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Deformable strut and tie model for the calculation of the plastic rotation capacity

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Abstract

When a plastic analysis, a non-linear analysis or a linear analysis followed by redistribution of bending moments is used to predict the structural behaviour of beams, the critical sections should have the necessary plastic rotation capacity to allow the predicted behaviour at failure. When some doubts may arise, then an explicit calculation of this capacity must be carried out. This paper presents a theoretical model for the calculation of plastic rotation, considering the influence of the main factors. Some results are presented on the basis of the model, and conclusions are drawn.

Keywords: Non-linear analysis; Plastic rotation capacity; Reinforced concrete beams

1. Introduction

This is an extended and revised version of reference [1].

Reinforced concrete beams follow a non-linear behaviour for high levels of loading and the distribution of bending moments in continuous beams may differ from the elastic distribution. This non-linearity is due, firstly, to the cracking of concrete and, later, for higher loads, to the yielding of steel. At the sections where the yielding of steel takes place, the stiffness is so lowered that a plastic hinge can be assumed to have taken place.

The evaluation of the ductility of reinforced concrete beams is very important, since it is essential in order to avoid a fragile collapse of the structure by ensuring adequate deformation at ultimate load. Ductility is defined as the capacity of a material, section or structure to suffer considerable plastic deformation without significant loss of strength. Ductile elements show signs of failure in the plastic phase as the collapse load is approached; in addition to serious deformation they also exhibit severe cracking. The concept of ductility is related to the moment redistribution capacity and, consequently, to the safety of the structure. In ductile beams the process of moment redistribution is slow and gradual, with no instantaneous transmission of forces that could eventually cause an abrupt collapse of the structure.

One of the procedures used to quantify the ductility is based on the plastic rotation capacity (θ_{pl}). The knowledge of the plastic rotation capacity of certain regions of the structure is important in a plastic analysis or an analysis based on moments redistribution (Fig. 1). The plastic rotation capacity of critical regions dictates the available degree of moment redistribution and the ability to exploit the additional resistance of hyperstatic structures.

Plastic rotation may be calculated as the integral of curvatures after reinforcement yielding in the plastified area, see Eq. (1). However, this equation is not easy to apply because the curvature has a strongly non-linear

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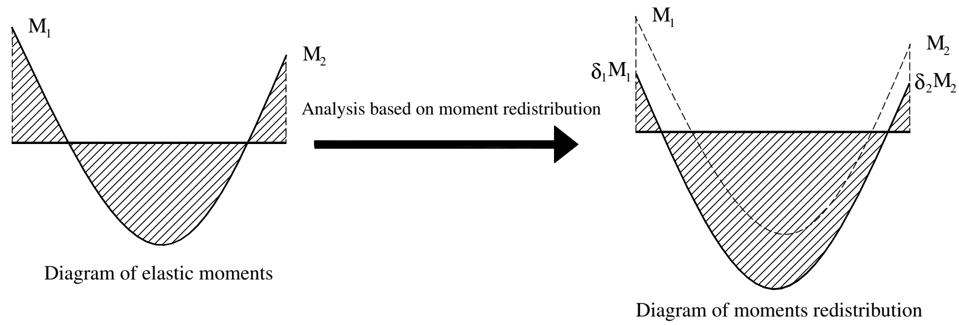

Fig. 1. Analysis based on moments redistribution.

development along the length of the beam due to the variation of the bending stiffness between the cracked and non-cracked sections.

$$\theta_{pl} = \int_{l_{pl}} \frac{1}{r}.\mathrm{d}x = \int_{l_{pl}} \frac{\varepsilon_s - \varepsilon_{sy}}{d - x}.\mathrm{d}x \tag{1}$$

where $1/r$ is the curvature; ε_s the tensile strain in the steel; ε_{sy} the yielding strain; d the effective depth of a cross-section; x the neutral axis depth and l_{pl} the length of the plastic zone.

The plastic rotation calculations proposed by codes of practice are either very simple or very difficult to carry out. Those very difficult are based on the definition of the moment–curvature or moment–rotation relationships. Those very simple are based on a graph relating the rotation capacity with x/d, but only takes into account the steel type and the slenderness of the member (for example, see Model Code 1990 or Eurocode 2) [2,3]. The complexity of the phenomenon is not sufficiently represented by the simple calculation procedure, and there is a need for a new intermediate method that combines the simplicity of the calculations with a good approximation to the actual behaviour of the structures.

The plastic rotation capacity depends on several interrelated factors, which makes the analysis quite difficult. For linear elements in bending the main factors are: the concrete compressive strength, the size and shape of the cross-section, the compressive reinforcement ratio, the shear reinforcement ratio, the slenderness of the element, and the shear force. But the most important factors are the tensile longitudinal reinforcement ratio, the reinforcement ductility and the bond–slip relationship [4].

Understanding and quantifying how the plastic rotation capacity varies with the above factors allows a global structural analysis to be made, where the predicted moment distribution will be closer to reality. As a consequence, structures will be more economical because the design of the elements will be optimised.

The theoretical procedure presented in this paper models the actual behaviour of structural concrete members. Plastic rotation capacity of critical regions can be worked out by taking into account some parameters, namely, steel type, concrete strength, rate of longitudinal steel, shear force and the confining effect on compressed concrete exercised by the shear reinforcement. Beyond the determination of the plastic rotation capacity, the behaviour of the critical regions is also characterized by means of a non-linear moment–rotation relationship.

