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Dealing with uncertainty in Decision Support Systems: Recent trends (2000-2011)

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Abstract. This paper reviews research in relation with modelling uncertainty within Decision Support Systems (DSS) from 2000 to 2011. It specifically addresses software that has been built or prototyped with the purpose of supporting actual decision making, which is able to explicitly deal with uncertainty (widely understood) on the corresponding model parameters and/or data. The main DSS features analysed are the underlying decision support methodology, the type of uncertainty modelling approach used, the DSS type, and the application area. We appreciate that there is an increasing interest in dealing with uncertainty in real decision support, with prevailing interest in probabilistic approaches and, when linguistic imprecision is involved, fuzzy approaches. We have also recognized an increasing variety of perspectives adopted.

1. Introduction

This paper reviews a set of recent Decision Support Systems (DSS) that were built to deal with decision making problems in which uncertainty, understood in a broad sense, is a major concern. Decision aiding based on DSS requires the input of data (economic, engineering, geographical, etc.) that can be difficult to obtain for a number of reasons: some data can be missing (e.g., from databases or time series); other data may refer to unknown future outcomes (e.g., future oil prices); other data may be subject to variability according to a statistical distribution to be estimated (e.g., the number of nonconforming items in a production batch); other data are known to be subject to physical or statistical measurement error (e.g., the quantity of oil in a well, or the Gross Domestic Product of a country); other data are controversial or contradictory (e.g. data from clinical trials).

In many cases, there are also modelling choices and parameters to be set that involve the subjective judgment of Decision Makers (DMs). For instance, the decision on how to measure comfort or how to assess a company's performance to encompass also environmental or social aspects is, to some extent, arbitrary and subjective. Prior distributions model expert judgements within a Bayesian setting. Many models incorporate parameters related with the DM's preferences, such as those that define how important each criterion is in a multi-criteria evaluation. In purely cognitive terms, some parameters are artefacts whose semantics may be difficult to understand for the DM, and there are well-documented biases related with the way judgmental questions are posed, e.g., (Schoemaker & Waid 1982). If the DSS is supporting a group decision or negotiation process, it must cope also with the potential lack of consensus as the opinions and preferences of the DMs about parameter values and data may differ. Note that our categorization of nature and sources of uncertainty largely follows that of (Morgan & Henrion 1990), who provide a detailed discussion.

It is not easy to define a set of keywords able to encompass all types of uncertainty mentioned in the previous paragraphs. This paper used as keywords the logical expression ("dss" *OR* "decision support system") AND ("uncertainty" OR "robustness") for a search in the SCOPUS bibliographic database. Results were limited to papers published from 2000 to 2011 in a group of 50 journals (Table 1), mostly from the areas of Computer Science and Operations Research. This search was complemented with another one focusing on the Intelligent Decision Technologies journal, as it is not yet indexed in this database, as well as a few other papers the authors were aware of. This resulted in a set of over 300 papers, which was then reduced to meet a number of a priori defined criteria for inclusion in this review.

The first criterion was that the paper should describe a DSS in the form of a well-defined software implementation, either fully deployed or as a working prototype, usually having a name given by its authors. This included not only standalone applications, but also software modules that run on other platforms, such as add-ons or plug-ins. Therefore, we have not included papers that are of a philosophical nature, or just presenting a framework or method. The same applies to papers that present algorithms implementing a method just to perform computational experiments, or comparisons with other methods, or to obtain results for an illustrative example. We neither include papers with the mere purpose of providing suggestions or guidelines for DSS development, or that discuss projects not yet implemented. Finally, papers presenting surveys or comparative studies of previously published systems are not included either.

A second criterion for inclusion in this review was that the DSS would require human intervention in the actual decision making process. This excluded papers about systems supposed to work without, or with very little, human intervention, such as industrial control systems, intelligent agents, or autonomous vehicles and robots. Systems that fundamentally aim at knowledge discovery, such as pattern recognition, information fusion, data mining, and diagnosis tools were also considered out of the scope for our review.

Finally, we sought to include only DSS that explicitly deal with uncertainty models for decision making. This left out several papers concerning neural networks and expert systems without any explicit modelling of uncertainty. Other excluded papers used an indicator to measure risk or uncertainty as a function of deterministic characteristics (e.g., considering a risk criterion in a multi-criteria evaluation).

Note that, by and large, we conform to the DSS definition in (French et al. 2009, p. 83) , that is a "computer-based systems that support the decision-making process, helping DMs to understand the problem before them and to form and explore the implication of their judgements, and hence to make a decision based upon understanding" and, more specifically, to what are called level 2 and 3 DSS there, thus focusing on actual decision support.

Table 1. Number of selected papers per journal (total = 83)

Journal	**No. of papers**
Expert Systems with Applications	15
Decision Support Systems	12
Environmental Modelling and Software	7
J. of the Operational Research Society, Knowledge Based Systems, Intelligent Decision Technologies	4
Computers and Electronics in Agriculture, Int. J. of Computational Intelligence Systems, Int. J. of Production Economics	3
Advanced Engineering Informatics, Annals of Operations Research, Computers and Operations Research, Engineering Applications of Artificial Intelligence, Fuzzy Sets and Systems , Int. J. of Geographical Information Science , Omega	2
Applied Mathematics and Computation, Artificial Intelligence in Medicine, European J. of Operational Research, Expert Systems, Int. J. of Decision Support System Technology, Interfaces, Journal of Multi-Criteria Decision Analysis, J. of Optimization Theory and Applications, Medical Decision Making, Military Operations Research, Operations Research, Stochastic Environmental Research and Risk Assessment, Technological Forecasting and Social Change, Trans. in GIS	1
Applied Soft Computing J., Cochrane Database of Systematic Reviews Online, Computer Methods and Programs in Biomedicine, Computers and Industrial Engineering, IEEE Trans. on Neural Networks, IEEE Trans. on Systems Man and Cybernetics (Part A, Part B, Part C), Infor J., Information Systems Frontiers, Int. J. of Approximate Reasoning, Int. J. of Computer Applications in Technology, Int. J. of Computers Communications and Control, Int. J. of Management and Decision Making, Int. J. of Production Research, Lecture Notes in Computer Science (including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), Optimization Methods and Software, Reliability Engineering and System Safety, Risk Analysis, Studies in Computational Intelligence, Transp. Planning and Technology, Transp. Research (Part A, Part C), Transp. Research Record	0

Even with our possibly narrow notions of DSS and uncertainty models, 83 papers still remained in our review, with the journal distribution presented in Table 1. As would be expected, the two journals with higher number of papers are devoted to decision support technologies, but surprisingly the third journal with more papers is devoted to applications in the environmental area. According to Figure 1, the number of papers is comparatively low but has a clear increasing trend, suggesting an increasing interest in this topic. The complete list of papers is presented in Table 2.

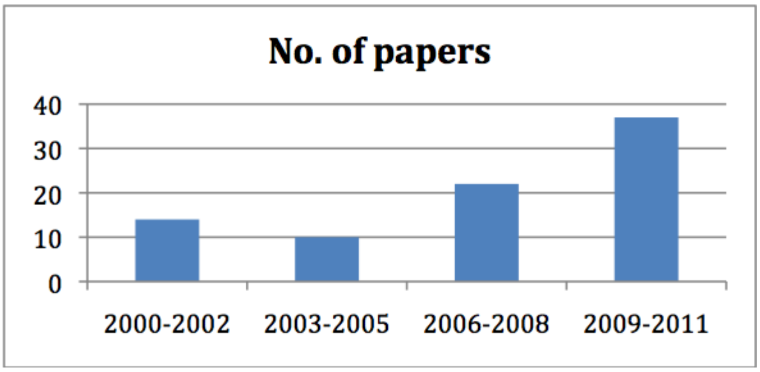

Figure 1. Evolution of published DSS papers dealing with uncertainty and robustness

