, whereas the unit cost per travelled km by car was estimated to be 0.5 €/km. These values were projected based on the survey's data, the reports of the transit operator SMTUC and by the Portuguese Statistical Institute (INE, 2015). The average speed for each mode in Coimbra was set to 35 km/h and 17 km/h for car and transit, respectively; the unit cost of time was set to 8€/h, the discomfort cost of transit was considered to be 0.23€/km and the vehicle costs linked to car were set to 0.324€/km. Finally, the parameters of the logit function were estimated based on the insights provided by the survey, taking the values $\theta = 0.7$ and $\beta = 5.5$.

We estimated that the transit operator had a deficit of 6.31×10^6€/year (SMTUC, 2013), which includes the necessary adjustments in what concerns the trips reported by the transit operator and the trips included in this analysis. With these reports it was also stated the relationship between the revenues and the operational costs of the transit operator, where the latter is the double of the former in their absolute values, and the relationship between the fixed and the variable operating costs. In this sense, it was estimated that the unitary variable operating cost was 0.1 €/pax·km (i.e., $M_B^V = 0.1$). In what concerns the estimation of the operating deficits of the parking systems, we took into consideration that the operating costs of the parking systems are half of revenues collected by the parking operator. In this sense, it was acknowledge a negative operating deficit of 0.52×10^6€/year, which was estimated by having a variable operating cost of 0.01€/pax (assuming that 70% of the operating costs are fixed).

Table ?? summarizes the current situation for the municipality of Coimbra taking into account the required adjustments. We highlight that the deficits of the transit operator are currently subsidize by the city council and that the profit collected by the parking operator is not enough to cover the amount that is subsidized, i.e., the joint operating deficits remain positive.

Table 3.8: Current situation.

Transit Fare (€/trip)		0.6
Parking fee (€/parking)		$\{0.01, 0.02, 0.14, 0.17\}$
Operating deficit (10^6 €/year)	Transit	6.31
	Parking	-0.52

Total	5.79	

	Transit	38.6 (25%)
Number of trips (10^3/day)	Parking	114.6 (75%)
	Total	153.2

3.5.2 Integrated transit-parking planning solutions

We aim at providing insights to infer if the integration of transit and parking is a good option to decrease the subsidize levels required by the transit operator in Coimbra (as shown in Table **??**). To such end, the following 3 scenarios are considered:

Scenario C_1: a single transit fare and a single parking fee;

Scenario C_2: a single transit fare and two parking fee levels;

Scenario C_3: a single transit fare and a single parking fee such that the joint operating deficits are as close as possible to zero.

The optimal pricing schemes for each one of these 3 scenarios are shown in Table 3.9, along with the main features achieved by setting the optimal pricing schemes for each scenario. The distribution of the parking fee levels achieved in Scenario C_2 for each zone is detailed in Figure 3.10.

Table 3.9: Optimum solution features for each scenario.

Scenario		C_1	C_2	C_3
Transit fare (€)		2.3	2.3	0.2
Parking fee (€)		1.9	1.5, 2.1	0.8
	Transit	-7.78	-8.24	11.29
Operating deficit (10^6 €/year)	Parking	-35.17	-35.47	-11.29
	Total	-42.95	-43.71	0
	Transit	31.1 (21%)	31.9 (21%)	57.6 (38%)
Number of trips (10^3/day)	Parking	118.3 (79%)	117.4 (79%)	95.0 (62%)
	Total	149.4	149.3	152.6
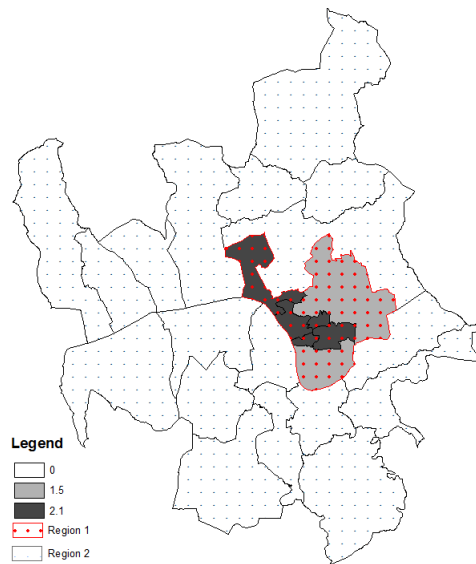
Figure 3.10: Optimum parking prices according to zones for Single optimum Transit fares and two optimal parking fees.

Transit fares change from 0.6€/trip to 2.3€/trip on both Scenarios C_1 and C_2, and the parking fees are increased to 1.9€/parking, and 1.5€/parking and 2.1€/parking on Scenarios C_1 and C_2, respectively. These changes on prices lead to a number of trips that are not made higher than in the current situation (Table 3.9 and **??**, respectively), and a decrease on the transit modal share from its current proportion of 25% to 21% in both Scenarios C_1 and C_2. The integration of transit and parking lead to highly profitable transit and parking systems, if no restrictions are considered for the operating deficit (as in Scenario C_3), where the joint deficits are decreased from the current value of 5.79 $\times 10^6$€/year to -42.95$\times 10^6$€/year and -43.71$\times 10^6$€/year in Scenarios C_1 and C_2, respectively. In the case of the operating deficits of the transit operator, the values change from 6.31$\times 10^6$€/year to -7.78$\times 10^6$€/year and -8.24$\times 10^6$€/year in Scenario C_1 and C_2, respectively. The difference between the joint operating deficits of the parking operator is almost negligible for these two scenarios, decreasing from the current operating deficits of -0.,52$\times 10^6$€/year to -35.17$\times 10^6$€/year in Scenarios C_1 and C_2.

The results achieved for Scenario C_3 are softer towards the current situation (Table 3.8) than the results of Scenario C_1 and C_2. If on one hand, a decrease occurs in the transit (from 0.6€ to 0.2€), on the other the parking fee is increased from the current values (Figure 3.10) to 0.8€. It is intended with this scenario to create awareness regarding the difference between determining prices that minimize the operating deficits, as it was aimed in Scenario C_1 and C_2, and determining prices that create closest to zero operating deficits. As expected, this Scenario C_3 improves the transit modal-share while it decreases the proportion of car trips. In general, the total number of trips that are made is increased in Scenario C_3 when compared to Scenario C_1 and C_2, even though the total number of trips that are made remain below the number of trips currently done (Table 3.9 and 3.8, respectively).

It must be noted that the model considers only the average values of the transit fare and parking fees.

As before, the value added to the current transit fare may be split among several transit fares possibilities in a way that minimizes the public reaction this average increase. In the case of parking, these average parking fees do not reflect that parking will be more expensive for those already paying for this good. In fact, it might induced a decrease on both personal parking costs while increasing the number of paid parking spots.

In summary, the solutions of these three scenarios allow the municipality to stop subsidizing the transit operator, either because the transit operator collects enough revenue to cover its operational costs (Scenarios C_1 and C_2) and no further funding forms are needed, either because the parking operator collects the revenues needed to cover the deficits of the transit operator (Scenario C_3).

3.6 Conclusion

This chapter presented an optimization model to assist local governments in the implementation of integrated transit-parking policies, where transit fares and parking fees are optimized so that transit systems' deficits are minimized by using the revenues collected by parking operators as transit operators' source of funding.

We started by analyzing the results of applying the model to an hypothetical city, where it was assessed the improvements of jointly managing the prices of both transit and parking systems when compared to only managing the prices of the transit operator (i.e., transit fares). This was followed by applying the model to the municipality of Coimbra, showing how transit and parking systems can became highly profitable by optimizing their transit fares and parking fees. These results were detailed taking into account three different scenarios, showing that transit systems can became self-sufficient in a financial stand point if, for instance, their transit fares are increased from current value of 0.6€ to 2.3€ per trip along with an increase on the parking fees from their current value to 1.9€, as detailed in the first scenario explored in subsection 3.5.2. These substantial increases on the prices applied by each operator might not be not be well accepted by the users. Taking this into account, the transit fares and parking fees were optimized to achieve a non-profit and without deficit integrated transit and parking system, resulting on smother changes on the prices and having the parking operator as the only funding source to cover the transit deficits.

Based on these results, it might be advocate an excessive emphasis on the deficit perspective. To overcome this, and as future work, changes on the objective function might be considered, e.g. the maximization of social welfare or transit share, given a threshold for the operating deficit; or adding additional social welfare measures to the optimization model. Furthermore, transit supplies considerations and the parking capacity adjustments should also be consider, so that operating costs can also be optimize from a supply point of view. The need to develop an efficient heuristic to solve the model for larger cities

should also be considered, where the existence of having exact solutions for midsize cities should to assess the quality of the solutions provided by the heuristic that will be developed. Finally, the accuracy of the model regarding speed and congestion variations and their inter-dependency towards modal-shares has been address by embedding macroscopic fundamental diagrams in an optimization model capable of handling pricing schemes so that the joint deficits are minimize, as detailed in the following chapter.

Chapter 4

Integration of an aggregated dynamic traffic model with optimization techniques for strategic transit-parking planning

4.1 Introduction

The role of transit (public transport) as essential for the mobility in urban areas is well recognized and world-wide promoted. As a matter of fact, public transport systems are characterized as attractive solutions to overcome the current environmental drawbacks linked to transport (e.g., fossil fuel consumption, pollutant emissions, and traffic congestion) without constraining people's mobility (Schiller et al., 2010; Miller, 2014). Additionally, transit systems are expected to be affordable, reliable and safe. These expectations of users towards the service provided by transit operators usually lead to severe financial problems, because the costs of providing such service are usually not fully recovered by the revenues collected (mostly in the form of fares) (APTA, 2003). Therefore, government and/or public agencies intervene to balance deficits, playing the important role of being the source of transit funding through subsidization (Vuchic, 2005).

To balance these deficits and to decrease transit's dependency on subsidies, we introduce an approach where transit and parking operators cooperate by using parking fees to fund transit deficits. Having this purpose in mind, an optimization model is developed for assisting local governments to establish pricing policies for the transit networks and the parking areas under their control, directly or indirectly (through concessions). This optimization model has a mixed-integer nonlinear formulation and embeds a network level based on the macroscopic fundamental diagram (MFD), which we called MFD model, with the aim of maximize the joint profit of both transit and parking operators by optimizing their respective transit fares and parking fees. This MFD model provides insights into the speed and congestion conditions in the road network (Geroliminis and Daganzo, 2008). We assumed the division of a city into zones with known dynamic demand between each two zones. Recently different clustering techniques

have been developed to partition a city in homogeneous clusters with well-defined MFDs (see for example Ji et al., 2014; Saeedmanesh and Geroliminis, 2016; Lopez, 2017). Each trip in these zones can be made by car (when available) or by transit, or not made if the (generalized) travel costs are considered too high by the traveler. These modal choices are described by two logit models of the generalized cost of each alternative, to distinguish the behavior of users when car is or is not available.

To the best of our knowledge, the integration of transit and parking through pricing features has been addressed relatively rarely in the literature. In general, this integration is approached through road pricing schemes (e.g. Tsekeris and Voß, 2009), and is usually analyzed in the light of micro-economic models, with the main concern of maximizing social welfare features of competitive transit/highway systems (e.g. Tabuchi, 1993; Huang, 2000; 2002; Danielis and Marcucci, 2002; Mirabel and Reymond, 2011; Gonzales and Daganzo, 2012). However, road pricing schemes are rarely applied in the real world due to their unpopularity and to their difficult implementation (Schlag and Schade, 2000; Jaensirisak et al., 2005; Ison and Rye, 2005; Santos and Fraser, 2006). This is one of the reasons why we decided to use parking fees instead of road pricing schemes to help dealing with transit deficits. Despite being considered "imperfect" substitutes of first-best pricing strategies (Verhoef et al., 1995), parking fees have been proven to be a good alternative to road pricing schemes (Inci, 2015). This is justified not only by their capability of influencing transit ridership and city dynamics, but also by being more easily accepted by users and more straightforward to implement (Dueker et al., 1998; Shoup, 2005; Balcombe et al., 2004; Albert and Mahalel, 2006; Marsden, 2006; Beirão and Cabral, 2007).

The optimization model presented in this chapter is the first optimization model in the literature that embed MFD features, at least to the best of our knowledge. This is also the first work that integrates this relationship between flow and density in a model that optimizes the transit fares and the parking fees, so that the deficits of the transit operator can be minimized by jointly maximizing the profits of transit and parking operators. Other distinctive features of our work is the inclusion of traffic directions in the MFD model, which not only influences the traffic network conditions but also the generalized costs of each alternative; the inclusion of cruising for parking, either in the MFD model either by including its cost into the car generalized cost, which influences the mode-choice; and the possibility of having different routes for different time periods, which are mainly dependent on the generalized cost of each alternative. Due to the complexity of having a mixed-integer-non-linear optimization model embedded with an MFD model, a greedy algorithm was developed to solve the optimization model. This algorithm also includes a traffic-equilibrium algorithm that defines the relationship between traffic dynamics and the modal-choices for known transit fares and parking fees. The capability of this optimization model to accommodate the reality is provided by application to the case study inspired in the Portuguese medium-sized city of Coimbra, where the lack of real-world applicability is usually a short come of the models that attempt at solving the financial problems of transit systems, as previously mentioned.

This chapter is organized as follows. Section 4.2 presents the modeling framework, where a detailed explanation of the optimization model is developed. The solving method, which comprehends a traffic-equilibrium algorithm and a greedy algorithm developed in the light of the benign configuration of the model's feasible region, is detailed in Section 4.3. The case study inspired in Coimbra, a medium-sized city in Portugal, is presented in Section 4.4. Insights towards the MFD model, the application of the developed optimization model, and conclusions regarding the developed solution method are presented in Section 4.5 for the detailed case study. A discussion on the suitability of this approach to decrease the subsidize levels of transit systems and further conclusions are developed at the end of the chapter.

4.2 Modeling Framework

The main goal of this work is to develop an optimization model for managing transit and parking systems in an integrated way, where transit fares and parking fees are optimized to maximize the joint profits of transit and parking operators. This optimization model has a mixed integer nonlinear formulation and includes an MFD model that provides a relationship between vehicle density and traffic flow in urban areas with small spatial density heterogeneity considering the number of vehicles (accumulation) in the traffic network (Geroliminis and Daganzo, 2008). MFD modeling has been integrated in a few traffic management frameworks, such as perimeter control (see for example Kouvelas et al., 2017; Ramezani et al., 2015; Haddad, 2015), congestion pricing (Simoni et al., 2015; Zheng and Geroliminis, 2016; Zheng et al., 2016), network design (Knoop et al., 2014; Ortigosa et al., 2015), and route guidance (Sirmatel and Geroliminis, 2017; Yildirimoglu et al., 2015; Leclercq et al., 2015).

Trips between each pair of zones can be made either by car (by people who can use this mode) or by transit, or not made at all if (generalized) travel costs are considered too high by the traveler. Modal choice in the city is described by two logit models. The first one describes the behavior of people who cannot use a car, and distinguishes the transit and the non-travel alternative. The second one applies to people who can use a car, and differentiates between transit, car and the non-travel alternative. For both logit models, choices depend on the modes' travel costs. In the case of transit, time costs, access costs, discomfort costs and transit fares are considered. In the case of car, these costs consist of in-vehicle time, vehicle depreciation, fuel and maintenance, cruising for parking costs and parking fees.

The optimization model has the objective of maximizing the joint profit of both transit and parking operators, considering transit fares and parking fees as decision variables of the model. The transit fares are defined according to transit zones, which are zones grouped by OD-pairs; that is, the amount paid by transit users depend on the origin and destination of their trips. With respect to parking fee structure, we consider that parking fees are applied considering parking zones that have different parking fee levels. The parking zones configuration is determined within the model. A detailed explanation of the

ingredients and the formulation of the optimization model are given in subsection 4.2.1.

The MFD model that is embedded in the optimization model is detailed in subsection 4.2.2. The MFD provides a relationship between network space-mean vehicle density and flow in urban areas with small spatial density heterogeneity. The MFD model integrates the traffic dynamics of car and transit together with route choice. While the traffic dynamics have been studied before in the MFD literature, their integration in an optimization framework with iterative procedure for equilibrium conditions is new. For example, multi-region car-only MFD traffic is modeled in Ramezani et al. (2015), route choice is modeled in Yildirimoglu et al. (2015), and transit-car interactions are modelled in Zheng and Geroliminis (2013; 2016).

For simplicity, only car travel demand is included as exogenous factor at contributing to the number of vehicles (accumulation) in each zone. For the same reason, a unimodal, continuous and concave MFD function is estimated for each region, considering characteristics of the road network (such as free flow speed, jam density and congested wave speed) and the utilization of part of the road capacity by transit vehicles. The MFD model we use is directional, i.e, it accounts for the accumulation observed for the two different directions of the road network. In this sense, the expected average speed to travel from a point A to a point B might be different from the estimated average speed to travel in the opposite direction (i.e., from B to A). This is included in the model by considering that accumulation depends on the traffic direction, that are dependent on the trip routes. As we will show, this influences the distribution of congestion in the city compared to a non-directional model.

We considered two different time frames to acknowledge that mode choices are more stable than speed changes over time. Therefore, one wider time frame is used to update the modal shares according to their generalized travel costs and the routes followed by car users for each OD-pair. This time frame is discretized and it is used to set the different speeds, congestion levels and cruising for parking effects in the MFD model.

These constituents are explained in detail in the following subsections. We start by exploring the optimization model that determines the transit fare and parking fee values that maximize the joint profits. The MFD model is explained afterwards, together with its embedding into the optimization model.

4.2.1 Optimization model

In this subsection, we present the formulation of the optimization model by describing the model structure, notation and assumptions.

Let $\mathbf{Z}$ be the set of trip zones, whose elements are connected by an oriented road network, and the set of time periods $\mathbb{T}_1$. The demand, or the number of potential travelers that *intend* to make a given trip, from an origin o to a destination d with departure time during period t_1, assuming that transit fares and parking fees are zero, and it is denoted by $\hat{Q}_{od}(t_1)$. These demand values are then split according to

users having car as a potential travel mode or not. In this sense, the demand that has car as alternative is denoted by $Q_{od}(t_1)$, while the demand that do not have car as an option is represented by $\widetilde{Q}_{od}(t_1)$. Travelers have the possibility of choosing to do their trip by car (A), transit (B), or not making the trip, which is expressed as *no-travel* alternative (O).

The mode choice is described by two logit models of the generalized travel costs to distinguish the cases where car is or is not an alternative. The first logit model is given by equations (4.1) and accounts for the behavior of those to whom the car alternative is available, distinguishing between demand for transit trips ($q_{odB}(t_1)$), car trips ($q_{odA}(t_1)$) and no-travel ($q_{odO}(t_1)$). The second logit model is given by equations (4.2) and expresses the behavior of those who cannot chose to travel by car, distinguishing between transit trips ($\widetilde{q}_{odB}(t_1)$) and no-travel ($\widetilde{q}_{odO}(t_1)$). These equations (4.1) and (4.2) account for the generalized cost of each alternative m ($C_{odm}(t_1)$) and the sensitivity of potential travelers to these generalized costs, which is dependent on the possibility of choosing car or not (θ and $\widetilde{\theta}$, respectively).

$$q_{odm}(t_1) = \frac{e^{-\theta \cdot C_{odm}(t_1)}}{e^{-\theta \cdot C_{odA}(t_1)} + e^{-\theta \cdot C_{odB}(t_1)} + e^{-\theta \cdot C_{odO}(t_1)}} \cdot Q_{od}(t_1), \quad m \in \{B, A, O\}, o, d \in \mathbf{Z}, t_1 \in \mathbb{T}_1 \tag{4.1}$$

$$\widetilde{q}_{odm}(t_1) = \frac{e^{-\widetilde{\theta} \cdot C_{odm}(t_1)}}{e^{-\widetilde{\theta} \cdot C_{odB}(t_1)} + e^{-\widetilde{\theta} \cdot C_{odO}(t_1)}} \cdot \widetilde{Q}_{od}(t_1), \quad m \in \{B, O\}, o, d \in \mathbf{Z}, t_1 \in \mathbb{T}_1 \tag{4.2}$$

The generalized cost of traveling from o to go to d departing during period t_1 using transit is given by equation (4.3). This cost is the result of summing the in-vehicle time cost ($C^T \cdot \sum_{m \in R_{odB}(t_1)} (D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})$ $/V_{mB}^{t_1}(H_m^{R_{odB}(t_1)})))$, the access cost $AC_{od}(t_1)$, the discomfort cost $DC_{od}(t_1)$, and the transit fare p_{odB}.

$$C_{odB}(t_1) = C^T \cdot \sum_{m \in R_{odB}(t_1)} \left(\frac{D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})}{V_{mB}^{t_1}(H_m^{R_{odB}(t_1)})} \right) + AC_{od}(t_1) + DC_{od}(t_1) + p_{odB}, \quad o, d \in \mathbf{Z}, t_1 \in \mathbb{T}_1 \tag{4.3}$$

The in-vehicle time cost is calculated by multiplying the unit time cost C^T with the total in-vehicle time that is needed to go from zone o to zone d ($\sum_{m \in R_{odB}(t_1)} (D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})/V_{mB}^{t_1}(H_m^{R_{odB}(t_1)})))$. This total in-vehicle time is equal to summing the total in-vehicle time spent in each zone m, while departing at period t_1 from o to d along the shortest-time path using the transit network. This path is given by the ordered set of zones $R_{odB}(t_1)$ crossed in the trip and by the direction followed in each zone m of this shortest path, which is designated by $H_m^{R_{odB}(t_1)}$ from the set $\{1, 2\}$. The distance travelled in a zone m of this shortest time path depends on the traffic direction and is given by $D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})$. In particular, $D_{mB}^{t_1}(H_m^{R_{odB}(t_1)}) = 0$ if the selected least-cost path does not pass through zone m. Finally, the in-vehicle time spent in a given zone m that belongs to the shortest path $R_{odB}(t_1)$ with direction $H_m^{R_{odB}(t_1)}$ is obtained by dividing the distance travelled in that zone $D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})$ by the average transit speed $V_{mB}^{t_1}(H_m^{R_{odB}(t_1)})$, which is estimated by the MFD model.

The transit network results from grouping all the transit routes, i.e., all the paths followed by a transit vehicle that serves a defined and known sequence of zones. Typically, a transit route is made a certain number of times per period according to a pre-defined and fixed schedule (frequency). The frequency

of transit route during period t_1 is $F_k(t_1)$. $\hat{T}$ is the set of transit routes. The relationship between the shortest path selected by the user and the transit network is specified by the binary variable $N_{odlk}(t_1)$, which takes the value 1 if the shortest path from o to d with departure time period t_1 includes transit route k and passes through zone l. Otherwise, it takes the value zero. The user's transit frequency is the minimum frequency across all transit routes included in the user's path, i.e. $\min_{k \in \hat{T}, l \in \mathbf{Z}: N_{odlk}=1}\{F_k(t_1)\}$. This value will be useful to estimate the access costs and the discomfort costs assigned by users to a transit trip.

The access cost is the cost associated with the walking time to/from transit stops and the waiting time at the first transit stop. The first component of the access cost function given by equation (4.4) corresponds to the cost of the average walking time to and from the transit stop. This value is calculated by multiplying the unit walking time cost C^W with the average walking time D_{odB}^W. For the calculation of the waiting time, we followed the approach adopted by Tirachini et al. (2010), but assuming the perspective of time losses instead of time savings. The average waiting time cost at the transit stop and the schedule delay (i.e., the difference between the actual and the preferred transit departure time) are captured in parameter SW. It is also considered that the average waiting time is half of the headway (time between vehicles doing the same route in a transit system), if the headway is constant. Finally, if none of the transit departures scheduled by the transit operator occurs at the user's most preferred time, then this SW parameter also accounts for the schedule displacement penalty, which increases with increasing headway. Knowing that headways are inversely proportional to frequency, the total average waiting time is estimated by the second component of equation (4.4).

$$AC_{od}(t_1) = C^W \cdot D_{odB}^W + \frac{SW}{\min_{k \in \hat{T}, l \in \mathbf{Z}: N_{odlk}=1}\{F_k(t_1)\}} \tag{4.4}$$

The discomfort cost is the extra travel cost that users assign to transit travel time due to their different time perception when traveling by transit compared to traveling by car. The estimation of this cost is based on Corporation (1996), and its value is intrinsically related to the level of crowdedness of the transit vehicles. This level, for a given time period (say, t_1) is estimated by the transit occupation rate, i.e., the number of transit riders per zone m belonging to each transit routes k (i.e., $(q_{ijB}(t_1) + \tilde{q}_{ijB}(t_1)) \cdot N_{ijmk}$, $i, j \in \mathbf{Z}$) divided by the transit capacity provided by that route ($S^B \cdot F_k(t_1)$). Therefore, the difference between the perceived in-vehicle time of transit and car increases with the number of passengers using transit. The discomfort cost is then the result of multiplying the total in-vehicle time with the transit occupation rate calibrated by parameters ψ_B and ρ_B, as shown in equation (4.5). For simplicity, we do not explicitly consider the number of transit riders that travel inside the vehicle more than one period, even though they should be included in the number of passengers contributing to increase the level of crowdedness felt in a transit vehicle. Instead, we calibrate the parameters ψ_B and ρ_B so that this additional discomfort is included.

$$DC_{od}(t_1) = \psi_B \cdot C^T \cdot \sum_{m \in R_{odB}(t_1)} \left(\frac{D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})}{v_{mB}^{t_1}(H_m^{R_{odB}(t_1)})} \right) \cdot \left(\frac{\sum\limits_{i,j \in \mathbf{Z}} (q_{ijB}(t_1) + \widetilde{q}_{ijB}) \cdot N_{ijmk}}{S^B \cdot F_k(t_1)} \right)^{\rho_B}, o, d \in \mathbf{Z}, t_1 \in \mathbb{T}_1 \quad (4.5)$$

The costs of traveling by car are dependent on the route followed by the driver, which we take to be the same for each OD pair and departure time. However, two drivers going from the same origin to the same destination can follow different routes if their departure time is different. It is assumed that drivers follow the least-cost route. Let $R_{odA}(t_1)$ be the route selected by drivers to go from o to d in period t_1. The car speed and the travel distance are dependent on the route and the traffic direction. In this sense, the direction followed in zone $m \in R_{odA}(t_1)$ is given by $H_m^{R_{odA}(t_1)}$. The traffic direction variables take the values 1 or 2, in line with the transit traffic directions. Let $D_{mA}^{t_1}(H_m^{R_{odA}(t_1)})$ and $V_{mA}^{t_1}(H_m^{R_{odA}(t_1)})$ be the travel distance and the average car speed for zone $m \in R_{odA}(t_1)$, respectively. Like the average transit speed, the car speed $V_{mA}^{t_1}(H_m^{R_{odA}(t_1)})$ is a function of the vehicle density in each zone and is determined by the MFD model. The unit time cost is given by C^T and the cost related to vehicle depreciation, fuel and maintenance is given by C^D. Let $CP_d(t_1)$ be the costs of cruising for parking, i.e., the costs for drivers of, after arriving to their destination, looking for a parking place, and p_{dA} be the parking fee. Then the generalized costs of driving a car from o to d with departure time t_1 ($C_{odA}(t_1)$) include the time costs, the vehicle depreciation, fuel and maintenance costs, the cruising-for-parking costs and the parking fees, as expressed by equation (4.6).

$$C_{odA}(t_1) = C^T \cdot \sum_{m \in R_{odA}(t_1)} \left(\frac{D_{mA}^{t_1}(H_m^{R_{odA}(t_1)})}{V_{mA}^{t_1}(H_m^{R_{odA}(t_1)})} \right) + C^D \cdot \sum_{m \in R_{odA}(t_1)} \left(\frac{D_{mA}^{t_1}(H_m^{R_{odA}(t_1)})}{\tau_{od}} \right) + CP_d(t_1) + \frac{p_{dA}}{\tau_{od}}, o, d \in \mathbf{Z}, t_1 \in \mathbb{T}_1$$

$$(4.6)$$

While the time costs ($C^T \cdot \sum_{m \in R_{odA}(t_1)} \left(D_{mA}^{t_1}(H_m^{R_{odA}(t_1)}) / V_{mA}^{t_1}(H_m^{R_{odA}(t_1)}) \right)$) and the cruising-for-parking costs ($CP_d(t_1)$) are passenger related, the costs linked to vehicle depreciation, fuel and maintenance are vehicle dependent ($C^T \cdot \sum_{m \in R_{odA}(t_1)} \left(D_{mA}^{t_1}(H_m^{R_{odA}(t_1)}) / \tau_{od} \right)$), which justifies the normalization of these cost by the average number of people in a car including the driver, which is given by parameter τ_{od}.

The cruising-for-parking cost is a function of parking occupation and is expressed by equation (4.7) (Gallo et al., 2011). In this equation α_1 and α_2 are calibration parameters, S_d^A is the parking capacity of trip zone d and $V_d(t_1)$ is the average number of free parking places in zone d during period $t_1 \in \mathbb{T}_1$. This last value $V_d(t_1)$ is dynamically determined by the MFD model.

$$CP_d(t_1) = \alpha_1 \cdot \left(\frac{S_d^A - V_d(t_1)}{S_d^A} \right)^{\alpha_2}, \quad d \in \mathbf{Z}, t_1 \in \mathbb{T}_1 \quad (4.7)$$

Finally, the generalized cost of not traveling ($C_{odO}(t_1)$) accounts for the sensitivity of users towards changes on transit fares and parking fees, attempting to capture the maximum average value that users

are willing to spent to do a given trip.

The objective function is expressed in equation (4.8), and accounts for the joint profit collected by the transit operator and the parking operator. In this expression, the revenue collected by the transit operator is the result of the collected transit fares. The revenue of the parking operator is computed by the product between the number of car trips and the parking fee paid by each car. The depreciation, maintenance and operating costs of the transit system, M_B, and of the parking system, M_A, are assumed to be fixed (i.e., independent of the actual utilization of each system).

$$OF = \sum_{t_1 \in \mathbb{T}_1} \left(\sum_{o,d \in \mathbf{Z}} \left(q_{odB}(t_1) + \widetilde{q}_{odB}(t_1) \right) \cdot p_{odB} + \sum_{o,d \in \mathbf{Z}} q_{odA}(t_1) \cdot p_{dA} \right) - M_B - M_A \tag{4.8}$$

We assume that transit fares vary according to pre-defined transit zones. Each OD pair is assigned to exactly one transit zone r from the set of known transit zones $\hat{\mathbf{R}}$, whereas each transit zone may correspond to multiple OD pairs. Let $\widetilde{P}_{odr}$ be a binary variable that is equal to 1 if od belongs to transit zone r, and equal to zero 0 otherwise, and let $\hat{p}_{rB}$ be the transit fare for transit zone r. The transit fare from trip zone o to trip zone d (p_{odB}) is given by equation (4.9).

$$p_{odB} = \sum_{r \in \hat{\mathbf{R}}} \widetilde{P}_{odr} \cdot \hat{p}_{rB}, \qquad o,d \in \mathbf{Z} \tag{4.9}$$

The parking operator can charge a fixed number of different parking fee levels. The number of different parking fee levels $|\hat{P}_A|$ is assumed to be fixed whereas the parking fee level value $\hat{p}_{uA}$ corresponding to a level $u \in \hat{P}_A$ is optimized by the model. We also define the binary decision variable w_{ud} that takes value 1 if the parking fee level d is assigned by the model to trip zone d, and zero otherwise. A trip zone will be assigned with a parking fee level if and only $Y_d = 1$, otherwise users will park for free in that zone and this variable will be equal to zero ($Y_d = 0$). These features are included in the optimization model through constraints (4.10) and (4.11).

$$p_{dA} = \sum_{u \in \hat{P}_A} \hat{p}_{uA} \cdot w_{ud}, \qquad d \in \mathbf{Z} \tag{4.10}$$

$$\sum_{u \in \hat{P}_A} w_{ud} = Y_d, \qquad d \in \mathbf{Z} \tag{4.11}$$

Equations (4.12) ensure that the seating capacity of transit vehicles is not exceeded in the zones crossed by the transit routes. Equation (4.13) guarantees that a certain level of transit supply is provided to the users by setting an upper bound U_0 for the potential trips that are not made. This constraint also accounts for the need of providing affordable transit to those who do not have car as an alternative. Parameters ψ and $\widetilde{\psi}$ ensure that this concern is addressed by the model, and the value of U_0 should be set accordingly to these parameters.

$$\sum_{o,d \in \mathbf{Z}} \left(q_{odB}(t_1) + \widetilde{q}_{odB}(t_1) \right) \cdot N_{odlk} \leq S_B \cdot F_k(t_1), \qquad l \in \mathbf{Z}, k \in \hat{t}, t_1 \in \mathbb{T}_1 \tag{4.12}$$

$$\sum_{t_1 \in \mathbb{T}_1} \sum_{o,d \in \mathbf{Z}} \left(\psi \cdot q_{odB}(t_1) + \widetilde{\psi} \cdot \widetilde{q}_{odB}(t_1) \right) \leq U_0 \tag{4.13}$$

Finally, constraints (4.14) ensure that each trip zone with paid parking is linked to a parking fee level while constraints (4.15) state that transit fares and parking fees are non-negative.

$$w_{ud} \in \{0,1\}, u \in \widetilde{P}_A, d \in \mathbf{Z} \tag{4.14}$$

$$\hat{p}_{rB}, \hat{p}_{uA} \in \hat{\mathbb{R}}_0^+, r \in \widetilde{P}_B, u \in \widetilde{P}_A \tag{4.15}$$

The optimization model for this integrated transit-parking planning problem allows determining the values of decision variables $\hat{p}_{rB}$ and $\hat{p}_{uA}$ that maximize the joint profit of transit and parking operators (4.8) while ensuring that constraints (4.1)-(4.2) and (4.9)-(4.15) are satisfied. The traffic dynamics in the city is embedded in this model through the MDF model explained in the following subsection, which defines the variables related to transit and car average speeds for each zone, traffic direction and time period ($V_{mB}^{t_1}(H_m^{R_{odB}(t_1)}$ and $V_{mA}^{t_1}(H_m^{R_{odA}(t_1)})$, respectively) and the average number of free parking spaces in each trip zone and time period ($V_d(t_1)$).