2. Proposed model: deformable strut-and-tie model

2.1. General description

The deformable strut-and-tie model (DST model) is based on the truss analogy. DST model is a mechanical model that consists of dividing a continuous structure into a hinged structure composed by its components, capable of both representing the internal mechanisms of force transmission as well as the overall deformation behaviour of the structure. One of the advantages of this model is to help the understanding of the internal behaviour of the concrete member. Michalka completed a study on the model in 1986 [5]. He used the strut-and-tie model to calculate the rotation capacity of plastic hinges, by taking into consideration the non-linear behaviour of the tension ties and compression struts.

To discretize a region, the designer should divide it according the path of the forces throughout a structure. However, the identification and separation of the components may cause some difficulties, since they could interfere with each other. Normally, three zones can be identified: compressive, tension and shear zones. The compressive zone is formed by the compressed concrete and by the compressed bars. The tension zone is formed by the tensioned bars and by the concrete between cracks. The shear zone is formed by the stirrups and by concrete. The different sources of deformability are modelled by struts and ties. This struts and ties are associated in order to represent the compressive and tensile stress fields, respectively. The truss model ensures the compatibility of the deformations and therefore the static equilibrium. The transfer of the individual components to the moment–rotation curve, M–θ, of the region studied was thus achieved. Once the behaviour of the plastic hinge is characterized by the M–θ curve, it is easy to determine the plastic rotation capacity. The rotation is calculated based on the deformability of the tension and compressed zones, with the tension zone being the most important.

To characterize the behaviour of the various struts and ties (strength and deformability) the mechanical properties of materials and the geometric characteristics of the cross-section must be known.

2.2. Behaviour of the reinforced tie

In a cracked section, the tension forces are supported by the steel bars alone. However, if a certain length of a cracked member is considered, then there are cracked sections and non cracked zones between the cracks, where the concrete resistance is not negligible. These tension stresses taken by concrete increases the stiffness of the tie. This is the so called "tension stiffening effects" (Fig. 2).

Modelling the deformability of the concrete tie follows the procedure described in Model Code 1990. This model was chosen because it considers the influence of the tension stiffening effect, modifying the σ–ε relationship of the steel. To overcome the difficulty of steel strain being variable along the length of the beam (when the concrete is cracked), the steel strain is described in terms of average value (Fig. 3).

(i) Uncracked phase, $0 < \sigma_s \leqslant \sigma_{sr1}$:

$$\varepsilon_{s,m} = \varepsilon_{s1} \tag{2}$$

(ii) Crack formation phase, $\sigma_{sr1} < \sigma_s \leqslant \sigma_{srn}$:

$$\varepsilon_{s,m} = \varepsilon_{s2} - \frac{\beta_t(\sigma_s - \sigma_{sr1}) + (\sigma_{srn} - \sigma_s)}{\sigma_{srn} - \sigma_{sr1}}(\varepsilon_{sr2} - \varepsilon_{sr1}) \tag{3}$$

(iii) Stabilized cracking phase, $\sigma_{srn} < \sigma_s \leqslant f_{yk}$:

$$\varepsilon_{s,m} = \varepsilon_{s2} - \beta_t(\varepsilon_{sr2} - \varepsilon_{sr1}) \tag{4}$$

(iv) Post yielding phase, $f_{yk} < \sigma_s \leqslant f_{tk}$

$$\varepsilon_{s,m} = \varepsilon_{sy} - \beta_t(\varepsilon_{sr2} - \varepsilon_{sr1}) + \delta\left(1 - \frac{\sigma_{sr1}}{f_{yk}}\right)(\varepsilon_{s2} - \varepsilon_{sy}) \tag{5}$$

where $\varepsilon_{s,m}$ is the mean steel strain; ε_{s1} strain of reinforcement in uncracked concrete; ε_{s2} the strain of reinforcement in the crack; ε_{sr1} the steel strain at the point of zero slip under cracking forces reaching f_{ctm}; ε_{sr2} the strain of reinforcement at the crack under cracking forces reaching f_{ctm}; $\Delta\varepsilon_{sr}$ the increase of steel strain in the cracking state; β_t the factor related with the percentage of concrete cracked, for short-term loading (pure tension) $\beta_t = 0.40$, for long-term or repeated loading (pure tension) $\beta_t = 0.25$; ε_{sy} the strain at yield strength; ε_{su} the steel strain (unembedded) reaching f_{tk}; ε_{smu} the mean steel strain reaching f_{tk}; σ_s the steel stress; σ_{sr1} the steel stress in the crack, when first crack has formed; σ_{srn} the steel stress in the crack, when stabilized crack pattern has formed (last crack), for normal cases $\sigma_{srn} = 1.3.\sigma_{sr1}$; f_{yk} the characteristic yield strength of reinforcement and f_{tk} the characteristic tensile strength of reinforcement. The coefficient δ is related with the type of steel taking into account the ratio f_{tk}/f_{yk} and the yield stress f_{yk}. For ductile steel (type A) with $f_{yk} = 500$ MPa the value δ is equal to 0.8.

In this model the bond between steel and concrete is considered through the value of β_t, which is related with the mean spacing between cracks. This subject is developed with more detail in Model Code 1990. It should be pointed out that the tension stiffening effect is greater before yielding of the steel and less significant after. In theory, therefore, a large tension stiffening effect should have a greater influence on the deformation capacity of beams with a high tensile reinforcement ratio than on that of beams with a low tensile reinforcement ratio.

