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Some statistical results on autoregressive conditionally heteroscedastic models

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Abstract

The aim of this paper is to present some statistical aspects of an order 1 autoregressive model with errors following a stationary and ergodic generalized threshold ARCH process. So, to analyse the precision of forecasts obtained with these models a probabilistic study will be done. Moreover, a consistent test for a general AR(1) model with errors following an ergodic white noise of null conditional median will be developed and adapted to our stochastic process.

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1. Introduction

Conditionally heteroscedastic models, proposed by Engle (1982) and generalized, in different ways, by other authors (Bollerslev, 1986; Nelson, 1991; Zakoian, 1990), are, under their stationarity conditions, white noises; so, they could be taken to describe the error process in general time series modelling.

In these models, the form of the conditional heteroscedasticity is known and this fact will certainly lead to improved theoretical statistical results. Therefore, Sections 2 and 3 of the study will concentrate on some statistical aspects of an order 1 autoregressive model with errors following a stationary and ergodic generalized threshold ARCH process (Rabemananjara and Zakoian, 1993; Gonçalves and Mendes Lopes, 1994).

In Section 2, a probabilistic study is developed to analyse the precision of forecasts obtained with these models. Unlike in the classical case, this precision is not independent of the current and past states of the process; so, this kind of modelling allows us to detect subperiods of stronger or weaker volatility. Expressions of the conditional

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variance and of the variance of the successive forecasts will be established, thereby generalizing the corresponding results of Zakoian (1990).

A test for those models will be proposed, taking into account that they are well-described by their conditional distribution in the past. So, in Section 3, a consistent test is implemented for the autoregression coefficient of an AR(1) model with errors following an ergodic white noise of null conditional median. The test definition follows the idea present in Gonçalves et al. (1996), in which a consistent test was obtained for stationary and ergodic ARMA models with conditionally Gaussian errors.

We point out that the consistence of the test is now established under very weak hypotheses. In fact, we only require the strong stationarity, the ergodicity and the nullity of the conditional median of the error process. Even the hypothesis of second-order integrability of the process, usual for classical tests, is not demanded in this case. To prove the consistence of the test we will use a non-traditional methodology, particularly useful in test theory (Tassi and Legait, 1990, p. 250), which is based on the asymptotic separation of the model distributions obtained under each one of the hypotheses to be tested.

The test proposed here may also be considered as a test for the equality to zero of the conditional median of the observed process.

In this paper, the following notations for a general process $X=(X_t,\ t\in\mathbb{Z})$ will be used: $X_t^+=\max(X_t,0)$, $X_t^-=\min(X_t,0)$ and $\underline{X}_{t-1}$ the σ-field generated by $(X_{t-i},\ i\geqslant 1)$.

2. The AR(1)-Gtarch(1,1) model: the forecast error

Let us consider a real stochastic process $Y=(Y_t,\ t\in\mathbb{Z})$ satisfying the autoregression equation

$$Y_t-\varphi Y_{t-1}=\varepsilon_t,\quad \varphi\in\,]-1,1[, \tag{1}$$

where $\varepsilon=(\varepsilon_t,\ t\in\mathbb{Z})$ follows the generalized threshold Arch (1,1) model given by

$$\begin{aligned}&\varepsilon_t=\sigma_t Z_t,\\ &\sigma_t=\alpha_0+\alpha_{1,+}\varepsilon_{t-1}^{+}-\alpha_{1,-}\varepsilon_{t-1}^{-}+\beta_1\sigma_{t-1},\end{aligned}\qquad (\alpha_0>0,\alpha_{1,+}\geqslant 0,\alpha_{1,-}\geqslant 0 \text{ and } \beta_1\geqslant 0),$$

with $(Z_t,\ t\in\mathbb{Z})$ a sequence of independent and identically distributed real random variables, with zero mean and unit variance, such that Z_t is, for every $t\in\mathbb{Z}$, independent of the σ-field $\underline{\varepsilon}_{t-1}$.

