

Assessment of Eurocode Methodologies for Verification of Flexural and Lateral Torsional Buckling of Prismatic Beam-Columns

Author: Gulzaib Anwar

Supervisors: Prof. Luís Simões da Silva Liliana Marques Trayana Tankova

University: University of Coimbra



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Abstract

At present, EC3 provides several methodologies for the stability verification of members and frames. Each of these methodologies has its own scope of applicability. The rules provided in clause 6.3.1 are for flexural buckling of uniform members and are based on the famous Ayrton-Perry type of formulation which is of straight-forward application and it has a clear mechanical background. Clause 6.3.2 of the Eurocode deals with lateral torsional buckling of uniform members. In this thesis focus is given to prismatic beam-columns. They can be designed using clause 6.3.3 which gives the interaction formulae for stability verification of a member subject to bending moment and axial force; as an alternative – in clause 6.3.4 the General Method is suggested. However, application of this method has been shown not to be reliable in many scientific studies – either due to the lack of mechanical consistency or due to the lack of clarification in adopting certain decisions for non-standard situations.

It was the purpose of this study to review the application of existing procedures for verification of flexural and lateral-torsional buckling of simply supported prismatic beamcolumns. This included recent proposals for lateral-torsional buckling of beams (Taras, 2010), the consideration of cross-section classification along member length (Greiner et al, 2011) and safety assessment and comparison of existing procedures in EC3-1-1 by a parametric study consisting of I-shaped cross sections ranging from class 1 to class 3, with different lengths, loading types (uniaxial bending with axial force) and height over width (h/b) ratios.

Results from clauses 6.3.1 to 6.3.3, general method and GMNIA were calculated for 4144 cases. In comparison with results from 6.3.1 to 6.3.3, the general method is more unsafe. Its results were found to vary with increasing cross sectional slenderness and increasing length. On the other hand results from clauses 6.3.1 to 6.3.3 are more conservative. In comparison, the general method was found to have a higher spread as indicated by the standard deviation of its results than from 6.3.1 to 6.3.3. Results from general method varied when calculated from different alternatives in the general method. When interpolation criteria was used in the general method, the results were more un-conservative in comparison to the minimum criteria. The spread of the general method included both safe and unsafe results whereas the results from 6.3.1 to 6.3.3 remained mostly conservative.



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Table of Contents

Abstract	2
Acknowledgements	3
Table of Contents	4
Index of Symbols	6
List of Figures	8
List of Tables	10
1 Introduction	11
1.1 Introduction	11
1.2 Objectives	11
1.3 Structure of the Thesis	12
2 The Eurocode Methodologies	13
2.1 Introduction	13
2.2 Pure Flexural Buckling of Uniform Members	13
2.3 Pure Lateral Torsional Buckling of Uniform Members	14
2.4 Recent proposal for Lateral Torsional Buckling Curves(Taras A, 2010)	16
2.5 Uniform Members in Bending and Axial Compression	16
2.6 Class and Resistances according to SEMICOMP+	18
2.7 General Method	20
2.8 Problems with General Method	21
3 Numerical Model	22
3.1 Introduction	22
3.2 The Model	22
3.3 Material Non-Linearity	22
3.4 Imperfections	23
3.5 Supports	24
3.6 Loading	24
3.7 Meshing	25
4 Methodology	26
4.1 Introduction	26
4.2 Catalogue for Parametric Study	26
4.3 Factor <i>\phi</i>	27
4.4 Resistances from clause 6.3.1 to 6.3.3 of EC3-1-1	27

4.5 Resistances from General Method	
4.6 GMNIA	
4.7 Self Defined Variables for Calculation of Assessment of Results	
4.8 Summary	
5 Results and Analysis	
5.1 Introduction	
5.2 Overview of the results	
5.2 Variation of Results with h/b Ratio	
5.3 Variation of Results by $\boldsymbol{\phi}$	40
5.4 Variation of Results by $\overline{\lambda} z$	46
5.5 Variation of Results with Different Loading Cases	
5.6 Summary	63
6 Conclusions	64
References	66

Index of Symbols

Area of cross section
Effective area of cross section
Flange width
Co-efficient for calculation of M _{cr}
Equivalent uniform moment factors
Modification factor for lateral torsional buckling reduction factor
Yield strength
Shear Modulus
Height of cross section
Second moment of area about relevant axis
Wrapping constant
Torsion constant
Correction factor for moment distribution
Interaction factors to be evaluated either from Annex A or Annex B of EC3-1-
1
Elastic critical moment for lateral torsional buckling
Resistance obtained from in plane GMNIA
Resistance obtained from in plane GMNIA for $\phi=0$
Design plastic resistance to axial force
Design plastic resistance to bending moment, y-y axis
Design plastic resistance to bending moment for class 3 section, y-y axis
Ratio of design axial force to design plastic resistance to axial force
Resistance obtained from the method
Resistance obtained from GMNIA
Thickness of flange
Utilization factor
Elastic section moduli
Plastic section moduli
Class 3 section modulus
Co-efficient defined for calculation of resistance
Ratio of resistance obtained from the method and resistance obtained from
GMNA
Imperfection factors

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$\alpha_{cr,op}$	Minimum load amplifier for in plane design loads to reach elastic critical load in lateral or lateral torsion buckling mode without taking into account in		
	nlane flexural buckling		
$\alpha_{ult,k}$	Minimum load amplifier for design loads to reach characteristic resistance of		
	most critical cross section considering only the in plane behavior		
<i>Ү</i> мо, <i>Ү</i> м1	Partial safety factors		
arphi	Load diagram factor		
ϕ	Ratio of α_{pl}^{My} to α_{pl}^{N}		
ϕ , $\phi_{\scriptscriptstyle LT}$	Intermediate factors for determination of χ , χ_{LT}		
ψ	Between maximum and minimum bending moment		
ψ	Stress ratio		
$\overline{\lambda} \overline{\lambda}$	Non-dimensional slenderness		
$\overline{\lambda}_{op}$	Global non-dimensional slenderness in general method		
$\chi_{y}, \chi_{z}, \chi_{LT}$	Reduction factors for relevant buckling mode		
XLT,mod	Modified reduction factor for lateral torsional buckling		
Xop	Reduction factor for non-dimensional slenderness		
Xmethod	Self defined reduction factor to obtain resistance obtained from the method		
	from cross sectional resistance keeping same ratio of applied actions.		
x - x	Axis along member length		
y - y	Axis parallel to flanges		
z - z	Axis perpendicular to flanges		
n	Number of cases		
SD	Standard deviation		
CoV	Co-efficient of variation		

