Nested Optimization Approach for the Capacity Expansion of Multiquality Water Supply Systems under Uncertainty

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Abstract

A nested optimization approach is proposed to solve capacity expansion problems of multiquality water supply systems. The problem to be solved consists of determining the infrastructure that should be built and/or rehabilitated at a specific time. This decision should be taken in a long-term planning perspective. It should consider how the operation will be performed to satisfy demand and water quality requirements by using multiple sources with different water quality at the source, take into account the temporal and spatial distribution of the water resources available and remain aware of the environmental impacts. In addition, decision processes which do not appropriately consider inherent uncertainties (e.g., hydrological, demographic, and technological uncertainties) can lead to suboptimal solutions. Here, uncertainty is handled using scenario planning with the aim of finding expansion solutions that can be expected to perform well under a set of possible future situations (or scenarios). The solution method combines simulated annealing with nonlinear programming to determine the solution to the nested optimization problem.

Keywords: Water supply, Capacity expansion, Nested optimization approaches

1. INTRODUCTION

Nested optimization approaches place one optimization task inside of another. The solution of the outer optimization task, usually referred to as upper level, depends on the solution of the inner optimization task, usually referred to as lower level.

Nested optimization approaches are often associated with bilevel programming or bilevel optimization (Vicente and Calamai 1994; Colson et al. 2007). Bilevel programming emerged from the classic Stackelberg problem (1952) in the field of game theory. The strategic game conceptualized by Stackelberg comprises a hierarchical planning process with a leader and a follower. The leader solves the problem of finding their optimal strategy assuming that they can anticipate the optimal response of the follower to their own actions. In this framework, the leader's optimization problem contains an inner optimization task that corresponds to the follower's optimization problem. Sinha et al. (2013a) give examples of a number of practical bilevel problems studied in the literature in the domain of transportation, economics, management and engineering. The hierarchical structure can introduce difficulties such as non-convexity and disconnectedness, even for simple bilevel linear programming problems (Sinha et al. 2013a).

But nested optimization approaches do not result from a hierarchical planning process as bilevel. Examples in the field of water resources include Schmitz et al. (2007), who presented a nested approach for improving irrigation efficiency where optimizing the irrigation schedule (i.e., number and date of applications) was the "outer task" and optimizing the control of each single water application (i.e., intensity and irrigation time) was the "inner task". The inner optimization task is performed by obtaining the inverse solution of a numerical irrigation model. Ricciardi et al. (2007) developed a nested optimization approach for the design of groundwater remediation systems. Scenario planning is used to represent the uncertainty of hydraulic conductivity. The inner optimization includes a series of deterministic linear mathematical programs, one for each scenario, for determining pumping systems that minimize operation and maintenance cost subject to hydraulic-gradient constraints. A penalty value is then added to the cost of the pumping system obtained for each scenario. The penalty term is given by a weighted sum of violation of hydraulic-gradient constraints that occur when the pumping system designed for each scenario is applied to all other scenarios considered to represent the uncertainty of hydraulic conductivity. Finally, the outer optimization examines which pumping system results in the minimum sum of the design cost with the penalty value.

Classic optimization techniques, including the Karush-Kuhn-Tucker approach, Branch-and-Bound techniques and the use of penalty functions, have been employed to solve problems formulated as nested but with limited application to simple cases. Alternatively, modern heuristics or a combination of modern heuristics with classical optimization methods have proved to be successful at handling complex problems formulated as nested (Schmitz et al. 2007; Ricciardi et al. 2009; Sinha et al. 2013a, 2013b).

The nested optimization approach described in this work was developed to support decision-making in the capacity expansion (or redesign) of multiquality water supply systems. In general, the first failures in the desired performance of the water systems caused by increased demand, reduced supply or the imposition of new regulation are handled with corrective actions using the existing infrastructure. However, more serious failures may require the construction of new infrastructure and/or the rehabilitation of what is in place (Hsu et al. 2008). Capacity expansion decisions should be taken in a long term perspective and consider how the systems will be operated in an uncertain environment. It has long been recognized, that failing to incorporate uncertainty in the planning process may result in solutions that do not meet needs in the immediate future, solutions that will become obsolete in the short/medium term or solutions that turn out to be oversized. One common approach to deal with uncertainty in optimization planning models is its representation by a set of scenarios that may be defined as structured representation of the uncertain model parameters. Scenario planning was used for example by Rosenberg and Lund (2009), Kang and Lansley (2013), Ray et al. (2014), or Lan et al. (2015) in recent studies dealing with the development of optimization models for multiquality water-supply systems with explicit representation of uncertainty.

But none of the previous optimization models include an explicit representation of water quality as in the nested optimization approach presented in this paper. This aspect can be relevant since different water quality at the source often determines differences in the water quality for the end-users. The explicit representation of water quality introduces nonlinear constraints to reflect the real physical conditions but those nonlinearities difficult the solution of the optimization models (Yang et al. 2000). For solving the nested optimization model detailed in the sections that follow, the authors have also developed a solution method that allows to deal efficiently with scenario planning and the explicit representation of water quality.

Section 2 briefly describes the nested optimization approach developed and its solution method to support capacity expansion solutions. Section 3 presents the results of applying it to a real-world case study. Some conclusions are drawn in Section 4. The work presented here is described in more detail in Vieira (2014).

2. NESTED OPTIMIZATION APPROACH

The decision to be taken in each situation corresponds to the infrastructure to be built and/or rehabilitated at a specific time for the capacity expansion of a multiquality water supply system. The nested optimization approach developed is intended to identify capacity expansion solutions provided by solving the outer optimization task and that depend on a set of solutions obtained from the inner optimization task. The solution of each inner optimization task enables us to determine the optimal operation of the redesigned water system in each scenario, which can be defined here as one single realization of the parameters defined as non-deterministic during the system operation. The consideration of multiple scenarios also makes it possible to define the approach developed as proactive. The explicit incorporation of some knowledge of uncertainty during system operation is intended to find capacity expansion solutions that are less sensitive to the non-deterministic parameters. Mathematically, the capacity expansion problem to be solved can be defined as follows:

$$\underset{Y}{\text{Maximize}}\; F(Y,X_1,X_{\ldots},X_{NS}) \tag{1}$$

where $F(Y,X_1,X_{\ldots},X_{NS})$ is the value of a mathematical function determined by solving the following NS mathematical programs (one for each scenario) with the capacity expansion solution Y={0, 1} as input data

$$\underset{X_s}{\text{Min}}\; f(X_s) \tag{2}$$

subject to (s.t): $g(X_s) = 0$

$X_s \geq 0$

where $s = 1,\ldots,NS$

The optimization problem defined by (1) sets the outer optimization task while the NS optimization problems defined by (2) set the inner optimization task. The vector Y represents the capacity expansion solutions, the vectors X_s define the operating decisions in each scenario s and NS is the total number of scenarios. Section 2.1 and 2.2 describe the optimization problems included in the two optimization tasks. Section 2.3 presents the method used to derive the capacity expansion solutions.

