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Lowering the critical temperature with eight-quark interactions

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Abstract

It is shown that eight-quark interactions, which are needed to stabilize the ground state of the combined three flavor Nambu–Jona-Lasinio and 't Hooft Lagrangians, play also an important role in determining the critical temperature at which transitions occur from the dynamically broken chiral phase to the symmetric phase.

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For a long time already, a phase transition which almost certainly occurs in hadronic matter at finite temperature is at the centre of attention of many researches and reviews [1–4]. It may be very important in the context of heavy-ion collisions and the evolution of the early Universe. At present, experimental facilities with ultra-relativistic heavy-ion reactions aiming at studying the QCD phase diagram have triggered even more intense studies from the theoretical side, ranging from effective models to calculations on the lattice. Recent lattice results predict for vanishing chemical potential and massive quarks a non-singular crossover for the QCD transition which lead to different critical temperatures, depending on the considered observables. The transition related to the renormalized chiral susceptibility yields $T_c = 151(3)(3)$ MeV, whereas for instance the inclusion of the Polyakov loop increases this value by ~ 25 MeV [5].

An effective chiral model which has been widely used to extract the low energy characteristics of hadrons involving the u, d, s quarks is the well known Nambu–Jona-Lasinio model [6], generalized to the $SU(3)_L \times SU(3)_R$ chiral symmetry and to include the $U(1)_A$ symmetry breaking of low energy QCD in form of the $2N_f$ determinantal interaction of 't Hooft (N_f denoting the number of flavors) [7]. This combined Lagrangian (we denote it by NJLH) has been first analyzed in [8] and [9] to study dynamical breakdown of chiral symmetry and related meson spectra in the vacuum, and since then in further numerous applications [10–14]. In more recent calculations the model is used to obtain the thermodynamic properties of hadrons and restoration of chiral and $U(1)_A$ symmetries, both at zero and finite chemical potential [15], and extended to include the colored diquark channels, of importance at large baryonic densities [16]. Calculations performed with the NJLH at zero chemical potential yield a critical temperature for the light quarks around $T_c \sim 200$ MeV, where the condensate undergoes rapid changes. This value clearly overestimates the lattice prediction.

In the present Letter, we argue that by extending the NJLH model to include interactions of eight quarks, the critical temperature is lowered considerably. The eight quark vertices have been introduced first in [17] to stabilize the scalar effective potential derived in the presence of ($2N_f = 6$) 't Hooft interactions [18] and have been discussed at length in [19].

We consider the following Lagrangian density

$$\mathcal{L}_{\text{eff}} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right)q + \mathcal{L}_{4q} + \mathcal{L}_{6q} + \mathcal{L}_{8q} + \cdots. \qquad (1)$$

Quarks q have color ($N_c = 3$) and flavor ($N_f = 3$) indices which are suppressed here. We suppose that multi-quark inter-

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actions $\mathcal{L}_{4q}$, $\mathcal{L}_{6q}$, $\mathcal{L}_{8q}$ are

$$\mathcal{L}_{4q} = \frac{G}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right], \tag{2}$$

$$\mathcal{L}_{6q} = \kappa(\det\bar{q}P_L q + \det\bar{q}P_R q), \tag{3}$$

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \tag{4}$$

where the flavor space matrices λ_a, $a = 0, 1, \ldots, 8$, are normalized such that $\mathrm{tr}(\lambda_a\lambda_b) = 2\delta_{ab}$, and $\lambda_0 = \sqrt{\frac{2}{3}}1$, λ_a at $a \neq 0$ are the standard $SU(3)$ Gell-Mann matrices. Chiral projectors are $P_{L,R} = (1 \mp \gamma_5)/2$, the determinant in the 't Hooft Lagrangian is over flavor indices. The eight-quark interactions are given by

$$\mathcal{L}_{8q}^{(1)} = 8g_1\left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)\right]^2, \tag{5}$$

$$\mathcal{L}_{8q}^{(2)} = 16g_2(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_j)(\bar{q}_j P_R q_k)(\bar{q}_k P_L q_i). \tag{6}$$

Here the summation over repeated flavor indices is assumed. One sees that the most general spin zero eight-quark interaction is composed of two chiral invariant combinations of products of four quark bilinears with couplings g_1, g_2, of which one, $\mathcal{L}_{8q}^{(1)}$, is OZI violating.

The model has eight parameters: The couplings of multi-quark interactions G, κ, g_1, g_2; three masses of current quarks, m_i, and the cutoff Λ (the model being not renormalizable).