List of Figures

Figure 3.1 Figure 3.2	Modeled Stress Strain Curve Minimum yield stress for different thickness of cross section according to EN10025	22 23
Figure3.3	First buckling mode for HE300B under uniform axial compression ($\overline{\lambda}_{z} = 1$)	23
Figure 3.4	Imperfection shape for section with loading conditions given in figure 3.3	24
Figure 3.5	Material Imperfections for h/b<1.2 and for h/b > 1.2	25
Figure 3.6	Model for loads	25
Figure 5.1	Scatter plot of self defined reduction factor χ for different methods against χ GMNIA	34
Figure 5.2	Position of mean and standard deviation for all results calculated from 6.3.1 to 6.3.3-GC	37
Figure 5.3	Position of mean and standard deviation for all results (all bending moment shapes) calculated from 6.3.1 to 6.3.3-SC	37
Figure 5.4	Position of mean and standard deviation for all results (all bending moment shapes) calculated from 6.3.1 to 6.3.3-Taras	38
Figure 5.5	Position of mean and standard deviation for all results (all bending moment shapes) calculated from GM-Int,GC	38
Figure 5.6	Positionof mean and standard deviation for all results (all bending moment shapes) calculated from GM-Int,SC	39
Figure 5.7	Position of mean and standard deviation for all results (all bending moment shapes) calculated from GM-Min,GC	39
Figure 5.8	Position of mean and standard deviation for results calculated from GM-Min,SC for different ϕ	40
Figure 5.9	Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b<1.2 for different ϕ	41
Figure 5.10	Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with 1.2< h/b<2 for different ϕ	43
Figure 5.11	Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2.tf<40mm for different ϕ	44
Figure 5.12	Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2.tf>40mm for different ϕ	45
Figure 5.13	Mean and standard deviations for the ratio of results from different methods	47
	with GMNIA for sections with h/b<1.2 for different values of $\overline{\lambda}_{z}$	
Figure 5.14	Mean and standard deviations for the ratio of results from different methods	48
	with GMNIA for sections with 1.2 <h <math="" b<2="" different="" for="" of="" values="">\lambda</h>	
Figure 5.15	Mean and standard deviations for the ratio of results from different methods	50
	with GMNIA for sections with h/b>2,tf<40mm for different values of λ_z	

rureor

Figure 5.16	e 5.16 Mean and standard deviations for the ratio of results from different method		
	with GMNIA for sections with h/b>2,tf>40mm for different values of $\overline{\lambda}_{z}$		
Figure 5.17	Mean along with range between mean \pm 1 standard deviation of α for general method when only axial force is present	52	
Figure 5.18	Mean along with range between mean \pm 1 standard deviation of α calculated for 6.3.1 when only axial force is present	53	
Figure 5.19	Mean and standard deviations for the ratio α from different methods for sections with h/b<1.2	55	
Figure 5.20	Mean and standard deviations for the ratio α for sections with 1.2 <h b<1.2<="" td=""><td>58</td></h>	58	
Figure 5.21	Mean and standard deviations for the ratio α from different methods for sections with h/b>2,tf<40mm	60	
Figure 5.22	Mean and standard deviations for the ratio α for sections with h/b>2,tf>40mm	63	

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List of Tables

Table 2.1	Limits for internal compression parts according to EC3-1-1 and SEMICOMP+	19
Table 2.2	Difference in cross sectional resistance in EC3-1-1 and SEMICOMP+ for class 3	19
	cross sections for relevant cases	
Table 3.1	Relative error (%) in Eigen-values for models with divisions 80 and 120	25
Table 4.1	Definition of catalogue for parametric study	26
Table 4.2	Different set of results by general method	31
Table 4.3	Summary of methods with abbreviations	32
Table 5.1	Statistical parameters for α for all cases	35
Table 5.2	Statistical parameters for values of α calculated from all results	36



1 Introduction

1.1 Introduction

Section 6.3 of EC3-1-1 deals with buckling resistance of structural members. Buckling verification of columns (members with axial compression only) can be done using section 6.3.1 whereas buckling verification for beams (members with bending moment only) could be done using section 6.3.2. The rules provided in clause 6.3.1 for flexural buckling are based on the famous Ayrton-Perry type of formulation which is of straight-forward application and it has a clear mechanical background. For buckling verification of beam-columns (members with both axial force and bending moment) EC3-1-1 provides two interaction formulae in section 6.3.3. Satisfaction of both formulae leads to both in plane and out of plane buckling verification. Application of 6.3.3 along with 6.3.1 and 6.3.2 are only valid for uniform members.

For structural components that do not come under the scope of these sections, Eurocode provides an alternative method for out of plane buckling verification in section 6.3.4 – the general method. However recent investigations have shown it not to be reliable in many scientific studies – either due to the lack of mechanical consistency or due to the lack of clarification in adopting certain decisions for non-standard situations.

This study aims to assess both these methodologies found in Eurocode.

1.2 Objectives

Following objectives have been set for this thesis:

- i. Review the application of existing procedures for verification of flexural and lateraltorsional buckling of simply supported prismatic beam-columns. This includes recent proposals for lateral-torsional buckling of beams (Taras, 2010) and the consideration of cross-section classification along member length (Greiner et al, 2011).
- ii. Provide the scope of validation to existing procedures, class 1 to class 3 cross sections, with different lengths, loading types (uniaxial bending with axial force) and I-shaped cross-section with different h/b ratios;
- iii. Safety assessment and comparison of the existing procedures in EC3-1-1: interaction formulae (clause 6.3.3) and general method (clause 6.3.4).

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1.3 Structure of the Thesis

The thesis consists of six chapters. Chapter 2 discusses and describes the different methods presented in EC3-1-1 for stability verification for beam columns. Rules provided for limiting case are also presented. In this context clauses 6.3.1 to 6.3.4 are discussed in Chapter 2. Chapter 3 discusses the numerical model used for numerical simulations; both the model and the assumptions are discussed. Chapter 4 presents the defined parametric study, and describes how resistances were calculated using different methods. Results and analysis is presented in chapter 5 and finally in chapter 6 conclusions are given.