2.1 *Inner optimization task*

The NS optimization problems included in the inner optimization task have the main characteristics of the model developed by Vieira et al. (2011) to optimize the operation of large-scale multiquality water supply systems dependent on surface water and groundwater sources. The application of the model requires the representation of a given infrastructure as a flow network composed of arcs and nodes, the characterization of the demand and a time series of inflows to reservoirs and of the aquifer recharge. The optimized decisions provided by the solution of the model include the volume of withdrawals from each water source, the operation of the treatment and pumping facilities and the water allocation from each source to the demand nodes. These operating decisions are discretized in monthly periods $t = \{1, 2, ..., NT\}$ over the entire operational planning time horizon, which includes NT time steps. Usually, monthly time steps are adequate for describing in detail the operation of large-scale water supply systems for planning purposes. Such discretization allows describing the most important intra-annual variations in both supply (e.g., storage in reservoirs and piezometric levels in aquifers) and water demand thereby avoiding overly simple representation of phenomena given by annual time steps.

Water quality is explicitly represented in the description of the water transport using the multicommodity network flow approach (Fig. 1). This approach requires that water from a different source, or simply of a different quality, is regarded as a separate commodity $k = 1,...,NK$ sharing a common distribution network. The water flows are modeled by the variable $x^k_{pq,t,s}$ representing a non-negative flow of water type identified by the index k in the network arc (p,q) from node p to node q in period t in scenario s. Due to their miscibility, it is justifiable to assume that waters modeled as different commodities are perfectly mixed when the time scale used for planning purposes is larger than one day (Yang et al. 2000).

[Insert Fig. 1 approximately here]

The objective function $f(...)$ in (2) adds the variable operating costs to a set of three penalty functions. The variable operating costs include all the abstraction, treatment and pumping costs that depend on the quantity of water supplied. The penalty functions are used when solving the model to avoid deviations from the objectives of *i*) to satisfy the demand and *ii*) to deliver water of the appropriate quality as specified, in terms of volumetric water blending ratios. One last penalty function is added as an artifice to avoid unnecessary spills from reservoirs. Weight factors in the penalty functions allow prioritization of the objectives for each situation.

The constraints $g(...)$ in (2) include mathematical functions that simulate the water balance in reservoirs, the groundwater flow in aquifers, and the water flow and quality in a distribution network. Legal water rights and environmental concerns (such as minimum discharges from reservoirs for downstream ecosystem maintenance and minimum piezometric levels in aquifers to prevent problems related to the over-exploitation of the groundwater resources) are also modeled as constraints.

The optimization problem is nonlinear. The penalty functions in $f(...)$ are quadratic so that greater deviations from the objectives are more heavily penalized. The multiplication of decision variables in the perfect mixing condition (Fig. 1) included in the model constraints introduces a high degree of nonlinearity and may make the solution of the inner optimization task (i.e., the optimal operation of the water system in the different scenarios) quite complex and time consuming.

2.2 *Outer optimization task*

The mathematical function $F(...)$ in (1) integrates two metrics – the performance index (*PI*) and the total solution cost (*PVC*) – that are used to evaluate each capacity expansion solution.

The performance index (*PI*) corresponds to the aggregation in a single value of the information given by three performance criteria in the scale [0,1] – reliability (*Rel*), vulnerability (*Vul*) and the water quality criterion (*VBld*). *Rel* and *VBld* are related to the water quantity and express the general characteristics for these criteria proposed by Hashimoto et al. (1982). Here, *Rel* is the volume of water supplied divided by the target demand (also known in the literature as volumetric reliability), and *Vul* is the maximum deficit relative to the demand in all time periods. The use of together *Rel* and *Vul* as the two water quantity criteria included in the performance index should guarantee its suitability for evaluations also related with the sustainability of the water systems (Kjeldsen and Rosbjerg 2004). *VBld* is the water quality criterion measuring the worst water quality conditions defined in terms of volumetric water blending ratios at the demand nodes in all time periods. By minimizing the *VBld*, the worst water quality conditions should be mitigate and, simultaneously, a higher volume of water with the best quality to the extent possible should be available *ceteris paribus*. Finally, the value of the performance index is calculated as the simple average of the three performance criteria:

$$PI = \frac{Rel + (1 - Vul) + (1 - VBld)}{3} \tag{3}$$

The terms $(1 - Vul)$ and $(1 - VBld)$ are used so that the objective is to maximize *Rel* and to minimize *Vul* and *VBld*. The value of *PI* is also a non-negative number taken as one or lower. Loucks (1997) and Zongxue et al. (1998) had initially proposed two different indexes to evaluate the performance of water systems and support decision-making. Other authors have come to examine and propose alternative formulation from those two initial indexes in more recent studies about the evaluation of performance of water systems (e.g., Sandoval-Solis et al. 2011; Hajiabadi and Zarghami 2014; Ray et al. 2014; Tseng et al. 2015). In any of these studies water quality is not included as a criterion for evaluating system performance. The inclusion of a water quality criterion for evaluating the performance of multiquality water-supply systems can be justified as different water quality at the source often determines differences in the water quality for the end-users (Yang et al. 2000). Sandoval-Solis et al. (2011) also argue that water quality criteria can be included in performance indexes for evaluating municipal water use.

The total solution cost (PVC) includes initial construction costs for system redesign (CC) and operating costs (OC) spread over the project lifetime. The operating costs are divided into fixed and variable costs (FOC and VOC) with the quantity of water supplied:

$$PVC = CC + FOC + VOC \tag{4}$$

The PVC reports the total solution cost up to the "present" at a certain discount rate.

The function $F(...)$ in (1) was inspired by the field of robust optimization introduced by Mulvey et al. (1995) and followed many others (e.g., Rosenberg and Lund 2009; Kang and Lansey 2013; Ray et al. 2014; Lan et al. 2015). The objective function of the outer optimization task allows the explicit balancing of the trade-offs between solution robustness and cost. Solution robustness is defined by a mean-variance formulation of the PI in all the scenarios, and the objective function $F(...)$ can be written as follows:

$$F(...) = \underbrace{\sum_{s=1}^{NS} p_s PI_s}_{\mathrm{E}(PI_s)} - \varphi \underbrace{\sum_{s=1}^{NS} p_s \left(PI_s - \sum_{s=1}^{NS} p_s PI_s \right)^2}_{\mathrm{Var}(PI_s)} - \omega PVC \tag{5}$$

where p_s is the probability of scenario s, $\mathrm{E}(PI_s)$ and $\mathrm{Var}(PI_s)$ are the expected value and the variance of the performance index over all scenarios, and φ and ω are weights representing the relative importance assigned to the variability of system performance and to the solution cost. One capacity expansion solution can be obtained from the solution of the nested optimization problem for each pair of values φ and ω. The mean-variance formulation addresses risk-averse behavior and higher values of φ reduce the chances of solutions with low performance in some scenarios being selected. Higher values of ω favor reduced cost solutions. The PVC is also an expected cost given that VOC are determined by an average value over all scenarios.