Table 2. List of papers considered in this revision

Authors	*Name*	*Uncertainty*	*Underlying models/methods*	*DSS type*	*Application*
Disney et al. 2000	APIOBPCS	Parametric	optimization (GA), simulation	Model	inventory control
Ríos Insua et al. 2000	MOIRA	Parametric	MCDA (MAVT)	Model	restoration of radionuclide contaminated aquatic ecosystems
Mackay & Robinson 2000	(not provided)	Fuzzy	application-specific	Geography	ecological and hydrological processes
Karacapilidis & Pappis 2000	HERMES	Fuzzy	similarity measurement (in argumentation)	Group	(none)
Li 2000	EXSYS	Fuzzy	neural networks, weighted sum, rules	Knowledge	marketing strategy development
Dias & Clímaco 2000	VIP Analysis	Parametric	MAUT	Group	(none)
Zeng & Zhou 2001	REGIS	Fuzzy	rules, sensitivity analysis (correlation)	Geography	Real state retailing
Mateos et al. 2001	MOIRA	Parametric	MCDA (MAVT)	Model	restoration of radionuclide contaminated aquatic ecosystems
Beynon et al. 2001	DS/AHP	Dempster–Shafer theory	AHP	Model	Real estate appraisal
Tarantola et al. 2002	(not provided)	Probabilistic	Fourier Amplitude Sensitivity Test	Geography	land depletion assessment
Ades & Cliffe 2002	WinBUGs	Probabilistic	Bayesian Markov chain Monte Carlo	Model	medical decision
Kristensen & Rasmussen 2002	(not provided)	Probabilistic	Bayesian network, application specific	Model	growing malting barley
Wilby et al. 2002	SDSM	Probabilistic	application specific, statistical analysis	Data	assessment of regional climate change impacts
Völkner & Werners 2002	GEPSIS	Fuzzy	rules (approximate reasoning), simulation	Model	business process planning
Jiménez et al. 2003	MOIRA	Parametric	MAUT, Monte Carlo	Model	selection of a supplier for cleaning services
Lou & Huang 2003	DRACO	Fuzzy	rule-based, neural networks	Knowledge	quality control in automotive coating operations
Aerts et al. 2003	(not provided)	Probabilistic	geostatistical simulation	Geography	optimal location for a ski run
Borges & Antunes 2003	Fuzzy MOLP	Fuzzy, Parametric	multiple objective linear programming	Model	(none)

Dias & Mousseau 2003	IRIS	Parametric	ELECTRE TRI	Model	(none)
Tan et al. 2004	POLCAGE 1.0	Possibility theory	possibilistic compromise programming	Model	life-cycle assessment of alternative transportation fuels
Kirkwood et al. 2005	(not provided)	Probabilistic	MAUT	Model	supply-chain-reconfiguration
Packham et al. 2005	IGAS	Accuracy of algorithm (GA)	Nonlinear optimization (GA), Clustering	Model	engineering design problem: rainfall-runoff modelling
Contesse et al. 2005	OBDSS (extension)	Probabilistic	mixed-integer programming , robust optimization	Model	Gas Purchase and Transportation
Li et al. 2005	(not provided)	Probabilistic, Fuzzy	application-specific, Rules	Geography	Typhoon insurance pricing
Lin et al. 2006	(not provided)	Probabilistic, consensus-based modeling	rule-based	Knowledge	lower back pain diagnosis
Besharati et al. 2006	(not provided)	Probabilistic	MAUT (multiplicative), Monte Carlo	Model	product design selection
He et al. 2006	PRES	Probabilistic	Bayesian network , Rules	Knowledge	selection of remediation technologies for petroleum-contaminated sites
Jiménez et al. 2006	GMAA	Probabilistic, Parametric	MAUT (additive), Monte Carlo simulation	Model	selection of a technology for the disposition of surplus weapons-grade plutonium
Xu et al. 2007	Dutch Meuse DSS	Probabilistic	Monte Carlo simulation, application-specific	Model	river basin management
Gascón et al. 2007	(not provided)	Fuzzy	rule-based	Knowledge	demand and supply features of the market of pharmaceutical generics
Salling et al. 2007	CLG-DSS	Scenarios, Probabilistic	Cost-Benefit Analysis, value functions, Monte Carlo	Model	large transport infrastructure projects appraisal
Jiménez et al. 2007	GMAA	Parametric	MAUT, Monte Carlo	Model	selection of a supplier for cleaning services
Zack 2007	QDSS	missing information, contradictory information	Rules	Knowledge	price quoting in a leasing company
Lourenço & Costa 2007	Public Participation Support System	Probabilistic	frequency analysis	Group	e-participation processes
Könnölä et al. 2007	RPM Explorer	Parametric	robust portfolio optimization, MAVT	Group	fostering of innovation ideas
Gijsman et al. 2007	DSSAT	Probabilistic	Simulation	Data	Agriculture - crop simulation
de Kort & Booij 2007	FLOCODS	Probabilistic	application-specific, Monte Carlo	Model	water management (risks of flooding)
Olson et al. 2007	(not provided)	Possibility theory	system dynamics	Model	model the microeconomic environment of a Bulgarian winery

Augusto et al. 2008	RIMER	Evidential Reasoning	rule-based	Knowledge	monitoring and diagnosis in a smart home
Castelletti et al. 2008	(not provided)	Probabilistic	Stochastic optimal control, Multi-objective optimization	Model	water resources planning
Chin et al. 2008	(not provided)	Evidential Reasoning	AHP	Model	product project screening
Lee & Kwon 2008	NSS CAKES-NEGO	Fuzzy	Fuzzy Cognitive Maps (FCMs)	Knowledge	B2B negotiation
Gomes et al. 2008	THOR	Fuzzy	MCDA	Model	Solid waste management
Yazgı Tütüncü et al. 2008	BEKS	Fuzzy, Probabilistic	EOQ, simulation	Model	inventory control
Guitouni et al. 2008	CASAP	Fuzzy, Probabilistic	MCDA (PAMSSEM)	Group	military planning
Lu et al. 2008	WFGDSS	Fuzzy	AHP	Group	Critical situation management
Kong et al. 2009	IDS	Evidential Reasoning	rule-base inference	Knowledge	Medicine
Saenz de Ugarte et al. 2009	(not provided)	Real-time contingencies	genetic algorithm, simulation	Data	scheduling
Montmain et al. 2009	SINERGIE	Parametric	MCDA (MAUT)	Model	motorway maintenance
Guezguez et al. 2009	PIDT	Possibility theory	Influence diagrams	Model	(none)
Namen et al. 2008	Robus	Scenarios	Robustness Analysis PSM	Model	sustainable community development
Chou 2009	PILCES	Probabilistic	Regression, Generalized linear models , application-specific	Model	cost estimation in construction
Li & Li 2009	(not provided)	Fuzzy, Probabilistic	AHP, Monte-Carlo, approx. Reasoning	Model	strategic planning
Jiménez et al. 2009	MOIRA	Probabilistic, missing data	MCDA (MAUT)	Model	restoration of radionuclide contaminated aquatic ecosystems
Cai et al. 2009	UREM-IDSS	Interval, Scenarios	optimization (LP)	Model	regional energy management systems planning
Clímaco et al. 2009	D2VIP-A	Parametric	MAUT	Group	(none)
Weng et al. 2010	HWRDS	Scenarios, Fuzzy	multi-objective programming , Fuzzy aggregation, AHP	Model	water resources management
Zhou et al. 2010	APIOBPCS	Probabilistic	Order-up-To Algorithm	Model	inventory management
Louvieris et al. 2010	(not provided)	Probabilistic	Bayesian belief networks	Model	military
Gemici-Ozkan et al. 2010	(not provided)	Probabilistic	multistage stochastic program	Model	portfolio optimization
Zhang et al. 2010	FMCGDSS	Fuzzy	Fuzzy MCDM aggregation	Model	Power distribution system planning
Ma et al. 2010	Decider	Fuzzy	multi-criteria group decision	Model	(none)
Pereira & Ramli 2010	ACORDA/P-log	Probabilistic	Causal Bayes Nets	Knowledge	(none)
De Maio et al. 2011	(not provided)	Fuzzy	Fuzzy Cognitive Maps (FCMs)	Web	Emergency management

