Pseudoscalar neutral mesons in hot and dense matter ☆

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Abstract

The behavior of neutral pseudoscalar mesons π^0, η and η' in hot and dense matter is investigated, in the framework of the three flavor Nambu–Jona-Lasinio model. Three different scenarios are considered: zero density and finite temperature, zero temperature and finite density in a flavor asymmetric medium with and without strange valence quarks, and finite temperature and density. The behavior of mesons is analyzed in connection with possible signatures of restoration of symmetries. In the high density region and at zero temperature it is found that the mass of the η' increases, the deviation from the mass of the η being more pronounced in matter without strange valence quarks.
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1. Introduction

Different regions of the QCD phase diagram are object of interest nowadays. Understanding matter under extreme conditions is relevant to understand the physics of heavy-ion collisions, the early universe and neutron stars. The dominant degrees of freedom of QCD at high T and μ are supposed to be partons (a quark–gluon plasma) instead of hadrons and restoration of symmetries is expected to occur. While the phase transition at zero chemical potential and finite temperature is accepted to be second order or crossover, there are indications that the phase transition with finite chemical potential and zero temperature is first order [1]. Experimental and theoretical efforts have been done in order to explore the μ–T phase boundary. Recent lattice results indicate a critical "endpoint", connecting the first order phase transition with the crossover region at $T_E = 160 \pm 35$ MeV, $\mu_E = 725 \pm 35$ MeV [2]. Understanding the results of experiments at BNL [3] and CERN [4] provides a natural motivation for these studies.

Much attention has been paid to the question of which symmetries are restored. In the limit of vanishing quark masses the QCD Lagrangian has 8 Goldstone bosons, associated with the dynamical

breaking of chiral symmetry. In order to give a finite mass to the mesons, chiral symmetry is broken ab initio by giving current masses to the quarks. The mystery of the nonexistence of the ninth Goldstone boson, predicted by the quark model, was solved by assuming that the QCD Lagrangian has a $U_A(1)$ anomaly. Explicitly breaking the $U_A(1)$ symmetry, for instance by instantons, has the effect of giving a mass to η' of about 1 GeV. So the mass of η' has a different origin than the masses of the pseudoscalar mesons and this meson cannot be regarded as the remanent of a Goldstone boson.

The study of the behavior of mesons in hot and dense matter is an important issue since they might provide a signature of the phase transition and give indications about which symmetries are restored. According to Shuryak [5], there are two scenarios: only $SU(3)$ chiral symmetry is restored or both, $SU(3)$ and $U_A(1)$, symmetries are restored. The behavior of η' in medium or of related observables, like the topological susceptibility [6] might help to decide between these scenarios. A decrease of the η' mass in medium could lead to the increase of the η' production cross section, as compared to that for pp collisions [7].

Theoretical concepts on the behavior of matter at high densities have been mainly developed from model calculations. The Nambu–Jona-Lasinio [NJL] type models [8] have been extensively used over the past years to describe low energy features of hadrons and also to investigate restoration of chiral symmetry with temperature or density [8–17].

The NJL model is an effective quark model that, besides its simplicity, has the advantage of incorporating important symmetries of QCD and of exhibiting also the symmetry breaking mechanism. Since the model has no confinement, several drawbacks are well known. For instance, the η' mass lies above the $\bar{q}q$ threshold. The model describes this meson as $\bar{q}q$ resonance, which would have the unphysical decay in $\bar{q}q$ pairs, and the definition of its mass is unsatisfactory. For this reason, it has been proposed that in order to investigate the possible restoration of $U_A(1)$ symmetry, instead of $M_{\eta'}$, the topological susceptibility χ, a more reliable quantity in this model, should be used [6].

The behavior of $SU(3)$ pseudoscalar mesons, in NJL models, with temperature has been studied in [9,10,17]. Different studies have been devoted to the

behavior of pions and kaons at finite density in flavor symmetric or asymmetric matter [12–14,16].

This Letter is devoted to investigate the phase transition in hot and dense flavor asymmetric matter and the in medium behavior of the neutral $SU(3)$ pseudoscalar mesons, in the framework of the $SU(3)$ NJL model including the 't Hooft interaction, which breaks the $U_A(1)$ symmetry. We choose these mesons because they might carry information about the restoration of symmetries: pions, which have been studied extensively in this concern, are a privileged tool to appreciate the restoration of chiral symmetry, since, being the lightest pseudoscalar mesons, are the best Goldstone-like bosons. On the other side, the in-medium behavior of η and η' is expected to carry information about the restoration of $U_A(1)$ symmetry. At variance with the other mesons, there is not much information about the behavior of these mesons in medium. Concerning the η meson, the recent discovery of mesic atoms might provide useful information in this concern [18].