2.3 *Solution method*

The method implemented to find the capacity expansion solutions combines simulated annealing with nonlinear programming (Fig. 2). The basic concept of the solution method is in taking advantage from the nested structure defined in (1)-(2) by decomposing the global and highly complex model (including discrete and continuous variables, and nonlinear constrains) into smaller sub-models with lower level of complexity and that can be efficiently solved independently. The application of decomposition solution methods has a fairly recent origin in the water sector (Cai et al. 2001; Reis et al. 2005, 2006), and has been followed by other authors lately (e.g., Chen et al. 2013; Afshar et al. 2015, Li et al. 2015).

[Insert Fig. 2 approximately here]

The solution of the nested problem (1)-(2) begins with a random generation of a capacity expansion solution represented in vector Y. This allows to express the binary variables in Y as input data and to define univocally NS nonlinear optimization problems that can be solved independently to determine the optimal operating decisions for each scenario s (i.e., X_s for $s = 1,\ldots,NS$). The value of $F(\ldots)$ is determined after obtaining the optimal operating decisions for all scenarios. The solution method proposed here can be implemented using the simulated annealing proposed by Cunha (1999). A stop criterion included in the simulated annealing algorithm determines after calculating the value of $F(\ldots)$ at each iteration either the end of the solution process or if a new expansion solution should be generated.

The solution method includes a second decomposition when solving each of the nonlinear optimization problems in the inner optimization task. As stated in section 2.1, the perfect mixing condition (Fig. 1) introduces a high degree of nonlinearity and may make the solution of the NS nonlinear optimization problems in the inner optimization task quite complex and time consuming. To reduce the computational burden, the NS nonlinear optimization problems can be solved with the decomposition approach as described by Vieira and Cunha (2011). In step one, the perfect mixing condition is eliminated from the set of constraints. This set of nonlinear constraints is added only in the second step so that a solution of the complete nonlinear optimization problem is then found. Vieira and Cunha (2011) showed significant reduction of the computation time when solving a nonlinear optimization problem similar to the one handled here in the inner optimization task. Vieira and Cunha (2011) suggested that this efficiency gain could be extremely useful for reducing the computational burden in capacity expansion problems.

3. CASE STUDY

The proposed nested optimization approach was implemented to identify potential capacity expansion solutions for the Barlavento Water System (BWS) located in the Algarve region of Portugal. The BWS is regional and supplies water for urban use to 9 of the 16 municipalities in the Algarve from surface water and groundwater sources. Surface water is soft and groundwater is naturally hard. Previous studies have demonstrated that a volumetric blend of hard groundwater should be kept below 25% to avoid significant variations in drinking-water quality (Vieira et al. 2011).

The current water sources of the BWS are two surface water reservoirs (Odelouca and Bravura) and two groups of wells (Vale da Vila and Almádena). But the Odelouca reservoir is smaller than was initially planned, as determined by the environmental impact assessment procedure of this BWS water source. A report by Hidroprojecto and Ambio (2005) for the water utility that manages the BWS, already assumed the reduction in size determined later (in 2006) for the Odelouca reservoir, concluded there would be difficulties in meeting demand for the year 2025 (74.7 million m^3/year) and suggested that structural solutions were needed to expand the capacity of the BWS.

3.1 *Capacity expansion options*

Hidroprojecto and Ambio (2005) singled out two possible surface water transfers from neighborhood systems and the construction of one seawater desalination plant with three possible design sizes for the capacity expansion of the BWS. Vieira (2014) added to those possible options the rehabilitation of six groups of wells and the installation of nanofiltration systems to soften the groundwater in all the well groups (i.e., those in the current sources and those to be rehabilitated as investment options). The typical water recovery rate (i.e., ratio of permeate flow rate to feed flow rate) of the nanofiltration systems would be 85%. Table 1 lists the current sources (CS) and the investment options (IO) used in this case study. Each capacity expansion solution results from the selection of one or more investment options. In this case study, there were 589 824 different capacity expansion solutions, given by all the possible combinations of the investment options listed in Table 1. The combination of all possible investment options defines the solution space of this case study. A preliminary evaluation allowed to conclude that it would not be practicable and too much time consuming to solve this case study by total enumeration. The final statistics about the computation time are summarized in section 3.3.1.

[Insert Table 1 approximately here]

The maximum flows indicated in Table 1 depend solely on the pumping and treatment systems installed/to be installed, whereas the firm quantities also depend on limits set by the authorities. The installation of nanofiltration systems can reduce either the maximum flow and/or the maximum firm quantity of each group of wells. For example, the total pumping capacity of the Vale da Vila group as a current source of the BWS is 984 L/s, but in any case the water utility cannot extract more than 13 million m^3/year, as defined by the authorities. The maximum values indicated for the investment option H4.O2 result from combining the maximum flow and firm quantity indicated before for the Vale da Vila well group as a current source of the

BWS with the water recovery rate of the nanofiltration systems (85%). In the investment option H4.O1, the maximum flow of 350 L/s corresponds to the 11.05 million m^3 (firm quantity for H4.O2) distributed uniformly over one year.

Furthermore, the withdrawals from each source are also limited by the simulation of the water balance in each surface reservoir and the simulation of the groundwater flow in each aquifer. The groups of wells are located in two aquifers. The Almádena group is located in the Almádena-Odiáxere aquifer. All the other groups are located in the Querença-Silves aquifer.

3.2 *Hydrological scenarios*

The hydrologic scenarios in this test case were generated from a multivariate time series of monthly precipitation values from a 55 year record (October 1951 - September 2006) for each surface reservoir and aquifer. The monthly values of precipitation were transformed into reservoir inflows with a hydrological model, and into aquifer recharge using average recharge rates based on the hydro-geological formations.

In this application, ten hydrological scenarios capture some uncertainty associated with the reservoir inflows and the aquifer recharge. Each scenario corresponded to a five year multivariate data block sampled from the historic multivariate time series. Nine of the ten scenarios were sampled randomly using the moving blocks bootstrap method with partial block overlap while one scenario was chosen specifically as detailed next. The moving blocks bootstrap method (Vogel and Shallcross 1996) is a simple nonparametric method. Avoiding defining assumptions regarding the marginal probability distributions of the variables and the spatial and temporal covariance structure of the variables, one simply resamples randomly, with replacement, a set of multivariate data blocks sampled from the historic multivariate time series. The challenge is to resample the records in such a way as to assure that the temporal and spatial covariance structure of the original time series is preserved as well as also that the first values and the last values of each block are be nearly independent (Vogel and Shallcross 1996; Buishand and Brandsma 2001). The other scenario was chosen specifically so that the serious drought that afflicted the Algarve in 2004 and 2005 would have to be included covering the period October 2001 – September 2006. In a reference paper, Watkins and McKinney (1999) had previously used an approach similar to this by combining scenarios selected randomly and drought scenarios chosen specifically to generate a finite number of scenarios in a planning model.

