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Reliability-based optimum design of cable-stayed bridges

J.H.O. Negrão and L.M.C. Simões

Abstract This paper presents a procedure for finding the reliability-based optimum design of cable-stayed bridges. The minimisation problem is stated as the minimisation of stresses, displacements, reliability and bridge cost. A finite-element approach is used for structural analysis. It includes a direct analytic sensitivity analysis module, which provides the structural behaviour responses to changes in the design variables. An equivalent multicriteria approach is used to solve the non-differentiable, non-linear optimisation problem, turning the original problem into the sequential minimisation of unconstrained convex scalar functions, from which a Pareto optimum is obtained. Examples are given illustrating the procedure.

Key words structural optimisation, cable-stayed bridge, reliability-based design, multiple-objectives

1 Introduction

The optimisation of cable-stayed bridges can be stated as that of the minimisation of structural cost or volume, and the maximum stresses throughout the structure. Additional objectives are aimed at deflections or displacements and to guarantee that the design variables are at least specified minimum values. The work started with the shape and sizing optimisation by using a 2D finite-element model for the analysis. The problem was extended to three-dimensional analysis and the consideration of erection stages under static loading (Negrão and Simões 1997). Seismic effects were considered in the optimisation both by a modal-spectral approach and a time-history based procedure (Simões and Negrão 1999). In most of the previous studies, a grid solution was adopted for modelling the deck, with stiffening girders supporting transverse beams, although box-girder sections were employed (Negrão and Simões 1999). Pre-stressing design variables were also considered for the problems of optimal correction of cable forces during erection. Deterministic optimisation enhanced by reliability performance and formulated within the probabilistic framework is called reliability-based optimum design. These are considered important ingredients in the design of advanced structural systems. Wider applications still exhibit limitations mainly attributed to the deeply nested architecture of this procedures involving analysis with finite elements, reliability analysis, sensitivity analysis and optimisation.

The behaviour of complex structures is often analysed by means of finite element analysis (FEA). Stresses and deformations of the structure can be computed given the (deterministic) parameters of loads, geometry and material behaviour. Some structural codes specify a maximum probability of failure within a given reference period (lifetime of the structure). This probability of failure is ideally translated into partial safety factors and combination factors by which variables like strength and load have to be divided or multiplied to find the so-called design values. These design values are to be used as input for an FEA. The outcome of the calculations is compared with the limit states (for example, collapse or maximum deformation). The structure is supposed to have met the reliability requirements when the limit states are not exceeded. Reality is different. First of all, the level I (code-based design) method (JCSS 1981) using partial safety factors makes it only plausible that the reliability requirements are met for average structures. The second aspect is that safety factors are often based on experience only. A link with the required reliability on a theoretical basis often does not exist. The third aspect is the system behaviour of structures. The safety factors are often derived for components of the structure like girders and columns. A structure as a whole behaves like a system of these components. As a result, depending on the kind of system, the structure can be more or less reliable than its com-

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ponents. The advantage of the code type level I method (using partial safety factors out of codes) is that the limit states are to be checked (by means of an FEA) for only a relatively small number of combinations of variables. A disadvantage is the lack of accuracy. A code type level I method uses partial safety factors, which lead to sufficient reliability for average components of structures. These problems can be overcome by using more sophisticated reliability methods such as level II (first order second moment reliability methods, FOSM) (Hasofer and Lind 1974) and level III (Monte Carlo) reliability methods (Rubinstein 1981). The problem with these methods is the considerable computational effort when used in combination with FEA. The combination of directional sampling (Bjerager 1988) and a response surface (Faravelli 1989) method is a kind of importance directional sampling using a smaller number of limit state evaluations. The improvement to the standard directional sampling procedure lies in the use of FEA for the important directions and a response surface for less important directions. In practise this means that, after the response surface is constructed, only few FEA computations have to be performed.

In this work first order reliability methods were used and the sensitivity information was obtained analytically. In a forthcoming paper the advanced simulation method combined with the response surface method will be proposed.

2
Structural analysis

The finite element based open code MODULEF (INRIA 1991) was used as the basic tool for structural analysis, because code availability was a fundamental requirement for further developments. Out of the several element types included in the element library of the programme, only the FE required for two- and three-dimensional models of cable-stayed bridges were retained and adapted to specific needs. These were 2D and 3D bar and beam (Euler–Bernoulli formulation) elements and 4- and 8-noded serendipity plate-membrane (Reissner–Mindlin formulation) elements.

