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Simple dynamic location problem with uncertainty: a primal-dual heuristic approach

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In this paper the simple dynamic facility location problem is extended to uncertain realizations of the potential locations for facilities and the existence of customers as well as fixed and variable costs. With limited knowledge about the future, a finite and discrete set of scenarios is considered. The decisions to be made are where and when to locate the facilities, and how to assign the existing customers over the whole planning horizon and under each scenario, in order to minimize the expected total costs. Whilst assignment decisions can be scenario dependent, location decisions have to take into account all possible scenarios and cannot be changed according to each scenario in particular. We first propose a mixed linear programming formulation for this problem and then we present a primal-dual heuristic approach to solve it. The heuristic was tested over a set of randomly generated test problems. The computational results are provided.

Keywords: dynamic location problems, primal-dual heuristic, uncertainty, scenarios

AMS Subject Classification: 90C10; 90B80; 90C27; 49M29

1. Introduction

Facility location problems have been widely studied by many researchers, and from the literature it is obvious the diversity of situations considered and the corresponding diversity of models and solution techniques developed, reflecting the importance of such problems [see 7, 24, 32, and references therein]. In simple terms, a location problem can be seen as the problem of efficiently deciding where (and possibly when) to locate facilities that will serve a set of customers.

In this paper we revisit the classical uncapacitated facility location problem (UFLP), also known as the simple plant location problem, proposing a dynamic and uncertain version of this problem as well as a primal-dual heuristic approach inspired on classical works to solve it. The UFLP consists of deciding where to locate a number of facilities among a finite set of potential sites, in order to minimize total costs (fixed facility costs plus variable production costs and transportation costs to customers). Since the facilities are uncapacitated, all demands will be assigned to the nearest open facility and the size of an open facility is computed as the sum of the demands it serves. The UFLP has been extensively studied since [18] and many variations and extensions of this problem along with solution methods

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(mainly heuristic and approximation algorithms) can be found in several books and papers (e.g., [7, 16, 24]). In a dynamic setting, one of the earliest dynamic uncapacitated facility location problem (DUFLP) is proposed in [35]. The authors consider the problem of closing up to a pre-specified number of initially open and operating facilities as demand declines over a given multiperiod planning horizon. It is also presented in this work a branch and bound algorithm and near optimal heuristic algorithms to solve the problem. In [36] the model is generalized to solve a facility phase-in/phase-out problem (i.e., opening new facilities or closing initially opened ones). A related model is proposed in [42] that considers the possibility of removing and establishing facilities in each time period and additional restrictions on the maximum number of facilities to be removed in each period. As solution method the authors propose a dynamic programming approach. In [40] is also considered the DUFLP, where new facilities can be opened and initially opened facilities can be closed over the planning horizon. In this work the authors present a branch-and-bound procedure incorporating a heuristic dual ascent method, the latter initially developed in [5, 11] for the static UFLP. More recently, a new version of the DUFLP is presented in [9] that not only allows for the opening and closing of facilities over the time horizon but also their reopening, where fixed costs include also reopening costs. A primal-dual heuristic is proposed to solve the problem. Regarding the capacitated case, we refer to [12, 14], where not only introductions to such problems are given and additional difficulties that arise in the capacitated case are emphasized, but also earlier models and solution methods are discussed. More recently, models and solution methods for dynamic capacitated problems are suggested in [10, 38], where other references can be found.

As location problems envolve strategic decisions that are costly to revert and that have consequences in the medium and long term, decision makers should consider not only the present situation but also the future. Dynamic models are pursuing this goal as the time dimension and a time horizon are explicitly considered in such models. However, whenever it is necessary to explicitly consider a time horizon, uncertainty appears due to unknown future conditions that may lead to a limited knowledge about problem parameters [28]. If the parameters of dynamic location models change deterministically over time, then it is not possible to incorporate the uncertainty inherent in real-world location problems even though time dimension is explicitly represented. Considering both time and uncertainty in location models allows the consideration of more realistic situations, although the resulting models become more complex than static and deterministic ones. During the last decades there has been considerable interest in location under uncertainty and a large volume of work is now available in specialized papers and monographs. For extensive reviews on location problems under uncertainty we refer to [22, 37]. In [21] a stochastic version of the UFLP is derived, in which demands, variable production and transportation costs, and selling prices (incorporated in the model) can be random. The problem is formulated as a two–stage stochastic program with recourse, where the first–stage decisions are the location and the size (capacity) of the facilities to be established, and the second–stage or recourse decisions are the allocation of the available production to the most profitable demands. As opposed to the deterministic case, the choice of both the demands to be served and the size of the facilities to be established also becomes part of the decision process. Solution methods are later presented in [23]. The authors propose a heuristic dual–based procedure, inspired on the method developed in [11] for the classical (static and deterministic) UFLP. More recently, a two-stage stochastic version of the UFLP and an approximation algorithm to solve it is proposed in [31]. Here, demand and fixed costs are both random, and facilities may be opened in either the first or second stage. A related two-stage stochastic program is proposed in [41] in which service installation costs are also considered. The authors propose a primal-dual approximation algorithm to solve the optimization problem. The UFLP plays a central role in the location research field, not only by itself but also integrated in other problems. In [25] it is proposed a conceptual framework for robust supply chain design under demand uncertainty. The aim is to find a supply chain configuration (or a group of configurations) that provides robust performance under demand uncertainty. Uncertainty of demand is represented by discrete scenarios with known probabilities. First the authors define various performance measures of "robustness" (minimum total expected cost, minimum variance of total cost, minimum of maximum deviation, multiple criteria) emphasizing different perspectives of robust supply chain. As solution methods, the authors discuss explicit enumeration methods and stochastic programming (SP) methods. In the SP approach the problem is formulated as a classic two-stage stochastic program. The objective function is to minimize total expected cost, which includes fixed costs of opening plants and warehouses, expected shipping cost from plants to warehouses and from warehouses to customers, and expected outsourcing cost when customers demands cannot be satisfied from warehouses. The authors discuss the difficulties in using these approaches when the total number of scenarios is large and suggest that this number could be reduced by a sampling based approach. Various stochastic capacitated versions have been also proposed, among which are [19, 20]. Less works on a dynamic framework are known. We refer to the DUFLP under uncertainty proposed in [15], where the fixed and variable costs are described via a set of scenarios. To solve the dynamic and stochastic program, the authors use the scenario and policy aggregation described in [33]. A dynamic capacity acquisition and assignment problem under uncertainty is proposed in [2]. The problem seeks a capacity expansion schedule for a set of resources and the assignment of resource capacity to tasks over the multi-period planning horizon. The problem can be viewed as the planning of locations and capacities of distribution centers (DCs) and the assignment of customers to the DCs. The model explicitly incorporates uncertainty in task processing requirements and assignments costs via a set of scenarios. Although the problem is a multi-period one, the capacity planning decisions for all periods are made in period/stage one (thus, a two-stage stochastic programming approach is adopted). In [34] is considered a dynamic facility location problem with uncertain demand, described by scenarios. The problem seeks the optimal decisions for production, inventory and transportation, to serve the customers during a fixed number of periods. It is assumed that the production sites have limited storage capacities. The model is first solved by dynamic programming and then a heuristic is proposed, the Sample Average Approximation Method (SSA) adapted to the multi-period case. For other multi-period stochastic problems, we refer to [3, 13] and for facility location problems integrated in supply chain to [1, 27, 29, 30].

In this work we consider a dynamic location problem where uncertainty is explicitly incorporated, represented by a finite and discrete set of future scenarios. It is important to point out that the representation of uncertainty in optimization models, applied also to location models, has been widely debated in the literature (e.g., [8, 17, 26, 37, 39]). The scenario approach appears as "an extremely powerful, convenient and natural way to represent uncertainty" [8], especially under high uncertain conditions such as those that may occur during a multi-period location problem and, consequently, the available information may not be sufficient to support a stochastic programming approach. In our model, fixed and assignment costs are scenario dependent, as well as the set of customers and the set of potential locations for facilities. We formulate our problem as an integer linear program, that contains the UFLP and the DUFLP as particular problems (NP-hard problems [6]). We propose a primal–dual heuristic approach directly inspired on the approaches developed in [11] and [40], designed for the static and dynamic versions of the UFLP, respectively.

The remainder of this paper is organized as follows. In the following section, the notation used in this paper is introduced and our problem is described. In section 3 the primal-dual heuristic is described. In section 4 computational experiments with results are provided. Section 5 concludes this paper with some notes on future work.