Since there is some freedom in the model parameter choices that in the realistic case with $SU(2)_I \times U(1)_Y$ flavor symmetry and non zero current quark masses, $m_u = m_d \neq m_s$, lead to qualitatively similar spectra for the low lying pseudoscalars and scalars, we chose as input several sets of parameters at $T = 0$.

We put furthermore all current quark masses to zero to have clean signatures of the phase transitions. This will not affect the trends we are about to discuss in connection with the pivot temperature values T_i (see below) obtained with and without the stabilizing eight-quark interactions. On the other hand, this simplifies essentially our analysis.

Let us recall that at $T = 0$ the effective potential of the model as a function of the constituent quark mass, M, in the $SU(3)$ limit is [19]

$$V(M) = \frac{h^2}{16}\left(12G + \kappa h + \frac{27}{2}\rho h^2\right) - \frac{3N_c}{16\pi^2}\left[M^2 J_0(M^2) + \Lambda^4\ln\left(1 + \frac{M^2}{\Lambda^2}\right)\right], \tag{7}$$

with Λ being an ultraviolet covariant cutoff in the quark one-loop diagrams, $\rho \equiv g_1 + \frac{2}{3}g_2$, and

$$J_0(M^2) = \Lambda^2 - M^2\ln\left(1 + \frac{\Lambda^2}{M^2}\right). \tag{8}$$

The function $h(M)$ is a real solution of the stationary phase equation related to the integration over bosonic auxiliary variables (for details see e.g. [17])

$$M + Gh + \frac{\kappa}{16}h^2 + \frac{3}{4}\rho h^3 = 0. \tag{9}$$

This equation yields a one-to-one real-valued mapping $M \to h$ in the parameter region

$$\rho > 0, \qquad G > \frac{1}{\rho}\left(\frac{\kappa}{24}\right)^2 \tag{10}$$

which defines the stability conditions in the $SU(3)$ limit of the effective potential.

The gap equation, obtained from the extremum condition $dV/dM = 0$, relates further the function $h(M)$ to the quark condensate as

$$h(M) = -\frac{N_c}{2\pi^2}MJ_0(M^2) = 2\langle 0|\bar{q}q|0\rangle. \tag{11}$$

The curvature of the effective potential $V(M)$ at the origin, $M = 0$, in the chiral limit is determined at $T = 0$ by the value of

$$\tau = \frac{N_c}{2\pi^2}G\Lambda^2, \tag{12}$$

with $\tau > 1$, $\tau = 1$ and $\tau < 1$ indicating a maximum, a saddle point and a minimum, respectively.

At finite temperature, after introducing the standard non-covariant three-momentum cutoff λ for the quark loops, see e.g. [20], one obtains for the effective potential

$$V_T(M) = \frac{h^2}{16}\left(12G + \kappa h + \frac{27}{2}\rho h^2\right) - \frac{3N_c}{\pi^2}\int_0^\lambda dp\, p^2\left[2T\ln\left(1 + \exp\left(\frac{E(p)}{T}\right)\right) - E(p)\right], \tag{13}$$

with $E(p) = \sqrt{p^2 + M^2}$ and p denoting the magnitude of the 3-momentum.

The curvature at the origin is now a temperature dependent function

$$\tau(T) = \frac{2N_c}{\pi^2}G\lambda^2 F(t), \quad t = \frac{T}{\lambda}, \tag{14}$$

$$F(t) = -\frac{1}{2} + \frac{t^2}{6\pi^2} + 2t\ln(1 + e^t) + 2t^2 L_2[-e^t], \tag{15}$$

where

$$L_n[z] = \sum_{k=1}^{\infty}\frac{z^k}{k^n} \tag{16}$$

denotes the polylogarithm function.

The function $F(t)$ is depicted in Fig. 1. To guide the discussion we show in Fig. 2 the temperature dependence of the effective potential for the parameter set (e) of Table 1, for $T = 0$ (dashed line) and three "critical" values T_a, T_d, T_s, introduced as follows.

