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An interactive bi-objective shortest path approach: searching for unsupported nondominated solutions

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Abstract

In many network routing problems several conflicting objectives must be considered. Even for the bi-objective shortest path problem, generating and presenting the whole set of nondominated solutions (paths) to a decision maker, in general, is not effective because the number of these paths can be very large. Interactive procedures are adequate to overcome these drawbacks. Current et al. [1] proposed an interactive approach based on a NISE-like procedure to search for nondominated supported solutions and using auxiliar constrained shortest path problems to carry out the search inside the duality gaps. In this paper we propose a new interactive approach to search for unsupported nondominated solutions (lying inside duality gaps) based on a k-shortest path procedure. Both approaches are compared. © 1999 Elsevier Science Ltd. All rights reserved.

Scope and purpose

Network routing problems are generally multidimensional in nature, and in many cases the explicit consideration of multiple objectives is adequate. Objectives related to cost, time, accessibility, environmental impact, reliability and risk are appropriated for selecting the most satisfactory ("best compromise") route in many problems. In general there is no single optimal solution in a multiobjective problem but rather, a set of nondominated solutions from which the decision maker must select the most satisfactory. However, generating and presenting the whole set of nondominated paths to a decision maker, in general, is not effective because the number of these paths can be very large. Interactive procedures are adequate to overcome these drawbacks. This paper introduces an interactive procedure to assist the decision maker in identifying the "best compromise" solution for the bi-objective shortest path problem. The procedure incorporates an efficient k-shortest path algorithm to identify nondominated solutions lying inside duality gaps. Test problem results indicate that the procedure can be readily executed on a PC for large-scale instances of problems.

Keywords: Shortest path; Multiple criteria; Network routing

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1. Introduction

Shortest path problems arise in a wide variety of practical problem settings, both as stand-alone models and as subproblems in more complex problem settings. The shortest path problem is quoted as the most common problem in combinatorial operations research [2] due to its numerous applications as the largest capacity path problem, the quickest path problem, the most reliable path problem, the minimum cost–time ratio problem, the minimum cost–reliability ratio problem and various routing problems. Ahuja et al. [3] describe applications in the telecommunications and transportation industries (whenever a message or a vehicle must be sent between two geographical locations as quickly or as cheaply as possible) and in urban traffic planning (urban planners use complex optimization models for computing traffic flow patterns based on shortest paths from origins to destinations). The wayfinding in emergency evacuations [4], the location of collective facilities (where the accessibility is a main concern) in an urban or regional context and the traffic assignment problem in a transportation network [5] are other practical examples which include the evaluation of shortest paths. Elimam et al. [6] present other civil engineering applications using shortest path-based models: the study of optimal sequences of wastewater treatment processes reducing pollutants levels at minimum cost to an acceptable standard (hydraulics) and the determination of minimum cost, energy efficient composite wall and roof structures (building structures). Consequently, many algorithmic approaches for location problems, vehicle routing, urban traffic engineering and even other problems that appear to have very different structures (as wastewater treatment or structures design) rely on the solution of shortest path problems.

Expressions as "cheap and quickest", "cheap and more reliable", "cheap with acceptable standard" or "cheap and energy efficient" come frequently associated with the shortest path problem – they suggest that models can be more realistic if more than one criterion is explicitly considered. In fact, it is well-recognized that many network routing problems are multiobjective in nature [7]. This has led to significant research effort devoted to formulating and solving multi-objective shortest path problems. The criteria, in addition to total path length (or cost), which have been addressed in these problems include: accessibility to the path [8, 9], travel time [10, 14] demand satisfaction [15], environmental protection [16], risk minimization [17], and reliability [18].

Efficient exact algorithms exist for the single objective shortest path problem [19]. Unfortunately, the multiobjective case (including the bi-objective case) is NP-complete [20]. Although the calculation of the whole set of the nondominated solutions in the bi-objective case can be done easily, it must be remarked that the number of the nondominated solutions can be very large. So, this is not, in general, an effective way of presenting alternative choices to a decision maker.

Note that in multiobjective shortest path problems the nondominated solutions are those paths where the values of the objective functions are such that it is not possible to find another feasible path better than the current one in at least one objective function without worsening the value of at least another objective.