A general definition of Gtarch(p,q) models can be found in Rabemananjara and Zakoian (1993).

The stationarity and the ergodicity of these models have been studied in Gonçalves and Mendes-Lopes (1994,1996). For the particular model considered here, these properties are equivalent to the following condition on the coefficients:

$$(\alpha_{1,+})^2E(Z_t^+)^2+(\alpha_{1,-})^2E(Z_t^-)^2+\beta_1^2+2\beta_1[\alpha_{1,+}E(Z_t^+)-\alpha_{1,-}E(Z_t^-)]<1. \tag{2}$$

In this kind of models, the form of the conditional variance to the past is known; therefore, a better insight into the accuracy of forecast intervals may be expected, if we replace the non-conditional variance by the conditional one. In fact, taking into account the particular form of σ_t, forecast intervals may be obtained, the widths of which are a function of the past variability of the data.

The importance of the study of the conditional variance of the forecast error may be also illustrated, if we suppose that the law of ε_t given $\underline{\varepsilon}_{t-1}$ is Gaussian with zero mean and variance σ_t^2. As the law of $Y_t/\underline{Y}_{t-1}$ is Gaussian with mean φY_{t-1} and variance σ_t^2, the one-step-ahead forecast intervals, with a confidence level equal to α, have the closed form:

$$]\varphi Y_{t-1} - a\sigma_t, \varphi Y_{t-1} + a\sigma_t[,$$

where $a = F^{-1}((1+\alpha)/2)$ and F is the distribution function of the $N(0,1)$ law. But, if we want to compute other forecast intervals, as the law of $\varepsilon_{t+s}/\underline{\varepsilon}_t$ for $s>1$ is not necessarily a Gaussian one, we can only obtain approximations using, in particular, moment inequalities, such as those of Bienaymé–Tchebycheff or Bernstein (Fréchet, 1950), from which forecast intervals may be deduced taking into account the conditional variance of the forecast error.

These comments lead us to the study of this conditional variance.

The forecast formula for the instant $t+s$, $s \in \mathbb{N}$, at instant t of the model (1) is

$$E(Y_{t+s}/\underline{Y}_t) = \varphi^s Y_t$$

and so, the conditional variance of the forecast error is

$$V(Y_{t+s}/\underline{Y}_t) = \sum_{i=1}^{s} \varphi^{2(s-i)} E(\sigma_{t+i}^2/\underline{\varepsilon}_t).$$

As the conditional variance of the ε-process is known, this expression may be made explicit. In fact, the following result is obtained:

Theorem 1. *If Y is the process defined by* (1) *and if we suppose that the law of Z_t is symmetrical with respect to the origin then*

$$V(Y_{t+s}/\underline{Y}_t) = \sum_{i=1}^{s} \varphi^{2(s-i)} (b^{i-1}\sigma_{t+1}^2 + \sigma_{t+1} f_i(\alpha_0, b, c) + g_i(\alpha_0, b, c)),$$

with

$$b = \tfrac{1}{2}\left((\alpha_{1,+})^2 + (\alpha_{1,-})^2 + 2\beta_1^2 + 4k\beta_1(\alpha_{1,+} + \alpha_{1,-})\right); \qquad c = \beta_1 + k(\alpha_{1,+} + \alpha_{1,-}),$$

$$f_i(\alpha_0, b, c) = \frac{2c\alpha_0}{b-c}(b^{i-1} - c^{i-1}),$$

$$g_i(\alpha_0, b, c) = \alpha_0^2 \frac{1-b^{i-1}}{1-b} + \frac{2c\alpha_0^2}{1-c}\left[\frac{b^{i-1}-1}{b-1} - \frac{b^{i-1}-c^{i-1}}{(b-c)}\right],$$

and $k = E(Z_t^+) = E(-Z_t^-)$.

