Simple procedures of choice in multicriteria problems without precise information about the alternatives' values

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Abstract

The additive model of multiattribute value theory is widely used in multicriteria choice problems. However, often it is not easy to obtain precise values for the scaling weights or the alternatives' value in each function. Several decision rules have been proposed to select an alternative under these circumstances, which require weaker information, such as ordinal information. We propose new decision rules and test them using Monte-Carlo simulation, considering that there exists ordinal information both on the scaling weights and on the alternatives' values. Results show the new rules constitute a good approximation. We provide guidelines about how to use these rules in a context of selecting a subset of the most promising alternatives, considering the contradictory objectives of keeping a low number of alternatives yet not excluding the best one.

Key words: Multi-Criteria Decision Analysis, MAUT / MAVT, imprecise / incomplete / partial information, ordinal information, simulation.

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1 Introduction

There exist many approaches to rank a set of decision alternatives or to select the best(s) one(s) taking into account multiple criteria (e.g., see [5,16] for recent comprehensive reviews of multicriteria evaluation approaches). In this paper we address multiattribute utility theory (MAUT) / multiattribute value theory (MAVT) [23], a popular type of approach that yields a global value assessment for each alternative. According to this technique, it is necessary to begin by building a value function for each criterion, which expresses on a cardinal scale the value associated with each level of the scale used to measure performance in that criterion. In the case of the utility function it is also possible to model different attitudes towards risk. The value function may be increasing or decreasing as the level increases (e.g., decreasing in the case of a cost). The most popular model for aggregating multiple value functions is the additive model: under some assumptions [23], the overall value of an alternative is the sum of value functions (one for each evaluation criterion), each of them weighted by a scale coefficient. We will refer to these scaling coefficients simply as "weights", although noting that they do not reflect directly the intuitive notion of importance of each criterion, as they are contingent on the range for which the value function was defined. This is one of the most well-known methods among practitioners and researchers, it is simple to understand, and its theoretical properties are well studied (e.g., see [23,44,45]).

Usually it is assumed that the exact values for the parameters of multiattribute evaluation models are known or can be elicited from a decision maker. However, in many cases, this assumption is unrealistic or, at least, there are advantages in working with less precise information. Several reasons justify why a decision maker might prefer to provide incomplete information [12,13,25,46]: the decision might need to be taken under pressure of time and lack of data; the decision maker might not feel confident in providing precise values for intangible or non-monetary parameters (e.g., parameters reflecting environmental impacts); the decision maker might have limited capacity to process information; the decision maker might not want to reveal his preferences in public or might not want to set his preferences (as they could change during the process); the evaluation of the alternatives in some criteria might result from inaccurate statistics or measurements; the decision maker might consider it is difficult to translate qualitative judgements into precise numerical values; the performance of some alternatives might depend on variables whose value is not known at the time of analysis; the information that would set the value of some parameters might be incomplete, not credible, contradictory or controversial. Some of these factors could be reduced expending time, discussions or money, but the decision maker may want to avoid incurring in these costs. Working with models which require less effort from the decision makers is a way of fostering the adoption of formal methods for decision aiding. Namely, the decision maker might indicate only qualitative or ordinal information, instead of providing exact values for all parameters.

The concern of working with incomplete information arises, naturally, in the context of the use of multiattribute value (or utility) functions. Most of the proposed methods deal with imprecision on weights, considering the value of each alternative in each criterion is precisely known (e.g., [2,13,20,40,42,46]). There are also methods that address imprecision on performance values (e.g., [21]), or are able to deal with imprecision on weights and on performance values simultaneously (e.g., [32,35,39,47]).

The work presented in this paper is motivated by the difficulty of eliciting a precise value for each alternative in each criterion. It addresses imprecision on performance values, both with and without imprecision on weights at the same time. Eliciting incomplete information about weights and about the value of each alternative in each criterion, although not precise, might be sufficient to increase the knowledge of the decision maker about the issue under analysis, leading to the identification of the most promising alternatives.

A research question that arises in this context is to know how good the proposed rules to select an alternative in the context of lack of precise information are, compared with an ideal situation in which the value of all parameters of the model is known. Usually this is studied using Monte-Carlo simulations: generating randomly a large number of problems (criteria weights and value of each alternative in each criterion), determining the alternative with highest multiattribute value, and comparing this alternative with the alternative chosen by the rule being studied, which uses only part of the information. As examples of such comparisons we can cite [4,40,41,42]. However, these references consider that only the weights are unknown, and it is important to extend this idea to the case where the values of the alternatives in each criterion are also unknown.

This paper presents new rules and simulation studies comparing different rules for choice when information about the weight of the criteria and about the value of each alternative in each criterion is imprecise. We consider that the available information has an ordinal nature, which Larichev et al. [28] considered can be more confidently elicited. For example, the decision maker can indicate that one alternative has higher value than another alternative in one criterion, without quantifying how much. The decision rules we will compare are based on the concept of Rank Order Centroid ([4,43]), which has been found to perform very well when there is ordinal information on the weights and known cardinal information for the alternatives' values [4,42]. The main purpose of our comparisons is then to assess how much the quality of the results degrades when we consider ordinal information on the alternatives' values.

Unlike most previous research, we will not only focus on the best alternative when comparing rules. Rather than using a rule to identify a single alternative, our aim is to test how the rules behave in a strategy of progressive reduction of the number of alternatives [13]. Our aim is to test rules as screening procedures that identify a subset of promising alternatives, trying to conciliate the contradictory objectives of maintaining a minimum number of alternatives while ensuring that the chosen subset contains the best alternative. These experiments are designed to be comparable with previous studies. Hence, we test similar problem dimensions.