Interactive procedures are adequate to overcome these drawbacks. Current et al. [1] proposed an interactive approach for the bi-objective shortest path problem. It starts by identifying a subset of the supported nondominated solutions using a NISE-like algorithm [21]. The search for unsupported nondominated solutions is carried out solving shortest path instances with additional linear constraints. Due to the computational complexity of the constrained shortest path problem

Current et al. in [1] propose an interactive search of the previously chosen duality gap(s) based on the constrained shortest path algorithm of Handler and Zang [22]. To improve the computational efficiency of the approach, Clímaco et al. [23] and Coutinho-Rodrigues et al. [24] proposed a different related (to Current et al. [1]) approach using a k-shortest algorithm for searching nondominated solutions inside duality gaps.

In this paper we compare the two approaches mentioned above: computational results show that the k-shortest path-based approach is much faster than the constrained shortest path-based approach.

2. Mathematical formulation and unsupported nondominated solutions searching procedure (GAPS)

Several authors use a weighted sum of the objective functions (NISE-like or other approaches) to calculate supported nondominated solutions for the bi-objective shortest path problem. These can be identified by solving the following single-objective shortest path problem [SP], whose objective function is a convex combination of the original two objective functions, Z_1 and Z_2 [21, 25]. It must be emphasized that in many real-world applications the objective functions are conflituous. For instance, assume that the first objective Z_1, is to minimize route cost, and the second objective, Z_2, is to minimize total travel time. Points A, B, C, S and R in Fig. 1 represent the objective function values for five nondominated routes.

$$[\text{SP}] \quad \text{Minimize} \quad \hat{Z} = w_1 Z_1 + w_2 Z_2 \tag{1}$$

subject to

$$\sum_{j \in N} X_{1j} = 1, \tag{2}$$

$$\sum_{i \in N} X_{ij} - \sum_{k \in N} X_{jk} = 0 \quad (\text{for all } j \in N \mid j \neq 1, n), \tag{3}$$

$$\sum_{i \in N} X_{in} = 1, \tag{4}$$

$$X_{ij} \in (0, 1), \quad \text{for all } i, j \text{ pairs}, \tag{5}$$

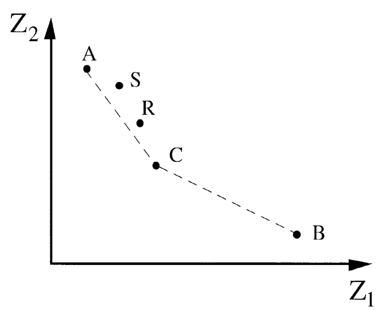

Fig. 1. A, B, C, R and S constitute the nondominated solutions set for a bi-objective (Z_1, Z_2) shortest path problem.

where

$$Z_1 = \sum_{i \in N} \sum_{j \in N} c_{ij} X_{ij}, \tag{6}$$

$$Z_2 = \sum_{i \in N} \sum_{j \in N} d_{ij} X_{ij}, \tag{7}$$

$$w_1 = \left| \frac{Z_{2\alpha} - Z_{2\beta}}{Z_{1\alpha} - Z_{1\beta}} \right|, \tag{8}$$

$$w_2 = 1, \tag{9}$$

$Z_{p\alpha}$ = the value of the pth objective function for nondominated solution α (α and β are candidates to adjacent nondominated solutions) (10)

$$X_{ij} = \begin{cases} 1 & \text{if arc } i, j \text{ is on the path,} \\ 0 & \text{otherwise,} \end{cases}$$

c_{ij}, d_{ij} = nonnegative arc "costs",
N = set of nodes in network,
node 1 is the source node,
node n is the sink node,
α, β – supported nondominated solutions.

Unfortunately these procedures do not identify unsupported nondominated solutions such as routes corresponding to S and R [21, 26].

In this paper we present a procedure to identify nondominated solutions in duality gaps (GAPS). "GAPS" incorporates a k-shortest algorithm, which, in the worst case, terminates when an upper bound is reached. The "GAPS" procedure is explained below. Computational results showing the efficiency of "GAPS" are presented in the next section.

"GAPS" is based upon solving a k-shortest path procedure [KSP] where w_1 was defined in (8) of [SP] and α and β (here used) are true adjacent nondominated supported solutions of the original bi-objective shortest path problem (e.g. solutions B and C in Fig. 2). In this case "CPB" is the duality gap where new nondominated solutions must be searched. The point P (Fig. 2) and the values $Z_{1\mathrm{P}}$ and $Z_{2\mathrm{P}}$ according to Eq. (10) enable us to obtain the first upper bound for the k-shortest search (i.e. $w_1 Z_{1\mathrm{P}} + w_2 Z_{2\mathrm{P}}$) in the duality gap defined by the points B, C and P. Note that $Z_{1\mathrm{P}} = Z_{1\mathrm{B}}$ and that $Z_{2\mathrm{P}} = Z_{2\mathrm{C}}$ (for details see Current et al. [1] or Cohon [21]).

w_1 and w_2 were defined in order to obtain $\hat{Z}$ [in (SP)] such that the line passing though $\overline{\mathrm{BC}}$ is a constant cost line for $\hat{Z}$. The optimal solution is obtained for B and C. The k-shortest path algorithm starts by looking for the first solution inside the duality gap ("CPB"). Suppose it is D in Fig. 3.