As described above, nine of the ten scenarios were sampled randomly from a historic time series in line with the hypothesis of stationary conditions. But this assumption is opened to wide discussion in recent times from the announced climate change scenarios. Many authors have discussed the “death” of stationary of the hydrologic processes (Milly et al. 2008; Matalas et al. 2012; Salas et al. 2012). In recent papers, Kasprzyk et al. (2012, 2013) and Herman et al. (2016) developed different approaches that include a special attention to drought scenarios and their impact on water resources planning given that these phenomena could become more frequent under climate change. In this paper, one used a simple approach by giving to all scenarios the same probability p_s in Eq. (5), including to the drought scenario. This represents giving to the drought scenario an importance higher than that related directly to its frequency from the historic time series, and envisaging some non-stationary about the hydrological processes in testing the nested optimization approach presented in this paper.

3.3 *Results and discussion*

3.3.1. Solution robustness and cost

The capacity expansion solutions presented next allow to explicit balancing the trade-offs between solution robustness and cost. The results presented next were obtained after solving the nested problem (1)-(2) with five different pair of values φ and ω [Eq. (1) is detailed in (5)]. The best solution for each pair of values φ and ω was found in tens of hours by searching always just less than 0.5% of the solution space. The statistics about the computation time and the search of the solution space confirmed that it would not be practicable and too much time consuming to solve this case study by total enumeration.

The five pair of values φ and ω in Eq. (5) were defined after setting $\varphi = 1$, $\omega = \omega^{*} / PVC_{Sup}$, and and $\omega^{*} = 0.1$, 0.5, 1, 5, 10. The PVC_{Sup} corresponds to the total cost defined by Eq. (4) of a particular capacity expansion solution designated as *Sup*. In this case study, the solution *Sup* was the one with the highest fixed costs ($CC + FOC$) in Table 1 (i.e., the solution *Sup* included the selection of investment options H1, H2, H3.O3, H4.O2, H5.O2, H6.O3, … , H10.O3). The discussion that follows also includes the analysis of the results obtained in Solution ∅, the "do nothing" solution that retains the current sources. The results for Solution ∅ and Solution *Sup* were obtained in a single iteration of the solution process described in section 2.3.

Table 2 shows that the expansion solutions determined with the three highest values of the weight balancing the cost ($\omega^{*} = 1$, 5 and 10) do not include any of the investment options found by Hidroprojecto and Ambio (2005). In these three cases, the capacity expansion of the BWS is achieved by rehabilitating the groups of wells, with or without including the installation of nanofiltration systems to soften groundwater. Table 3 shows that the cost of those three solutions is lower – total cost between 158.6 and 177.6 million euros (€) – but there is less impact on the system performance, as $\mathrm{E}(PI_s)$ is lower and $\mathrm{Var}(PI_s)$ is higher. These results are due to two main factors. First, apart from the Almádena group of wells, all the groups are located in the Querença-Silves aquifer. This means that too often the withdrawals in the Querença-Silves aquifer are limited by model constraints that become active because of minimum piezometric levels in selected locations. Second, the rehabilitation of groups of wells may not be enough to reverse reductions in the maximum flows and/or total firm quantity from the installation of nanofiltration systems in the Vale da Vila and/or Almádena groups of wells (see section 3.1). Both factors contribute to the demand not being fully met in more than one scenario. The results also show that the poorest values of the performance criteria and the performance index are for the scenario specifically included here (i.e., scenario 2001-2006 – see section 3.2), thus the serious drought that afflicted the Algarve in 2004 and 2005 was always included in this case study.

[Insert Table 2 approximately here]

[Insert Table 3 approximately here]

Table 2 also shows that the same capacity expansion solution was found with ω^* = 0.1 and 0.5. It corresponds to the capacity expansion of the BWS by prescribing new infrastructure for the transfer of surface water from the Santa Clara system. Table 3 shows that for this capacity expansion solution E(PI_s) almost equals one (i.e., maximum) and Var(PI_s) is virtually null indicating a nearly optimal system performance in all scenarios.

All metrics [*CC*, *PVC*, E(PI_s) and Var(PI_s)] are in the range defined by the values for Solution ∅ and Solution *Sup* (Table 3). These results support the hypothesis that Solution ∅ and Solution *Sup* should be those of minimum and maximum robustness, respectively.

In brief, and as expected, the variation of ω^* allowed the identification of a trade-off between solution robustness and cost. Lower cost solutions were found by increasing ω^* as this weight corresponds to a cost penalty. But lower cost solutions are also less robust solutions as E(PI_s) decreased and Var(PI_s) increased when ω^* was increased. Significant improvements in solution robustness necessarily imply higher costs. A robust solution was found with a reduced penalization of cost.

3.3.2. Detailed evaluation of system performance

A more detailed evaluation about system performance can be drawn from Table 4. This table shows besides the average value of the performance index (*PI*) over all scenarios (already in Table 3) its minimum and maximum value between all scenarios as well as the same statistics for the three performance criteria (*Rel*, *Vul* and *VBld*) that define the performance index. The last two performance criteria are represented in Table 4 by $(1 - Vul)$ and $(1 - VBld)$ such that for all the performance indicators the minimum corresponds to the worst value and the maximum to the best value, respectively.

From section 2.2, the *PI* is defined by Eq. (3); the reliability *Rel* represents the volume of water supplied divided by the target demand; the vulnerability *Vul* is the maximum deficit relative to the demand in all time periods; and *VBld* is the water quality criterion representing the worst water quality conditions at all demand nodes in all time periods. *PI*, *Rel*, *Vul* and *Vul* are non-negative taken as one or lower. From the introduction to this case study, the ratio *HGW/TW* [hard groundwater supplied/total water (soft + hard) supplied] should be kept below 0.25 (or 25%) to avoid significant variations in drinking water quality. In this case study, the *VBld* measured specifically the difference between the highest ratio *HGW/TW* at all demand nodes in all time periods and the volumetric blending objective of 0.25, but only above that value: $VBld = \max[(HGW/TW - 0.25), 0]$.