In the next section we will present some of the existing approaches in the literature to deal with the use of ordinal information and other types of incomplete information. The rules tested are presented in detail in section 3, which also introduces the mathematical notation. In section 4 the conducted simulations are described, and the corresponding results are presented in section 5. Section 6 presents some conclusions and some ideas for future research.

2 A review

There are many methods that allow working with ordinal information, see for example [7]. The decision maker may indicate that a criterion is more important than some of the others, or that an alternative has better performance than another in a certain criterion, but not quantifying how much. This concern arises not only in MAVT methods, but also in methods based on different principles. For instance, in the context of outranking methods Bisdorff [6] extended the principle of concordance to the context of ordinal information about criteria weights. The methods QUALIFLEX [34], and ORESTE [37] also allow considering rankings in several criteria and a ranking of the relative importance of these criteria.

Other ordinal information methods not based on the idea of a multiattribute value function are Verbal Decision Analysis (VDA) [33], the TOMASO method [31], and distance-based approaches, to cite rather diverse examples. VDA methods (ZAPROS [26] for ranking problems, ORCLASS [27] for classification problems) are designed for problems with a large number of alternatives and a small number of criteria, making very few assumptions about the way the decision maker aggregates preferences. The TOMASO method can also be used for sorting or ranking alternatives evaluated on ordinal scales, based on Choquet integrals. Distance-based approaches attempt to find a ranking that is as close as possible (according to some distance) to a set of rankings provided as an input. As examples we can cite [10,11,18].

The indirect elicitation of preferences is used in the paradigm of ordinal regression. According to this paradigm, initially information regarding holistic preferences concerning a set of reference alternatives is obtained and then the parameters for the model that maximize compatibility with this information are inferred. The inferred parameter values are then used to rank the alternatives. In the context of MAVT, this class of methods includes UTA [22], MACBETH [3], and GRIP [17]. Rather than inferring precise parameter values, VIP Analysis [13] can use ordinal information to infer constraints on MAVT weights, and then finds the set of conclusions that is compatible with these constraints (robust conclusions). The SMAA method [24] takes the reverse perspective by finding parameter values compatible with potential results, e.g., allowing to find out what type of parameter values makes an alternative the best one. The SMAA-O [25] is a variant of SMAA for problems in which criteria are measured in ordinal scales.

To reconstitute the judgement of a decision maker concerning some alternatives provided as examples it is not necessary to infer numerical constraints or values. Greco et al. [19] presented the Decision Rule Approach, in which preferences are shaped in terms of "if ..., then ..." rules, based on the Dominance-based Rough Set Approach. Also based on the concept of dominance, Iyer [21] explored the idea of extending dominance-based decision-making to problems with noisy evaluations. The author's idea was to eliminate alternatives which are dominated by any other alternative according to the multi-criteria evaluations, without assuming the aggregation method was known.

Much work has been developed for the case of MAVT/MAUT with incomplete information, which includes ordinal information as a particular case. Sage and White [39] proposed the model of imprecisely specified multiattribute utility theory (ISMAUT), in which precise preference information about both weights and utilities is not assumed. Malakooti [30] suggested a new algorithm for ranking and screening alternatives when there exists incomplete information about the preferences and the value of the alternatives. An extended version of Malakooti's work was presented by Ahn [1]. Park, Kim, and colleagues [15,29,35] provided linear programming characterizations of dominance and potential optimality for decision alternatives when information about values and/or weights is not complete, extended the approach to hierarchical structures [29], and developed the concepts of weak potential optimality and strong potential optimality [35]. White and Holloway [47] considered an interactive selection process: a facilitator asks a decision maker questions and obtains responses that will be used to decide on the next question, aiming to eventually identify the most preferred alternative.

Finally, some methods follow decision rules to rank alternatives under incomplete information on the weights. Barron and Barret [4] studied algebraic formulas such as equal weights and the use of ROC (rank order centroid) weights to select a representative weights vector w from a set of admissible weights W, with the purpose of using w to evaluate the alternatives. These authors concluded that ROC weights provide a better approximation than the other weighting vectors they considered. Another type of rules that have been proposed imply solving optimization problems [40]: the maximin rule consists in evaluating each alternative for its minimum (worst case) value; the minimax regret rule consists in evaluating each alternative for the maximum loss of value relatively to a better alternative (the "maximum regret"); the central values rule consists in evaluating each alternative for the midpoint of the range of possible values. Although none of these rules ensures that the alternative indicated as being the best one is the same that would result if precise values for weights were elicited, simulations show that in general the alternative selected is among the best ones (e.g., [42]).

The work in this paper belongs to this last group of approaches of using rules based on information easy to elicit. Our objective is to rank the alternatives, or to select one alternative, without requiring precise information from the decision maker. We will propose two new rules, based on the ideas of the ROC weights rule, to deal with incomplete information in the value of each alternative in each criterion.

3 Notation and decision rules

3.1 Notation

Let us denote by $A = \{a_1, ..., a_m\}$ a discrete set of m alternatives. Let $X = \{x_1, ..., x_n\}$ denote a set of n criteria (attributes) for evaluating these alternatives. Let $v_i(.)$ denote the value function (or utility function - the difference here is not important) corresponding to criterion x_i. Consequently, $v_i(a_j) \in [0, 1]$ denotes the value of the alternative a_j according to criterion x_i.