In the zero temperature limit the curvature reduces to $\tau(0) = N_c G\lambda^2/\pi^2$. From this we see that if we wish to adjust our zero of temperature curvature so that $\tau(0) = \tau$ we must choose $\Lambda^2 = 2\lambda^2$. The function $F(t)$ decreases monotonically to zero as $T \to \infty$, starting from $F(0) = 1/2$ and therefore, if the curvature $\tau > 1$ at $T = 0$, the system will undergo at some critical temperature $T = T_a$ a change of curvature at the origin, starting

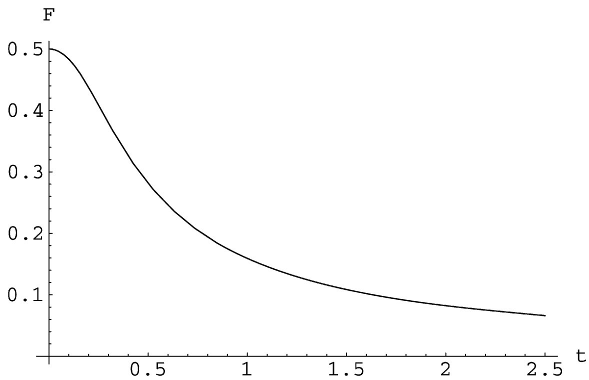

Fig. 1. The function $F(t)$ related with the curvature of the effective potential at the origin, in dimensionless units $t = T/\lambda$.

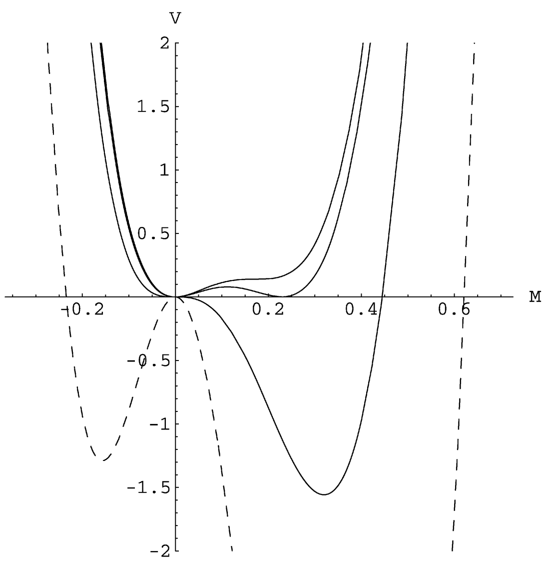

Fig. 2. The effective potential V (in units $(10\ \mathrm{GeV})^{-4}$) for the parameter set (e) as function of M (in GeV). Full curves: lowest for T_a, middle for T_d and upper for T_s. Dashed line: $T = 0$ case, the physical minimum occurs outside the plotting range.

Table 1
Parameters of the model at $T = 0$: G (GeV^{-2}), Λ (MeV), κ (GeV^{-5}), g_1, g_2 (GeV^{-8}). Indicated are also the temperatures T_a, T_d, T_s (MeV) (see text)

Sets	Λ	G	$-\kappa$	g_1	g_2	T_a	T_d	T_s
a	820	13.5	1300	0	0	192	202	204
d	839	12.16	1082	0	0	174	183	185
e	839	11.28	1083	1500	327	143	161	163
f	839	8.92	1083	6000	327	–	111	135

from a saddle point at T_a and entering a region of local stability near this point. Increasing further the temperature the system undergoes changes reminiscent of a first order transition, passing through a configuration with two degenerate minima at $T = T_d$ until at some higher temperature T_s a further saddle point appears, and for $T > T_s$ only the symmetric phase survives. Obviously if $\tau < 1$ the system is at $T = 0$ already in the coexistence or even in the symmetric phase.

In the following we select a few parameter sets that have been obtained previously with and without inclusion of the eight-quark interactions to determine the mass spectra of low lying pseudoscalar and scalar nonets, related weak decay constants and quark condensates. In Table 1, the model parameters Λ, G, κ, g_1, g_2, are taken from [21], set (a), and from [19], sets (d, e, f), at $T = 0$. This input is kept fixed in the finite temperature evolution of the effective potential. One sees that the eight-quark interactions present in sets (e, f) reduce considerably the temperatures T_a, T_d, T_s, as compared to the cases with $g_1 = g_2 = 0$. In the last set (f) the curvature τ is slightly below 1, so the system has already a minimum at the origin at $T = 0$.

Let us discuss now the reasons for the obtained decrease of T_c. The underlying mechanism is quite simple. The critical temperature is correlated with the value of $\tau(0)$: The closer $\tau(0)$ is to 1, the lower T_c. The eight-quark interactions decrease the effective value of $G\Lambda^2$ (to see this one should consider the mass spectrum of meson states [19]), and therefore lower the transition temperature. In the parameter sets shown we have: $\tau(0) = 1.4$ (a), $\tau(0) = 1.3$ (d), $\tau(0) = 1.2$ (e) and $\tau(0) = 0.95$ (f).