Note that to identify solutions inside the duality gap the solutions obtained using the k-shortest algorithm must verify the upper bounds of the duality gap. The first solution obtained in the duality gap is nondominated (by definition of the nondominated solutions).

By definition of nondominated solutions new eventual nondominated solutions inside the duality gap must be in the shadow area of Fig. 4. An improved upper bound for $\hat{Z}$ can be obtained from P′ in Fig. 4. It is clear how to continue the search step by step in the duality gap. It may stop

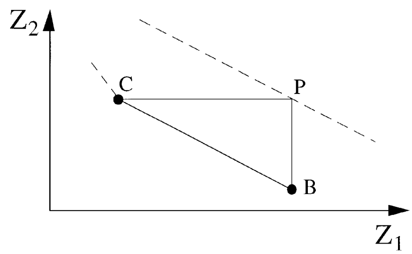

Fig. 2. The duality gap "CPB".

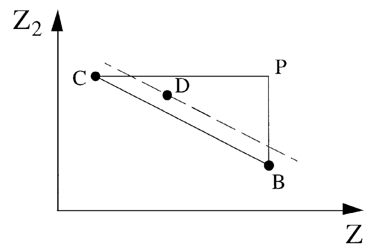

Fig. 3. The first solution, D, inside the duality gap.

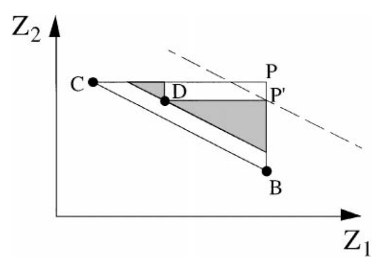

Fig. 4. Upper bound for $\hat{Z}$ and area for eventual nondominated solutions.

when no more nondominated solutions exist inside the duality gap or alternatively where the decision maker decides to stop because she/he is satisfied.

The decision support tool we developed enables a graphical representation of the gap (see Coutinho-Rodrigues et al. [24]) with the identification of the nondominated solutions and of the shadow areas corresponding to each state of the search. So, an interactive search inside the gap is available alternatively with the automatic calculation of the whole set of nondominated solutions inside the gap.

3. Computational results

To check the efficiency of "GAPS" 39 randomly generated networks were tested. These networks were generated considering that every pair of nodes in the network is connected by at least one