According to the additive aggregation model, the global (multi-attribute) value of an alternative $a_j \in A$ is:

$$v(a_j) = \sum_{i=1}^{n} w_i v_i(a_j) \tag{1}$$

where w_i represents the scale coefficient or "weight" associated with v_i. For these parameters we have:

$$w_1, ..., w_n \geq 0 \text{ and } \sum_{i=1}^{n} w_i = 1 \quad (2)$$

Without loss of generality we will consider that criteria weights are indexed by descending order, given ordinal information provided by a decision maker, for example, comparing "swings" from the worst to the best performance in each value function [14,44] (we assume the worst level corresponds to value 0 and the best level corresponds to value 1). Thus, the set of all vectors of weights compatible with this information is:

$$W = \{(w_1, w_2, ...w_n) : w_1 \geq w_2 \geq ... \geq w_n \geq 0, \sum_{i=1}^{n} w_i = 1\} \quad (3)$$

Let V denote the set of the $n \times m$ matrices, having as elements all values $v_i(a_j)$ $(i = 1, ..., n;\ j = 1, ..., m)$ compatible with ordinal information provided by the decision maker. We will consider that we have a ranking of the value of each alternative in each criterion, and also possibly a ranking of the difference of values between consecutive alternatives in each criterion.

3.2 Decision rules

3.2.1 Ordinal information on the weights

Since criteria weights are usually the parameters that are harder to elicit [38], research efforts have mostly focussed on the case in which incomplete information refers only to the weights. Past studies have verified that some decision rules based on ordinal information about the weights lead to good results [2,40,41,42]. In [42], among other experiments, a set of Monte-Carlo simulations was carried out in order to see how different rules (ROC weights, maximin rule, minimax regret rule and central values rule) compared on different indicators. The rules were compared, for example, in terms of their "hit rate", which indicates the proportion of times in which the alternative chosen with a vector of supposedly true weights (i.e., the vector that would be obtained if precise weights were elicited) coincides with the alternative indicated by the rule. The results indicate that, given a ranking of the weights, the ROC weights are the best rule (having a hit rate between 79% and 88%, for problem dimensions similar to those considered in this paper).

In this work we will also use ROC weights when the ordinal information refers to the weights. ROC weights are computed from the vertices of polytope W (3). This polytope corresponds to a simplex whose vertices are $(1, 0, ..., 0)$, $\left(\frac{1}{2}, \frac{1}{2}, 0, ..., 0\right)$, $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, ..., 0\right)$,...,$\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$. The coordinates of the centroid are found by averaging the corresponding coordinates of the defining vertices. ROC weights can be easily calculated using the following formula (recall the indices of criteria reflect their order, w_1 is the highest weight and w_n is the lowest one):

$$w_i^{(ROC)} = \frac{1}{n}\sum_{j=i}^{n}\frac{1}{j},\ \ i = 1, ..., n. \tag{4}$$

As referred by Butler and Olson [9], if ties exist, extreme points will coincide. To obtain the centroid, only one of the tied values is included. For instance, if in a case with three objectives the decision maker states that $w_1 \geq w_2$ and $w_2 = w_3$, then the vertices are $(1, 0, 0)$, and $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, hence $w_1 = \frac{1}{2}\left(1 + \frac{1}{3}\right) = \frac{2}{3}$ and $w_2 = w_3 = \frac{1}{2}\left(0 + \frac{1}{3}\right) = \frac{1}{6}$. Solymosi and Dombi [43] described the process of generalizing the centroid approach to cases that include weak orders or partial orders.

3.2.2 Ordinal information on the value of each alternative and value differences in each attribute

If the decision maker states that it is difficult to indicate the exact value of each alternative in each attribute, a natural idea is to ask him for a ranking, e.g., "considering the attribute x_1, alternative a_1 is the best one, followed by a_2 as the second best, and then a_3". In this work, we assume that for each attribute the worst level corresponds to value 0 and the best level corresponds to value 1. This is a usual convention that is legitimate if the weights are set taking these levels into account.

Similarly to the case of ordinal information on the weights, we deem that in this case it is also possible to use an algebraic formula to choose a vector of values for each attribute, able to approximately represent all vectors values compatible with the ordinal information. One possibility is to use ROC values for each attribute, i.e., the centroid of the polytope defined by the ranking of the values on that attribute. Since attributes are normalized in such a way that the highest value in each attribute is 1 and the lowest value is 0, the centroid corresponds to equally spaced values in the interval $[0, 1]$. Hence, for attribute x_i, the ROC values are defined as follows $(i = 1, ..., n)$:

$$v_i^{(ROC)}(a_j) = \frac{m - r_i(a_j)}{m - 1},\ \ j = 1, ..., m. \tag{5}$$

where $r_i(a_j)$ represents the rank position of alternative a_j considering the attribute x_i and $r_i(a_j) < r_i(a_k) \Rightarrow v_i(a_j) \geq v_i(a_k)$. Suppose, for example, that we have 5 alternatives, and for criterion x_i, $1 = v_i(a_1) \geq v_i(a_2) \geq v_i(a_3) \geq$

$v_i(a_4) \geq v_i(a_5) = 0$. The centroid of this simplex is $\left(1, \frac{3}{4}, \frac{2}{4}, \frac{1}{4}, 0\right)$ since the vertices are $(1,1,1,1,0)$, $(1,1,1,0,0)$, $(1,1,0,0,0)$ and $(1,0,0,0,0)$. In a case with m alternatives, the centroid of the simplex defined by $1 = v_i(a_1) \geq v_i(a_2) \geq ... \geq v_i(a_m) = 0$ is equal to $\left(1, \frac{m-2}{m-1}, ..., \frac{1}{m-1}, 0\right)$.