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2 The Eurocode Methodologies

2.1 Introduction

EC3-1-1 provides two methods for stability verification of beam-columns. First one being an interaction formula based on a combination of the resistance ratios between columns and beams while the other one named as "General Method" is provided in clause 6.3.4 – the General Method allows for safety assessment of out of plane stability of structural components that do not come under the scope of applicability of clauses 6.3.1, 6.3.2, 6.3.3. In this chapter both of these methods are presented along with limiting cases of column and beam (pure flexural and pure lateral torsional buckling case).

Limiting case of column and beam are first discussed in sections 2.2 and 2.3 of this chapter. Recent proposal for lateral torsional buckling in Taras (2010) is presented in section 2.4. Section 2.5 discusses the interaction formulae for beam-columns. Section 2.6 presents the general method which is followed by a discussion on problems in general method in section 2.7.

2.2 Pure Flexural Buckling of Uniform Members

Column buckling is well studied; its verification procedure given in Eurocode is based on second order elastic theory and is well established (ECCS TC8, 2006). Clause 6.3.1 of EC3-1-1 gives the procedure for verification against such buckling for uniform members.

Verification is done by ensuring

$$\frac{N_{Ed}}{N_{b,Rd}} \le 1$$
(2.1)

where

N_{Ed}=Design value of compressive force

N_{b,Rd}=Design value of buckling resistance

Design value of buckling resistance can be calculated as follows:

For class 1, 2 and 3 sections

$$\overline{\lambda} = \sqrt{\frac{\mathrm{Af}_{\mathrm{y}}}{\mathrm{N}_{\mathrm{cr}}}}$$
(2.2)

for class 4 sections

$$\overline{\lambda} = \sqrt{\frac{A_{eff}f_y}{N_{cr}}}$$
(2.3)

$$\phi = 0.5[1 + \alpha \left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^{2}]$$
(2.4)

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \overline{\lambda}^2}} \le 1$$
(2.5)

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and finally, for class 1, 2 and 3 sections

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}}$$
(2.6)

whereas, for class 4 sections

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$$
(2.7)

Where,

 $\lambda = Non-dimensional \ slenderness$

 N_{cr} = elastic critical force

 α = Imperfection factor

 $\chi = Reduction factor$

Imperfection factor α can be obtained from table 6.1 and 6.2 of EC3-1-1. It depends upon type of cross section, thickness of cross section's plates, buckling plane and yield strength.

2.3 Pure Lateral Torsional Buckling of Uniform Members

In a case where a structural member is only subjected to bending moment, it may buckle in lateral torsional buckling mode. Buckling resistance check for uniform members in such a loading is provided in clause 6.3.2 of EC3-1-1.

The verification for lateral torsional buckling can be performed by

$$\frac{M_{Ed}}{M_{b,Rd}} \le 1 \tag{2.8}$$

Where

M_{Ed}=Design value of moment

M_{b.Rd}=Design value of buckling resistance which is given by

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$
(2.9)

 $W_y = W_{pl,y}$ for class 1 and 2 sections, $W_{el,y}$ for class 3 and $W_{eff,y}$ for class 4 sections Non dimensional slenderness for lateral torsional buckling is defined as

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$$
(2.10)

M_{cr} could be calculated from the following relevant expression

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$$
(2.11)

Two lateral torsional buckling curves are provided in EC3 under clause 6.3.2.2; Lateral torsional buckling curves-General Case and clause 6.3.2.3; lateral torsional buckling curves for rolled sections or equivalent welded sections-Special Case.

In the general case, equations that represent buckling curves are similar to those for column buckling case. These buckling curves are intended to serve as lower bound in cases with uniform bending moment in constant cross sections where more suitable lateral torsional buckling curves cannot be applied (ECCS TC8, 2006). These curves are represented by the following equations:

$$\phi_{LT} = 0.5[1 + \alpha_{LT} \left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^{2}]$$
(2.12)

$$\chi_{LT} = \frac{1}{\varphi_{LT} + \sqrt{\varphi_{LT}^2 - \overline{\lambda}_{LT}^2}} \le 1$$
(2.13)

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Recommended values for imperfection factor α_{LT} are given in table 6.3 and table 6.4 of EC3-1-1.

The second set of buckling curves is intended for rolled sections or welded sections of equivalent shapes. These curves are derived such that higher buckling curves than column buckling curves could be provided to include the favorable effects of torsional and lateral torsional rigidity present (ECCS TC8, 2006).

$$\phi_{LT} = 0.5[1 + \alpha_{LT} \left(\overline{\lambda}_{LT} - \overline{\lambda}_{LT,0}\right) + \beta \overline{\lambda}_{LT}^{2}]$$
(2.14)

$$\chi_{LT} = \frac{1}{\varphi_{LT} + \sqrt{\varphi_{LT}^2 - \beta \overline{\lambda}_{LT}^2}} \le 1$$
(2.15)

and

$$\chi_{\rm LT} \le \frac{1}{\overline{\lambda}_{\rm LT}^2}$$
(2.16)

with recommended maximum value of $\overline{\lambda}_{LT,0} = 0.4$ and recommended minimum value of $\beta = 0.75$. Table 6.5 of EC3-1-1 gives recommendations for buckling curve.

The favorable effect on buckling resistance due to variable bending moment should be included by dividing χ_{LT} with f to obtain $\chi_{LT,mod}$ which is then used to calculate $M_{b,Rd}$.

$$\chi_{\rm LT,mod} = \frac{\chi_{\rm LT}}{f} \le 1 \tag{2.17}$$

$$f = 1 - 0.5(1 - k_c)[1 - 2(\overline{\lambda}_{LT} - 0.8)^2] \le 1$$
 (2.18)

where, k_c is the correction factor whose values are to be obtained from table 6.6 of EC3-1-1.

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2.4 Recent proposal for Lateral Torsional Buckling Curves(Taras A, 2010)

The Eurocode approach to lateral torsional buckling is to treat flexural and lateral torsional buckling behaviors alike assuming a similar behavior in the sense that it uses buckling curves for flexural buckling with some modifications to be used for lateral torsional buckling (Taras A, 2010). These expressions for buckling curves have not been derived for lateral torsional buckling but instead they were adopted from flexural buckling case. Taras(2010) however proposes expressions for lateral torsional buckling curves; these in contrast have been derived in a consistent manner for lateral torsional buckling. These give results much better than current Eurocode curves when compared with GMNIA results (Marques L, 2012).