Proof. We have

$$Y_{t+s}=\varphi^{s}Y_{t}+\sum_{i=1}^{s}\varphi^{s-i}\varepsilon_{t+i}$$

and so

$$(A)\quad E(Y_{t+s}^{2}/\underline{Y}_{t})=\varphi^{2s}Y_{t}^{2}+\sum_{i=1}^{s}\varphi^{2(s-i)}E(\sigma_{t+i}^{2}/\underline{\varepsilon}_{t}).$$

Using the definition of σ_t, the general properties of the conditional expectation and taking into account that $\varepsilon_t=\sigma_t Z_t$, we show:

(i) $E(\sigma_{t+i}^{2}/\underline{\varepsilon}_{t})=\alpha_0^2+bE(\sigma_{t+i-1}^{2}/\underline{\varepsilon}_{t})+2\alpha_0 cE(\sigma_{t+i-1}/\underline{\varepsilon}_{t})$

and

(ii) $E(\sigma_{t+i}/\underline{\varepsilon}_{t})=\alpha_0\sum_{j=0}^{i-2}c^{j}+c^{i-1}\sigma_{t+1}$.

The result is obtained replacing, recursively, (i) and (ii) in expression (A). □

Let us study now the behaviour of the conditional precision as s grows to infinity.

Theorem 2. *Under the general hypotheses of Theorem* 1 *and supposing that* ε *is a weakly stationary Gtarch* (1,1) *process, we have*

$$V(Y_t)=\frac{\alpha_0^2(1+c)}{(1-b)(1-\varphi^2)(1-c)}.$$

Proof. The convergence, when $s\to\infty$, of the series present in Theorem 1 occurs, as $b>0$ and $c>0$, if

$$b<1 \text{ and } c<1,$$

We point out, firstly, that as the law of Z_t is symmetrical, the condition $b<1$ is equivalent to the condition (2) of stationarity of ε.

Let us prove now that the condition $b<1$ implies $c<1$. To make the analysis easier, we take

$$\alpha_{1,+}=x,\quad \alpha_{1,-}=y \quad\text{and}\quad \beta_1=z.$$

We need to study the intersection of the conditions $z+k(x+y)<1$ and $x^2+y^2+2z^2+4kz(x+y)<2$.

The equation $(E1)$ $kx+ky+z=1$ describes the plane passing by the points $(1/k,0,0)$, $(0,1/k,0)$ and $(0,0,1)$, while the equation $(E2)$ $x^2+y^2+2z^2+4kzx+4kzy-2=0$ describes an ellipsoid of revolution (see Fig. 1 for $k=\frac{1}{3}$).

Let us analyse the relative position of the curves $(E1)$ and $(E2)$ when $z=h$, for h arbitrarily fixed $(0\leqslant h\leqslant 1)$.

In the plane $z=h$, the equation $(E1)$ is the straight line given by $kx+ky+h-1=0$ and the equation $(E2)$ is the circumference with centre $(-2kh,-2kh)$ and radius $\sqrt{8k^2h^2+2-2h^2}$. The distance between the centre of the circumference and the