The formula to approximate the values using ROC values can also be used if there are ties concerning the value of the alternatives in some criteria. In cases with one tie in one criterion, the problem is solved decreasing one dimension to the number of alternatives of the problem, i.e, considering $m-1$ value levels instead of m in that criterion. In cases with two ties the problem is solved decreasing two dimensions to the problem, and so on. We assume that there is no attribute for which all alternatives have the same value (in practice, such an attribute could be discarded).

In order to obtain richer information about the alternatives' values, besides a ranking of the alternatives in each attribute, it is also possible to ask the decision maker to provide a ranking of the differences of value between consecutive alternatives. Suppose for instance that considering an attribute x_i, a decision maker draws his rough idea of the relative position of 4 alternatives a_1, a_2, a_3, a_4 concerning their value according to x_i as depicted in Figure 1. From this drawing we could ask the decision maker to confirm that not only $v_i(a_4) \geq v_i(a_3) \geq v_i(a_2) \geq v_i(a_1)$, but also to confirm that the ranking of the consecutive value differences $\Delta_{i1} = v_i(a_2) - v_i(a_1)$, $\Delta_{i2} = v_i(a_3) - v_i(a_2)$, $\Delta_{i3} = v_i(a_4) - v_i(a_3)$ is $\Delta_{i1} \geq \Delta_{i3} \geq \Delta_{i2}$. This type of drawings-based elicitation has been previously proposed to obtain parameter values, for instance in [36], but in our case the objective is not to read an exact value for each alternative: we only consider ordinal information about the position of alternatives and about the difference of value between consecutive alternatives. Of course, the rank-order of the consecutive value differences might also be asked to the decision maker directly without using a graphical representation.

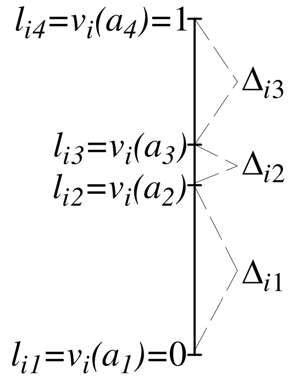

Figure 1. Example of a rough drawing on a scale for attribute x_i.

Based on this type of information, we can propose a rule to approximate the alternatives values, which we will call ΔROC rule. Let us consider an attribute x_i and let us denote s the number of different value levels in this attribute implied by the ordinal information provided by the decision maker, including the value levels $l_1 = 0$ and $l_s = 1$, which bound all other levels. For instance, if the ordinal information is $v_i(a_4) \geq v_i(a_2) \geq v_i(a_1) \geq v_i(a_3)$, then there are four different levels, which are by decreasing order $l_4 = v_i(a_4) = 1$, $l_3 = v_i(a_2)$, $l_2 = v_i(a_1)$ and $l_1 = v_i(a_3) = 0$. Let us denote the value difference between two consecutive levels as $\Delta_{ik} = l_{k+1} - l_k$, for $k = 1, ..., s-1$. From these definitions, it is easy to check that:

$$\Delta_{i1}, ..., \Delta_{i(s-1)} \geq 0 \text{ and } \sum_{k=1}^{s-1} \Delta_{ik} = 1 \tag{6}$$

Since the values differences are ranked, are positive, and add up to 1, an expression similar to the formula used to derive ROC weights can be used. The approximation to the values of each alternative in each criterion can be obtained using the following algorithm:

Step 1 Ask the decision maker to provide a ranking of the alternatives in each criterion (possibly with ties). Label the resulting different levels as $l_1, ..., l_s$, ranked from lowest to highest, with $l_1 = 0$ and $l_s = 1$. Each level will correspond to the value of one alternative (or more than one, in case of ties).

Step 2 Ask the decision maker to provide a ranking of the difference of values between consecutive levels $\Delta_{i1}, ..., \Delta_{i(s-1)}$. For each $k = 1, ..., s-1$, let $rank(k)$ denote the rank of Δ_{ik} within the set $\{\Delta_{i1}, ..., \Delta_{i(s-1)}\}$. This rank is an integer between 1 and $s-1$, with rank 1 denoting the highest difference.

Step 3 Determine a rank order centroid for $s-1$ variables:

$$\Delta_{[k]} = \frac{1}{s-1} \sum_{j=k}^{s-1} \frac{1}{j}, k = 1, ..., s-1. \tag{7}$$

Step 4 For each $k = 1, ..., s-1$, set the the values provided by the centroid approximation: $\Delta_{ik}^{(ROC)} = \Delta_{[rank(k)]}$.

Step 5 The approximate values for the levels in attribute x_i, are then defined as follows:

$$l_1 = 0$$

$$l_j = \sum_{k=1}^{j-1} \Delta_{ik}^{(ROC)}, \ j = 2, ..., s. \tag{8}$$

Step 6 $v_i^{(\Delta ROC)}(a_j)$, the approximate values in attribute x_i for alternative a_j $(j = 1, ..., m)$ based on the Δ ROC values rule, is equal to the approximate value of the respective level, according to the values to levels correspondence created in step 1.

For the example presented in Figure 1, the ROC for three differences of value (as there are 4 alternatives) is $\Delta_{[1]} = \frac{11}{18}$, $\Delta_{[2]} = \frac{5}{18}$, and $\Delta_{[3]} = \frac{2}{18}$. Taking into account the order $\Delta_{i1} \geq \Delta_{i3} \geq \Delta_{i2}$, we obtain $\Delta_{i1}^{(ROC)} = \frac{11}{18}$, $\Delta_{i2}^{(ROC)} = \frac{2}{18}$, and $\Delta_{i3}^{(ROC)} = \frac{5}{18}$. Therefore, the ΔROC values vector is $\left(0, \frac{11}{18}, \frac{13}{18}, 1\right)$:

- $v_i^{(\Delta ROC)}(a_1) = 0$,
- $v_i^{(\Delta ROC)}(a_2) = \Delta_{i1}^{(ROC)} = \frac{11}{18}$,
- $v_i^{(\Delta ROC)}(a_3) = \Delta_{i1}^{(ROC)} + \Delta_{i2}^{(ROC)} = \frac{11}{18} + \frac{2}{18} = \frac{13}{18}$,
- $v_i^{(\Delta ROC)}(a_4) = \Delta_{i1}^{(ROC)} + \Delta_{i2}^{(ROC)} + \Delta_{i3}^{(ROC)} = \frac{11}{18} + \frac{2}{18} + \frac{5}{18} = 1$.