Proposed expression for reduction factor is given as

$$\chi_{LT} = \frac{\phi}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \phi \overline{\lambda}_{LT}^2}} \le 1$$
(2.19)

where

$$\phi_{\rm LT} = 0.5[1 + \varphi \left(\left(\frac{\overline{\lambda}}{\frac{LT}{\overline{\lambda}}} \right)^2 \alpha_{\rm LT} (\overline{\lambda}_{\rm e,0} - 0.2) \right) + \overline{\lambda}_{\rm LT}^2]$$
(2.20)

 φ is the load diagram factor and it takes into account the effect of variable moment across the cross section and it has a value of 1.05 for parabolic, 1.11 for triangular and $1.25 - 0.1\psi - 0.15\psi^2$ for trapezoidal bending moment where ψ is the ratio between maximum and minimum bending moment.

In Eurocode, imperfection factors for lateral torsional buckling are defined for groups of h/b<2 and h/b > 2. This makes possible to take into account the favorable effect of torsional rigidity but

by doing this we do not remain consistent with residual stress definition. $\left(\frac{\overline{\lambda}}{\overline{\lambda}_z}\right)^2$ term in the

above equation accounts for torsional rigidity and thus makes possible to maintain grouping for value of imperfection factor at h/b<1.2 and h/b >1.2. For hot rolled I sections with h/b>1.2, $\alpha_{LT} = 0.12\sqrt{W_{y,el}/W_{z,el}} \le 0.34$ whereas for h/b ≤ 1.2 , $\alpha_{LT} = 0.16\sqrt{W_{y,el}/W_{z,el}} \le 0.49$ is proposed. For welded I sections, $\alpha_{LT} = 0.21\sqrt{W_{y,el}/W_{z,el}} \le 0.64$ is proposed.

2.5 Uniform Members in Bending and Axial Compression

Stability verification according to EC3-1-1 for uniform members under bending and compression can be performed using the interaction formulas from its clause 6.3.3. The formulae were derived based on second order theory with end fork conditions on simply supported single span member and these are only valid for doubly symmetric uniform members that are not susceptible to

distortional deformations. These formulae have linear additive form such that effects of axial force and bending moment are linearly added in a proportion defined by interactions factors that incorporate non linearity.

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The two equations given in EC3-1-1 are

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$$\frac{\frac{N_{Ed}}{\chi_{y}N_{Rk}}}{\frac{N_{Ed}}{\gamma_{M1}}} + k_{yy}\frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\frac{\chi_{LT}M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$

$$\frac{N_{Ed}}{\chi_{z}N_{Rk}} + k_{zy}\frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}M_{y,Rk}}}{\chi_{LT}M_{y,Rk}} + k_{zz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rk}} \le 1$$

$$(2.21)$$

γм1

Where, N_{Ed} , $M_{y,Ed}$ and $M_{z,Ed}$ are design values of applied actions. $\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ are additional moment in class 4 sections due to shift of centroid axis. N_{Rk} , $M_{y,Rk}$ and $M_{z,Rk}$ are plastic or elastic capacities according to the class of the cross section. χ_y , χ_z , χ_{LT} are reduction factors for relevant buckling to be evaluated according to 6.3.1 for first two factors and from 6.3.2 for last one as described in section 2.2 and 2.3 of this chapter. k_{yy} , k_{yz} , k_{zy} and k_{zz} are interaction factors to be evaluated either from Annex A or Annex B of EC3-1-1.

γм1

The code proposes two different methods for the calculation for interaction factors given in Annex A (Method 1) and Annex B (Method 2). Any one of them can be chosen. Method 1 is much more comprehensive in the sense that it accounts for each physical phenomena separately whereas in contrast; in method 2 simplicity prevails. Separate accounting of each phenomenon in method 1 can be useful in design optimization whereas method 2 has the practical advantage by being simpler in nature.

In both methods a distinction has been made in sections which are susceptible to torsional deformation and those that are not. Method 1 gives a criterion when these would or would not be susceptible in the form of two equations. In both methods, for members susceptible to torsional deformation a similar approach to clause 6.3.2 is employed by adopting flexural buckling equations for lateral torsional buckling by calibrating the modified interaction factors. In method 2 each equation presented above corresponds to in or out of plane buckling verification check but in method 1 only after satisfaction of both equation in plane or out of plane buckling is verified.

The effect of material non linearity is included for class 1 and class 2 sections. In method 1 this is done by variable C_{ij} in interaction factors and using plastic capacities as characteristic resistances in interaction equations. Presence of instability prevents the achievement of total plastic capacity and this is what C_{ij} aims to include. In method 2 this is included in k_{ij} .

The C_m factors in method 1 are included to avoid the determination of cross section that is most heavily loaded. This is done by replacing bending moment with a fictitious sinusoidal bending moment that produces the same amplified bending moment. C_m values of method 2 takes uniform bending moment as the reference case. The values of Cm are derived using GMNIA results for different bending moments and that of uniform moment. In our case, the interaction factors k_{yy} and k_{zy} have to be calculated. These were calculated using method B given in Annex B of EC3-1-1. These depend upon the member class.

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For class 1 and 2, k_{yy} for members suspectible to torsion is given as

$$k_{yy} = \min(C_{my}\left(1 + \left(\overline{\lambda}_{y} - 0.2\right)\frac{N_{Ed}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right), C_{my}\left(1 + 0.8\frac{N_{Ed}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right))$$
(2.23)

whereas, for class 3

$$k_{yy} = \min(C_{my}\left(1 + \left(0.6\overline{\lambda}_{y}\right)\frac{N_{Ed}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right), C_{my}\left(1 + 0.6\frac{N_{Ed}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right))$$
(2.24)

Similarly, k_{zv} for class 1 and 2 is given by

$$k_{zy} = \max\left(\left(1 - \left(\frac{0.1 \overline{\lambda}}{(C_{mLT} - 0.25)}\right) \frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M_1}}}\right), \left(1 - \left(\frac{0.1}{(C_{mLT} - 0.25)}\right) \frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M_1}}}\right)\right)$$
(2.25)

whereas for class 3

$$k_{zy} = \max\left(\left(1 - \left(\frac{0.05 \overline{\lambda}}{(C_{mLT} - 0.25)}\right) \frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}\right), \left(1 - \left(\frac{0.05}{(C_{mLT} - 0.25)}\right) \frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}\right)\right)$$
(2.26)

2.6 Class and Resistances according to SEMICOMP+

Member and cross sectional resistance depend upon the class of the cross section. EC3-1-1 gives four sectional classes. The class of the cross section is determined by the slenderness of the plates of the section given by c/t (width to thickness) ratio.