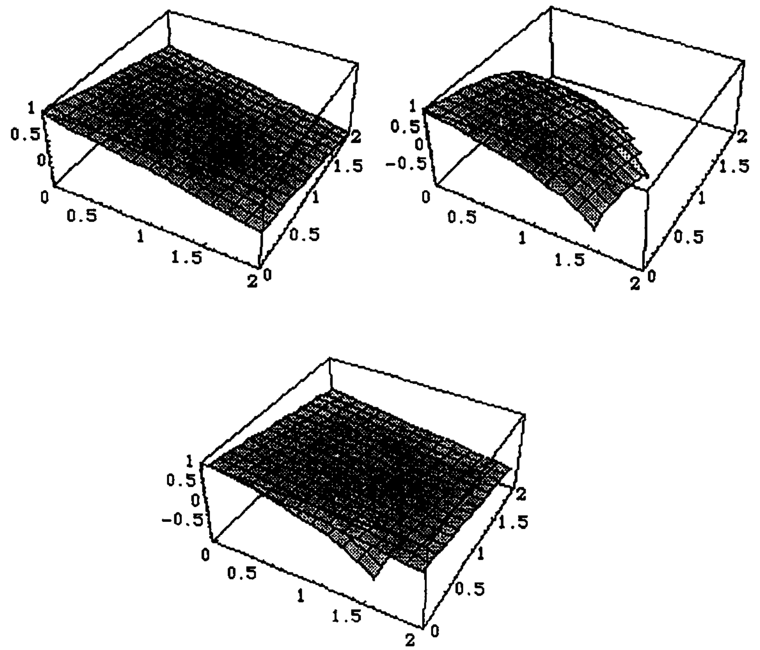

Fig. 1. Curves $(E1)$ and $(E2)$ for $k=\frac{1}{3}$.

straight line is

$$d=-\frac{k(-2kh)+k(-2kh)+h-1}{\sqrt{k^2+k^2}}=\frac{4k^2h-h+1}{\sqrt{2k^2}}$$

and it is clearly greater than the radius of the circumference if $k<\frac{1}{2}$. As Z_t has a symmetrical distribution of unit variance, it is easy to prove that $k\leqslant\frac{1}{2}$. Moreover, the case $k=\frac{1}{2}$ corresponds to a degenerated distribution for $|Z_t|$.

So under our general hypothesis of stationarity for the ε-process, and excluding the extreme case $k=\frac{1}{2}$, the referred series is convergent to $[\alpha_0^2(1+c)]/[(1-b)(1-\varphi^2)(1-c)]$. □

It should be noted that, in spite of the dependence on the past of the forecast for a finite lead time, when this lead time grows indefinitely, we obtain a result independent of the forecast origin t, as in the classical case.

The latter equality allows us to obtain the variance of the error process

$$V(\varepsilon_t)=\frac{\alpha_0^2(1+c)}{(1-b)(1-c)},$$

which generalizes the result of Zakoian (1990) when ε follows a conditionally Gaussian TARCH (1) process.

3. A test for an AR(1) model

3.1. The test consistence

Let $X=(X_t,\ t\in\mathbb{Z})$ be a real stochastic process such that

$$X_t=\varphi X_{t-1}+\varepsilon_t, \quad \varphi\in\,]-1,1[,$$

where $\varepsilon=(\varepsilon_t,\ t\in\mathbb{Z})$ is a strongly stationary and ergodic process such that $\varepsilon_t\neq 0$, a.s., for every $t\in\mathbb{Z}$. We assume that the median of the conditional law of ε_t given $\underline{\varepsilon}_{t-1}$ is unique and zero, for every $t\in\mathbb{Z}$.

Considering $T+1$ observations of X, denoted by $x_0,x_1,\dots,x_T$, a test may be proposed for the hypotheses

$$\mathrm{H}_0\colon \varphi=0 \text{ against } \mathrm{H}_1\colon \varphi=\beta \quad (\beta\neq 0 \text{ arbitrarily fixed}).$$

This test is constructed, following the idea of Gonçalves et al. (1996), using the sets

$$D_t=\left(]0,+\infty[\times\left]-\infty,\frac{\beta}{2}x_{t-1}\right[\right)\cup\left(]-\infty,0[\times\left]\frac{\beta}{2}x_{t-1},+\infty\right[\right) \quad \text{if } \beta>0,$$

$$D_t'=\left(]-\infty,0[\times\left]-\infty,\frac{\beta}{2}x_{t-1}\right[\right)\cup\left(]0,+\infty[\times\left]\frac{\beta}{2}x_{t-1},+\infty\right[\right) \quad \text{if } \beta<0$$

for every $t\in\{1,\dots,T+1\}$.