2.3. Behaviour of the concrete strut

To model the deformability of the compressive zone of a member under flexion through a strut, the constitutive law for concrete needs to be known. To obtain a more realist

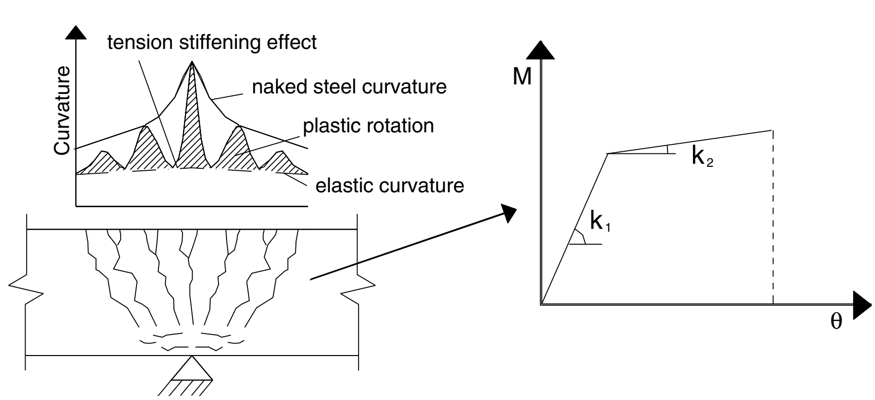

Fig. 2. Moment–rotation curve, M–θ, of the plastic hinge.

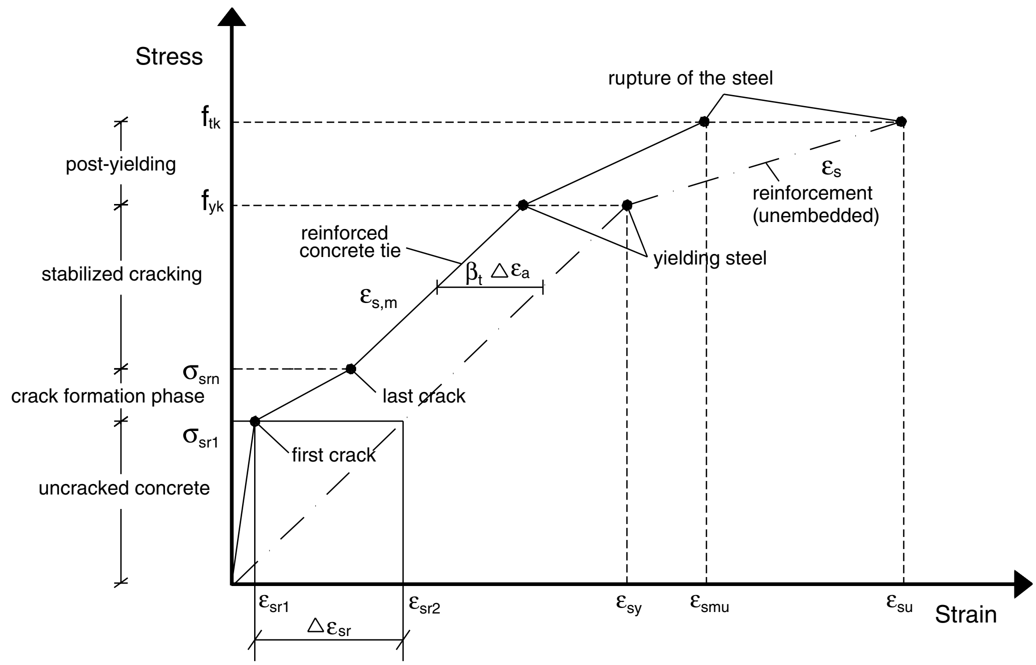

Fig. 3. Stress–strain relationship used for the reinforced concrete tie.

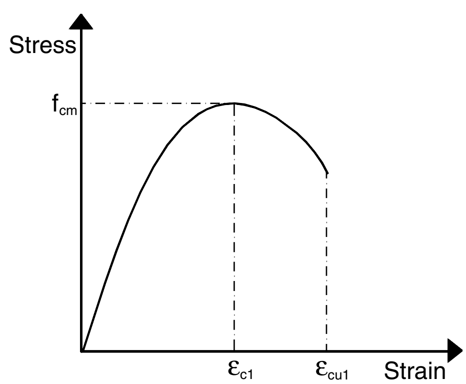

Fig. 4. Stress–strain relationship used to model the concrete.

model, a non linear σ–ε relationship should be considered. The ultimate strain plays an important role if rupture takes place by concrete crushing. To adopt a constitutive curve, the authors followed Eurocode 2 and Model Code 1990 (Fig. 4). For each strength class of concrete, an equation $\sigma_c = f(\varepsilon_c)$ is defined:

$$\frac{\sigma_c}{f_{cm}} = \frac{k \cdot \eta - \eta^2}{1 + (k-2) \cdot \eta} \tag{6}$$

where $\eta = \varepsilon_c/\varepsilon_{c1}(\varepsilon_c < 0)$; ε_{c1} is the strain at peak stress, $\varepsilon_{c1}(‰) = -0.7.f_{cm}^{0.31}$ and $f_{cm} = f_{ck} + 8$ MPa; $k = -1.1.E_{cm}.\varepsilon_{c1}/f_{cm}$ and $E_{cm} = 22 \cdot [(f_{cm})/10]^{0.3}$; Eq. (6) is valid for $0 < |\varepsilon_c| < |\varepsilon_{cu1}|$, where ε_{cu1} is the nominal ultimate strain; for concretes with $f_{ck} < 55$ MPa, $\varepsilon_{cu1} = -3.5$ (‰); for concretes with $f_{ck} \geqslant 55$ MPa, $\varepsilon_{cu1} = -2.8–27 \cdot [(98 - f_{cm})/100]^4$ (‰).