2. Problem description

Consider a planning horizon represented by a discrete set of time periods $\mathcal{T} = \{1, ..., t, ..., T\}$. The *future* will be one of a finite set of possibilities, represented by a discrete set of *scenarios* $\mathcal{S} = \{1, ..., s, ..., S\}$, where each scenario characterizes the value of all uncertain elements. Suppose that each $s \in \mathcal{S}$ will occur with probability p^s such that $\sum_{s\in\mathcal{S}} p^s = 1$.

Let the set of potential facility sites be denoted by $J = \{1, ..., j, ..., M\}$ and the set of possible customer locations (or demand points) by $I = \{1, ..., i, ..., N\}$. In reality, these sets include all the potential facility locations and all the potential customers for all possible scenarios, despite the fact that for each scenario in particular possibly only a subset of potential locations and a subset of customers is considered. The reason for this is that we consider uncertainty associated not only with the fixed and variable costs, but also associated with the existence of customers and the future existence of potential locations. Let us define δ_{it}^s as equal to 1 if customer i has a demand that has to be fulfilled during period t for scenario s, and 0 otherwise. Then we have to guarantee that all customers such that $\delta_{it}^s = 1$ are assigned to an open facility, for all $(t, s) \in \mathcal{T} \times \mathcal{S}$.

In terms of costs, the model considers not only fixed costs (opening and operating), but also variable costs associated with the assignment of customers to the facilities. For $(j, t, s) \in J \times \mathcal{T} \times \mathcal{S}$, let f_{jt}^s be the fixed cost of establishing (opening) facility j at the beginning of period t plus the operating and subsequent costs in period t, under scenario s; for $(i, j, t, s) \in I \times J \times \mathcal{T} \times \mathcal{S}$, c_{ijt}^s represents the assignment cost of customer i to facility j in period t and under scenario s. If it is not possible to open facility j at the beginning of time period t, under scenario s, then the corresponding fixed cost will be considered equal to $+\infty$. We assume that once a facility is opened, it stays open until the end of the planning horizon.

The decisions to be made are where and when to locate new facilities, and how to assign the existing customers over the whole planning horizon and under each scenario. Thus, we define the following binary decision variables: x_{jt} equals 1 if facility j is opened at the beginning of period t, and 0 otherwise; y_{ijt}^s equals 1 if customer i is assigned to facility j in period t and under scenario s, and 0 otherwise. As a matter of fact, assignment decisions are considered to be taken a period at a time, so they can be changed according to the scenario that came true. Location decisions are hard to revert, so we have to live with the decision taken whatever the scenario that came to occur. Our aim is to make the best location decisions, considering the uncertainty associated with the future. Several different objective functions could be considered, but in this paper we consider the minimization of expected total costs.

We can formulate the problem as follows:

$$\min \quad \sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{s\in\mathcal{S}} p^s f^s_{jt}\, x_{jt} + \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\sum_{i\in I}\sum_{j\in J} p^s c^s_{ijt}\, y^s_{ijt} \tag{1a}$$

subject to

$$\sum_{j\in J} y^s_{ijt} = \delta^s_{it} \qquad \forall i\in I, t\in\mathcal{T}, s\in\mathcal{S}, \tag{1b}$$

$$\sum_{\tau=1}^{t} x_{j\tau} - y^s_{ijt} \geq 0 \qquad \forall i\in I, j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \tag{1c}$$

$$\sum_{t\in\mathcal{T}}(-x_{jt}) \geq -1 \qquad \forall j\in J, \tag{1d}$$

$$x_{jt}\in\{0,1\} \qquad \forall j\in J, t\in\mathcal{T}, \tag{1e}$$

$$y^s_{ijt}\in\{0,1\} \qquad \forall i\in I, j\in J, t\in\mathcal{T}, s\in\mathcal{S}. \tag{1f}$$

The objective function (1a) minimizes the expected total costs (fixed plus variable costs). Constraints (1b) require that, under each scenario and in every time period, an existing customer is assigned to exactly one facility. Constraints (1c) impose that an existing customer can only be assigned to open facilities. A customer can be assigned to different facilities at different time periods and different scenarios. Constraints (1d) ensure that each facility is opened at most once during the time horizon (located at the same site in all scenarios). Finally, (1e)–(1f) restrict the decision variables to be binary.

The above formulation contains the UFLP ($|\mathcal{T}| = |\mathcal{S}| = 1$) and the DUFLP ($|\mathcal{T}| > 1, |\mathcal{S}| = 1$) as particular problems, and has $|J|\,|\mathcal{T}| + |J|\,|I|\,|\mathcal{T}|\,|\mathcal{S}|$ binary variables and $|I|\,|\mathcal{T}|\,|\mathcal{S}| + |J|\,|I|\,|\mathcal{T}|\,|\mathcal{S}| + |J|$ restrictions (not counting the zero-one constraints). Even for moderate dimensions of these sets, (1a)–(1f) becomes a quite large integer linear program.

3. Heuristic approach

We propose a primal-dual heuristic based on the approaches developed in [11, 40]. The main idea of the approach is to obtain good solutions from the dual problem of the corresponding LP relaxation of the original problem, more precisely from the so called condensed dual problem. The various procedures are designed to reduce the duality gap between dual and primal function values. The dual ascent procedure constructs a dual solution and an associated set of candidate facility locations. The primal procedure yields a corresponding candidate primal solution. If the dual and primal solutions satisfy all complementary slackness conditions, then the solutions are optimal. If not, the heuristic continues with the adjustment procedures in order to improve these solutions.

In order to describe the heuristic, we begin by formulating the dual problem, the condensed dual problem and the complementary slackness conditions between the dual and primal problems.

3.1. *Dual problem and complementary slackness conditions*

Consider the LP relaxation of the primal problem defined by (1a)–(1d) and where restrictions (1e) and (1f) are replaced by nonnegativity constraints. Defining in (1a) $\mathcal{C}_{ijt}^s = p^s c_{ijt}^s$ and $\mathcal{F}_{jt}^s = p^s f_{jt}^s$, and considering dual variables v_{it}^s, w_{ijt}^s and u_j associated with the restrictions (1b), (1c) and (1d), respectively, the dual problem is given by:

$$\max \quad \sum_{i\in I}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \delta_{it}^s v_{it}^s - \sum_{j\in J} u_j \tag{2a}$$

subject to

$$v_{it}^s - w_{ijt}^s \le \mathcal{C}_{ijt}^s \qquad \forall i\in I, j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \tag{2b}$$

$$\sum_{i\in I}\sum_{s\in\mathcal{S}}\sum_{\tau=t}^{T} w_{ij\tau}^s - u_j \le \sum_{s\in\mathcal{S}} \mathcal{F}_{jt}^s \qquad \forall j\in J, t\in\mathcal{T}, \tag{2c}$$

$$w_{ijt}^s \ge 0 \qquad \forall i\in I, j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \tag{2d}$$

$$u_j \ge 0 \qquad \forall j\in J. \tag{2e}$$

For each (i,j,t,s), by constraints (2b) and (2d), we may set

$$w_{ijt}^s = \max\{0, v_{it}^s - \mathcal{C}_{ijt}^s\} \qquad \forall i,j,t,s, \tag{3}$$

to obtain the condensed dual problem:

$$\max \quad \sum_{i\in I}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \delta_{it}^s v_{it}^s - \sum_{j\in J} u_j \tag{4a}$$

subject to

$$\sum_{i\in I}\sum_{s\in\mathcal{S}}\sum_{\tau=t}^{T} \max\{0, v_{i\tau}^s - \mathcal{C}_{ij\tau}^s\} - u_j \le \sum_{s\in\mathcal{S}} \mathcal{F}_{jt}^s \qquad \forall j,t, \tag{4b}$$

$$u_j \ge 0 \qquad \forall j. \tag{4c}$$

The corresponding slack variables π_{jt} for constraints (4b) are given by:

$$\pi_{jt} = \sum_{s\in\mathcal{S}} \mathcal{F}^s_{jt} - \sum_{i\in I}\sum_{s\in\mathcal{S}}\sum_{\tau=t}^{T} \max\{0, v^s_{i\tau} - \mathcal{C}^s_{ij\tau}\} + u_j \qquad \forall j,t. \tag{5}$$