Let $g\colon \mathbb{R}\to\mathbb{R}^+$ be a strictly positive and symmetrical function, increasing on $\mathbb{R}^+$; we define the acceptance regions of H_0 by

$$\forall T\in\mathbb{N},\quad A_T=\begin{cases}\{x^{(T)}:\sum_{t=1}^{T} g(\beta x_{t-1})[2\mathbb{1}_{D_t}(x_{t-1},x_t)-1]\geqslant 0\}, & \beta>0,\\ \{x^{(T)}:\sum_{t=1}^{T} g(\beta x_{t-1})[2\mathbb{1}_{D_t'}(x_{t-1},x_t)-1]\geqslant 0\}, & \beta<0,\end{cases}$$

where $x^{(t)}=(x_t,x_{t-1},\dots)\in\prod_{-\infty}^{t}\mathbb{R}$.

The idea behind the definition of these regions may be easily illustrated. Let us consider, e.g., the case $\beta>0$ and $x_{t-1}>0$. Suppose that $\sum_{t=1}^{T}[2\mathbb{1}_{D_t}(x_{t-1},x_t)-1]>0$. In this case, the random variables

$$2\mathbb{1}_{D_t}(x_{t-1},x_t)-1=2\mathbb{1}_{\{x_t<\beta(x_{t-1})/2\}}-1, \quad t=1,2,\dots,T,$$

take the value 1 more often than the value -1; so, as the conditional median of the law of X_t given X_{t-1} is φx_{t-1}, this seems to imply that $\varphi=0$. Otherwise, if that expression is negative, $\varphi=\beta$ seems more likely. Moreover, when $\beta(x_{t-1})/2$ is very close to zero, the events $X_t<\beta(x_{t-1})/2$ or $X_t>\beta(x_{t-1})/2$ have almost the same probability. So, it is natural to introduce a function weighting the large values of βx_{t-1}.

Denoting the model distribution and the corresponding expectation when the autoregressive parameter is equal to φ by P_φ and by E_φ, respectively, and supposing g P_φ-integrable, we will prove the following result:

Theorem 3. *Under the previous conditions, the sets A_T, $T\in\mathbb{N}$, are the acceptance regions of a convergent sequence of tests of* H_0 *against* H_1.

Proof. We will only consider the case where $\beta > 0$ because the study for $\beta < 0$ is analogous. Let us define

$$\overline{\Psi}_T = \frac{1}{T}\sum_{t=1}^{T} \Psi(x_{t-1}, x_t),$$

where $\Psi(x_{t-1}, x_t) = \Psi_t = g(\beta x_{t-1})[2 1_{D_t}(x_{t-1}, x_t) - 1]$.

Using the ergodic theorem (Azencott and Dacunha-Castelle, 1984) we know that $\lim_{T\to+\infty} \overline{\Psi}_T = E_\varphi[\Psi(X_0, X_1)]$, a.s.

We will study the sign of this limit under each one of the two hypotheses; in this way we deduce the corresponding asymptotic separation and, as a consequence, we obtain the test convergence (Geffroy, 1980; Moché, 1989).

We have

$$E_\varphi(\Psi_1) = E_\varphi(E_\varphi(\Psi_1/\underline{X}_0)) = E_\varphi\{g(\beta X_0)[2E_\varphi(1_{D_1}(X_0, X_1)/\underline{X}_0) - 1]\}.$$

Moreover,

$$E_\varphi[(1_{D_1}(X_0, X_1)/\underline{X}_0)]$$
$$= 1_{]0,+\infty[}(X_0)P_\varphi\left(X_1 < \frac{\beta}{2}X_0/\underline{X}_0\right) + 1_{]-\infty,0[}(X_0)P_\varphi\left(X_1 > \frac{\beta}{2}X_0/\underline{X}_0\right).$$