The lowering of the critical temperature T_c has also been recently observed in the $N_f = 2$ case with eight-quark interactions [22].

We conclude that the presence of the eight-quark interactions in the hadronic vacuum results in an observable effect which deserves a more detailed study. The arguments presented above lead to the expectation that the tendency will be the same when one considers the realistic case with physical current quark masses. Work in this direction is in progress.

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References

[1] R.D. Pisarski, F. Wilczek, Phys. Rev. D 29 (1984) 338.
[2] F.R. Brown, et al., Phys. Rev. Lett. 65 (1990) 2491.
[3] H. Meyer-Ortmanns, Rev. Mod. Phys. 68 (1996) 473.
[4] J.T. Lenaghan, D.H. Rischke, J. Schaffner-Bielich, Phys. Rev. D 62 (2000) 085008, nucl-th/0004006.
[5] Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, Nature 443 (2006) 675, hep-lat/0611014;
Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B 643 (2006) 46, hep-lat/0609068.
[6] Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122 (1961) 345;
Y. Nambu, G. Jona-Lasinio, Phys. Rev. 124 (1961) 246;
V.G. Vaks, A.I. Larkin, Zh. Eksp. Teor. Fiz. 40 (1961) 282, Sov. Phys. JETP 13 (1961) 192.
[7] G. 't Hooft, Phys. Rev. D 14 (1976) 3432;
G. 't Hooft, Phys. Rev. D 18 (1978) 2199.
[8] V. Bernard, R.L. Jaffe, U.-G. Meissner, Phys. Lett. B 198 (1987) 92;
V. Bernard, R.L. Jaffe, U.-G. Meissner, Nucl. Phys. B 308 (1988) 753.
[9] H. Reinhardt, R. Alkofer, Phys. Lett. B 207 (1988) 482.
[10] S. Klimt, M. Lutz, U. Vogl, W. Weise, Nucl. Phys. A 516 (1990) 429;

U. Vogl, M. Lutz, S. Klimt, W. Weise, Nucl. Phys. A 516 (1990) 469;
U. Vogl, W. Weise, Prog. Part. Nucl. Phys. 27 (1991) 195.
[11] M. Takizawa, K. Tsushima, Y. Kohyama, K. Kubodera, Nucl. Phys. A 507 (1990) 611.
[12] S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649.
[13] T. Hatsuda, T. Kunihiro, Phys. Rep. 247 (1994) 221.
[14] V. Bernard, A.H. Blin, B. Hiller, U.-G. Meißner, M.C. Ruivo, Phys. Lett. B 305 (1993) 163, hep-ph/9302245;
V. Dmitrasinovic, Nucl. Phys. A 686 (2001) 379, hep-ph/0010047;
K. Naito, M. Oka, M. Takizawa, T. Umekawa, Progr. Theor. Phys. 109 (2003) 969, hep-ph/0305078.
[15] P. Costa, M.C. Ruivo, C.A. de Sousa, Yu.L. Kalinovsky, Phys. Rev. D 70 (2004) 116013, hep-ph/0408177;
P. Costa, M.C. Ruivo, C.A. de Sousa, Yu.L. Kalinovsky, Phys. Rev. D 71 (2005) 116002, hep-ph/0503258.
[16] S.B. Ruster, V. Werth, M. Buballa, I.A. Shovkovy, D.H. Rischke, Phys. Rev. D 72 (2005) 034004, hep-ph/0503184.
[17] A.A. Osipov, B. Hiller, J. da Providência, Phys. Lett. B 634 (2006) 48, hep-ph/0508058.
[18] A.A. Osipov, B. Hiller, V. Bernard, A.H. Blin, Ann. Phys. 321 (2006) 2504, hep-ph/0507226;
B. Hiller, A.A. Osipov, V. Bernard, A.H. Blin, SIGMA 2 (2006) 026, hep-ph/0602165.
[19] A.A. Osipov, B. Hiller, A.H. Blin, J. da Providência, hep-ph/0607066, Ann. Phys. (2006), doi:10.1016/j.aop.2006.08.004, in press.
[20] W. Florkowski, Acta Phys. Pol. B 28 (1997) 2079.
[21] A.A. Osipov, B. Hiller, J. Moreira, A.H. Blin, Eur. Phys. J. C 46 (2006) 225, hep-ph/0601074.
[22] K. Kashiwa, H. Kouno, T. Sakaguchi, M. Matsuzaki, M. Yahiro, nucl-th/0608078.