Patiniotakis et al. 2011	Fuzzy UTASTAR	Fuzzy	UTA	Model	Transportation
Papadopoulos et al. 2011	(not provided)	Fuzzy, Sensitivity analysis	Rules	Knowledge	agriculture
Yang et al. 2010	IDS	Evidential Reasoning	rule-based functions	Knowledge	marketing
Li et al. 2011	WebDigital	Fuzzy	Rules if-then, Monte Carlo simulation	Knowledge	digital marketing
Mouzakitis et al. 2011	Axios	Fuzzy	PROMETHEE	Web	investment
Qi & Altinakar 2011	(not provided)	Probabilistic	Monte Carlo Simulation, event tree analysis	Geography	flood management
Beraldi et al. 2011	(not provided)	Probabilistic	Simulation, Stochastic programming	Model	asset allocation
Rees et al. 2011	(not provided)	Fuzzy	Genetic algorithm (optimization)	Model	Cybersecurity risk planning
Leu & Adi 2011	MDS	Probabilistic	Hidden Markov Model , autoregressive time series	Model	drainage water tunnel
Noor-E-Alam et al. 2011	(not provided)	Fuzzy	ME-MCDM	Model	supplier evaluation
Chen et al. 2011	CEDSS	Probabilistic, Fuzzy, Indicator-based	multi-criteria analysis (additive v.f.)	Geography	river catchment management
Ting et al. 2011	HKSMP	Probabilistic	Bayesian reasoning, case-based reasoning (CBR)	Knowledge	medical prescription
Zhang et al. 2011	FICMDSS	Fuzzy, Interval	inexact programming	Model	water quality management in agricultural systems
Damghani et al. 2011	(not provided)	Fuzzy	mathematical programming , Rules	Model	investment selection
Wang et al. 2011	MPVFDSS	Vague sets	application-specific	Model	Wafer manufacturing
Kala et al. 2011	SANE	Probabilistic	neural networks, sum integration	Knowledge	medical diagnosis
Papageorgiou 2011	(not provided)	Fuzzy	Fuzzy Cognitive Maps (FCMs) , DEMATEL (MCDA)	Knowledge	modelling medical knowledge
Loboda et al. 2010	(not provided)	Probabilistic	Bayesian network	Knowledge	generating real-time suggestions to improve users performance
Chen et al. 2011	CEDSS	Probabilistic	MCDA (MAVT)	Geography	river catchment management

We shall analyse the papers reviewed based on four key issues in relation with uncertainty within DSSs: how uncertainty is modelled within the incumbent DSS; what is the underlying decision aiding methodology used that accommodates uncertainty; what type of DSS is actually used from an architectural point of view; and, finally, the application area. Other generic relevant features in DSSs are described in (Burstein & Holsapple 2008).

2. Strategies to deal with uncertainty

In this section, we provide a brief introduction to various types of strategies and models used in the reviewed literature, as far as uncertainty is concerned. The treatment is necessarily brief for space reasons, but we shall provide pointers to the literature where further information may be seen on various approaches. Clearly, several of the strategies employed are intimately related, as outlined below, and indeed may be used in combination. For example, within a probabilistic setting it is convenient to undertake a sensitivity analysis to check the impact of probabilities and utilities on the DSS recommendation, see (Rios Insua & Ruggeri 2000).

- Probabilities

Probabilities constitute the most widely used and best known formalism for quantifying uncertainty. Probabilities have well-defined mathematical properties based on Kolmogorov's axioms, with various interpretations, including the frequentist and the subjective, the latter being the most general. In this last interpretation, probabilities are described as a measure of the degree of belief in the occurrence of an event, and have behavioural axiomatic foundations. Such foundations lead to a natural way of updating beliefs in the light of new evidence based on Bayes theorem (French & Rios Insua 2000). Procedures to elicit beliefs are described, e.g., in (O'Hagan et al. 2006).

Complex probability models may be sometimes described through graphical models, of which the most popular ones are Bayesian networks, which are also sometimes called causal networks, belief nets or probabilistic influence diagrams. If the graphical model includes decision and value nodes, they fully describe the decision problem at hand.

Some of the DSSs reviewed incorporate probabilistic elements to standard optimization problems, either on constraints or objective terms, leading to stochastic programming problems, which may be solved with various strategies described, e.g. in (Birge & Louveaux 2011).

A frequent criticism to the probabilistic approach lays in the difficulty of building the incumbent probability distributions, therefore leading to other uncertainty paradigms.

- Imprecise probabilities

A natural extension of the probability model is the imprecise probability model. The underlying principle is that, normatively, we should build a unique probability modelling the DM's beliefs. However this requires ultrafine discerning capabilities on the DM, who may not be able to provide such precise information. It may be also the case that we need to deal with several DMs (precise or not) and imprecise probabilities emerge as a way to model the common knowledge grounds of all participants.

One natural way of thinking about imprecise probabilities is through upper and lower probabilities. For a given event, relevant in the decision problem at hand, we are not able to provide the precise probability, but rather upper and lower bounds for such probability. If this occurs for all relevant events, we have an upper and lower probability model, provided that the bounds satisfy certain coherence requirements. A well-known model of upper and lower probabilities is Dempster-Shafer evidence theory, see (Shafer 1976), which uses belief and necessity measures as bounds. A good overview of these theories may be seen in (Miranda 2008).

We may handle, however, more general constraints on probabilities based on inequality constraints and others, which generally lead to the concept of convex sets of probabilities, which have an axiomatic behavioural support. Clearly, based on the convex set of probabilities, we may define an upper and lower probability model.

The operationalization of this approach faces two problems. The first one refers to the updating of information in the light of new evidence. The natural idea is to consider a class of standard models and update each of them through Bayes' formula. However, this may be difficult to implement except under very stringent structural conditions and we may need to use approximations and simulation, see (Rios Insua & Ruggeri 2000) for an overview. Some of the paradigms, like Dempster-Shafer's approach which uses Dempster's combination rule, lead to their own updating rules.

The second problem refers to the recommendations to be provided by the DSS. Again, it is natural to view the issue as a class of standard decision problems and solve them individually and either show the whole set of solutions or try to summarise somehow such set of solutions based on their common grounds. By doing this, we are performing some kind of sensitivity analysis. This may be too involved computationally and we may go for computing solutions which are robust in some sense. Typically, these are solutions with a certain guarantee of performing reasonably well under all relevant probability models or which perform reasonably well for a sufficiently large class of probability models. By reasonably well, we refer to a large number of criteria being considered, such as attaining a minimum target expected utility or minimizing the entailed regret (of not being the actual best alternative).

Note that we have focused this discussion on the belief part of the DSS model, but similar issues may be mentioned about the preference part, i.e. we might have imprecision about preferences, e.g. to be dealt with through classes of utility functions.

Also, some of the methods have been combined to provide somewhat different approaches. For example, utility theory, Dempster-Shafer's theory of evidence, statistical tools, and information technology are combined in the so-called evidential reasoning approach, as in (Xu et al. 2006).

- Set inclusion

Several approaches focusing on imprecision (or incompleteness), mainly in preferences, deal in reality only with the sets of values that the parameters of interest (weights of a utility function, probabilities of events, group member weights, etc.) may adopt. The approach in this case is clearly parametric and the ideas outlined above concerning sensitivity analysis are relevant here. These include the undertaking of what-if analysis (e.g., what is the optimal solution if the parametric setting is a given one); the computation of worst outcomes for each alternative to find out the alternative with best worst outcome, as a concept of robust solution; or the computation of the volume of the parametric set under which a given solution is optimal, as a way to ascertain its robustness.

The idea of set inclusion is used as we consider sets of parameters and the smaller the sets, the more precise the information is. Indeed, under the normative ideal, we shall typically have a singleton parametric set, e.g. with the corresponding unique utility function weights. Moreover, several set inclusion based procedures incorporate mechanisms to suggest directions or areas in which the parameter set may be reduced to attain higher robustness and increase the DM's confidence on the results of the analysis. Interval numbers may be included within this heading.

(Vilkkumaa et al., in print) include an overview on approaches in which set inclusion ideas are relevant.

- Scenario development and analysis

Scenario analysis may be contemplated as well within the set inclusion paradigm. It refers mainly to problems of strategic nature and long-term effects, in which we are able to identify relevant events or developments for the future, the so-called scenarios. However, for reasons such as lack of knowledge or too much uncertainty, we are not capable or it does not seem sensible trying to assess the corresponding probabilities. Sometimes these are called problems under severe uncertainty, see e.g. (Wright & Goodwin 1999). There is a clear resonance of the classical distinction between decision making under risk and under uncertainty, see e.g. (French 1986). Therefore, the standard solutions from decision making under uncertainty have been used in this case, including providing equal weights to various scenarios, minimum regret solutions, as well as variants which try to cater for various degrees of pessimism and risk aversion.

- Linguistic imprecision

Many of the papers reviewed in this area used some kind of fuzzy set based approach. Fuzzy set based concepts were introduced to cope with imprecision in language. Rather than using standard (crisp) sets, in which we may say that an element belongs or not to the set, fuzzy sets are used in which a degree of membership, between 0 and

1, is given to each element. Based on such ideas, standard set concepts are fuzzified. Similarly, standard decision support tools and methodologies have been fuzzified , e.g fuzzy cognitive maps, fuzzy aggregation rules or fuzzy MCDM approaches, see (Ross 2010).

Possibility theory was introduced as an extension of fuzzy sets and fuzzy logic by (Zadeh 1978). The key concept is that of a possibility measure which aims at modelling the possibility of an event happening. The distinctive feature of possibility measures is that the possibility of the union of two events is the maximum of the possibility of both events. Possibility theory uses also the necessity of an event, which is one minus the possibility of the complementary set. Recommendations are based on extensions of integral concepts (with respect to possibility measures) like Sugeno's integral.