Then, the complementary slackness conditions are:

$$\pi_{jt}\; x_{jt} = 0 \qquad \forall j,t, \tag{6a}$$

$$v^s_{it}\left(\sum_j y^s_{ijt} - \delta^s_{it}\right) = 0 \qquad \forall i,t,s, \tag{6b}$$

$$w^s_{ijt}\left(\sum_{\tau=1}^{t} x_{j\tau} - y^s_{ijt}\right) = 0 \qquad \forall i,j,t,s, \tag{6c}$$

$$u_j\left(1 - \sum_t x_{jt}\right) = 0 \qquad \forall j, \tag{6d}$$

$$y^s_{ijt}\;\left(v^s_{it} - \mathcal{C}^s_{ijt} - w^s_{ijt}\right) = 0 \qquad \forall i,j,t,s. \tag{6e}$$

3.2. ***Primal-Dual heuristic***

For ease in the exposition, let us reindex, for each scenario s, $\mathcal{C}^s_{ijt}$ for each (i,t) in nondecreasing order as $\mathcal{C}^{s(k)}_{it}$, for $k = 1,2,...,k^s_{it}$, where k^s_{it} denotes the number of facility-to-customer links for (i,t) under scenario s. Thus, $\mathcal{C}^{s(1)}_{it} = \min_{j\in J}\{\mathcal{C}^s_{ijt}\}$. For convenience, we also include $\mathcal{C}^{s(k^s_{it}+1)}_{it} = +\infty, \forall\ (i,t,s)$.

Let I^+ be the set of *pseudo customers* (i,t,s) corresponding to the dual variables v^s_{it} that the procedure will try to increase. Initially, I^+ will be equal to all possible combinations $(i,t,s) \in I\times\mathcal{T}\times\mathcal{S}$, except those such that $\delta^s_{it} = 0$. Later, I^+ will be set within the respective procedures. We note that a customer without demand does not contribute to the improvement of the dual objective function value and does not also contribute to any violation of the complementary slackness conditions. Thus, these customers are excluded from the ascent procedures.

The steps of the heuristic are as follows:

(1) Set $v^s_{it} = \mathcal{C}^{s(1)}_{it}$, $\forall\ (i,t,s)$, and $u_j = 0, \forall\ j$.
Set $I^+ = \{(i,t,s) \in I\times\mathcal{T}\times\mathcal{S} : \delta^s_{it} = 1\}$.
(2) Execute the dual ascent procedure.
(3) Execute the primal procedure. If an optimal solution is found, then stop.
(4) Execute the primal–dual adjustment procedure.

The heuristic stops when the optimal solution is found or when there are no primal or dual improvements after a given number of trials within the adjustment procedure.

3.2.1. Dual ascent procedure

This procedure, that may start with any feasible solution, will try to increase the values of variables v_{it}^s belonging to set I^+. The increase of such variables will lead to an increase of the dual objective function value and, simultaneously, to the decrease of some slacks' values (see step 6). The maximum value that variables v_{it}^s can take is limited by restrictions (4b). Equivalently, we can also consider slacks defined by (5) and acknowledge that these slacks have to remain nonnegative. Instead of increasing the value of each dual variable v_{it}^s as much as possible in one single step, the procedure follows an iterative approach: in each iteration, the algorithm will try to increase a dual variable v_{it}^s to the smallest $\mathcal{C}_{ijt}^s$ that is greater than or equal to the current v_{it}^s value. If this is not possible, due to the fact that at least one slack would become negative, than the variable is increased as much as possible guaranteeing that all slacks remain nonnegative (steps 4, 5 and 6). The procedure is repeated until it is not possible to increase the value of any variable v_{it}^s because of the slacks that are already equal to zero: if any variable was further increased, there was at least one slack that would have to take a negative value. The slacks that are equal to zero will define the set of candidate facility locations.

In what follows, $(i,t,s)_q$, with $q \leq |I \times \mathcal{T} \times \mathcal{S}|$, represents a given, but arbitrary, sequence of pseudo customers.

(1) Consider any dual feasible solution $\{v_{it}^s\}$ such that $v_{it}^s \geq \mathcal{C}_{it}^{s(1)}, \forall\ (i,t,s)$, and $\pi_{jt} \geq 0, \forall\ (j,t)$.
For each (i,t,s) define $k(i,t,s) = \min\{k : v_{it}^s \leq \mathcal{C}_{it}^{s(k)}\}$. If $v_{it}^s = \mathcal{C}_{it}^{s(k(i,t,s))}$, then $k(i,t,s) \leftarrow k(i,t,s)+1$.
(2) $(i,t,s) \leftarrow (i,t,s)_1$ and $q \leftarrow 1$; $r = 0$.
(3) If $(i,t,s) \notin I^+ \vee \delta_{it}^s = 0$, then go to step 7.
(4) Set $\Delta_{it}^s = \min_j\{\pi_{j\tau} : v_{it}^s - \mathcal{C}_{ijt}^s \geq 0, \tau \leq t\}$.
(5) If $\Delta_{it}^s > \mathcal{C}_{it}^{s(k(i,t,s))} - v_{it}^s$, then $\Delta_{it}^s = \mathcal{C}_{it}^{s(k(i,t,s))} - v_{it}^s$; $r = 1$; $k(i,t,s) \leftarrow k(i,t,s)+1$.
(6) For all $j \in J$ with $v_{it}^s - \mathcal{C}_{ijt}^s \geq 0$, set $\pi_{j\tau} = \pi_{j\tau} - \Delta_{it}^s, \tau \leq t$; set $v_{it}^s = v_{it}^s + \Delta_{it}^s$.
(7) If $q < |I^+|$, then $q \leftarrow q+1$, $(i,t,s) \leftarrow (i,t,s)_q$, and return to step 3.
(8) If $r = 1$, then return to step 2, otherwise stop.

3.2.2. Primal procedure

From the dual ascent procedure results the dual feasible solution $\{v_{it}^{s+}\}$ with an objective function value v_D^+, and associated slacks $\{\pi_{jt}^+\}$. A corresponding primal feasible solution, $\{x_{jt}^+\}$ and $\{y_{ijt}^{s+}\}$, can be constructed, with an objective function value v_P^+.

In order to describe the primal procedure, let us first define the following sets:
$J^* = \{(j,t) \in J \times \mathcal{T} : \pi_{jt}^+ = 0\}$;
$J_t^* = \{j \in J : (j,\tau) \in J^*, \tau \leq t\}$, $\forall t \in \mathcal{T}$;
$J_t^+ = \{j \in J : \text{facility } j \text{ is open at time } t\}$, $\forall t \in \mathcal{T}$.
In addition, define $t_1(j) = \min\{\gamma : j \in J_\gamma^+\}$ and $t_2(j) = \max\{\gamma \leq t_1(j) : (j,\gamma) \in J^*\}$. Then,
$J^+ = \{(j,t_2(j)) \in J \times \mathcal{T} : j \in J_\tau^+ \text{ for some } \tau\}$.

The set J^* corresponds to all (j,t) such that j can be opened at the beginning of t without violating (6a); set J_t^* corresponds to all j that can be opened up to t; set J_t^+ corresponds to all j that are actually open during t; set $J^+ \subseteq J^*$ corresponds to all j that open at the beginning of t, i.e., J^+ dictates what facilities are actually opened and when (location decisions).

The facilities that are considered first (step 2) are the ones that at a given time t should be assigned to a given customer (i, s), according to conditions (6c), called *essential* facilities. Other facilities are only opened if strictly necessary (step 3). If a facility j needs to be open at some time period(s) and the first time period when it needs to be open is t, then it will be opened at the beginning of time period $t_2(j)$, defined as being the time period closest to t such that the corresponding slack is equal to zero. It should be noted that, as we are dealing with an uncapacitated location problem, there will always be an admissible solution that can be built in this way: we can be sure that there exists at least one facility j such that π_{j1} is equal to zero (at least one facility can be opened at the beginning of the first time period). If this was not true, then it would still be possible to improve the dual solution by increasing at least one v^s_{i1} dual variable.

