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One-loop fermion determinant with explicit chiral symmetry breaking

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Abstract

We use a proper-time regularization to define the real part of the one-loop fermion determinant for the case in which *explicit* chiral symmetry breaking takes place. We demonstrate that beyond the chiral limit the standard definition of $\mathrm{Re}[\ln\det D]$ by $\ln|\det D|$ deforms the symmetry breaking pattern of the basic fermionic Lagrangian. We show how to obtain the correcting functional in order to arrive at the fermion determinant whose transformation properties are consistent with the general symmetry requirements. As an example it is shown how the fundamental symmetries and the explicit chiral symmetry breaking pattern associated with the ENJL model are consistently preserved in this way. © 2000 Published by Elsevier Science B.V.

It is known [1] how to derive an exact non-perturbative representation for the chiral fermion determinant which (modulo anomalies) is manifestly chiral gauge covariant. In particular this technique has been widely used in the literature [2,3] to derive the low-energy effective action of an extended Nambu–Jona-Lasinio (ENJL) model with the explicit chiral symmetry breaking term in the Lagrangian. The central object in the calculation of the effective action is the quantity $\ln\det D$, where the differential Dirac operator D depends on collective meson fields which have well defined transformation laws with respect to the action of the chiral group. If one neglects the current quark mass term in D the combination $D^\dagger D$ transforms covariantly. This fact ensures that the definition of the real part of $\ln\det D$ in terms of a proper time integral

$$\ln|\det D| = -\frac{1}{2}\int_0^\infty \frac{dT}{T}\rho(T,\Lambda^2)\mathrm{Tr}\left(e^{-TD^\dagger D}\right) \qquad (1)$$

cannot destroy the symmetry properties of the basic Lagrangian.

In this note we observe that in the presence of the explicit chiral symmetry breaking term this is not any longer true. The naive definition of the real part of $\ln\det D$ in terms of a proper time integral (1) modifies the chiral symmetry breaking pattern of the original quark Lagrangian and needs to be corrected in order to lead to the fermion determinant whose transformation properties exactly comply with the symmetry content of the basic Lagrangian. The necessary modification can be done by adding a func-

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tional P in the collective fields and their derivatives to the right hand side of Eq. (1), i.e. we require that

$$\mathrm{Re}(\ln\det D) = \ln|\det D| + P. \tag{2}$$

In the limit $\hat{m}=0$, where $\hat{m}$ is a current quark mass, $P=0$ and the old result (1) emerges as a part of our definition. This strategy formally reminds Gasser and Leutwyler's correcting procedure which they used however for a different purpose, namely to restore the standard result (1) for the real part of the fermion determinant defined by the heat kernel $\mathrm{Tr}[\exp(-T\bar{D}^2)]$ where $\bar{D}=\gamma_5 D$, especially chosen to include anomalies [4]. We consider here the ENJL model as an instructive example of our general idea. In this case the functional P must be determined in such a manner that the real part of the effective Lagrangian for the bosonized ENJL model $\mathscr{L}_{\mathrm{eff}}$ will have the same transformation laws as the basic quark Lagrangian $\mathscr{L}$. In addition it should not change the 'gap'-equation. These requirements together completely eliminate the freedom inherent to the definition of this functional. Let us stress that in our case P cannot be fixed by the requirement that the determinant remains unchanged when axial-vector and pseudoscalar fields are switched off, like, for instance, in [4]. As a consequence the effective potential of the NJL model gets $\hat{m}$-corrections from P at every step of the heat kernel expansion. This result is one of the cases where we meet another essential difference between our and Gasser and Leutwyler's approach. We have P being a functional as opposed to a polynomial in [4]. This is a general feature related to the non-renormalizability of the NJL model. Using formula (2) together with the way we propose to fix P, one can systematically take into account the effect of explicit symmetry breaking in the ENJL model. The correct description of this effect is evidently necessary in order to obtain realistic mass formulae and meson dynamics.

Consider the effective quark Lagrangian of strong interactions which is invariant under a global colour $SU(N_c)$ symmetry

$$\mathscr{L} = \bar{q}\left(i\gamma^\mu\partial_\mu - \hat{m}\right)q + \frac{G_S}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau_i q)^2\right] - \frac{G_V}{2}\left[(\bar{q}\gamma^\mu\tau_i q)^2 + (\bar{q}\gamma^\mu\gamma_5\tau_i q)^2\right]. \tag{3}$$

Here q is a flavour doublet of Dirac spinors for quark fields $\bar{q}=(\bar{u},\bar{d})$. Summation over the colour indices is implicit. We use the standard notation for the isospin Pauli matrices τ_i. The current quark mass matrix $\hat{m}=\mathrm{diag}(m_u,m_d)$ is chosen in such a way that $m_u=m_d$. Without this term the Lagrangian (3) would be invariant under global chiral $SU(2)\times SU(2)$ symmetry.