2. Model and formalism

We use a version of the $SU(3)$ NJL model described by the Lagrangian:

$$L = \bar{q}(i\partial \cdot \gamma - \hat{m})q$$
$$+ \frac{g_S}{2} \sum_{a=0}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q}(i\gamma_5)\lambda^a q)^2\right]$$
$$+ g_D \left[\det[\bar{q}(1-\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\right]. \quad (1)$$

Here $q = (u, d, s)$ is the quark field with three flavors, $N_f = 3$, and three colors, $N_c = 3$. λ^a are the Gell-Mann matrices, $a = 0, 1, \ldots, 8$, $\lambda^0 = \sqrt{2/3}\,\mathbf{I}$. The explicit symmetry breaking part (1) contains the current quark masses $\hat{m} = \mathrm{diag}(m_u, m_d, m_s)$. The last term in (1) is the lowest six-quark dimensional operator and it has the $SU_L(3) \otimes SU_R(3)$ invariance but breaks the $U_A(1)$ symmetry. This term is a reflection of the axial anomaly in QCD. For the general reviews on the three flavor version of the NJL model see references in Refs. [8–10].

To obtain a four quark interaction from the six quark interaction we make a shift $(\bar{q}\lambda^a q) \to (\bar{q}\lambda^a q) + \langle\bar{q}\lambda^a q\rangle$, where $\langle\bar{q}\lambda^a q\rangle$ is the vacuum expectation

value, and contract one bilinear ($\bar{q}\lambda^a q$). The hadronization procedure can be done by the integration over quark fields in the functional integral, leading to the following effective action:

$$
\begin{aligned}
&W_{\text{eff}}[\varphi, \sigma] \\
&= -\frac{1}{2}\big(\sigma^a S_{ab}^{-1}\sigma^b\big) - \frac{1}{2}\big(\varphi^a P_{ab}^{-1}\varphi^b\big) \\
&\quad - i\,\text{Tr}\ln\big[i(\gamma_\mu\partial_\mu) - \hat{m} + \sigma_a\lambda^a + (i\gamma_5)\big(\varphi_a\lambda^a\big)\big].
\end{aligned}
\tag{2}
$$

The fields σ^a and φ^a are scalar and pseudoscalar meson nonets. We have introduced projectors

$$
S_{ab} = g_S\delta_{ab} + g_D D_{abc}\langle\bar{q}\lambda^c q\rangle,
\tag{3}
$$

$$
P_{ab} = g_S\delta_{ab} - g_D D_{abc}\langle\bar{q}\lambda^c q\rangle.
\tag{4}
$$

The constants D_{abc} are such that they coincide with the $SU(3)$ structure constants d_{abc} for $a, b, c = (1, 2, \ldots, 8)$ and $D_{0ab} = -\frac{1}{\sqrt{6}}\delta_{ab}$, $D_{000} = \sqrt{2/3}$.

A straightforward generalization of the model for finite temperature and density can be done by using the Matsubara technique (see [17,19]). The first variation of the action (2) leads to the well-known gap equations. By expanding the effective action (2) over meson fields and keeping the pseudoscalar mesons only, we have the effective action:

$$
\begin{aligned}
W_{\text{eff}}^{(2)}[\varphi] &= -\frac{1}{2}\varphi^a\big[P_{ab}^{-1} - \Pi_{ab}(P)\big]\varphi^b \\
&= -\frac{1}{2}\varphi^a D_{ab}^{-1}(P)\varphi^b,
\end{aligned}
\tag{5}
$$

where $D_{ab}^{-1}(P)$ is the inverse unnormalized meson propagator and $\Pi_{ab}(P)$ is the polarization operator, which in the momentum space has the form

$$
\begin{aligned}
&\Pi_{ab}(P) \\
&= iN_c\int\frac{d^4 p}{(2\pi)^4} \\
&\quad \times \text{tr}_D\big[S_i(p)\big(\lambda^a\big)_{ij}(i\gamma_5)S_j(p+P)\big(\lambda^b\big)_{ji}(i\gamma_5)\big].
\end{aligned}
\tag{6}
$$