3
Design variables

The structural response of a cable-stayed bridge is conditioned by a large number of parameters, concerning cross-sectional shapes and dimensions, overall bridge geometry, applied pre-stressing forces, deck-to-pylon connections, etc. Some of them play only a limited role in the bridge behaviour while others, such as the cable pattern and pre-stressing forces, are of major importance for both safety and serviceability purposes. Three types of design variables were considered: sizing, shape and mechanical. Sizing design variables are cross-sectional characteristics of bar, beam and plate elements, such as web height, flange width, plate thickness, etc. Changes of such variables do not imply the need for remeshing. Shape design variables produce geometry changes that require nodal co-ordinates updating or even complete remeshing. Other design variables can be characterised as hybrid, because they define both the box-girder cross-section shape and the deck geometry, requiring co-ordinates updating only. Finally, the fixed-end pre-stressing force is a mechanical design variable not related to any geometric quantity. The currently available types are shown in Fig. 1.

All these types play complementary roles in the process of design optimisation. Sizing design variables directly provide for cost/volume decrease. Shape and mechanical design variables have a neglectable direct relation to structural cost but allow for better stress distributions, which in turn lead to further decreases in sizing variables. Pre-stressing force design variables are essential for achieving acceptable solutions when deflections are considered in the dead load condition.

The final behaviour of cable-stayed bridges is deeply related to the erection. Among the various methods used for bridge erection, cantilevering method has become the most popular, due to its suitability for building large spans under strict clearance demands. For the solution of this problem it was assumed that the chronological sequence, corresponding to the erection stage set, might be thought of as a set of independent sub-structures, each corresponding to an erection stage. This is done automatically by the mesh and variable linking generator. The

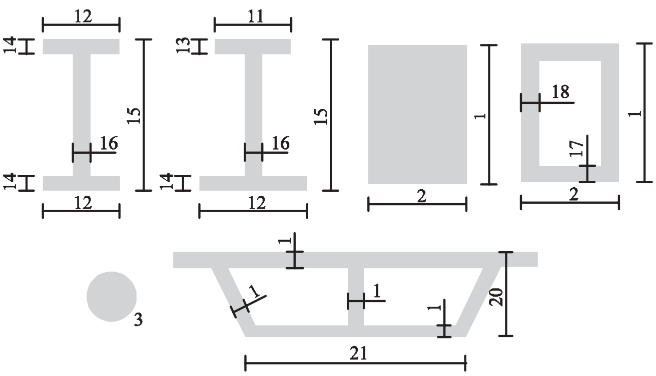

Fig. 1 Sizing and hybrid design variables

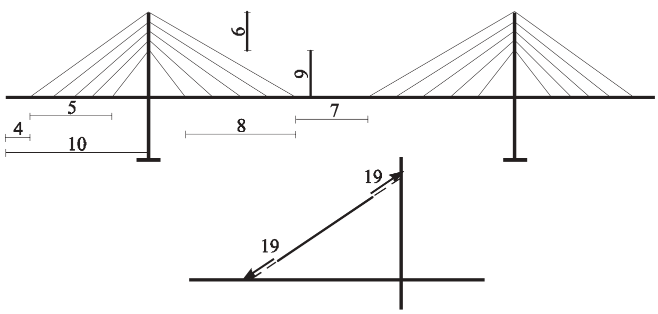

Fig. 2 Shape and mechanical design variables

number of Right-hand sides will be usually that of the final structure, due to load combination involving wind, earthquake and live load, acting in the several positions of the span.

4
Reliability-based optimisation

A failure event may be described by a functional relation, the limit state function, in the following way

$$F = \{g(x) \leq 0\} \tag{1}$$

The probability of failure may be determined by the following integral

$$p_F = \int_{g(x)<0} f_x(\mathbf{x})\,dx \tag{2}$$

where $f_x(\underline{\mathbf{x}})$ is the joint probability density function of the random variables $\underline{\mathbf{x}}$. This integral is, however, non-trivial to solve. Various methods for the solution have been proposed including numerical integration techniques, Monte Carlo simulation and asymptotic Laplace expansions (Ghanem and Spanos 1991). Numerical integration techniques become inefficient for increasing dimensions of the vector $\underline{\mathbf{x}}$. Monte Carlo simulation techniques may be used, but in the following the focus will be on the first order approximation to FOSM (FORM), which are consistent with the solutions obtained by asymptotic Laplace integral expansions. In the case the limit state function $g(\underline{\mathbf{x}})$ is a linear function of the normally distributed basic random variables $\underline{\mathbf{x}}$ the probability of failure can be written in terms of the linear safety margin M as:

$$P_F = P\{g(x) \leq 0\} = P(M \leq 0)\,, \tag{3}$$

which reduces to the evaluation of the standard normal distribution function

$$P_F = \Phi(-\beta) \tag{4}$$

where β is the reliability index given as

$$\beta = \mu_M/\sigma_M \tag{5}$$

The reliability index has the geometrical interpretation as the smallest distance from the line (or the hyperplane) forming the boundary between the safe domain and the failure domain. The evaluation of the probability of failure reduces to simple evaluations in terms of mean values and standard deviations of the basic random variables.

When the limit state function is not linear in the random variables $\underline{\mathbf{x}}$, Hasofer and Lind (1974) suggested to perform the linearization of the limit state function in the design point of the failure surface represented in normalised space $\underline{\mathbf{u}}$.

$$u_i = (x_i - \mu_{x_i})/\sigma_{x_i} \tag{6}$$

As one does not know the design point in advance, this has to be found iteratively in a number of different ways. Provided that the limit state function is differentiable, the following simple iteration scheme may be followed:

$$\alpha_i = -\partial g(\beta\alpha)/\partial u_i \left[\sum_{j=1}^{n} \partial g(\beta\alpha)^2/\partial u_i\right]^{-1/2} \tag{7}$$

$$G(\beta\alpha_1, \beta\alpha_2, \ldots \beta\alpha_\nu) = 0, \tag{8}$$

which will provide the design point $\underline{\mathbf{u}}^*$ as well as the reliability index β.

The reliability assessment requires an enumeration of the reliability indices associated with limit state functions to evaluate the structural system probability of failure. Collapse modes are usually correlated through loading and resistances, so an exact evaluation of the probability is impractical, or even impossible to perform numerically. For this reason, several investigators considered this problem by either finding bounds for p_F or approximating solutions. In general, the admissible failure probability for structural design is very low. A first estimate of p_F can be found through well-known first-order bounds proposed by Cornell (1967):

$$\underset{\text{all } k}{\text{Max}}\lfloor P_r(Z_k)\rfloor \leq p_F \leq \sum_{k=1,m} P_r\,[Z_k \leq 0]. \tag{9}$$

The lower bound, which represents the probability of occurrence of the most critical mode (dominant mode) is obtained by assuming the mode failure events Z_k to be perfectly dependent, and the upper bound is derived by assuming independence between mode failure events. Hence, approximation by Cornell's first-order upper bound is very conservative because it neglects the high correlation between failure modes. Improved bounds can be obtained by taking into account the probabilities of joint failure events such as $P(F_i \cap F_j)$, which means the probability that both events F_i and F_j will simultaneously occur. The resulting closed-form solutions for the lower and upper bounds are as follows:

$$p_F \geq (F_1) + \sum_{i=2}^{m} \text{Max}\left\{\left[P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j\right]; 0\right\} \tag{10}$$

$$p_F \leq \sum_{i=1}^{m} P(F_i) - \sum_{i=2}^{m} \underset{j<i}{\text{Max}}\, P(F_i \cap F_j)\,. \tag{11}$$

The above bounds can be further approximated using Ditlevsen's method of conditional bounding (Ditlevsen 1979) to find the probabilities of the joint events. This is accomplished by using a Gaussian distribution space in which it is always possible to determine three numbers β_i, β_j and the correlation coefficient ρ_{ij} for each pair of collapse modes F_i and F_j.

Improved bounds can also be obtained by using Vanmarcke's concept of failure mode decomposition (Vanmarcke 1971) which takes into account the conditional probability that the $(i-1)$ mode survives given that mode i occurs. By assuming that the probability of occurrence of the ith mode $P(F_i)=\phi(\beta_i)$ depends on β_i only, the conditional probability $P(S_j|F_i)$ is evaluated in terms of the safety indices β_i and β_j and the coefficient of correlation ρ_{ij} between the failure modes F_i and F_j. A different approximate method which avoids calculating conditional probabilities resulting from conditions leading to failure via pairs of failure modes is the PNET (Ang 1982). This method also requires the evaluation of the coefficients of correlation between any two failure modes i and j and is based on the notion of demarcating correlation coefficient ρ_0 assuming those failure modes with high correlation ($\rho_{i,j} \geq \rho_0$) to be perfectly correlated and those with low correlation ($\rho_{i,j} < \rho_0$) to be statistically independent. This method is not very convenient because the solutions will be heavily dependent on the assumed demarcating coefficient ρ_0. A discrete reliability sensitivity analysis is derived and used in the optimisation algorithm.