The transformation law for the quark fields is the following

$$\delta q = i(\alpha+\gamma_5\beta)q, \quad \delta\bar{q} = -i\bar{q}(\alpha-\gamma_5\beta), \tag{4}$$

where parameters of global infinitesimal chiral transformations are chosen as $\alpha=\alpha_i\tau_i$, $\beta=\beta_i\tau_i$. Therefore our basic Lagrangian $\mathscr{L}$ transforms according to the law

$$\delta\mathscr{L} = -2i\hat{m}(\bar{q}\gamma_5\beta q). \tag{5}$$

It is clear that nothing must destroy this symmetry breaking requirement of the model (we are not considering anomalies here).

Following the standard procedure we introduce colour singlet collective bosonic fields in such a way that the action becomes bilinear in the quark fields and the quark integration becomes trivial

$$Z = \int \mathscr{D}q\mathscr{D}\bar{q}\mathscr{D}s\mathscr{D}p_i\mathscr{D}V_\mu^i\mathscr{D}A_\mu^i \times\exp\left\{i\int d^4x\left[\mathscr{L} - \frac{1}{2G_S}(s^2+p_i^2) + \frac{1}{2G_V}\left(V_{\mu i}^2 + A_{\mu i}^2\right)\right]\right\}. \tag{6}$$

We suppress external sources in the generating functional Z and assume summation over repeated Lorentz (μ) and isospin ($i=1,2,3$) indices. One has to require from the new collective variables that

$$\delta(s^2+p_i^2)=0, \quad \delta\left(V_{\mu i}^2 + A_{\mu i}^2\right)=0 \tag{7}$$

in order not to destroy the symmetry of the basic Lagrangian $\mathscr{L}$. After replacement of variables

$$s = \sigma - \hat{m} + G_S(\bar{q}q), \tag{8}$$

$$p_i = \pi_i - G_S(\bar{q}i\gamma_5\tau_i q), \tag{9}$$

$$V_\mu^i = v_\mu^i + G_V(\bar{q}\gamma_\mu\tau_i q), \tag{10}$$

$$A_\mu^i = a_\mu^i + G_V(\bar{q}\gamma_\mu\gamma_5\tau_i q), \tag{11}$$

these requirements together with (4) lead to the transformation laws for the new collective fields

$$\delta\sigma = -\{\beta,\pi\}, \quad \delta\pi = i[\alpha,\pi] + 2(\sigma - \hat{m})\beta, \tag{12}$$

$$\delta v_\mu = i[\alpha, v_\mu] + i[\beta, a_\mu],$$
$$\delta a_\mu = i[\alpha, a_\mu] + i[\beta, v_\mu]. \tag{13}$$

We have introduced the notation $\pi = \pi_i\tau_i$, $v_\mu = v_{\mu i}\tau_i$, $a_\mu = a_{\mu i}\tau_i$. Therefore the transformation law of the quark fields finally defines the transformation law of the bosonic fields.

The Lagrangian in the new variables has the form

$$\mathscr{L} = \bar{q}\left[i\gamma^\mu\partial_\mu - \sigma + i\gamma_5\pi + \gamma^\mu(v_\mu + \gamma_5 a_\mu)\right]q - \frac{(\sigma - \hat{m})^2 + \pi_i^2}{2G_S} + \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V}. \tag{14}$$

The subsequent integration over quark fields shows that the effective potential has a non-trivial minimum and that spontaneous chiral symmetry breaking takes place. Redefining the scalar field $\sigma \to \sigma + m$ we come finally to the effective action

$$S_{\text{eff}} = -i\ln\det D - \int d^4x\left[\frac{(\sigma + m - \hat{m})^2 + \pi_i^2}{2G_S} - \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V}\right], \tag{15}$$

where the Dirac operator D is equal to

$$D = i\gamma^\mu\partial_\mu - m - \sigma + i\gamma_5\pi + \gamma^\mu(v_\mu + \gamma_5 a_\mu). \tag{16}$$

In this broken phase the transformation law of the pion field changes to

$$\delta\pi = i[\alpha,\pi] + 2(\sigma + m - \hat{m})\beta \tag{17}$$

in full agreement with the variable replacement $\sigma \to \sigma + m$ for the scalar field in (12). Let us comment on the importance of this step in the process of calculating the functional P. If one would first try to derive P in the symmetric phase and then perform the shift $\sigma \to (\sigma + m)$ it would be necessary to calculate all orders of the proper-time expansion. All of them would contribute (after the shift of σ) as a factor with a certain power of m to a fixed order in the fields. It is more constructive to calculate directly in the broken phase. This way one achieves a resummation of an infinite number of terms of the symmetric phase. It crucially simplifies the problem and points to the essential difference between the treatment of models with and without spontaneous symmetry breakdown.