The model is fixed by coupling constants g_S, g_D, the cutoff parameter Λ which regularizes momentum space integrals and current quark masses. We use the parameter set $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, $g_S\Lambda^2 = 3.67$, $g_D\Lambda^5 = -12.36$ and $\Lambda = 602.3$ MeV, that has been determined by fixing the conditions: $M_{\pi^0} = 135.0$ MeV, $M_K = 497.7$ MeV,

$f_\pi = 92.4$ MeV and $M_{\eta'} = 960.8$ MeV. We also have: $M_\eta = 514.8$ MeV, $\theta(M_\eta^2) = -5.8°$, $g_{\eta\bar{u}u} = 2.29$, $g_{\eta\bar{s}s} = -3.71$, $M_{\eta'} = 960.8$ MeV, $\theta(M_{\eta'}^2) = -43.6°$, $g_{\eta'\bar{u}u} = 13.4$, $g_{\eta'\bar{s}s} = -6.72$.

Although, due to the lack of confinement in the model, the η'-meson is here described as a resonance state of $\bar{q}q$, we use $M_{\eta'}$ as an input parameter.

To consider the diagonal mesons π^0, η and η' we take into account the matrix structure of the propagator in (5) (See Refs. [9,17,19] for details). In the basis of π^0–η–η' system we write the projector P_{ab} and the polarization operator Π_{ab} as matrices:

$$
P_{ab} = \begin{pmatrix} P_{33} & P_{30} & P_{38} \\ P_{03} & P_{00} & P_{08} \\ P_{83} & P_{80} & P_{88} \end{pmatrix} \quad \text{and}
$$

$$
\Pi_{ab} = \begin{pmatrix} \Pi_{33} & \Pi_{30} & \Pi_{38} \\ \Pi_{03} & \Pi_{00} & \Pi_{08} \\ \Pi_{83} & \Pi_{80} & \Pi_{88} \end{pmatrix}.
\tag{7}
$$

It should be noticed that in media with different densities of u and d quarks the nondiagonal matrix elements P_{30}, $P_{38} \propto (\langle\langle\bar{q}_u q_u\rangle\rangle - \langle\langle\bar{q}_d q_d\rangle\rangle)$ and Π_{30}, $\Pi_{38} \propto (I_2^{uu}(P) - I_2^{dd}(P))$ are nonvanishing and correspond to π^0–η and π^0–η' mixing (here $\langle\langle\bar{q}_i q_i\rangle\rangle$ are the in medium quark condensates and $I_2^{ii}(P)$ are written below). Here we did the approximation of neglecting these elements, whose effects we found to be negligible by means of a simple estimation. The nonvanishing elements of Π_{ab} depend on the regularized integrals [19]:

$$
\begin{aligned}
&I_2^{ii}(P_0, T, \mu_i) \\
&= -\frac{N_c}{2\pi^2}\mathcal{P}\int\frac{\text{p}^2 d\text{p}}{E_i}\frac{1}{P_0^2 - 4E_i^2}\big(n_i^+ - n_i^-\big) \\
&\quad - i\frac{N_c}{4\pi}\sqrt{1 - \frac{4M_i^2}{P_0^2}}\left(n_i^+\left(\frac{P_0}{2}\right) - n_i^-\left(\frac{P_0}{2}\right)\right),
\end{aligned}
\tag{8}
$$

where $n_i^\mp = n_i^\mp(E_i)$ are the Fermi distribution functions of the negative (positive) energy state of the ith quark. Notice that, as the temperature and/or density increase these integrals may acquire an imaginary part, as long as the meson masses cross the $\bar{q}q$ threshold. This is also true for η' in the vacuum, as already mentioned.

3. Discussion and conclusions

We present our results for the nature of the phase transition and for the masses of π^0, η and η' in three scenarios: zero density and finite temperature, zero temperature and finite density and finite temperature and density. Two types of asymmetric quark matter are considered: for the first case (case I), that might be formed temporarily in heavy-ion collisions, we fix densities by $\rho_d = 2\rho_u$ and $\rho_s = 0$; for the second one (case II), that might exist in neutron stars, we impose the condition of β equilibrium and charge neutrality through the following constraints, respectively on the chemical potentials and densities of quarks and electrons $\mu_d = \mu_s = \mu_u + \mu_e$ and $\frac{2}{3}\rho_u - \frac{1}{3}(\rho_d + \rho_s) - \rho_e = 0$, with $\rho_i = \frac{1}{\pi^2}(\mu_i^2 - M_i^2)^{3/2}\theta(\mu_i^2 - M_i^2)$ and $\rho_e = \mu_e^3/3\pi^2$.