4.2.2 Macroscopic Fundamental Diagram model (MFD model)

The MFD model embedded in the optimization model described in the previous subsection considers two geographical levels of analysis - zones and regions ($\mathbf{Z}$ and $\mathbf{R}$)-, and three levels of car traffic analysis. The first level (route level) determines car accumulations and traffic flows based on the routes chosen by drivers for the different trip OD pairs and departure time periods. In the second level, designated as zone level, the accumulations and flows at the route level are summed up for the different trip zones. This allows estimating the level of cruising for parking in each zone as a function of parking occupation rates. The average car speed is estimated at the region level. This third level relates the MFD function for each region with the car traffic flow and car accumulation in that same region, which are determined by summing the flows and accumulations for the zones included in each region. The interaction between these three levels is detailed in flow chart shown in Figure 4.1.
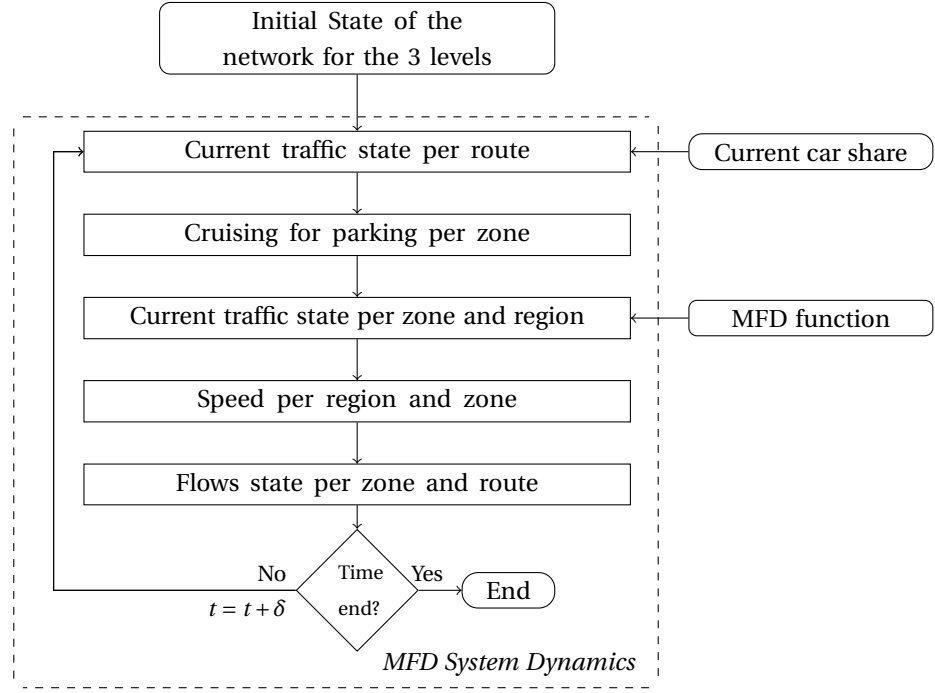
Figure 4.1: Flow chart of the MFD model.

The following relationships are the results of having two time frames, $\mathbb{T}_1$ and $\mathbb{T}_2$, which discretized one another (i.e., $\mathbb{T}_2$ is a discretization of $\mathbb{T}_1$). The total demand for a period $t_1 \in \mathbb{T}_1$ (i.e. $Q_{od}(t_1)$, or $\widetilde{Q}_{od}(t_1)$) is obtained by summing the demand for the periods $t_2 \in \mathbb{T}_2$ ($Q^*_{od}(t_2)$ or $\widetilde{Q}^*_{od}(t_2)$) that discretize $t_1 \in \mathbb{T}_2$, i.e., $Q_{od}(t_1) = \sum_{t_2 : t_2 \in t_1} Q^*_{od}(t_2)$. The number of car trips included as inputs of the MFD model for a period $t_2 \in \mathbb{T}_2$ results from the product of the logit proportion for the period in $\mathbb{T}_1$ that includes $t_2 \in \mathbb{T}_2$ by the total demand estimated for time period $t_2 \in \mathbb{T}_2$. This connection is described in equation (4.16). The route followed to travel from o to d with departure at period t_2 is assumed to be the same as the route chosen in the correspondent period t_1, i.e., $R_{odA}(t_2) = R_{odA}(t_1)$ if and only if $t_2 \in t_1$. The same is assumed for the traffic directions $H_m^{R_{odA}(t_1)}$.

$$q^*_{odA}(t_2) = \frac{e^{-\theta \cdot C_{odA}(t_1)}}{e^{-\theta \cdot C_{odA}(t_1)} + e^{-\theta \cdot C_{odB}(t_1)} + e^{-\theta \cdot C_{odO}(t_1)}} \cdot Q^*_{od}(t_1), \quad o, d \in \mathbf{Z}, t_2 \in \mathbb{T}_2 : t_2 \in t_1, t_1 \in \mathbb{T}_1 \tag{4.16}$$

4.2.2.1 Route level

The first level of traffic analysis is the route level. Let $n_r^{odt_2'}(t_2)$ be the accumulation of cars in zone r during time period $t_2 \in \mathbb{T}_2$ with the origin of their trip at o and destination at d with departure time during time period t_2', and let $m_r^{odt_2'}(t_2)$ and $m_{r^-}^{odt_2'}(t_2)$ respectively be the outgoing flow from zone r to the next zone and from the zone before r (i.e., r^-) to r in the sequence of zones that describe the route $R_{odA}(t_2)$.

The car accumulation is estimated by equations (4.17), which ensure the conservation of traffic flow in each route (similar to Zheng and Geroliminis, 2013 , 2016). If the zone and period under analysis match the origin and departure time of the trip, then the accumulation of this first level is equal to the

number of car trips that start during the period under analysis (i.e., $\frac{q^2_{odA}(t_2)}{\tau_{od}}$), while if time period t_2 is after the departure time period t'_2 (i.e., $t'_2 < t_2$), the accumulation results from excluding the flow that comes from this zone to the following zone in the route $R_{odA}(t'_2)$ during the previous time period ($m_r^{odt'_2}(t_2-1)$) from the accumulation on this zone provided by the trips from o to d that started during t'_2. If is some other zone rather than the origin o, the route-based accumulation is equal to summing the accumulation on the previous time period ($n_r^{odt'_2}(t_2-1)$) with the flow received by the previous zone in the route $R_{odA}(t'_2)$ during the previous time period ($m_{r^-}^{odt'_2}(t_2-1)$) and excluding the accumulation sent to the following zone ($m_r^{odt'_2}(t_2-1)$).

$$n_r^{odt'_2}(t_2) = \begin{cases} \frac{q^*_{odA}(t_2)}{\tau_{od}} & \text{if } r = o \text{ and } t'_2 = t_2 \\ n_r^{odt'_2}(t_2-1) - m_r^{odt'_2}(t_2-1), & \text{if } r = o \text{ and } t'_2 < t_2 \\ n_r^{odt'_2}(t_2-1) + m_{r^-}^{odt'_2}(t_2-1) - m_r^{odt'_2}(t_2-1), & \text{if } r \neq o \text{ and } t'_2 < t_2 \\ 0, & \text{if } r \neq o \text{ and } t'_2 > t_2 \end{cases} \qquad \begin{matrix} r \in R_{odA}(t'_2) \\ t'_2, t_2 \in \mathbb{T}_2 \\ o, d \in \mathbf{Z} \end{matrix} \qquad (4.17)$$

The outgoing flow $m_r^{odt'_2}(t_2-1)$ depends on whether zone r is the destination d or not. In the case it is, the outgoing flow is equal to the number of vehicles that reach the destination, where this flow is afterwards transferred to cruising for parking (as explained in the details in the following subsection). If it is not, the outgoing flow is the minimum between two terms: (1) the proportion of outgoing flow of this route towards the total outgoing flow of the zone under analysis, and (2) the boundary capacity of next zone r^+ of the route $R_{odA}(t'_2)$ (i.e., $BC_{r^+}^{odt'_2}(t_2)$). The outgoing flow is described by equations (4.18), where $N_r^{H_r^{R_{odA}(t'_2)}}(t_2)$ and $M_r^{H_r^{R_{odA}(t'_2)}}(t_2)$ are the accumulation and the outgoing flow for the zone-based level, respectively, which are detailed in the following subsection (subsection 4.2.2.2).

$$m_r^{odt'_2}(t_2) = \begin{cases} \min\left\{ \frac{n_r^{odt'_2}(t_2)}{N_r^{H_r^{R_{odA}(t'_2)}}(t_2)} \cdot M_r^{H_r^{R_{odA}(t'_2)}}, BC_{r^+}^{odt'_2}(t_2) \right\}, & \text{if } r \neq d \\ n_r^{odt'_2}(t_2), & \text{if } r = d \end{cases} \qquad \begin{matrix} r \in R_{odA}(t'_2) \\ t'_2, t_2 \in \mathbb{T}_2, \quad o, d \in \mathbf{Z} \end{matrix} \qquad (4.18)$$

We assumed that the relationship between route flows and zone flows follows a FIFO logic ("first-in-first-out"). In this sense, the travelers that first initiate their trip are the first ones to arrive to the following zone and the first ones to depart to the next zone, independently of their origin. For the sake of simplification, we do not explicitly write this assumption in expressions (4.17) and (4.18).

4.2.2.2 Zone level

The zone level is the second level of traffic analysis. In this level, the car accumulation and the outgoing flow are the result of summing the route level considering the zones that belong to each route and

the traffic direction followed in these zones according to the chosen route (i.e., $R_{odA}(t_2')$ and $H_r^{R_{odA}(t_2')}$, respectively). In this sense, the traffic flow conservation in a zone r is given by equations (4.19). There, $CP_r(t_2)$ denotes the number of cars cruising for parking in zone r during period t_2, α_r^i represents the contribution of these vehicles to the accumulation in zone r and $I_{i=H_r^{R_{odA}(t_2')}}$ is a binary variable that takes value 1 if $i = H_r^{R_{odA}(t_2')}$ and zero otherwise.

$$N_r^i(t_2) = \sum_{o \in \mathbf{Z}} \sum_{\substack{d \in \mathbf{Z} \\ d \neq r}} \sum_{\substack{t_2' \leq t_2 \\ t_2' \in \mathbb{T}_2}} I_{i=H_r^{R_{odA}(t_2')}} \cdot n_r^{odt_2'}(t_2) + \alpha_r^i \cdot CP_r(t_2), \qquad r \in \mathbf{Z}, t_2 \in \mathbb{T}_2, i \in \{1,2\} \tag{4.19}$$

We assume that parking occurs in the end of a period whereas cruising for parking starts in the beginning of a period. This assumption signifies that drivers will be cruising for parking, at least, one period, which corresponds to the time necessary to find an available parking space and to do the parking maneuver. Parking spaces are assumed to become available in the beginning of a period. Taking this into account, the number of vehicles cruising for parking in a zone r during period t_2 is given by equations (4.20). This formulation includes the number of car trips that reach their destination d ($\sum_{\substack{t_2' \leq t_2 \\ t_2' \in \mathbb{T}_2}} n_r^{odt_2'}(t_2)$), the number of free parking spaces $E_d(t_2)$ that will be occupied in the end of time period t_2, and the number of vehicles that were already cruising in the previous time period ($CP_d(t_2 - 1)$). The number of parking spaces that will be occupied is expressed by equation (4.21), where $V_d(t_2)$ represents the total number of free parking spaces during period t_2. This value depends on the number of free parking spaces available in the previous period ($V_d(t_2 - 1)$), the number of car trips that are started in the destination ($\sum_{k \in \mathbf{Z}} \frac{q*_{dkA}(t_2)}{\tau_{dk}}$) and the number of occupied parking spaces in the previous time period ($E_d(t_2 - 1)$), as expressed in equation (4.22).

$$CP_d(t_2) = \sum_{o \in \mathbf{Z}} \sum_{\substack{t_2' \leq t_2 \\ t_2' \in \mathbb{T}_2}} n_r^{odt_2'}(t_2) - E_d(t_2 - 1) + CP_d(t_2 - 1), \qquad d \in \mathbf{Z}, t_2 \in \mathbb{T}_2 \tag{4.20}$$

$$E_d(t_2) = \begin{cases} CP_d(t_2 - 1), & \text{if } V_d(t_2) \geq CP_d(t_2 - 1), \\ V_d(t_2), & \text{otherwise} \end{cases} \qquad d \in \mathbf{Z}, t_2 \in \mathbb{T}_2 \tag{4.21}$$

$$V_d(t_2) = V_d(t_2 - 1) + \sum_{k \in \mathbf{Z}} \frac{q_{dkA}^*(t_2)}{\tau_{dk}} - E_d(t_2 - 1), \qquad d \in \mathbf{Z}, t_2 \in \mathbb{T}_2 \tag{4.22}$$

Let L_r^i be the average trip length done by car in zone r with direction i, which is OD independent, and $v_{rA}^i(t_2)$ be the average car speed in zone r during period t_2. The internal trip completion rate follows as expressed in equation (4.23), where the outgoing follow (which is the same as the internal trip completion rate) is equal to the number of vehicles per average trip length times the speed or equal to the total accumulation. These two options depend on the quotient between the accumulation and the average trip length exceeding the accumulation of car in the zone, to ensure that the number of outgoing

vehicles is at maximum equal to the number of accumulation cars .

$$M_r^i(t_2) = \begin{cases} v_{rA}^i(t_2) \cdot \dfrac{N_r^i(t_2)}{L_r^i}, & \text{if } v_{rA}^i \cdot \dfrac{N_r^i(t_2)}{L_r^i} \leq N_r^i(t_2) \\ \\ N_r^i(t_2), & \text{otherwise} \end{cases} \tag{4.23}$$

4.2.2.3   Region level

The third level concerns the region-based level where the average speed is estimated. Following the same logic as before, the accumulation of vehicles in a region is equal to sum of the accumulations of the zones that belong to that region. In this sense, the accumulation per region is determined as shown in equations (4.24), for the two traffic directions. Let $\hat{P}_{\hat{r}}(\cdot)$ be the production of region $\hat{r}$, i.e., the product of average flow and network length, which is a function of the region accumulation $\hat{N}_{\hat{r}}^I(t_2)$. The average speed in traffic direction i is estimated for a region $\hat{r}$ as expressed by equations (4.25).

$$\hat{N}_{\hat{r}}^i(t_2) = \sum_{\substack{r \in \mathbf{Z}, \\ r \in \hat{r}}} N_r^i(t_2), \qquad \hat{r} \in R, t_2 \in \mathbb{T}_2, i \in \{1,2\} \tag{4.24}$$

$$\hat{v}_{\hat{r}}^i(t_2) = \frac{\hat{P}_{\hat{r}}(\hat{N}_{\hat{r}}^i(t_2))}{\hat{N}_{\hat{r}}^i(t_2)}, \qquad \hat{r} \in R, t_2 \in \mathbb{T}_2, i \in \{1,2\} \tag{4.25}$$

We assume that the average speed $\hat{v}_{\hat{r}}^i(t_2)$ estimated for a given region r during period t_2 sets the average speed for the zones that belong to region r, i.e., $\hat{v}_{mA}^i(t_2) = \hat{v}_{\hat{r}A}^i(t_2)$ if zone m is part of region $\hat{r}$. These average car speeds will define the average transit and car speed values for each zone that are included in routes considered in the generalized costs of each mode (4.3) and (4.6). This relationship is given by $v_{mA}^i(t_1) = \sum_{t_2 \in t_1} \frac{\hat{v}_{mA}^i(t_2)}{\bar{t}_{1\_2}}$, where $\bar{t}_{1\_2}$ is the number of time periods from time frame $\mathbb{T}_2$ that belong to t_1. The value of the transit speed $v_{mB}^i(t_1)$ is determined by the product between the average car speed $v_{mA}^i(t_1)$ and the parameter γ_m^i.

4.3   Solution Method

In this section, we present an algorithm capable of finding near-optimal (or even optimal) values for transit fares and parking fees so that the joint profits of both transit and parking operators are maximized. This solution method is justified by the high complexity of solving the optimization model. This algorithm includes the following components: (1) a traffic equilibrium algorithm that calculates the equilibrium for known transit fares and parking fees, and (2) a greedy algorithm to wisely find good (close-to-optimum or even optimum) solutions for both transit fares and parking fees so that the maximization of the joint profits takes place.

4.3.1 Traffic-Equilibrium algorithm

The traffic-equilibrium algorithm finds the traffic equilibrium in the city for given transit fares and parking fees. If on one hand, the number of car trips (car demand-share) implies that all the generalized costs are known (transit fares and parking fees included) to choose between the different alternatives; on the other hand, being aware of these generalized costs also requires that the average car speed is known, which is determined by the MFD model that needs the number of car trips as exogenous traffic flow demand.

This method assumes that transit fares and parking fees are known, as well as the initial values for average transit and car speeds, transit and car routes and the generalized costs of each alternative, which are estimated for the time frame $\mathbb{T}_1$ according to equations (4.1)-(4.2) and their ingredients. With these values, the number of car trips for time frame $\mathbb{T}_2$ are calculated and the MFD model is applied using these new car trip values as exogenous traffic flow demand (see Figure 4.1). The generalized costs of each alternative are updated by the new average transit and car speeds estimated by the MFD model, and compared to the ones used in the first place. If the maximum difference between the oldest and the newest generalized costs is lower than a given value ε, this component of the solving method ends and the equilibrium is assumed to be found, otherwise the new modal shares are re-estimated considering the average between the oldest and the new generalized costs. This solution method is repeated until a convergence condition is met. A schematic flowchart of this traffic-equilibrium algorithm is shown in Figure 4.2.
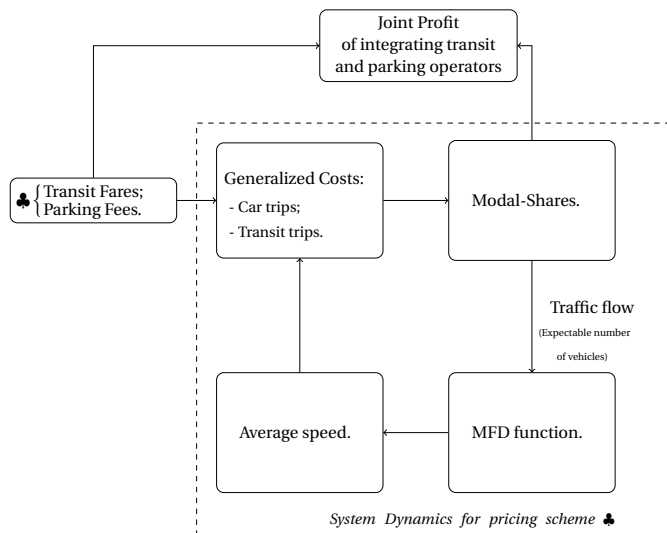
Figure 4.2: Traffic-Equilibrium algorithm flow chart.

4.3.2 Greedy Algorithm

The second component of the solving method is a greedy algorithm. This choice lies not only on the easy-to-implement characteristics of this algorithm, but also on the benign configuration of the feasible region of the model exemplified in Figure 4.3. This feasible region was achieved for the small-sized city, and a similar behavior was found for the case study detailed in the following section (Figure 4.10).
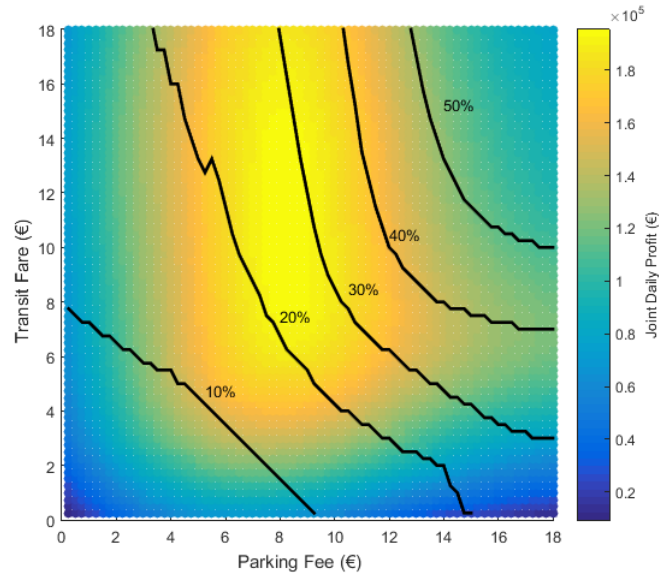
Figure 4.3: Solution values as a function of parking fees and transit fares (results developed for a 10-zone city), where the black lines are the threshold of the share of trips that are not made.

By simplification, we assume that each price will be defined within a limited number of decimal places, leading to a discrete searching space. This assumption is compatible with a real-world application, since prices are necessarily defined within a maximum of 2 decimal places. It is worth noting that this assumption does not simplify the complexity of the optimization model, or whatsoever replaces the need to develop a heuristic.

In general terms, the greedy algorithm consists on jumping from the best solution so far to the following best solution until no further improvements are possible. This approach consists on the following. We start by selecting initial values for the transit fares and the parking fees (x_{0B} and x_{0A}) and assess the initial objective function value $\overline{OF}_0$ considering the solution method previously described (subsection 4.3.1). For each one of the two pricing schemes, a set of incremental parameters (δ_B and δ_A for transit and parking, respectively) and proportional parameters lower than 1 (ϖ_B and ϖ_A for transit and parking, respectively) are defined. With these sets, the new set of candidate solutions is selected and each new solution is analyzed, creating a stage of the greedy algorithm. The best solution of each stage is selected and a new set of candidate solutions is created based on this new best solution, leading to a new stage. In the process of building a new set of candidate solutions, we consider two different approaches according to the following criterion. If the best solution found so far results from the previous stage, the

following set of solutions will be build based on the incremental parameters' set; otherwise the new set of solutions will be created using a new set of incremental parameters, which are generated by multiplying the elements of this old set by the proportional parameters ϖ_B and ϖ_A. This prevents the algorithm to get stuck in a local minimum and to search in a smart-way the space of solutions by taking advantage of its configuration (Figure 4.3). This iterative mechanism will end when the best solutions found in $\widetilde{k}$ sequential stages are the same and the elements from the incremental set are lower than $\widetilde{\varepsilon}$. A pseudo code of this greedy algorithm is schematically shown in Figure 4.4.

This implementation requires an efficient mechanism to generate the candidate solutions in step 5a., which is as follows.

Let $\overline{x}_{t-1B}$ be the best solution found so far for the transit fares, which satisfies $\overline{x}_{t-1B} = \left(p_{1B}, p_{2B}, \ldots, p_{nB}\right)$, where n represents the number of transit fare levels. The first solution that is generated results from adding to $\hat{p}_{1B}$ the first element of δ_B, while keeping the remaining transit fares at their current value. The second is equal at adding the second element of δ_B to p_{1B}. This logic is kept until all the possible combinations between $\overline{x}_{t-1B}$ and δ_B are generated. To this set, it is also added the solutions that result from having all transit fares equal to adding each single value of $\overline{x}_{tB}$ with each element of δ_B. In this sense, X_{tB} is compound by all the distinct elements reached by these two procedures. This procedure is schematically explained in the pseudo code shown in Figure 4.4.

To generate the parking fees the logic is slightly different, because besides selecting the optimum pricing fee levels, we also need to select the zones to which this same parking fee level will be assigned to. In this sense, let $\overline{x}_{t-1A}$ be the best parking fee values found so far for each zone, i.e., $\overline{x}_{t-1A} = \left(p_{1A}, p_{2A}, \ldots, p_{\#A}\right)$. The first solution that is generated results from adding to p_{1A} the first element of δ_A. If by changing the value of p_{1A} to $p_{1A} + \delta_A(1)$, we keep the number of different parking fees below the number of parking fee levels $|\widetilde{P}_A|$, we proceed to generate the second solution. Otherwise, all the p_{kA} equal to p_{1A} will have their value changed to $p_{1A} + \delta_A(1)$ while the remaining values are kept. The same approach will be considered for the remaining elements of the incremental set δ_A, and this procedure will be developed for all the elements of $\overline{x}_{t-1A}$. The pseudo-code of this procedure of generating parking fee solutions is detailed in Figure 4.4. This set is complemented by the set of generated solutions achieved by considering the result of having equal parking tariffs for every charged zone. This is the result of adding to each unique element of $\overline{x}_{t-1A}$, each different element of δ_A and assigned it to $p_{tA}, \forall t$. Finally, X_{tA} is compound by all the distinct generated solutions achieved by these two procedures.

A. Greedy Algorithm

1. Choose initial parking fees and transit fares values (x_{0B} and x_{0A});

2. Choose initial elements for the incremental parameters (δ_B and δ_A) and proportional parameters (ϖ_B and ϖ_A);

3. Update the values of the best solution found so far, i.e. $\overline{x}_{0B} \leftarrow x_{0B}$ and $\overline{x}_{0A} \leftarrow x_{0A}$;

4. Set the stage number to 1 ($t \leftarrow 1$) and the value of the stop condition to zero ($stop \leftarrow 0$)

5. **While** ($stop \neq 1$)

   a. Define the new set of prices schemes $X_{tB} = \{x_{t_1 B}, \ldots, x_{t_{n'} B}\}$ and $X_{tA} = \{x_{t_1 A}, \ldots, x_{t_{n'} A}\}$ for stage t by taking into account the incremental parameters δ_B and δ_A and the best solution found so far ($\overline{x}_{t-1B}, \overline{x}_{t-1A}$);

   b. Compute all the possible combinations of element of sets X_{t_B} and X_{tA}, excluding repetitions. For each element resultant from this procedure, verify if this combination was previously analyzed. If it was, continue for the following combination, otherwise compute the solution method explained in subsection 4.3.1;

   c. Compute the objective function value $OF(x_{t_l B}, x_{t_{l'} A})$ and the penalization for not satisfying constraints (4.12) and (4.13) $CS(x_{t_l B}, x_{t_{l'} A})$, for each element of $x_{t_l B}, x_{t_{l'} A} \in X_t$;

   d. Set the best objective function value $\overline{OF}_t$ as expressed by equation (4.26);

   $$\overline{OF}_t = \max\{\overline{OF}_{t-1}, \{OF(x_{t_l B}, x_{t_{l'} A}) - CS(x_{t_l B}, x_{t_{l'} A}) : x_{t_l B}, x_{t_{l'} A} \in X_t\}\} \tag{4.26}$$

   e. Update the best solution values $\overline{x}_{tB}$ and $\overline{x}_{tA}$ according to equation (4.27).

   $$(\overline{x}_{tB}, \overline{x}_{tA}) = \{(x_{t_l B}, x_{t_{l'} A}) : OF(x_{t_l B}, x_{t_{l'} A}) - CS(x_{t_l B}, x_{t_{l'} A}) = \overline{OF}_t\} \tag{4.27}$$

   f. **If** $\overline{OF}_t = \overline{OF}_{t-1} = \ldots = \overline{OF}_{t-k}$ **then** multiply all the incremental parameters of set δ_B and δ_A by the proportional parameters ϖ_B and ϖ_A, respectively.

   g. **If** the absolute value of all elements of sets δ_B and δ_A are lower than ε **and** $\overline{OF}_t = \overline{OF}_{t-1} = \ldots = \overline{OF}_{t-k}$ **then** $stop = 1$ and $\overline{OF}_t$ and $(\overline{x}_{tB}, \overline{x}_{tA})$ are returned;

   f. $t \leftarrow t + 1$

6. **End**

B. Generate transit fare solutions:

1. $k \leftarrow 1$

2. **For** $j = 1, \ldots, \#Z$ **do**

   a. **For** $i = 1, \ldots, \#\delta_B$ **do**

      i. $y \leftarrow \overline{x}_{t-1B}$ (x_{t-1B}: best solution found so far);

      ii. $p_{jB} \leftarrow p_{jB} + \delta_B(i)$;

      iii. $x_{t_k A} \leftarrow y$

      iv. $k \leftarrow k + 1$

   End-For

   End-For

C. Generate parking fee solutions:

1. $k \leftarrow 1$

2. **For** $j = 1, \ldots, \#Z$ **do**

   a. **For** $i = 1, \ldots, \#\delta_A$ **do**

      i. $y \leftarrow \overline{x}_{t-1A}$ (x_{t-1A}: best solution found so far);

      ii. $p_{jA} \leftarrow p_{jA} + \delta_A(i)$;

      iii. **if** $\#unique(p_{1A}, \ldots, p_{jA} \ldots, p_{\#ZA}) > |\tilde{P}_A|$ **then**

      $$p_{tA} \leftarrow p_{jA} \text{ iff } p_{tA} == p_{jA} - \delta_A(i)$$

      iv. $x_{t_k A} \leftarrow y$

      v. $k \leftarrow k + 1$

   End-For

   End-For

Figure 4.4: Pseudo code of the greedy-algorithm, of the generation of transit fare solutions and of the generation of parking fee solutions.

4.4 Case Study

A case study inspired in the municipality of Coimbra in central Portugal is designed to illustrate the behavior of the optimization model. This example is used instead of the real city due to the lack of data to calibrate and estimate all the ingredients involved in the model. This case study was developed to be as close as possible to the reality observed in the municipality of Coimbra. In fact, whenever it was possible, the calibration of the generated data and of the parameters was based on: 1) the data collected by a mobility survey conducted in Coimbra during 2009, 2) the road network data, and 3) the existent public transport network.

The case study has a total of 156 000 inhabitants and it is divided into 25 zones (Figure 4.5) with a density of about 450 inhabitants/km^2. Each one of these 25 zones has a correspondent zone in Coimbra, which was also divided into 25 zones according to the geographic boundaries and social similarities observed on the mobility survey data conducted in 2009. 7 out of the 25 zones are considered as city center zones (Figure 4.5), with an estimated average number of 10600 employees and 9100 inhabitants. An average of 5150 people live in each one of the remaining 18 zones, which is substantial higher than the average number of 1780 people working on these zones. These values were achieved by considering the inhabitants and employers estimated for the municipality of Coimbra. In this city, two regions are considered for the MFD model: a Region 1 that is compound by the city center zones and a Region 2 that correspond to the residential zones, as displayed in Figure 4.5.
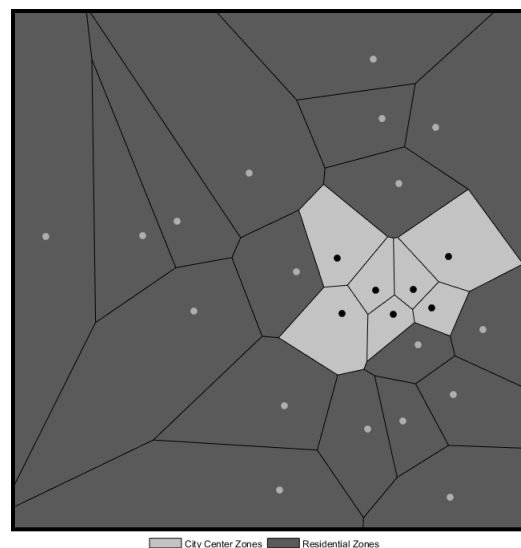
Figure 4.5: Zoning for the city inspired in Coimbra.

A centroid is built in each zone intending at representing the geographic location on each zone with main social activities, which also considered the correspondent centroid estimated for the municipality of Coimbra (Figure 4.5). We assumed that all zones are connected by a road network that accounts for the co-existence of car and transit, although some connections might not be possible by transit to reproduce

the Coimbra's transit network. This assumption requires a slight change of the logit model that includes the car alternative, considering that some destinations can be reached only by car, where the transit mode is omitted from the logit expression (equation (4.1)).

The number of trips made between OD pairs is estimated for the morning period (between 7.30 a.m. and 12.30 p.m.). We start by identifying the number of trips that people would do if the transit fare and the parking fee were set to zero during this morning period. This value was estimated to be 0.49, leading to a total of 76862 trips. These trips were distributed across all OD pairs based on the proportional distributions estimated through the mobility survey. The method used to estimate the number of trips if both prices were set to zero is detailed the Appendix.

We proceed by smoothly distributing the number of trips between OD pairs across time by using the time distribution observed for the municipality of Coimbra. This distribution was estimated considering the equivalent regions for the municipality as the ones displayed in Figure 4.5 and assuming that time frame $\mathbb{T}_2$ is composed by the 5-minutes intervals between 7.30 a.m. and 12.30 p.m., whereas $\mathbb{T}_1$ considers the correspondent intervals of 30-minutes. These functions are drawn in Figure 4.6., and they were considered as the distribution of the trips between each OD pair across time by relating the OD zones with the regions to whom they belong to. Finally, these trips were split between those that could be made by car (Q_{od} and Q_{od}^*) and those who cannot ($\widetilde{Q}_{od}$ and $\widetilde{Q}_{od}^*$) based on the Coimbra's reality, which leads to a proportional relationship of 75%-25%, respectively.
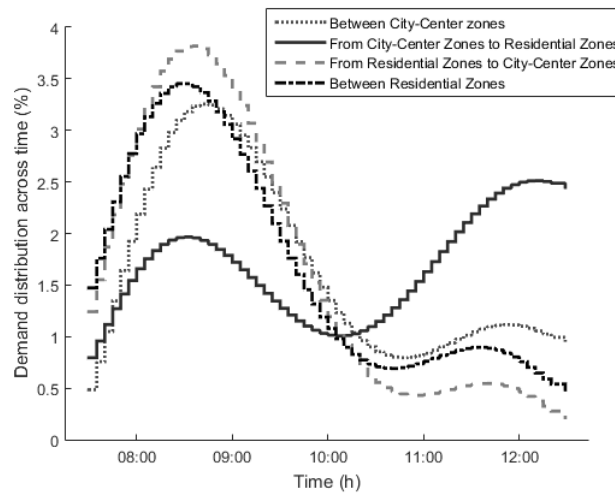
Figure 4.6: Proportional distribution of trips across time based on Coimbra and the inner and outer regions.

The transit system was designed in the model under the assumption that to go from one zone to another it would be necessary at most one transfer. The transit network is the result of merging all the transit routes, and coincides with the road network on the links included in the transit routes. The frequency of each designed route is generated between 1 and 6 for each period of $\mathbb{T}_1$, which is based on the expectable demand variation during the morning period. Because transit routes require grater travel

distances than the ones determined by a simple Euclidean distance, the transit distance for a zone m with direction i is also multiplied by a random number generated from the interval [1.5,2.5]. The values of variables $D_{mB}^{t_1}(H_m^{R_{odB}(t_1)})$, $R_{odB}(t_1)$, $H_m^{R_{odB}(t_1)}$ and N_{odlk} are determined based on the shortest path criterion and on the routes that were designed inspired on the transit network system of Coimbra. The capacity of each transit vehicle was set to 60 pax/veh.

The average car distance between two adjacent zones was calculated by multiplying the Euclidean distance between the two centroids, by crossing the mid-point of the boundary, with a factor in the interval [1,2] that was chosen so that these distances would be as close as possible to the ones observed for the municipality of Coimbra. We opt for this approach instead of using the true values determined by the road network of Coimbra for the sake of ensuring congruence among the several ingredients of the model (e.g., car and transit distances, road network topology or the MFD function included into the MFD model). The road network direction to go from a zone o to an adjacent d is randomly selected from the set $\{1,2\}$. As expected, if the direction to go from o to d is 1, then the direction of going from d to o will be assigned to 2.

The route selected by car users to go from each origin to each destination (R_{odA}) is estimated based on the fastest path, which requires the average car speeds assessed by the MFD model. The parking capacity of each zone is inspired on the municipality of Coimbra, leading to a proportion towards the total expected demand for each zone between 0.1 and 4. These proportions account for the parking spaces that are occupied during the whole morning period.

The parameters θ and $\widetilde{\theta}$ that capture the sensitivity of travelers towards the generalized cost are set to 0.86 and 0.25, respectively, based on the municipality of Coimbra towards traveling costs. These values are the result of an attempt at reproducing the reality observed for Coimbra in what concerns the modals-shares determined based on the generalized costs of each mode (see equations (4.1) and (4.2)), detailed as below, accounting for those who have and those who do not have the car alternative to do their trip.