Note that this algorithm can account for the existence of ties in the value of the alternatives in each criterion. Ties in the ranking of consecutive value differences in step 2, on the other hand, can be dealt with using the procedures proposed for the case of ties in the criteria weights [9,43].

4 Simulation

In the previous section we presented simple rules that can be used to generate approximate values for the weights and for the alternatives values in each attribute, given ordinal information about these parameters. The parameter values derived by these rules can then be used to select a promising subset of alternatives. However, the decision maker should have an idea of how good these proposed alternatives are, when compared to the results that would have been achieved if an elicitation of precise cardinal values for all the parameters had been conducted (admitting such a precise elicitation was possible). In this section we describe a sequence of experiments using Monte Carlo simulation to compare the results provided by rules with the results that are obtained under precise cardinal information.

Each generated random problem is characterized by a (precise) value matrix and a (precise) weights vector. Using the additive model on the generated random data we then compute the overall value of each alternative, and we obtain the corresponding ranking of the alternatives. This is what we call the supposedly true ranking, i.e., the ranking that would be obtained if cardinal information was elicited. On the other hand, each of the rules is applied considering only the ordinal information contained in the generated data. Using the approximate parameter values derived from a rule, we can build the ranking of the alternatives that the rule yields. Comparing the ranking of the alternatives according to the supposedly true parameters with the ranking of the alternatives according to a decision rule, we consider the following results:

- The position that the best alternative according to the true ranking reaches in the ranking generated by the decision rule: this allows us to know how many alternatives from the top of the ranking provided by the rule must be kept to include the supposedly best alternative in the set of selected alternatives.
- The position that the best alternative in the ranking generated by the rule reaches in the supposedly true ranking: this allows us to know how good the alternative chosen by the rule is in terms of the supposed true ranking.

Similarly to Barron and Barret [4] we also calculated the "value loss", i.e., the difference of multiattribute value between the alternative selected by a decision rule and the true best alternative, considering the supposedly true parameter values.

In these experiments we have considered situations with 5, 10, and 15 attributes, and 5, 10, and 15 alternatives. Similarly to [40] and [42], we have generated 5000 random problems for each problem dimension (after verifying that using a greater number of problems did not affect significantly the results). The uniform distribution was considered for all generated parameters, as in most comparable previous experiments ([2,40,42]).

The scaling weights were generated according to an uniform distribution in W using the process described in [8]. To obtain weights for n attributes, we generate $n-1$ independent random numbers from a uniform distribution on $(0,1)$ and rank these numbers. Let the ranked numbers be $r_{(n-1)} \geq ... \geq r_{(2)} \geq r_{(1)}$. The following differences can then be obtained: $w_n = 1 - r_{(n-1)}$, $w_{n-1} = r_{(n-1)} - r_{(n-2)}$,..., $w_1 = r_{(1)} - 0$. Then, the set of numbers $(w_1, w_2, ..., w_n)$ adds up to 1 and is uniformly distributed on the simplex defined by the rank-order constraints (3), after relabelling.

The single-attribute values $v_i(a_j)$ were generated from a uniform distribution in the interval [0,1] and then normalized attribute-wise in such a way that the highest value in each attribute is 1 and the lowest value is 0. For each criterion, let v_i^{lo} and v_i^{hi} denote the lowest and highest values among the m values generated. Then, the normalized value of $v_i(a_j)$ is equal to $(v_i(a_j) - v_i^{lo})/(v_i^{hi} - v_i^{lo})$.

The first simulations were performed considering known cardinal weights and ordinal information on the values (with and without ordinal information on the value differences). Additional simulations were performed considering simultaneously ordinal information on weights and ordinal information on the values (again, with and without ordinal information on the value differences). The situation with known cardinal values and ordinal information on the weights was already studied in [42]. Results are presented in the next section.

5 Results

5.1 Incomplete information concerning the value of each alternative in each attribute

In this set of experiments we considered the precise weights of the criteria (henceforth referred to as TRUE weights) were known, but we supposed that the decision maker indicated incomplete information about the value of each alternative in each criterion. We tested the ROC values rule (assuming that the decision maker ranked the alternatives) and the ΔROC values rule (assuming that the decision maker ranked the alternatives and also ranked the difference between consecutive alternatives) to derive approximate values for the alternatives in each criterion.

A first set of experiments was carried out in order to see how the different rules compare if the analysis aims at selecting only the best alternative according to a rule. These experiments indicate the position reached by the alternative suggested by the ROC values and ΔROC values rules on the supposedly true ranking. Detailed results are presented in tabular form in Table 1 (in this table TRUE ΔROC indicates the use of TRUE weights and ΔROC values and TRUE ROC indicates the use of TRUE weights and ROC values). This table shows, for each rule and for each size, the average position on the supposedly true ranking (the minimum position was always 1) and the proportion of cases where the position reached is 1, 2, 3, 4, or higher. Note that the proportion of cases where the reached position is equal to 1 corresponds to the hit rate.