2.4. Behaviour of the confined concrete strut

The most practical way of considering the lateral confining effect on concrete is by modifying the constitutive law used for short duration axial loads. The usual way of confining the concrete is to use high shear reinforcement ratios. The concrete only exhibits the lateral confinement effect when the applied stress is close to the maximum axial strength. The transverse deformations are higher for those stresses, owing to the progressive internal cracking, which forces the concrete against the shear reinforcement. Then the shear reinforcement exerts a confinement action relative to the concrete core. This action is, therefore, a passive confinement.

Section 3.5.2 of MC90 defines a model for analysing elements to the ultimate limit states under axial loads. In the absence of more precise data a linear relationship between f_{cc}^* and f_{cc} can be used (Fig. 5)

$$f_{cc}^* = f_{cc} \cdot (1.000 + 2.50 \cdot \alpha \cdot \omega_w) \quad \text{if} \quad \sigma_2/f_{cc} < 0.05 \tag{7}$$

$$f_{cc}^* = f_{cc} \cdot (1.125 + 1.25 \cdot \alpha \cdot \omega_w) \quad \text{if} \quad \sigma_2/f_{cc} > 0.05 \tag{8}$$

$$\varepsilon_{c1}^* = \varepsilon_{c1} \cdot (f_{cc}^*/f_{cc})^2 \tag{9}$$

$$\varepsilon_{c,85}^* = \varepsilon_{c,85} + 0.1\alpha\omega_w \tag{10}$$

where f_{cc} is the unconfined strength; f_{cc}^* the confined strength; ω_w the volumetric mechanical ratio of confining steel; α the effectiveness of confinement, equal to $\alpha_n \cdot \alpha_s$, α_n depends on the arrangement of stirrups in the cross section and α_s depends on the spacing of the stirrups; and σ_2 the effective lateral compression stress due confinement. The CEB Bulletin of Information no. 228 proposes alterations to the above equations when the concrete compressive strength is higher than 50 MPa. Eq. (7) is modified

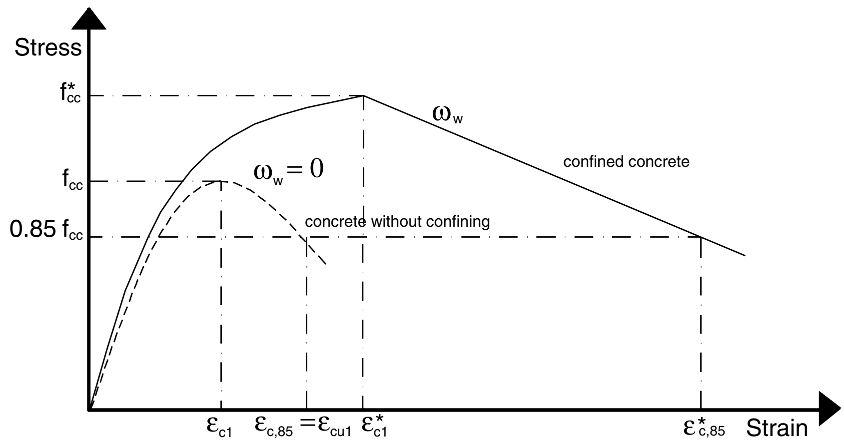

Fig. 5. Stress–strain relationship for confined concrete according to MC90.

as shown in Eqs. (11), (8) disappears and Eq. (10) is modified as shown in Eq. (12)

$$f^*_{cc} = f_{cc} \cdot (1.000 + 1.50 \cdot \alpha \cdot \omega_w) \tag{11}$$

$$\varepsilon^*_{c,85} = \varepsilon_{c,85} + 0.05\alpha\omega_w > \varepsilon^*_{c1} \tag{12}$$

The application of this model, defined for elements under axial loads, to the compressive zone of elements under bending-moment can introduce some mistakes because, in the latter situation, the reinforcement ratio confines the concrete less efficiently. The compressive stresses due to bending are not uniform throughout the compressive zone, with the compressive stress varying from a maximum at the most extreme fibre to zero at the neutral axis, and so the lateral strain are also variable. As a consequence, the lateral confinement stress provided by shear reinforcement will also not be constant throughout the compressive zone. However, the approach of considering the lateral confinement stress to be constant seems to be acceptable because the compressive zone in beams close to failure is relatively small and because the confining effect is larger in fibres where the concrete strains are greater. It is in those fibres that the confining action is most important. MC90 also contains a comment that leaves open the hypothesis of applying this confining model to the concrete in beams. In Section 3.5.2.1 of MC90 there is a reference to the case of the compressive zone of beams, in which the neutral axis may be considered as a ‘‘solid’’ border limiting lateral expansion.

2.5. Length of the plastic hinge

Several researchers confirmed experimentally the assumption that the plastic hinge has a length equal to the effective depth of the section seems to be the most correct one [4,6]. Eurocode 2 makes a reference in Section 5.6.3 that supports the options of this paper: the length of the plastic hinge is approximately 1.2 times the depth of section. It is important to emphasise that the hypothesis of a constant plastic hinge length, adopted in this study, being an approximation to the actual behaviour, is widely accepted as sufficiently accurate for practical proposes. However, for particular situations, beams with low tensile