The cost of time was based on the Portuguese reality (INE, 2015) with a unit cost of $C^T = 8$ €/h. By assuming that people live mainly in the area near the centroid of each zone, we support our choice of estimating the walking distance to and from a transit stop as a randomly percentage between 5% and 25% of the average distance between the centroid and the mid-point of every side border. The average walking speed was estimated to be 5 km/h (Mohler et al., 2007). The parameters used to estimate the discomfort cost linked to ride a transit were set to 1.5 and 2.6 ($\psi_B = 1.5$ and $\rho_B = 2.6$). Based the Coimbra's public transport scheduling, we set the parameter included on the access to take 5 ($SW = 5$). The unit maintenance cost of a car was estimated to be 0.324€/km (Association, 2016; Litman, 2009). The parameters included into the cruising for parking cost function are set to 9 and 42 for α_1 and α_2 respectively, allowing at varying the cruising of parking cost between 0€ to 9€ depending on the parking saturation (Gallo et al., 2011). The car occupation rate was estimated to be 1.2 pax/veh according to the mobility

survey conducted in Coimbra.

The relationship between car average speed and transit average speed given by parameter γ_m^i for a zone m with direction i is estimated based on the municipality of Coimbra. In this sense, the average transit commercial speed for the city center zones is on average 57% of the average car speed for these zones, whereas for the residential zones this value increases to 68%. These values were achieved by comparing the average transit commercial speed in each region with the average car speed. The average transit commercial speed was determined by analysing the annual reports of the transit operator that manages the transit system in Coimbra, whereas the average car speed was based on the survey conducted in Coimbra.

In what regards the dynamic features, one MFD function is estimated for each one of the two regions. It is assumed the same MFD function for both directions in the same region. Based on the road network existent in Coimbra, the road network topology has a total of 185 km and 590 km in each direction of the road network that belongs to the city center and to the residential zones, respectively. Two unimodal fundamental diagrams for each road link were designed considering the road network characteristic of Coimbra and the average road capacity occupied by buses. Note that these average speeds account for the contributing of the number of vehicles cruising for parking towards the accumulation of cars in each zone. In this sense, we assumed that each cruising vehicle was equal to 1.5 vehicles that are not cruising, and this amount was therefore divided between the two road directions ($\alpha_r^i = 0.75, i \in \{1, 2\}, r \in \mathbf{Z}$). The trip length of each zone is the average distance travelled by each car user inside the zone based on the followed road direction.

We assume that travelling by transit will have a fixed cost of 0.5 €, independently of the OD pairs, and parking in the city center will require a payment of 2€ per parking, whereas parking in residential zones will remain free as it is in the prevalent scenario.

The operating costs assigned to each system, M_B and M_A, are determined based on real world operating costs for the two services, mixing the reality of Coimbra with the average farebox recovery ratio (Guerra, 2011; Lindquist et al., 2009; Litman, 2010).

We proceed with the implementation of the solution method to have insights on the dynamics, modal-shares and financial features of the case study. This solution method took 225 seconds, which sustains the need to use a heuristic capable of finding near-optimal solutions with the lowest number of explored solutions possible for the sake of having good results within a reasonable computation time. With these features, the prevalent financial and modal-shares achieved for each one of the two operators are described on Table 4.1.

Table 4.1: Financial features for both transit and parking operators, for the morning period (M.P.).

Transit fare (€/trip)		0.5
Parking fee (€/parking)		2
	Transit	-15.3
Profit (10^3 €/ M.P.)	Parking	13.8
	Total	-1.5
	Transit	21.09 (29%)
Number of trips (10^3 /M.P.)	Car	52.16 (71%)
	Total	73.25

The transit system reveals a deficit during the morning period of 15.3×10^3€/day. This value is in line with what has been observed world-wide, corresponding to the levels of subsidization required by the transit operator that operates the transit system of Coimbra. The revenue collected by the transit operator covers around 41% of the operating costs of the transit system operating in this city inspired in Coimbra. A different scenario occurs for the parking operator that collects more 27% in revenues than the operating costs of the parking lots.

The dominance of car use is noticeable on the current modal-shares. In fact, from a total of around 73.25×10^3 trips made during the morning period (M.P.), 71% are made by car. Furthermore, being aware that 25% of the trips only have the option of travelling by transit, we can realize that a greater part of the transit trips is made by those to whom car is not available. This highlight the importance of constraint (4.13) and why we decided to set the parameters ψ and $\widetilde{\psi}$ to take the values 0.5 and 1.5, respectively. In this sense, we will try to ensure that the new transit fares will remain affordable to those who do not have other transport alternative. The value of U_0 was set accordingly, taking 7220, which truly corresponds to a maximum of 9607 trips that are not made and correspond to increase the prevalent 5% to 12.5% for the non-travel alternative.

Finally, the average car speed values of each region estimated for the morning period are displayed on Figure 4.7. These values result from the application of the solution method with the the ingredients previously detailed. Observing Figure 4.7, we can conclude that this city does not present high levels of congestion, where the lowest average speed value is mainly observed for the time periods between 8a.m. and 9a.m., which is in line with Coimbra's reality. Furthermore, the directions followed by each car user in the network influence the observed speed, showing that this improvement of the classical accumulation-based MFD approach is an important asset to be included in the model.
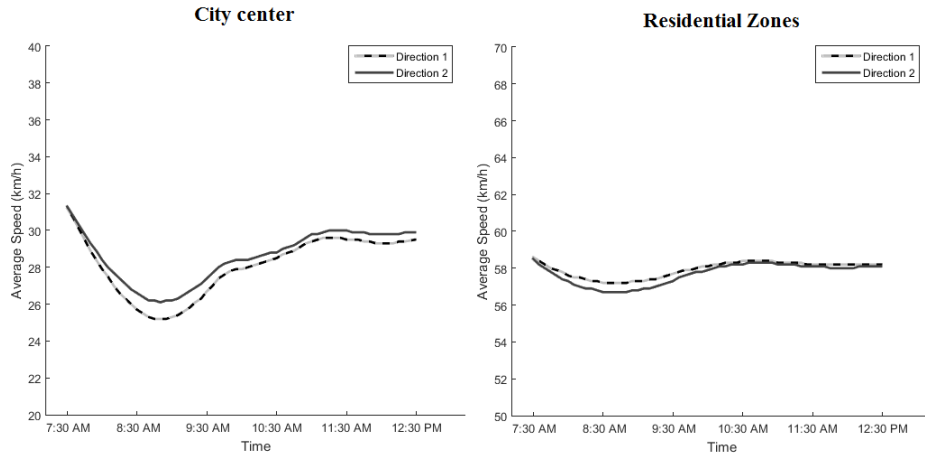
Figure 4.7: Average car speed values for each region.

In the following section, we assess the traffic dynamics of this case study (see subsection 4.5.1), and the implementation of the optimization model using the solution method detailed in Section 4.3. Insights towards the algorithm performance and further conclusions regarding how the integrated approach influences the finances of both transit and parking operators, how the configurations of both transit fee and parking fees are helpful at mitigating the public transport deficits and how this integration impacts the whole dynamic of the city are also provide.

4.5 Study Results

In this section, we first present the results for both the MFD model, where the effects of changing the transit fares and parking fees are assessed along with the importance of including cruising for parking features. This is followed by exploring the solutions given by the optimization model and the capability of this approach at minimizing the transit deficits.

4.5.1 MFD model

Numerical studies are now presented to assess the traffic dynamics of the case study. The first one analyses the impact of transit fares and parking fee towards the road network dynamics. It is followed by the insights on how the total demand and the cruising for parking features included into the MFD dynamic model impact the traffic dynamic and the user behavior.

Considering the prevalent transit fare and parking fee values charged by each operator, i.e., 0.5€ and 2€, respectively (Table 4.1), the average speed as shown in Figure 4.7 is achieved and it corresponds to the MFDs displayed in Figure 4.8 for Region 1 with direction 1 (similar conclusions can be taken for Region 1 with direction 2).

To understand the impact of transit fares and parking fee towards the dynamics of the city, we mixed and match the values 0.5€ and 4€ for the single transit fare and 2€ and 8€ for the single parking fee

charged in the city center zones (Table 4.2). Computing the solution method, the average speed values and the estimated MFD are as shown in Figure 4.8 for Region 1 with direction 1, considering the correspondent modal-shares expressed in Table 4.2. Note that the MFD dynamic model is only influenced by the car traffic but the estimation of the MFD function accounts for the transit contribution to the accumulation versus flow relationship.
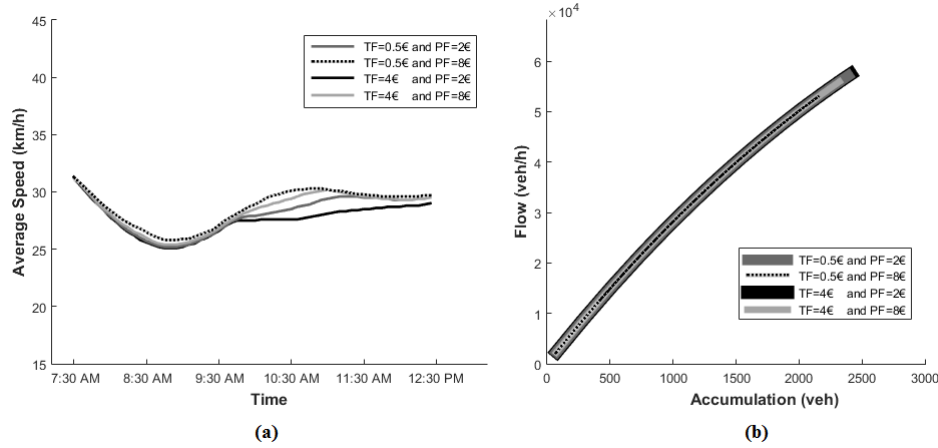
Figure 4.8: (a) Average Speed and (b) MFD Function with different transit fares and parking fees, for Region 1 with direction 1.

Table 4.2: Financial features for both transit and parking operators, for the morning period (M.P.) with different transit fares and parking fees.

Transit fare (€/trip)		0.5	0.5	4	4
Parking fee (€/parking)		2	8	2	8
Number of trips (10^3 /M.P.)	Transit	21.09 (29%)	24.2 (34%)	17.29 (24%)	18.96 (27%)
	Car	52.16 (71%)	46.96 (66%)	54.21 (76%)	51.22 (73%)
	Total	73.25	71.16	71.51	70.18

The first major conclusion taken from these results is that the total accumulation observed in Figure 4.8 has its maximum for each pricing combination in line with the number of car trips made (Table 4.2). This is an expectable result that ensures that the MFD dynamic model changes with changes on the car demand values. Comparing the number of trips made with the pricing applied by each operator, it is possible to conclude that the higher the parking fee, the greater the number of users that decide to make their trip by transit instead of car. The same is observed when the transit fares are increased while keeping constant the parking fee. When the prices are increased significantly (e.g., 4€/trip and 8€/parking for transit fares and parking fees, respectively), the total number of trips decrease, revealing the importance of considering a no-travel alternative. Finally, it is possible to conclude that higher accumulation leads to lower average speed, which supports the suitability of the estimated expression for the MFD function. However, in none of these results the significant levels of congestion were achieved. In fact, we conclude that low levels of congestion are observed due to the average speed values and the maximum accumulation observed in the MFD plot.

To draw conclusions about the reactions of the dynamic dealt by the MFD model with higher levels of demand, in particular car demand, we proceed with the implementation of the solution method with double values of the total number of trips, aiming at analyzing how this change influences the dynamic of the city (Scenario I). No further adjustments are made on the traffic network and on the transit supply. This analysis is then extended to go over the influence of the number of vehicles cruising for parking on the total flow assessed by the MFD model. This is developed by considering two different approaches besides the doubled demand values: (1) setting to zero the parameters that introduces the number of vehicles cruising for parking effect into the MFD dynamic model (denominated as Scenario II); and (2) defining the parking capacity of each zone sufficiently high exclude the existence of cruising for parking (denominated as Scenario III). These scenarios were computed using the solution method and assuming the prevalent values for both transit fares (0.5€/trip) and parking fees (2€/parking). Figure 4.9 shows the final average speeds and the MFD function for Region 1 with direction 1. Table 4.3. summarizes the modal-shares for each described scenario.
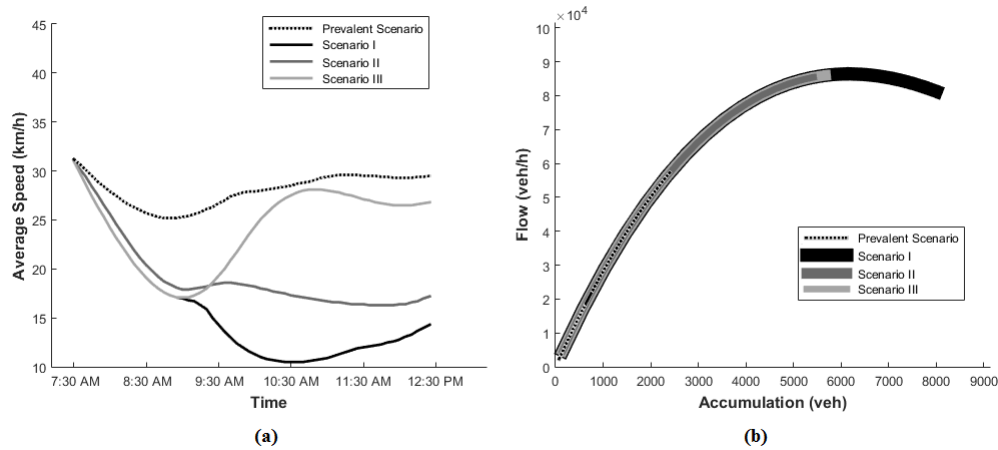
Figure 4.9: (a) Average Speed and (b) MFD Function for the 3 scenarios, for Region 1 with direction 1.

Table 4.3: Number of trips per mode for the morning period (M.P.) for the 3 scenarios, for Region 1 with direction 1.

Scenario		Prevalent	I	II	III
	Transit	21.09 (29%)	26.45 (19.5%)	27.72 (20.2%)	28.05 (20.2%)
Number of trips (10^3 /M.P.)	Car	52.16 (71%)	109.49 (80.5%)	109.19 (79.8%)	110.47 (79.8%)
	Total	73.25	135.93	136.91	138.51

As observed in the previous results, the scenario with the curve with higher accumulation values, and therefore the lowest average speed values, is also the one with the higher number of car trips (scenario I). This is followed by scenario II that excludes the consideration of cruising for parking in the MFD model. The importance of including the number of vehicles cruising for parking is highlighted by these results. In fact, when the parameter that sets the contribution of one cruising vehicle to the road network accumulation is set to zero (Scenario II), the levels of congestion are significantly decreased and the cri-

tical part of the MFD function for this region is not reached. This is also observed on the average speed and on the number of trips made by car. Note that improving the average car speed also improves the average transit speed, which justifies the increase on the number of transit riders when comparing Scenario I and II. Furthermore, although the effect of cruising for parking has been excluded from the MFD dynamic model (Scenario II), the cruising cost remains on the user's generalized cost, which justifies the small change on the number of car users when comparing these two scenarios.

When the parking capacity is set to values high enough to accommodate all the potential parking demand (Scenario III), an expectable increase on the number of car trips is observed (Table 4.3) along with an increase on the number of transit riders. The peaks of the average speed curve are coincident with the shape of the demand across the morning period (Figure 4.6), which highlights that the congestion observed at 10.30 am in Scenario I is a result of having vehicles cruising for parking because the parking capacity is not enough to accommodate all the parking demand. However, we recall that users stick to their choice of using the car over transit because the transit capacity was not improved and the transit speed is lower than the average car speed. By excluding the problems raised by having a lack of parking capacity, we recall the importance of including the cruising effect into the MFD dynamic model, together with scenario II.

These scenarios allowed to assess the impact of different demand values (either due to varying the transit fares and/or parking fee values, either by changing the demand values and the features related to cruising effects) into the MFD model and how this model is a suitable approach to determined dynamic speeds for a given time period. In the following subsection, we explore how to improve the finances of the transit operator by optimizing the transit fares and parking fees. These pricing changes will have an influence on the modal-choice and on the dynamic of the case study, which is assessed by embedding the MFD model into the optimization model, as shown in Figure 4.8 and in Table 4.2.

4.5.2 Optimization Model

In this subsection, insights that can help at inferring the suitability of the integration of transit and parking operators to decrease the subsidize levels required by the transit operator (as shown in Table 4.1) are provided. A small analysis about the greedy algorithm is also detailed, including its performance and capability of providing optimal or near-optimal solutions within a reasonable computation time.

The results shown in this subsection are the ones achieved by the greedy algorithm detailed in subsection 4.3.2. We considered that the two set of incremental parameters were equal to $\delta_B = \delta_A = -10, 0, 10$ and the proportional parameters were set to 0.5 (i.e., $\varpi_B = \varpi_A = 0.5$). For simplification, we assume that the elements resultant by multiplying the incremental set by the proportion parameters should be multipliers of 0.25. In the case of achieving a value that is not a multiple of 0.25, this value is replaced by the closest value multiple of 0.25, and the algorithm continues.

The first approach aims at finding the optimal combination of a single transit fare and a single parking fee. These new prices for the transit fare and parking fee, 3.25€ and 7€, respectively, are the global optimum of the optimization model, with the features described in Table 4.4. We ensure the optimality of these values through the feasible region (similar to the one displayed in Figure 4.1), which was developed through an exhaustive search procedure that explored the values shown in Figure 4.10 and took the computation time shown in Table 4.4. In Figure 4.10, the red square corresponds to the optimum value achieved by this procedure, which matches to the transit fare of 3.25€ and the parking fee of 7€. The transit fares and parking fees analyzed during this procedure are multiple of 0.25, to ensure congruency between the exhaustive search and the greedy algorithm results.

As expected, the number of solutions analyzed by the greedy algorithm are significantly lower than the ones analyzed by the exhaustive procedure. This reveal the suitability of the greedy algorithm not only at achieving the optimum solution in a small amount of time, but also that the analysis was developed in a smart way, which lead to mainly assess potential good solutions instead of poor ones. Note that both approaches analyze infeasible solutions. Furthermore, the inclusion of a mechanism that verifies if a solution has already been analyzed by the solution method decreases significantly the computation time required by the greedy algorithm. This small detailed will be extremely important when non-single transit fares and/or parking fees are considered. However, it is worth noting that for these scenarios, the exhaustive procedure will not be conducted due to its extremely high computation time requirements (e.g., we will need to analyze a total of 2^7 combinations for each 2 different parking fee levels and 1 transit fare level, assuming that each solution requires an average of 4 minutes, we will need almost 8 hours to assess two different parking fee levels and one transit fare).

Table 4.4: Optimum solution features for a single transit fare and a single parking tariff (Scenario I).

Transit fare (€/trip)		3.25
Parking fee (€/parking)		7
Profit (10^3 €/ M.P.)	Transit	37.68
	Parking	132.12
	Total	169.8
Number of trips (10^3 /M.P.)	Transit	19.53 (27.5%)
	Car	51.38 (72.5%)
	Total	70.91
Computation time (min)	Greedy algorithm	207
	Exhaustive procedure	2954
Number of generated solutions	Greedy algorithm	120
	Exhaustive procedure	768
Number of analyzed solutions	Greedy algorithm	49
	Exhaustive procedure	768
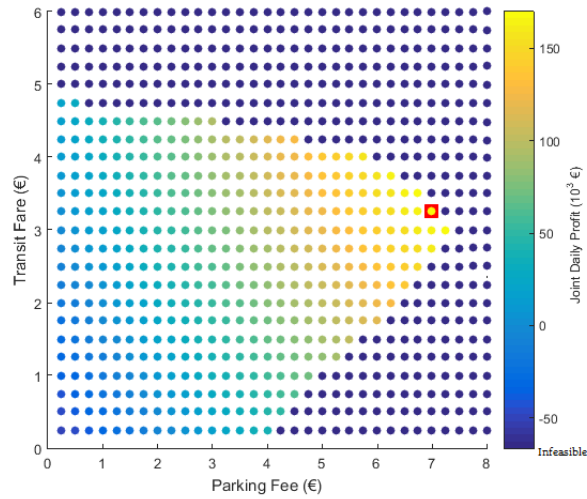
Figure 4.10: Solutions analyzed by the exhaustive search procedure for Scenario I.

Comparing the results achieved by implementing the optimal single transit fares and single parking fee, a significant improvement of the transit operator profit is achieved. In fact, by increasing the transit fare to 3.25€ along with charging 7€ per parking in the city center, the prevalent transit deficit of 15.3×10^3€/day is transformed to a positive profit of 37.68×10^3€/day. In this sense, the parking operator do not need to cover the transit losses, because they do not exist with this new pricing scheme. The parking operator also improves its profit, from the prevalent 13.8×10^3€/day to 132.12×10^3€/day. As expected, these new prices also influence the number of trips assigned to each alternative, leading to a decrease on both transit and car trips when compared to the prevalent number of trips of each mode.

In order to analyze how different pricing configurations might influence the finances of both operators and the number of trips assigned to each mode, we proceed this analysis by considering the following 5 scenarios:

(II) single transit fares and two parking tariffs;

(III) single transit and three parking tariffs;

(IV) two transit fares and single parking tariff;

(V) two transit fares and two parking tariffs;

(VI) two transit fares and three parking tariffs.

For the scenarios with two transit fares (IV, V, VI), it is assumed that the trips with origin and destination in Region 1 will have a lowest transit fare whereas the remaining transit trips will have a higher transit fare. This assumption is plausible with what has been observed in the Portuguese reality.

The results for each one of these scenarios are shown in Table 4.5, while the parking fees charged in each zone are displayed in Figure 4.11 for Scenarios II, III, V and VI. These values were achieved by

computing the greedy algorithm using the prevalent values of transit fares and parking fees as initial solutions of the algorithm.

Table 4.5: Optimal solution features for each Scenario.

Scenario		II	III	IV	V	VI
Transit fare (€/trip)	R1 - R1	3.25	3.25	1.75	1.75	1.75
	Remaining			6.5	6.5	6.5
Parking fee (€/parking)	Level 1	7	7		7.25	7.25
	Level 2	7.25	7.25	7.25	7.5	7.5
	Level 3		7.25			7.5
Profit (10^3 €/ M.P.)	Transit	37.75	37.75	65.11	65.12	65.12
	Parking	133.78	133.78	132.7	132.99	132.99
	Total	171.53	171.53	197.81	198.11	198.11
Number of trips (10^3 /M.P.)	Transit	19.55 (27.6%)	19.55 (27.6%)	20.35 (28.6%)	20.36 (28.6%)	20.36 (28.6%)
	Car	51.33 (72.4%)	51.33 (72.4%)	50.85 (71.4%)	50.82 (71.4%)	50.82 (71.4%)
	Total	70.88	70.88	71.2	71.18	71.18
Computation time (min)	Greedy algorithm	1712.8	2567.4	1013.8	6060.7	8484.2
Number of generated solutions		744	1153	523	3212	4504
Number of analyzed solutions		374	575	215	1513	2149

(a) 7 €/parking 7.25€/parking   (b) 7.25€/parking 7.5€/parking
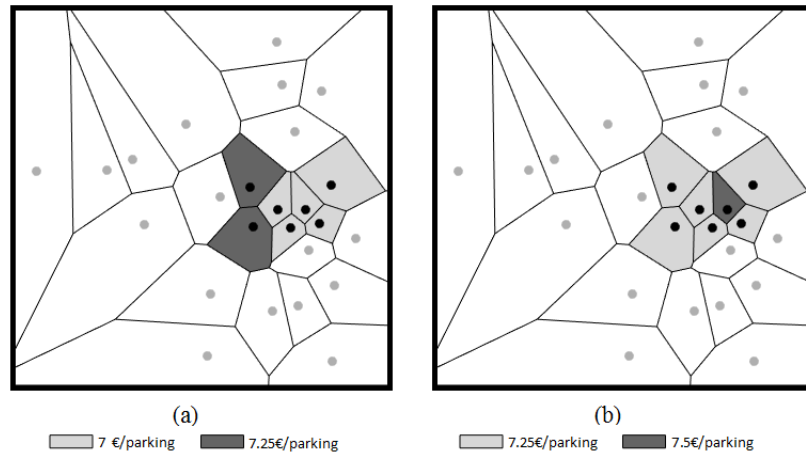
Figure 4.11: Solutions analyzed by the exhaustive search procedure: (a) one transit fare – Scenario I and II; (b) two transit fares – Scenario IV and V.

The results shown in Table 4.5 highlight the good performance of the developed greedy algorithm. In fact, the results of scenario II are better than the results shown for scenario I in Table 4.4, because the joint profits are increased. It can be considered that scenario I set a *lower bound* for all the scenarios under analyzes, in particular for scenario II and IV. The concept behind this *lower bound* characteristic assigned to some scenarios is as follows. If one scenario results from another scenario by increasing the number of transit zones and/or increasing the number of parking fee levels, the simplest scenario sets

the lower bound for the scenario with higher pricing complexity. In other words, the worst case for the scenario with higher pricing complexity is to be equal to the scenario with simplest pricing schemes. This boundary allows to ensure that these are good results, even though an exhaustive search has not been made to prove that these solutions are optimal or near-optimal. Comparing scenarios II and III and scenarios V and VI, we observe that even though a different pricing configuration is possible, the results are the same. In fact, scenario II can set a lower bound for scenario III, whereas scenarios III and V define the lower bounds for scenario VI.

As expected, the computation times are proportional to the number of solutions that are analyzed. Moreover, the number of solutions, generated and analyzed, increases with the complexity inferred to the pricing schemes. However, the number of solutions also vary depending on the complexity being assigned to the transit fares or to the parking fees. This is an expected result because while the configuration of the transit zones is known, the parking zones configuration is also optimized by the model. This justifies why the computation times of scenarios II and IV are different, being higher for the former. Since the distribution of the prices are known in advance for scenario IV, it is expectable (and verified in Table 4.5) that the number of solutions that are generated and analyzed to be lower than these values for scenario II, which leads to the computation times' relationship. It is also observed that the proportion of solutions that are generated and the solutions that are analyzed is quite similar across the several scenarios, showing the importance of verifying if a generated solution was or not analyzed by the solution method because this procedure is less time consuming than the solution method.

The influence of the pricing configurations towards the profits collected by each operator is mainly observed when the transit operator considers two different transit fares instead of a single transit fare (e.g., scenarios I and IV). The profits of the parking operator are almost the same for all scenarios, showing that the small changes observed in the parking operator's profit results from minor adjustments of the parking fee configurations, instead of being the result of changes on transit fare pricing values and configuration defined by the transit operator. Similar conclusions are taken for the transit operator in what concerns the effect of parking fees on the profits collected by the transit operator.

The number of trips assigned to each mode are quite similar for all the six scenarios. However, the number of transit riders increases when 2 transit fares are considered instead of having a single transit fare. We recall that the transit zones assumed that shortest trips (trips with origin and destination in the city center) would have cheapest transit fares than the remaining trips. The scenarios with the transit fare values with this configuration, i.e., 1.75€ and 6.5€ for trips with origin and destination in the city center and for the remaining OD pairs, respectively, are the ones with the highest number of transit users when the prices are optimized. In this sense, users are willing to pay more for longer transit trips, whereas for shortest transit trips the transit fare should be decrease so that users can chose transit even when car is an alternative.

We conclude that the best option among the 6 scenarios analyzed is the one with 2 different transit fares, 1.75€ for inner city-center trips and 6.5€ for the remaining one, and two parking fee levels, 7.25€ and 7.5€, assigned to each trip zone as shown in Figure 4.11-b). This scenario improves the transit deficit from its prevalent losses of 15.3×10^3€/M.P. to a positive profit of 65.12×10^3€/M.P, which represents a gain of around 40% when compared to having an optimized single transit fare (e.g., scenario I). Furthermore, if two parking fares are considered instead of a single one, the number of transit riders is the highest one among all scenarios and the closest one to the number of riders in the prevalent scenario. (see the results for the prevalent scenario, scenario I, and scenario V). Similar conclusions cannot be made for the parking operator, whose profits are slightly smaller in this scenario V than for scenarios with a single transit fare and at least two levels of parking fees (scenario II and III). However, this is the scenario with the best joint profit, which sustains our choice. Furthermore, if the two operators agree that two different transit fares should be applied, this scenario reveals to be the best option for the parking operator.

These scenarios allow to end the subsidization of transit systems, showing that transit systems can be profitable and self-sufficient in a financial perspective when their management is integrated with the management of parking systems.

4.6 Conclusion

This chapter has presented an optimization model to circumvent transit financial problems by managing transit systems and parking systems in an integrated manner, using parking fees to fund transit deficits, if needed. This model includes a network level aggregate traffic model based on the macroscopic fundamental diagram (MFD), which was designated as MFD model. Due to the complexity of the optimization model, which has a combinatorial and nonlinear formulation, we developed a solution method to deal with the dynamics embedded into the optimization by the MFD model and a greedy algorithm to solve the optimization model.

This model was applied to a case study inspired in the municipality of Coimbra, a medium-size city in the center of Portugal, showing the suitability of the model to deal with the financial problems of transit systems. The influence of demand towards the road dynamic, which is assessed by the MFD model, is shown by the changes on speed along with the demand variations. This case study also provides insights for the role of cruising for parking in the average speed of the road network, and the importance of decreasing such externality. The suitability of the solution method at solving the optimization model was demonstrated in this example, along with the analysis of how different pricing configurations might affect the mode-choice and the joint profits of the two operators. In the end, insights into how an unlucrative transit system can become profitable by only managing transit fares and parking fees while

ensuring reasonable levels of service were provided.

Based on the case study's results, we can advocate that an excessive emphasis on the deficit perspective has been made. To overcome this feature, and as future work, some changes regarding the objective function should be considered, e.g. the maximization of social welfare or transit share, given a limit for the operating deficit. Furthermore, adjustments to the supply of each system, such as transit frequencies and/or parking capacity, should also be addressed as well as the consideration of dynamic parking fees. The inclusion of a more detailed cruising for parking model (e.g. Geroliminis, 2015) into the MFD model is also another research priority.

Appendix

Let C_{odB} and C_{odA} respectively be the generalized costs of taking a trip from to using transit and car, as given by equations (4.3) and (4.6), respectively, without any time concerns. To estimate these costs, we attempt at reproducing in an aggregated way what was observed for Coimbra, by considering the prices charged by each operator, the ingredients included on these generalized costs as displayed on (4.3) and (4.6), respectively. The average daily speed for both transit and car modes was estimated considering the data collected by the survey, as well as the discomfort costs (4.5) and the cruising for parking cost (4.7). The method that estimates this demand is based on the classical gravity model formulation so that plausible number of trips can be ensured.

A first estimation for the number of trips between a pair of zones od is assigned to $\widetilde{T}_{od}$ through expression (4.28), where the parameters α_{GM}, μ_{GM}, ε_{GM} and θ are estimated according to the reality observed for Coimbra. PC_o and EC_d are the inhabitants and employers for zones o and d, respectively.

$$\widetilde{T}_{od} = \alpha_{GM} \cdot \frac{PC_o^{\mu_{GM}} \cdot EC_d^{\varepsilon_{GM}}}{exp\left(\theta \cdot \frac{C_{odB}+C_{odA}}{2}\right)}, \quad o,d \in \mathbf{Z} \tag{4.28}$$

To estimate the total number of trips that would be done if both transit fares and parking fees were zero ($\hat{T}_{od}$), new generalized costs are estimated for trips done by transit or by car. In this sense, the values of $\hat{C}_{odB}$ and $\hat{C}_{odA}$ are equal to $\widetilde{C}_{odB} - p^0_{odB}$ and $\widetilde{C}_{odA} - p^0_{odA}$, respectively, where p^0_{odB} and p^0_{odA} are the transit fare and parking fee values implemented by each operator. By replacing the values of C_{odB} and C_{odA} with these new generalized costs into expression (4.28), a new number of trips $\hat{T}_{od}$ is determined. This amount includes the trips that are not done due to the costs of transit fares and parking fee. Let $\widetilde{LT}_{od}$ be the number of trips that are not done by people due to these costs, which is equal to $\widetilde{LT}_{od} = \hat{T}_{od} - T_{od}$. The estimation of the cost of not making the trip, $\widetilde{C}_{odO}$, is displayed in equation (4.29), and the final number of trips that are lost by the system LT_{od} due to the pricing of transit and parking are estimated as expressed by equation (4.30).

$$\widetilde{LT}_{od} = \alpha_{GM} \cdot \frac{PC_O^{\mu_{GM}} \cdot EC_d^{\varepsilon_{GM}}}{exp\left(\theta \cdot \widetilde{C}_{odO}\right)} \Rightarrow \widetilde{C}_{odO} = \frac{ln\left(\alpha_{GM} \cdot \frac{PC_O^{\mu_{GM}} \cdot EC_d^{\varepsilon_{GM}}}{\widetilde{LT}_{od}}\right)}{\theta}, \quad o,d \in \mathbf{Z} \tag{4.29}$$

$$LT_{od} = \alpha_{GM} \cdot \frac{PC_O^{\mu_{GM}} \cdot EC_d^{\varepsilon_{GM}}}{exp\left(\theta \cdot \widetilde{C}_{odO}\right)}, \quad o,d \in \mathbf{Z} \tag{4.30}$$

Finally, the total demand for a OD pair is given by $\hat{Q}_{od} = T_{od} + LT_{od}$. To accommodate further pricing changes and to include inclusion of cruising for parking and discomfort costs, the cost $\widetilde{C}_{odO}$ is disturbed by a random variable with values between 1 and 3, which leads to the values C_{odO} for each OD pair included in the model. Note that this procedure was conducted due to the lack of information on the users' elasticities towards transit fares and parking fee.

Chapter 5

Computational study of a hybrid simulated-annealing cross-entropy algorithms for facility location problems

5.1   Introduction

Simulated Annealing (SA) algorithms have been shown to provide near-optimal or even optimal solutions to many optimization models. These algorithms have become especially attractive due to their well-known global optimum convergence properties (Kirkpatrick et al., 1983; Lundy and Mees, 1986; Locatelli, 2002). If properly designed, SA algorithms have the capability of escaping from local optima and finding a global optimum by uphill and downhill moves. Otherwise, their performance may be weak, either because they return low-quality solutions or because the computation time taken to find good solutions is excessively long. The reason for this to happen may be because they spend too much effort in the evaluation of solutions of poor quality. If each iteration can be performed very quickly, this may not be a problem. In contrast, if each iteration is time-consuming, then the computational effort necessary to run the algorithm may become prohibitive in the case of large model instances.

The study we present in this chapter was performed with the purpose of finding a heuristic that could substantially reduce the number of iterations required to complete the execution of a SA algorithm, while at the same time, increasing – or, at the least, not decreasing – the quality of the best solution that is returned. This decrease on the number of analyzed solutions is truly important when solving problems with time consuming objective functions, which are the main motivation to develop this word. Specifically, what we did was to hybridize the SA algorithm with the Cross Entropy (CE) algorithm so that the probability of selecting a low-quality solution in each iteration is decreased (without being zero); i.e., the elements typically presented in good solutions have a higher probability of being chosen than elements that rarely belong to good solutions. CE algorithms have been proposed by Rubinstein (1999, 2001) as a generic Monte Carlo technique for solving complicated simulation and optimization problems

(see, e.g., De Boer et al., 2005; Rubinstein and Kroese, 2013).