5 Sensitivity analysis

The analytic direct method was adopted for the purpose of sensitivity analysis, given the availability of the code, the discrete structural pattern and the large number of constraints under control. For ordinary linear statics problems, derivatives of kinematic constraints (displacements) are provided by solving a structural system with pseudo-loading.

The stress derivatives are accurately determined from the chain derivation of the finite element stress matrix:

$$\underline{\sigma} = \underline{D}\,\underline{B}_e\,\underline{u}_e \tag{12}$$

$$\frac{\partial \underline{\sigma}}{\partial x_i} = \frac{\partial (\underline{D}\,\underline{B}_e)}{\partial x_i}\underline{u}_e + \underline{D}\,\underline{B}_e \frac{\partial \underline{u}_e}{\partial x_i}\,. \tag{13}$$

The first term of right-hand side may be directly computed during the computation of element contribution for the global system, on the condition that derivative expressions are pre-programmed and called on that stage. The second term on the right-hand side is somewhat more difficult to compute because an explicit relation between displacement vector and design variable set does not exist. Pre-programming and storing the stiffness matrix and right-hand side derivatives in the same way as described for the stress matrix, the displacement derivatives may be computed by the solution of N pseudo-load right-hand sides. The stress derivatives are then computed in a straightforward way. The explicit form of matrix derivatives depends on the type of element. For 2D and 3D bar and beam elements their calculation is a straightforward task. For plate-membrane elements, the differentiation of the whole finite element formulation is required.

6 Optimisation

Pareto's economic principle (Pareto 1893) is gaining increasing acceptance to multi-objective optimisation problems. In minimisation problems a solution vector is said to be Pareto optimal if no other feasible vector exists that could decrease one objective function without increasing at least another one. The optimum vector usually exists in practical problems and is not unique. In regards to reliability-based design, several alternative formulations exist. A comprehensive review can be found in Thoft-Christensen (1991). The material cost together with maximum probability of failure and measures of the structural performance and imposed by manufacturing and technical considerations are the objectives to be minimised. Size, shape, material configuration and loading parameters may be allowed to vary during the optimisation process. Bounds must be set for average cross-sectional and geometric design variables in order to achieve executable solutions and required aesthetic characteristics. The overall objective of cable-stayed bridge design is to achieve an economic and yet safe solution. In this study it is not intended to include all factors influencing the design economics. One of the factors conventionally adopted is the cost of material used. A second set of goals arises from the requirement that the stresses should be as small as possible. The optimisation method requires that all these goals should be cast in a normalised form. Another set of goals arises from the imposition of lower and upper limits on the sizing variables, namely minimum cable cross sections to prevent topology changes and executable dimensions for the stiffness girder and pylons cross sections. Similar bounds must be considered for the geometric design variables. Additional bounds are set when geometric design variables are considered, to ensure that no geometry violation occurs when these design variables are updated. Additional goals may be established in order to ensure the desired geometric requirements during the optimisation process (mesh discretisation, ratios of variation of cable spacing on deck and pylons, etc). For these the chosen approach was to initially supply all the necessary information, by means of a *geometry coefficients set* describing such conditions.

The objective is to minimise all of these objectives over sizing and geometry variables $\underline{\mathbf{X}}$. This problem is discontinuous and non-differentiable and is therefore hard to solve. However, by using an entropy-based approach, Templeman (1993) has shown that its solution is equivalent to that of an unconstrained convex scalar function, which may be solved by conventional quasi-Newton methods. This function depends only on one control parameter, ρ, which must be steadily increased through the optimisation process. The scalar function is very similar to that of Kreisselmeyer–Stainhauser (Haftka and Gurdal 1992), derived for control problems:

$$\mathrm{F}\left(\underline{\mathrm{x}}\right)=\frac{1}{\rho}.\ln\left[\sum_{\mathrm{j}=1}^{\mathrm{M}}\mathrm{e}^{\rho(\mathrm{g}(\underline{\mathrm{x}}))}\right]. \qquad (14)$$