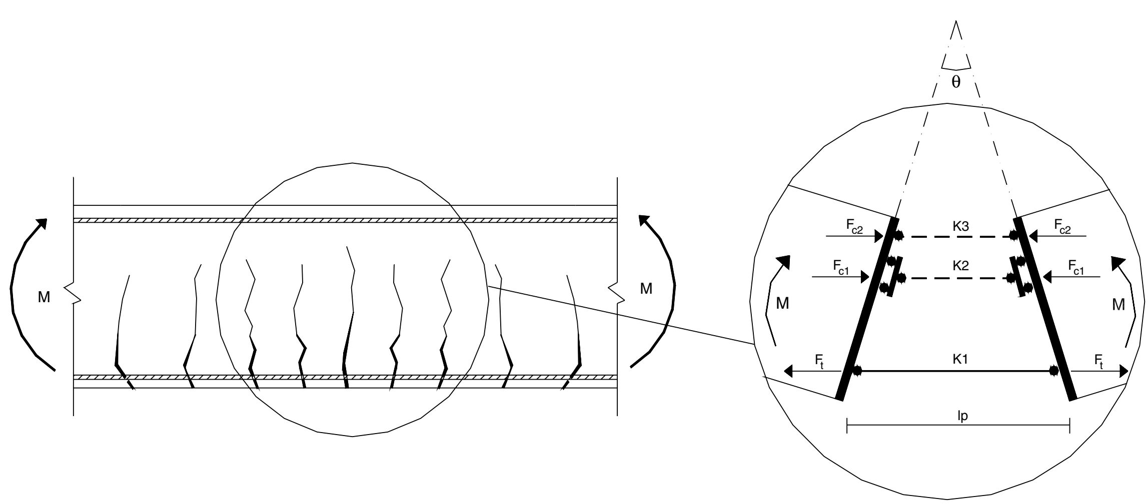

Fig. 6. Division of the critical zone into sub-regions (pure bending).

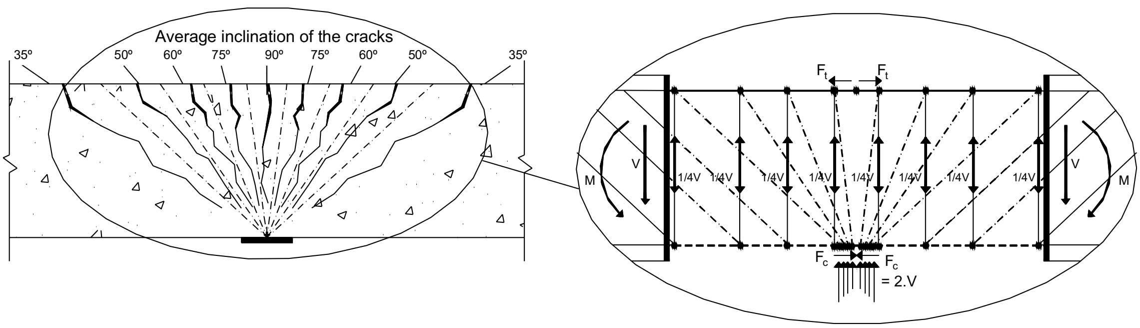

Fig. 7. Idealized truss for simple bending case.

reinforcement ratios containing low ductile steel, this assumption could be a gross approach and could overestimate the plastic rotation capacity. In these members the plastic deformations are located near the critical crack over a very short length of bar.

In the case of simple bending, the region in study should be divided in small portions with short lengths to enable the evaluation of the influence of the bending moment/shear force ratio and the variation of the length of plastic hinge during the load history.

3. Application of the model to the case of pure bending

In this case the critical zone is divided into two subregions: a compression zone (concrete and compressed steel bars) and a tension zone behaving as a reinforced concrete tie (Fig. 6). Where k_1 represents the tension tie; k_2 the concrete strut which do not have a fixed position, since the diagram of the concrete stresses is not constant (a slide was provided to allow vertical displacements of the strut); k_3 the steel bars in compressive zone and l_{pl} the length of the plastic zone.

For a particular moment value, the total rotation, θ_{total}, might be worked out, accordingly to Eq. (13), by calculating the total deformation of the tensioned concrete tie, $\delta_{\mathrm{tension}}$, and the total deformation of the compressed strut, δ_{compr} (this is the total deformation at the level of the resultant of the compressive stresses). As usual, z is the lever arm.

$$\theta_{\mathrm{total}} = \frac{|\delta_{\mathrm{tension}}| + |\delta_{\mathrm{compr}}|}{z} \tag{13}$$

To quantify the forces installed in struts and ties, some basic assumptions need to be assumed for the behaviour of reinforced concrete members in flexure. A classical assumption is the Bernoulli hypothesis which considers that plane sections remain plane after deformation. This assumption is valid if bonding between concrete and steel is perfect. However, this condition is not real when the sections are close to the ultimate load. The compatibility condition is defined by Eq. (14)

$$\frac{\varepsilon_{\mathrm{c}}}{\varepsilon_{\mathrm{s}}} = \frac{|x|}{d - |x|} \tag{14}$$

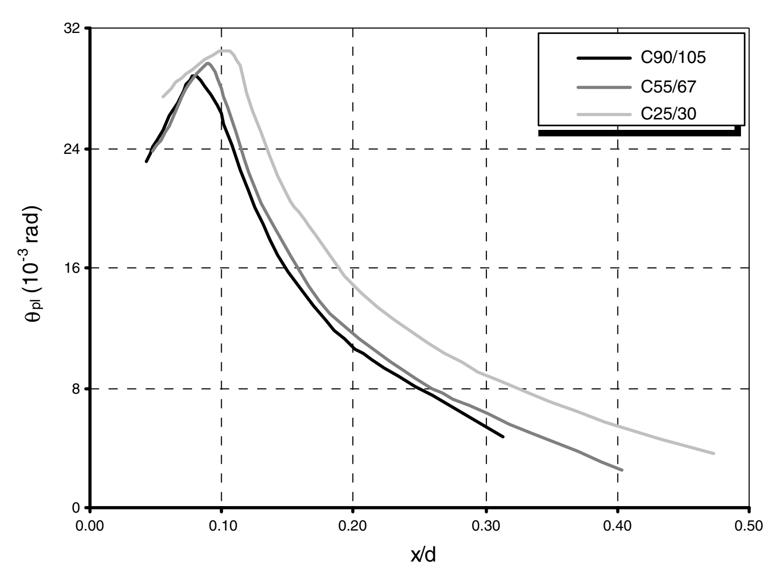

Fig. 8. Relation between $\theta_{\mathrm{pl}}e$ x/d (class S of steel).