Several efforts are reported in the literature concerning the hybridization of SA algorithms with other global and local heuristics. Examples of algorithms that have been hybridized with SA algorithms include greedy algorithms (Geng et al., 2011; Leung et al., 2012), tabu search (Osman, 1993; Mousavi and Tavakkoli-Moghaddam, 2013; Küçükoğlu and Öztürk, 2015; Lin et al., 2016) and genetic algorithms (Wong, 2001; Ganesh and Punniyamoorthy, 2005; Casas-Ramírez and Camacho-Vallejo, 2017). However, to the best of our knowledge, the hybridization of SA algorithm with CE algorithm has not been addressed in the literature.

The optimization model we have selected for testing the proposed hybrid SA-CE algorithm and for comparing the performance of this algorithm with the classic SA algorithm is a facility location model. This type of optimization model is amongst the most studied in the literature since the 1960s, and has been dealt with in fields such as operational research, supply chain management and regional science (see, e.g., Drezner and Hamacher, 2001; ReVelle and Eiselt, 2005; Melo et al., 2009; Teye et al., 2017; Amin and Baki, 2017). One of the best-known facility location models represents the following problem: given a set of potential sites for locating facilities, a set of demand centers with known demand values for the services provided in the facilities, and the distances between the sites and the demand centers, the aim is to determine where p facilities should be open so that the sum of the distances from facility users to their closest open facilities is minimized (Hakimi, 1964; 1965; ReVelle and Swain, 1970). Exact and heuristic methods have been proposed in the literature to solve the classic p-median model, as well as its multiple extensions (for a comprehensive review see, e.g., Mladenović et al. (2007)). In this work, we considered the Capacitated p-median model with single and Closest Assignment constraints (CPM-CA), which is an extension of the classical p-median model for which the facilities are required to comply with given capacity limits and the demand centers are required to be fully served by the closest open facility. The decision of tackling this extension of the p-median model was because of its practical relevance (Gerrard and Church, 1996) and because this model is much harder to solve than the classical p-median model.

The organization of this chapter is as follows. In the next two sections, we present the mathematical formulation of the CPM-CA model (Section 5.2) and the stages of the methodological approach followed in this study (Section 5.3). This is proceed by a detailed explanation of the classical SA algorithm in Section 5.4 and of the hybrid SA-CE algorithm in Section 5.5. The instances that have been generated, partly at random, for testing the performance of the algorithms are presented in Section 5.6, and the set of indicators used for assessing and comparing the performance of the two detailed algorithms are described in Section 5.7. In Section 5.8 we explain how the parameters of the algorithms were calibrated. The results obtained in the comparison of the algorithms are presented in section 5.9. Further insights into the performance of the hybrid algorithm are provided in Section 5.10. A summary of our main conclusions is presented in the final section.

5.2 Optimization model

We present in this section the mathematical formulation of the Capacitated p-Median Model with Closest Assignment Constraints (CPM-CA).

The notation we will use is as follows:

Sets:

I - set of potential users (demand centers);

J - set of m sites where facilities can be located (potential facility locations).

Parameters:

B_j^{min} and B_j^{max} - minimum and maximum capacity values for the facilities if located in $j \in \mathbf{J}$;

C_{ij} - total travel cost for serving all the users from demand centers $i \in \mathbf{I}$ at site $j \in \mathbf{J}$, which is given by $U_i \cdot D_{ij}$;

D_{ij} - distance between center $i \in \mathbf{I}$ and site $j \in \mathbf{J}$;

P – total number of located facilities;

U_i - total number of users for demand center $i \in \mathbf{I}$.

Decision variables:

x_{ij} - binary variable that takes 1 if the demand center $i \in \mathbf{I}$ is fully served by the facility located at $j \in \mathbf{J}$, and zero otherwise;

y_j - binary decision variable that takes 1 if a facility is located at site $j \in \mathbf{J}$, and zero otherwise.

Given this notation, the formulation of the CPM-CA is as follows:

$$\text{Max} \quad C = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} C_{ij} \cdot x_{ij} \tag{5.1}$$

subject to

$$\sum_{j \in \mathbf{J}} x_{ij} = 1, \quad , \forall i \in \mathbf{I} \tag{5.2}$$

$$x_{ij} \le y_j, \quad , \forall i \in \mathbf{I}, j \in \mathbf{J} \tag{5.3}$$

$$\sum_{j \in \mathbf{J}} y_j = P \tag{5.4}$$

$$\sum_{i \in \mathbf{I}} U_i \cdot x_{ij} \le B_j^{\max} \cdot y_j, \quad , \forall i \in \mathbf{I} \tag{5.5}$$

$$\sum_{i \in \mathbf{I}} U_i \cdot x_{ij} \geq B_j^{\min} \cdot y_j, \qquad , \forall i \in \mathbf{I} \tag{5.6}$$

$$\sum_{k \in \mathbf{J}: d_{ik} \leq d_{ij}} x_{ik} \geq y_j, \qquad , \forall i \in \mathbf{I}, j \in \mathbf{J} \tag{5.7}$$

$$x_{ij} \in \{0, 1\} \qquad , \forall i \in \mathbf{I}, j \in \mathbf{J} \tag{5.8}$$

$$y_j \in \{0, 1\} \qquad , \forall j \in \mathbf{J} \tag{5.9}$$

The objective function (5.1) minimizes the total travel costs. Constraints (5.2) and (5.3) state that all demand centers have to be fully served by open or located facilities, respectively. The total number of facilities that are open is set to P by constraint (5.4). The minimum and maximum capacity constraints are expressed by (5.5) and (5.6), respectively. The closest and single assignment properties required from the solutions are enforced by constraints (5.7) and (5.8).

Extensive analyses of alternative formulations for the closest assignment constraints were conducted by Cánovas et al. (2007) and Espejo et al. (2012). The former authors have shown that constraints (5.7) are mathematically dominated by constraints (5.10). However, after comparing the computation times for both formulations on the test instances described in the following section, we concluded that the model performs faster with constraints (5.7) than with constraints (5.10), which justifies our formulation choice.

$$\sum_{k \in \mathbf{J}: d_{ik} \leq d_{ij}} x_{ik} + \sum_{k \in \mathbf{J}: d_{ik} \leq d_{ij}, d_{ak} > d_{aj}} x_{ak} + y_j \leq 1, \qquad , \forall i, a \in \Pi, j \in \mathbf{J} \tag{5.10}$$

5.3 Methodological Approach

We developed an approach consisting of seven steps to assess and draw conclusions regarding the introduction of features of the CE algorithm to speed-up the SA algorithm.

We start by adjusting the classical SA algorithm to be suitable to solve the CMP-CA model, as detailed in Section 5.4. This procedure enables to exemplify the capability of the SA algorithm to handle facility location problems and also to draw conclusions about the implementation in what concerns the quality of the solutions and the computation effort required to achieved those same solutions.

This is followed by explaining how to implement the CE algorithm in order to provide guidance to the solutions assessed by the SA algorithm, i.e., how the hybridization takes place and how this new algorithm should be adjusted to be suitable to solve CPM-SA models (Section 5.5). As before, the capability of this new model to handle facility location problems will also be assessed, along with the achieved solutions and their computational effort.

As in any other heuristic, both algorithms have parameters that, for the sake of finding good solutions with the lowest computational effort possible, should be calibrated. Therefore, and to provide insight regarding the quality of the solutions and their computational effort, not only test instances should be

generated but also performance indicators should be established.

The generation of test instances takes into account the data included in the CPM-CA model, and ensures the existence of feasible solutions (see Section 5.6 for details). Two different sets are generated, one with several medium sized test instances (100 sites and 100 centers) and another set with variable sized test instances (number of sites and centers randomly selected from set $\{100, 150, 200, 250, 300\}$). The sets consider instances with variable capacity limits and variable demand, and fulfill different purposes. While the set with medium sized test instances serves to calibrate the two algorithms and to draw conclusions regarding the quality of their solutions and their computational effort following the established performance indicators, the second set is used to improve the analysis of the algorithm that provides better values for the identified performance indicators, which are detailed in Section 5.7.

In other words, after drawing conclusions regarding the calibration of the parameters of each algorithm (as detailed in Section 5.8), the comparison of the two algorithms takes place, which allow to conclude which one of the two algorithms had the best performance. This comparison is based on the results achieved for the performance indicators, as explored in Section 5.9.

To provide further details for the algorithm with the better performance indicators, a final analysis is developed. This analysis considers the second set of test instances, where different sizes are analyzed, with its main focus on large sized instances. This final analysis contributes to improve the insight for potential applications of the algorithm, such as its potential suitability to efficiently solve problems with time consuming objective functions with parameters calibrated for smaller test instances.

5.4   Simulated Annealing

A SA algorithm is a local search method inspired by the physical annealing process studied in statistical mechanics, where a solid material is heated until melting and then chilled till getting the lowest energy level according to an appropriate scheduling.

This algorithm starts by selecting an initial solution, which plays the role of being the current solution. A set of configurations or perturbations are defined to obtain candidate solutions from the current one, and the current solution's neighborhood is defined. If one generated candidate solution is better than the current one, then the current solution is replaced by the candidate one. In the opposite scenario, the change of the current solution by the candidate one occurs with some probability that depends on a parameter θ, called temperature. The lowest the temperature, the lowest the probability of replacing the current solution by a worse candidate solution. The implementation of this algorithm to solve the CPM-CA model is as follows.

It begins with an initial solution Y_1^1 and an initial temperature (θ_I), which is the "high temperature". A solution, either candidate y_{it}^j either current y_{it}^*, is characterized by the neighborhood ($N(y_{it}^j)$) and

$N(y_{it}^*)$), the objective function value (S_j^{it} and S_{it}^*) and the penalty term that accounts for the level of infeasibility of the solution in respect to the set of constraints of the CPM-CA model (5.2)-(5.9) (T_j^{it} and T_j^*).

Each iteration it of the SA algorithm analyzes at least NS candidate solutions at the same temperature ($j = 1,\dots,NS$). A candidate solution is accepted according to the following criteria: (1) it improves the value of the objective function added with the penalty term or (2) the difference between the objective function with the penalty factor of the two solutions satisfies the probability condition defined according to the Boltzmann distribution. This second acceptance condition of candidate solutions is influenced by the degree of degradation of the objective function with the penalty factor (the smaller the degradation, the greater the acceptance probability) and the temperature (θ). By including the temperature as a parameter of the Boltzmann distribution, which sets the probability of changing the current solution by a candidate solution, the algorithm becomes more selective at accepting worse solutions by progressively decreasing the temperature (Kirkpatrick et al., 1983).

The temperature parameter changes according to the improvement of solutions from one iteration to the following one. This change of the temperature results from multiplying its current value θ by the cooling rate ϕ, occurring if neither the current solution is changed in two successive iterations nor the average objective function of the candidate solution of following iteration is better than the average objective function value of the former candidate solutions. The cooling rate is known to be critical to the efficiency of SA.

With this procedure, the implementation of SA ends by setting a stop condition that, for instance, is defined by a minimum value for the temperature (i.e., a final temperature θ_F). This adaptation of the general SA algorithm to solve the CPM-SA problem explored in this chapter is detailed in the pseudo code of Figure 5.1, where the role of the set of parameters NS, θ_I, θ_F and ϕ (i.e., number of candidate solutions explored in an iteration, initial temperature, final temperature and cooling rate) is specified, along with the penalty factor T_j^{it} that affects the objective function if the candidate solution is unfeasible.

---

(0)   Determine the initial binary solution Y_1^1 and the set of parameters $\{\theta_I, \theta_F, \phi, NS\}$;

(1) $it \leftarrow 0; \theta_{it} \leftarrow \theta_I$ and $Improving \leftarrow 0$

(2)   **While $\theta_{it} \geq \theta_F$ do**

   (2.1)   The following operations are executed for $j \leftarrow 1, ..., NS$:

       i.   The solution y_{it}^j is randomly chosen from the neighborhood $N(y_{it}^*)$ as follows:

         A)   Randomly select an opened facility from y_{it}^*, and close it;

         B)   Randomly select an closed facility from y_{it}^*, and open it;

       ii.   Determine the objective function value S_j^{it}, which includes a penalization factor T_j^{it} that depends on the unfeasibility of the solution.

       iii.   Choose $p \in [0, 1]$

       iv.   **If $p \leq \min\left\{1, \frac{exp(S_{it}^* - S_j^{it})}{\theta_{it}}\right\}$ then:**

         A) $S_{it}^* \leftarrow S_j^{it}$;

         B) $y_{it}^* \leftarrow y_j^{it}$;

       End-if

   (2.2) $S_{it}^* \leftarrow \min_j \{S_j^{it}\}$ and $y_{it}^* \leftarrow \{y_{it}^m : S_{it}^m = y_{it}^*, m = 1, ..., NS\}$

   (2.3)   **If $S_{it}^* < S^*$ then**

       i. $S^* \leftarrow S_{it}^*$

       ii. $y^* \leftarrow y_{ij}^*$

       iii. $Improving \leftarrow 1$

   End-if

   (2.4)   **If the average value of $\{S_j^{it}\} >$ average value of $\{S_j^{it-1}\} >$ and $Improving == 0$ then**

       i. $\theta_{it+1} \leftarrow \theta_{it} \cdot \phi$

   End-if

   (2.5)   Pass to the next iteration ($it \leftarrow it + 1$)

   End-While

Figure 5.1: Pseudo code of the applied SA algorithm.

5.5   Hybrid Simulated-Annealing Cross-Entropy Algorithm

In this section, we present how the hybridization of the SA with features of the CE algorithm was made, in order to decrease the number of poor candidate solutions that are tested while running the SA algorithm. In this sense, features of the CE algorithm are introduced in the SA algorithm to help guiding the generation of candidate solutions.

The CE algorithm was developed as an adaptive technique for estimating probabilities in the context of rare events in complex stochastic networks. As explained by De Boer et al. (2005), the main idea behind the CE algorithm is to solve the transformation of an original optimization model into an associated stochastic model, which is afterwards efficiently tackled by an adaptive algorithm. This allows to construct random sequences of solutions that will probabilistically converge to the optimal or near-optimal solution of the original model. The two phases that are explored by the CE algorithm after

defining the associated stochastic model, are the following (De Boer et al., 2005):

1. *Generate a random data sample (trajectories, vectors, etc.) according to a specified mechanism.*

2. *Update the parameters of the random mechanism based on the data to produce a "better" sample in the next iteration.*

Based on the explanations of De Boer et al. (2005), we developed a procedure to guide the process of choosing a new candidate solution in the SA algorithm (see Section 5.4). To such end, we start by defining the probability distribution linked to each facility to be open in a given site. Let $W = [P_1, P_2, \ldots, P_W]$ be this probability, which is assessed based on a proportion γ of the best candidate solutions analyzed so far, allowing at generating new candidate solutions. In this sense, the guidance provided by the CE algorithm is divided into two different stages: (1) the estimation of a new solution y_{it}^{j}, and (2) the update of the probability distribution. These two stages will take place in step i. and in step (2.2) of the SA pseudo code shown in Figure 5.1, respectively. The pseudo code for these two stages are detailed in Figure 5.2, where the role of the parameters W and γ, the probability distribution and the proportion of the best candidate solutions, respectively, are specified.

Stage 1.

    i. The solution y_{it}^{j} is randomly chosen from the neighborhood $N(y_{it}^{*})$ as follows:

        A. Select an opened facility from y_{it}^{*} and close it, according to the probability distribution generated by normalizing the opposite probability of all open facilities;

        B. 2. Select a closed facility from y_{it}^{*} and open it, according to the probability distribution generated by normalizing the opposite probability of all close facilities;

Stage 2.

(2.2) $S_{it}^{*} \leftarrow \min_{j}\{S_{j}^{it}\}$ and $y_{it}^{*} \leftarrow \{y_{it}^{m} : S_{it}^{m} = y_{it}^{*}, m = 1, \ldots, NS\}$ and update the probability scores of each site as follows:

    i. Define the vector O with the ascending order all the objective functions $S_{j}^{it}, \forall j, it$;

    ii. Select the first $\gamma\%$ elements of set i.;

    iii. Determine the number of times that a given site of y is open according to the set defined in ii.;

    iv. Assign the minimum and maximum scores P_1 and P_w to the minimum and maximum values found in iii. for the number of times that a facility is opened;

    v. Assign an element of W to each site according to its rank in the number of times that a facility is opened in the set selected in ii., taking into account the boundaries P_1 and P_w, as defined in iv..

Figure 5.2: Pseudo code of the changes introduced on the SA algorithm by embedding characteristics of the CE algorithm.

5.6 Test Instances

We now describe how the test instances considered in this study were generated and how we use them in our experimental analysis. As explained in the methodological approach (Section 5.3), the deve-

lopment of test instances is required to calibrate and assess the performance of the algorithms detailed in this chapter.

A first set of 10 instances is set up for a total of 100 sites and 100 centers to calibrate both the SA algorithm and the hybrid algorithm, as well as drawing conclusions regarding their performance. The calibration of the parameters included in each algorithm is needed due to their influence on both the quality of the returned solutions and on the computational effort required to find those solutions.

A second set of 25 instances is generated considering different sizes for the total number of sites and centers (5 test instances for each one of the following sizes $\{100, 150, 200, 250, 300\}$). This second set of test instances will be used to assess the robustness and the performance of the algorithm that have the best results for the first set of test instances, for both the quality of the returned solutions and the effort of computing them.

After setting the number of sites (m), we proceed by defining the number of facilities P that must be located. This value is uniformly generated between 10% and 30% of the total number of sites. The site coordinates are uniformly generated within a square with side m. Assuming that sites and centers are placed coincidently, the Euclidian distance between them is determined (i.e., D_{ij}). Demands U_i for each center are randomly generated in the interval [1, 100].

The capacity limits of the facilities are generated as follows for all sites. An element of the set $\{0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5\}$ is randomly selected and subtracted/added to one, this value is then multiplied by the quotient $\sum_{I \in \mathbf{I}} \frac{U_i}{P}$, which sets the minimum/maximum capacity limits for each site.

The existence of feasible solutions for each generated instance should be ensured. To such end, we used one of the top-quality software packages available – XPRESS Version 7.7 (FICO, 2014) – to solve the CPM-CA optimization models for each generated test instance. Note that even though the existence of feasible solutions for the generated instances is guaranteed, it does not mean that the SA and the hybrid algorithms will find a feasible solution.

5.7   Performance Indicators

The assessment and comparison of the performance of the algorithms, as well as the calibration of their parameters, took into consideration the performance indicators that follows.

The first performance indicator (#1) is the number of feasible solutions returned by the algorithms when a set of test instances with similar characteristics are computed. Note that even though the existence of an optimum solution is ensured while generating the test instances, the capability of an algorithm to find a feasible solution depends mainly on the values that are selected for the algorithm's parameters and the quality of the candidate solutions that are generated, which highlights the importance of assessing this performance indicator.

The second performance indicator (#2) accounts for the objective function of the returned solution, if this solution is feasible. The returned solution is the current solution of the algorithm when the run of an algorithm ends.

The third performance indicator (#3) concerns the computation time needed to solve a test instance taking into account a set of parameters. It is aimed with this indicator to provide insights about the performance of the algorithms for different sets of parameters in what concerns the computation time required to return a good feasible solution. With this indicator, we are also aware of the cost of using a SA algorithm and the cost of adding features of the CE algorithm to the SA algorithm.

Finally, to evaluate and compare these algorithms in what concerns their utility and capability of solving problems when a single solution is extremely time consuming, four performance indicators are also considered: (#4) number of iterations needed to find the solution returned; (#5) number of different solutions analyzed; (#6) the iteration where the returned solution is examined for the first time; and (#7) the computation time needed to achieve the returned solution. These performance indicators also help at assessing the robustness of the algorithm in what concerns the random ingredients that are included in their implementation.

These detailed seven performance indicators are summarized in Table 5.1.

Table 5.1: Summary of the most important parameters used for the computational experiments.

Indicator	Definition
#1	Returned solution is feasible.
#2	Objective function of the returned solution (if feasible).
#3	Computational time of the returned solution (if feasible).
#4	Number of iterations needed to find the returned solution.
#5	Number of different solutions analyzed by the algorithm.
#6	Number of iterations needed to analyzed for the first time the returned solution.
#7	Computation time to analyze for the first time the returned solution.

5.8 Algorithms Calibration

Appropriate calibration of the algorithms' parameters is essential to have good algorithms' performances, guarantying the quality of the returned solutions within reasonable computational effort. This process, as well as the remaining analysis developed through this chapter, was coded in Matlab R2015a on a computer with a 3.5 GHz i7 processor with 32 GB RAM, and a Windows 10 64-bit operating system.

5.8.1 Simulated Annealing

The SA algorithm requires the calibration of 4 parameters: the initial temperature θ_I , the penalty factor $T_{Y'}$ the number of candidate solutions analyzed in the same temperature NS and the cooling rate

ϕ. The final temperature θ_F is fixed and equal to 1E-04.

The calculation of the initial temperature is as follows. A value from the set $\alpha \in \{0.05, 0.1, 0.2, 0.3, 0.4\}$ is randomly selected and an initial candidate solution Y' is randomly generated. Assessing the objective function value $S_{Y'}$ and the penalty factor $T_{Y'}$ as explain below, the initial temperature is given by $\theta_I = \left\| \frac{S_{Y'} + T_{Y'}}{log(\alpha)} \right\|$.

Considering a random element selected from the set $\beta \in \{1, 2, 3, 4, 5, 10\}$, the maxima travel distances D_{ij}, the average demand $\overline{U} = \frac{\sum_{l \in \mathbf{I}}}{P}$ and the number of constraints that are unsatisfied, let us say $UC_{Y'}$ the calculation of the penalty factor $T_{Y'}$ is given by $max_{i \in \mathbf{I}, j \in \mathbf{J}}\{D_{ij}, 0\} \cdot \overline{U} \cdot UC_{Y'} \cdot \beta$.

The number of candidate solutions explored in an iteration, i.e. NS, is the result of multiplying the number of sites (m) by a randomly chosen element from the set $\{2, 4, 5, 7, 10\}$, whereas the cooling rate ϕ is randomly selected from the set $\{0.5, 0.6, 0.7, 0.8, 0.9\}$.

Taking this into account, the calibration of the SA algorithm takes place by considering the generation of 100 different combinations of these 4 parameters. The algorithm was run considering each one of the 100 parameters' set for the first 10 test instances detailed in Section 5.6, with 100 sites and 100 centers. The performance indicator #1 (returning a feasible solution), and the average and standard deviation of the performance indicators #2 and #3 (objective function and computation time, respectively), are determined for each set of parameters taking into the 10 test instances, as displayed in Table 5.2 for some sets.

The first performance indicator and the average and standard deviation values of the remaining two performance indicators across the 10 test instances are used to select the best values for the 4 parameters under consideration. This selection was as follows. We started by selecting select the set of parameters with at least 9 returned feasible solutions from the total 10 (performance indicator #1). This led to 8 out of the 100 analyzed set of parameters. From these 8 sets, the first 4 sets displayed in Table 5.2 were selected by considering the best averages for parameter #2. These 4 set of parameters will be used from now on in all the analysis developed considering the SA algorithm. In Table 5.2 we also show the 10 best set of parameters, which include the 4 selected sets, and the worst 10 sets of parameters, along with the performance indicator #1 and the average and standard deviation (std. dev.) values of performance indicators #2 and #3 used to choose the set of parameters included in the following applications of the SA algorithm.

Table 5.2: Set of parameters selected to calibrate the SA algorithm.

Set of parameters	θ	NS	α	β	Indicator #1	Avg indicator #2 (10^3) (std. dev. 10^3)	Avg indicator #3 (10^3) (std. dev. 10^3)
1	0.9	2	0.4	10	1	41.38 (12.45)	46.44 (1.88)
2	0.9	4	0.4	5	1	41.4 (10.91)	144.56 (4.47)

3	0.9	7	0.05	1	1	41.71 (11.85)	151.77 (6.16)
4	0.9	7	0.4	1	0	41.86 (12.87)	90.64 (4.79)
5 (Not Selected)	0.9	10	0.1	2	1	42.2 (11.25)	208.02 (7.75)
6 (Not Selected)	0.9	5	0.4	2	1	42.32 (11.3)	116.01 (5.57)
7 (Not Selected)	0.8	5	0.4	2	1	43.01 (12.17)	55.23 (3.22)
8 (Not Selected)	0.8	4	0.3	10	0	43.33 (10.47)	43.47 (2.54)
9 (Not Selected)	0.9	7	0.2	10	2	40.97 (11.32)	158.65 (11.64)
10 (Not Selected)	0.9	7	0.4	10	2	41.1 (11.01)	159.73 (12.45)
				...			
91 (Not Selected)	0.6	2	0.2	1	7	43.21 (9.97)	8.85 (0.64)
92 (Not Selected)	0.5	5	0.2	2	7	44.09 (13.35)	16.73 (1.16)
93 (Not Selected)	0.5	7	0.2	4	8	44.29 (13.85)	25.51 (2.36)
94 (Not Selected)	0.6	10	0.05	10	8	45.59 (13.39)	48.44 (4.34)
95 (Not Selected)	0.5	7	0.1	5	7	45.74 (12.83)	25.82 (3.46)
96 (Not Selected)	0.6	2	0.2	10	6	45.87 (12.5)	9.82 (0.64)
97 (Not Selected)	0.5	7	0.3	2	8	45.97 (13.06)	24.11 (2.17)
98 (Not Selected)	0.6	2	0.05	2	7	46.34 (13.13)	8.9 (1)
99 (Not Selected)	0.5	10	0.3	5	7	46.45 (15.74)	38.27 (3.28)
100 (Not Selected)	0.6	2	0.1	4	7	47.87 (15.6)	9.18 (0.82)

5.8.2 Hybrid Algorithm

In what concerns the calibration of the hybrid algorithm, the parameters also calibrated in the SA algorithm were kept within the ranges previously detailed, i.e. the initial temperature θ_I, penalty factor $T_{Y'}$ the number of candidate solutions analyzed in the same temperature NS and the cooling rate ϕ (see subsection 5.8.1 for details). The two additional parameters required for the guidance process of generating new solutions, i.e. the set of probabilities W and the proportion of solutions γ used to set these probabilities, were calculated as follows.

The set of probabilities W was defined with 3 different scores: low, medium and high. The medium score was assigned to probability 0.5, because a given site with the characteristics of being half open in the γ proportion of solutions and the remaining half times closed, has 50/50 chances to belong to the optimum solution. The highest and lowest scores are set together considering the same distance to the average value. Therefore, the set of values {0.1/0.9; 0.2/0.8; 0.25/0.75; 0.3/0.7; 0.4/0.6} for the lowest/highest

scores are considered.

The parameter γ that defines the proportion of elements analyzed to define the guidance of the algorithm is randomly selected from the set $\{0.05, 0.1, 0.15, 0.2, 0.25\}$.

The calibration procedure for this hybrid approach is similar to the one developed for the calibration of the SA algorithm. I.e., 100 combinations of the 6 parameters introduced in this hybrid algorithm are randomly selected. The algorithm is then applied to each one of the 10 test instances. The three performance indicators (#1, #2 and #3) are then determined for each set of parameters and test instances and the final sets of parameters are selected. The final values of performance indicator #1 and the average and standard deviation of parameters #2 and #3 are displayed for some sets of parameters in Table 5.3.

As to the calculation of the parameters of the SA algorithm, the set of parameters with at least 9 feasible returned solutions from a total of 10 possibilities was selected. This led to 97 out of the 100 analyzed set of parameters, which is a significant improvement when comparing the results achieved by the SA algorithm without introducing the CE guidance (8 out of the 100 analyzed set of parameters, see Table 5.2 for further details). Due to this high number of 97 set of parameters with at least 9 out of 10 feasible solutions, we decided to pick representative sets in terms of both parameters and indicators values, taking into account the following criteria. The first two sets were selected considering being the best ones in the average of the second performance indicator (objective function values) with an average computation time below 1000 seconds (sets 1 and 2, shown in Table 5.3). The set of parameters 3 and 4 displayed in Table 5.3 were selected because they were the best ones for the second criteria with an average computation time below 50 seconds.

Table 5.3: Set of parameters selected to calibrate the hybrid algorithm application.

Set of parameters	θ	NS	α	β	γ	Probabilities W	Indicator #1	Avg indicator #2 (10^3) (std. dev. 10^3)	Avg indicator #3 (10^3) (std. dev. 10^3)
1	0.7	2	0.05	10	0.1	[0.4,0.5,0.6]	0	36.08 (10.51)	447.7 (17.43)
2	0.9	10	0.4	4	0.25	[0.3,0.5,0.7]	0	36.17 (10.53)	215.87 (18.35)
3	0.8	4	0.1	1	0.1	[0.2,0.5,0.8]	0	36.38 (10.44)	45.1 (4.57)
4	0.6	5	0.2	2	0.05	[0.25,0.5,0.75]	0	36.45 (10.26)	14.14 (1.15)
5 (Not Selected)	0.7	10	0.3	2	0.15	[0.4,0.5,0.6]	0	36.1 (10.55)	5073.08 (325.51)
6 (Not Selected)	0.5	7	0.3	5	0.15	[0.25,0.5,0.75]	0	36.15 (10.52)	2947.49 (133.19)
7 (Not Selected)	0.9	5	0.05	4	0.25	[0.2,0.5,0.8]	0	36.2 (10.71)	329.4 (14.4)
8 (Not Selected)	0.8	2	0.4	5	0.05	[0.25,0.5,0.75]	0	36.27 (10.44)	200.85 (10.27)
9 (Not Selected)	0.9	4	0.4	2	0.1	[0.4,0.5,0.6]	0	36.32 (10.25)	276.89 (8.42)
10 (Not Selected)	0.5	10	0.4	4	0.1	[0.25,0.5,0.75]	0	36.32 (10.33)	291.82 (17.18)

...									
91 (Not Selected)	0.9	4	0.2	5	0.1	[0.3,0.5,0.7]	1	37.42 (9.82)	55.09 (4.05)
92 (Not Selected)	0.9	7	0.2	5	0.05	[0.1,0.5,0.9]	1	37.44 (9.85)	9.24 (0.98)
93 (Not Selected)	0.8	2	0.05	1	0.25	[0.1,0.5,0.9]	0	37.49 (10.15)	55.14 (3.76)
94 (Not Selected)	0.7	10	0.4	10	0.15	[0.3,0.5,0.7]	1	37.7 (11.77)	11.71 (0.82)
95 (Not Selected)	0.9	4	0.1	2	0.1	[0.2,0.5,0.8]	1	37.74 (11.34)	10.1 (0.6)
96 (Not Selected)	0.6	10	0.1	4	0.2	[0.2,0.5,0.8]	0	37.84 (9.47)	10.87 (0.76)
97 (Not Selected)	0.6	4	0.4	5	0.2	[0.2,0.5,0.8]	0	37.87 (11.7)	9.55 (0.86)
98 (Not Selected)	0.8	4	0.4	4	0.2	[0.1,0.5,0.9]	2	37.72 (11.78)	10.06 (1.03)
99 (Not Selected)	0.7	10	0.2	10	0.1	[0.3,0.5,0.7]	2	36.77 (10.35)	9.12 (0.75)
100 (Not Selected)	0.5	4	0.3	1	0.25	[0.4,0.5,0.6]	2	37.02 (11.09)	11.97 (1.37)

5.9 Comparative Results

We now aim at comparing the two algorithms and drawing conclusions regarding the potential improvements of introducing the CE guidance into the SA algorithm.

At first, the robustness of each algorithm is analyzed by considering their 4 best sets of parameters (see Section 5.8 for further details). By robustness it is understood the sensitiveness of each algorithm towards their random components, and it is assessed by the performance indicators #4, #5, #6 and #7. This analysis was conducted by running the algorithms 5 times in order to solve the 10 instances used in the calibration procedure and the final 4 sets of parameters detailed in Table 5.2 and Table 5.3, for the SA algorithm and the hybrid algorithm, respectively.

These results were also used in a second analysis to draw conclusions about which model is more suitable at providing good solutions for problems with time consuming objective functions within a reasonable amount of computation time.

We start the comparison of the two algorithm by exploring the average and standard deviation of the performance indicator #2 and the box plot drawn for the results achieve for the performance indicator #3, which concern the objective function values and the computation time. These values are shown in Table 5.4 and Figure 5.3, respectively. Note that the red line presented in each boxplot displayed in Figure 5.3 represents the average of performance indicator #3, and the blue structure accounts for the dispersion of the computation time values around their average values.

Table 5.4: Performance indicator #2 for the SA algorithm and the Hybrid algorithm.

Set of Parameters	SA algorithm:	Hybrid algorithm
1	44.13 (10.08)	38.2 (10.36)
2	45.27 (11.7)	36.44 (9.98)
3	43.45 (10.85)	36.88 (10.08)
4	44.6 (11.7)	36.95 (10.00)

a) SA Algorithm
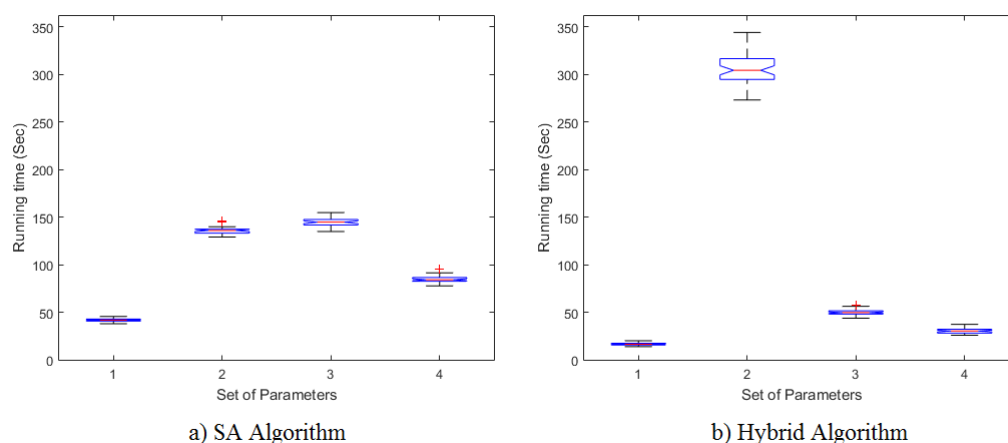
b) Hybrid Algorithm

Figure 5.3: Performance indicator #3 for the (a) SA algorithm and the (b) Hybrid algorithm.

In general, the guidance provided by the CE algorithm to the SA algorithm (hybrid algorithm) improves significantly the quality of the solutions (Table 5.4). However, a trade-off between the quality of the solutions and the computation times is highlighted when these results are crossed with their computation times (Figure 5.3). This is specially observed in the results of the hybrid algorithm with the calibration of the set of parameters 2. Excluding this set of parameters for the hybrid algorithm analysis, we also conclude that the hybrid algorithm reveals a less time consuming performance than the SA algorithm.