From Table 4, the minimum value of *Rel* in the hypothesis “do nothing” (Solution ∅) represents that the satisfaction of the demand at the system level (given by the ratio total water supplied/total water demand) has the minimum value of 0.818 or 81.8% in one of the ten scenarios. Also for Solution ∅ and from the minimum value of $(1 - Vul) = 0.738$, it is possible to conclude that there is at least one time period in which the ratio total water supplied/total water demand is not higher than 73.8%. Finally, the minimum value of

$(1 - VBld)$ reveals that there is at least one demand node and one time period in which all the water supplied has a ratio *HGW/TW* equal to 1 or 100%. [here, when the ratio *HGW/TW* = 1, $VBld = 0.75$, and $(1 - VBld) = 0.25$]. All the minimum values of the performance criteria were recorded in the same scenario (the scenario October 2001 – September 2006 that covers that the serious drought that afflicted the Algarve in 2004 and 2005) whereby the minimum value of the *PI* in all scenarios for Solution ∅ (= 0.602) can be determined directly from the minimum values of the performance criteria shown in Table 4.

The increase in the value of $E(PI_s)$ from Solution ∅ to the expansion solution found with $\omega^* = 10$ (in Table 3 or Table 4) is more closely related with the evolution of $(1 - VBld)$. The *VBld* is the water quality criterion, and the installation of nanofiltration systems (NFS) in Almádena wells group (option H5.O2 – see Table 2) has a significant impact in this performance criteria. On the contrary, there is no significant improvement in the performance criteria related with water quantity (*Rel* or *Vul*). In Solution ∅, the withdrawals from the Querença-Silves aquifer were too often limited by minimum piezometric levels in the inner optimization task. In the capacity solution found with $\omega^* = 10$, there are new water sources in the Querença-Silves aquifer (options H7.O1 and H10.O1) but it is not possible to increase significantly the total abstractions in that aquifer.

The installation of NFS in Vale da Vila wells group for hardness removal in the capacity expansion solutions found with $\omega^* = 1$ and 5 (option H4.O1 – see Table 2) contributes significantly to guarantee good water quality in any scenario as $(1 - VBld)$ is always maximum (Table 4). But the installation of the NFS decreases the statistics of the performance criteria related with water quantity in comparison with the capacity solution found with $\omega^* = 10$. This happens given that it is not possible to use all the water pumped from the aquifer but only 85% which is the water recovery rate of the NFS (section 3.1).

In the capacity expansion solution found ω^* = 0.1 or 0.5, the target demand would be totally satisfied in any scenario, and there would be only a minimal deviation to the objective of supplying water with a volumetric blending of hard groundwater lower than 25%. The maximum volumetric blending of hard groundwater was 26.7% in the already mentioned serious drought scenario.

Finally, Fig. 3 shows the variation of the *PI* in all scenarios for Solution ∅ and for the capacity expansions solutions identified in Table 2. The scenario that covers the serious drought that afflicted Algarve in 2004 and 2005 is scenario #10. But other scenarios lead to a lower system performance in Solution ∅ and in the lower cost solutions found ω^* = 1, 5 and 10, particularly, in scenarios #2, #4, #7 and #8. These scenarios include other less serious droughts that afflicted the Algarve in 1950s, 1970s, 1980s and 1990s (Vieira 2014). As shown in Fig. 3, only in the solution found with ω^* = 0.1 or 0.5 and that implies a higher investment, it would be possible to have nearly optimal performances in all scenarios, thereby reducing some impact about the uncertainty associated to the natural hydrology.

[Insert Fig. 3 approximately here]

4. CONCLUSIONS

The application of the nested optimization approach to the selected case study indicated that it could potentially support decision-making in real-world problems. The modeling approach presented here allows the evaluation of the trade-offs between system robustness and cost, explicitly considering uncertain factors during the system operation. The capacity expansion solution identified here as robust is associated with an initial investment of 28.3 million euros and a total cost of less than 200 million euros. That capacity expansion solution costs more than other solutions that show a good performance in some historic scenarios but that fail in other historic scenarios. Even without considering other sources of uncertainty (e.g., non-stationarity of hydrologic series, population growth, cost factors) arriving at a definitive decision on the capacity expansion solution of the BWS will never be straightforward. In general, if the decision taken is to make significant investments in the capacity expansion of the water systems and extreme situations do not then occur, it can be always claimed that unnecessary investments were made. However, relatively modest investments might not be enough to limit the negative impacts of extreme events to an acceptable level. The ability of the proposed modeling approach to generate a restricted set of potential capacity expansion solutions that can be studied in more detail before reaching a final decision has been effectively demonstrated.

REFERENCES

Afshar A, Masoumi F, Solis SS (2015) Reliability based optimum reservoir design by hybrid ACO-LP algorithm. Water Resour Manag 29(6):2045-2058. doi:10.1007/s11269-015-0927-9

Buishand TA, Brandsma T (2001) Multisite simulation of daily precipitation and temperature in the Rhine basin by nearest-neighbor resampling. Water Resour Res 37(11):2761-2776.

Cai XM, McKinney DC, Lasdon LS (2001) Solving nonlinear water management models using a combined genetic algorithm and linear programming approach. Adv Water Resour 24(6):667-676

Chen YW, Chang LC, Huang CW, Chu HJ (2013) Applying genetic algorithm and neural network to the conjunctive use of surface and subsurface water. Water Resour Manag 27(14):4731-4757. doi:10.1007/s11269-013-0418-9

Colson B, Marcotte P, Savard G (2007) An overview of bilevel optimization. Ann Oper Res 153:235-256

Cunha MC (1999) On solving aquifer management problems with simulated annealing. Water Resour Manag 13(3):153-169

Hajiabadi R, Zarghami M (2014) Multi-objective reservoir operation with sediment flushing; Case study of Sefidrud reservoir. Water Resour Manag 28(15):5357-5376. doi:10.1007/s11269-014-0806-9

Hashimoto T, Stedinger JR, Loucks DP (1982) Reliability, resiliency, and vulnerability criteria for water-resource system performance evaluation. Water Resour Res 18(1):14-20

Herman JD, Zeff HB, Lamontagne JR, Reed PM, Characklis GW (2016). Synthetic drought scenario generation to support bottom-up water supply vulnerability assessments. J Water Resour Plan Manag doi:.1061/(ASCE)WR.1943-5452.0000701

Hidroprojecto, Ambio (2005) Expansion of the Water Supply to the Algarve – Technical-Economic Study. Lisbon (in Portuguese)

Hsu NS, Cheng WC, Cheng WM, Wei CC, Yeh WWG (2008) Optimization and capacity expansion of a water distribution system. Adv Water Resour 31(5):776-786

Kang D, Lansey K (2013) Scenario-based robust optimization of regional water/wastewater infrastructure. J Water Resour Plan Manag 139(3):325-338. doi: 10.1061/(ASCE)WR.1943-5452.0000236

Kasprzyk JR, Reed PM, Characklis GW, Kirsch BR (2012) Many-objective de Novo water supply portifolio under deep uncertainty. Environ Modell Softw 34:87-104. doi:10.1016/j.envsoft.2011.04.003