The results indicate that the use of supposedly TRUE weights and ΔROC values leads to a hit rate of at least 90%. It is also possible to observe that the hit rate increases with the number of alternatives. Using TRUE weights and ROC values the hit rate varies between 76% and 81%. Since the use of the ΔROC rule requires more information than the use of the ROC rule, it is not surprising that the former performs better. However, it is noteworthy that the use of ΔROC values leads to a considerable increase in the hit rate when compared with the use of ROC values.

Instead of using a rule to select a single alternative, the analysis can aim at retaining a small subset of promising alternatives for further analysis, in a strategy of progressive reduction of the number of candidates. In such cases, the objective is to retain a subset of alternatives that is as small as possible, yet without eliminating the best one. An interesting question is to know how many alternatives should be retained when a rule based on ordinal information is used. To answer this question, we studied in our simulations the position of the supposedly best alternative in the ranking produced by each rule. Table

2 shows, for each problem size, the average position of the supposedly best alternative in the ranking provided by each rule (the minimum position was always 1), as well as the proportion of cases where the position is 1, 2, 3, 4, or higher. The probability of retaining the supposedly best alternative increases naturally with the number of alternatives that are retained. In all cases, selecting two alternatives would suffice to keep the supposedly best one in 93% of the cases, while selecting three alternatives would suffice in 97% of the cases. We can see that the additional information required from the decision maker by the ΔROC values rule is compensated by clearly superior results when compared with the ROC values rule. Indeed, in at least 99% of the cases the supposedly best alternative was one of the two best ranked alternatives according to the ΔROC values rule.

In Table 3 it is possible to see the value loss implied by selecting the top-ranked alternative obtained by the different rules. In this table ROC TRUE refers to the use of ROC weights and TRUE values, ROC ΔROC refers to the use of ROC weights and ΔROC values, ROC ROC refers to the use of ROC weights and ROC values, TRUE ΔROC refers to the use of TRUE weights and ΔROC values, and TRUE ROC refers to the use of TRUE weights and ROC values. Considering the weights are known and using ΔROC values (TRUE ΔROC columns), the average value loss varies between 0.0070 and 0.0316. The maximum value loss is a value between 0.0580 and 0.2455. Considering the weights are known and using ROC values (TRUE ROC columns), the average value loss varies between 0.0051 and 0.0139. The maximum value loss is a value between 0.0998 and 0.3123. In average, when considering the weights are known, using the ΔROC values leads to roughly half of loss of value incurred by using the ROC values. Note however that in both cases the average value loss can be considered very small.

		TRUE ΔROC						**TRUE ROC**					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.10	91.00	8.20	0.72	0.08	0.00	1.25	78.78	17.44	3.36	0.40	0.02
5	10	1.09	91.82	7.36	0.68	0.14	0.00	1.28	78.66	16.38	3.64	1.04	0.28
5	15	1.08	93.32	5.88	0.62	0.18	0.00	1.29	79.02	15.30	4.14	1.06	0.40
10	5	1.10	90.76	8.46	0.74	0.04	0.00	1.24	80.94	14.92	3.38	0.70	0.06
10	10	1.09	91.58	7.54	0.76	0.12	0.00	1.32	77.38	16.02	4.50	1.58	0.52
10	15	1.08	93.00	6.28	0.64	0.08	0.00	1.31	77.76	16.04	4.50	1.16	0.54
15	5	1.10	90.80	8.36	0.80	0.04	0.00	1.26	79.20	16.64	3.38	0.70	0.08
15	10	1.09	91.88	7.12	0.92	0.06	0.02	1.31	76.84	16.7	4.64	1.26	0.56
15	15	1.08	92.80	6.40	0.64	0.14	0.00	1.30	79.00	14.74	4.40	1.44	0.42

Table 1
Position of the best alternative according to the ROC values and ΔROC values rule in the supposedly true ranking (n denotes the number of criteria and m the number of alternatives).

		TRUE ΔROC						**TRUE ROC**					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.10	91.00	8.12	0.80	0.08	0.00	1.25	78.78	17.60	3.28	0.30	0.04
5	10	1.09	91.82	7.50	0.64	0.04	0.00	1.28	78.66	16.44	3.68	0.94	0.28
5	15	1.07	93.32	5.96	0.66	0.04	0.02	1.29	79.02	14.92	4.40	1.20	0.30
10	5	1.10	90.76	8.46	0.76	0.02	0.00	1.24	80.94	15.10	3.22	0.64	0.01
10	10	1.10	91.58	7.26	1.10	0.04	0.02	1.32	77.38	16.06	4.80	1.24	0.52
10	15	1.08	93.00	6.26	0.72	0.02	0.00	1.31	77.76	16.36	3.72	1.42	0.74
15	5	1.10	90.80	8.38	0.76	0.06	0.00	1.26	79.20	16.56	3.40	0.80	0.04
15	10	1.09	91.88	7.10	0.90	0.06	0.00	1.31	76.84	16.82	4.44	1.36	0.54
15	15	1.08	92.80	6.48	0.64	0.06	0.02	1.29	79.00	14.88	4.58	1.12	0.42

Table 2
Position of the supposedly best alternative in the ranking induced by the ROC values and ΔROC values rule.