The steps of the primal procedure are as follows:

(1) Set $J^+ = J^+_t = \emptyset$, $\forall t$. Build J^* and J^*_t, $\forall t$.
(2) For each $t \in \mathcal{T}$, if $j \in J^*_t$ such that $\exists (i, s) : v^{s+}_{it} \geq \mathcal{C}^s_{ijt}$ and $v^{s+}_{it} < \mathcal{C}^s_{ij't}, \forall j' \in J^*_t \backslash \{j\}$, then $J^+_\tau = J^+_\tau \cup \{j\}$, $\forall \tau \geq t$.
(3) For each (i, t, s), if $\nexists j \in J^+_t$ with $v^{s+}_{it} \geq \mathcal{C}^s_{ijt}$, then $J^+_\tau = J^+_\tau \cup \Big\{ j \in J^*_t : \mathcal{C}^s_{ijt} = \min\{\mathcal{C}^s_{ij't} : v^s_{it} \geq \mathcal{C}^s_{ij't}\} \Big\}$, $\forall \tau \geq t$.
(4) Build J^+.
(5) Update J^+_t, $\forall t$. Assign each (i, t, s) to facility $j \in J^+_t$ with lowest $\mathcal{C}^s_{ijt}$.

3.2.3. *Primal–Dual adjustment procedure*

The primal–dual adjustment procedure will try to enforce the conditions (6c) that are still being violated by the current solution. The violation of these conditions means that, for a given scenario s, time period t and customer i, there are at least two variables w^s_{ijt} different from zero such that the corresponding facilities j are both open in period t. The only way of satisfying (6c) would be to assign customer i to more than one opened facility, which is not admissible from the primal problem point of view. This procedure will try to change the current dual solution, by decreasing the value of at least one variable v^s_{it} (and thus decreasing the value of the corresponding w^s_{ijt}). At least two slacks will be increased with this operation, that may lead to the increase of other dual variables and to a better solution.

In order to describe the primal–dual procedure, let us first consider the additional sets:

$J^{s*}_{it} = \{j : \exists \tau \leq t \mid (j, \tau) \in J^* \text{ and } v^s_{it} \geq \mathcal{C}^s_{ijt}\}$, $\forall (i, t, s)$;
$J^{s+}_{it} = \{j : \exists \tau \leq t \mid (j, \tau) \in J^+ \text{ and } v^s_{it} > \mathcal{C}^s_{ijt}\}$, $\forall (i, t, s)$;
$I^+_{jt} = \{(i, \tau, s) : J^{s*}_{i\tau} = \{j\} \text{ for } \tau \geq t\}$, $\forall (j, t)$.

In addition, we denote a best source and a second-best source for (i, t, s) in J^+_t by $j(i, t, s)$ and $j'(i, t, s)$, respectively:

$\mathcal{C}^s_{ij(i,t,s)t} = \min_{j \in J^+_t} \{\mathcal{C}^s_{ijt}\}, \forall (i, t, s)$;
$\mathcal{C}^s_{ij'(i,t,s)t} = \min_{j \in J^+_t, j \neq j(i,t,s)} \{\mathcal{C}^s_{ijt}\}, \forall (i, t, s)$ for $|J^{s+}_{it}| > 1$.

And we define, $\mathcal{C}^{s-}_{it} = \max_j \{\mathcal{C}^s_{ijt} : v^s_{it} > \mathcal{C}^s_{ijt}\}$.

For a given (i, t, s), the set J^{s*}_{it} represents all facilities j that can be open at period t (because a slack $\pi_{j\tau}$ is equal to zero for some $\tau \leq t$) and such that if j is open then customer i can be assigned to j at period t under scenario s. Similarly, for a given (i, t, s), the set J^{s+}_{it} considers all facilities that are in operation during period t in the current primal solution, and such that customer i would have to be assigned to j in period t under scenario s to guarantee the satisfaction of (6c).

Table 1. Parameters used in the random generation of the test problems.

S	2	5	10	20
T	5	10	15	—
M	5	10	20	50
N	20	50	100	200

If $|J_{it}^{s+}| > 1$, for some (i,t,s), then a complementary slackness condition (6c) is violated. In such case, the decrease of the variable v_{it}^s causes the increase of at least two slacks $\pi_{j\tau}$, associated with distinct facilities (step 4). Set I_{jt}^+ corresponds to all variables $v_{i\tau}^s$ whose value can be increased with the increase of slacks $\pi_{j\tau}, \tau \leq t$, and that must be constructed to the execution of the dual ascent procedure (step 5).

The steps of the primal-dual adjustment are:

(1) $(i,t,s) \leftarrow (i,t,s)_1, q \leftarrow 1$; set $v_D = v_D^+$ and $v_P = v_P^+$; set $r = 0$.
(2) If $|J_{it}^{s+}| \leq 1$, then go to step 9.
(3) If $I_{j(i,t,s)t}^+ = \emptyset$ and $I_{j'(i,t,s)t}^+ = \emptyset$, then go to step 9.
(4) For each (j,τ), with $\tau \leq t$ and $v_{it}^s > \mathcal{C}_{ijt}^s$, set $\pi_{j\tau} = \pi_{j\tau} + v_{it}^s - \mathcal{C}_{it}^{s\,-}$; set $v_{it}^s = \mathcal{C}_{it}^{s\,-}$.
(5) a) Set $I^+ = I_{j(i,t,s)t}^+ \cup I_{j'(i,t,s)t}^+$ and execute the dual ascent procedure.
b) Set $I^+ = I^+ \cup \{(i,t,s)\}$ and execute the dual ascent procedure.
c) Set $I^+ = I \times \mathcal{T} \times \mathcal{S}$ and execute the dual ascent procedure.
(6) If v_{it}^s is changed, then return to step 2.
(7) Execute the primal procedure.
(8) If neither $v_D^+ > v_D$ nor $v_P^+ < v_P$, then $r \leftarrow r+1$; otherwise $r \leftarrow 0$ and update v_D and v_P.
(9) If $v_D \geq v_P$, or $r = r_{max}$ or $q = |I \times \mathcal{T} \times \mathcal{S}|$, then stop; otherwise $q \leftarrow q+1, (i,t,s) \leftarrow (i,t,s)_q$, and return to step 2.

4. Computational experiments

This section is devoted to the presentation and discussion of the computational experiences carried out to evaluate the performance of the heuristic both in terms of solution quality and time. As we are not aware of the existence of benchmark problem instances that could be easily adapted to the presented model, we have chosen to randomly generate different problem instances, by varying the number S of scenarios, number T of time periods, number M of possible facility locations and number N of possible customers according to Table 1. As we are in the presence of a dynamic problem under uncertainty, data must change simultaneously over time and among the different scenarios. For each combination of (S,T,M,N), with $N > M$, five instances were randomly generated according to the procedure that is described in Appendix A. Different random seeds were used for each of the instances. We have, in total, 780 instances, that were solved by the heuristic and by a general solver (LpSolve). We decided to stop the solver if its solution time exceeded 7200 seconds (s). We note that the smallest instance considered has 1025 variables with 1205 constraints but the largest has 3000750 variables with 3060050 constraints.

Data for all test problems are available from the authors. The proposed heuristic approach was coded in C–language and LPSolve v5.5.2.0 [4] was used as LP solver[1]. The computational experiences have been carried out on a AMD Turion(tm) X2 Dual–Core Mobile RM–70 processor at 2.00 GHz with 3.00 GB of RAM.

In tables 2–5 we summarize the computational results obtained. Each table corresponds to a given number of scenarios. We report the minimum and maximum number of opened facilities (dimension of the set J^+) as well as the minimum, average and maximum gap on the five instances solved for each combination of (T, M, N). Gap is given by $(f_P - f_{LB})/f_{LB}$, in percentage, where f_P represents the primal objective function value found by the heuristic and f_{LB} is the best known lower bound on the optimal value, which is equal to the optimum solution provided by the solver (for problems the solver was able to solve), or is equal to the dual objective function value found by our heuristic. The primal–dual heuristic was able to solve all the 780 instances. The following tables also show the solution times (in seconds) of the heuristic and the solver. We report the minimum, average and maximum time spent by our algorithm and by the solver to solve each group of five instances. We note that time results do not include the time required to read the problems, only the time to solve them. As far as the solver results are concerned, the solver could not solve some of the five instances, due to lack of memory to read the problem or the execution time has exceeded 7200 s. We report these cases and statistics refer only to those instances that were solved. Whenever the solver was not able to solve any of the five instances, the solver time is given as '*'. Only on the larger instances, with $(S, T, M, N) = (20, 15, 50, 200)$, the heuristic exceeded the time limit established à priori. In terms of solution quality, the worst gap, 4.02%, was observed with instances with 20 scenarios and with $T = 15$, $M = 50$ and $N = 100$. Within each S-scenario problems, in average, the larger gaps were observed in instances with largest M and N.