The principle of equivalence should also be considered. There should exist an equivalence between strains, stresses, and internal forces.

4. Application of the model to the case of simple bending

In the presence of shear force, the behaviour of the critical region can be modelled by a truss designed to resist the internal forces recorded in the maximum load situation. At the beginning of the 20th century, Mörsch was the first researcher to develop the theory of the truss analogy that allows the design of members with shear reinforcement. The truss mechanism admits that concrete between diagonal cracks works as a compressed strut, that the stirrups are tension ties and the bending tension/compressed zone (in the top and bottom of the beam) are also truss struts and ties.

The first difficulty in the strut-and-tie model conception for these regions is defining the angle between concrete compression struts and the main tension ties. Bearing in mind that the strut-and-tie model is not applied to whole length of the beam, but only to the plastic hinge region, the question of the struts' angle is particularly important. In the design model codes, Mörsch's truss is usually taken to have compressed struts with a 45° angle with the beam axis. In some situations that supposition gives rise to conservative results. The latest version of EC2 allows an angle other than 45° to be adopted, with limit values of 22° and 45° being recommended.

The strut angle inclination is approximately equal to the inclination of the cracks and it depends on the ratio between the shear force and bending-moment. Studies accomplished by Graubner show that crack inclination decreases as the distance to the support increases [7]. In regions close to the sections where the concentrated forces are applied the diagonal cracks stop being parallel to one another and start to converge directly towards the compressed zone.

The strut-and-tie model proposed in Fig. 7 is based on the crack inclination observed in experimental tests, on the main stress direction obtained in a finite elements model and on studies carried out by other researchers, notably Michalka [4,5,8–10]. Regarding the statements mentioned above, it should be noted that the angle between the concrete compression struts and the horizontal axis is a fundamental factor in the study of the shear force influence on the plastic rotation capacity. On the one hand, the presence of the shear force makes the beam work as a truss and causes a tension force shift in the longitudinal bars, and on the other, the presence of the shear force prevents the average strain from being constant, in comparison with the pure bending situation, causing it to have a decreasing variation. This implies smaller rotations for the same moment. Taking into account these considerations, it can be seen that shear force can increase or reduce the rotation capacity of the critical beam regions (increase or reduce the length of the plastic hinge), depending, essentially, on the angle assumed for the diagonal struts.

5. Parametric study

On the basis of the model presented some extrapolations can be made with respect to plastic rotation capacity. In the following graphs the plastic rotation capacity is related to several variables. In this study the main parameters are: a constant cross section of 400×250 mm, concrete strength classes C25/30, C55/67 and C90/105, and steel ductility classes B, A and S (according to Model Code 1990). The calculations were carried out by considering the characteristic values for the materials.

5.1. Influence of the concrete strength

From the analysis of the graph on Fig. 8 it can be observed that, for sections with the same neutral axis

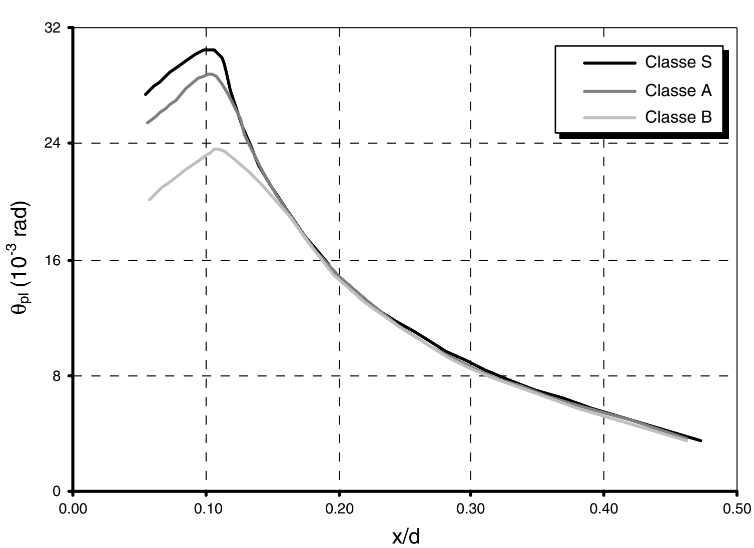

Fig. 9. Relation between $\theta_{pl}e\ x/d$ (C25/30 of concrete).

depth, x/d, the increase of the concrete strength originates a decreasing of the plastic rotation. The peak point corresponds to sections where the failure occurs simultaneously by the reinforcement and by the concrete. The right zone of the peak point corresponds to section failures by crushing of the concrete and the left zone corresponds to section failures by the steel reinforcement. For different concrete classes the differences are more noticeable in the right zone. For example, a shift of 4×10^{-3} rad may be found when comparing C25/30 to C90/105 concretes. This might be explained by the fact that ε_{cu} decreases with the increasing of the concrete strength, consequently conditioning the value of the maximum rotation. Furthermore, the θ_{pl}–x/d curves corresponding to higher strength concretes suffer a slight translation to the left relatively to the C25/30 curve. This is explained by the fact that the use of high strength concretes, for the same amount of tensile reinforcement, results in reduced neutral axis depth when compared to normal strength concrete.