According to Eurocode, class 3 cross sections are assumed to provide only elastic level of resistance which is in contrast to the fact that their actual resistances may be considerably higher because of internal plastic redistribution (Greiner R. et al, 2011). The plastic behavior of class 1 and class 2 cross sections is proceeded by elastic behavior of class 3 sections thus forming a discontinuity. Recent research project SEMICOMP aimed to remove this discontinuity and provide a continuous transition between class 2 and class 3 sections. A complementary project SEMICOMP+ aims to disseminate the results of this research project.

SEMICOMP+ not only provides a procedure for evaluation of design resistances for class 3 sections but also gives classification procedure for member and cross sectional class along with proposal of new c/t limits for internal compression parts.

Table 2.1 gives the new c/t limits for internal compression parts according to SEMICOMP+.

Clas	S	In bending	In compression	In bending and compression
1	EC3-1-1	$c/t \leq 72\varepsilon$	$c/t \leq 33\varepsilon$	when $\alpha > 0.5$: $c/t \le \frac{396\varepsilon}{13\alpha - 1}$ 36ε
				when $\alpha \leq 0.5$: $c/t \leq \frac{1}{\alpha}$
	SEMICOMP+	$c/t \leq 72\varepsilon$	$c/t \leq 28\varepsilon$	when $\alpha > 0.5$: $c/t \le \frac{126\varepsilon}{5.5\alpha - 1}$
				when $\alpha \le 0.5$: $c/t \le \frac{36\varepsilon}{\alpha}$
2	EC3-1-1	$c/t \le 83\varepsilon$	$c/t \leq 38\varepsilon$	when $\alpha > 0.5$: $c/t \le \frac{456\varepsilon}{13\alpha - 1}$
				when $\alpha \leq 0.5$: $c/t \leq \frac{41.5\varepsilon}{\alpha}$
	SEMICOMP+	$c/t \le 83\varepsilon$	$c/t \leq 34\varepsilon$	when $\alpha > 0.5$: $c/t \le \frac{188\varepsilon}{6.53\alpha - 1}$
				when $\alpha \leq 0.5$: $c/t \leq \frac{41.5\varepsilon}{\alpha}$
3	EC3-1-1	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	when $\psi > -1$: $c/t \le \frac{42\varepsilon}{0.67 + 0.33\psi}$
				when $\psi \leq -1$: $c/t \leq 62\varepsilon(1-\psi)\sqrt{-\psi}$
	SEMICOMP+	$c/t \le 124\varepsilon$	$c/t \leq 38\varepsilon$	when $\psi > -1$: $c/t \le \frac{38\varepsilon}{0.653 + 0.347\psi}$
				when $\psi \leq -1$: $c/t \leq 62\varepsilon(1-\psi)\sqrt{-\psi}$
Wh	ere, $\varepsilon = \sqrt{235}$	\overline{fy} , αc is portion	on in compression	, ψ is the ratio of tensile stress to compressive stress

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at the end of the internal part.

Table 2.1: Limits for internal compression parts according to EC3-1-1 and SEMICOMP+. The relevant difference in cross sectional resistance calculations for this thesis between existing procedures and SEMICOMP+ for class 3 cross sections is given in table 2.2.

Case	EC3-1-1	SEMICOMP+
Mono-axial	$\frac{M_{Ed}}{M_{Ed}} < 1$	$M_{Ed} < 1$
Bending	$W_{el}f_{y/\gamma_{M0}} \leq 1$	$W_3 f_{y/\gamma_{M0}} $
Mono-axial	$\frac{N_{Ed}}{M_{Ed}} + \frac{M_{Ed}}{M_{Ed}} < 1$	$\frac{N_{Ed}}{N_{Ed}} < 1$
Bending and	$N_{el,Rd}$ ' $M_{el,Rd}$ = 1	$N_{pl,Rd}$ -
Compression		$\frac{M_{Ed}}{M_{Ed}} < 1$
		$M_{N.3.y,Rd} = 1$
		$M_{N.3.y,Rd} = M_{3.y,Rd}(1-n)$
		$M_{3.y,Rd} = \frac{W_3 f_y}{\gamma_{M0}}$
		$n = \frac{N_{Ed}}{N_{pl,Rd}}$

Table 2.2: Difference in cross sectional resistance in EC3-1-1 and SEMICOMP+ for class 3 cross sections for relevant cases.

Utilization factors (UF) have to be calculated for 11 points along the member. Utilization factor is defined as

$$UF = \frac{R_{Ed}}{R_{Rd}}$$
(2.27)

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Where R_{Ed} represents given loading and R_{Rd} represents resistance. R_{Rd} is obtained by increasing the loading (axial force and moments) in the same proportion until resistance R_{Rd} . R_{Rd} depends upon the class of the cross section. It has to be ensured that utilization factors at all 11 points are less than 1.

The class at the highest utilization factor is the member class and is decisive for member checks.

2.7 General Method

General method presented in clause 6.3.4 of EC3-1-1 allows for verification of resistance of out of plane stability. The methods presented above and detailed in clauses 6.3.1 to 6.3.3 are applicable to uniform members. General method may be used as an alternative for sections that do not fall under the applicability of these clauses.

According to general method the out of plane resistance can be checked by satisfying

$$\frac{\chi_{\rm op}\alpha_{\rm ult,k}}{\gamma_{\rm M1}} \ge 1 \tag{2.28}$$

where

 $\alpha_{ult,k}$ is the minimum load amplifier for design loads to reach characteristic resistance of most critical cross section considering only the in plane behavior with effects from in plane global and local geometrical deformations and imperfections.