The average results for all S–scenario problems are reported in the last row of the corresponding tables. We can see that the number of scenarios considered do not result in markedly different solution qualities. However, the execution times required by the solver are clearly higher than those required by the heuristic, especially for large sized problems. In most of the test problems with large dimensions the solver could not solve them in less than 7200 s. The heuristic time can vary a lot, even for problems with the same size. For example, for instances with $(S, T, M, N) = (10, 15, 20, 200)$ the execution time ranges from 0.28 to 1231.29 s, in average 508.18 seconds.

The computational results show that the heuristic is capable of finding very good quality solutions in reasonable computational times, clearly outperforming the general solver.

As it is well known, when solving integer programming problems general solvers tend to reach a good admissible (sometimes optimal) solutions fast, and then spend a lot of time trying to improve this solution or proving that the solution is optimal. So comparing the computational time of a dedicated heuristic to that of a general solver can be seen as unfair to the general solver. That is why we have repeated all the computational tests but now using the general solver as an heuristic procedure: for each set of instances, we have limited the maximum computational time spent by the general solver considering this maximum time equal to the maximum time spent by the heuristic and then compare the quality of the solutions found by the two approaches. When this time limit was considered, and for all test problems, the solver was not able to find any admissible solution (upper and lower bounds of

[1] mantemos isto? Há um que não gosta do LP solver, mas é...optimizer?

Table 2. Computational results for 2–scenario problems.

T	M	N	$\|J^+\|$		gap (%)			Heur. time			Solver time		
			min	max	min	aver	max	min	aver	max	min	aver	max
5	5	20	2	4	0.00	0.00	0.00	0.00	0.01	0.02	0.08	0.11	0.16
5	5	50	2	4	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.64	0.83
5	5	100	4	5	0.00	0.00	0.00	0.00	0.00	0.00	2.23	2.64	3.32
5	5	200	5	5	0.00	0.00	0.00	0.00	0.04	0.09	7.85	8.62	9.91
5	10	20	3	4	0.00	0.11	0.44	0.00	0.02	0.06	0.19	0.38	0.53
5	10	50	5	7	0.00	0.13	0.36	0.00	0.06	0.17	1.48	2.33	3.42
5	10	100	5	7	0.00	0.03	0.14	0.00	0.08	0.27	5.51	7.36	8.81
5	10	200	7	9	0.00	0.00	0.00	0.00	0.37	1.81	18.24	25.13	31.51
5	20	50	5	9	0.00	0.41	1.52	0.05	0.13	0.30	4.56	6.57	9.66
5	20	100	8	10	0.00	0.05	0.15	0.03	0.83	1.51	20.65	23.25	27.02
5	20	200	10	13	0.00	0.01	0.04	0.02	3.24	12.29	74.54	101.52	121.56
5	50	100	13	16	0.19	0.64	1.85	0.48	3.23	5.13	75.04	169.58	264.31
5	50	200	18	22	0.11	0.33	0.67	6.29	13.41	19.44	391.73	471.97	620.62
10	5	20	3	4	0.00	0.00	0.00	0.00	0.00	0.02	0.30	0.37	0.45
10	5	50	4	5	0.00	0.00	0.00	0.00	0.00	0.02	1.79	2.43	3.03
10	5	100	5	5	0.00	0.00	0.00	0.00	0.03	0.11	8.14	8.53	9.24
10	5	200	5	5	0.00	0.00	0.00	0.02	0.03	0.08	24.16	31.05	43.01
10	10	20	3	6	0.00	0.03	0.12	0.00	0.01	0.02	0.86	1.34	1.89
10	10	50	6	8	0.00	0.00	0.00	0.00	0.01	0.03	4.62	5.61	7.27
10	10	100	7	10	0.00	0.00	0.00	0.00	0.01	0.02	16.91	19.99	21.40
10	10	200	9	10	0.00	0.00	0.00	0.00	0.02	0.08	72.24	87.83	109.93
10	20	50	8	12	0.00	0.06	0.32	0.08	0.58	1.25	13.43	23.38	33.29
10	20	100	11	15	0.00	0.04	0.20	0.11	0.98	2.26	71.04	82.84	101.03
10	20	200	16	19	0.00	0.01	0.06	0.09	1.72	6.77	233.77	270.59	361.19
10	50	100	19	23	0.37	1.08	2.39	1.95	6.33	11.25	398.89	546.24	746.12
10	50	200	26	30	0.19	0.35	0.61	40.17	52.61	90.46	1510.53	1737.18	1880.55
15	5	20	3	5	0.00	0.00	0.00	0.00	0.01	0.06	0.70	0.91	1.28
15	5	50	4	5	0.00	0.00	0.00	0.00	0.00	0.00	4.09	5.09	6.19
15	5	100	5	5	0.00	0.00	0.00	0.00	0.04	0.09	16.65	19.49	22.07
15	5	200	5	5	0.00	0.00	0.00	0.00	0.34	1.64	71.09	80.89	91.23
15	10	20	4	7	0.00	0.00	0.00	0.00	0.00	0.02	1.72	2.70	3.67
15	10	50	7	9	0.00	0.00	0.00	0.00	0.01	0.03	11.00	12.75	14.56
15	10	100	8	10	0.00	0.00	0.00	0.00	0.03	0.11	37.30	49.85	67.16
15	10	200	10	10	0.00	0.00	0.00	0.02	0.02	0.02	155.06	215.19	247.49
15	20	50	9	12	0.00	0.03	0.13	0.31	1.02	1.97	47.00	54.76	71.79
15	20	100	14	16	0.00	0.07	0.21	0.02	1.62	7.47	114.54	168.61	217.79
15	20	200	17	20	0.00	0.00	0.00	0.27	1.16	3.23	620.72	696.22	878.47
15	50	100	23	28	0.35	0.76	1.31	2.39	5.40	9.50	1064.62	1768.31	2946.97
15	50	200[a]	32	37	0.00	0.52	2.28	58.62	106.15	210.16	2699.81	3370.90	3957.47
		Aver			**0.03**	**0.12**	**0.33**	**2.84**	**5.12**	**9.94**	**200.09**	**258.54**	**331.95**

[a] Solver was unable to solve one of the instances with $T = 15$, $M = 50$ and $N = 200$.

the primal objective function value were equal to '$+\infty$' and '$-\infty$', respectively). It should be noted that the minimum times presented by the solver (see tables 2–5) are greater than the maximum times spent by the heuristic to compute the solution for the same problems.

5. Conclusions and future work

In this paper we have described an uncapacitated discrete dynamic location problem that considers uncertainty in many of the problems' parameters. The uncertainty is represented by a set of possible future scenarios. An efficient primal-dual heuristic was developed that is able to calculate very good quality solutions in reasonable computational times, even for large dimension instances. In this model, an objective function that considers the minimization of the expected cost is being considered. In fact, if the decision maker is considered as being risk neutral, this objective function value will probably be the most adequate one. But when we are dealing with uncertainty, different decision makers with different risk profiles will probably consider different solutions as optimal: they can also be interested in minimizing the maximum regret, or considering the best solution in the worst

Table 3. Computational results for 5–scenario problems.