Kasprzyk JR, Nataraj S, Reed PM, Lempert RJ (2013) Many objective robust decision making for complex environmental systems undergoing change. Environ Modell Softw 42:55-71. doi:10.1016/j.envsoft.2012.12.007

Kjeldsen TR, Rosbjerg D (2004) Choice of reliability, resilience and vulnerability estimators for risk assessments of water resources systems. Hydrolog Sci J 49(5):755-767

Lan FJ, Lin WH, Lansey K (2015). Scenario-based robust optimization of a water supply system under risk of facility failure. Environ Modell Softw 67:160-172

Li CL, Zhou JZ, Ouyang S, Wang C, Liu Y (2015) Water resources optimal allocation based on large-scale reservoirs in the upper reaches of Yangtze River. Water Resour Manag 29(7):2171-2187. doi:10.1007/s11269-015-0934-x

Loucks DP (1997) Quantifying trends in system sustainability. Hydrolog Sci J 42(4):513-530

Matalas NC (2012) Comment on the announced death of stationarity. J Water Resour Plan Manag 138(4):311-312

Milly PCD, Betancourt J, Falkenmark M, Hirsch RM, Kundzewicz ZW, Lettenmaier DP, Stouffer RJ (2008) Climate change - Stationarity is dead: Whither water management? Sci 319(5863):573-574. doi: 10.1126/science.1151915

Mulvey JM, Vanderbei RJ, Zenios SA (1995) Robust optimization of large-scale systems. Oper Res 43(2):264-281

Ray PA, Watkins Jr DW, Vogel RM, Kirshen PH (2014) Performance-based evaluation of an improved robust optimization formulation. J Water Resour Plan Manag 140(6):04014006

Reis LFR, Walters GA, Savic D, Chaudhry FH (2005) Multi-reservoir operation planning using hybrid genetic algorithm and linear programming (GA-LP): An alternative stochastic approach. Water Resour Manag 19(6):831-848

Reis LFR, Bessler FT, Walters GA, Savic D (2006) Water supply reservoir operation by combined genetic algorithm-linear programming (GA-LP) approach. Water Resour Manag 20(2):227-255. doi:DOI 10.1007/s11269-006-8049-z

Ricciardi KL, Pinder GF, Karatzas GP (2007) Efficient groundwater remediation system design subject to uncertainty using robust optimization. J Water Resour Plan Manag 133(3):253-263

Ricciardi KL, Pinder GF, Karatzas GP (2009) Efficient groundwater remediation system designs with flow and concentration constraints subject to uncertainty. J Water Resour Plan Manag 135(2):128-137

Rosenberg DE, Lund JR (2009) Modeling integrated decisions for a municipal water system with recourse and uncertainties: Amman, Jordan. Water Resour Manag 23(1):85-115

Salas JD, Rajagopalan B, Saito L, Brown C (2012) Special section on climate change and water resources: Climate nonstationarity and water resources management. J Water Resour Plan Manag 138(5):385-388

Sandoval-Solis S, McKinney DC, Loucks DP (2011) Sustainability index for water resources planning and management. J Water Resour Plan Manag 137(5):381-390

Schmitz F, Wöhling T, de Paly M, Schütze N (2007) GAIN–P: A new strategy to increase furrow irrigation efficiency. The Arab J Sci Eng 32(1C):103-114

Sinha A, Malo P, Deb K (2013a) Efficient evolutionary algorithm for single-objective bilevel optimization. arXiv:1303.3901v2 [cs.NE]

Sinha A, Malo P, Frantsev A, Deb K (2013b) Multi-objective Stackelberg game between a regulating authority and a mining company: a case study in environmental economics. 2013 IEEE Congress on Evolutionary Computation (CEC-2013), Cancun, pp. 478-485

Stackelberg H (1952) The theory of market economy. Oxford University Press, Oxford

Tseng HW, Gan TY, Yu PS (2015) Composite drought indices of monotonic behaviour for assessing potential impact of climate change to a water resources system. Water Resour Manag 29(7):2341-2359. doi:10.1007/s11269-015-0945-7

Vicente LN, Calamai PH (1994). Bilevel and multilevel programming: A bibliography review. J Global Optim 5(3):291-306

Vieira J (2014) Decision models for the management of multisource water supply systems. PhD Thesis, University of Coimbra (in Portuguese)

Vieira J, Cunha MC (2011) A two steps solution approach to solving large nonlinear models: application to a problem of conjunctive use. Water Sci Technol 64(12):2492-2499

Vieira J, Cunha MC, Nunes LM, Monteiro JP, Ribeiro L, Stigter TY, Nascimento J, Lucas H (2011) Optimization of the operation of large-scale multisource water-supply systems. J Water Resour Plan Manag 137(2):150-161

Vogel RM, Shallcross AL (1996) The moving blocks bootstrap versus parametric time series models. Water Resour Res 32(6):1875-1882

Watkins DW, McKinney DC (1999). Screening water supply options for the Edwards aquifer region in central Texas. J Water Resour Plan Manag 125(1):14-24

Yang SL, Sun YH, Yeh WWG (2000) Optimization of regional water distribution system with blending requirements. J Water Resour Plan Manag 126(4):229-235

Zongxue X, Jinno K, Kawamura A, Takesaki S, Ito K (1998) Performance risk analysis for Fukuoka water supply system. Water Resour Manag 12(1):13-30

LIST OF FIGURES

LIST OF TABLES

Table 1 Summary of the current sources (CS) and the investment options (IO)

Water source			Investment ID	Firm quantity (× 10^6 m^3/year)	Costs: *CC* (×10^6 €)	*FOC* (×10^3 €/year)	*VOC* (€/m^3)
CS	Odelouca reservoir			257.20	NA	NA	0.106
	Bravura reservoir			6.00	NA	NA	0.190
	Vale da Vila wells group*			13.00	NA	NA	0.090
	Almádena wells group+			3.47	NA	NA	0.023
IO	Inter-system water transfer						
	Santa Clara		H1	20.00	28.31	443.3	0.122
	Sotavento		H2	18.42	35.45	348.1	0.113
	Sea-water desalination plant		H3.O1/ H3.O2/H3.O3	7.88/ 15.77/23.65	23.03/ 41.60/56.37	1152.8/ 2004.7/2847.7	0.266/ 0.263/0.261
	Installation of nanofiltration systems (NFS) in current wells group						
	Vale da Vila		H4.O1/H4.O2	11.05/11.05	6.67/16.14	135.1/202.1	0.137/0.133
	Almádena		H5.O1/H5.O2	1.61/1.95	1.09/1.96	34.2/39.7	0.140/0.137
	Rehabilitation of wells groups with local disinfection (LD) or installation of nanofitration systems (NFS)						
	Paderne*	LD NFS	H6.O1/ H6.O2/H6.O3	7.27/ 3.09/6.18	1.41/ 3.37/5.35	100.0/ 112.6/148.3	0.037/ 0.150/0.147
	Torrinha*	LD NFS	H7.O1/ H7.O2/H7.O3	3.15/ 1.34/2.68	0.18/ 1.03/1.89	16.4/ 44.0/54.3	0.023/ 0.141/0.137
	Marco*	LD NFS	H8.O1/ H8.O2/H8.O3	6.53/ 2.78/5.55	0.73/ 2.48/4.26	56.9/ 79.0/104.5	0.029/ 0.143/0.140
	Ferrarias*	LD NFS	H9.O1/ H9.O2/H9.O3	1.86/ 0.79/1.58	0.12/ 0.62/1.13	9.7/ 36.3/43.1	0.023/ 0.145/0.140
	Medeiros*	LD NFS	H10.O1/ H10.O2/H10.O3	2.52/ 1.07/2.14	0.17/ 0.85/1.54	12.9/ 40.0/48.7	0.023/ 0.145/0.138