Interestingly enough, possibility (and necessity) measures may be seen as particular cases of upper and lower probabilities.

Figure 2 provides a frequency distribution of the various approaches used in dealing with uncertainty within DSS. Probabilistic approaches are the most frequent, followed closely by those based on fuzzy concepts, including possibility measures. The other approaches appear much less frequently. Note that probabilistic approaches, followed by fuzzy approaches have prevailed over the years (Figure 3). However, in recent years the variety of uncertainty modelling strategies has increased.

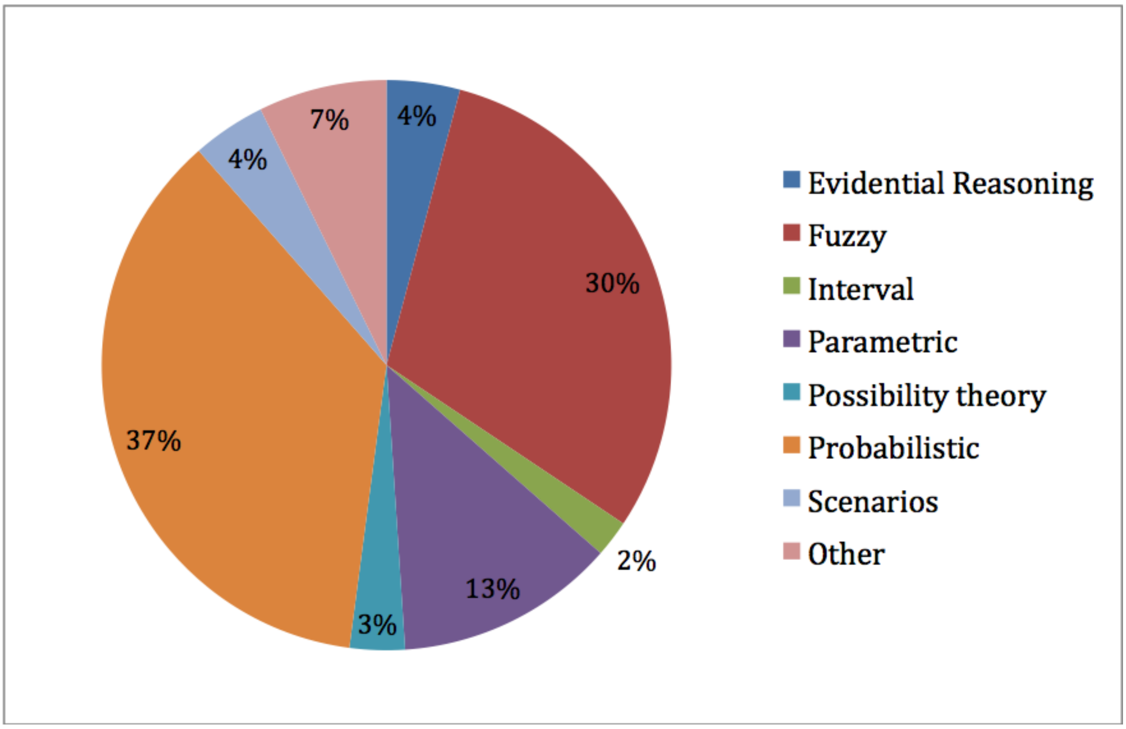

Figure 2. Frequency of different uncertainty modelling strategies in this review.

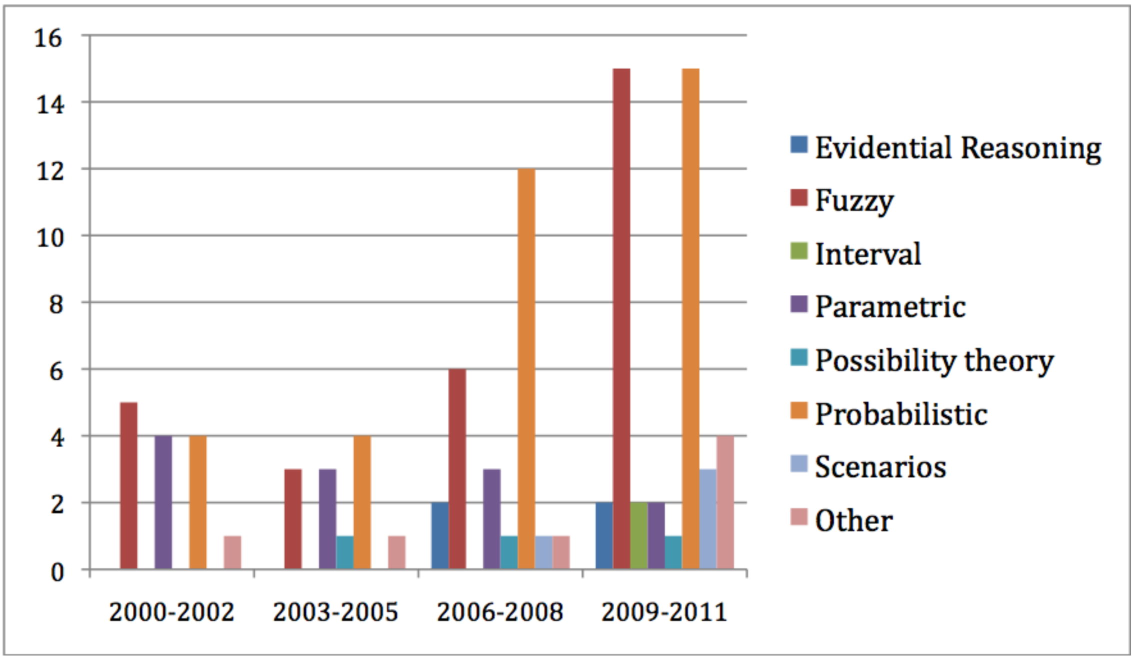

Figure 3. Evolution of strategies to model uncertainty.

3. Underlying decision support methodology

Concerning the underlying decision support methodology within which the treatment of uncertainty is considered, we have identified the following categories in this review: optimization, multi-criteria decision analysis (MCDA), rules, simulation, cognitive maps, artificial neural networks and Bayesian networks. The main features of these approaches are briefly described below.

- Optimization

Optimization-based DSS generally use an optimization engine for (linear, non-linear, integer) mathematical programming models, which may be built upon a commercial solver or is specifically tailored, for instance, to make the most of special algorithmic features taking advantage of structural properties of the problem at hand. In particular, to deal with complex combinatorial or/and strongly non-linear models meta-heuristics are also used, which may be hybridized with mathematical programming approaches (generally under the designation math-heuristics), such as genetic/evolutionary algorithms, simulated annealing, or tabu search (Gendreau & Potvin 2010).

One of the most important features required for optimization-based DSS, particularly in dealing with large-scale optimization models, is the capability of problem generation, in general using some kind of algebraic modelling languages. These are high-level programming languages offering a similar syntax to the usual mathematical notation for optimization problems. This permits a clear separation between the model structure, the data to be supplied to the model and the specific solution method. Furthermore, this enables model changes and maintenance to be carried out in a gradual manner over the system lifetime. However, the use of such algebraic modelling languages has not been reported in the set of papers reviewed.

Moreover, due to the intrinsic complexity of optimization models, optimization-based DSS should provide results analysis and reporting capabilities to assist DMs who are knowledgeable regarding the application domain but to whom expertise on mathematical models and algorithms cannot be required.

Techniques to cope explicitly with uncertainty in optimization models include stochastic programming (in which uncertainty is modelled through discrete or continuous probability distributions), fuzzy programming (in which coefficients are modelled through fuzzy numbers, and constraints and objective functions for which a goal is specified are dealt with as fuzzy sets), interval programming (in which the coefficients are interval numbers, that is, they are unknown but bounded with no need to specify probabilistic distributions as in stochastic programming or possibilistic distributions as in fuzzy programming). The establishment of plausible ranges in which model coefficients and parameters may drift is also considered by robust optimization, which seeks to compute solutions compatible with all those values. That is, solution robustness is measured in terms of its best performance against all possible realizations of the coefficients and parameters values. Min-max and min-max regret are usual formulations in the realm of robust programming (Kouvelis & Yu 1987), in which it is also possible to establish, for instance, a probability for which the solution is required to satisfy specific constraints.

- Multi-criteria / multi-objective models (MCDA)

A multi-criteria decision support system (MCDSS) is based on multi-criteria models and methods. In general, the label multi-criteria encompasses models in which the set of potential solutions are implicitly defined by a set of constraints (multi-objective) or they are explicitly known a-priori (multi-attribute).