Table 1
Summary of test problem results

Prob #	Network dimensions		# Non-dominated Solutions inside the Gap	Total times searching the gap			Ratios of total search times in the gap			Search times for the best solutions optimizing $\hat{Z}$	
	# Nodes	# Arcs		AlgA (sec)	AlgD (sec)	AlgC (sec)	AlgD / AlgA	AlgD / AlgD	AlgC / AlgA	AlgA (sec)	AlgD (sec)
1	200	800	4	0.47	1.32	11.68	2.8	8.9	25.0	0.38	0.33
2	250	800	3	0.75	2.50	18.58	3.3	7.4	24.8	0.58	0.13
3	250	800	4	0.63	1.25	15.72	2.0	12.6	24.8	0.53	0.15
4	250	800	2	0.62	2.15	12.43	3.5	5.8	20.2	0.55	0.50
5	275	1000	3	0.70	2.72	18.17	3.9	6.7	26.0	0.65	0.60
6	275	1000	5	0.70	1.85	27.75	2.6	15.0	39.6	0.65	0.58
7	275	1000	4	0.72	2.75	24.93	3.8	9.1	34.8	0.63	0.17
8	275	1000	4	0.65	1.82	18.62	2.8	10.2	28.6	0.62	0.60
9	300	800	4	0.92	1.90	26.63	2.1	14.0	29.1	0.82	0.15
10	300	800	4	0.77	1.88	24.25	2.5	12.9	31.6	0.75	0.68
11	300	1000	4	0.75	1.57	21.43	2.1	13.7	28.6	0.72	0.18
12	300	1000	6	0.85	3.10	41.23	3.6	13.3	48.5	0.75	0.68
13	300	1000	4	0.78	1.82	21.13	2.3	11.6	27.0	0.70	0.68
14	300	1000	6	0.93	3.85	40.22	4.1	10.4	43.1	0.75	0.70
15	300	1000	3	0.85	2.50	22.87	2.9	9.1	26.9	0.75	0.68
16	400	1000	5	1.28	2.70	39.63	2.1	14.7	30.9	1.22	1.17
17	400	1000	2	1.25	0.83	18.28	0.7	21.9	14.6	1.25	0.20
18	400	1000	4	1.32	2.72	34.57	2.1	12.7	26.3	1.25	1.17
19	400	1200	6	1.25	3.15	48.15	2.5	15.3	38.5	1.20	1.18
20	400	1200	4	1.37	3.90	43.03	2.9	11.0	31.5	1.28	1.17
21	400	1200	3	1.23	3.10	27.30	2.5	8.8	22.1	1.22	1.18
22	500	1300	4	1.90	3.18	47.35	1.7	14.9	24.9	1.90	1.80
23	500	1300	5	1.97	4.05	74.37	2.1	18.4	37.8	1.92	1.78
24	500	1300	4	1.93	3.80	48.47	2.0	12.8	25.1	1.87	1.80
25	1000	6000	4	7.20	11.58	181.28	1.6	15.7	25.2	7.15	6.98
26	1000	6000	8	7.82	58.27	528.85	7.5	9.1	67.7	7.25	7.00
27	1000	6000	6	7.40	15.08	342.43	2.0	22.7	46.3	7.20	0.88
28	1000	6000	5	7.27	18.77	242.60	2.6	12.9	33.4	7.10	6.98
29	1000	6000	6	7.47	27.15	323.87	3.6	11.9	43.4	7.12	0.88
30	2000	8000	4	27.58	39.77	817.93	1.4	20.6	29.7	27.47	26.55
31	2000	8000	3	27.45	32.13	642.75	1.2	20.0	23.4	27.42	26.55
32	2000	8000	4	27.52	43.52	848.77	1.6	19.5	30.8	27.42	26.55
33	2000	8000	3	27.57	43.17	669.02	1.6	15.5	24.3	27.43	26.55
34	2000	8000	4	27.35	42.55	693.90	1.6	16.3	25.4	27.23	27.63
35	4000	8000	3	108.10	120.08	2005.25	1.1	16.7	18.5	108.03	105.38
36	4000	8000	3	109.17	128.43	2509.52	1.2	19.5	23.0	109.08	105.40
37	4000	8000	2	108.52	118.92	1510.12	1.1	12.7	13.9	108.47	105.27
38	6000	8000	3	242.07	257.18	4471.03	1.1	17.4	18.5	242.00	236.40
39	6000	8000	2	243.02	248.62	3334.52	1.0	13.4	13.7	243.00	236.37

AlgA – Algorithm with Azevedo et al. [28–30] "*k*-shortest".
AlgD – Algorithm with Dreyfus [27] "*k*-shortest".
AlgC – Algorithm of Current et al. [1] with dual using lagrangean relaxation.

path. This condition seems realistic for many real-world applications such as routing problems in transportation and communication systems. Networks of various sizes and densities were generated. Networks from 200 nodes and 800 arcs to 6000 nodes and 8000 arcs were used. Arc node ratios vary from 1.35 to 6. The whole set of the nondominated solutions was identified in one "ad hoc" chosen gap for each test network. All tests were performed on a *Macintosh Ci* computer with a 68030 *Motorola* CPU and a math coprocessor.

The GAPS procedure was tested with two k-shortest path algorithms: Dreyfus [27] and Azevedo et al. [28–30]. The Dreyfus algorithm was encoded according to the suggestions of Lawler [31]. They are referred to as AlgD and AlgA, respectively. In addition, all the nondominated solutions were identified in the selected gaps using the constrained shortest path algorithm of Handler and Zang [22] as suggested by Current et al. [1]. This approach is referred to as AlgC.

The results of these tests are presented in Table 1. Column 1 identifies the problem and columns 2 and 3 list the number of nodes and arcs, respectively. Column 4 gives the number of nondominated solutions within the searched gap. Columns 5–7 report (in CPU seconds) the total time required to search inside the gap via AlgA, AlgD and AlgC, respectively. Columns 8–10 report the ratios of total search time for AlgD : AlgA; AlgC : AlgD; and AlgC : AlgA, respectively. Columns 11 and 12 report (in CPU seconds) the time to calculate the best solution of [SP] using AlgA and AlgD.