The nature of the phase transition in NJL models at finite T and/or μ has been discussed by different authors [10,11,14–16]. At zero density the phase transition is a smooth crossover one; at nonzero densities different situations occur. Here we analyze this question by examining the curve of the pressure as a function of density and temperature (Fig. 1), defined as $P(\rho, T) = -[\Omega(\rho, T) - \Omega(0, T)]$, where $\Omega(\rho, T)$ is the thermodynamic potential [11,14,20]. At zero temperature and finite density, as it has been shown in [14,15], there is a region of densities where the pressure is negative, meaning that a phase of low density and broken symmetry coexists with a phase of high density and partially restored chiral symmetry (see Fig. 1). A suitable choice of parameters leads to the first zero of the pressure at $\rho \sim 0$ and the second zero, (at $\rho_c = 2.25$, for case I and case II) corresponds to a minimum of the energy per particle, which means that a stable a hadronic phase exists at these densities, consisting of droplets of quarks surrounded by a non-trivial vacuum. As pointed out in [16] different features occur in the quark phase (for $\rho > \rho_c$) according to we consider or not matter in beta equilibrium. In the last case the mass of the strange quark becomes lower than the chemical potential at densities above $\sim 3.8\,\rho_0$ what implies the occurrence of strange quarks in this regime. As it will be discussed below, this fact leads to meaningful differences in the behavior of η, η'.

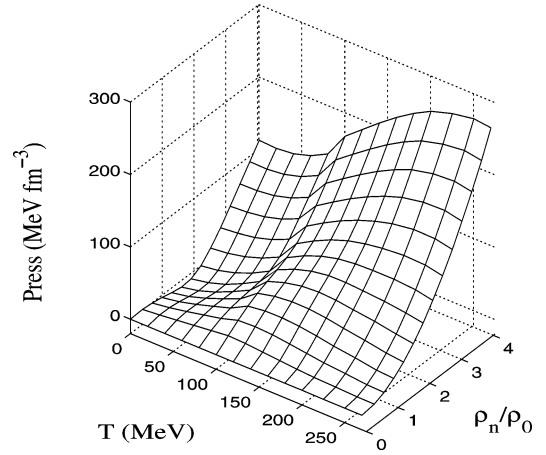Now we discuss the nature of the phase transition at finite density and temperature by studying the pressure

Fig. 1. Pression as function of density and temperature.

in the T–ρ that is shown in Fig. 1. One can see that there is a region of the surface with negative curvature that corresponds to the region of temperatures and densities where the phase transition is first order, since the pressure and/or compressibility are negative. The point $T = 56$ MeV, $\rho_n = 1.53\,\rho_0$, were the pressure is already positive and the compressibility has only one zero, is identified, as usual, as the critical endpoint, that connects the first order and the second order phase transition regions. Above that point, we have a smooth phase transition, corresponding to the regions of positive curvature of the surface.

Unlike at $T = 0$, we cannot talk anymore about a stable hadronic phase in the first order phase transition region because the absolute minimum of the energy per particle is now at zero density. In spite of these drawbacks, we think it is illustrative to plot the masses of π^0, η and η' in the whole region of the ρ–T plane.

Let us now analyze the in medium behavior of the mesons under study. To understand our results, it is useful to study the limits of the Dirac sea (denoted by ω_i) of this mesons. They can be obtained by looking the limits of the regions of poles in the integrals $I_2^{ii}(P_0)$ (see (8)): $\omega_u = 2\mu_u$, $\omega_d = 2\mu_d$ and $\omega_s = 2\mu_s$. (At finite temperature, we will have $\omega_u = 2M_u$, $\omega_d = 2M_d$ and $\omega_s = 2M_s$.)