In multi-objective models, the multiple axes of evaluation of potential solutions are operationalized through objective functions to be optimized, usually conflicting and incommensurate, in the feasible region defined by the set of constraints. The aim of the decision process may be characterizing as extensively as possible the non-dominated solution set in order to learn about the underlying trade-offs in different regions of the search space or supporting the DM in recognizing a compromise solution providing an acceptable balance between the competing objective functions. This might be accepted as the final outcome.

Uncertainty in multi-objective models usually concerns the coefficients of the objective functions and/or the constraints. It may also concern the decision (control) variables values, e.g. assessing the degrading of the objective function values if the decision variables drift from their optimal values within a certain range. The approaches mentioned for optimization (stochastic programming, fuzzy programming, and interval programming) are also applied for multi-objective models. In this case, instead of evaluating the behaviour of the optimal solution in face of uncertainty, the behaviour of a non-dominated solution (or a sample of the non-dominated solution set or even the entire non-dominated frontier) is assessed.

In multi-attribute models three types of problems are generally considered: choice (selecting the best alternative or a reduced sub-set of alternatives for further screening), sorting (assigning the alternatives to pre-defined ordered categories of merit), or ranking (generating a complete or partial ranking of the alternatives from the best to the worst, possibly accepting ties). Since in these models there is not a prominent solution due to the conflicting nature of the criteria, the involvement of the DM in providing information about his/her preferences is of paramount importance. MCDSS should be designed to help the DM getting a better understanding of complex decision problems, through an interactive process with a constructive framework ranging from problem structuring to shaping the decision model and alternatives.

Uncertainty in multi-attribute models usually concerns the performances of the alternatives on the different criteria and/or the preference-related parameters (e.g., criterion weights). The uncertain performances of the alternatives are usually addressed using scenarios, probability distributions, or fuzzy sets. The difficulties of setting parameter values are usually addressed performing parametric analyses (sensitivity analysis, robustness analysis) of sets of acceptable model versions, scenarios (namely to take into account multiple perspectives), or using linguistic quantifiers.

- Rules

If-then rules are used to structure information with a semantic content about a specific domain, allowing relationships to be defined between the data. Usually, an inference engine combines the rules from the knowledge base with new data to provide a recommendation. This type of DSS is based on rules elicited from human domain experts that imitate reasoning of a human expert in that domain, expectedly at a comparable level. Caution must be taken since imitating human thinking and its efficient heuristic principles, which is well described e.g. in (Gigerenzer et al 1999), may also lead to imitate its flaws. These systems are generally endowed with some explanatory capabilities to justify why a particular recommendation has been given.

The treatment of uncertainty may be made using fuzzy rules, whose membership functions are aimed at capturing, for instance, linguistic variables through statement such as: “if control variable x is low, then radiation should increase”. Bayesian networks, described below, may be viewed as if-then rule-based systems with probabilities.

- Simulation

Simulation generally refers to approaches that replicate computationally the behaviour of an actual or projected human or physical system. Typically in a simulation-based DSS, several simulation runs are executed and their aggregate results lead to recommendations. The decision variables in the model are the inputs that are manipulated in the test.

Simulation-based DSS are mostly used for dynamic analysis of system operations, for predicting and exploring the system behaviour (i.e., assessing the effects of specific events and actions). Agent-based simulation has recently gained an increasing importance due to the availability and affordability of computational power, which makes possible to adopt a bottom-up system perspective, i.e. focusing on the system components as the essential units (which have very simple behaviour) and complexity gradually emerges as the analysis progresses to upper system levels. Simulation-based DSS are often coupled with visual tools, which enable visual information feedback about the system behaviour.

Uncertainty may be related with information arising from the random behaviour of physical systems (dealt with statistical and probabilistic methods) and from human perception and cognition processes (for which fuzzy logic and neural networks are often used).

Simulation-based DSS have been used to deal with problems in production scheduling, manpower planning, for instance in call-centres, inventory planning and control, queuing systems, designing overbooking policies, hedging against financial risks, etc.

- Cognitive maps

Cognitive maps display a representation of human thinking about a specific domain, by graphically mapping concepts and their inter-connections, identifying causes and effects, and explaining causal links. Cognitive maps may be used to unveil mental models of DMs, particularly shaped by the ways in which they anticipate events.

Fuzzy cognitive maps have been used to model dynamic systems with uncertain, imprecise and incomplete causal information (Kosko 1992; Glykas 2010). A fuzzy cognitive map is a fuzzy signed oriented graph, in which nodes represent concepts and directed graphs interconnecting nodes represent causal relationships. Each concept has state values, which reflect the degree with which the concept is active at a particular time. The state space of nodes is modelled as a fuzzy set to represent the concept. The weight associated with the directed arc measures the strength of the causal relationship. In general, this weight lies between -1 (strong negative causality between the concepts) to 1 (strong positive causality), while 0 indicates there is no causal relationship. The inference process in fuzzy cognitive maps is an iterative process consisting of updating the state vector values according to a weight matrix and initial conditions, in a discrete time mode. The descriptive approach of standard fuzzy cognitive maps may be complemented with probabilistic information to consider the impacts of randomness (caused by occurrence of random events) besides fuzziness (model of inexactness mostly due to human judgment).

- Artificial neural networks

Artificial neural networks, the analogy of which is emulating the functioning of human brain, consist of neurons - highly distributed interconnected adaptive nonlinear processing units - and synapses - structural and functional units that mediate the interactions between neurons (Kosko 1992). Knowledge is acquired by means of a learning process and synapse weights are used to store it. According to their architecture, neural networks may be classified as non-adaptive (no feedback loop exists), unsupervised (network weights are changed according to some specified set of rules - self-organization) and supervised (an external function provides a measure of the output quality). The hybridization between fuzzy logic and neural networks is generally aimed at capturing cognitive uncertainty, in which the latter are used to design and tune fuzzy membership functions to produce better output decisions. As weights are used in some MCDA methodologies to reflect, up to a certain extent, the DM’s preferences, neuron inputs are weighted to represent the relative importance of each input to a processing element. Therefore, the ability to learn and generalize can be interesting in MCDA-based DSS, for instance, by using former decisions (in similar contexts) to tune a network of methods capable of replicating decisions (thus recognizing patterns of decisions) (Antunes & Tsoukiàs 1997).

- Bayesian networks

Bayesian networks provide a graphical representation of causality relationships between random variables using a directed acyclic graph, in which nodes represent variables and arcs represent the causality relationship between those variables. Each variable has a finite set of mutual exclusive states and a conditional probability table is assigned to each variable and its parents in the graph, thus leading to a joint probability distribution over the variables in the graph. Bayesian networks enable to integrate uncertainty in DSS, namely those based on expert knowledge and/or data measurements, in terms of probability of occurrence of an event knowing that some particular event occurred or to derive an a-priori unknown relationship between events through an inference and learning process. See (Jensen & Nielsen 2010) for a full description.

Algorithms for computing posterior and predictive probabilities include belief propagation and junction trees, as well as statistical sampling techniques in large Bayesian networks. Influence diagrams may be viewed as an extension of Bayesian networks that can represent and solve decision problems under uncertainty by adding utility nodes (holding a table of utility values for all value configurations of the parent nodes) and decision nodes. They typically provide a more compact description than that provided by decision trees and are solved with a combination of probabilistic manipulations and dynamic programming. This allows performing decision related tasks, such as computing the expected utility and finding the optimal decision. Sometimes handling influence diagrams is too complex and we may need to use Monte Carlo simulation, possibly based on Markov chains (MCMC), see (Bielza et al. 1999).

About two-thirds of the papers included in this review use MCDA, optimization and rule-based approaches (see Figure 4). Figure 5 displays a trend revealing the growing importance of multi-criteria approaches to capture not just the conflicting and incommensurate nature of the axes of evaluation of the merits of the courses of action but also to link this aspect to cope with uncertainty. It is also noticeable that Bayesian networks and fuzzy cognitive maps have not been much used in DSS until the more recent years, because efficient algorithms to perform the required computations were developed only lately.