 χ_{op} is the reduction factor for non-dimensional slenderness $\overline{\lambda}_{op}$

 $\overline{\lambda}_{op}$ is to be determined from

$$\overline{\lambda}_{\rm op} = \sqrt{\frac{\alpha_{\rm ult,k}}{\alpha_{\rm cr,op}}}$$
(2.29)

where, $\alpha_{cr,op}$ is the minimum load amplifier for in plane design loads to reach elastic critical load in lateral or lateral torsional buckling mode without taking into account in plane flexural buckling.

By using this λ_{op} , the value of χ_{op} is to be calculated. For χ_{op} either a minimum from χ and χ_{LT} from 6.3.1 and 6.3.2 or an interpolated value between from χ and χ_{LT} by using the formula for $\alpha_{ult,k}$ corresponding to critical cross section may be used.

Finite element analysis can be used to calculate $\alpha_{ult,k}$ and $\alpha_{cr,op}$.

In contrast to 6.3.3, the general method does not make a separation between different types of actions but takes the member as a whole. The method also has a much wider range of applicability e.g. uniform sections, non-uniform sections, mono-symmetric sections built up or not etc. The method could also be used to analyze frames or sub frames composed of these members which may have any irregular support condition. However method does not cover cases with biaxial bending moment.

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2.8 Problems with General Method

General method has some problems with its consistency, reliability and applicability.

It has been shown in Simões da Silva et al(2010) that inconsistent results with clause 6.3.1 for columns are obtained when in plane local imperfections are included for the calculation of $\alpha_{ult,k}$. An upper bound to resistance is provided in the form of $\alpha_{ult,k}$ that is obtained by considering inplane effects. In-plane effects do not in all cases have such pronounced effect on out of plane buckling resistance.

A problem of inconsistency in safety level is observed as out of plane resistance may be calculated from two different ways for the calculation of χ_{op} . This gives two different safety levels.

Finite element analysis may be used for the evaluation of amplification factors in general method. However in practice, problems could arise due to lack of proper guidance e.g. in definition of magnitude and shape of imperfections etc. This may also be time consuming.

General method is proposed as an alternative for cases where clauses 6.3.1 to 6.3.3 do not apply whereas the method in itself makes use of reduction factors taken from these clauses whose expressions were derived from assumptions made in these clauses.

The usage of first option for calculation of reduction factor that is the minimum of reduction factors for flexural and lateral torsional buckling yields discontinuity of results (Simões da Silva, 2010). Usage of the other option mitigates this problem. It must be noted that ECCS TC8 (2006) has recommended the use of first option only.

The conservativeness of results by the general method may partly be due to the conservativeness of values of reduction factor used. Thus any betterment in expressions for buckling curves e.g. in Taras (2010), also calls for check of reliability of the method again.

3 Numerical Model

3.1 Introduction

Numerical models were generated for comparison of results from different methodologies and for the calculation of results from the general method of Eurocode. In the former case, results of geometrically and materially non-linear analysis (GMNIA) of these models were assumed to be representatives of real behavior. This chapter describes the models and the assumptions therein.

3.2 The Model

Commercial finite element software ABAQUS version 6.13 was used for generation of numerical models. Four nodal linear shell elements (S4) were used with six degree of freedom per node.

Elasto-plastic constitutive law was used. Newton Raphson iterations were used to solve the equilibrium equations within each increment of the load stepping routine; the increment size being based on convergence criteria.

3.3 Material Non-Linearity

Von-Mises yield criteria is used for non-linearity of material.

Steel was modeled as an elastic perfectly plastic material with a Poisson ratio of 0.3 and modulus of elasticity of 210GPa.



Fig 3.1: Modeled stress strain curve

Minimum yield stress provided in product standard EN10025 for corresponding thickness was used as yield stress.



Fig 3.2: Minimum yield stress for different thickness of cross section according to EN10025

3.4 Imperfections

Geometrical imperfections proportional to the Eigen-mode obtained from LBA (Linear Bifurcation Analysis) were included with a maximum value of L/1000.

For example for the Eigen-mode shape given in figure 3.3, imperfection shape will correspond to figure 3.4.



Fig 3.3: First buckling mode for HE300B under uniform axial compression ($\lambda = 1$).



Fig3.4: Imperfection shape for section with loading conditions given in figure 3.3. In cases where global and other buckling modes interacted, appropriate web and flange stiffeners had to be applied to get uncoupled buckling modes for introduction of imperfections. Care is maintained such that imperfections are in a direction such that least favorable results are obtained.

Following membrane residual stresses were considered as material imperfections in the models.



Fig 3.5: Material Imperfections for h/b < 1.2 and for h/b > 1.2.

3.5 Supports

Simply supported end fork boundary condition is adopted for the models, that is vertical and transverse displacements are prevented at both ends at a node situated at the center of the web; whereas longitudinal displacement is only prevented at one end. Ends are modeled to remain straight but they are allowed to wrap.

3.6 Loading

Axial force and end moments are applied as concentrated force/moment at the ends on nodes at mid-height of the web whereas parabolic moment distribution is modeled as distributed loading at the center of the web at nodes along the length of member. Concentrated vertical loading

corresponding to a triangular moment distribution of the shape of isosceles triangle is applied on the node at the center of the member at mid-height of web.



3.7 Meshing

Divisions for meshing were set after mesh convergence. Mesh convergence was done setting the number of divisions along the web and flange as 16 and varying the divisions along the length of the members for $\lambda_z=1$ for each section and doing a Linear Bifurcation Analysis with only compressive axial force. 120 divisions along the length were set for for $\lambda_z=1$. For all other values of λ_z an extrapolated number of divisions were used. Table 3.1 shows the percentage relative error in Eigen-values for first buckling modes of sections with divisions 80 and 120.

Sections	Relative Error (%)
HE300A	0.13
HE300B	0.17
HD400x347	0.23
HE300C	0.22
HE340B	0.18
HE400A	0.16
HE500B	0.20
IPE100	0.18
IPE160	0.17
IPE180	0.17
HE600x337	0.22
IPE360	0.15
HE650B	0.19
HE650x343	0.21
HE300A	0.13

Table 3.1: Relative error (%) between Eigen-values for models with 80 and 120 divisions.