T	M	N	$\lvert J^+\rvert$		gap (%)			Heur. time			Solver time		
			min	max	min	aver	max	min	aver	max	min	aver	max
5	5	20	1	2	0.00	0.00	0.00	0.00	0.00	0.02	0.47	0.51	0.56
5	5	50	2	3	0.00	0.00	0.00	0.00	0.01	0.02	3.42	4.29	5.54
5	5	100	4	4	0.00	0.00	0.00	0.00	0.01	0.02	10.48	14.21	18.70
5	5	200	4	5	0.00	0.00	0.00	0.00	0.01	0.03	38.05	51.95	61.87
5	10	20	2	3	0.00	0.00	0.00	0.00	0.02	0.06	1.50	2.06	3.42
5	10	50	3	5	0.00	0.07	0.36	0.00	0.19	0.55	8.80	11.93	17.44
5	10	100	5	7	0.00	0.00	0.00	0.00	2.63	10.19	35.65	41.94	53.42
5	10	200	7	8	0.00	0.00	0.00	0.00	0.24	0.66	138.92	176.28	204.44
5	20	50	5	6	0.00	0.39	1.41	0.08	0.74	1.89	33.79	51.27	66.67
5	20	100	7	8	0.00	0.19	0.56	0.02	5.38	10.78	93.54	184.33	240.07
5	20	200	9	13	0.00	0.08	0.26	2.14	34.26	52.57	602.52	840.71	1084.33
5	50	100	10	12	0.00	0.15	0.49	4.57	14.99	23.76	687.40	984.75	1292.27
5	50	200	14	18	0.16	0.24	0.34	49.97	94.49	188.82	3258.87	4243.81	5243.82
10	5	20	2	4	0.00	0.06	0.29	0.00	0.04	0.20	2.26	2.50	3.00
10	5	50	4	5	0.00	0.06	0.31	0.00	0.14	0.50	10.95	15.89	21.92
10	5	100	4	5	0.00	0.00	0.00	0.02	0.02	0.03	44.06	46.75	51.28
10	5	200	5	5	0.00	0.00	0.00	0.02	0.03	0.05	201.49	226.94	273.97
10	10	20	3	4	0.00	0.29	1.46	0.00	0.31	1.11	6.68	9.70	11.59
10	10	50	4	7	0.00	0.10	0.33	0.00	0.86	3.48	36.16	51.28	65.13
10	10	100	7	8	0.00	0.00	0.00	0.02	0.17	0.56	154.46	185.67	238.81
10	10	200	9	10	0.00	0.00	0.00	0.03	0.04	0.05	364.87	566.97	853.41
10	20	50	7	9	0.00	0.25	0.57	1.45	4.93	8.81	128.76	205.82	276.32
10	20	100	8	13	0.00	0.02	0.12	0.27	9.71	27.44	489.92	688.11	914.27
10	20	200	13	18	0.00	0.01	0.01	2.18	19.64	68.11	1766.34	2640.57	3348.20
10	50	100	15	19	0.30	0.74	1.34	11.22	50.01	82.74	3048.36	4795.48	7152.59
10	50	200	20	24	0.83	1.05	1.40	210.62	344.60	432.31	*	*	*
15	5	20	2	4	0.00	0.00	0.00	0.00	0.01	0.02	5.65	5.99	6.13
15	5	50	4	5	0.00	0.00	0.00	0.02	0.02	0.03	29.97	33.62	41.12
15	5	100	4	5	0.00	0.00	0.00	0.03	0.07	0.19	107.89	126.81	140.43
15	5	200	5	5	0.00	0.00	0.00	0.05	0.06	0.09	493.69	554.62	653.95
15	10	20	3	5	0.00	0.00	0.00	0.22	0.68	1.95	15.91	18.10	20.97
15	10	50	6	7	0.00	0.00	0.00	0.00	0.04	0.11	96.13	124.75	148.18
15	10	100	8	9	0.00	0.01	0.04	0.03	1.97	8.94	444.77	489.07	561.88
15	10	200	10	10	0.00	0.00	0.00	0.06	0.32	1.36	1187.18	1471.82	1701.38
15	20	50	7	9	0.00	0.11	0.39	2.81	10.82	25.55	316.88	353.42	404.52
15	20	100	9	15	0.00	0.13	0.41	4.99	23.75	48.55	1043.98	1300.18	1491.25
15	20	200	14	18	0.00	0.01	0.03	1.75	53.01	156.41	4576.93	5245.83	6506.93
15	50	100[a]	17	24	0.68	1.47	2.72	23.43	60.40	120.53	6564.31	6882.34	7200.37
15	50	200	24	30	0.42	1.30	1.87	20.58	338.01	639.04	*	*	*
		Aver			**0.06**	**0.17**	**0.38**	**8.63**	**27.50**	**49.17**	**690.82**	**882.44**	**1091.36**

[a] Solver was unable to solve three of the instances with $T = 15$, $M = 50$ and $N = 100$.

scenario. This means that other objective functions should be considered, that can better represent the attitude towards risk of different decision-makers. This will certainly be one of the developments to be followed. We also intend to consider capacity constraints, as well as considering the possibility of closing already opened facilities, to increase the range of applicability of this model.

References

[1] E. Aghezzaf, *Capacity planning and warehouse location in supply chains with uncertain demands*, Journal of the Operational Research Society 56 (2005), pp. 453–462.

[2] S. Ahmed and R. Garcia, *Dynamic capacity acquisition and assignment under uncertainty*, Annals of Operations Research 124 (2004), pp. 267–283.

[3] M. Albareda-Sambola, A. Alonso-Ayuso, L.F. Escudero, E. Fernández, and C. Pizarro, *On solving the multi-period location-assignment problem under uncertainty*, Tech. Rep. WP09/07, Dpto. de Estadística e Investigación Operativa, Universidad Rey Juan Carlos, Spain, 2009.

[4] M. Berkelaar, K. Eikland, and P. Notebaert, *Open source (mixed–integer) linear programming system* http://lpsolve.sourceforge.net/5.5/.

[5] O. Bilde and J. Krarup, *Sharp lower bounds and efficient algorithms for the simple plant location problem*, Annals of Discrete Mathematics 1 (1977), pp. 79–97.

[6] G. Cornuejols, G.L. Nemhauser, and L.A. Wolsey, *The uncapacitated facility location problem*, in *Discrete Location Theory*, P.B. Mirchandani and R.L. Francis, eds., John Wiley&Sons, 1990, pp. 119–171.

Table 4. Computational results for 10–scenario problems.

T	M	N	$\|J^+\|$		gap (%)			Heur. time			Solver time		
			min	max	min	aver	max	min	aver	max	min	aver	max
5	5	20	2	3	0.00	0.07	0.36	0.00	0.22	0.53	2.89	3.19	3.84
5	5	50	3	3	0.00	0.00	0.00	0.00	0.12	0.58	12.62	17.11	23.32
5	5	100	4	4	0.00	0.00	0.00	0.00	0.01	0.02	58.38	65.14	73.29
5	5	200	4	5	0.00	0.00	0.00	0.02	0.03	0.03	207.31	230.73	243.10
5	10	20	1	4	0.00	0.02	0.11	0.00	0.36	0.62	6.96	9.89	12.84
5	10	50	3	5	0.00	0.03	0.14	0.00	1.93	6.94	45.43	66.04	109.22
5	10	100	4	5	0.00	0.00	0.00	0.09	5.44	23.95	148.47	255.05	351.09
5	10	200	6	8	0.00	0.02	0.10	0.05	4.33	10.64	795.18	1038.60	1442.13
5	20	50	4	6	0.00	0.14	0.46	1.89	5.10	8.74	155.02	226.56	356.30
5	20	100	6	8	0.00	0.25	1.27	2.40	8.23	17.85	541.68	796.70	909.26
5	20	200	8	12	0.00	0.03	0.10	26.57	164.40	341.20	2121.63	2988.89	4074.88
5	50	100	8	12	0.00	0.17	0.57	34.94	79.29	121.93	3436.65	4215.17	5468.21
5	50	200	14	19	1.22	2.07	3.25	418.86	634.12	946.89	*	*	*
10	5	20	2	3	0.00	0.00	0.00	0.00	0.01	0.02	7.89	11.65	16.07
10	5	50	3	5	0.00	0.00	0.00	0.02	0.03	0.08	50.67	65.89	95.52
10	5	100	4	5	0.00	0.00	0.00	0.03	0.04	0.05	197.23	221.54	247.67
10	5	200	5	5	0.00	0.00	0.00	0.06	0.09	0.14	810.00	868.02	992.83
10	10	20	2	4	0.00	0.00	0.00	0.02	0.32	1.25	27.33	32.99	43.54
10	10	50	4	6	0.00	0.06	0.30	0.03	9.70	34.91	182.36	223.63	320.38
10	10	100	6	8	0.00	0.00	0.00	0.06	1.24	3.78	696.07	877.82	961.08
10	10	200	8	10	0.00	0.00	0.00	0.11	0.12	0.13	2308.36	2687.90	3046.52
10	20	50	6	9	0.00	0.33	0.97	6.51	27.74	49.41	584.74	954.05	1261.76
10	20	100	9	11	0.00	0.19	0.67	7.22	73.58	205.44	2551.49	3135.98	3598.62
10	20	200	13	15	0.00	0.04	0.12	1.79	243.75	460.86	*	*	*
10	50	100	13	17	0.62	1.66	2.27	73.26	225.40	334.34	*	*	*
10	50	200	18	25	0.50	1.23	2.30	1091.86	1871.35	2703.61	*	*	*
15	5	20	3	4	0.00	0.00	0.00	0.02	1.33	6.54	21.09	30.78	38.45
15	5	50	4	5	0.00	0.00	0.00	0.05	0.07	0.13	149.46	168.39	188.90
15	5	100	4	5	0.00	0.00	0.00	0.08	0.20	0.41	545.28	595.69	690.44
15	5	200	5	5	0.00	0.00	0.00	0.14	0.44	1.47	1953.11	2107.77	2261.94
15	10	20	3	5	0.00	0.07	0.37	0.28	1.62	4.68	65.30	88.62	126.95
15	10	50	4	7	0.00	0.00	0.00	0.05	1.25	3.65	447.81	497.29	550.57
15	10	100	7	9	0.00	0.00	0.00	0.11	10.99	41.96	1472.00	1997.70	2838.22
15	10	200[a]	9	10	0.00	0.00	0.00	0.17	0.21	0.23	5932.49	6218.71	6353.24
15	20	50	6	8	0.00	0.18	0.35	0.78	17.82	40.72	1374.77	1757.89	2792.24
15	20	100[b]	8	12	0.00	0.23	0.69	8.19	70.07	115.46	4948.38	5251.97	5518.58
15	20	200	14	18	0.00	0.02	0.05	0.28	508.18	1231.29	*	*	*
15	50	100	17	23	1.14	1.95	2.85	187.43	427.05	785.32	*	*	*
15	50	200	22	28	0.41	1.23	1.91	526.03	1771.76	3170.56	*	*	*
		Aver			**0.10**	**0.26**	**0.49**	**61.27**	**158.15**	**273.75**	**995.56**	**1178.35**	**1406.59**