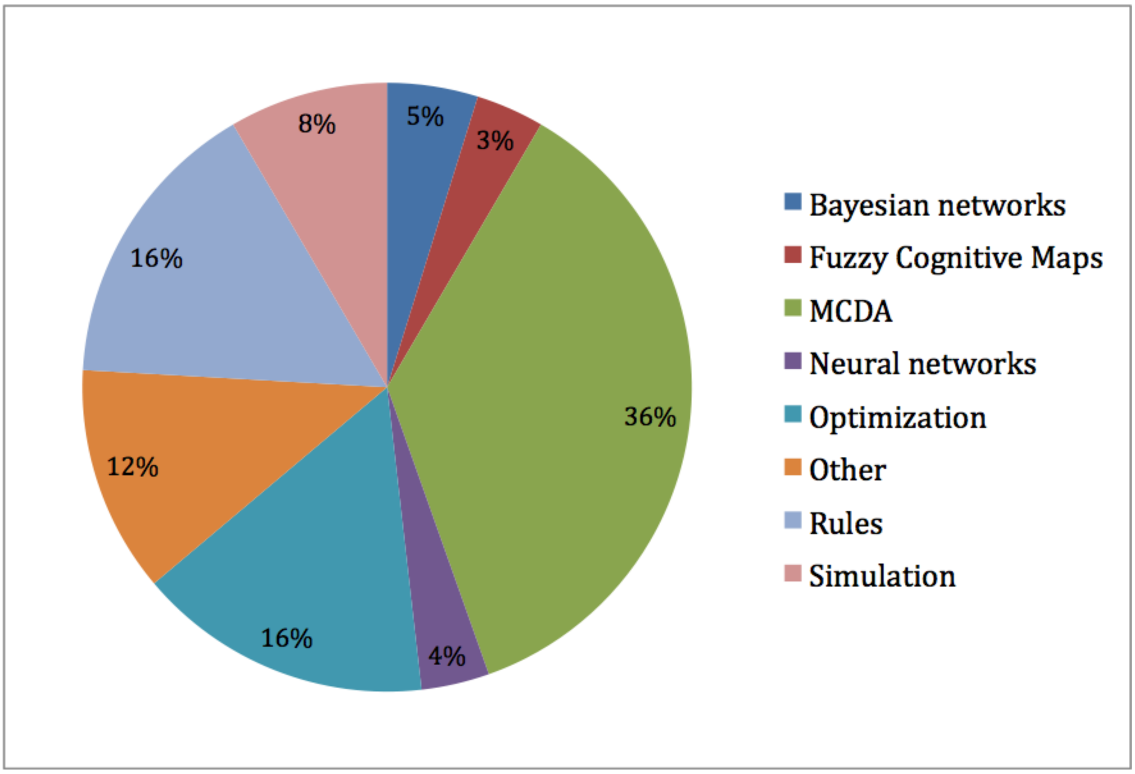

Figure 4. Frequency of underlying decision support methodology in this review.

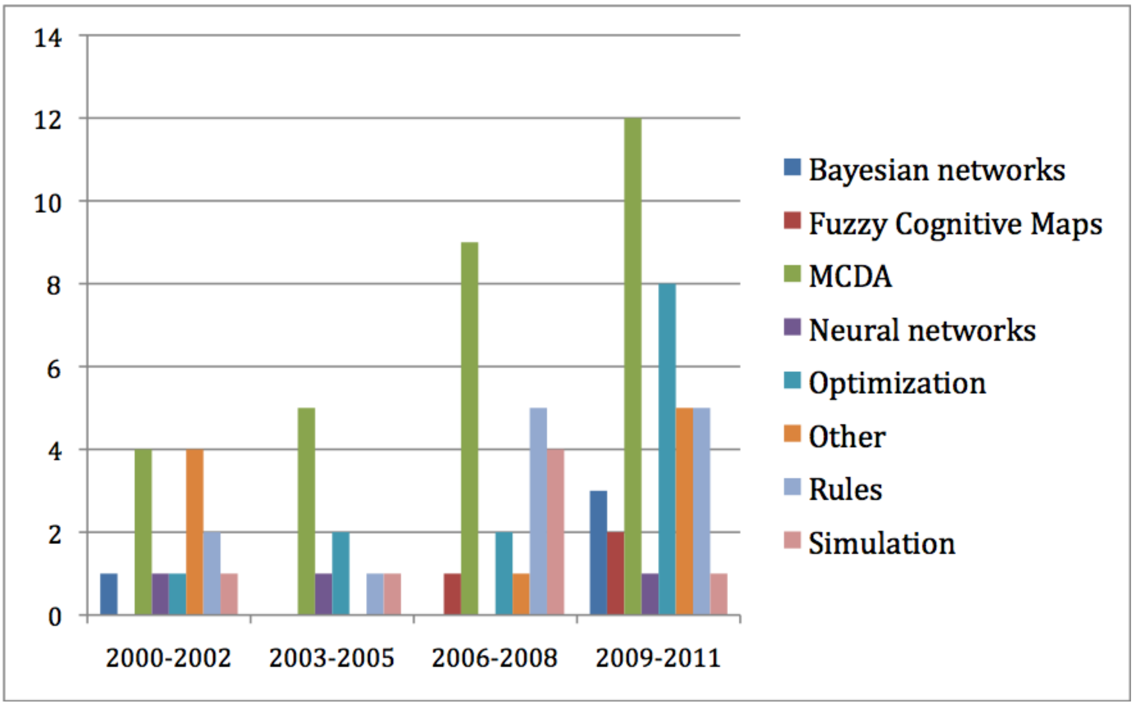

Figure 5. Evolution of underlying decision support methodology used.

4. DSS type

As far as the DSS type is concerned, we consider the following categories: group-oriented, data-oriented, geography-oriented, knowledge-oriented, model-oriented, and web-oriented. Note that our categorization closely follows that of (Power 2004).

- Group-oriented

Group Decision Support Systems (GDSS) aim at improving the quality of decision processes whenever multiple DMs, often with conflicting goals, are involved generally in a (local or distant, synchronous or asynchronous) meeting environment. In these processes, in which negotiations and the establishment of compromises are required, GDSS should play a role in dealing with avoiding miscommunication, resolution of conflicts and generation of new ideas to overcome stalemates. The term Collaborative DSS also appears in the literature mostly referring to assist teams of DMs in the solution of ill-structured problems, in which emphasis is placed on mutual cooperation (rather than conflict resolution). According to the functionalities provided, this type of DSS may range from simple communication schemes to some form of more sophisticated collaborative computing, either using synchronous or asynchronous communications. Note that in this type of systems a key uncertainty source arises because of possible discrepancies among experts and/or stakeholders.

- Data-oriented

Data-oriented DSS emphasize access to and manipulation of large amounts of data, in general, historical data stored in data warehouse systems. The distinction between data-oriented and model-oriented DSS sometimes appears in the literature, although it is practically impossible to design data management procedures, especially for large collections of data, without also designing data-intensive analytical models and methods in an inter-dependent manner. In large companies and the public sector, the policy analysis and design processes largely rely on such data-oriented DSS, well beyond simple data-retrieval systems. Data oriented DSS are clearly plagued with uncertainty issues, for reasons outlined in our introduction.

- Geography-oriented

Spatial DSS are designed to assist users in decision-making processes involving spatial problems, such as the location of desirable, semi-desirable or obnoxious infrastructures. Modelling and analytical capabilities are coupled with geographic information systems (GIS) capabilities using a range of spatial (land use, water reservoirs, etc.) and non-spatial information (social or economic indicators, etc.) to analyse plausible scenarios and provide decision support in shaping decisions with spatial impact, such as the location of a new airport or routing the transportation of hazardous materials. In these problems multiple criteria evaluation of the merit of different alternatives (e.g., routes, areas) are generally at stake and must be explicitly taken into account in problem structuring, model building and analysis phases. For this type of DSS, built in procedures based on spatial statistics are clearly relevant for the type of uncertainty typically involved in spatial data, see e.g. (Ripley 2004).

- Knowledge-oriented

A knowledge-oriented DSS assists the decision making process using a knowledge base, which may be constructed according to different procedures of knowledge extraction, coupled with analytical methodologies. Since information resources on which knowledge is built are generally very heterogeneous, knowledge management should rely on specific formal languages through the entire life-cycle of knowledge generation, codification, sharing, and utilization. Ontologies (formal, explicit specifications of a shared conceptualization) are often used for knowledge representation in DSS because they facilitate the computational representation of background knowledge about complex domains. Due to the lack of ability of ontologies to deal explicitly with uncertainty, Bayesian networks have been proposed for probabilistic knowledge representation under uncertainty regarding both structure and numerical information, as described in Section 3.

- Model-oriented

According to (Power & Sharda 2007), model-driven DSS are distinct from decision analysis or operations research computer-supported tools in making models accessible to non-technical users and being intended for frequent utilization in the same or similar decision situations. In model-oriented DSS, quantitative models are the core element of the DSS architecture and the level of functionality depends on the type of model - (multi-objective) optimization, (multi-criteria) decision analysis, simulation, etc. Model-oriented DSS are not, in general, data intensive but they usually require sophisticated forms of preference information elicitation to be supplied in particular to multi-criteria decision analysis or multi-objective optimization models.