4 Methodology

4.1 Introduction

For the assessment of different methodologies, a parametric study was carried out. Resistances were calculated for cases according to the parametric study by clauses 6.3.1 to 6.3.3 of EC3-1-1, general method of EC3-1-1 and by geometric and materially non-linear analysis. This chapter aims to coherently describe the procedure. This chapter describes the catalogue for parametric study and the methodology for calculation of resistances through these methods.

4.2 Catalogue for Parametric Study

Parametric study consisted of 4144 cases. Following table summarizes the parametric study conducted.

Sections			$\Phi = \frac{\alpha_{pl}^{My}}{\alpha_{pl}^{N}}$	$\overline{\lambda}_{z}$	Му
			∞	0.4	
Sections	h/b	tf(mm)	5.67	0.6	
HEA300	0.97	14	2.74	0.8	
HEB300	1.00	19	1.73	1	
HD400x347	1.01	43.7	1.19	1.2	\sim
HE300C	1.05	29	0.83	1.4	
HEB340	1.13	21.5	0.57	1.6	
HEA400	1.30	19	0.36	1.8	
HEB500	1.67	28	0.17		
IPE100	1.82	5.7	0		
IPE160	1.95	7.4			
IPE180	1.98	8			
HE600x337	2.04	46			
IPE 360	2.12	12.7			
HEB650	2.17	31			
HE650x342	2.20	46			
14	•		9 + 1	8	4

Table 4.1: Definition of catalogue for parametric study

$$\alpha_{pl}^{My} = \frac{M_{pl,y,Rd}}{M_{y,Ed}} \qquad \qquad \alpha_{pl}^{N} = \frac{N_{pl,Rd}}{N_{Ed}}$$
(4.1)

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Only hot rolled sections with S325 steel grade were considered. All sections are at the most class 3 for pure compression and class 2 for bending. Only uniform axial force was considered.

4.3 Factor ϕ

Resistances calculated from GMNIA, general method and through clauses 6.3.1 to 6.3.3 are calculated such that applied loading i.e. axial force and moment about y-axis are in a proportion defined by the factor ϕ .

 ϕ is defined as

$$\phi = \frac{\frac{M_{pl,y,Rd}}{M_{Ed}}}{\frac{N_{pl,Rd}}{N_{Ed}}}$$
(4.2)

Resistances were calculated using different values of ϕ , as defined in the parametric study.

4.4 Resistances from clause 6.3.1 to 6.3.3 of EC3-1-1

In this thesis, member and cross sectional classes and design resistances for class 3 sections were determined according to SEMICOMP+ as explained in section 2.6. All sections considered in the parametric study were either class 1,2 or 3.

The minimum of cross sectional resistance and buckling resistance is considered as the design resistance.

For $\phi = \infty$ (only axial force), the cross sectional resistance in EC3-1-1 is given by and was calculated using

$$N_{Rd} = \frac{Af_{y}}{\gamma_{M0}} \tag{4.3}$$

For $\phi = 0$ (only bending moment), the cross sectional resistance for class 1 and 2 is given by

$$M_{Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} \tag{4.4}$$

and for class 3 according to SEMICOMP it is given by

$$M_{Rd} = \frac{W_3 f_y}{\gamma_{M0}} \tag{4.5}$$

For class 1 and 2 cross sections when $\phi = (0, \infty)$, the cross sectional resistance is obtained from clause 6.2.9.1 of EC3-1-1. Equation 6.36 of EC3-1-1 is for hot rolled sections when both bending and axial force are present.

$$M_{N,y,Rd} = M_{pl,y,Rd} \frac{(1-n)}{1-0.5a}$$
(4.6)

where $n = N_{Ed}/N_{pl,Rd}$ and $a = (A - 2bt_f)/A$ but $a \le 0.5$.

Using equations 4.2 and 4.6 for a given ϕ , N_{Rd} could be calculated by

$$N_{Rd} = \frac{1}{\left[\frac{1-0.5a}{\phi N_{pl,Rd}} + \frac{1}{N_{pl,Rd}}\right]}$$
(4.7)

Using equation $4.2, M_{Rd}$ could then be calculated by

$$M_{Rd} = \frac{M_{pl,y,Rd}}{\phi} * \frac{N_{Rd}}{N_{pl,Rd}}$$
(4.8)

For class 3 sections, expressions from SEMICOMP+ (2011) are used

$$M_{N.3.y,Rd} = M_{3.y,Rd}(1-n)$$
(4.9)

where $n = N_{Ed} / N_{pl,Rd}$.

Using a similar procedure, N_{Rd} and M_{Rd} are calculated using expressions

$$N_{Rd} = \frac{M_{3.y,Rd}}{\left[\frac{M_{pl,y,Rd}}{\phi_{N_{pl,Rd}}} + \frac{M_{3.y,Rd}}{N_{pl,Rd}}\right]}$$
(4.10)

$$M_{Rd} = \frac{M_{pl,y,Rd}}{\phi} * \frac{N_{Rd}}{N_{pl,Rd}}$$
(4.11)

For $\phi = \infty$ (only axial force), the buckling resistance is calculated using

$$N_{Rd} = \frac{\chi A f_y}{\gamma_{M0}} \tag{4.12}$$

where $\chi = \min(\chi_y, \chi_z)$ and was calculated using expression 2.5 for y and z axis. For $\phi = 0$ (only bending moment), the buckling resistance was calculated using

$$M_{Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}}$$
(4.13)

 χ_{LT} was calculated using proposals from Taras A. (2010) as explained in section 2.4 and from the general case and special case in clause 6.3.2 of EC-1-1.

For the calculation of $\overline{\lambda}_{LT}$, elastic critical moment M_{cr} was calculated using equation 2.11. when $\phi = (0, \infty)$, the buckling resistance has to be calculated using

$$\frac{\frac{N_{Ed}}{\chi_{y}N_{Rk}}}{\frac{\gamma_{M1}}{\gamma_{M1}}} + k_{yy}\frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}M_{y,Rk}}}{\frac{\gamma_{LT}M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$
(4.14a)

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \le 1$$
(4.14b)

In our case, the interaction factors k_{yy} and k_{zy} have to be calculated. These were calculated using method 2 given in Annex B of EC3-1-1. These depend upon the member class. These factors are presented in equations 2.23 to 2.26.