Note: * – Wells group locate in Querença-Silves aquifer; + – Wells group located in Álmádena-Odiáxere aquifer

Table 2 Solutions obtained with $\boldsymbol{\varphi}$ = 1 and $\boldsymbol{\omega}^* = 0.1$, 0.5, 1, 5 and 10

Weight φ	Weight ω^*	Investment options selected
	10	H5.O2, H7.O1, H10.O1
	5	H4.O1, H5.O1, H10.O1
1	1	H4.O1, H5.O1, H7.O3, H9.O3, H10.O1
	0.5	H1
	0.1	H1

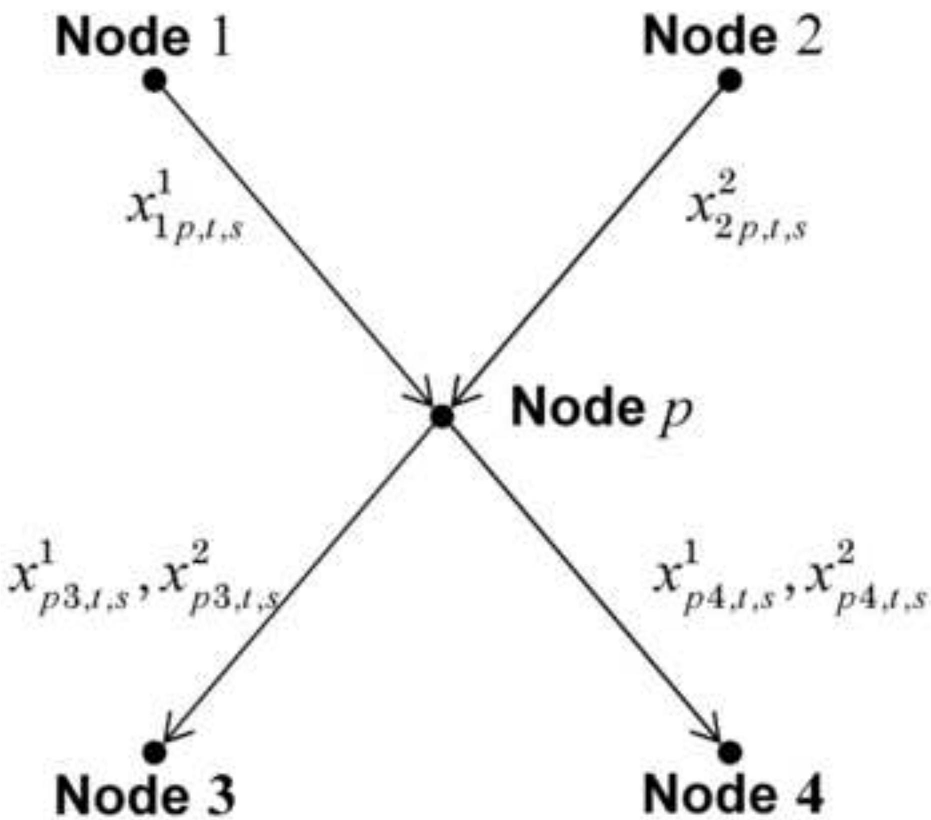
Table 3 Summary of results for Solution ∅, Solution *Sup* and solutions found with $\boldsymbol{\omega}^* = 0.1$, 0.5, 1, 5 and 10 (indicated in Table 2)

Solution	E(PI_s)	Var(PI_s)	CC (×10^6 €)	PVC (×10^6 €)
∅	0.861	0.233	0	154.7
ω^*= 10	0.929	0.135	2.31	158.6
ω^*= 5	0.966	0.024	7.93	169.7
ω^*= 1	0.980	0.015	10.94	177.6
ω^*= 0.5 and 0.1	0.999	≈0	28.31	194.7
Sup	1	0	152.4	389.4

Table 4 Minimum (min.), average (E) and maximum (max.) values of the performance index and the performance criteria for Solution ∅, Solution *Sup* and solutions found with $\boldsymbol{\omega}^* = 0.1$, 0.5, 1, 5 and 10

Solution	PI_s			Rel_s			(1-Vul_s)			(1-$VBld_s$)		
	min.	E(…)	max.	min.	E(…)	max.	min.	E(…)	max.	min.	E(…)	max.
∅	0.602	0.861	1.000	0.818	0.972	1.000	0.738	0.950	1.000	0.250	0.663	1.000
ω^*= 10	0.649	0.929	1.000	0.876	0.984	1.000	0.822	0.973	1.000	0.250	0.829	1.000
ω^*= 5	0.844	0.966	1.000	0.806	0.964	1.000	0.726	0.935	1.000	1.000	1.000	1.000
ω^*= 1	0.878	0.980	1.000	0.849	0.978	1.000	0.785	0.961	1.000	1.000	1.000	1.000
ω^*= 0.5 and 0.1	0.994	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.983	0.998	1.000
Sup		1.000			1.000			1.000			1.000	