Problem (14) is unconstrained and differentiable which, in theory, gives a wide choice of possible numerical solution methods. However, since the goal functions $g_j(x,z)$ do not have an explicit algebraic form in most cases, the strategy adopted was to solve (14) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(x,z)$ truncated after the linear term. This gives:

$$\min\ \mathrm{F}\left(\underline{\mathrm{x}}\right)=\frac{1}{\rho}.\ln\left[\sum_{\mathrm{j}=1}^{\mathrm{M}}\mathrm{e}^{\rho\left(\mathrm{g_o}(\underline{\mathrm{x}})+\sum_{\mathrm{i}=1}^{\mathrm{N}}\frac{\partial \mathrm{g_{oj}}(\underline{\mathrm{x}})}{\partial \mathrm{x_i}}\mathrm{dx_i}\right)}\right] \qquad (15)$$

where N and M are, respectively, the number of sizing plus geometric design variables and the number of goal functions g_{oj} and $\partial g_{oj}/\partial x_i$ are the goals and their derivatives evaluated for the current design variable vector (x_o,z_o), at which the Taylor series expansion is made. The inaccuracy arising from truncating the goals after the first order term is controlled by using a move limits strategy throughout the process.

Solving (15) for particular numerical values of g_{oj} forms only one iteration of the complete solution of problem (14). The solution vector (x_1,z_1) of such an iteration represents a new design that must be analysed and gives new values for g_{1j}, $\partial g_{1j}/\partial x_i$ and (x_1,z_1), to replace those corresponding to (x_o,z_o) in (15). Iterations continue until changes in the design variables become small. During these iterations the control parameter ρ must not be decreased to ensure that a multiobjective solution is found.

7 Numerical example

The model represented in Fig. 3 was considered. It consists of a symmetric three-span cable-stayed bridge. Monosymmetric I-shaped cross-sections are prescribed for the stiffening girders, while the pylons are made up of steel plates defining a rectangular hollow cross section. I-shaped transverse beams support the wearing surface. A numerical example with sizing design variables only and without erection stages will be presented next. Consistent to the traditional ultimate limit state design (level 1 approach), design stresses of 275 MPa and 700 MPa were considered for the deck and pylon and stay elements, respectively. With a safety factor for structural steel of 1.15 and an assumed coefficient of variation of 0.10, these correspond to mean values of 345 and 877 MPa. Three load combinations involving both dead and live load were considered, corresponding to live load on the whole deck, in side spans or in central span only. For the sake of simplicity, each of these live load distributions was assumed as an independent event. Design and mean values were derived by using the characteristic value of $4\ \mathrm{kN/m^2}$, prescribed in the portuguese code (RSA). A safety factor of 1.50 and a coefficient of variation of 0.40 were adopted. The safety factor and coefficient of variation of the dead load are 1.35 and 0.20, respectively. Shape and sizing parameters referred to in Table 1 were also considered as random variables, with coefficients of variation of 0.01. However, the results show no significant differences from the situation in which these parameters were considered as deterministic. Although randomness of Youngs modulus also plays an important role in the structural reliability, this was not considered here for the sake of simplicity. In this example, the probability of failure will be connected with critical stresses throughout the structure, induced by the loadings. However, other failure modes or criteria could be used as well, such as the excessive deflection or cable under-stressing.