So, under H_0,

$$E_0[(1_{D_1}(X_0, X_1)/\underline{X}_0)] = \begin{cases} P_0[(X_1 < \frac{\beta}{2}X_0/\underline{X}_0)] & \text{if } X_0 > 0, \\ P_0[(X_1 > \frac{\beta}{2}X_0/\underline{X}_0)] & \text{if } X_0 < 0, \\ 0 & \text{if } X_0 = 0. \end{cases}$$

As the conditional law of ε_t given $\underline{\varepsilon}_{t-1}$ has a unique median of zero value and as $X_0 \neq 0$ a.s., we obtain

$$E_0[(1_{D_1}(X_0, X_1)/\underline{X}_0)] > \tfrac{1}{2}, \quad \text{a.s.}$$

So,

$g(\beta X_0)[2E_0(1_{D_1}(X_0, X_1)/\underline{X}_0) - 1] > 0$, a.s., and then $\lim_{T\to+\infty}$ (a.s.) $\overline{\Psi}_T > 0$ which implies

$$P_0(\{\overline{\Psi}_T \geqslant 0\}) \to 1 \quad \text{as } T \to +\infty.$$

On the other hand, under H_1, we have

$$E_\beta[(1_{D_1}(X_0, X_1)/\underline{X}_0)] = \begin{cases} P_\beta(X_1 < \frac{\beta}{2}X_0/\underline{X}_0), & X_0 > 0, \\ P_\beta(X_1 > \frac{\beta}{2}X_0/\underline{X}_0), & X_0 < 0, \\ 0, & X_0 = 0, \end{cases}$$
$$= \begin{cases} P_\beta(\varepsilon_1 < -\frac{\beta}{2}X_0/\underline{\varepsilon}_0), & X_0 > 0, \\ P_\beta(\varepsilon_1 > -\frac{\beta}{2}X_0/\underline{\varepsilon}_0), & X_0 < 0, \\ 0, & X_0 = 0, \end{cases}$$

as, under the considered hypotheses, $\underline{\varepsilon}_t = \underline{X}_t$, for every $t \in \mathbb{Z}$.

Taking into account that the conditional median is unique and null,

$$E_\beta[(1_{D_1}(X_0,X_1)/\underline{X}_0)] < \tfrac{1}{2}, \quad \text{under } H_1.$$

Then $g(\beta X_0/\sigma_1)[2E_\beta(1_{D_1}(X_0,X_1)/\underline{X}_0) - 1] < 0$; so, we obtain $\lim_{T\to+\infty}$ (a.s.) $\overline{\Psi}_T < 0$ and, finally,

$$P_\beta(\{\overline{\Psi}_T \geqslant 0\}) \to 0 \quad \text{as } T \to +\infty.$$

The asymptotic separation of the two hypotheses is then proved. □

3.2. Simulation experiments

To illustrate the suitability of this test for models with conditionally heteroscedastic errors, a simulation study will be done in which a real process $Y=(Y_t,\ t\in\mathbb{Z})$ following the model (1) is considered. In this model, the white noise $\varepsilon=(\varepsilon_t,\ t\in\mathbb{Z})$ is the strongly stationary TARCH (1) process (Zakoian, 1990) defined by

$$\varepsilon_t = \sigma_t Z_t,$$

$$\sigma_t = 1 + 0.5|\varepsilon_{t-1}|,$$

where Z_t, $t\in\mathbb{Z}$, are i.i.d. N(0, 1) random variables.

In order to analyse the importance of the weight function in the test statistics, we shall consider the case in which the observations are equally weighted ($g(x)=1$) and that which corresponds to the weight function $g(x)=|x|$.

Moreover, to investigate the possible advantages of adopting our test, a comparison study is done with the Box–Pierce classical test, with a size fixed on 5%, for an AR(1)-TARCH (1) model (Gouriéroux, 1992); in order to do this we have to ensure the existence of the fourth-order moments of the TARCH (1) model. The coefficients of the model presented here were chosen in order to satisfy such a condition.