- Web-oriented

Web-based and web-enabled DSS are aimed at making the most of the web capabilities to facilitate decision support to managers, for instance to deal with group decision and negotiation problems in which the participants are geographically separated or in spatial planning problems using web GIS. Web-based DSS are implemented using web specific technologies having a web server as the central component. In web-enabled DSS some components are located in a remote legacy system and a browser can be used to access the full DSS functionalities. A present trend is DSS designed as stand-alone systems being migrated to web-enabled DSS adding the necessary web technology components, which involves lower costs and faster redevelopment processes than redesigning and implementing a full web-based system. This trend is witnessed, for instance, in moving mapping and GIS functionalities to the internet, thus profiting from the potential to make distributed geographic information and applications widely available using a browser. A popular strand in this respect is the new generation of e-participation systems that facilitate in various forms group decision support over the web, see (Rios Insua & S. French 2010) for a description.

Model-oriented DSS are by far the most reported in this review, followed by knowledge-oriented DSS also displaying a growing trend (Figures 6 and 7). This trend recognizes the need to use well-structured decision models, and combine them with expert knowledge, to deal with several sources of uncertainty and reach robust conclusions.

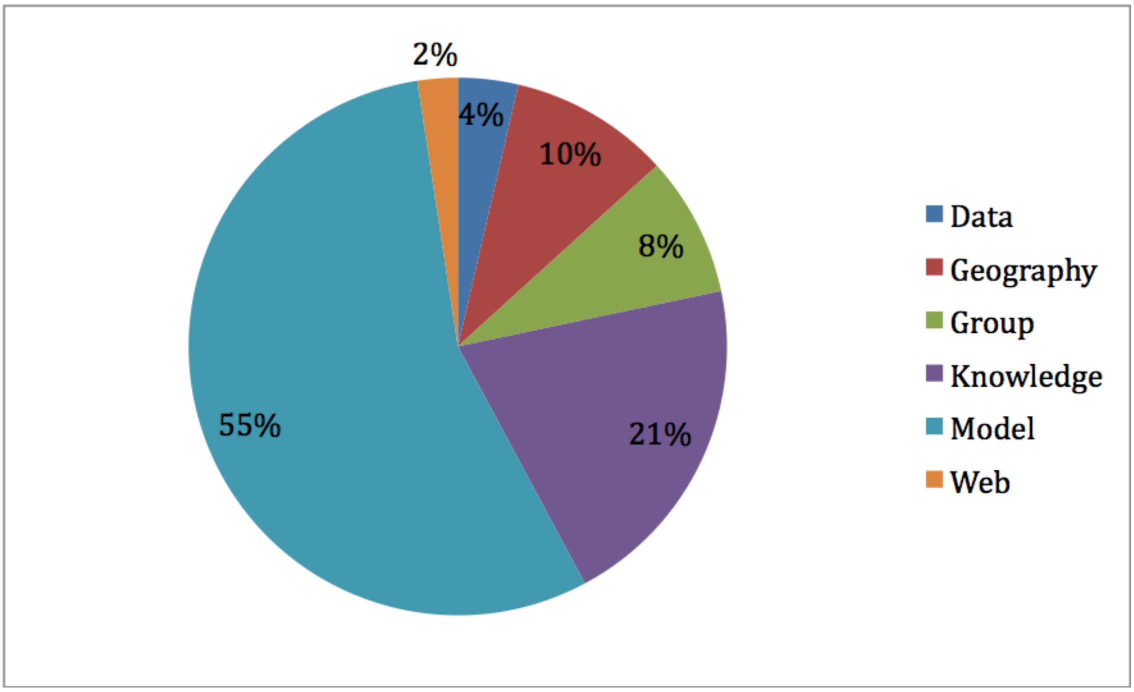

Figure 6. **Frequency of different DSS types in this review.**

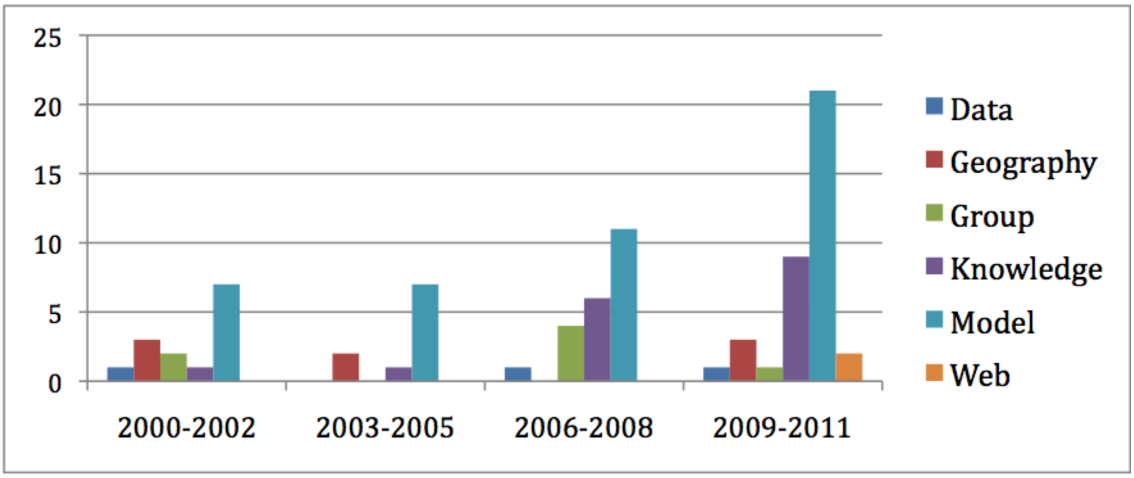

Figure 7. **Evolution of DSS types used.**

5. Application areas

The scope of applications that emerged in this review is quite diverse, suggesting the wide applicability of the methods, strategies and architectures proposed. The most represented application areas in this review are marketing/strategy, finance/economics, environment, water resources, medical, and operations management, which account for about three-quarters of the applications reported in the literature reviewed (Figure 8). Somewhat more in detail, we may say that:

- Topics in marketing/strategy include marketing and strategy development, business process planning, fostering innovation, supplier selection, real state retailing and appraisal and negotiation. Typical uncertainties would include selling prices or degree of success of innovation initiatives.

- Topics in finance/economics include portfolio optimization, asset allocation, investment/project evaluation, cost estimation, price quoting, and insurance pricing. Typical sources of uncertainty would include market stability and asset prices.
- Topics in environmental management include restoration of contaminated ecosystems, land depletion assessment, assessment of regional climate change impacts, solid waste management, and life-cycle assessment of alternative technologies. Typical uncertainties would stem from changes in local weather patterns and evolution of contaminated ecosystems.
- Topics in water resources include ecological and hydrological processes, flood risk assessment and management, evaluation of drainage water tunnels, river basin planning and management, and water quality management. Typical uncertainties would include water demand for various purposes or water inflows at relevant points of the river basin.
- Topics in the medical area typically intend to model expert knowledge for medical diagnosis and prescription, with uncertainties in relation with the incumbent illnesses, their impact for instance in work productivity or retirement age, and the relevant risk factors.
- Topics in operations management include inventory management and control, scheduling, and maintenance, with uncertainties such as the product demand, availability of production machines, and cost estimates.

The evolution panorama (Figure 9) shows two noteworthy aspects: the range of application areas has been steadily growing, especially in connection with water resources management.