		ROC TRUE		ROC Δ ROC		ROC ROC		TRUE Δ ROC		TRUE ROC	
n	m	average	maximum	average	maximum	average	maximum	average	maximum	average	maximum
5	5	0.0589	0.4222	0.0663	0.4926	0.0816	0.5351	0.0316	0.2455	0.0655	0.3123
5	10	0.0433	0.2999	0.0482	0.3177	0.0668	0.5143	0.0171	0.0994	0.0459	0.2377
5	15	0.0383	0.3009	0.0400	0.2364	0.0543	0.4223	0.0076	0.1755	0.0391	0.2105
10	5	0.0391	0.3053	0.0459	0.3605	0.0662	0.3808	0.0238	0.1195	0.0535	0.2602
10	10	0.0314	0.3092	0.0339	0.3466	0.0480	0.2845	0.0119	0.0705	0.0367	0.1996
10	15	0.0276	0.2158	0.0292	0.2780	0.0412	0.2804	0.0100	0.0607	0.0292	0.1599
15	5	0.0285	0.2217	0.0365	0.4018	0.0534	0.3167	0.0207	0.0991	0.0447	0.2341
15	10	0.0236	0.2448	0.0280	0.1751	0.0346	0.1946	0.0111	0.0693	0.0311	0.1922
15	15	0.0207	0.1610	0.0215	0.1869	0.0355	0.2080	0.0070	0.0580	0.0243	0.0998

Table 3
Value loss implied by selecting the best alternative according to a rule based on ordinal information.

5.2 Incomplete information concerning weights and concerning the value of each alternative in each attribute

In this section we consider the criteria weights and the value of each alternative in each criterion are unknown. The decision maker indicates only ordinal information about the weights and about the value of alternatives in each criterion, possibly adding ordinal information about differences of value between consecutive alternatives in each criterion. We tested the rule of using ROC weights together with ROC values, as well as the rule of using ROC weights together with ΔROC values. Table 4 shows the position in the supposedly true ranking of the best alternative obtained by each rule (in this table ROC ΔROC means the use of ROC weights and ΔROC values and ROC ROC means the use of ROC weights and ROC values). Using ROC weights and ΔROC values the hit rate (column %1) decreases with the number of alternatives.

Table 5 shows the position of the supposedly true alternative in the ranking induced by the ROC weights / ROC values and ROC weights / ΔROC values rules. In the previous experiments that considered TRUE weights and ΔROC values, results indicated the hit rate was higher than 90%. If we consider that we also do not know the weights, and use ROC weights, the results are still fairly good (the hit rate is greater than 78%). If the two top-ranked alternatives obtained using ROC weights and ΔROC values are kept, instead of a single one, then the supposedly best alternative is one of these two in at least 94% of the cases.

The results are obviously worse than in the case with known weights, as in this situation the rules use less information. However, it should be noted that combining ROC weights with ΔROC values yields results very close to those obtained in [42] assuming that the values of the alternatives were known (see Table 6). Once again the additional information requested from the decision maker by the ΔROC values rule is compensated by superior results when compared with the ROC values rule. These results suggest that using the ROC weights and ΔROC values rule to facilitate the elicitation of information leads to a rapid identification of the most promising alternatives.

In Table 3 it is possible to see the value loss of the different rules. Considering the weights are unknown (using ROC weights) and using ΔROC values the average value loss varies between 0.0215 and 0.0663. The maximum value loss is a value between 0.1869 and 0.4926. Considering the weights are unknown and using ROC values the average value loss increases to values between 0.0355 and 0.0816, and the maximum value loss is a value between 0.1946 and 0.5351. Considering both the weights and the values of each alternative in each attribute are unknown, the average value loss is still small. Comparing the third and fifth columns of Table 3 indicates that when there is only ordinal information about the weights and ROC weights are used, the average loss of value using ΔROC values is very similar to the average loss of value implied by the ROC weights alone (i.e., considering TRUE values for the alternatives).

		ROC ΔROC						**ROC ROC**					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.21	83.12	13.40	3.06	0.42	0.00	1.32	74.98	19.22	4.74	0.98	0.08
5	10	1.29	79.44	14.84	3.88	1.42	0.42	1.41	72.92	17.80	6.00	2.42	0.90
5	15	1.31	78.98	14.56	4.18	1.34	0.60	1.47	71.26	17.86	6.52	2.70	0.92
10	5	1.21	83.02	13.90	2.66	0.38	0.04	1.31	75.98	18.14	4.62	1.18	0.08
10	10	1.25	81.42	13.82	3.42	1.02	0.32	1.42	72.26	18.24	60.6	2.26	1.18
10	15	1.29	80.50	13.90	3.48	1.20	0.92	1.46	71.84	17.92	6.08	2.36	1.80
15	5	1.19	84.40	12.44	2.76	0.40	0.00	1.32	76.20	17.52	4.92	1.12	0.24
15	10	1.23	83.24	12.72	2.74	0.94	0.36	1.39	74.02	17.40	5.48	2.12	0.98
15	15	1.26	81.40	13.30	3.76	1.02	0.52	1.39	75.22	15.80	5.74	1.86	1.38

Table 4

Position of the best alternative according to the ROC weights / ROC values and ROC weights / ΔROC values rules in the supposedly true ranking.

		ROC ΔROC						**ROC ROC**					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.21	83.12	13.54	2.90	0.44	0.00	1.32	75.00	19.28	4.46	1.10	0.16
5	10	1.28	79.44	15.16	3.70	1.14	0.56	1.43	72.92	17.84	5.46	2.18	1.60
5	15	1.31	78.98	14.66	4.08	1.28	0.68	1.47	71.26	17.84	6.66	2.48	1.08
10	5	1.20	83.02	14.08	2.58	0.30	0.02	1.32	75.98	17.26	5.42	1.24	0.10
10	10	1.25	81.42	13.66	3.50	1.10	0.32	1.43	72.26	18.26	5.86	2.36	1.26
10	15	1.28	80.50	13.82	3.72	1.32	0.64	1.46	71.84	17.48	6.72	2.26	1.70
15	5	1.19	84.40	12.60	2.54	0.42	0.04	1.31	76.20	18.18	4.42	1.04	0.16
15	10	1.22	83.24	12.86	2.78	0.88	0.24	1.39	74.02	17.32	5.74	1.96	0.96
15	15	1.25	81.40	13.54	3.76	0.90	0.40	1.38	75.22	16.22	5.20	2.44	0.92

Table 5

Position of the supposedly best alternative in the ranking induced by the ROC weights/ ROC values and ROC weights / ΔROC values rules.