5.2. Influence of the steel ductility

Steel ductility has a key influence on plastic rotation when the section fails by the reinforcement. Steel ductility is usually evaluated through 2 parameters: the f_t/f_y ratio, and the strain of steel at maximum load, ε_{su}. It is normally accepted that, smaller values of the f_t/f_y ratio lead to shorter lengths of the plastic hinge. As far as the effects of variation of ε_{su} are concerned, larger values of ε_{su} result

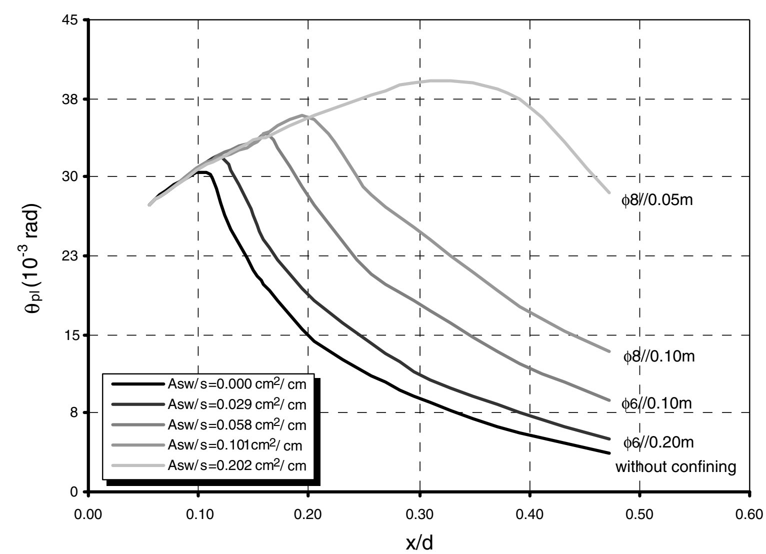

Fig. 10. Relationship between θ_{pl} and the parameter x/d for concrete class C25/30 and steel class S (different shear reinforcement ratios).

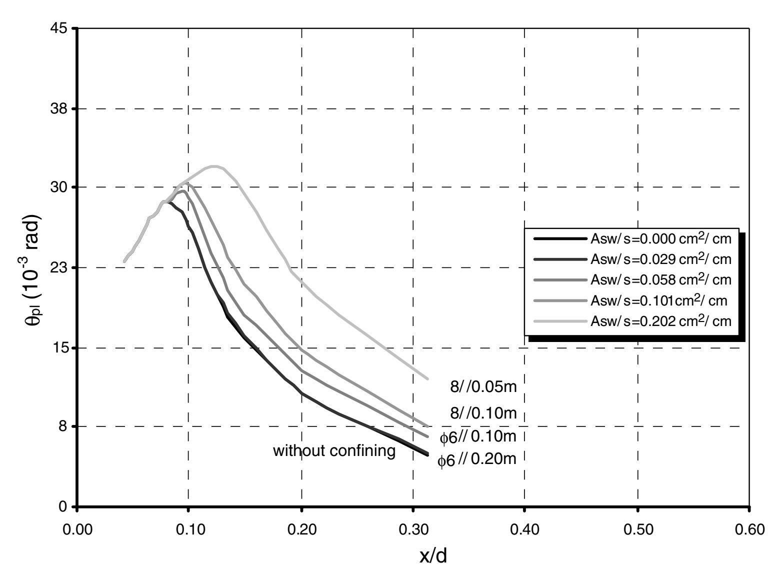

Fig. 11. Relationship between θ_{pl} and the parameter x/d for concrete class C90/105 and steel class S (different shear reinforcement ratios).

in higher maximum curvature values and, consequently, to larger rotation capacity values.

As it would be expected, the steel class has a considerable effect on the capacity of plastic rotation when the section fails by the reinforcement and has a null influence when the section fails by concrete crushing (Fig. 9). It can be observed that the more ductile the steel is, the higher the rotation capacity turn out to be.

5.3. Influence of the shear reinforcement ratio

The graphs in Figs. 10 and 11 clearly show that plastic rotation capacity grows with increasing shear reinforcement ratio. This increase is only relevant in the descending part of the θ_{pl}–x/d curves because the concrete confinement is only advantageous when the section fails by concrete crushing. Another aspect of the comparisons in the graphs in Figs. 10 and 11 is that, the benefit in terms of increased plastic rotation capacity is greater in normal strength concretes than in high strength concretes.

For class C25/30 concrete the plastic rotation capacity increases about 16×10^{-3} rad when it passes from a situation of no transverse reinforcement to a situation where the stirrups have a diameter of 8 mm and are 100 mm apart. For the same variation of shear reinforcement but for class C90/105 concrete, the increment of plastic rotation capacity is about 5×10^{-3} rad. This difference is caused by the theoretical model considering a different

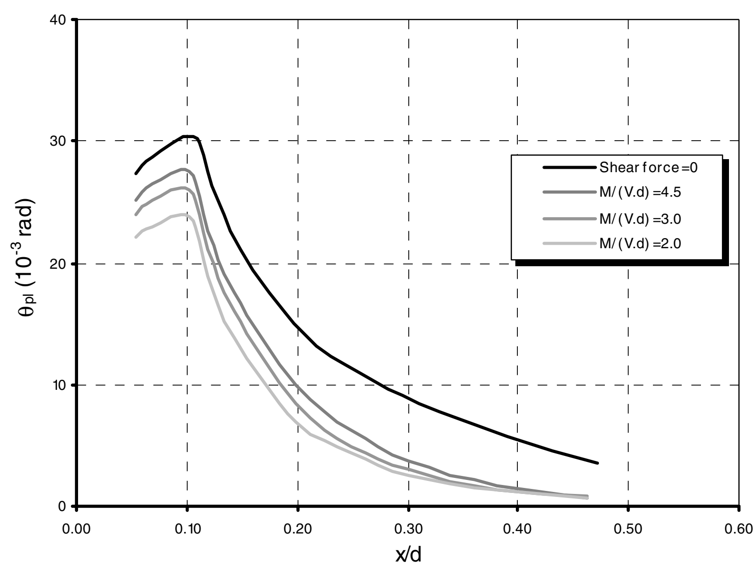

Fig. 12. Relationship between θ_{pl} and the parameter x/d for concrete class C25/30 and steel class S (different levels of shear force).