The $\ln|\det D|$ is conveniently calculated using the heat kernel method or even more directly in the way suggested in [1]. The result of these calculations on the basis of formula (1) is well known, see for example [2]. We give it here using our notation and restricting to the second order heat coefficient,

$$\begin{aligned}\mathscr{L}_{\text{eff}} = {}& \frac{v_{\mu i}^2 + a_{\mu i}^2}{2G_V} - \frac{1}{2G_S}\left[(\sigma + m - \hat{m})^2 + \pi_i^2\right] \\ & + \frac{N_c J_0}{4\pi^2}\left(\sigma^2 + 2m\sigma + \pi_i^2\right) - \frac{N_c J_1}{8\pi^2} \\ & \times\left[\tfrac{1}{6}\mathrm{tr}\left(v_{\mu\nu}^2 + a_{\mu\nu}^2\right) - \tfrac{1}{2}\mathrm{tr}\left((\nabla_\mu\pi)^2 + (\nabla_\mu\sigma)^2\right) \right. \\ & \left. + \left(\sigma^2 + 2m\sigma + \pi_i^2\right)^2\right],\end{aligned} \tag{18}$$

where trace is to be taken in isospin space. Here we have used the notation

$$v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu], \tag{19}$$

$$a_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu], \tag{20}$$

$$\nabla_\mu\sigma = \partial_\mu\sigma - i[v_\mu, \sigma] + \{a_\mu, \pi\}, \tag{21}$$

$$\nabla_\mu\pi = \partial_\mu\pi - i[v_\mu, \pi] - \{a_\mu, \sigma + m\}. \tag{22}$$

The functions J_n represent the integrals which appear in the result of the asymptotic expansion of the heat kernel

$$J_n = \int_0^\infty \frac{dT}{T^{2-n}} e^{-Tm^2}\rho(T, \Lambda^2), \quad n = 0,1,2\ldots \tag{23}$$

We consider a class of regularization schemes (proper-time regularizations) which can be incorporated in this expression through the kernel $\rho(T, \Lambda^2)$. These regularizations allow to shift in loop momenta.

A typical example is the covariant Pauli-Villars cut-off [5]

$$\rho(T,\Lambda^2) = 1 - (1 + T\Lambda^2)\, e^{-T\Lambda^2}. \tag{24}$$

The Lagrangian (18) obtained on the basis of formula (1) does not fulfill the transformation law (5) and should be modified. Indeed, if one uses the classical equation of motion for the pion field, $\pi_i = iG_S \bar{q}\gamma_5\tau_i q$, see (14), one can rewrite Eq. (5) in terms of meson fields

$$\delta\mathcal{L} = -\frac{2\hat{m}}{G_S}(\beta_i\pi_i). \tag{25}$$

Now it is a simple task to see that the Lagrangian (18) has a different transformation law. It is necessary to take into account the corresponding contribution from the functional P to correct this unsuccessful result. In the considered approximation[2] it is a polynomial $P'(\sigma,\pi,v_\mu,a_\mu)$. We have calculated it. Let us note that P' is unique up to a chirally invariant polynomial. One can always choose P' in such a manner that the 'gap'-equation is not modified, i.e. using this chiral symmetry freedom to avoid from P' terms linear in σ. As a result the Lagrangian (18) gets an additional contribution

$$\mathcal{L}_{\text{eff}} \to \mathcal{L}_{\text{eff}} + \Delta\mathcal{L}_{\text{eff}}, \tag{26}$$

where

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff}} = &-\frac{\hat{m}^2(\sigma^2+\pi_i^2)}{2m(m-\hat{m})G_S} + \hat{m}\frac{N_c J_1}{2\pi^2} \\ &\times\left[(2m-\hat{m})\sigma^2 + \sigma(\sigma^2+\pi_i^2)\right] \\ &-\hat{m}\frac{N_c J_1}{4\pi^2}\text{tr}\left\{(2m-\hat{m})a_\mu^2 - a_\mu\partial_\mu\pi\right. \\ &\left.+ia_\mu[v_\mu,\pi] + 2\sigma a_\mu^2\right\}. \end{aligned} \tag{27}$$

Having established this counterterm one may look for kinetic and mass terms of the composite meson fields and extract the physical meson masses by bringing the kinetic terms to the canonical form by means of field renormalizations. These field redefinitions are standard and we are not going to discuss them here, leaving this issue for a forthcoming more detailed and longer paper. Let us only point out that after redefinitions the symmetry breaking part takes the form [6]

$$\delta\mathcal{L}_{\text{eff}} = -2m_\pi^2 f_\pi(\beta_i\pi_i'), \tag{28}$$

which leads to the well known PCAC relation. Here π_i' denotes the physical pion field.

In conclusion we have analyzed in this work the effect of explicit chiral symmetry breaking in a bozonized version of the $SU(2)$ ENJL model. We have shown that one cannot naively infer that the standard proper-time procedure formulated in terms of $D^\dagger D$ to evaluate the fermion determinant in the chiral symmetric case can be as well applied in the presence of explicit symmetry breaking terms. This procedure which is commonly used in the literature is misleading. We have found that the most appropriate way to trace the symmetry breaking pattern during the bosonization procedure is to use the definition of the one-loop fermion determinant through the expression (2) with the functional $P(v,a,\sigma,\pi)$ to be fixed by the symmetry requirements and the form of the 'gap'-equation. This way one gets correctly the chiral invariant result as well as the symmetry breaking part.

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[2] Let us remind that the result (18) includes only the contributions from the first three Seeley-DeWitt coefficients.