However, none of these analyses provided insight capable of drawing conclusions about which algorithm was more appropriate at solving problems with time consuming objective functions. In the light of assessing our suspicions about the improvement provided by the hybridization, the average and standard deviation of the four performance indicators linked to the algorithms' robustness were considered (i.e., #4, #5, #6 and #7). As before, these average and standard deviation values were calculated concerning each set of parameters and taking into account the 5 experiences for each one of the 10 test instances under consideration (Table 5.5).

The performance indicator #4 determines the number of analyzed solutions by each algorithm. This indicator is complemented by the performance indicator #5, since this last one does not allow the coun-

ting of repeated solutions. The analysis of performance indicator #5 is motivated by the fact that in time consuming objective function problems, it might be sometimes preferable to verify if a solution was previously analyzed than re-analyzing its contribution to the problem. This statement supports the idea that the best approach is the one that consumes less time. However, this issue is not put at stake in the CPM-CA case, where all solutions were systematically analyzed. Finally, to improve the insight regarding which part of the algorithm is costlier, the procedure of looking for a good solution or to ensure that a good solution is actually good or even optimal, the final two performance indicators explained in Section 5.7 are considered. Therefore, the average and standard deviation of the iterations where the returned solution was examined for the first time (performance indicator #6) are considered along with the average and standard deviation of the computation time needed to achieve such solutions (performance indicator #7). The average and standard values of the 4 performance indicators are displayed in Table 5.5.

Table 5.5: Performance indicators for each algorithm considering its best calibration parameters set.

Algorithm	Set Parameters	Avg indicator #4 (std. dev.)	Avg indicator #5 (std. dev.)	Avg indicator #6 (std. dev.)	Avg indicator #7 (std. dev.)
SA	1	86.44 (3.59)	27.22 (3.9)	64.8 (14.68)	34.6 (6.75)
	2	268.49 (7.44)	58.93 (7.34)	189.99 (48.54)	112.4 (22.48)
	3	284.76 (8.35)	76.65 (7.56)	191.56 (55.02)	110.52 (26.26)
	4	167.92 (5.8)	46.74 (6.92)	124.13 (29.95)	69.93 (14.66)
Hybrid	1	27.72 (1.99)	12.68 (3.04)	14.05 (4.02)	8.14 (2.39)
	2	432.54 (18.88)	158.42 (20.56)	168.78 (27.05)	95.75 (15.46)
	3	80.18 (3.89)	30.13 (4.8)	33.02 (6.14)	18.64 (3.67)
	4	50.65 (4.3)	24.37 (4.84)	26.68 (5.81)	15.23 (3.71)

The analysis of these results is as follows. In general, the SA algorithm analyzes (Table 5.5) more solutions than the hybrid algorithm, even though the final solutions returned by the former are worse than the ones returned by the later (Table 5.4). This conclusion is especially relevant when the performance indicator #5 is analyzed. In fact, the hybrid algorithm analyzes fewer number of solutions (#4) with a smaller number of repetitions (#5). These conclusions reveal that the purpose of introducing the CE algorithm as guidance mechanism for SA algorithms is met. The standard deviations calculated for these two performance indicators show that SA algorithm results are generally more subjected to the calibration procedure than the hybrid algorithm. This is a central issue when the analysis of each solution is extremely time consuming, because the calibration mechanism requires the running of the algorithm several times.

The two last performance indicators stress the following. SA algorithm puts more effort on the search procedure than in closing the gap that ensures that the solution final solution is in fact good (in average, the returned solution is reached after about 70% of solutions have been analyzed). This suggests the reliance of the solutions on the parameters included and calibrated on the SA algorithm, which is in

accordance with the results shown in Table 5.4, where the SA solutions are quite far from the optimal solutions.

The opposite is observed in the solutions achieved by the hybrid algorithm, highlighting the suitability of this algorithm to deal with the problems with time consuming objective functions. In fact, the hybrid algorithm is faster at finding good solutions than the SA algorithm. In average, the returned solution is achieved by the hybrid algorithm in the first half of the analyzed solutions, while the same is not stated in the performance indicators drawn for the SA algorithm (Table 5.5). This suggests that in cases where the computation time is more important than achieving the optimum solution, the calibration procedure might be developed so that an earlier stop condition can be integrated in the algorithm.

These conclusions are sustained by the candidate solutions analyzed by both algorithms, using the 3rd set of parameters, which were applied to solve the CPM-SA problem for one of the 10 generated cities. In Figure 5.4, the objective function value of each analyzed solution is shown for the two algorithms, along with the solutions accepted by the Boltzmann distribution. By comparing the candidate solutions generated for each algorithm, it is observed that the SA algorithm accepts worst solutions than the hybrid algorithm (the higher the objective function value of the grey dots, the worst the candidate solution is). In what concerns the best solutions found by each algorithm, the hybrid algorithm reveals a smoother search than the SA algorithm. This ensures that the inclusion of features of the CE algorithm into the SA algorithm serves the purpose of guiding the search procedure, corroborating the conclusions drawn for the relationship between the time that each algorithm spends on analyzing a neighborhood of a good solution or pursuing to find a good solution.
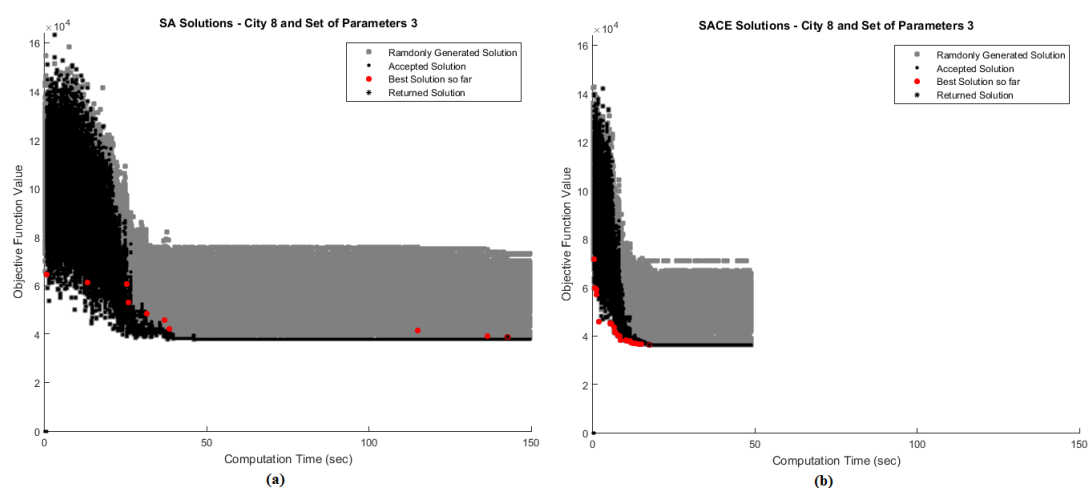
Figure 5.4: Solutions explored for each one of the two algorithms: (a) SA and (b) Hybrid.

Summing up the conclusions drawn so far, the hybrid algorithm seems to be more efficient than the SA algorithm into solving optimization models, with and without time-consuming objective functions. It is also safe to argue that the introduction of a guidance procedure will not jeopardize the SA algorithm,

leading to an algorithm that is at least as good as the classical one.

5.10   Computational Study

The hybrid algorithm incorporates the good characteristics of SA algorithms and attempts to improve its short comes in what concerns the generation of candidate solutions. As it was shown, the guided generation of candidate solutions has revealed to be a good option at improving the quality of the candidate solutions explored by the SA algorithm. To improve our knowledge about the designed hybrid algorithm, we solve the CPM-CA model with the second set of randomly generated test instances (see Section 5.6). This set of new instances includes 5 different instances for each element of the set $\{100, 150, 200, 250, 300\}$.

The use of the hybrid algorithm includes the four sets of parameters estimated in subsection 5.8.2 (see Table 5.3) to solve each one of the 25 problems. Although these parameters were the result of a calibration procedure that took place for test instances with size 100, we also want to assess if parameters estimated for smaller instances are suitable at solving larger instances. This conclusion along with the capability of the algorithm at finding good solutions for problems with different sizes are the main focus of this section.

We start our analysis by considering the performance indicator #1, which is displayed by way of proportion in Figure 5.5. This performance indicator reveals that the calibration procedure is suitable until a certain extent of the relationship between the size of the problems used to do the calibration of the parameters and the size of the problems being solved by that same calibrated parameters. Analyzing these results, a maximum of 95% of the solutions returned by the set of parameters 2 and 4 for the instances with size 250 were feasible. However, observing the solutions returned for 300 sized-instances, it is possible to conclude that all the solutions returned by these two sets of parameters were feasible. In this sense, no further conclusions can be drawn by using this performance indicator due to the fact of the results do not reveal a pattern, which require the analysis of the remaining performance indicators.
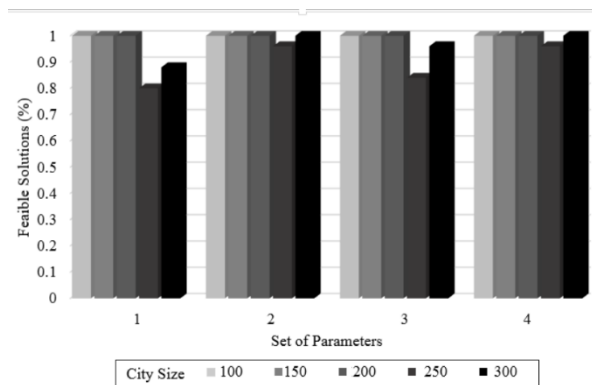
Figure 5.5: Proportion of the number of feasible solutions returned by the SA-CE algorithm for each instance and set of parameters.

To raise awareness regarding the quality of the feasible solutions, the performance indicators #2 and #3 were estimated. These two performance indicators are displayed in Figure 5.6 for each set of parameters and each test instance size, where a boxplot was drawn for each case. The red line presented in each boxplot represents the average of each performance indicator, and the blue structure accounts for the dispersion of the objective function values and the computation time values around their average values.

Objective Function Values (Performance Indicar #2)

Computation time (Performance Indicator #3)
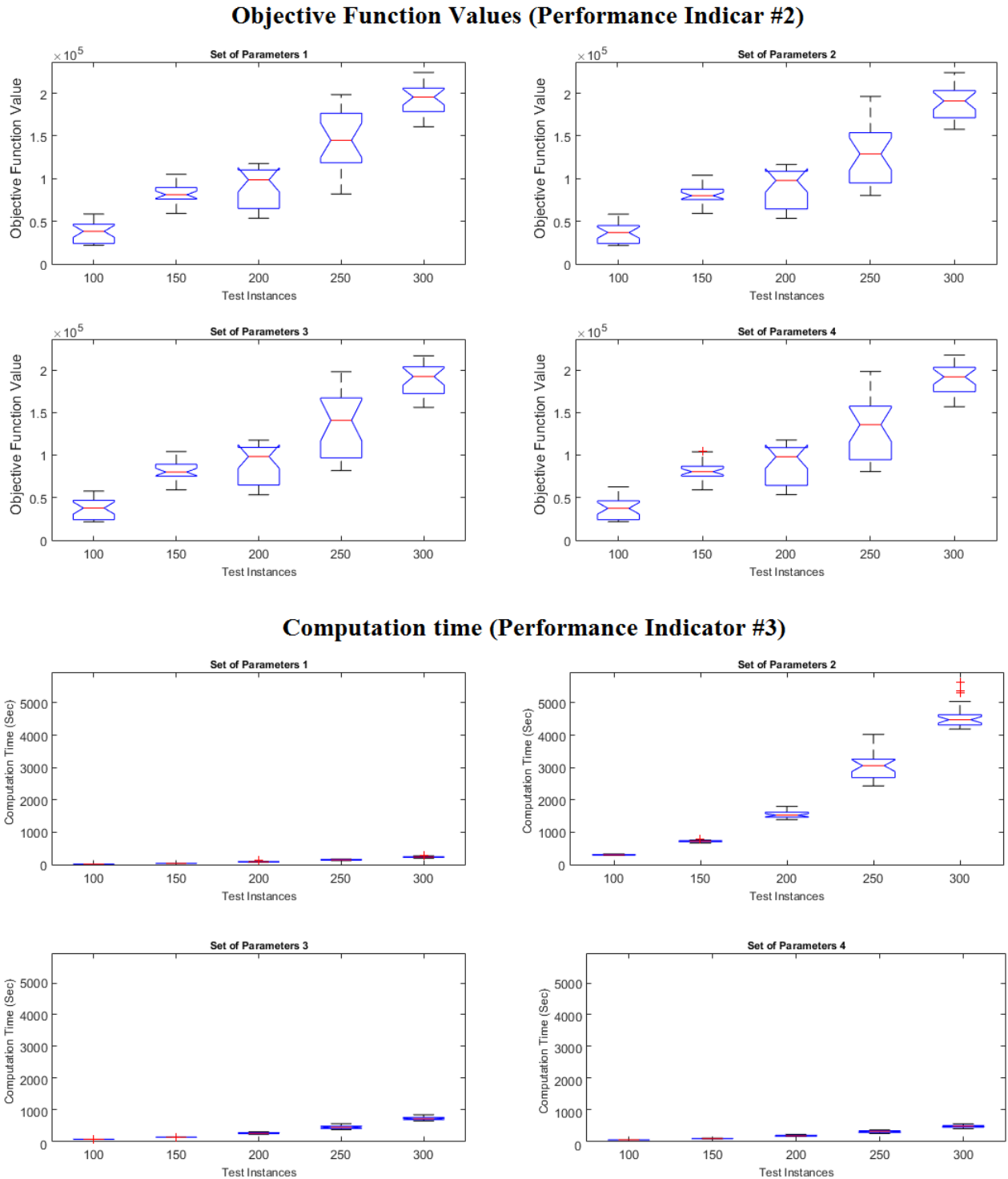
Figure 5.6: Performance indicators #2 and #3 for each test instance and set of parameters.

Analyzing the results shown in Figure 5.6, we conclude that average solutions achieved by the algorithm are quite similar across the sets of parameters under consideration for the test instance with size not higher than 200. For the remaining 2 test instances, although smooth, the objective function values achieved by the set of parameters 2 and 3 revealed better results when compared to the remaining parameters set. However, the same cannot be argued for the computation times, where the set of parameters 2 revealed higher computation times than the remaining set of parameters. This trade-off between the quality of the solution and the computation time should be carefully analyzed while applying this heuristic.

We also provide the outcomes for the average and standard deviation of the remaining four performance indicators to raise awareness in what concerns the applicability of this hybrid algorithm at solving problems with time-consuming objective functions and to provide insight towards the trade-off previously mentioned regarding the solution's quality and the computation time. These results are shown in Figure 5.7.

As before, the set of parameters 2 is the one that analyzes more solutions, either in total (#4) or unique (#5). The set of parameters 1 is the one with less computational time in what concerns the generation and analysis of the returned solutions and the one that analysis fewer solutions. This may give clues about the suitability of this set of parameters to solve our problem. If the purpose of solving the time-consuming objective function problem is to find a good solution in a short amount of time, the use of this set of parameters might be a good option. Furthermore, the calibration procedure can be performed for simpler problems (whatever this might mean in a particular context of time-consuming objective function problems), and then used to solve more complex problems. For instance, smaller problems might be used to calibrate the heuristics, and the resultant parameters used to solve larger problems. These conclusions are in line with the conclusions taken so far.

As we expected, the size of the problem influences the number of solutions that were analyzed as well as the time needed to find and return the final solution (Figures 5.6 and 5.7). I.e., the greater the size of the problem, the higher the amount of potential solutions are analyzed and the more time consuming the algorithm is.

We recall that the calibration procedures were conducted for problems with the size of the smaller test instance (100), even though they seem to be suitable at solving problems with double of their calibration problems' size. A re-calibration of the parameters that account for larger problems might also be developed. in order to assess the quality of the solutions achieved by the parameters resultant from the calibration of smaller instances are good. The presented results reveal evidence pointing in this direction. However, this procedure will be developed and assessed in our future work, because it is beyond the scope of this work, which mainly concerns the comparison of the classical SA algorithm with a hybridization of the classical SA algorithm with a guidance mechanism inspired on the CE algorithm.
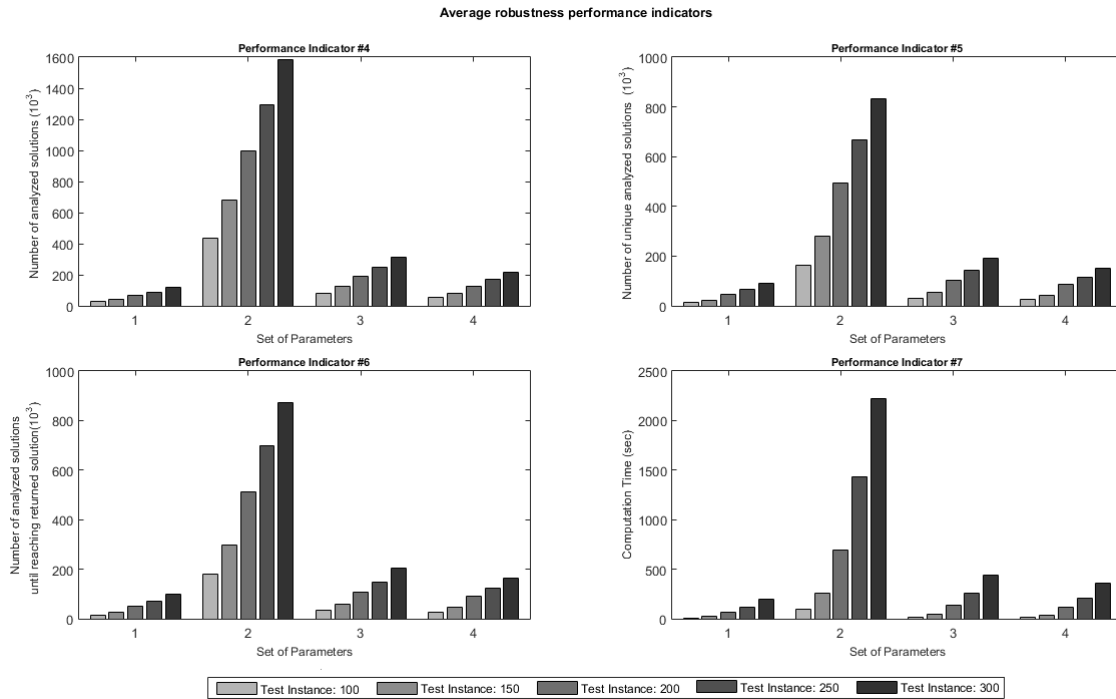
Figure 5.7: Average robustness performance indicators for the 4 set of parameters set.

5.11 Conclusion

In this chapter, we presented a guided procedure to help the classical Simulated Annealing algorithm at choosing good candidate solutions instead of blindly generated poor ones to be analyzed. This guidance is developed by introducing ingredients from the classical Cross Entropy algorithm into the SA algorithm, leading to a hybrid Simulated-Annealing-Cross-Entropy algorithm. The motivation of this hybridization is to develop a good solving method for problems where the analysis of the objective function is extremely time consuming, which usually are harder to be optimally solved within reasonable computational times. To provide details and take conclusions about the introduction of this guidance procedure, we opted to implement the two algorithms in the light of a capacitated p-Median problem with closest assignment constraints.

We started our study by explaining in detail the two algorithms. We proceed by developing a procedure capable of randomly generate test instances for the CMP-CA problem and by selecting meaningful performance indicators. This is followed by properly calibrating the two algorithms, so that a meaningful and adequate comparison could be properly conducted. This is the context of what followed, where the comparison of the two algorithms' performance and suitability for a specific size of CMP-CA problems was developed in the light of the all the explained ingredients. This analysis led to the conclusion that introducing a guidance procedure improves the SA performance, and that the inclusion of features of the CE algorithm were a good guidance mechanism. In fact, these improvements were observed on the quality of the solution, in the computation times and on the total number of solutions that were analy-

zed. This last outcome is especially important when it comes to solving problems with time-consuming objective functions, which are the main motivation of this work. Furthermore, the small reliance of the hybrid algorithm on the calibration procedure when compared to the SA algorithm was also highlighted. Our final concern was to provide understanding on the suitability of this hybrid algorithm at finding good results for larger problems, even when the calibrated parameters were the result of a calibration method for smaller instances. A broader analysis should be conducted for further analysis regarding these results, where the solutions of the algorithm using the sets of calibrated parameters for each problem size should be compared and assessed against the parameters resultant from a calibration procedure conducted for smaller instances of the problem.

Chapter 6

Game-theoretic Approach to Transit and Parking Planning under Competition

6.1 Introduction

Transit systems (or public transport systems) are often advocated as a good alternative to automobiles. They are characterized as attractive solutions to overcome the current environmental issues linked to transport networks (e.g., fossil fuel consumption, pollutant emissions, and traffic congestion) without constraining people's mobility (Schiller et al., 2010; Miller, 2014). The expectations from transit systems to be reliable, affordable and safe often led to a high degree of government regulation of transit systems in many parts of the world. However, these regulations often caused severe financial problems for these systems, primarily because the costs of providing such high level of service were not fully recovered by the fare revenues collected by the transit operators (Vuchic, 2005).

This situation raises the question of what would happen if a transit system makes profitability as the main objective. In this chapter, we explore this extreme opposite scenario where pricing and supply of the transit system are managed purely on profit-maximization objectives. That is, we attempt to characterize the scenario where these systems are fully deregulated, without any social welfare-related constraints on prices and service levels.

A review of the existing literature aimed at solving the financial problems of transit systems indicates that the societal importance of these systems plays a central role in these studies. The approaches taken by these previous studies can be classified into two main streams: microeconomic approaches and operations research approaches. The first stream is usually more focused on establishing the optimal transit pricing and subsidization rules for stylized cities (e.g. Mohring, 1972; Vickrey, 1980; Small et al., 2007; Van Reeven, 2008; Basso and Jara-Díaz, 2010; 2012). Studies in the second stream often focus on real-world scenarios, but the interactions between prices and demands, and between transit and other transportation alternatives, are not explicitly taken into account. For further details on this transit litera-

ture stream, we suggest the reviews by Guihaire and Hao (2008), Kepaptsoglou and Karlaftis (2009), and Ibarra-Rojas et al. (2015).

In this chapter, we introduce a modeling and algorithmic framework capable of analyzing what would happen in the extreme scenario of fully deregulated transit systems. Furthermore, we also intend to provide a versatile framework that could be quickly adjusted by the city planners to accommodate any other concerns regarding the desired characteristics of the transit systems or any population needs. For example, the framework is flexible enough to easily add extra constraints related to minimum frequency or maximum fare on certain routes, etc. In this sense, any number of intermediate scenarios between the two extremes (a fully government-controlled welfare-oriented system and a fully privatized profit-maximizing system) can also be handled by the approach presented in this chapter. Given that driving (and subsequently parking) personal vehicles is the main alternative to using transit in many cities, especially for daily commuting needs, our approach consists in a game-theoretic model of the interactions between the decisions of the transit and parking operators. Under this approach, transit operator and parking operator are both modeled as profit maximizers whose optimal decisions depend on each other's decisions.

To the best of our knowledge, the topic of interactions between transit and parking planning under competition has not been explored in the existing literature. However, game-theoretic concepts have been broadly used in other transportation literature to deal with multiple decision makers whose objectives might be at odds with each other's, either fully or partially (Hollander and Prashker, 2006; Zhang et al., 2010). In these approaches, certain variants and refinements of the pure strategy Nash equilibrium (Nash, 1951) have been shown to be good solution concepts to solve non-cooperative transportation problems (e.g. Zhang et al., 2008; Martín and Román, 2003; Adler, 2001; 2005; Arnott, 2006; Vaze and Barnhart, 2012; 2015; Harder and Vaze, 2017). In particular, they have been used to analyze the impacts of pricing and transit network design on transit systems (e.g. Zhou et al., 2005; Sun and Gao, 2007).

We propose a two-stage game-theoretic framework where both operators optimize their capacity decisions in the first stage and their pricing decisions in the second stage. The first stage of our game deals with decisions regarding the level of supply by each operator. In the case of the parking operator, the parking capacity offered in each paid parking lot is the decision made in the first stage. In the case of the transit operator, the frequency value for each transit route and the corresponding adjustments to the fleet size are decided in the first stage while assuming that all remaining components of transit network design (e.g., network structure or set of routes) are fixed. The second stage addresses decisions made over a shorter time horizon. Each operator maximizes its own profit by adjusting its own prices (i.e., the transit fares or the parking fees) for a given set of prices of the other operator. We will use the solution concept called subgame perfect pure strategy Nash equilibrium (SPPSNE) for solving this problem. This solution concept offers multiple advantages. First, it is a more intuitive refinement of the general notion of a

pure strategy Nash equilibrium (PSNE) for extensive form (i.e., multi-stage) games. Second, in a recent study involving transportation capacity and pricing competition, this solution concept has been shown to have promising mathematical, computational and empirical properties (Harder and Vaze, 2017).

This chapter makes four major contributions. First, ours is the first study to model the competition between transit and parking operators using a game-theoretic model, and we do so by developing a two-stage game involving capacity decisions in the first stage and pricing decisions in the second stage. Second, under simplifying assumptions, we provide theoretical proofs of the existence of a PSNE for our game. The unicity of the pure Nash equilibrium is shown through numerical experiments for some case-studies, because the analytical proof was extremely hard to compute given the complexity of the analytical expressions. These experiments provided results that corroborate our assumption regarding the uniqueness of the pursued Nash equilibria. Third, we develop a new semi-approximate approach for solving the two-stage transit-parking game to a SPPSNE in a computationally efficient manner. Finally, we perform a thorough evaluation of our modelling and solution approach using a series of case studies that are inspired by real-world data. Our results show the suitability of our approach to model the extreme scenario of fully deregulated profit-maximizing transit systems, assuming a competitive environment between transit and parking operators. We highlight that this approach is also capable of accommodating additional constraints to assess solutions corresponding to intermediate states of regulation, without jeopardizing the effectiveness of the solution method or the Nash equilibrium's existence and its potential uniqueness properties.

This chapter is organized as follows. In Section 6.2, the game-theoretic models are described. In Section 6.3, we explore the proof for the existence of a PSNE at each of the two stages of the game under certain simplifications and assumptions made to the payoff functions, along with the numerical experiments developed to assess the uniqueness of the PSNE. The method developed for solving the two-stage game is detailed in Section 6.4. Section 6.5 presents the process of designing our computational experiments based on actual characteristics of real-world cities. This computational framework is then used to design case studies to evaluate the performance of the model and the solution approach. These computational results are presented in Section 6.6, while Section 6.7 details the insights obtained from our framework in terms of the main effects of the fully deregulated solutions for three different cities. Finally, in Section 6.8, a summary of our main conclusions and the directions for further research are provided.

6.2 Decision Models

Since the main focus is to improve the financial performance of the fully deregulated transit operator, we model it as a profit maximizing agent. However, changes in transit capacity and fares can affect the passengers' trip decisions, which in turn influence the financial performance of the parking operator as

well. So, the parking operator is expected to respond to these changes by adjusting its own capacity and pricing decisions. Thus, the overall financial performances of both operators are dependent on these interrelated decisions, which we aim to model with the following game-theoretic approach.

We use a two-stage game-theoretic framework. The first stage incorporates a long-term (e.g., months or even years before operations) perspective. In the first stage, the parking operator decides its parking capacity in terms of the number of parking spots in various paid parking locations. On the other hand, the transit operator, in the first stage, decides its frequency of service on each transit route and the corresponding fleet size needed to provide that frequency. In the second stage, pricing decisions are made by each operator over a short-term horizon (e.g., days or hours before operations). The SPPSNE is used as the solution concept for this game. Due to the extremely large size of the operators' overall decision spaces, we develop a semi-approximate solution approach. It consists in: 1) finding a PSNE of the second-stage game, 2) approximating the second-stage game's equilibrium payoffs using more tractable functions of the first-stage decisions, and 3) using these second-stage payoff approximations to find the first-stage game's PSNE.

The structure, notation and assumptions of the short-term and the long-term decision models are explained in subsections 6.2.1 and 6.2.1 respectively. In these subsections, the units of all variables and parameters are provided in square brackets immediately following the variable or parameter notation.

6.2.1   Short-Term Pricing Decisions

In this subsection, we describe the model structure, notation, and assumptions for the short-term pricing decisions that constitute the second stage. Let $\mathbf{Z}$ be the set of trip zones into which the city is divided. Travel demand within the city is assumed to be aggregated at the level of zone pairs as origin-destination pairs. This unconstrained demand between each ordered pair of zones (i, j) is denoted by Q_{ij} [passengers]. Each user can select any one of the three alternatives, namely, traveling by car (A), traveling by transit (B), or not making the trip at all, which is also known as the no-travel alternative (O).

The mode choice is described by a logit model of the generalized travel costs. Thus, the number of users that travel from origin zone i to destination zone j using mode m ($q_{ijm}, \forall i, j \in \mathbf{Z}, m \in \{A, B, O\}$) is given by equation (6.1), where C_{ijm} [€] is the generalized cost of alternative m, and parameter θ captures the sensitivity of travelers to this generalized travel cost.

$$q_{ijm} = \frac{e^{-\theta \cdot C_{ijm}}}{e^{-\theta \cdot C_{ijA}} + e^{-\theta \cdot C_{ijB}} + e^{-\theta \cdot C_{ijO}}} \cdot Q_{ij}, \qquad i, j \in \mathbf{Z}, m \in \{A, B, O\} \tag{6.1}$$

The generalized cost of travelling from origin i to destination j using transit is given by equation (6.2). This cost is the sum of the in-vehicle time cost $C^T \cdot D_{ijB}^T$, the transit fare p_{ijB} [€/trip], the accessibility cost AC_{ij}, and the discomfort cost DC_{ij}.

$$C_{ijB} = C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij} + DC_{ij}, \qquad i, j \in \mathbf{Z} \tag{6.2}$$

The in-vehicle time cost is obtained by multiplying the unit time cost C^T [€/h] and the total in-vehicle time that users need to spend to go from zone i to zone j by transit (D_{ijB}^T [h]). This total in-vehicle time is given by $D_{ijB}^T = \sum_{l \in \mathbf{Z}} D_{ijlB}^T$, where D_{ijlB}^T equals the amount of time spent in zone l while traveling from zone i to zone j along the shortest path using the transit network. $D_{ijlB}^T = 0$ if the shortest path from zone i to zone j using the transit network does not pass through zone l. Thus the in-vehicle time calculation assumes that the transit users will always take the shortest path (defined as the one with the minimum travel time) from their origins to their destinations along the transit network. Such a shortest path may involve some transfers among different transit routes. Here, a transit route is defined as the sequence of stops served by a transit vehicle (such as a bus). Typically, a particular transit route will be served multiple times a day. The transit network can be thought of as a collection of all such transit routes. We denote the set of transit routes by $\hat{T}$.

The frequency f_k [veh/h] of a transit route is defined as the average number of transit vehicle trips per unit time serving a transit route $k \in \hat{T}$. Binary parameter N_{ijlk} takes value 1 if any part of route k is used in the shortest path from i to j and if that part of route k passes through zone l. Otherwise it takes value 0. Then the transit frequency from the perspective of a transit user with origin at zone i and destination at zone j is given by $\min_{k \in \hat{T}, l \in \mathbf{Z}: N_{ijlk} = 1} \{f_k\}$, which corresponds to the minimum frequency across all transit routes used by the user's shortest path.

The accessibility cost is the sum of the costs associated with the walking time to and from the transit stop, and the total waiting time at the transit stop. It is given by equation (6.3). The first component of this accessibility cost function corresponds to the average walking time cost, which is the product of the cost per unit walking time, C^W [€/h], and the average walking time, D_{ijB}^W [h]. Average walking time is the sum of the average walking time to the origin transit stop from the user's actual origin and the average walking time from the destination transit stop to the user's actual destination. The second component of the accessibility cost function in equation (6.3) is inspired by the total waiting time formulation developed by Tirachini et al. (2010). In their work, the authors expressed this value as time savings, whereas we simplified their expression to account for time losses. Parameter SW is estimated to account for: (1) the average waiting time at the transit stops, and (2) the average cost to the user due to the transit departure times being different from the one that a user prefers the most. As explained by Tirachini et al. (2010), if the headway (time between vehicles in a transit system) is constant, then the average waiting time is half of the headway. If none of the departures are scheduled at the user's most preferred time, then this parameter SW also accounts for the schedule displacement penalty (sometimes called the schedule

delay cost) that also increases with increasing headway. Since the headways are inversely proportional to the frequency, the total average waiting time is as expressed by the second component of equation (6.3).

$$AC_{ij} = C^W \cdot D_{ijB}^W + \frac{SW}{\min_{k \in \hat{T}, l \in \mathbf{Z}:N_{ijlk}=1}\{f_k\}}, \quad i,j \in \mathbf{Z} \tag{6.3}$$

The discomfort cost represents the extra cost assigned by users to transit travel time because of their different time perceptions of traveling by transit when compared to car. This cost is dependent on the crowding of the transit vehicles, which is an increasing function of the number of passengers using transit and a decreasing function of the transit frequency. Our approach to calculate the discomfort cost is given by equation (6.4), and it is inspired by the one proposed by Corporation (1996). Here, ψ_B and ρ_B are constant positive parameters and S^B is the passenger carrying capacity per transit vehicle.

$$DC_{ij} = \sum_{l \in \mathbf{Z}} \sum_{k \in \hat{T}} D_{ijlB}^T \cdot C^T \cdot \psi_B \cdot \left(\frac{\sum_{o,d \in \mathbf{Z}} q_{odB} \cdot N_{odlk}}{S^B \cdot f_k}\right)^{\rho_B}, \quad i,j \in \mathbf{Z} \tag{6.4}$$

Let D_{ijA}^T [h] be the car travel time. Let D_{ijA}^K [km] be the total distance traveled by car to go from zone i to zone j, and C^K [€/km] be the vehicle depreciation, fuel and maintenance costs per unit distance. Also, let CP_{jA} [€] be the cost of cruising for parking and p_{jA} [€] be the parking fee. Here, the term "cruising" refers to the process of drivers driving around on roads in certain parts of the city or driving through a large parking garage looking for vacant parking spots. Then the generalized cost (C_{ijA}) of driving a car from i to j includes the vehicle depreciation, fuel and maintenance costs, travel time costs, cruising time costs, and parking fees, as expressed in equation (6.5).

$$C_{ijA} = C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + CP_{jA} + p_{jA}, \quad i,j \in \mathbf{Z} \tag{6.5}$$

Here τ_{ij} [trips/veh] is the car occupancy rate that links car-trips and person-trips, and is estimated as the average number of people in a car including the driver. The cruising cost is a function of parking saturation and is as expressed in equation (6.6) (Gallo et al., 2011). In this equation α_1 and α_1 are constant positive parameters and s_j^A [veh] is the parking capacity of zone j. The numerator inside the parentheses ($\sum_{i,j \in \mathbf{Z}} \frac{q_{ijA}}{\tau_{ij}}$) is a measure of the total number of cars using a parking location in trip zone j and the denominator s_j^A is the number of parking spots in zone j.

$$CP_{jA} = \alpha_1 \left(\frac{\sum_{i \in \mathbf{Z}} \frac{q_{ijA}}{\tau_{ij}}}{S_j^A}\right)^{\alpha_2} \quad i,j \in \mathbf{Z} \tag{6.6}$$

The generalized cost of not making the trip is estimated as a function of the generalized cost of making the trip by transit or by car. This cost is determined so as to account for the sensitivity of users toward not making the trip due to large increases in transit fares and parking fees. We assume this cost

to be OD dependent so that, similar to the generalized costs of making the trip by transit or car, the generalized cost of not making the trip is also an increasing function of the OD distance.