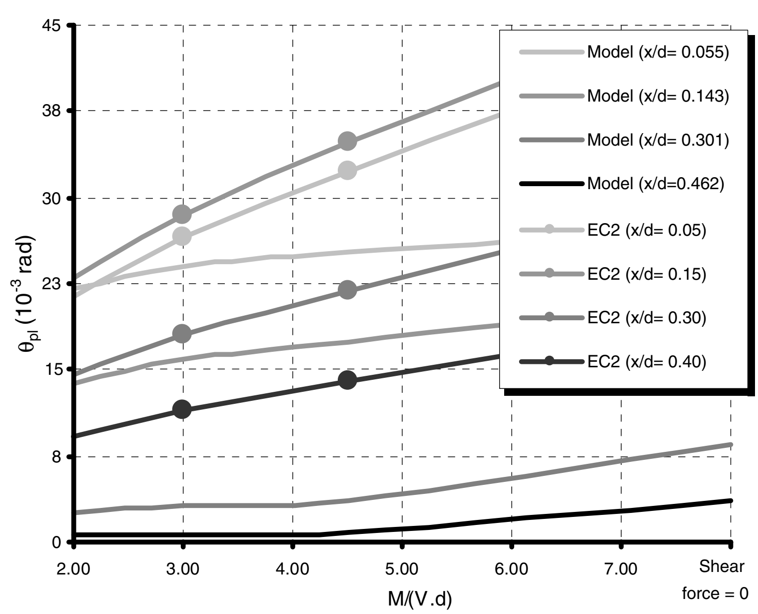

Fig. 13. Relationship between θ_{pl} and $M/(V \cdot d)$ for concrete class C25/30 and steel class S (different levels of shear force).

confinement action for concretes whose strength is above 50 MPa. The increase in plastic rotation capacity with increasing shear reinforcement ratio is essentially due to the enlargement of the ultimate concrete strain. In Figs. 10 and 11, ϕ is the diameter of a reinforcing bar; A_{sw} the transverse reinforcement area and s the distance between stirrups.

5.4. Influence of the shear force

Fig. 12 shows that the larger the shear force, V, (smaller $M/(V \cdot d)$ relationship) the smaller the plastic rotation capacity. The shift from a pure bending situation to a simple bending situation with a $M/(V \cdot d)$ relationship equal to 2.0, the plastic rotation capacity reduction is approximately 5×10^{-3} rad. These results contradict those found by other researchers (Pommerering and Graubner), who concluded that shear force exerts a favourable effect on plastic rotation capacity due to a tension force shift in the longitudinal reinforcement. The model proposed here does not ascribe this beneficial effect to the shear force, because of the angle assumed for the inclined struts. This model effectively contemplates the tension force shift but, for the angles of inclination adopted, there is also a decrease in the variation of the tension force in sections close to point where the moment is maximum.

MC90 does not consider the influence of the shear force on the plastic rotation capacity, the θ_{pl}-x/d curves recommended depend solely on steel ductility. With respect to this subject, EC2 is more complete. It defines a correction factor $k_\lambda = \sqrt{\lambda/3}$ when the $M/(V \cdot d)$ relationship is different of 3. In Fig. 13, the results of the theoretical model are compared with the recommendations in EC2. This graph relates the plastic rotation capacity to $M/(V \cdot d)$ for curves with different x/d values. There was found to be a similar tendency relative to the shear force influence, namely, the smaller the shear force the higher the plastic rotation capacity. However, that influence is quite muted according to the model and considerably more preponderant according to EC2.

6. Conclusions

This paper presents a theoretical model for the evaluation of the M–θ relationship of the plastic hinges in reinforcement concrete beams. The proposed model is a physical model which is able to evaluate the rotation capacity of a critical zone of a structural concrete member. This is an advantage when compared to the classical strut-and-tie model. This model takes into account some parameters that are fundamental for deformability study (concrete strength, steel ductility, shear reinforcement ratio and shear force). The consideration of high strength concrete is important, since the material is less ductile than normal strength concrete.

The influence of the concrete strength and the influence of the steel ductility in the plastic rotation capacity were studied. It was found that for sections with the same x/d value, the increasing f_{ck} originates a decrease of plastic rotation capacity and the steel ductility only influences the value of plastic rotation capacity when the section fails by the reinforcement (low values of x/d).

It was concluded that the increase in the plastic rotation capacity with increasing shear reinforcement ratio is greater in normal strength concretes than in high strength concretes. It was also verified that, larger the shear force result in a lesser plastic rotation capacity. In this area there was found to be a slight difference between the model's results and the EC2 recommendations. The proposed model indicates that the influence of shear forces is weaker.

The DST model was applied to simple situations (pure and simple bending) and it needs to be generalised for other situations. This model could be a powerful tool for the study of discontinuities regions in structural concrete members.

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