The behavior of these mesons at $\rho = 0$ and $T \neq 0$ has been study by [9,17]. We reproduce these results for the sake of comparison with our new findings at $T = 0$, $\rho \neq 0$ (see Fig. 2(a)). The mesonic masses
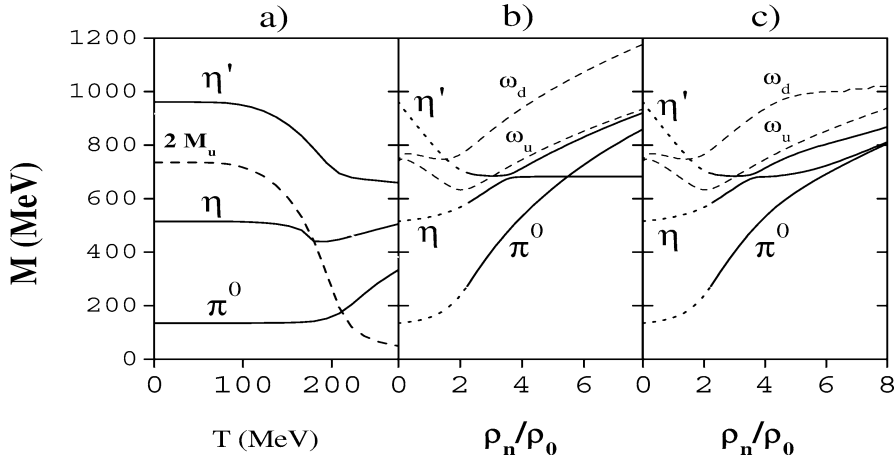
Fig. 2. η, η' and π^0 masses as function of temperature (a) and density: (b) case I; (c) case II.

change smoothly and when they exceed the sum of the masses of the constituent quarks the mesons become unbound (Mott temperature). Usually, the critical temperature is defined as the Mott transition temperature for the pion. The masses of η and η' show a tendency to become degenerated, even after the critical temperature. The decrease of the mass of η' is not generally considered enough to give an indication of restoration of $U_A(1)$ symmetry in hot media [6,7].

Now, we analyze the results for the masses of η and η' at finite densities and zero temperature, that are plotted in Fig. 2(b), (c). Up to the critical density, they exhibit a tendency to became degenerated, but, after that point they split again, the splitting being more pronounced in case I. These results show that if a degeneracy of η, η' is an indication of the restoration of $U_A(1)$ this is unlikely at high densities. In case II the system shows a tendency to the restoration of flavor symmetry, which is related with the presence of strange quarks in the medium that occurs at about $\rho \sim 3.8\,\rho_0$ and, as shown in [16], manifests itself in other observables. As we can see, the η'-meson lies above the quark–antiquark threshold for $\rho_n < 2.5\rho_0$ and it is a resonant state. After that density, the η' becomes a bound state. We checked that the behavior in matter with $\rho_u = \rho_d$, $\rho_s = 0$ is qualitatively similar to case I.

In order to understand the difference between the results in the high density region for cases I and II, it is convenient to study the in medium behavior of the mixing angle, θ, that is plotted in Fig. 3. As it is well known, the quark content of η and η' depend on the

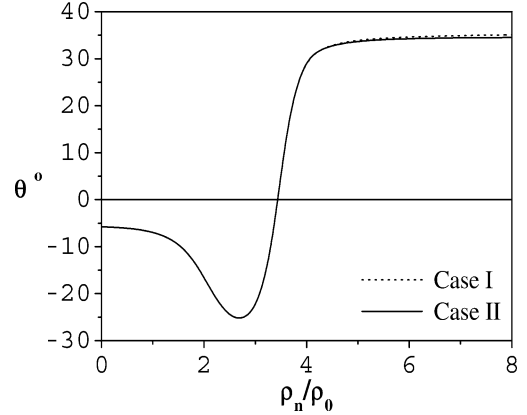
Fig. 3. Mixing angle, θ, as function of density.

mixing angle in the form:

$$|\eta\rangle = \cos\theta \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - 2\bar{s}s\rangle$$
$$- \sin\theta \sqrt{\frac{2}{3}} |\bar{u}u + \bar{d}d + \bar{s}s\rangle, \tag{9}$$

$$|\eta'\rangle = \cos\theta \sqrt{\frac{2}{3}} |\bar{u}u + \bar{d}d + \bar{s}s\rangle$$
$$+ \sin\theta \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - 2\bar{s}s\rangle. \tag{10}$$

Above $\rho \simeq 3.5\,\rho_0$ the angle becomes positive and increases rapidly and the strange quark content, $y = (\bar{u}u + \bar{d}d)/\bar{s}s$, of the mesons changes: at low density, the η' is more strange than the η but the opposite occurs at high density. Since in case I there are no

strange quarks in the medium (what implies that the strange quark mass is almost unaffected), the η mass should stay constant in the region of densities where its content is dominated by the strange quark. This explains the larger splitting between η and η' in case I.