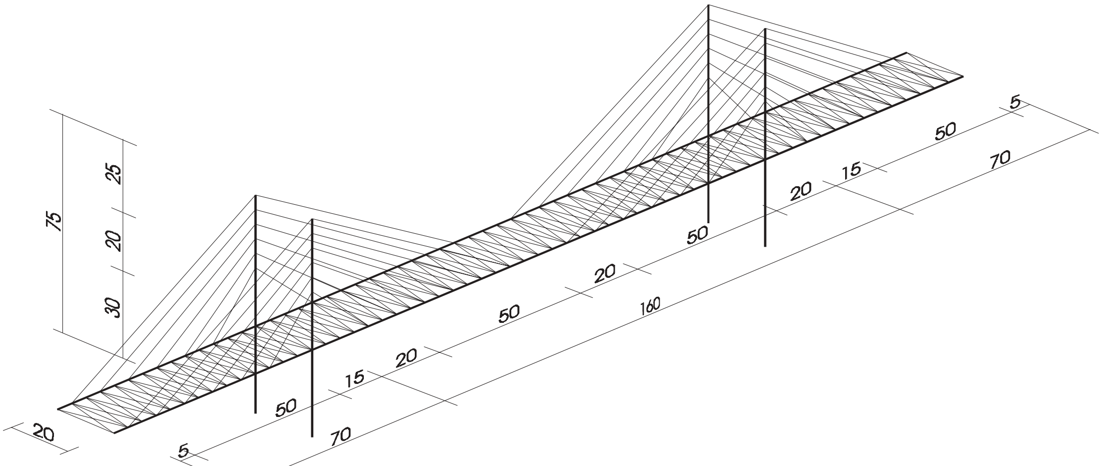

Fig. 3 Geometry of the bridge model

Table 1 Sizing and geometry design variables

DV	Description	DV	Description
1	Width of top flange of side span girders	22	Longitudinal wall thickness of pylons under the deck
2	Width of bottom flange of side span girders	23	As #19, above the deck
3	Thickness of top flange of side span girders	24	As #20, above the deck
4	Thickness of bottom flange of side span girders	25	As #21, above the deck
5	Height of side span girders	26	As #22, above the deck
6	Thickness of web of side span girders	27	Cross sectional area for stays #1 and 24
7	As #1, for girders close to the pylons	28	Cross sectional area for stays #2 and 23
8	As #2, for girders close to the pylons	29	Cross sectional area for stays #3 and 22
9	As #3, for girders close to the pylons	30	Cross sectional area for stays #4 and 21
10	As #4, for girders close to the pylons	31	Cross sectional area for stays #5 and 20
11	As #5, for girders close to the pylons	32	Cross sectional area for stays #6 and 19
12	As #6, for girders close to the pylons	33	Cross sectional area for stays #7 and 18
13	As #1, for central span girders	34	Cross sectional area for stays #8 and 17
14	As #2, for central span girders	35	Cross sectional area for stays #9 and 16
15	As #3, for central span girders	36	Cross sectional area for stays #10 and 15
16	As #4, for central span girders	37	Cross sectional area for stays #11 and 14
17	As #5, for central span girders	38	Cross sectional area for stays #12 and 13
18	As #6, for central span girders	39	Width of flanges of transverse beams
19	Cross-section width of pylons below the deck	40	Height of transverse beams
20	Cross-section height of pylons below the deck	41	Wall thickness of transverse beams
21	Transverse wall thickness of pylons under the deck	–	–

Table 2 Initial and final (optimised) values of design variables

DV	Starting value	Optimised value	DV	Starting value	Optimised value
1	1	0.77329	2	1	1.5
3	0.03	0.02165	4	0.03	0.03224
5	3	4	6	0.03	0.01771
7	1	0.59049	8	1	1.5
9	0.03	0.01771	10	0.03	0.03715
11	3	3.65071	12	0.03	0.02056
13	1	1.02242	14	1	0.97554
15	0.02	0.0165	16	0.02	0.01548
17	2.5	1.47622	18	0.02	0.015
19	3	2	20	3	3.74356
21	0.025	0.015	22	0.025	0.01506
23	2.5	2	24	2.5	2
25	0.02	0.015	26	0.02	0.015
27	0.009	0.01001	28	0.007	0.00288
29	0.005	0.00188	30	0.005	0.00574
31	0.005	0.00466	32	0.005	0.00592
33	0.005	0.00201	34	0.005	0.00479
35	0.005	0.00314	36	0.005	0.00473
37	0.005	0.00275	38	0.007	0.00775
39	0.5	0.4	40	0.8	0.99502
41	0.02	0.015			

The initial design was optimised through using the multicriteria approach described in Sect. 6 to compare the reliability index and bimodal bounds in both cases. Starting and optimised values of design variables described in Table 1 are listed in Table 2. The overall achieved cost reduction was about 25%, considering the same cost factors for all types of structural elements.

For the starting design, a minimum reliability index of $\beta = 4.332$, with an associate failure probability of $\Phi(-\beta) = 7.4 \times 10^{-6}$ and second order bounds of

$7.4 \times 10^{-6} \leq \mathrm{P_f} \leq 1.68 \times 10^{-5}$ were found. The corresponding values for the optimised solution were $\beta = 4.563$, $\Phi(-\beta) = 2.5 \times 10^{-6}$ and $2.5 \times 10^{-6} \leq \mathrm{P_f} \leq 6.0 \times 10^{-6}$. The bound interval shows that most of the nearly 1000 limit states considered are highly correlated.

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