Short-term pricing decisions made by each operator in Stage II of the game are modelled by a PSNE which assumes that each operator maximizes its own profits (or minimizes its own losses). Expression (6.7) computes the revenue (OF_m^{SS}) of an operator m in the form of transit fares and parking fees. Since the total system ownership, maintenance, and operating costs are assumed to be independent of the actual number of passengers using either system, costs are not affected by the Stage II decisions. Therefore, maximizing profit in Stage II is equivalent to maximizing revenues, and hence revenue is the objective function for each operator when making the Stage II decisions. Note that we ignore the per-user variable costs (if any) to both the operators.

$$OF_m^{SS} = \sum_{i,j \in \mathbf{Z}} q_{ijm} \cdot p_{ijm} \tag{6.7}$$

Transit fares are assumed to vary according to pre-defined transit zones (not to be confused with trip zones described earlier in this subsection). This assumption is general enough to include the possibilities of implementing both flat and non-flat transit fares, which have been explored in the literature and applied in real-world transit systems (e.g. Farber et al., 2014; El-Geneidy et al., 2016). Each OD pair ij is assumed to belong to exactly one transit zone r, while each transit zone may correspond to multiple OD pairs. The transit fare for passengers in all OD pairs belonging to the same transit zone is assumed to be the same. We will assume that the assignment of an OD pair to a transit zone is fixed. Optimizing this assignment may bring additional benefits to the transit operator, but that is left as a future research direction. R is the set of all transit zones. So the transit fare from trip zone i to trip zone j (p_{ijB}) is given by equation (6.8). L_{ijr} is a binary parameter that is equal to 1 if the OD pair ij belongs to the transit zone r, and equal to 0 otherwise. $\hat{p}_{rB}$ [€/trip] is the transit fare for transit zone r.

$$p_{ijB} = \sum_{r \in R} \hat{p}_{rB} \cdot L_{ijr} \qquad i,j \in \mathbf{Z} \tag{6.8}$$

The parking operator is allowed to charge a fixed number of different parking fee levels in the zones where the parking is operated by it. These zones are identified by setting the binary constant parameter $\gamma_j, \forall j \in \mathbf{Z}$ to take the value 1, and 0 otherwise. For zones $j \in \mathbf{Z}$ such that $\gamma_j = 0$, we assume that sufficient supply of free parking is available and that this parking is not operated by the parking operator. Let PL be the set of distinct parking fee levels. The number of different parking feel levels ($|PL|$) is assumed to be fixed whereas the fee (pl_{dA} [€/veh]) corresponding to each parking level d as well as the assignment of the trip zones to different parking fee levels can be optimized by the parking operator. A *parking zone* is defined as the set of all trip zones corresponding to the same parking fee level. The assignment of trip zones to parking zones is optimized through the parking operator's profit maximization

problem. The binary decision variable w_{jd} is set to 1 if a trip zone j belongs to parking zone d, and is 0 otherwise. Equations (6.9) and (6.10) describe the relationships between parking decision variables w_{jd}, p_{jA} [€/trip] and pl_{dA}, and constant parameters γ_j.

$$p_{jA} = \sum_{d \in PL} \frac{pl_{dA} \cdot w_{jd}}{\tau_{ij}}, \quad j \in \mathbf{Z} \tag{6.9}$$

$$\sum_{d \in PL} w_{jd} = \gamma_j, \quad j \in \mathbf{Z} \tag{6.10}$$

Equation (6.11) is the parking capacity constraint for each trip zone, while equation (6.12) is the transit vehicle seating capacity constraint for each transit route within each trip zone through which the transit route passes.

$$\sum_{i \in \mathbf{Z}} \frac{q_{ijA}}{\tau_{ij}} \leq s_j^A, \quad j \in \mathbf{Z} \tag{6.11}$$

$$\sum_{i,j \in \mathbf{Z}} q_{ijB} \cdot N_{ijlk} \leq S_B \cdot f_k, \quad l \in \mathbf{Z}, k \in \hat{T} \tag{6.12}$$

Finally, constraints (6.13) and (6.14) enforce lower and upper limits on the allowable transit fares and parking fees. The lower limit of 0 ensures non-negativity of prices while the upper limits ($pl_{dA}^{max}, \forall d \in PL$, and $p_{rB}^{max}, \forall r \in R$) ensure that arbitrarily large prices cannot be charged by the transit and parking operators. Having upper limits on operator prices is practically reasonable because arbitrarily large prices are not practical for such applications.

$$0 \leq pl_{dA} \leq PL_{dA}^{\max}, \quad d \in PL \tag{6.13}$$

$$0 \leq p_{rB} \leq P_{rB}^{\max}, \quad r \in R, k \in \hat{T} \tag{6.14}$$

Below is a summary of all the decision variables within the Stage II model.

q_{ijA} – number of car trips made from trip zone i to j. This is a variable affected by both operators' Stage II decisions.

q_{ijB} – number of transit trips made from trip zone i to j.. This is a variable affected by both operators' Stage II decisions.

q_{ijO} – number of trips belonging to the no-travel alternative from trip zone i to j.. This is a variable affected by both operators' Stage II decisions.

p_{rB} – transit fare for transit zone r. This is a variable affected by only the transit operator's Stage II decisions.

p_{jA} – parking fee for trip zone j. This is a variable affected by only the parking operator's Stage II decisions.

pl_{dA} – parking fee for parking zone d. This is a variable affected by only the parking operator's Stage II decisions.

w_{jd} – binary variable indicating whether the trip zone j belongs to parking zone d. This variable is affected by only the parking operator's Stage II decisions.

CP_{jA} – cruising cost. This is a variable affected by both operators' Stage II decisions.

DC_{ij} – transit discomfort cost. This is a variable affected by both operators' Stage II decisions.

Solving for the Stage II PSNE corresponds to optimizing over these decision variables for simultaneous solution of the optimization problems of the two operators. Each optimization problem corresponds to maximizing the revenue expressed by equation (6.7) while satisfying constraints (6.1) to (6.6), and (6.8) to (6.14).

6.2.2 Long-Term Capacity Decisions

In this section, we describe the model structure, notation, and assumptions for the long-term capacity decisions that constitute Stage I of our game-theoretic model. The long-term decisions are related to the supply side for each operator. For the transit operator, Stage I decisions include the transit frequency on each transit route and the transit vehicle fleet size, while the parking operator's Stage I decisions include the number of parking spots in each trip zone with paid parking.

6.2.2.1 Transit Operator

In this first stage of our game, the transit operator intends to maximize its profit for the known and fixed transit network by adjusting the route frequencies and by purchasing or selling vehicles in the transit fleet. For a given set of Stage I decisions by both operators, the transit revenues are obtained from the Stage II PSNE. Let $\overline{OF_B^{SS}}$ denote the revenue of the transit operator corresponding to this Stage II PSNE. It is a function of 1) transit frequency (f_k^{EV}) on each route $k \in \hat{T}$ using the transit operator's existing fleet of vehicles, 2) transit frequency (f_k^{PV}) on each route $k \in \hat{T}$ using the transit operator's additionally purchased fleet of vehicles, and 3) parking capacity (s_j^A) allocated in each trip zone $j \in \mathbf{Z}$ with paid parking.

When calculating the costs, the model accounts for the possibility that the ownership, maintenance and operating costs of existing and newly purchased vehicles in the fleet could be different, for example due to improved technology in the newer vehicles. Let M_B^S be the ownership and maintenance cost per day of operation that is saved by selling a transit vehicle from the existing fleet. Similarly, let M_B^P be the additional cost of ownership and maintenance per day of a newly purchased transit vehicle. The transit operating cost per day includes two components: a fixed component per service frequency and a variable component that is proportional to the distance travelled. The variable component corresponding to

each transit route is the product of the route frequency (f_k^{EV} using the existing fleet and f_k^{PV} using the newly purchased fleet), the distance travelled for each route $k \in \hat{T}$ ($\hat{D}_{kB}^K$), and the operating cost per unit distance (M_B^{EV} for the existing fleet and M_B^{PV} for the newly purchased fleet). So the variable component of the operating cost is equal to $M_B^{EV} \cdot \sum_{k \in \hat{T}} \left(f_k^{EV} \cdot \hat{D}_{kB}^K \right) + M_B^{PV} \cdot \sum_{k \in \hat{T}} \left(f_k PV \cdot \hat{D}_{kB}^K \right)$. The fixed component of the operating cost is simply the product of a unit cost per trip (M_B^{EVF} for the existing fleet and M_B^{PVF} for the newly purchased fleet) and the number of trips per day. So, the total fixed component of the operating cost is given by $M_B^{EVF} \cdot \sum_{k \in \hat{T}} f_k^{EV} + M_B^{PVF} \cdot \sum_{k \in \hat{T}} f_k^{PV}$.

Then the transit operator's overall profit per day is given by equation (6.15), where v^{PV} and V^{SV} respectively represent the number of vehicles that are purchased and sold.

$$
\begin{aligned}
OF_B^{FS} =& \overline{OF}_B^{SS} \left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in \mathbf{Z}} \right) - M_B^{EV} \cdot \sum_{k \in \hat{t}} F_K^{EV} \cdot \hat{D}_{kB}^K - M_B^{PV} \cdot \sum_{k \in \hat{T}} f_k^{PV} \cdot \hat{D}_{kB}^K \\
& - M_B^{EVF} \cdot \sum_{k \in \hat{T}} f_k^{EV} - M_B^{PVF} \cdot \sum_{k \in \hat{T}} f_k^{PV} - M_B^{PV} \cdot v^{PV} + M_B^{SV} \cdot v^{SV}
\end{aligned}
\tag{6.15}
$$

Stage I objective function of the transit operator is to maximize daily profits. For this optimization problem, the set of constraints is given by (6.16)-(6.21). Transit vehicles cannot be used entire day to perform transit trips due to vehicle maintenance, cleaning, and refueling time requirements, drivers' rest requirements, and lack of demand (e.g., late at night). Let μ denote that fraction of a day for which a vehicle is actually available to make transit trips. This is sometimes called the maximum allowable usage intensity for the transit vehicle. Let V^0 denote the existing number of vehicles in the fleet. Then constraints (6.16) and (6.17), respectively, ensure that the number of vehicles in the newly purchased and the existing fleets are sufficient to provide their corresponding frequencies across all transit routes. Constraint (6.18) ensures that the number of vehicles to be sold does not exceed the number of vehicles in the existing fleet, and the frequency for each route is defined by constraint (6.19). Constraints (6.20) and (6.21) ensure that the frequencies and number of vehicles take only non-negative integer values. Finally, constraints (6.22) and (6.23) enforce any upper and lower limits on the main decision variables. If there are no upper limits to be enforced, then we can use sufficiently large positive integers instead. If there are no lower limits to be enforced, then we can use zeros as lower limits.

$$
v^{PV} \cdot \mu \geq \sum_{k \in \hat{T}} f_k^{PV} \cdot \hat{D}_{kB}^K
\tag{6.16}
$$

$$
(V^0 - v^{SV}) \cdot \mu \geq \sum_{k \in \hat{T}} f_k^{EV} \cdot \hat{D}_{kB}^k
\tag{6.17}
$$

$$
v^{SV} \leq V^0
\tag{6.18}
$$

$$
f_k = f_k^{EV} + f_k^{PV}, \quad k \in \hat{T}
\tag{6.19}
$$

$$
v^{VN}, v^N \in \mathbb{Z}_0^+
\tag{6.20}
$$

$$
f_k, f_k^{EV}, f_k^{PV} \in \Gamma_B, k \in \hat{T}
\tag{6.21}
$$

$$V_{min}^{PV} \le v^{PV} \le V_{max}^{PV}, \; V_{min}^{SV} \le v^{SV} \le V_{max}^{SV} \tag{6.22}$$

$$F_{k,min} \le f_k \le F_{k,max}, \; F_{k,min}^{EV} \le f_k^{EV} \le F_{k,max}^{EV}, F_{k,min}^{PV} \le f_k^{PV} \le F_{k,max}^{PV}, \quad k \in \hat{T} \tag{6.23}$$

The Stage I optimization problem for the transit operator thus consists of obtaining the optimal values of decision variables v^{PV}, v^{SV} and $f_k^{EV}, f_k^{PV} \forall k \in \hat{T}$ to maximize the transit operator profit given by equation (6.15) while ensuring that the constraints (6.16)-(6.23) are satisfied.

6.2.2.2 Parking Operator

The parking operator maximizes its overall profit by deciding the parking capacity across all trip zones with paid parking in Stage I of the game. Overall profit is obtained by subtracting the parking system operating costs and the costs of capacity addition from the parking fee revenues. The parking fee revenues are given by the Stage II PSNE. Let $\overline{OF}_A^{SS}$ denote the revenue of the parking operator corresponding to this Stage II PSNE. Similar to the revenue ($\overline{OF}_B^{SS}$) of the transit operator described earlier in this subsection, $\overline{OF}_A^{SS}$ is also a function of 1) transit frequency (f_k) on each route $k \in \hat{T}$, and 2) parking capacity (s_j^A) allocated in each trip zone $j \in \mathbf{Z}$ with paid parking.

Let y_j^U denote the number of parking spots added to parking capacity of trip zone $j \in \mathbf{Z}$. Let y_j^P be a binary decision variable that equals 1 if the parking capacity of trip zone $j \in \mathbf{Z}$ is increased, and is 0 otherwise. Also, let x_j be the number of parking spots removed from zone $j \in \mathbf{Z}$. The operating costs are obtained by multiplying the unit operating cost (M_A^V) per parking spot by the total parking capacity managed by the parking operator (i.e., the number of parking spots in all trip zones with $\gamma_j = 1$). The cost of parking capacity addition has two components: a variable component obtained by multiplying the number of added spots in each zone ($y_j^U, \forall j \in \mathbf{Z}$) by the unit cost ($E_A^V$) per added spot, and a fixed component (E_A^F) per trip zone for which the decision to expand capacity is made (that is, $y_j^P = 1$). The parking operator may also decide to decrease the parking capacity by x_j spots for a given zone j. Such decision does not entail any cost by itself, but helps in decreasing the operator's operating expenses, while also potentially reducing its parking revenues.

Then the parking operator's overall profit per day is given by equation (6.24).

$$OF_A^{FS} = \overline{OF}_A^{SS} \left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in Z} \right) - M_A^V \cdot \sum_{j \in Z} s_j^A \cdot \gamma_j - E_A^V \cdot \sum_{j \in \mathbf{Z}} y_j^U - E_A^F \cdot \sum_{j \in \mathbf{Z}} y_j^P \tag{6.24}$$

The objective function of the parking operator is to maximize daily profit. For this optimization problem, the set of constraints is given by (6.25)-(6.32). Let S_j^{A0} be the existing parking capacity of trip zone $j \in \mathbf{Z}$. Constraints (6.25) relate the value for the decision variable s_j^A, with the existing parking capacity, the number of added parking spots and the number of removed parking spots in that zone. Constraints (6.26) and (6.27) together ensure that decision variable y_l^P is set to 1 if and only if decision

variable y_j^U is positive. Constraints (6.28) and (6.29) guarantee that only the zones where parking is managed by the parking operator (i.e., the zones with $\gamma_j = 1$) will have the possibility of having their parking capacities adjusted. Constraints (6.30) ensure that the binary variables y_l^P can only take a value of 0 or 1, while constraints (6.31) ensure that the variables y_j^U and x_j related respectively to the addition and removal of capacity can only take non-negative integer values. Finally, constraints (6.32) enforce any upper and lower limits on the main decision variables. If there are no upper limits to be enforced, then we can use sufficiently large positive integers instead. If there are no lower limits to be enforced, then we can use zeros as lower limits.

$$s_j^A = S_j^{A0} + y_j^U - x_j, \quad j \in \mathbf{Z} \tag{6.25}$$

$$y_j^U \le M \cdot y_l^P, \quad j \in \mathbf{Z} \tag{6.26}$$

$$y_j^U \ge y_l^P, \quad j \in \mathbf{Z} \tag{6.27}$$

$$y_l^P \le \gamma_j, \quad j \in \mathbf{Z} \tag{6.28}$$

$$x_j \le \gamma_j, \quad j \in \mathbf{Z} \tag{6.29}$$

$$y_j^P \in \{0, 1\}, \quad j \in \mathbf{Z} \tag{6.30}$$

$$s_j^A, y_j^U, x_j \in \mathbb{Z}_0^+, \quad j \in \mathbf{Z} \tag{6.31}$$

$$S_{j,\min}^A \le s_j^A \le S_{j,\max}^A, Y_{j,\min}^U \le y_j^U \le Y_{j,\max}^U, X_{j,\min} \le x_j \le X_{j,\max}, \quad j \in \mathbf{Z} \tag{6.32}$$

Stage I optimization problem for the parking operator thus consists of obtaining the optimal values of decision variables $y_j^P \in \{0, 1\}, j \in \mathbf{Z}$ and $y_j^U, x_j \in \mathbb{Z}_0^+, j \in \mathbf{Z}$ to maximize the parking operator profit given by equation (6.24) while ensuring that the constraints (6.25)-(6.32) are satisfied. The upper and lower bounds on the parking capacity variables ensure that the feasible space of the parking operator's Stage I optimization problem is a compact set. The values of the transit frequency ($f_k, k \in \hat{T}$) are held constant in this problem.

6.3 Theoretical Properties

In this section, we describe the theoretical properties of our model. We use the SPPSNE as the solution concept to solve our two-stage game due to its common usage in literature and its intuitive practical appeal. This solution concept requires the existence of a PSNE at each stage of the game. Moreover, if there are multiple PSNE for either stage of the game then that often reduces the credibility of the solution concept for characterizing real-world game situations. Therefore, we need to verify the existence and uniqueness properties of the PSNE at each stage.

Due to the complexity of our model, we are able to prove the existence of the PSNE only upon making some additional simplifications. Specifically, we make two additional simplifications. First, we ignore the

discomfort cost of transit and the cruising cost of parking, i.e., we set $\psi_B = \alpha_1 = 0$. Second, we assume that, if we ignore the integer domain of the Stage I decision variables, then the Stage I payoff functions of both operators are concave in their own decision variables and the corresponding second derivatives exists. Note that we don't need to relax the domain of the decision variables for the proofs to hold, we simply need concave payoffs with well-defined second derivatives when ignoring the integer domain of the variables.

The uniqueness of the PSNE at each stage is demonstrated through numerically experiments. A PSNE is a set of decisions by all players such that each player's decision is a best response to the decisions of other players, as defined in Definition 6.1.

Definition 6.1. *(Best-response): Let $N = \{1, \ldots, n\}$ be the set of players. Let $a = (a_1, a_2, \ldots, a_n) \in A = A_1 \times A_2 \times \ldots \times A_n$ be a strategy profile (or an action profile) where A_i is the set of all possible actions of player i. Let $u = (u_1, u_2, \ldots, u_n)$ be the profile of utility functions with the player i's utility function given by $u_i : A \to \mathbb{R}$. Finally, for each player i, we denote by index $-i$, the set of all other players. Then, $a_i^* \in A_i$ is a best response to $a_{-i} \in A_{-i}$ if $\forall a_i \in A_i, u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$*

In our game model, the transit operator and the parking operator are the two players in each stage. Player strategies for Stage II are their respective pricing decisions, whereas the Stage I strategies are the transit frequencies and the parking capacities respectively. Finally, the Stage II payoff functions are given by equation (6.7) for both players, and Stage I payoff functions are given by equations (6.15) and (6.24) for transit and parking operator respectively. These functions relate the payoff of each player with both players' strategies.

We invoke the general theorem by Tian (2015) to prove the existence of a PSNE in each stage of our game. In this work, the author establishes a single condition, called *recursive diagonal transfer continuity*, which is a necessary and sufficient condition for the existence of a PSNE in games with arbitrary compact strategy spaces and arbitrary payoff functions. An *upsetting relationship* between two action profiles x and y expressed by $y \succ x$, is said to hold if and only if $\exists i \in N$ s.t. $u_i(y_i, x_{-i}) > u_i(x)$. In that case, strategy profile y is said to upset strategy profile x. Thus, $x^* \in A$ is a PSNE if and only if there does not exist any strategy $y \in A$ that upsets x^*. Let $U : A \times A \to \mathbb{R}$ be the aggregator function defined as $U(y, x) = \sum_{i \in N} u_i(y_i, x_{-i}), \forall (x, y) \in A \times A$. The property of *recursive diagonal transfer continuity*, which is defined in Definition 6.3, is the necessary and sufficient condition to ensure the existence of a PSNE as detailed in Theorem 6.1 (Tian, 2015). But before that, we define the term 'recursively upset', in Definition 6.2, because it is needed for the definition of recursive diagonal transfer continuity.

Definition 6.2. *A strategy profile $y^0 \in A$ is said to be recursively upset by $z \in A$ if and only if there exists a finite set of deviation strategy profiles $\{y_1, y_2, \ldots, y_{m-1}, z\}$ such that $U(y_1, y_0) > U(y_0, y_0), U(y_2, y_1) > U(y_1, y_1), \ldots, U(z, y_{m-1}) > U(y_{m-1}, y_{m-1})$.*

Definition 6.3. *(Recursive diagonal transfer continuity, Tian (2015)): A game $G = (A_i, u_i)_{i \in N}$ is said to have the recursive diagonal transfer continuity property iff, whenever $U(y, x) > U(x, x)$ for $x, y \in A$, there exists a strategy profile $y^0 \in A$ (possibly $y^0 = x$) and a neighbourhood $V_x \subset A$ of x such that $U(z, x') > U(x', x') \forall x' \in V_x$ for any z that recursively upsets y^0.*

Theorem 6.1. *(Tian (2015)): Let A be a compact set and u_i be arbitrary functions with codomain in $\mathbb{R}$. Then the game $G = (A_i, u_i)_{I \in N}$ possesses a PSNE if and only if it has the recursive diagonal transfer continuity property.*

6.3.1 The existence of a Nash equilibrium for Stage II

We proceed by showing that, if we set $\psi_B = \alpha_1 = 0$, then our Stage II subgame has the recursive diagonal transfer continuity property. With $\psi_B = \alpha_1 = 0$, the Stage II payoff functions simplify to those shown in equations (6.33) and (6.34)) for the transit and parking operators, respectively.

$$SOF_B^{SS} = \sum_{i,j \in \mathbf{Z}} \frac{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)}}{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)} + e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)} + e^{-\theta \cdot C_{ijO}}} \cdot Q_{ij} \cdot p_{ijB} = \sum_{i,j \in \mathbf{Z}} \pi_{ijB} \qquad (6.33)$$

$$SOF_A^{SS} = \sum_{i,j \in \mathbf{Z}} \frac{e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)}}{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)} + e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)} + e^{-\theta \cdot C_{ijO}}} \cdot Q_{ij} \cdot p_{ijA} = \sum_{i,j \in \mathbf{Z}} \pi_{ijA} \qquad (6.34)$$

Here π_{ijB} and π_{ijA} are defined as follows:

$$\pi_{ijB} = \frac{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)}}{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)} + e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)} + e^{-\theta \cdot C_{ijO}}} \cdot Q_{ij} \cdot p_{ijB} \qquad (6.35)$$

$$\pi_{ijA} = \frac{e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)}}{e^{-\theta \cdot \left(C^T \cdot D_{ijB}^T + p_{ijB} + AC_{ij}\right)} + e^{-\theta \cdot \left(C^T \cdot D_{ijA}^T + \frac{C^K \cdot D_{ijA}^K}{\tau_{ij}} + p_{jA}\right)} + e^{-\theta \cdot C_{ijO}}} \cdot Q_{ij} \cdot p_{ijA} \qquad (6.36)$$

SOF_B^{SS} and SOF_A^{SS} are both continuous functions defined over compact sets. This guarantees the existence of maximum and minimum values for these functions. Let $p_B^* = (p_{ijB})_{i,j \in \mathbf{Z}}$ and $p_A^* = (p_{jA})_{j \in \mathbf{Z}}$ be the price vectors that maximize SOF_B^{SS} and SOF_A^{SS}, respectively. Appendix B, proves that the functions SOF_B^{SS} and SOF_A^{SS} are concave, sustaining the hypothesis that p_B^* and p_A^* are local maxima of each payoff function SOF_B^{SS} and SOF_A^{SS}, respectively.

Let us now suppose that the aggregator function verifies $U(x, y) > U(x, x)$ for some $x = (x_1, x_2)$ and $y = (y_1, y_2)$ that satisfy constraints (6.13)-(6.14), where e.g. $(x_1, x_2) = \left((p_{ijB})_{i,j \in \mathbf{Z}}, (p_{jA})_{j \in \mathbf{Z}}\right)$. Let $y^0 = (p_B^*, p_A^*)$ and V_x be a neighbourhood of x such that (p_B^*, p_A^*) is the unique local maximum in V_x. Since $U(y, y^0) < U(y^0, y^0)$, for all y, it is impossible to find any securing strategy y^1 (i.e., a strategy in the neighborhood V_x with a strictly higher payoff) such that $U(y^1, y^0) > U(y^0, y^0)$. Hence the recursive diagonal

transfer continuity holds. And, by Theorem 6.1, this subgame has a PSNE.

6.3.2 The existence of a Nash equilibrium for Stage I

We proceed by showing that by ignoring the assumptions of having non-negative integer domain of the Stage I decision variables frequencies $\left([f_k^{EV}, f_k^{VP}]_{k \in \hat{T}}\right)$ and parking spaces a $\left([s_j^A]_{j \in \mathbf{Z}}\right)$, we have concave payoff functions in their own decision variables and that the second derivative exists. This is guaranteed by defining the revenues $\overline{OF_B^{SS}}\left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in \mathbf{Z}}\right)$ and $\overline{OF_A^{SS}}\left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in \mathbf{Z}}\right)$ of the payoff functions (6.15) and (6.24), respectively, as concave functions with continues second derivatives. Note that the second derivative of payoff functions (6.15) and (6.24), with compact sets of $\mathbb{R}^{\hat{T} \times \#\mathbf{Z}}$ as domain, are equal to the second derivative of the revenue functions $\overline{OF_B^{SS}}\left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in \mathbf{Z}}\right)$ and $\overline{OF_A^{SS}}\left([f_k]_{k \in \hat{T}}, [s_j^A]_{j \in \mathbf{Z}}\right)$ with the same domains, respectively. The existence of local maxima values for $\widetilde{f}_k^{EV}$, $\widetilde{f}_k^{PV}$ and $\widetilde{s}_j^A$ is ensured by these assumptions.

Let $\overline{f}_k^{EV}$, $\overline{f}_k^{PV}$ and $\overline{s}_j^A$ be the non-negative integer frequencies and parking capacities values closest to $\widetilde{f}_k^{EV}, \widetilde{f}_k^{PV}$ and $\widetilde{s}_j^A$, respectively. Assuming that the aggregator function verifies $U(x, y) > U(x, x)$ for some $x = (x_1, x_2)$ and $y = (y_1, y_2)$ that satisfy constraints (6.23) and (6.32), where e.g. $(x_1, x_2) = \left([f_k^{EV}, f_k^{PV}], [s_j^A]_{j \in \mathbf{Z}}\right)$. Let $y^0 = \left([\overline{f}_k^{EV}, \overline{f}_k^{PV}], \overline{s}_k^A\right)$ and V_x be a neighbourhood of x such that $\left([\overline{f}_k^{EV}, \overline{f}_k^{PV}], \overline{s}_k^A\right)$ is unique in V_x. Since $U(y, y^0) < U(y^0, y^0)$, for all y that satisfies constraints (6.23) and (6.28), it is impossible to find any securing strategy y^1 (i.e., a strategy in the neighborhood V_x with a strictly higher payoff) such that $U(y^1, y^0) > U(y^0, y^0)$. Hence the recursive diagonal transfer continuity holds. And, by Theorem 6.1, this subgame has a PSNE.

6.3.3 The uniqueness of a Nash equilibrium for Stage II and Stage I

The unicity of a Nash equilibrium is explored for the two stages by taking into account a numerical experiment, because it is quite challenging to analytical proven this characteristic for the previously detailed functions with complex expressions.

These numerical experiments took into consideration features that are detailed in the following section, and they were developed as follows:

1. Randomly generate a case study setup as detailed in Section 6.5;

2. Randomly select initial values for the following variables: transit fares and parking fees for Stage II and transit frequencies and parking fees for Stage I;

3. For each initial values of each stage, apply an iterative response chain taking into account the optimization models that integrate each stage. This chain considers that each player iteratively optimizes its own payoff function by reacting to the other's player decisions, and it ends when

neither player is able to further improve its profit by changing its own decisions, achieving a PSNE. In particular, the functions $\overline{OF_B^{SS}}$ and $\overline{OF_A^{SS}}$ are detailed for Stage I as explained on the second step of the Solution Method developed in Section 6.4.

For simplicity, we considered the cities explored in Section 6.5 as the ones required by step 1 and applied this 3-step small procedure to draw conclusions regarding the unicity of the Nash equilibrium. By comparing the several returned PSNE, we conclude that the final combination of prices for the Stage II and capacities for Stage I were systematically the same regardless the set starting point. Therefore, the unicity of a Nash equilibrium in each one of the two stages of our model is confirmed.

It is worth highlighting that this procedure does not assumed any of the simplified considerations made to show the existence of PSNE. In this sense, we showed that the inclusion of both discomfort cost functions and cruising for parking cost functions into the payoff functions of Stage II do not jeopardize the existence of a PSNE in the analyzed case-studies. In fact, this procedure shows that the use of PSNE concepts is a suitable approach to solve the developed two stage game-theoretic model.

6.4   Solution Method

Due to the extremely large size of the players' overall decision spaces and the complexity of their profit functions, we develop an approximate solution method consisting of the following three steps:

1) Solve for a pure Nash equilibrium of Stage II for several randomly generated combinations of the decisions made during Stage I by both operators;

2) Approximate revenues estimated for Stage II during Step 1 for each operator as functions of the decisions made by each operator during Stage I;

3) Use these revenue approximations to find PSNE of Stage I.

Step 1 and 3 are quite similar, each one involving computationally finding a PSNE for a 2-player game. This is performed via a sequential optimization heuristic, which is a standard method that has been commonly used in prior transportation studies that use computational approaches to solve game-theoretic models (e.g. Martín and Román, 2003; Adler, 2001; 2005; Harder and Vaze, 2017). The idea is to implement an iterative response chain, where each player iteratively optimizes its own profit by reacting to the other player's decision. This response chain will stop when neither player is able to further improve its profit by changing its own decision. This is when a PSNE is achieved.

Step 2 links the two stages of the model through a polynomial regression. Profit of each operator in the Stage I game is a function of the capacity decisions of both operators. However, it is a complex function that manifests through a second stage PSNE, and does not have a closed form expression. So, in Step

2, we approximate it using a closed form expression of the transit frequencies and parking capacities and identify the best fit parameter estimates using a regression. The independent variables in this regression are the decision variables of the first stage game and the dependent variable is the profit of each player at the Stage II PSNE.

Step 1 and 3 requires repeatedly solving combinatorial and nonlinear optimization formulations. These are very challenging optimization problems (Burer and Letchford, 2012; Hemmecke et al., 2010), which only a few software products are capable of handling effectively. Baron Software (Tawarmalani and Sahinidis, 2005) is one of the most suitable software products to deal with nonlinearities and combinatorial formulations (Burer and Letchford, 2012; Bussieck and Vigerske, 2014). It uses a global optimization approach based on polyhedral branch-and-cut algorithms (Kesavan and Barton, 2000). In Step 2, we compared various candidate regression model specifications, and used a k-fold cross-validation approach to identify the best among all the specifications evaluated by us. More details on the regression model specification, estimation and validation process are provided in Section 6.7.

6.5 Case Study Setup

In this section we present our computational framework to generate instances of cities inspired by the real-world data. This framework uses real-world data in order to ensure that case studies match, behavior as closely as possible, with the demand-supply dynamics of real cities and their users'.

6.5.1 City Configuration

The first step in creating a representative instance of a city is to determine its size and the number of zones in it. For simplicity, we divided our cities into some number (N_{center}) of zones corresponding to the city center and some number (N_{suburbs}). The geographic configuration of real cities is often dependent on, among other factors, the number of inhabitants (PC), the number of employment opportunities (EC) and the population density of the city (DC). We generated these three values for each city by taking into account their ranges for medium-sized cities as listed in the recent reports from the Organization for Economic Co-operation and Development (OECD, 2016). For simplicity, we assumed the city instances to be square in shape with the length of each side given by $LC = \sqrt{\frac{PC}{DC}}$.

The coordinates, $(x_i, y_i), \forall I \in \mathbf{Z}$, of each zone's centroid are randomly generated as follows. For the zones in the city center, the centroids are randomly generated within a smaller square at the center of the city, with its length proportional to the city's length (see Figure 6.1). For the suburban zones, the centroids are randomly located within the city but outside the aforementioned smaller square. Once the coordinates for the centroids of all zones are determined, the border of each zone is determined by drawing Voronoy diagrams (Aurenhammer, 1991) around the centroids.

The distance between two adjacent zones is calculated as the sum of the Euclidean distances from each zones centroid to the mid-point of their shared border. This is exemplified by the lines showing the distances between zones A and B, and zones B and C, in Figure 6.1. Using this process, all pairs of adjacent zone centroids are connected. The distance between non-adjacent zones is calculated by determining the shortest-path along this network of line segments. For example, the path between the centroids of zones A and C passes through the centroid of zone B as shown in Figure 6.1.
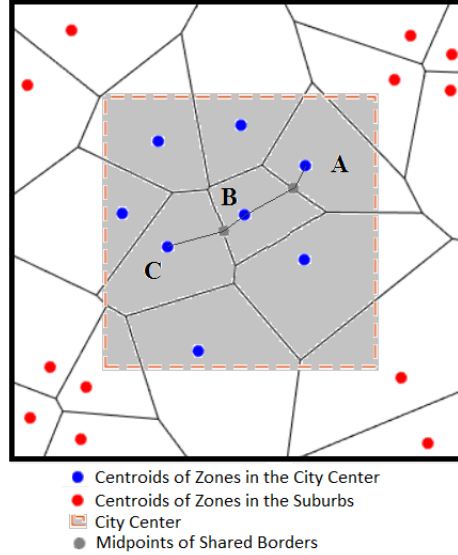
Figure 6.1: Example of a randomly generated city with 20 zones.