Considering now the finite density and temperature case (we plot the masses of the mesons for the case of quark matter with strange quarks in β equilibrium at finite T in Fig. 4), and similarly to the finite temperature and zero density case, the $\bar{q}q$ threshold for the different mesons is again at the sum of the

constituent quark masses, so the mesons dissociate at densities and temperatures close to the critical ones. A second feature to be noticed is that the mesons that have a Goldstone boson like nature show more clearly the difference between the chiral symmetric and asymmetric phase. The diagram for π^0 shows clearly a "line" that separates the chiral broken phase (region of negative curvature) from the chiral restored phase. This is not so evident for the η. A more smooth behavior is found for η' (Fig. 4), which is natural, since it is not a Goldstone boson associated with chiral symmetry.
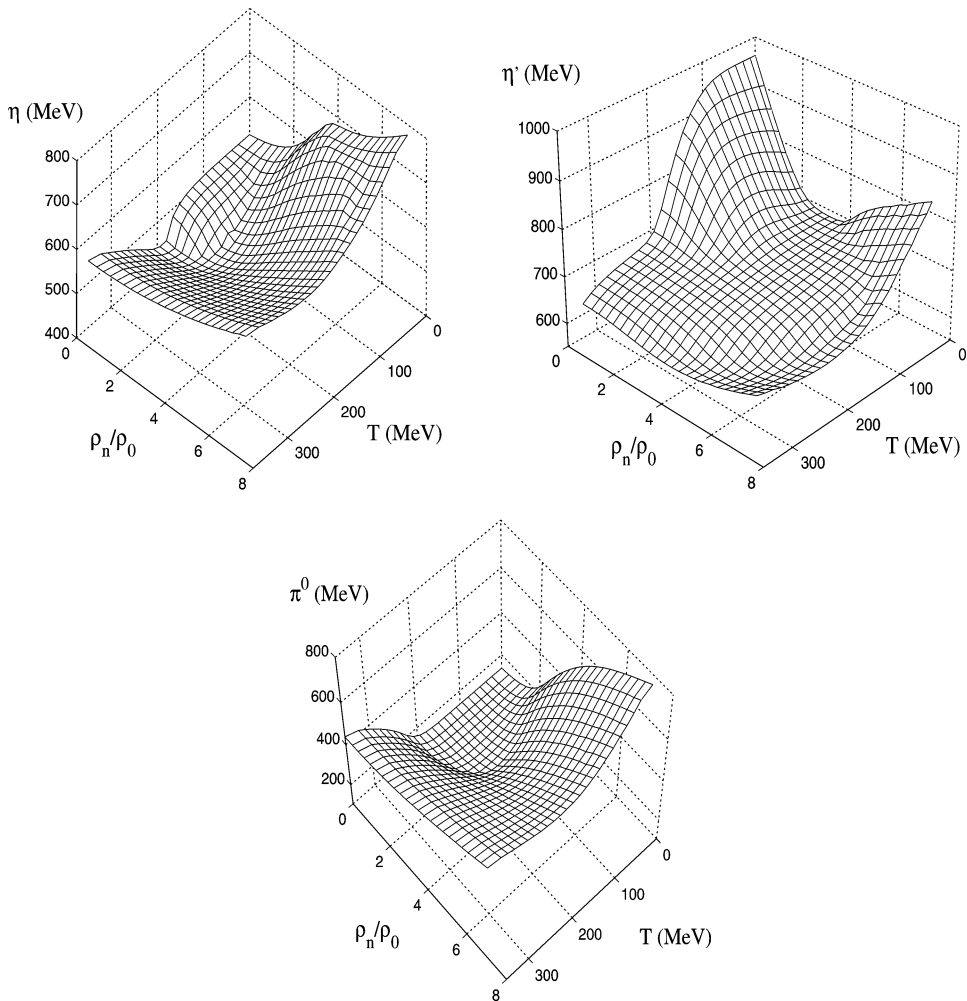

Fig. 4. Mesons masses as functions of temperature and density (case II). Upper panel: η and η' mesons. Lower panel: π^0 meson.

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