[a] Solver was unable to solve one of the instances with $T = 15$, $M = 10$ and $N = 200$.

[b] Solver was unable to solve two of the instances with $T = 15$, $M = 20$ and $N = 100$.

[7] M.S. Daskin, *Network and Discrete Location: Models, Algorithms and Applications*, John Wiley&Sons, 1995.

[8] R.S. Dembo, *Scenario optimization*, Annals of Operations Research 30 (1991), pp. 63–80.

[9] J. Dias, M.E. Captivo, and J. Clímaco, *Efficient primal-dual heuristic for a dynamic location problem*, Computers & Operations Research 34 (2007), pp. 1800–1823.

[10] J. Dias, M.E. Captivo, and J. Clímaco, *A dynamic location problem with maximum decreasing capacities*, Central European Journal of Operations Research 16 (2008), pp. 251–280.

[11] D. Erlenkotter, *A dual-based procedure for uncapacitated facility location*, Operations Research 26 (1978), pp. 992–1009.

[12] D. Erlenkotter, *A comparative study of approaches to dynamic location problems*, European Journal of Operational Research 6 (1981), pp. 133–143.

[13] P. Hernández, A. Alonso–Ayuso, F. Bravo, L.F. Escudero, M. Guignard, V. Marianov, and A. Weintraub, *A branch-and-cluster coordination scheme for selecting prison facility sites under uncertainty*, Computers & Operations Research 39 (2012), pp. 2232–2241.

[14] S.K. Jacobsen, *Multiperiod capacitated location models*, in *Discrete Location Theory*, P.B. Mirchandani and R.L. Francis, eds., John Wiley&Sons, 1990, pp. 173–208.

[15] K. Jornsten and M. Bjorndal, *Dynamic location under uncertainty*, Studies in Regional and Urban Planning 3 (1994), pp. 163–183.

[16] B. Korte and J. Vygen, *Facility location*, in *Combinatorial Optimization*, B. Korte and J. Vygen, eds., Vol. 21, Springer Berlin Heidelberg, 2012, pp. 593–628.

[17] P. Kouvelis and G. Yu, *Robust Discrete Optimization and its Applications*, Kluwer Academic Publishers, 1997.

[18] A.A. Kuehn and M.J. Hamburger, *A heuristic program for locating warehouses*, Management Science 9 (1963), pp. 643–666.

[19] G. Laporte, F.V. Louveaux, and L.V. Hamme, *Exact solution to a location problem with stochastic*

Table 5. Computational results for 20–scenario problems.

T	M	N	$\lvert J^+\rvert$		gap (%)			Heur. time			Solver time		
			min	max	min	aver	max	min	aver	max	min	aver	max
5	5	20	1	3	0.00	0.01	0.07	0.00	0.13	0.61	9.42	12.28	17.64
5	5	50	3	4	0.00	0.00	0.00	0.02	0.71	3.45	61.84	70.29	95.61
5	5	100	3	4	0.00	0.00	0.00	0.03	30.86	154.19	222.89	286.07	381.81
5	5	200	4	5	0.00	0.00	0.00	0.06	0.08	0.09	1001.93	1109.75	1302.44
5	10	20	2	4	0.00	0.22	1.08	0.02	2.53	4.96	37.78	45.14	57.60
5	10	50	3	4	0.00	0.08	0.39	1.26	14.10	23.99	257.40	291.35	309.33
5	10	100	5	6	0.00	0.00	0.00	3.09	71.90	245.59	877.80	1069.26	1361.01
5	10	200	6	8	0.00	0.03	0.15	0.09	145.20	638.04	2729.54	3547.23	4662.37
5	20	50	4	6	0.00	0.13	0.63	0.36	21.11	44.29	499.04	993.34	1600.09
5	20	100	6	8	0.00	0.19	0.82	19.64	101.85	222.91	2429.08	3547.42	4711.87
5	20	200	9	13	0.01	0.57	1.47	47.05	621.55	1342.97	*	*	*
5	50	100	8	14	1.28	2.01	2.75	202.60	310.82	359.07	*	*	*
5	50	200	16	20	1.85	2.47	3.33	383.79	1812.58	2803.42	*	*	*
10	5	20	2	3	0.00	0.00	0.00	0.03	0.55	1.78	34.05	47.44	67.78
10	5	50	3	4	0.00	0.00	0.00	0.06	0.07	0.09	256.78	266.85	277.40
10	5	100	4	5	0.00	0.00	0.00	0.13	0.15	0.19	1023.39	1138.34	1499.16
10	5	200	5	5	0.00	0.00	0.00	0.23	0.27	0.33	3540.65	3699.76	4046.42
10	10	20	2	4	0.00	0.23	1.16	0.05	2.61	6.44	128.87	157.65	201.02
10	10	50	4	5	0.00	0.00	0.00	0.08	23.42	45.74	791.93	949.63	1187.52
10	10	100	6	7	0.00	0.00	0.00	0.16	8.25	38.05	2891.10	3888.13	4534.98
10	10	200	9	10	0.00	0.00	0.00	0.30	1.62	3.76	*	*	*
10	20	50	5	8	0.00	0.12	0.56	34.94	140.44	225.34	2789.70	3035.98	3677.72
10	20	100	8	11	0.02	0.43	1.03	53.57	193.76	409.70	*	*	*
10	20	200	13	14	0.01	0.16	0.38	689.88	1786.88	3325.52	*	*	*
10	50	100	13	16	0.68	1.82	3.55	215.51	748.85	1289.93	*	*	*
10	50	200	18	21	0.62	1.10	2.25	1860.63	3639.29	4646.76	*	*	*
15	5	20	2	3	0.00	0.00	0.00	0.06	9.97	49.55	107.58	123.71	157.44
15	5	50	3	5	0.00	0.00	0.00	0.14	0.17	0.19	537.65	648.65	858.02
15	5	100	4	5	0.00	0.00	0.00	0.28	0.42	0.86	2195.15	2440.06	2643.19
15	5	200	5	5	0.00	0.00	0.00	0.50	0.57	0.70	*	*	*
15	10	20	2	5	0.00	0.14	0.72	0.06	12.31	43.73	296.65	414.25	614.06
15	10	50	5	6	0.00	0.00	0.00	0.17	79.82	297.60	1564.88	2319.46	2902.85
15	10	100	7	10	0.00	0.02	0.10	0.34	8.53	37.82	*	*	*
15	10	200	9	10	0.00	0.00	0.00	0.61	0.65	0.73	*	*	*
15	20	50	6	9	0.05	0.70	2.32	49.64	198.52	353.08	*	*	*
15	20	100	11	13	0.02	0.34	0.49	109.61	435.24	641.11	*	*	*
15	20	200	10	15	0.00	0.08	0.32	403.70	2561.07	5787.24	*	*	*
15	50	100	16	18	0.96	2.49	4.02	414.34	1973.06	3138.74	*	*	*
15	50	200	22	26	1.35	1.89	3.01	6442.44	13133.70	16098.22	*	*	*
		Aver			**0.18**	**0.39**	**0.78**	**280.40**	**720.35**	**1084.28**	**1055.87**	**1308.78**	**1615.97**

demands, Transportation Science 28 (1994), pp. 95–103.