Different zones in a city may have different characteristics. A zone is considered residential if housing, rather than commercial/industrial activity, dominates it. These characteristics identify if a zone is a trip generator (residential) or a trip attractor (commercial/industrial). This assignment of zones to types (residential versus commercial/industrial) is randomly performed while ensuring that the zones in the city center have a greater probability of being commercial/industrial zones while those in the suburbs have a greater probability of being residential zones. Using this rationale, the number of inhabitants (PC_i) and the number of employment opportunities (EC_i) for a given zone i are randomly generated as shown in equation (6.37), where $\mu_{[a,b]}$ is a random value between a and b. The values of constants C_1 and C_2 are chosen so that $\sum_{i \in \mathbf{Z}} PC_i = PC$ and $\sum_{i \in \mathbf{Z}} EC_i = EC$.

$$\begin{cases} PC_i = \begin{cases} \mu_{[0.5,1]} \cdot C_1, & \text{if } i \text{ is a residential zone} \\ \\ \mu_{[0,0.5]} \cdot C_1, & \text{otherwise} \end{cases} \\ \\ EC_i = \begin{cases} \mu_{[0,0.25]} \cdot C_2, & \text{if } i \text{ is a residential zone} \\ \\ \mu_{[0.75,1]} \cdot C_2, & \text{otherwise} \end{cases} \end{cases} \tag{6.37}$$

The average distance by car for each OD pair is calculated by multiplying the Euclidean distance between the origin and destination zone centroids with a uniform random number in the interval [1,2]. This accounts for the fact that cars often need to travel longer than the Euclidean distances to go from one geographical location to another. The average travel time by car, D_{ijA}^{T}, is estimated by the ratios of the distance traveled in each zone to the average daily speed for that same zone, which is then summed across all zones on the shortest path. These average daily speed values for each zone are generated randomly within the interval [20,40] km/h (Table 6.1), which is in line with the average speeds observed in 15 of the most congested European cities (Statista, 2008).

When designing the transit networks, we assumed the number of different routes in a city to be known and that going from one zone to another requires at most one transfer between transit routes. As explained in Section 6.6, we created and computationally tested multiple cities with varying number of transit routes. This transit network is iteratively built as follows. From the set of zones **Z**, two zones are randomly selected and the shortest path between them is calculated using the Dijkstra's Algorithm over the network of line segments joining zone centroids with the midpoints of their shared borders. This path will define a transit route. Next, two other zones are randomly selected from the set of zones that do not belong to any of the routes defined so far and the shortest path is obtained again. This procedure will continue until all the zones belong to some route or if only one zone remains at the end (depending on whether the city has odd or even number of zones). In the case of a single zone remaining at the end, this zone will be added to the route that causes minimum change with this inclusion. To choose this route it is assumed that some zone of the route is adjacent to the zone that will be added and that this route is the one whose length is the closest to the length of the route without adding this left out zone. If the number of routes generated by this process exceeds the pre-determined number of different transit routes, a readjustment is performed by combining the routes with the highest number of overlapping connected zones. If the number of generated routes is lower than the predetermined number, we selected the largest routes and split each of them into two routes at each route's midpoint zone. Note that these two adjustments are done while ensuring that at most one transfer is required for traveling between each origin and each destination. The transit network is the set of all the transit routes generated through this process. The transit distance results from multiplying the shortest path distance along the transit network by a random number in the interval [1.5,2.5]. This is intended to account for the fact that a transit route usually requires greater travel distance than its Euclidean distance. If a transfer is needed, an additional of 10% transit distance is added to account for transfer times and additional waiting times at the transfer. The transit travel times D_{ijB}^{T} and D_{ijlB}^{T} are calculated by taking into account the transit distances and the average transit speed for each zone, with the latter randomly selected from the interval [12,27] km/h (Neff and Dickens, 2013).

The average walking time to and from a transit stop is calculated using a walking distance equal to an

uniform random percentage between 5% and 25% of the average distance between the zone centroid and the mid-point of every side of the zone border, and an average walking speed of 5 km/h (Mohler et al., 2007). This percentage range of walking distances is motivated by the assumption that the residential locations of people in each zone are more concentrated in the area near the zone centroid.

6.5.2   Model Parameters

In this subsection, we describe the process used to simulate the various parameters used in the users' utility expressions. Moreover, we also describe the process to simulate the prevalent values of transit route frequencies, parking lot capacities, transit fares and parking fees. These values are assumed to prevail in these simulated cities before the implementation of decision support tools based on our game-theoretic models. These prevalent conditions thus serve as benchmarks for comparison to our results. To keep the discussion simple, we defer the detailed explanation of the process used to simulate the unconstrained demand values Q_{ij} and the generalized cost of not making the trip C_{ijO} to Appendix A. As before, all parameters and costs in this subsection are simulated such that the values are consistent with the real-world ranges of these parameters. Table 6.1 summarizes the most important parameters used in our computational experiments.

Table 6.1: Summary of the most important parameters used for the computational experiments.

Parameter	Interval
Average car speed (per zone)	[20, 40] km/h
Average transit speed (per zone)	[12, 27] km/h
Average walking speed (per pair of zones)	5 km/h
Unit cost of time (C^T)	[6,9] €/hour
Average number of trips per the day per person (π)	[2, 4] trips/day
Car occupancy rate (τ_{ij})(per pair of zones)	[1.2, 2] pax/car
Vehicle depreciation, fuel and maintenance costs per driving distance (C^K)	[0.3, 0.4] €/km
Transit capacity per vehicle (S^B)	[50, 70] pax
Accessibility cost parameter (SW)	[12, 160] €
Transit fare (p_{ijB})	[0.5, 3] €/transit trip
Parking fee for non-residential zones (p_{jA})	[1,6] €/parking

The parameter θ that captures the sensitivity of travelers toward the generalized cost is randomly generated in the interval [0.2,0.6] (Litman, 2004; Paulley et al., 2006; Sharaby and Shiftan, 2012).

The unit cost of travel time (C^T) is randomly generated in the interval [6,8] €/hour, following the

actual values for Portugal (INE, 2015). The average number of daily trips π is randomly generated in the range [2,4] for the generated city, according to the US and European trends (Santos et al., 2009; Pasaoglu, 2012). This parameter is used to estimate the unconstrained demand, as shown in Appendix A. The capacity of a transit vehicle is generated as a uniform random number in [50,70] passengers (Vuchic, 2005; Group et al., 2013). The parameter SW included in the accessibility cost function is generated as a uniform random number in [12,160] €/day (Wardman, 2004; Wardman et al., 2004; Vuchic, 2005; Psarros et al., 2011).

The cost per unit distance travelled by cars is generated as a uniform random number in [0.3,0.4] €/km and includes the costs related to vehicle depreciation, fuel and maintenance (Association, 2016; Litman, 2009). The car occupancy rate is randomly generated uniformly in the interval [1.2,2] pax/veh for each OD pair (Santos et al., 2011; Agency, 2010).

The prevalent transit fares on all routes are generated as uniform random numbers in the interval [0.5,3] €/trip, while the prevalent parking fees in all zones are generated as uniform random numbers in [1,6] €/parking (of Travel, 2010; Pasaoglu et al., 2012). The transit frequency for each route is randomly generated within the interval [13,156] trips/day. The parking capacity of each zone is set as a proportion of the total demand with that destination zone. In the case of a paid parking zone, this proportion is randomly generated in the interval [0.4,0.6], and in case of a free parking zone, in the interval [0.7,0.9].

6.6 Computational Study

In this section, we describe the set of computational experiments conducted using our two-stage game-theoretic model and highlight the major computational results. By applying our framework to several simulated cities, we draw conclusions regarding the suitability of our modeling approach and solution method. All computational experiments were conducted on a personal computer with a 3.5 GHz i7 processor with 32 GB RAM, and a Windows 10 64-bit operating system. All optimization problems were solved using the Baron Software (Tawarmalani and Sahinidis, 2005). We present a preliminary analysis of the computational run-times, and a heuristic enhancement to expedite the computations. Using the case study setup explained in Section 6.5, five cities (C1, C2, C3, C4, and C5) were designed, with the number of trip zones equal to 5, 10, 15, 20, and 25 respectively. Table 6.2 lists the number of trip zones, parking zones and transit routes for each city.

Table 6.2: Characteristics of each city's parking and transit supply .

City	C1	C2	C3	C4	C5
Number of Trip Zones	5	10	15	20	25
Number of Parking Zones	2	4	6	8	10
Number of Routes	2	3	4	5	5

We perform preliminary computational analysis of these five cities by solving for the Stage II PSNE using the iterative optimizations approach described in Section 6.4, for 20 different randomly generated combinations of transit frequencies and parking capacities for each city. The transit frequencies were varied between 13 and 156 daily trips, whereas the parking capacity in each trip zone with paid parking was varied between 500 and 8,000 parking spots. A summary of computational run times for this Stage II solution process is in Table 6.3.

Table 6.3: Computational run times for solving to Stage II PSNE directly using Baron Software.

City	C1	C2	C3	C4	C5
Average (min)	460.0	50.3	952.7	851.1	1099.3
Standard Deviation (min)	415.1	97.0	578.3	750.1	837.8
Minimum (min)	23.0	2.0	105.2	14.6	99.4
Maximum (min)	1460.8	462.3	2066.1	2969.5	3082.0

Table 6.3 demonstrates that the Stage II solution step itself is extremely time consuming. Even for small-sized cities like C1, across the 20 computational runs, it took between 23 and 1461 minutes to solve this step. For cities with somewhat larger sizes, like C5, the Stage II run times ranged between 99 and 3082 minutes across the 20 computational runs. In general, run times of several hours, or even days, are usually not a big issue when solving transportation planning problems that are strategic in nature. However, as will be shown in the next subsection, the solution to the overall two-stage game model requires repeating this solution process several hundred (or more) times for evaluating the operator revenues as functions of Stage I decisions. Therefore, the overall computational run times for a single instance of a city may take months and/or may require using several computers in parallel.

Upon further investigation, we find the main reason of these large computational times to be the two-way dependency between the number of users choosing transit and driving (q_{ijm} values given by equation (6.1)) on the one hand, and the transit discomfort costs (DC_{ij} values given by equation (6.4)) and the cruising costs for parking (CP_{jA} values given by equation (6.6)) on the other hand. The values of $q_{ijm} \forall i, j \in \mathbf{Z}, m \in \{A, B\}$ are affected by the values of DC_{ij}, $\forall i, j \in \mathbf{Z}$, and CP_{jA}, $\forall j \in \mathbf{Z}$, through equation (6.1); and the values of DC_{ij}, $\forall i, j \in \mathbf{Z}$ and CP_{jA}, $\forall j \in \mathbf{Z}$ are affected by those of q_{ijm}, $\forall i, j \in \mathbf{Z}$, $m \in \{A, B\}$ values through equations (6.4) and (6.6) respectively. Equations (6.1), (6.4) and (6.6) are all highly non-linear. Thus, for any particular set of pricing decisions by the operators, the optimization model needs to satisfy non-linear constraints with such two-way dependencies. By carefully observing the performance of the optimization solution process, we find that it often reaches close to the eventual optimal prices quickly, but then spends a large amount of additional time to make sure that equations (6.1), (6.4) and (6.6) are satisfied by adjusting the passenger flows and mode shares. Therefore, we developed the following two-step heuristic procedure to decrease the computational times for the Stage II solution step.

A) First, we set a maximum computational time of 5 minutes for Baron Software for solving each operator's Stage II optimization problem and obtain a set of pricing decisions.

B) Now we fix prices at the levels obtained in Step A and again solve the Stage II optimization problem for that same operator using Baron. In this step, we do not enforce any maximum computational time restrictions.

We implemented this procedure to solve for the Stage II PSNE for the same five instances of cities (C1 through C5) using the same inputs. Comparing the results of this two-step heuristic procedure with those obtained by delegating the optimization problem directly to Baron, we found that the exact same PSNE was maintained in each of the 20 computational instances for each of the five cities. The computational times for the two-step heuristic shown in Table 6.4, when compared to those in Table **??**, show a significant decrease in run-times (shown in square brackets in Table 6.4,) ranging between 91% and 97%. These results validate the use of this two-step heuristic as a much faster alternative solution method. This faster heuristic approach was used for conducting all computational experiments presented in Section 6.7.

Table 6.4: Computational run times for solving to Stage II PSNE using the two-step heuristic approach.

City	C1	C2	C3	C4	C5
Average (min) [% improvement]	13.1 [97]	4.3 [91]	46.3 [95]	52.2 [94]	89.9 [92]
Standard Deviation (min)	9.8	3.5	84.3	38.9	105.7
Maximum (min) [% improvement]	2.8 [88]	1.5 [25]	5.1 [95]	18.3 [-25]	29.7 [70]
Minimum (min) [% improvement]	35.3 [98]	17.7 [96]	347.2 [83]	157.6 [95]	452.4 [85]

6.7 Model Results

In this section, we present three different case studies generated by the case study setup detailed in Section 6.5, with 25, 10 and 5 zones, respectively (Figure 6.2). Let us denote these three cities by MC (for 'Medium City'), SC (for 'Small City') and TC (for 'Tiny City'), respectively. Subsection 6.7.1, provides a summary of their most important parameters. Then, subsections 6.7.2, 6.7.3 and 6.7.4 present the model results for MC, SC and TC respectively. For each city, we also present a series of results of sensitivity analyses to the model parameters.

6.7.1 Test City Configurations

The number of inhabitants and the number of daily trips in each city depend on the number of trip zones, which are shown in Figure 6.2. MC has 230,700 inhabitants, SC has 47,700, and TC has 27,770 inhabitants; while the number of daily trips equals 285,200, 71,900 and 50,700 respectively. We assumed

that the probability of being a residential zone is 0% for a zone in the city center, and 100% for a suburban zone. Thus, these three cities have all residential zones in the suburbs and all commercial/industrial zones in the city center.
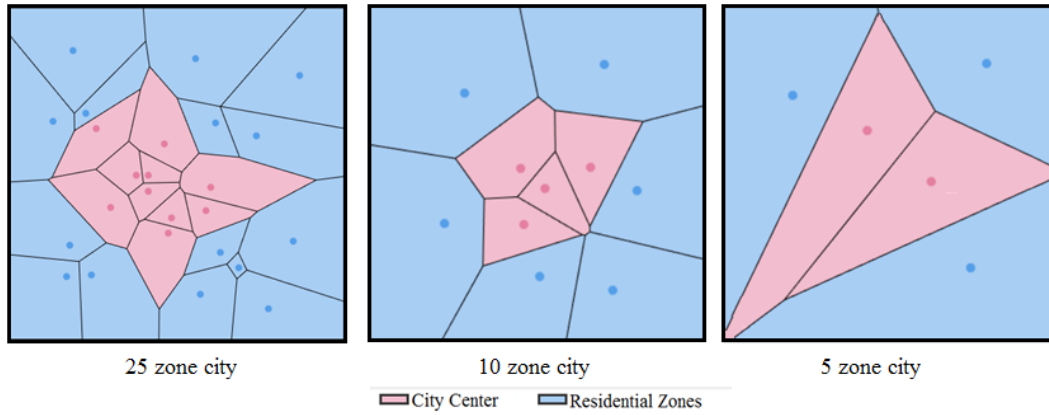
Figure 6.2: Trip zones in the three randomly-generated cities.

The average speeds of travel by cars and transit vehicles are 27.6 km/h and 17.4 km/h respectively for MC, 29.7 km/h and 19.5 km/h respectively for SC, and 28.3 km/h and 19.3 km/h respectively for TC. The hourly costs of time for MC, SC and TC are set to 6.4€, 6.1€ and 7.4€, respectively.

The vehicles in the transit fleet in MC and TC are assumed to have a capacity of 63 pax/veh each, while the capacity of the transit vehicles in SC is assumed to be 51 pax/veh. Parameter SW in the accessibility cost function takes values of 36, 154 and 19 in MC, SC and TC respectively, while the discomfort cost function parameter ψ_B takes values 1.9€, 2€ and 1.6€ respectively. This latter parameter indicates that if the transit vehicle crowding is at its maximum, then an hour spent inside the transit vehicle is equivalent, in terms of the associated discomfort, to $1 + \psi_B$ hours spent inside a car. The power parameter of this discomfort cost function is assumed to be the same, $\rho_B = 2.6$, for the three cities.

The average cost per kilometer of traveling by car and the car occupancy rates are the same for the three cities, taking values of 0.324 €/km and 1.2 pax/veh respectively. The parameters α_1 and α_2 of the cruising cost function are also the same for the three cities, and are assumed to take values 9 and 42 respectively (Gallo et al., 2011). The average costs of not taking the trip are equal to 9.32€, 9.5€ and 7.3€ for MC, SC and TC, respectively.

The parameter θ in the mode choice function (6.1) representing the travelers' sensitivity to the generalized trip costs is assumed to take values of 0.42, 0.49 and 0.48 for MC, SC and TC respectively. The transit routes are shown in Figures 6.3a, 6.3b and 6.3c, while the prevalent parking capacity for each trip zone is shown in Figures 6.3d, 6.3e and 6.3f. As shown in these figures, the transit routes allow users to travel from every origin trip zone to every destination trip zone with a maximum of one transfer. Each transit route is symmetric in the sense that it is assumed to have the same frequency of service in either direction. The prevalent frequency values in each direction are shown in Table 6.5.
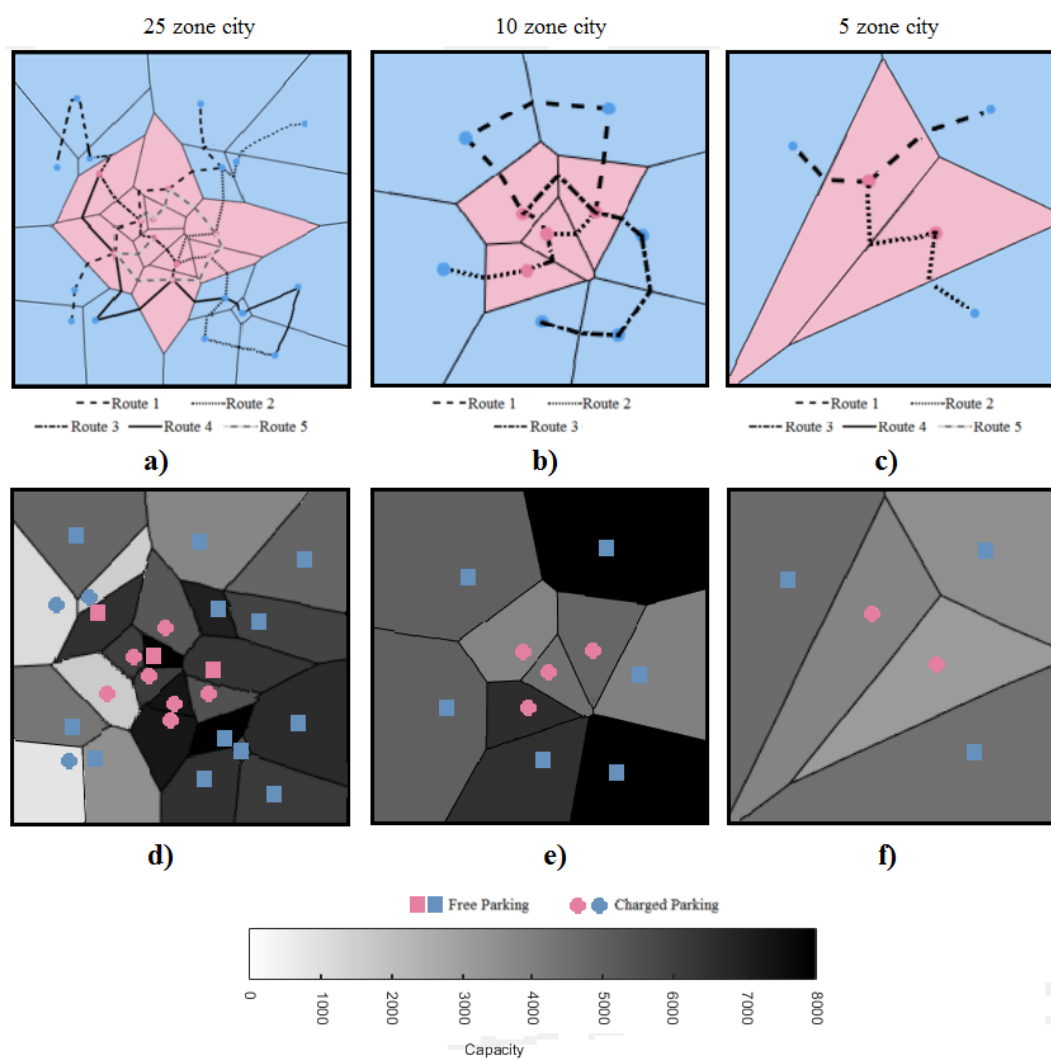
Figure 6.3: Transit routes (Figures 6.3a, 6.3b and 6.3c) and parking capacities (Figures 6.3d, 6.3e and 6.3f).

Table 6.5: Transit frequency per route and city.

City Route (one way)	1	2	3	4	5
MC (25 zones)	80	150	123	120	125
SC (10 zones)	80	128	122	-	-
TC (5 zones)	135	106	-	-	-

The prevalent financial situation for each operator is described in Table 6.6. These values are inspired by the real world values of operating costs and revenue-to-operating cost ratios (Guerra, 2011; Lindquist et al., 2009; Litman, 2010).

Table 6.6: Summary of operator finances.

City	MC	SC	TC
Transit fare (€/trip)	1.25	0.9	0.75
Parking fee (€/parking)	3.5	2.1	2.7

Transit operating costs	Variable component (€/km)	Prevalent	1.31	1.45	1.5
		Purchased	1.11	1.16	1.2
	Fixed component (€/trip)	Prevalent	22.75	22.5	20
		Purchased	19.34	16.88	17
Fleet: purchase cost (€/veh per day)			125	125	110
Fleet: selling price (€/veh per day)			15	20	10
Parking operating costs (€/parking space)			1.6	1.2	1.7
Parking Expansion cost	Variable (€/space)		0.3	0.25	0.2
	Fixed (€/expanded parking zone)		60	60	60
Profit (10^3 €/day)	Transit operator		-14.96	-26.43	-3.27
	Parking operator		54.67	16.75	24.28
Number of trips (10^3/day)	By transit		64.65 (33%)	16.01 (25%)	9.93 (38%)
	By car		129.78 (67%)	47.54 (75%)	16.07 (62%)
	Total		194.43	63.55	26.01

The numbers for the fixed and variable costs for transit are listed in Table 6.6 for the existing and purchased fleet of transit vehicles. The purchased transit vehicle costs were randomly generated so that variable and fixed costs are between 5% and 25% lower than those for the existing fleet of transit vehicles (Hallmark et al., 2012). However, the purchase of each new vehicle adds an extra daily payment generated as a uniform random number between 100€ and 150€ (Hallmark et al., 2012). Selling an existing vehicle saves a daily value randomly generated between 10€ and 20€. The randomly generated fixed cost of parking capacity expansion varies between 50 €/day and 70 €/day while the additional variable cost per added parking spot is set between 0.2 and 0.4 €/day (Litman, 2009).

For each city, a regression model is estimated in Step 2 of the solution approach (described in Section 6.3). The predictor variables in the regression model include each transit route's frequency and its square, parking capacity of each trip zone with paid parking and its square, and interaction terms given by the products of parking capacity of each trip zone with paid parking and the frequency of each transit route which crosses that trip zone. An example of such interaction term would be the product of transit route 1's frequency and parking capacity of the trip zone located in the north of the city center of city TC (see Figure 6.3).

Note that the total number of possible combinations of transit route frequencies and parking capacities for all trip zones with paid parking is very large. Hence solving Stage II game for even a tiny city (like TC) is computationally very challenging. Therefore, our regression relies on a sample of all possible

combinations. The sample size is determined by assuming a ratio of 15 data points per fitted parameter. This ratio is often deemed to be a reasonable lower bound to ensure that the estimated parameters are reliable. We varied the parking capacity in each trip zone with paid parking between 500 and 8,000 parking spots. Similarly, we varied the frequency for each transit route between 13 and 156 daily trips. A k-fold cross validation approach (Kohavi et al., 1995) was used to validate the predictive ability of the regression model. The overall sample was divided between a training/cross-validation dataset (with 80% of all data points), and a testing dataset (with 20% of all data points). The training/cross-validation dataset was divided into k=12, k=6 and k=3 folds for the cities MC, SC and TC respectively, and the k-fold cross validation method was applied. Several specifications were compared for the multiple regression model, and the best specification in terms of the highest out-of-sample goodness of fit (R^2 values) was selected. Appendix C reports the R^2 values as evaluated on the testing dataset by using this best model specification. Finally, this best model specification was applied to the entire dataset to estimate parameters that were subsequently used for Step 3 of the solution process.

6.7.2 Medium city Instance

In this subsection, we present computational results based on city MC and analyze the sensitivity of the results to the Stage I game parameters.

For our first analysis, the Stage II is solved to a PSNE for the prevalent values of transit frequencies and parking capacities. Table 6.7 summarizes the results related to transit fare, parking fee, operator profits and number of passenger trips along with change from the prevalent values measured as percentage of the absolute value of the prevalent values. Table 6.7 shows a substantial increase in transit fares and parking fees. This leads to an operating profit of 56.46 €/day for the transit operator, which is a significant improvement compared to the prevalent deficit of 14.96 €/day. However, the transit passenger trips decrease from 63.65/day to 38.67/day. The parking operator's operating profit increases by 43%, while the number of users choosing to travel by car increases slightly (by 0.5%). Mode share switches from 33% transit and 67% car to 23% transit and 77% car.

Table 6.7: Stage II PSNE summary.

		Value	Change (%)
Transit fare (€/trip)		3.94	215
Parking fee (€/parking)		5.16	47
Profit (10^3 €/day)	Transit operator	56.46	477
	Parking operator	81.98	50
Number of trips (10^3/day)	By transit	38.67	-40
	By car	130.49	1
	Total	169.16	-13

For our second analysis, we solve the entire two-stage game model. This requires estimating the regression model parameters. Following the solution method described in Section 6.4, the best regression models for the transit operator and for the parking operator have the R^2 values of 0.98 and 0.93, respectively.

Table 6.8 summarizes the Stage I decisions obtained by solving the two-stage game for five different cost scenarios. Each scenario corresponds to different combinations of costs included in the operator objective functions for Stage I. Note that the lower limits set for both transit frequencies and parking capacity can represent a regulation measure, showing that our framework can be easily adapted to include intermediate regulation measures.

Table 6.8: Capacity decisions for city MC under the SPPSNE.

	Scenario	Prevalent Capacities with Stage II Competition	1	2	3	4	5
Purchased transit	Variable (€/km)	-	0.98	1.11	1.24	0.98	1.24
fleet operating costs	Fixed (€/trip)	-	17.06	19.34	21.67	17.06	21.67
Fleet: purchase cost (€/veh per day)		-	100	125	150	100	150
Fleet: selling price (€/veh per day)		-	20	15	10	20	10
Parking expansion	Variable (€/space)	-	0.2	0.3	0.4	0.4	0.2
cost	Fixed (€/expanded parking zone)	-	50	60	70	70	50
	Zone 1	6343	5720	5716	5713	5720	5713
	Zone 2	7271	6060	6060	6060	6060	6060
	Zone 3	5455	4737	4737	4737	4737	4737
	Zone 5	6032	5075	5079	5082	5075	5082
Parking	Zone 7	6095	6078	6078	6078	6078	6078
Capacity	Zone 8	5272	3889	3889	3889	3889	3889
	Zone 9	1626	961	961	961	961	961
	Zone 11	854	500	500	500	500	500
	Zone 18	1524	1524	1524	1524	1524	1524
	Zone 24	1139	2286	2156	2025	2025	2286
	Existing Fleet	80	74	75	75	74	75
Route 1	New Fleet	0	0	0	0	0	0
	Total	80	74	75	75	74	75

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		Existing Fleet	150	20	25	29	20	29
	Route 2	New Fleet	0	0	0	0	0	0
		Total	150	20	25	29	20	29
		Existing Fleet	123	144	144	144	144	144
	Route 3	New Fleet	0	0	0	0	0	0
		Total	123	144	144	144	144	144
		Existing Fleet	120	13	13	13	13	13
	Route 4	New Fleet	0	0	0	0	0	0
		Total	120	13	13	13	13	13
		Existing Fleet	125	103	105	107	103	107
	Route 5	New Fleet	0	0	0	0	0	0
		Total	125	103	105	107	103	107
		Fleet (veh)	285	152	156	159	152	159
Profit		Transit Operator	56.46	85.88	85.32	84.77	86.07	84.58
$(10^3\ €/\text{day})$		Parking Operator	81.98	91.31	91.09	90.91	91.23	90.98

The first main observation from the results in Table 6.8 is the fact that the transit operator does not purchase any vehicle under any of the five scenarios, but actually decreases its fleet size significantly. A reduction in parking capacity and transit frequency is observed, but the variations of their values along with the variations of the cost parameters are small, revealing the low sensitivity of the equilibrium results to cost changes. There is a 46% average decrease in transit fleet size across the five cost scenarios, caused by a considerable reduction in transit frequencies on all routes except Route 3. For Route 3, the frequency increases from 123 trips/day to 144 trips/day under all cost scenarios. Similar broad trends hold for the parking capacity values. For all zones except zone 24, the parking capacity either decreases or remains constant under all five cost scenarios. The transit frequency for route 4 and parking capacity for zone 11 were assigned their lowest allowable values possible, suggesting their relative lack of profitability from the respective operators' perspectives.

These results highlight the interdependent nature of the transit and parking operators' decisions. For example, scenario I and scenario V have the same costs for the parking operator whereas the transit operator has significantly different costs. Specifically, the transit vehicle selling price for scenario I is twice that for scenario V, resulting in overall higher transit frequencies because of selling seven fewer vehicles. In response, the parking capacity is slightly readjusted by the parking operator, by decreasing zone 1 capacity by 7 parking spots and increasing zone 5 capacity by the same amount (and keeping

all others the same under both scenarios). Note that while the transit operator's costs have an indirect effect on parking operator's decisions, it is much smaller in magnitude than the direct effect on transit operator's decisions.

On the other hand, when the transit vehicle selling prices are the same (scenario III and scenario V), the transit operator chooses the same frequencies across its entire route network. However, due to the lower costs of parking capacity expansion in scenario V compared with scenario III, the parking operator has 201 extra parking spots in zone 24. This shows an example where the changes in parking operator costs fail to create any indirect effect on transit operator decisions, though there is a significant direct effect on parking operator's decisions.

In summary, these computational experiments using city MC provide three major takeaways. First, equilibrium results are found to have low sensitivity to changes in various cost parameters, even when they are varied over wide ranges. Second, compared to prevalent conditions, competition leads to lower capacity and higher prices leading to reduced number of trips and higher profits, especially for transit. Finally, the decisions of the two operators are shown to be highly interdependent, and changes in parameters of one operator may affect equilibrium decisions of both operators. Next, we will explore the effects of different pricing configurations using the small city case study.

6.7.3   Small City Instance

In this subsection, city SC is used to analyze different transit fare structures and the parking fee structures. Specifically, we consider the following four scenarios.

I. 1 parking fee level and 1 transit zone;

II. 1 parking fee level and 2 transit zones;

III. 2 parking fee levels and 1 transit zone;

IV. 2 parking fee level and 2 transit zones.

In scenarios II and IV involving 2 transit zones, all OD pairs with both origin as well as destination in the city center are considered to be belonging to the first transit zone while all other ODs are assumed to belong to the second one. In scenarios III and IV involving 2 parking levels, one parking level corresponds to trip zone 1 and the other corresponds to the remaining 3 trip zones with paid parking.

We again start by holding the transit frequencies, fleet size, and the parking capacities constant at their prevalent levels and solving for a Stage II PSNE.

Table 6.9: Stage II PSNE summary for each scenario.

Scenario		I	II	III	IV
Transit fare	OD pairs in the city center	3.44	3.43	3.44	3.43
(€/trip)	Remaining OD pairs		3.45		3.44
Parking fee	Zone 1	5.02	5.02	4.7	4.7
(€/parking)	Zones 2, 3 and 4			5.12	5.13
Profit	Transit operator	-4.65	-4.61	-4.73	-4.65
(10^3 €/day)	Parking operator	50.5	50.5	50.56	50.64
Number of trips	By transit	10.52 (19%)	10.52 (19%)	10.5 (19%)	10.5 (19%)
(10^3/day)	By car	45.88 (81%)	45.88 (81%)	45.91 (81%)	45.91 (81%)
	Total	56.4	56.4	56.41	56.41

Table 6.9 lists resultant prices, operator profits and numbers of trips under each scenario. Different configurations are shown to result in quite similar overall profitability results, and none of them mitigates the transit deficits. However, the transit losses are approximately 80% lower compared with the prevalent conditions described in Table 6.6. Moreover, the total number of trips stays almost constant across the four scenarios. Going from scenario I to scenario II, and from scenario III to IV, as the transit operator is afforded more flexibility in setting its fares, the transit operator's profit improves, while the parking operator's profit stays constant or increases. On the other hand, going from scenario II to IV, and from scenario I to III, when the parking operator is afforded more flexibility in setting its fees, the parking operator's profit goes up while that of the transit operator is reduced.

As before, transit profit maximization leads to a decrease in transit ridership from the prevalent value of 16,010 trips/day to an average of 10,510 trips/day across the four scenarios. This decrease is quite stable across the scenarios and is a direct effect of the increase in the transit fares from 0.9 €/trip to an average of 3.44 €/trip.

Next, we solve the entire two-stage model to its SPPSNE using the three-step approach described in Section 6.4. The Stage II payoff approximation step included the estimation of eight different regression models, corresponding to the payoffs of the two operators across the four different scenarios. These best performing regression model specifications included all linear, quadratic and interaction terms, with average testing R^2 values of about 0.950 and 0.945 for the transit operator and for the parking operator, respectively. The full list of individual R^2 values as evaluated on the testing datasets is shown in Appendix C.

Table 6.10: Capacity decisions for city SC under the SPPSNE.

	Scenario	Prevalent	I	II	III	IV
	Zone 1	4800	4139	4221	4109	4080
Parking Capacity	Zone 2	6600	5295	5229	5277	5236
	Zone 3	4500	4070	4034	3833	3870
	Zone 4	3900	3763	3617	3658	3665
	Route 1 Existing Fleet	80	116	109	114	116
	Route 1 New Fleet	0	0	0	0	0
	Route 1 Total	80	116	109	114	116
Transit	Route 2 Existing Fleet	128	119	106	114	119
Frequency	Route 2 New Fleet	0	0	0	0	0
(trips/day)	Route 2 Total	128	119	106	114	119
	Route 3 Existing Fleet	122	29	13	17	29
	Route 3 New Fleet	0	0	0	0	0
	Route 3 Total	122	29	13	17	29
	Fleet (veh)	101	82	71	76	82
Profit	Transit Operator	-26.43	11.81	11.74	11.4	11.66
(10^3 €/day)	Parking Operator	16.75	52.08	53.68	53.41	52.59

Results listed in Table 6.10 show that the transit operator significantly modifies all the route frequencies while also reducing its fleet size by selling many of the existing vehicles and not purchasing any new vehicles. Although smaller in magnitude, the changes made by the parking operator lead to a decrease in the parking capacity in all four zones under all four scenarios, but the drop in parking capacity in zone 2 is especially large.