The behaviour of our test when H_0 is true is evaluated by undertaking a simulation study for several values of T ($T=50, T=100, T=150$) letting $\varphi=0$ in the model. We then tested this model against four alternatives ($\beta=0.1, \beta=0.25, \beta=0.5, \beta=0.9$). The percentage of rejections of the white-noise hypothesis in 200 replications is presented in Table 1.

We observe a decreasing number of rejections of the null hypothesis as β or T increases; moreover, this decrease is, in general, stronger when the observations are really weighted.

Also recorded was the percentage of rejections of H_0 with the Box–Pierce test with a significance level fixed at 5%. As is clear from the definition of the test, the rejection probability of the test increases if one considers alternative hypotheses close to the null. Thus, our test does not favour the null in all circumstances, as is the case for traditional tests, in which the size is fixed a priori. The power simulations (see Table 2) indicate that this size property indeed implies better power properties.

Table 1
Percentage of rejections of H_0 where $\varphi = 0$

T		50			100			150	
β	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)
0.1	0.41	0.42	0.135	0.35	0.3	0.2	0.35	0.305	0.215
0.25	0.325	0.275	0.17	0.27	0.2	0.165	0.215	0.16	0.175
0.5	0.15	0.105	0.205	0.1	0.095	0.25	0.055	0.035	0.205
0.9	0.015	0.025	0.16	0.01	0.0	0.26	0.0	0.0	0.23

Table 2
Percentage of rejections of H_1 where $\varphi = \beta$ $(\beta > 0)$

T		50			100			150	
β	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)	$g(x) = 1$	$g(x) = \|x\|$	BP (5%)
0.1	0.51	0.475	0.775	0.415	0.405	0.72	0.36	0.35	0.67
0.25	0.295	0.28	0.58	0.2	0.24	0.345	0.185	0.185	0.165
0.5	0.155	0.125	0.105	0.085	0.05	0.015	0.02	0.02	0.0
0.9	0.005	0.005	0.0	0.0	0.0	0.0	0.0	0.0	0.0

In order to have some insight about the rate of convergence of the power function, we analyse its performance for several values of T. The test was constructed for several values of β ($\beta = 0.1, 0.25, 0.5$ and 0.9) and taking $\varphi = \beta$ in the true model. In each case we recorded the percentage of rejections of the alternative (true) hypothesis in 200 replications of the model. The results are presented in Table 2.

We note the decreasing number of rejections of H_1 produced by our test when β moves away from zero or as T increases. As mentioned before, we find indeed that the weaker size properties of our test for alternatives close to the null, imply better power properties for these alternatives. The particular simulation study presented here, therefore, shows that our test should be considered as an alternative to more classical ones. The importance of the weight function is also evident.

Finally, we point out that the simulations here presented are only intended to indicate the behaviour of our test and that more detailed Monte Carlo studies have been left for future research.

3.3. *Concluding remarks*

This test may be applied to strongly stationary and ergodic processes $X = (X_t,\ t \in \mathbb{Z})$ with the probability law well described by the conditional distributions of X_t given the σ-field $\underline{X}_{t-1}$; thus, beyond its application to classical AR(1) models, it can also be applied, as we have seen in the previous paragraph, to the AR(1) models with conditionally heteroscedastic errors if they are strongly stationary and ergodic.

Moreover, this test does not impose the existence of moments of the marginal distribution of X of order greater than one. It may then be applied to error processes that e conditionally heteroscedastic and strongly stationary but not necessarily weakly ıtionary.

A point of future research is naturally the study of the convergence rates of the /el and power functions. In particular, we expect to generalize the result of exponential decay of the level sequence obtained in Gonçalves et al. (1996) under stronger nditions than those here considered.

All the simulation studies were programmed in the statistical software CSS: egafile Manager.

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