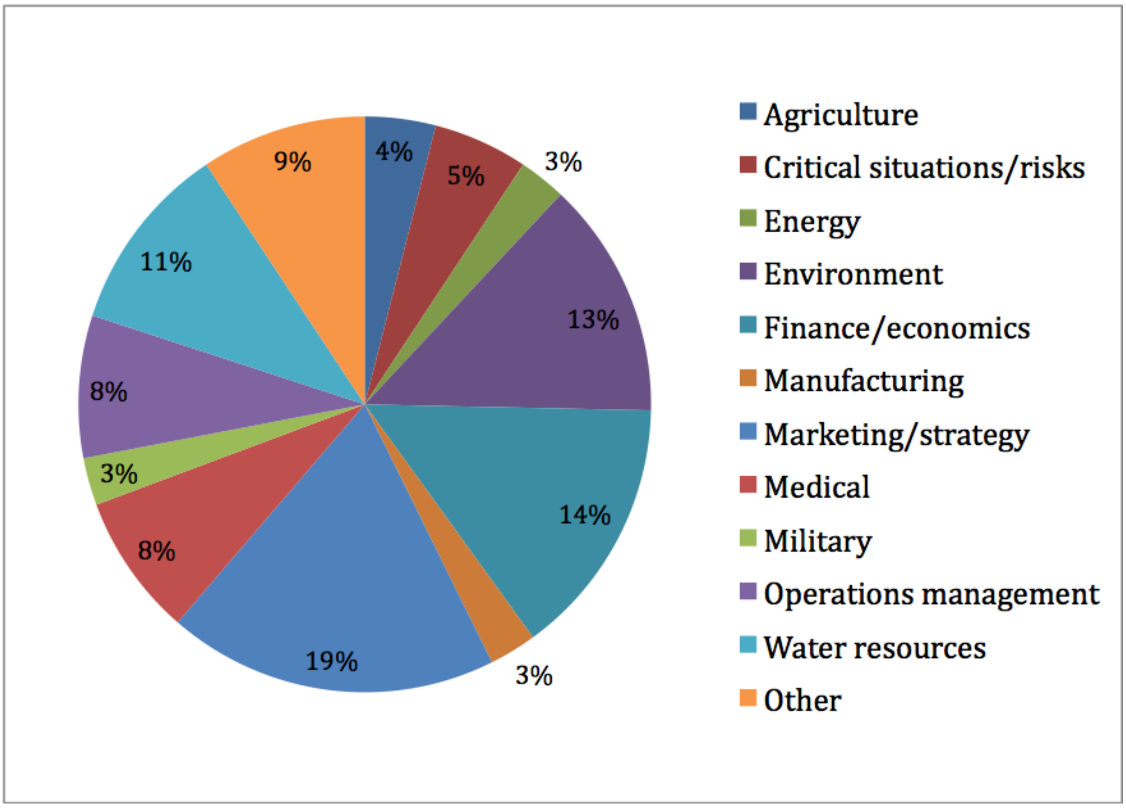

Figure 8. Frequency of application areas in this review.

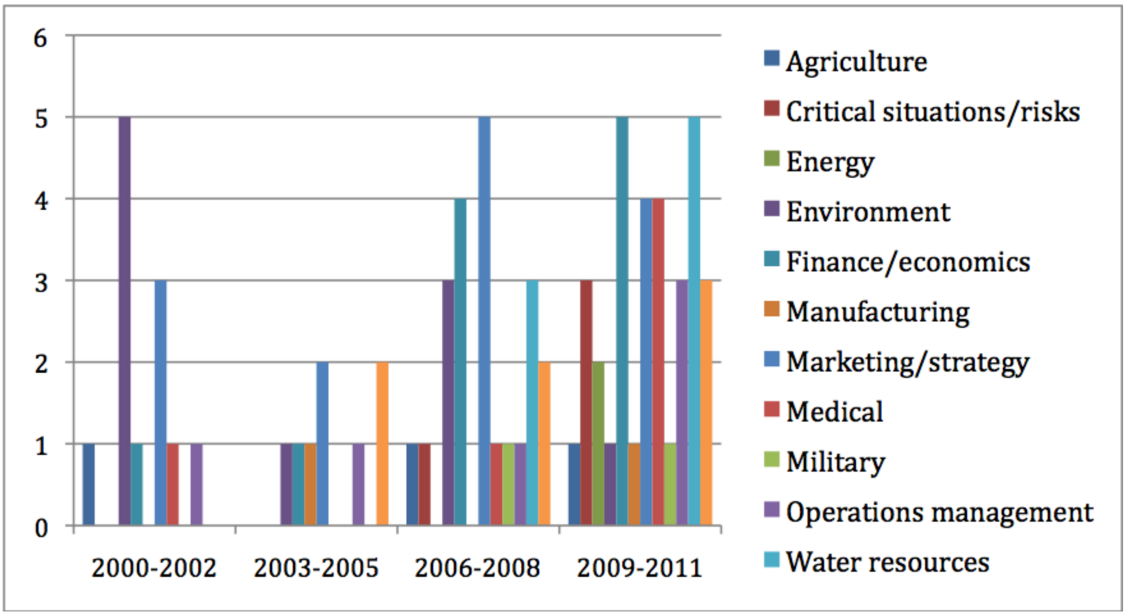

Figure 9. Evolution of application areas.

6. Conclusions

We have provided an overview of how issues in relation with uncertainty, whatever its source might be, have been dealt within the applied DSS literature over the last twelve years. The variety of strategies to deal with uncertainties has increased in recent years. An interesting conclusion is that uncertainty is being increasingly addressed in decision support, with probabilistic models and fuzzy based approaches the most frequent strategies. Probabilistic models refer mainly to uncertainty issues, whereas fuzzy based models refer mainly to imprecision. When both issues arise, one is frequently led to one of the imprecise probability approaches, including possibility measures.

Although other frameworks such as Bayesian networks and fuzzy cognitive maps have been increasingly used in the more recent years, the underlying approaches for dealing with uncertainties are mostly MCDA, optimization and rule-based approaches, with a strong increasing tendency for MCDA. This brings the issue of dealing with uncertainty about preference-related parameters, which contributes to having ten cases in which a parametric analysis is performed. MCDA is also often associated with probabilistic modelling (ten cases) and fuzzy modelling (twelve cases).

In terms of DSS type, model-oriented DSS prevail, followed by knowledge-oriented DSS also displaying a growing trend. As expected, model-oriented DSS tend to be associated with MCDA or optimization approaches, whereas knowledge-oriented DSS tend to be associated with rules. Geography-oriented DSS are in a distant third position, with similar numbers of MCDA, optimization, and simulation approaches.

Concerning application areas, the areas of management prevail: applications on marketing/marketing plus economics/finance represent one third of the total. Next, one can identify a group of applications concerned with environmental sustainability: agriculture, energy, environment, and water resources account for approximately 30% of the applications.

Probabilistic and fuzzy strategies to deal with uncertainty are used in all fields. Fuzzy strategies are dominant in marketing, whereas probabilistic strategies seem to dominate in medical, environment and water resources applications (Table 3). Concerning the underlying approach (see Table 4), MCDA dominates in environment and water resources. In finance/economics MCDA and optimization share the protagonism. In marketing/strategy, MCDA and rule-based approaches prevail. Operations management is an area in which optimization is the most used.

	Evidential Reasoning	Fuzzy	Interval	Parametric	Possibility theory	Probabilistic	Scenarios
Agriculture		1				2	
Critical situations/risks		3				1	
Energy		1	1				1
Environment		2		3	1	5	
Finance/economics	1	3			1	5	1
Manufacturing		1					
Marketing/strategy	1	8		3		3	
Medical	1	1				4	
Military		1				2	
Operations mgt.		1		2		3	

Water resources	3	1	6	1

Table 3. Application vs Strategy to deal with uncertainty.

	Bayesian networks	FCMs	MCDA	Neural networks	Optimization	Rules	Simulation
Agriculture	1					1	1
Critical situations/risks		1	1		1		1
Energy			1		1		
Environment			6			1	
Finance/economics			4		4	2	1
Manufacturing				1			
Marketing/strategy		1	6	1	1	5	
Medical		1		1		3	1
Military	1		1				
Operations mgt.			1		4		
Water resources			4		1		2

Table 4. Application vs Underlying approach

We may, therefore, conclude that uncertainty is of growing concern in actual decision support systems, as indeed should be, with an increasing number of DSS addressing issues in connection with uncertainty in many application domains. Given this vitality, which is witnessed in this review, some issues could be discussed as outlined below.

Most of the DSSs reviewed are ad-hoc in the sense that they are developed with a specific application in mind. It seems there is ground for the development of generic DSSs based on given methodologies. Some generic systems include WINBUGS, implementing Markov chain Monte Carlo methods in graphical models, or GeNIe to evaluate influence diagrams.

A methodological area of outmost importance for applications is the elicitation of judgments, both as far probabilities and other paradigms are concerned. Much remains to be done in the art and science of elicitation procedures. In this respect, an important problem refers to multiple experts and multiple models, one example being uncertainty in climate change. In this area, there are numerous competing models, most of them predicting global warming, differing, however, in the estimates of the extent of such effect and the uncertainty around it. We may deal with such issue through model averaging; however, most approaches assume independence among models and expert judgments, typically leading to less uncertainty than should be acknowledged.

The example of climate change also points out to another emerging area, that of severe uncertainty, which frames much of long-term policy making. This is frequently undertaken via scenario analysis and there is an increasing interest in combining this approach with MCDA methods.

Solving these, and other important challenges, will no doubt help in promoting even more the incorporation of uncertainty models into actual decision aiding and decision support systems. While writing this review, much of Europe is drowning on economic uncertainty amid discussions by politicians attempting to steer a continent with their actions (and inactions). We do not anticipate any shortage of opportunities to use uncertainty modelling methods and software.

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