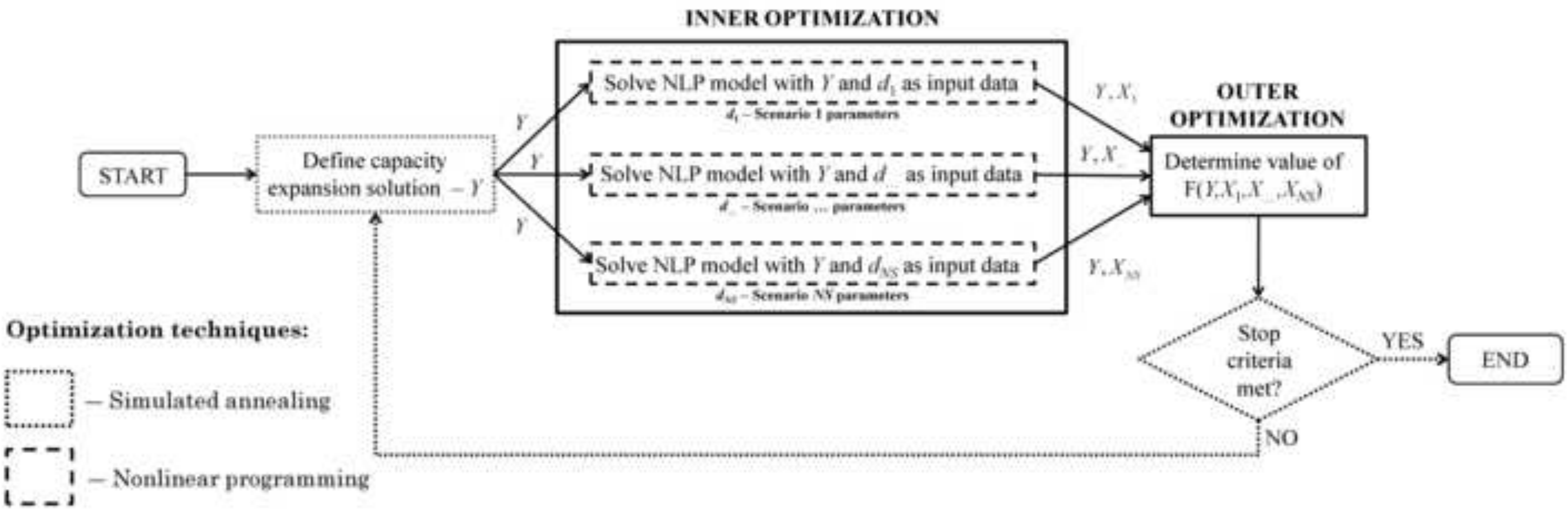
Fig1



Perfect mixing condition at node p:

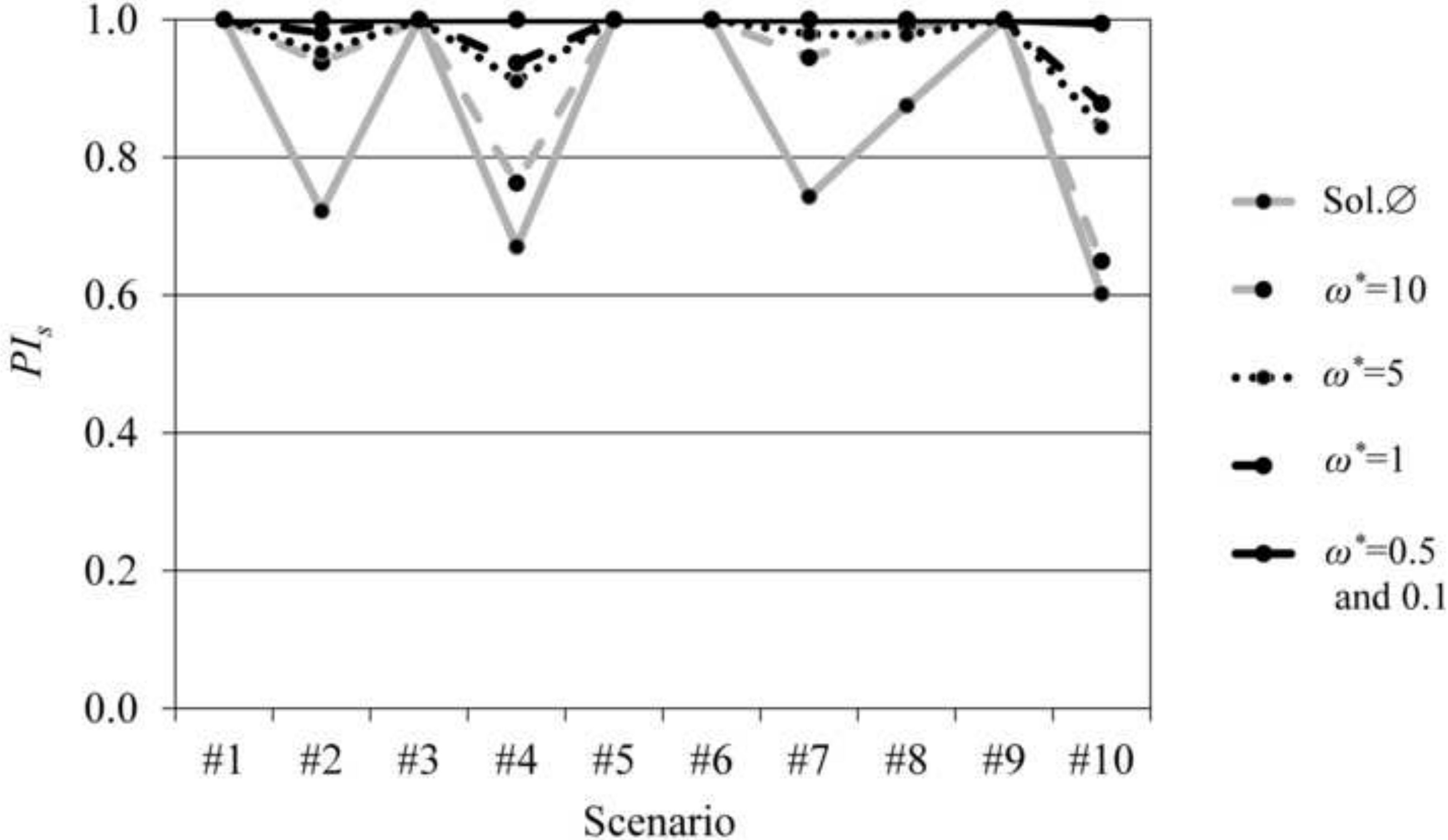
$$\begin{cases} \dfrac{x^1_{p3,t,s}}{x^1_{p3,t,s}+x^2_{p3,t,s}} = \dfrac{x^1_{p4,t,s}}{x^1_{p4,t,s}+x^2_{p4,t,s}} \\[2ex] \dfrac{x^2_{p3,t,s}}{x^1_{p3,t,s}+x^2_{p3,t,s}} = \dfrac{x^2_{p4,t,s}}{x^1_{p4,t,s}+x^2_{p4,t,s}} \end{cases}$$

Fig2



INNER OPTIMIZATION
START
Define capacity expansion solution – Y
Y
Solve NLP model with Y and d1 as input data
d1 – Scenario 1 parameters
Solve NLP model with Y and d as input data
Scenario ... parameters
Solve NLP model with Y and dNS as input data
dNS – Scenario NS parameters
Y, X1
Y, XNS
OUTER OPTIMIZATION
Determine value of
Stop criteria met?
YES
END
NO
Optimization techniques:
– Simulated annealing
– Nonlinear programming

Fig3


1.0
0.8
0.6
0.4
0.2
0.0
PI_s
#1
#2
#3
#4
#5
#6
#7
#8
#9
#10
Scenario
Sol.∅
ω^*=10
ω^*=5
ω^*=1
ω^*=0.5 and 0.1