		(A)						**(B)**					
n	m	average	% 1	% 2	% 3	% 4	% $\geq$ 5	average	% 1	% 2	% 3	% 4	% $\geq$ 5
5	5	1.18	84.96	12.26	2.40	0.34	0.04	1.18	84.96	12.16	2.66	0.22	0.00
5	10	1.25	81.76	13.22	3.58	1.10	0.34	1.25	81.76	13.36	3.54	1.00	1.34
5	15	1.30	79.72	14.14	3.90	1.36	0.88	1.29	79.72	14.40	3.90	1.24	0.74
10	5	1.17	85.58	11.98	2.18	0.20	0.06	1.17	85.58	12.00	1.94	0.42	0.06
10	10	1.24	82.46	13.10	3.20	0.78	0.46	1.24	82.46	12.76	3.36	1.02	0.40
10	15	1.27	80.80	13.58	3.84	1.26	0.52	1.27	80.80	13.78	3.94	0.90	0.58
15	5	1.14	87.92	10.08	1.76	0.24	0.00	1.15	87.92	9.92	1.82	0.26	0.04
15	10	1.20	84.46	12.10	2.64	0.50	0.30	1.21	84.46	11.92	2.66	0.66	0.30
15	15	1.24	82.96	12.34	3.30	1.08	0.32	1.23	82.96	12.94	2.74	0.88	0.48

Table 6

Position of the best alternative according to the ROC weights in the supposedly true ranking (A), and position of the supposedly best alternative in the ranking induced by the ROC weights (B), assuming the true values for the alternatives were known (results from [42]).

6 Conclusions

This work presented a sequence of Monte-Carlo simulations with the aim of assessing different decision rules for the context where there exists only ordinal information about the weights of the attributes and about the values of each alternative in each attribute. The rules studied in this paper consisted in computing approximate values for the parameters, which can then be used for ranking alternatives based on the multiattribute additive aggregation model.

Previous studies considered using this type of rules based on algebraic formulas to compute approximate values for the weights. If the decision maker provides a ranking of the weights, then the ROC weights rule was found to provide the best approximation [4], and was found to perform even better than more complex optimization-based rules [42]. The originality of our new study is the consideration of analogous decision rules for the case in which we have also ordinal information concerning the value of each alternative in each attribute. We proposed an adaption of the ROC weights rule for this purpose: the ROC values rule. We also proposed a new rule that requires a little more information (but easily elicited, for example, by a rough drawing): the ΔROC values rule.

Another noteworthy aspect of this study is that we tested strategies to select more than one alternative. This contrasts with the assumption that the decision maker uses these rules to select only the top alternative according to the rule, which would lead us to focus only on the calculation of hit rates and loss of value. Hence, we were also interested in finding out how many alternatives should be kept to ensure a good probability of not excluding the truly best one. The objective of this type of strategies is the simplification of the problem in terms of the number of alternatives, with the aim of studying the most promising ones in more detail, or with the aim of eliciting more information. This is particularly interesting when assessing the performance of all alternatives under all the criteria would imply significant costs, time, or effort.

As our experiments have shown, using ordinal information leads in general to good results in the identification of the most promising alternatives. The best rule presented for cases without any cardinal information was the combined use of ROC weights and ΔROC values. With this rule, the hit rate varies between 79% and 85%. This rule is also very interesting for selecting a subset of the most promising alternatives: selecting the two best alternatives according to this rule is sufficient in 94% of the cases or more, depending on the problem dimension, to retain the best alternative according to the true weights and values.

Another finding resulting from these experiments is that the ΔROC values rule leads to results that are clearly superior to the results of the ROC values rule, reducing the average value loss to roughly one half, and in some cases even less. Finally, we found out that the combined use of ROC weights and ΔROC values does not provide results significantly worse than when ROC weighs are used in a situation with known cardinal values.

The elicitation of ordinal information makes the cognitive task of the decision maker easier. Hence, given these results, we deem that the use of this type of rules to identify a small subset containing the most promising alternatives is an interesting possibility, whenever it is costly or difficult to obtain precise values for all parameters. Eliciting ordinal information about consecutive value differences requires a little additional effort, but the resulting increase in the quality of the results, in our opinion, justifies this extra step.

The conclusions presented in this paper should be read carefully, since the experiments were restricted to the case where the decision is based on a complete ranking of the criterion weights and on a complete ranking of the value of each alternative in each attribute. For the case where the set of acceptable weights and the set of acceptable values are defined by a set of general linear restrictions, it is possible that the ROC and ΔROC rules lose some power. However, as referred by [4], the ROC is a specific example of centroid values, which generalizes to any convex value set specified by linear inequalities, and for a large class of situations the centroid computations are not very difficult [43]. Testing the quality of centroid-based approximations for other types of constraints is an interesting subject for future research.

Another future research path is the use of this type of approximations in multi-actor settings, namely group decisions and negotiations, where eliciting order relations is less prone to disagreement than eliciting precise parameter values. Finally, we deem that this type of approximations might also be interesting to be studied for other multi-criteria methods besides additive MAVT.

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