Contrary to the previous observation based on fixed capacity scenarios, the transit operator's profits actually reduce going from scenario I to scenario II, while those of the parking operator increase. This implies that, in two-stage competition, the extra flexibility afforded to the transit operator in setting the fees might be exploited by the parking operator, hurting the profits collected by the parking operator. The trend when going from scenario III to scenario IV is reversed, however, as this time the extra flexibility afforded to transit helps transit by increasing its profit and hurts parking by decreasing its profit. The effects of parking price flexibility are also similarly complicated. When going from scenario I to III, transit profits decrease while parking profits increase. When going from scenario II to IV, both transit and parking profits decrease.

These results are in line with what was claimed so far. I.e., competition leads to higher profits for both operators, the decisions of the two operators are shown to be highly interdependent, and changes in one operator's pricing structure affects the profits of both operators. Flexibility helps an operator's profits when the capacity decisions are fixed. However, the two-stage model's SPPSNE results do not necessarily

suggest that more flexibility for an operator means higher profits for that operator or lower profits for its competitor. An important takeaway is that the two-stage competitive interactions are complicated, and flexible pricing structures may benefit or harm an operator depending on the setting.

6.7.4 Tiny City Instance

In this subsection, city TC is used to analyze the sensitivity of the results to the generalized cost parameters, by evaluating eight different scenarios. These eight scenarios are created by considering moderate changes in parameters θ, SW, α_2 and ρ_B. The changes are made one parameter at a time while holding all others fixed. The eight scenarios are as follows:

I. Moderate decrease in the users sensitivity to cost ($\theta = 0.35$).

II. Moderate increase in the users sensitivity to cost ($\theta = 0.61$).

III. Moderate decrease in the transit accessibility cost parameter ($SW = 17$).

IV. Moderate increase in the transit accessibility cost parameter ($SW = 21$).

V. Moderate decrease in the power in the cruising function ($\alpha_2 = 22$).

VI. Moderate increase in the power in the cruising function ($\alpha_2 = 62$).

VII. Moderate decrease in the power in the transit discomfort function ($\rho_B = 1.6$).

VIII. Moderate increase in the power in the transit discomfort function ($\rho_B = 3.6$).

The best performing linear regression model specifications estimated for each of these eight scenarios included all the linear, quadratic and interaction terms, having an average R^2 of 0.977 and 0.986 for the transit and parking operator, respectively.

Table 6.11: Summary of each operator finances for city TC under the prevalent conditions.

Scenario		With Default Values of All Parameters	I	II	III	IV	V	VI	VII	VIII
Profit	Transit Operator	-3.27	-3.25	-3.3	-3.25	-3.31	-3.22	-3.29	-4.1	-2.82
(10^3 €/day)	Parking Operator	24.28	24.25	24.33	22.09	25.92	23.99	24.38	24.81	23.54
Number of trips (10^3/day)	Transit	9.93 (-38%)	9.96 (38%)	9.9 (38%)	9.96 (40%)	9.88 (37%)	10.01 (39%)	9.91 (38%)	8.83 (35%)	10.54 (40%)
	Car	16.08 (62%)	16.07 (62%)	16.1 (62%)	15.11 (60%)	16.81 (63%)	15.95 (61%)	16.13 (62%)	16.32 (65%)	15.75 (60%)
	Total	26.01	26.02	26	25.07	26.69	25.96	26.03	25.14	26.29

The prevalent values of profits and the number of trips by each mode are shown in Table 6.11, while the results from calculation of the SPPSNE are provided in Table 6.12.

Table 6.12: Capacity decisions for city TC under SPPSNE.

Scenario		With Default Values of All Parameters	I	II	III	IV	V	VI	VII	VIII
Parking	Zone 1	1540	2100	1406	1535	1544	1910	1369	1848	1357
Capacity	Zone 2	3782	3533	3984	3780	3785	3837	3761	3919	3708
	Old Fleet	78	81	72	72	78	77	78	84	60
Route 1	New Fleet	4	6	4	6	4	5	4	6	10
Transit	Total	82	87	76	78	82	82	82	90	70
Frequency	Old Fleet	0	0	0	0	0	0	0	0	0
(trips/day) Route 2	New Fleet	103	101	103	101	103	102	103	115	97
	Total	103	101	103	101	103	102	103	115	97
Fleet (veh)		15	15	14	14	15	15	15	16	13
Profits		13.90	13.90	18.03	11.58	14.04	13.77	13.94	13.92	10.20	15.67
(10^3 €/day)		21.93	21.93	23.77	20.87	22.01	21.96	21.11	22.26	23.01	21.32

Table 6.12 results show that changes in Stage II game parameters affect Stage I results.

The increase of user's cost sensitivity (scenario II) leads to a decrease in transit operators' fleet, due to decreasing the frequency of Route 1 and maintaining the frequency in Route 2. The opposite is observed for Route 1 when the user's cost sensitivity is decreased, whereas the frequency in Route 2 is decreased, as displayed by the results achieved for Scenario I. Similar trends also occur for the parking operator's supply, with exception of the parking capacity of zone 2 that is increased in scenario II instead of kept unchanged. These changes are driven by the fact that when users are more sensitive to costs, it restricts the operators' ability to charge higher prices, which in turn makes it unprofitable to provide additional capacity.

The results of the changes in the parameters related to transit trips (namely, scenarios III, IV, VII and VIII) show that the transit parameters influence the equilibrium behavior of both operators. Specifically, if the cost of travelling by transit decreases (namely, scenarios III and VIII), both operators reduce their capacities. This is driven by the fact that such transit cost reduction allows transit operator to charge higher fares and offer lower frequencies, which in turn leads to a reduction in parking capacity as well.

From scenarios V and VI, we see that the higher the value of the cruising cost function exponent, the lower the capacity. In fact, the effect of cruising for parking is more relevant when the parameter is increased than when it is decreased.

These results highlight the importance of the features in the generalized costs besides the transit fares and parking fees. Although both operators became highly profitable under a deregulated market structure, these profits also depend on the importance assigned by users to features that the operators can only control indirectly. E.g., cruising for parking is only excluded from the users' generalized costs if the parking operator provides extremely high amounts of capacity, which would be extremely costly. On the other hand, smaller number of parking spaces might lead to higher parking fees and also higher cruising for parking costs to the users, which probably will severely decrease the number of car trips due to the lack of parking spaces and therefore the revenues collected by the parking operator will also decrease. Similar concerns can be addressed to the choices linked to the frequencies provided by the transit operator to decrease the discomfort of users when traveling by transit. In this sense, not only the prices charged by the operator should be accounted when setting the offer that maximizes the profit, but also the effects that these adjustment will have on features indirectly controlled by the operators. This model provides meaningful insights towards these two-way dependencies, showing that even in competition, without regulation and seeking to improve their own profits, the collateral effects of these changes should be taken into account by the operators when it comes to adjust both prices and supply.

6.8 Conclusion

In this chapter we presented a game-theoretic approach to reduce transit deficits by modeling competition between the transit operator and the parking operator. By managing the prices and the levels of supply provided to the transportation network users, the two operators attempt to maximize their own profits in a two-stage game. In the first stage, long-term decisions such as transit frequency, fleet, and parking capacity are made, while the pricing decisions are made in the second stage.

The simulated cities inspired by real-world data provide examples of the real-world applications of our approach. Through this small framework, 3 different cities were computed and the application of the game-theoretic model was conducted. Through the results drawn for each one of these 3 different case studies, it was shown that the extreme scenario of having a fully deregulated market can overcome the financial problems that often characterize transit systems. Furthermore, by exploring different city characteristics, such as users' sensitiveness towards congestion or transit occupation rate, the complexity and interdependency of the relationship between transit and parking operators were highlighted.

The integration of transit and parking systems with competition between transit and car provided a better understanding on how pricing schemes and the supply of each service influence not only the short-term financial features, but also the potential supply adjustments and expected long-term profits, without including any restriction or regulations for the service provided to users. Furthermore, the deregulation assumption managed through competition between transit operators and parking operators

led to a significantly decreased on the supply that is offered when compared to the prevalent scenario, which was usually designed with regulation market restrictions. Even though fully deregulation market solutions might not be the best solutions in what concerns the social responsibility usually assigned to transportation systems, this framework is easily adapted to accommodate intermediate regulation stages so that the knowledge towards the suitability of different market structures to solve the problems raised by providing affordable transit systems with reasonable levels of service can be improved.

The chameleon characteristic of the developed framework to deal with intermediate stages of regulations, or cases of having city planners with major concerns of social welfare policies over financial ones, will be addressed in our future work. This future analysis will also explore the influence of different definitions of transit zones and their resultant transit-shares and transit network traffic dynamics on the profit-maximization goals. Another extend that will be asses in the future is the integration of the two services in a deregulated market where transit will be financed by the revenues collected by the parking operator. In other words, although aiming at maximizing their own profit, the crossed-effects between transit and car will be taken to a next level where parking will subsidize transit by a proportion of its profits, without and with intermediate regulatory measures (e.g., minimum levels of service or lower bounds for the number of made trips).

Appendix A

Let C_{ijB}^0 and C_{ijA}^0 respectively be the generalized costs of taking a trip from i to j using transit and car, as given by equations (6.2) and (6.5), respectively. To estimate these costs, we assumed that a former and non-competitive prevalent scenario exists for the city. As any city in the world, a prevalent behavior exists that can be observed, analyzed and replicate. Due to the lack of data, we developed a framework capable of randomly-generate cities that take into account real-world features (see Section 6.5). In this sense, we consider that the cost of not making a trip depends on the prevalent costs that users are already paying (note that this assumptions can be made for either a generated either for a real-world city). The generalized costs C_{ijB}^0 and C_{ijA}^0 are therefore assessed by considering the prices charged by each operator, the ingredients included on these generalized costs as displayed on (6.2) and (6.5), respectively, and the equilibrium required by these equations in what concerns the discomfort costs (6.4) and the cruising for parking cost (6.6). It is based on this prevalent scenario that the cost of not making the trips is estimated, in an attempt to analyze where the threshold between collecting higher fees with lower levels of captured demand, or the other way around, is placed. In fact, the inclusion of these costs and the alternative related to them ensures that it would not be possible to charge extremely high fares and fees while assuming that people would be willing to pay those irrational values. We include the constant multiplier $\varepsilon_{ij} > 1$ so that the elasticity of users towards the change of transit fares and parking fees is included, as

a way to account for the extra cost that users are willing to pay to go from i to j. Then the expression for the generalized cost of not making the trip is assumed to be $C_{ijO} = \varepsilon_{ij} \cdot \frac{(C_{ijA}^0 + C_{ijB}^0)}{2}$, ensuring that none of the two operator can infinitely increase their prices.

Our method to estimate the unconstrained demand is based on the classical gravity model formulation so that plausible number of trips can be ensured for the generated case studies.

A first estimation for the number of trips between a pair of zones ij is assigned to $\widetilde{T}_{ij}$ through expression (6.38). In this expression: α_{GM}, μ_{GM}, and ε_{GM} are parameters randomly generated in $[0,1]$; θ is generated as explained in subsection 6.5.2; $\widetilde{T}_{ijB}$ and $\widetilde{T}_{ijA}$ are simplified versions of expressions (6.2) and (6.5), respectively, by setting the transit discomfort cost and the cruising cost for parking to zero while considering the remaining ingredients of these costs as explained in Section 6.2.

$$\widetilde{T}_{ij} = \alpha_{GM} \cdot \frac{PC_i^{\mu_{GM}} \cdot EC_j^{\varepsilon_{GM}}}{exp\left(\theta \cdot \frac{\widetilde{C}_{ijA} + \widetilde{C}_{ijA}}{2}\right)}, \qquad i, j \in \mathbf{Z} \tag{6.38}$$

Due to the fact that the total number of trips estimated through (A1) are not equal to the expected number of trips done (i.e., $\sum_{i,j\in\mathbf{Z}} \widetilde{T}_{ij} \neq \pi \cdot PC$), we proceed with the normalization of the number of trips estimated in (6.25) to meet the total number of daily trips ($\pi \cdot PC$) as shown in (A2). The number of trips done by each person is given by π, and this value is estimated as explained in subsection 6.5.2.

$$T_{ij} = \alpha_{GM} \cdot \frac{\pi \cdot PC}{\sum_{i,j\in\mathbf{Z}} \widetilde{T}_{ij}} \cdot \frac{PC_i^{\mu_{GM}} \cdot EC_j^{\varepsilon_{GM}}}{exp\left(\theta \cdot \frac{\widetilde{C}_{ijA} + \widetilde{C}_{ijA}}{2}\right)}, \qquad i, j \in \mathbf{Z} \tag{6.39}$$

To estimate the total number of trips that would be done if both transit fares and parking fees were zero ($\hat{T}_{ij}$), new generalized costs are estimated for trips done by transit or by car. In this sense, the values of $\hat{C}_{ijB}$ and $\hat{C}_{ijA}$ are equal to $\widetilde{C}_{ijB} - p_{ijB}^0$ and $\widetilde{C}_{ijA} - p_{jB}^0$, respectively, where p_{ijB}^0 and p_{jA}^0 are the transit fare and parking fee values implemented by each operator in the prevalent situation (without the introduction of competition between the two operators). By replacing the values of $\widetilde{C}_{ijB}$ and $\widetilde{C}_{ijA}$ with these new generalized costs into expression (6.39), a new number of trips $\hat{T}_{ij}$ is determined. This amount includes the trips that are not done due to the costs of transit fares and parking fee. Let $\widehat{LT}_{ij}$ be the number of trips that are not done by people due to these costs, which is equal to $\widehat{LT}_{ij} = \hat{T}_{ij} - T_{ij}$. The estimation of $\widetilde{C}_{ijO}$ as displayed in equation (6.40), and the final number of trips that are lost by the system L_{ij} due to the pricing of transit and parking are estimated as expressed by equation (6.41).

$$\widehat{LT}_{ij} = \widetilde{\alpha}_{GM} \cdot \frac{\pi \cdot PC}{\sum_{i,j\in\mathbf{Z}} \widetilde{T}_{ij}} \cdot \frac{PC_i^{\mu_{GM}} \cdot EC_j^{\varepsilon_{GM}}}{exp\left(\theta \cdot \widetilde{C}_{ijO}\right)} \Rightarrow \widetilde{C}_{ijO} = \frac{ln\left(\widetilde{\alpha}_{GM} \cdot \frac{\pi \cdot PC}{\sum_{i,j\in\mathbf{Z}} \widetilde{T}_{ij}} \cdot \frac{PC_i^{\mu_{GM}} \cdot EC_j^{\varepsilon_{GM}}}{\widehat{LT}_{ij}}\right)}{\theta}, \; i, j \in \mathbf{Z} \tag{6.40}$$

$$LT_{ij} = \widetilde{\alpha}_{GM} \cdot \frac{\pi \cdot PC}{\sum_{i,j\in\mathbf{Z}} \widetilde{T}_{ij}} \cdot \frac{PC_i^{\mu_{GM}} \cdot EC_j^{\varepsilon_{GM}}}{exp\left(\theta \cdot \widetilde{C}_{ijO}\right)}, \qquad i, j \in \mathbf{Z} \tag{6.41}$$

Finally, the total autonomous demand for a OD pair ij is given by $Q_{ij} = T_{ij} + LT_{ij}$. To accommodate

further pricing changes and also the inclusion of cruising for parking and discomfort costs, the cost $\widetilde{C}_{ijO}$ is disturbed by a random variable with values between 1 and 2, which leads to the values C_{ijO} for each OD pair included in our model. In this sense, the parameter that estimates the extra cost that users are willing to pay to go from i to j is calculated by $\varepsilon_{ij} = \frac{2 \cdot C_{ijO}}{C^0_{ijA} + C^0_{ijB}}$.

Appendix B

We now aim at proving the existence of values that maximize the functions SOF^{SS}_B and SOF^{SS}_A, $p^*_B = (p_{ijB})_{i,j \in \mathbf{Z}}$ and $p^*_A = (p_{jA})_{i,j \in \mathbf{Z}}$, and their uniqueness. For simplicity, let $\widetilde{C}_{ijm}$ be the variable that represent all the costs included on the generalize cost of these functions besides $(p_{ijB})_{i,j \in \mathbf{Z}}$ and $p_A = (p_{jA})_{i,j \in \mathbf{Z}}$, therefore a general expression for SOF^{SS}_B and SOF^{SS}_A is given by (6.42), where the decision variables are respectively p_{ijB} and p_{jA}. In this sense, the values of p_{jA} are fixed for SOF^{SS}_B and p_{ijB} is fixed for SOF^{SS}_A. Note that the assumption of setting $\psi_B = \alpha_1 = 0$ holds in this subsection, and that the domains of SOF^{SS}_B and SOF^{SS}_A are compact sets of $\mathbb{R}^{(\#\mathbf{Z})^2}$.

$$SOF^{SS}_m = \sum_{i,j \in \mathbf{Z}} \frac{e^{-\theta \cdot (\widetilde{C}_{ijm} + p_{ijm})}}{e^{-\theta \cdot (\widetilde{C}_{ijA} + p_{jA})} + e^{-\theta \cdot (\widetilde{C}_{ijB} + p_{ijB})} + e^{-\theta \cdot (\widetilde{C}_{ijO})}} \cdot Q_{ij} \cdot p_{ijm}, \quad m \in \{A, B\} \tag{6.42}$$

The first derivative of function (6.42) is defined by equation (6.43), where we represented $\frac{e^{-\theta \cdot (\widetilde{C}_{ijm} + p_{ijm})}}{e^{-\theta \cdot (\widetilde{C}_{ijA} + p_{jA})} + e^{-\theta \cdot (\widetilde{C}_{ijB} + p_{ijB})} + e^{-\theta \cdot (\widetilde{C}_{ijO})}}$ by $LF(p_{ijm})$ for the sake of simplifying the following expression. Let $p^*_B = (p_{ijB})_{i,j \in \mathbf{Z}}$ and $p^*_A = (p_{jA})_{j \in \mathbf{Z}}$ be the inflection points satisfying $\frac{\partial SOF^{SS}_m}{\partial p_{ijm}}(p^*_m) = 0$.

$$\frac{\partial SOF^{SS}_m}{\partial p_{ijm}} = Q_{ij} \cdot LF(p_{ijm}) \cdot \left[1 - \theta \cdot p_{ijm} \cdot \left(-LF(p_{ijm})\right)\right] \tag{6.43}$$

Since $\frac{\partial SOF^{SS}_m}{\partial p_{ijm}}$ is class C^∞ , the second derivative follows as shown in equation (6.44).

$$\frac{\partial^2 SOF^{SS}_m}{\partial p^2_{ijm}} = Q_{ij} \cdot \theta \cdot \left(1 - LF(p_{ijm}) \cdot \left(LF(p_{ijm}) \left[-2 + \theta \cdot p_{ijm} - 2 \cdot \theta \cdot p_{ijm} \cdot \left(LF(p_{ijm})\right)\right]\right.\right. \tag{6.44}$$

By acknowledging that $LF(p_{ijm}) < 1$ and that $Q_{ij} \cdot \theta \cdot (1 - LF(p_{ijm})) \cdot (LF(p_{ijm})) \geq 0$, we conclude that the second derivative is negative (i.e., $\frac{\partial^2 SOF^{SS}_m}{\partial p^2_{ijm}} \leq 0$). The proof of this statement is developed assuming that the second derivative is positive, which returns a contradiction. We recall that θ is a positive parameter and that the values of p_{ijm} should always be positive. The proof is as follows:

$$0 \leq \frac{\partial^2 SOF^{SS}_m}{\partial p^2_{ijm}} \Rightarrow 0 \leq = Q_{ij} \cdot \theta \cdot \left(1 - LF(p_{ijm}) \cdot \left(LF(p_{ijm}) \left[-2 + \theta \cdot p_{ijm} - 2 \cdot \theta \cdot p_{ijm} \cdot \left(LF(p_{ijm})\right)\right]\right.\right.$$

$$\Rightarrow 0 \leq \left[-2 + \theta \cdot p_{ijm} - 2 \cdot \theta \cdot p_{ijm} \cdot \left(LF(p_{ijm})\right)\right] \leq \left[-2 - p_{ijm}\theta\right]$$

$$\Rightarrow \frac{-2}{\theta} \geq p_{ijm} \Rightarrow p_{ijm} \leq 0 \vee \theta \leq 0 \qquad \textbf{\textit{contradiction!}}$$

In particular, $\frac{\partial^2 SOF^{SS}_m}{\partial p_{ijm} \partial p_{klm}} = 0$, for $i, j, k, l \in \mathbf{Z}$. In this sense, the hessian matrix of

SOF_m^{SS}, i.e. , $D^2 SOF_m^{SS}\left((p_{ijm})_{i,j\in\mathbf{Z}}\right)$, is a diagonal matrix, with $\frac{\partial^2 SOF_m^{SS}}{\partial p_{ijm}^2}$ as diagonal elements and size $(\#\mathbf{Z})^2 \times (\#\mathbf{Z})^2$. Therefore, the hessian matrix is negative semidefinite [4](NSD), which is straightforward conclusion from the definition of negative semidefinite.

Let $p_B^* = (p_{ijB})_{i,j\in\mathbf{Z}}$ and $p_A^* = (p_{jA})_{j\in\mathbf{Z}}$ be the inflection points satisfying $\frac{\partial SOF_m^{SS}}{\partial p_m^*} = 0$. Because the hessian matrix $D^2 SOF_M^{SS}\left((p_{ijm})_{I,j\in\mathbf{Z}}\right)$ is negative semidefinite, we conclude that these inflection points are local maxima. Furthermore, SOF_M^{SS} is a concave function (Theorem 6.1).

Theorem 6.1. *The (twice continuously differentiable) function $f : A \subset \mathbb{R}^n \leftarrow \mathbb{R}$ s convex if and only if $D^2 f(x)$ is NSD for every A.*

Appendix C

Table 6.13: R^2 values for each used regression to compute the first stage of the game-theoretic approach.

City	Scenario	Mode	R^2			
25	all	Transit	0.98			
		Car	0.93			
10	I	Transit	0.99			
		Car	0.95			
	II	Transit	0.83			
		Car	0.92			
	III	Transit	0.98			
		Car	0.95			
	IV	Transit	0.99			
		Car	0.96			
	Prevalent	Transit	0.98			
		Car	0.99			
	I	Transit	0.97			
		Car	0.98	II	Transit	0.98
		Car	0.99			
	III	Transit	0.98			
		Car	0.98			
	IV	Transit	0.98			

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[4] A $N \times N$ matrix M is negative semidefinite if $\forall z \in \mathbb{R}^N, z^T \cdot M \cdot z \leq 0$.

[5]

	Car	0.98
V	Transit	0.98
	Car	0.99
VI	Transit	0.98
	Car	0.98
VII	Transit	0.99
	Car	0.99
VIII	Transit	0.96
	Car	0.99

Chapter 7

Conclusion

Transit systems are acknowledged to be good alternatives to car use. However, because the revenues collected by transit operator are usually not enough to cover the costs of providing affordable transit systems with acceptable levels of service, the government and/or public agencies provide subsidies to cover those deficits. In this thesis, we proposed the integration of both transit and parking systems in order to decrease the extreme reliance of transit operators on these subsidies to maintain the operation of transit systems. This integration comprehends two different perspectives, a physical integration and a pricing scheme integration. The physical integration was developed by studying the best places to install park-and-ride facilities so that the traffic congestion observed in the central areas of a city could be decreased by introducing the possibility of users to switch from car to transit when achieving the periphery of the cities. In what concerns the pricing approach, two different standpoints are considered. One where the market is regulated and it was assumed that parking operators' revenues will help to mitigate the subsidize levels provided to transit operators; and another where a fully deregulated market is assumed, and profit-maximization goals are pursued by both transit and parking operators. The developed optimization models are useful or at helping local governments into selecting the best configuration of park-and-ride facilities or into establishing the pricing policies for both transit and parking systems under their control, directly or indirectly (e.g., through concessions) to solve the financial problems that characterize transit systems.

This thesis pursued three main global objectives. The first one was related to developing an optimization model capable of identifying the best places to install park-and-ride facilities. The other two objectives derived from the integration of transit and parking through pricing schemes and consisted of developing optimization models to bridge some gaps found in the literature. Therefore, we developed new methodological approaches to optimize the prices charged to users by both transit and parking operators so that the deficits of the transit systems could be decreased by being covered with the revenues collected by the parking operator, or by assuming a fully deregulated market where both transit and parking operators were competing to increase their own revenues. These objectives were fully accomplished

by the developed research.

In chapter 2, we developed an optimization model that deals with the physical integration of the transit system and the parking system. The developed mathematical model optimizes the best set of places to install park-and-ride facilities in the periphery of the city so that the total person-kilometer distance traveled by car in the city center is minimized. Previously, other authors developed mathematical models to address this facility location problem, but always in an incomplete way. For example, Faghri et al. (2002) developed a knowledge-based GIS to place park-and-ride facilities that has the capability of managing nonquantitative criteria via a designed expert system, but the final location solution did not affect the prespecified demand. Horner and Groves (2007) and Farhan and Murray (2008) sought for real world applications by minimizing the distance traveled by car in a monocentric city; and by maximizing the trips intercepted by park-and-ride facilities along with the minimization of travel and the maximization of the utilization of existing park-and-ride facilities, respectively; without properly addressing the users' mode-choice. Aros-Vera et al. (2013) and Chen et al. (2016) dealt with this short come by including mode-choice decisions according to logit functions in an optimization model that maximizes the number of car trips that are intercept by park-and-ride facilities, in the case of the former; and that maximize the number of rail-riders in a city, in the case of the later. Our model also addressed mode-choices as a discrete choice given by logit functions, accounting for the behavior of users in what concerns the deviations required to use a park-and-ride facility towards the least-cost route, and accounted for the minimizing of the person-kilometer distance traveled by car in the city center. It was acknowledged that by applying the developed model to the city of Coimbra, car use could be reduced by 19.1% with the installation of only 3 park-and-ride facilities. Our final analysis comprehended the comparison of this physical integration of transit and parking system towards the single improvements of these system according to a user point of view, allowing at drawing conclusions about the efficiency of park-and-ride facilities to decrease the number of kilometers travelled by car in the city center.

Although several improvements to the model might be advocated in what concerns the implementation of park-and-ride facilities and the features that should be considered while assessing the best locations to install them, namely parking capacity restrictions or the inclusion of changes on the traffic dynamic to account for the relieve on the congestion achieved by the use of the installed park-and-ride facilities; we diverted our research to the integration of the two systems in what concerns setting pricing schemes that minimizes the joint operating deficits (or maximize the joint profit) and leave the further improvements of this facility location model as future work.

The two chapters that followed dealt with the integration of transit and parking systems in a regulated market, whereas Chapter 6 introduced a framework that accounts for the opposite scenario (i.e., fully deregulated market). As previously mentioned, and to the best of our knowledge, this is the first work that addressed the integration of both transit and parking systems following a pricing scheme perspective.

Chapter 3 presented the first optimization model that integrated transit and parking systems. This model determines the transit fares and parking fees that minimize the joint operating deficits, i.e., the difference between the total operating costs and the total revenues collected by both transit and parking operators, while ensuring a reasonable level of service (by setting upper bounds to the number of potential trips that are not made). At first, this model was implemented to a hypothetical city where two cases were analyzed. The first case focused on assessing the capability of the transit operator to decrease their need of subsidization by only readjusting the transit fares (and by keeping unchanged the parking fees), which included the influence of these changes on the number of car users and affecting the revenues collected by the parking operator. The second step considered that both transit and parking operators were able to change their prices so that their joint deficits were minimized, showing that transit systems can be profitable if managed in an integrated manner. This second approach was also applied to the municipality of Coimbra, attempting at solving the financial problems faced by the transit operator that manages the transit system in this municipality. This implementation was developed considering three different scenario. Two where only the number of parking fee levels were changed, and a third one that optimized prices so that the joint operating deficits were as close as possible to zero. The first scenario improved significantly the joint deficits from 5.79×10^6€/year to -42.95×10^6€/year, by setting the transit fares from 0.6€ to 2.3€ and the parking fees to be 1.9€ per parking in the center of the municipality. In this scenario, as well as in the scenario with two parking fee levels that presented similar conclusions, the transit operator was capable of improving its operating deficits until the point of being profitable. However, this optimization model did not accounted for the relationship between demand and speed, assuming that the values of speed remained unchanged while the number of car trips changed with the optimization of transit fares and parking fees.

The short come of this model was improved on the mathematical model developed in Chapter 4 by including the variation of speed on the users' generalized costs and its relationship with the modal-share functions. This was accomplished by embedding traffic conditions of the network into the optimization model with resource to a network level aggregate traffic model based on the macroscopic fundamental diagram (MFD). A solution method was developed to solve this model due to its complexity, which included a traffic equilibrium algorithm that was incorporated in a greedy algorithm to provide good-solutions in a reasonable amount of time. Further improvements were also included in this optimization-dynamic model when compared to the static model developed in the Chapter 3. This improvements were the inclusion of cruising for parking effects, the differentiation of the modal-share functions to accommodate the division of the demand between those who can travel by car and those who cannot travel by car, and the inclusion of different routes for the same OD pair depending on the departure time. By applying this model to the morning period of a case study inspired in the municipality of Coimbra, we assessed the influence of the demand towards the road dynamic of the city, which is

also affected by the pricing schemes that were optimized by the model. In fact, the joint operating deficits growth from the current 1.5 x10³€/M.P. to -169.8 x10³€/M.P. if prices were updated from the current 0.6€ and 1.2€ to 3.25€ and 7€, for the transit fare and parking fee respectively, leading to a profitable transit system. This optimization model was solved by the developed solution method, whose capability of finding good solutions was also explored along with insights on how to turn a deficit transit system into a profitable one by only managing the transit fares and the parking fees under a regulated market structure.

As previously mentioned, the optimization model developed in Chapter 4 was solved by a solution method that included a greedy algorithm. However, this method was not our first attempt at developing a heuristic capable of finding good solutions in a reasonable amount of time for the optimization model detailed in Chapter 4. Our first approach was a hybrid simulated annealing-cross entropy algorithm, which was studied in Chapter 5. This hybrid algorithm was replaced by the solution method with the greedy algorithm due to the fact of not improving significantly the computation time when compared to solving the optimization model with resource to an exhaustive search for smaller case studies with single transit fares and single parking fees. Notwithstanding, the developed hybrid algorithm seemed to be a good improvement to what has been developed in the literature, along with being a good algorithm to solve problems with time-consuming objective functions. According to the literature, simulated annealing algorithms (SA) are good options at finding near-optimal solutions for several optimization models, where the properties of convergence are usually advocated as insurance due to the capability of the algorithm of escaping local optima solutions. The solutions analyzed by the SA algorithm are randomly generated, which allows the generation and analysis of poor quality solutions. This is not a problem if the computation time required by the analysis is really small. As opposite, having solutions with high time-consuming analysis will require excessively amounts of time to achieve a good and final solution when using the classic SA algorithm. In this sense, we developed a hybrid algorithm, where the general idea concerned the use of cross-entropy techniques to guide the procedure of finding good solutions that were assessed by the simulated annealing algorithm. With this hybridization, the probability of selecting a low-quality solution was decreased, because each element of the solution would have a higher probability of taking the values that compound solutions with good objective function values than the values that are linked to poor quality solutions. An improvement was clearly observed with the introduction of CE algorithm into the classical SA algorithm. This conclusion was sustained by comparing the solutions achieved by using randomly generated problems for a capacitated p-median with closest assignment constraints problem (CPM-CA problem). Furthermore, the reliance of both algorithms on the calibration was also analyzed, which lead to the conclusion that the SA algorithm solutions were more dependent on having a good calibration for the parameters than the hybrid algorithm. This is a relevant conclusion for cases where the analysis of a single solutions requires higher computation ti-

mes because besides the time that the algorithm takes at finding near-optimal solutions, it also requires higher computation times in the process of calibrating the parameters included in the algorithm.

Finally, the alternative of integrating transit and parking systems in a fully deregulated market was considered in Chapter 6, where a two-stage game-theoretic model was developed along with a framework capable of solving it. This model can also be quickly adjusted to accommodate further concerns regarding the characteristics of the city under analysis, namely requirements that need to be fulfilled by the transit system or characteristics linked to the population that need to be addressed, along with setting the pricing schemes and capacities of both transit and parking systems. The first stage of the game dealt with the optimization of the capacity provided by each operator in order to maximize the profit of each operator. These long-term decisions accounted for the revenues that were managed in the second stage of the game, where transit fares and parking fees were optimized in a short-term horizon to maximize the revenues collected by operator. An approximate solution method was also developed to solve this two-stage game, due to the extremely large size of the overall decisions that each operator can make and the complexity of their objective functions. The two stage game was implemented using this solution method on three different case studies, which were generated by a case study setup inspired in real-world features that was also detailed in Chapter 6. The interaction between each operator and how their decisions influence the other operator's outcome were explored and assessed in this chapter.

Globally, the models presented in this thesis were able, at least in theoretical scenarios, to decrease the congestion felt in the city center (in the case of Chapter 2) and to create transit systems without deficits, or even profitable, by including or not parking revenues as funding source instead of subsidies (in the case of Chapter 3, 4 and 6). Most of the research effort made in this thesis addressed the integration of transit and parking systems considering different point of views. From these, we have concluded that: (1) installing park-and-ride facilities can be a better option at decreasing the person-kilometers travelled in the city center when compared to measures such as increasing parking fees, decreasing transit fares or improving the transit service; (2) integrating the management of both transit and parking systems can significantly improve the financial results of each operator; (3) including traffic dynamic features into the users' generalized costs helps at assessing how pricing schemes help at alleviating the congestion in the city; and (4) assuming a deregulation of the market is an efficient solution to deal with the transit deficits but might jeopardize the social role of this system in the society, if proper requirements of minimum levels of service or further characteristics of the system pursued by transit users are excluded from the analysis.

A general conclusion that can be drawn from the results achieved in Chapters 3, 4 and 6 is the excessive emphasis on solving the transit deficit without including any social welfare measure. This might be solved by finding an equilibrium between the minimization of deficits (which is equivalent to profit maximization objectives) and, e.g., the maximization of the social welfare. However, this is beyond the

scope of the thesis and is left as future work. Furthermore, other principles to determine the zonation for different transit fares and parking fees values should also be tested, the inclusion of time-dependent pricing should also be addressed, as well as improving the realism presented in the treatment of cruising for parking (in particular, for the models developed in Chapters 3 and 6).

Due to the lack of data, most of the conclusions were drawn based on the results achieved for case studies inspired in real-word situations, where a great effort was made to design them or as close as possible to the reality observed for the municipality of Coimbra or by mixing and matching features of real-world data. Nevertheless, all the models described in this thesis should be useful to provide insights in the integration of transit and parking systems or from a physical or from a financial point of view, including regulated and deregulated markets, and should also be easily applied in practice in their current form or with minor adjustments to accommodate smaller characteristics of the reality under analysis.

Despite this, we believe that the methodologies introduced and applied in this thesis represent a notable contribution for the existing literature on facility location of park-and-ride facilities and on fulfilling the gap in the literature in what concerns the integration of transit and parking system according to financial stand points. The case studies, mostly inspired on real-world situations, clearly show the contribution of integrating transit and parking to overcome the need of subsidizing transit systems due to the deficits that characterized public transport services.

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