The ratios in columns 8–10 indicate that the k-shortest path algorithms (AlgA and AlgD) were clearly more efficient than the constrained shortest path algorithm (AlgC). AlgD required from 1/5 to 1/23 of the time required by AlgC and AlgA requiring from 1/13 to 1/68 of the time required by AlgC, to identify all the nondominated solutions in the gap. AlgA was significantly more efficient than AlgD in solving 38 of the 39 test problems.

In the last two columns of Table 2 we present the run times necessary to identify the first solution inside the gap for Alg A and AlgD. The comparison of those columns with columns 5 and 6 show that AlgA was particularly efficient in identifying gap solutions from the second best. In order to emphasize the comparison between AlgA and AlgC, Figs. 5 (ratios of total search times inside the gap) and 6 (ratios of average searching times from the second best) are presented.

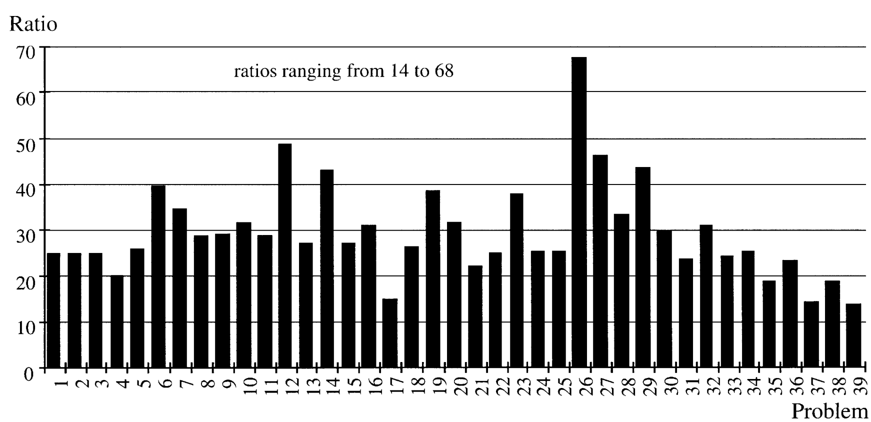

Fig. 5. Rations AlgC/AlgA of total search times inside the gap for the test problems.

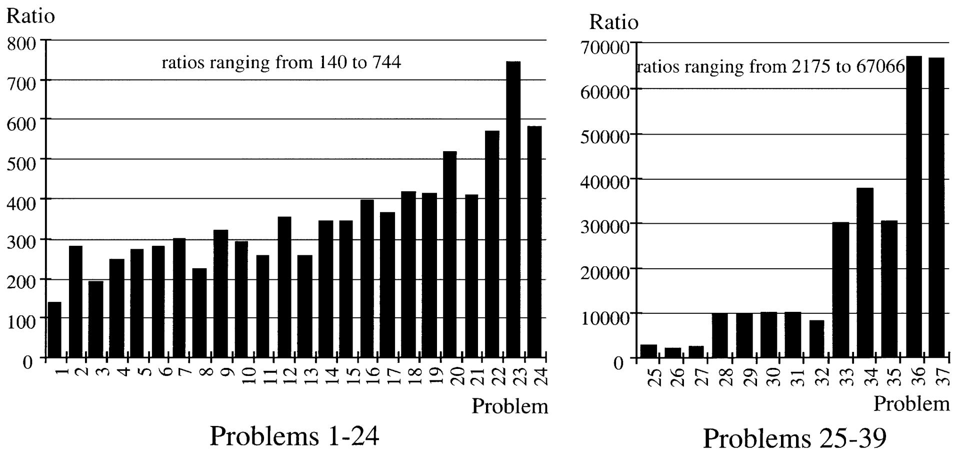

Fig. 6. Rations AlgC/AlgA: average searching times for identifying an additional nondominated solution inside the duality gap.

4. Conclusions

In this paper we compare two approaches for searching nondominated solutions inside duality gaps concerning bi-objective shortest path problems.

Two major points must be emphasized from the results:

– The approach using a *k*-shortest path algorithm is unquestionably faster than the approach using a constrained shortest path algorithm.
– Although our implementation of the algorithm by Azevedo et al. [28–30] does not take in account the most recent computational improvements proposed by its authors, it works in our problem clearly better than the *k*-shortest path algorithm by Dreyfus [27].

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