[20] C.K.Y. Lin, *Stochastic single-source capacitated facility location model with service level requirements*, International Journal of Production Economics 117 (2009), pp. 439–451.

[21] F.V. Louveaux, *Discrete stochastic location models*, Annals of Operations Research 6 (1986), pp. 23–34.

[22] F.V. Louveaux, *Stochastic location analysis*, Location Science 1 (1993), pp. 127–154.

[23] F.V. Louveaux and D. Peeters, *A dual-based procedure for stochastic facility location*, Operations Research 40 (1992), pp. 564–573.

[24] P.B. Mirchandani and R.L. Francis, *Discrete Location Theory*, John Wiley&Sons, 1990.

[25] Y. Mo and T.P. Harrison, *A conceptual framework for robust supply chain design under demand uncertainty*, Supply Chain Optimization (2005), pp. 243–263.

[26] J.M. Mulvey, R.J. Vanderbei, and S.A. Zenios, *Robust optimization of large-scale systems*, Operations Research 43 (1995), pp. 264–280.

[27] S. Nickel, F. Saldanha-da-Gama, and H. Ziegler, *A multi-stage stochastic supply network design problem with financial decisions and risk management*, Omega 40 (2012), pp. 511–524.

[28] S.H. Owen and M.S. Daskin, *Strategic facility location: A review*, European Journal of Operational Research 111 (1998), pp. 423–447.

[29] F. Pan and R. Nagi, *Robust supply chain design under uncertain demand in agile manufacturing*, Computers & Operations Research 37 (2010), pp. 668–683.

[30] B.S. Pimentel, G.R. Mateus, and F.A. Almeida, *Stochastic capacity planning in a global mining supply chain*, Computational Intelligence in Production And Logistics Systems (CIPLS), 2011 IEEE Workshop On (2011), pp. 1–8.

[31] R. Ravi and A. Sinha, *Hedging uncertainty: approximation algorithms for stochastic optimization problems*, Lecture Notes in Computer Science 3064 (2004), pp. 101–115.

[32] C. Revelle, H. Eiselt, and M. Daskin, *A bibliography for some fundamental problem categories in discrete location science*, European Journal of Operational Research 184 (2008), pp. 817–484.

[33] R.T. Rockafellar and R.J.B. Wets, *Scenarios and policy aggregation in optimization under uncertainty*, Mathematics of Operations Research 16 (1991), pp. 119–147.

[34] M. Romauch and R.F. Hartl, *Dynamic facility location with stochastic demands*, Lecture Notes in Computer Science 3777 (2005), pp. 180–189.
[35] G.M. Roodman and L.B. Schwarz, *Optimal and heuristic facility phase-out strategies*, AIIE Transactions 7 (1975), pp. 177–184.
[36] G.M. Roodman and L.B. Schwarz, *Extensions of the multi-period facility phase-out model: new procedures and application to a phase-in/phase-out problem.*, AIIE Transactions 9 (1977), pp. 103–107.
[37] L.V. Snyder, *Facility location under uncertainty: a review*, IIE Transactions 38 (2006), pp. 537–554.
[38] J.E. Torres-Soto and H. Uster, *Dynamic-demand capacitated facility location problems with or without relocations*, International Journal of Production Research 1 (2010), pp. 1–27.
[39] K. Van der Heijden, *Scenarios: The art of Strategic Conversation*, John Wiley&Sons, 1996.
[40] T.J. Van Roy and D. Erlenkotter, *A dual-based procedure for dynamic facility location*, Management Science 28 (1982), pp. 1091–1105.
[41] X. Wang, D. Xu, and X. Zhao, *A primal-dual approximation algorithm for stochastic facility location problem with service installation costs*, Frontiers of Mathematics in China 6 (2011), pp. 957–964.
[42] G.O. Wesolowsky and W.G. Truscott, *The multiperiod location-allocation problem with relocation of facilities*, Management Science 22 (1975), pp. 57–65.

Appendix A. Generation of test problems

Below we provide the approach used in the generation of test problems (in general). As far as scenario probabilities (p^s) are concerned, these were randomly generated such that the sum of all probabilities is equal to 1. Table A1 presents some input values that were considered and that must be known before the generation procedure.

For ease in the exposition, let us first consider the following additional notation:
J_t^s : Set of potencial facility locations that can be selected (opened) at the beginning of time period $t \in \mathcal{T}$ for scenario $s \in \mathcal{S}$,
I_t^s: Set of customer locations with demand during period $t \in \mathcal{T}$ for scenario $s \in \mathcal{S}$,
where $J_t^s \subseteq J$ and $I_t^s \subseteq I$.

(1) Random generation of $(x, y)-$coordinates in a rectangular area of size $MaxX \times MaxY$ corresponding to the location of $|J| + |I|$ nodes (potencial facility sites plus possible customer locations).
(2) Random generation of arcs between the network nodes with probability p_{arc}; afterwards, if there isn't an arc between two nodes "close" (the Euclidean distance between them is less than d), an arc is created between them with probability $p_{arcc} > p_{arc}$.
(3) For $s = 1$ (basic scenario):
   3.1 for $t = 1$: random generation of costs associated with arcs, according to a Uniform distribution $\mathcal{U}[lc, uc]$;
   for each $t \geq 2$, each arc cost is equal to the cost in period $t-1$ plus a changing factor randomly generated.
   3.2 for each $t \geq 1$:
      i. calculation of the shortest path between each possible customer location and each potential facility location—assignment costs—using the Floyd-Warshall algorithm.
      ii. random generation of set J_t^1, with $J_1^1 \neq \emptyset$, and fixed costs:
      each location j is included in J_t^1 with probability p_f^1;
         – if $j \in J_t^1$, then the fixed cost at j is randomly generated from a Uniform distribution $\mathcal{U}[lf, uf]$, and for each $\tau > t$ the fixed cost is increased by a changing factor randomly generated;
         – if $j \notin J_t^1$, then the fixed cost at j is set to $+\infty$.
      iii. random generation of set I_t^1: each customer i is included in I_t^1 with probability p_c^1; in addition, for $t \geq 3$, if i was included in I_{t-2}^1 and excluded from I_{t-1}^1, then i is included in I_t^1 with probability

Table A1. Input values.

p_{arc}	0.75
d	50
p_{arcc}	0.80
p^s_f	0.80 for $s = 1$ and 0.5 $\forall s \neq 1$
p^s_c	0.80 for $s = 1$ and 0.3 $\forall s \neq 1$
p_c	0.10
p^s_a	0.40
p^s_{cf}	0.60

$p_c < 0.5$.

(4) For $s \neq 1$ (other scenarios):

4.1 for $t = 1$, consider the data generated for the basic scenario and $t = 1$.

4.1 for each $t \geq 2$:

i. each arc cost that was generated for time period t of the basic scenario (basic cost) changes in time period t of scenario s with probability p^s_a; if a variation occurs, then the arc cost is equal to the basic cost plus a changing factor Θ_a randomly generated.

ii. calculation of the shortest path between each possible customer location and each potential facility location.

iii. random generation of set J^s_t and fixed costs:

each location j is included in J^s_t with probability p^s_f;

- if $j \in J^s_t \cap J^1_t$, then the fixed cost at j that was generated for time period t of the basic scenario (basic cost) changes in time period t of scenario s with probability p^s_{cf}; if a variation occurs, then the fixed cost is equal to the basic cost plus a changing factor Θ_f randomly generated;
- if $j \in J^s_t$ but $j \notin J^1_t$, then the fixed cost at j is randomly generated from a Uniform distribution $\mathcal{U}[lf, uf]$, and for each $\tau > t$ the fixed cost is increased by a changing factor randomly generated;
- if $j \notin J^s_t$, then fixed cost at j is set to $+\infty$.

iv. random generation of set I^s_t: the demand state of customer i that was generated for time period t of the basic scenario changes in time period t of scenario s with probability p^s_c.