For a given ϕ , equations 4.14a and 4.14b could be written as

$$XN_{Rd} + (C_{my} + AN_{Rd})(ZN_{Rd}) - 1 = 0$$
(4.15a)

$$YN_{Rd} + (1 - BN_{Rd})(ZN_{Rd}) - 1 = 0$$
(4.15b)

where

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$$A = \min\left(\left(\overline{\lambda}_{y} - 0.2\right) \frac{C_{my}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}, 0.8 \frac{C_{my}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right) for \ class \ 1 \ and \ 2$$
(4.16)

$$A = \min\left(\left(0.6\,\overline{\lambda}_{y}\right)\frac{C_{my}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}, 0.6\,\frac{C_{my}}{\frac{\chi_{y}N_{Rk}}{\gamma_{M1}}}\right) for \ class \ 3$$
(4.17)

$$B = \min\left(\left(\frac{0.1\,\overline{\lambda}}{(C_{mLT} - 0.25)}\right)\frac{1}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}, \left(\frac{0.1}{(C_{mLT} - 0.25)}\right)\frac{1}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}\right) \text{for class 1 and 2}$$
(4.18)

$$B = \min\left(\left(\frac{0.05\,\overline{\lambda}}{(C_{mLT} - 0.25)}\right)\frac{1}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}, \left(\frac{0.05}{(C_{mLT} - 0.25)}\right)\frac{1}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}\right) for \ class \ 3$$
(4.19)

$$X = \frac{1}{\frac{\chi_y N_{Rk}}{\gamma_{M_1}}} \tag{4.20}$$

$$Y = \frac{1}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}$$
(4.21)

$$Z = \frac{W_{pl,y}/\gamma_{M0}}{\frac{\chi_{LT}\phi A W_{class} f_y}{\gamma_{M1}}}$$
(4.22)

 $W_{class} = W_{pl,y}$ for class 1 and 2 whereas $W_{class} = W_{3,y}$ for class 3. Equations 4.15a and 4.15b are quadratic equations and these could be solved to calculate N_{Rd} . Sensible result out of the two roots is chosen using simple conditions $N_{Rd} > 0$ and $N_{Rd} < N_{pl}$. Minimum N_{Rd} from the equations 4.15a and 4.15b is the buckling resistance N_{Rd} . M_{Rd} could then be calculated by

$$M_{Rd} = \frac{M_{pl,y,Rd}}{\phi} * \frac{N_{Rd}}{N_{pl,Rd}}$$
(4.23)

4.5 Resistances from General Method

For the calculation of out of plane buckling resistance through the general method, a Linear Bifurcation Analysis (LBA) and an in plane GMNIA was done. These were done for the calculation of different amplification factors required in the general method. In plane GMNIA was done by applying out of plane restraints of members. The in plane amplification factor required in the general method needs to be calculated considering effects from in plane imperfections. An in plane bow imperfection was applied to all sections.

The global non dimensional slenderness $\overline{\lambda}_{op}$ in the general method is calculated using

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$$\overline{\lambda}_{\rm op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \tag{4.24}$$

where $\alpha_{ult,k}$ is the in plane amplification factor which was calculated using in plane GMNIA. $\alpha_{cr.op}$ is obtained from LBA.

For $\phi = \infty$, the out of plane buckling resistances is given by

$$N_{Rd} = \chi_{op} N_{inplane} \tag{4.25}$$

where $N_{inplane}$ is the resistance obtained from in plane GMNIA.

 χ_{op} is the reduction factor that is to be calculated using λ_{op} . For this case ($\phi = \infty$), χ_{op} was calculated using equations for χ for flexural buckling.

For $\phi = \infty$, the out of plane buckling resistances is given by

$$M_{Rd} = \chi_{op} M_{inplane} \tag{4.26}$$

where $M_{inplane}$ is the resistance obtained from in plane GMNIA.

For this case ($\phi = 0$), χ_{op} was calculated using equations for χ_{LT} for lateral torsional buckling using $\overline{\lambda}_{op}$. Two different set of equations are provided in EC3-1-1 under clause 6.3.2.2-general case and clause 6.3.2.3-special case. Both methods were used to get two set of buckling resistances for cases with $\phi = 0$.

For the case $\phi = (0, \infty)$, the out of plane buckling resistance was calculated using

$$N_{Rd} = \chi_{op} N_{inplane} \tag{4.27}$$

where $N_{inplane}$ is the resistance obtained from in plane GMNIA.

 M_{Rd} could then be calculated by

$$M_{Rd} = \frac{M_{pl,y,Rd}}{\phi} * \frac{N_{Rd}}{N_{pl,Rd}}$$
(4.28)

EC3 gives two different alternatives for calculation of calculation of χ_{op} , either the minimum of χ and χ_{LT} or an interpolated value between χ and χ_{LT} . Results were obtained from both. Interpolation was done using following equation given in Simões da Silva et al (2010).

$$\chi_{op} = \frac{\phi + 1}{\frac{\phi}{\chi} + \frac{1}{\chi_{LT}}}$$
(4.29)

 χ_{LT} was also calculated using both the general case and the special case. In total four different set of results were obtained. Table 4.3 describes the different set of results obtained by general method.

ϕ	Xop	χ_{LT}
ω	χ	-
0	XLT	General Case
	ΧLT	Special Case
(0,∞)	Minimum b/w χ and χ_{LT}	General Case
	Minimum b/w χ and χ_{LT}	Special Case
	Interpolated b/w χ and χ_{LT}	General Case
	Interpolated b/w χ and χ_{LT}	Special Case

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Table 4.2: Different set of results by general method

4.6 GMNIA

GMNIA was done to compare the results obtained from the previous two methods. GMNIA were assumed to be representative of real behavior. Models for GMNIA were created using the numerical model as explained in chapter 3.

First a linear bifurcation analysis was performed to get the buckling shape. In cases where local and global buckling modes interacted, appropriate stiffeners were added in LBA to get a global mode only. Imperfections in the GMNIA models were taken from these LBAs.

4.7 Self Defined Variables for Calculation of Assessment of Results

 χ_{method} and α_{method} were defined for assessment of the results. These are defined as follows

$$\chi_{method} = \frac{\sqrt{N_{method}^2 + M_{method}^2}}{\sqrt{N_{cs}^2 + M_{cs}^2}}$$
(4.30)

where N_{method} , M_{method} are the values of results obtained from the method and N_{cs} , M_{cs} represent the cross sectional capacity at the same ϕ value.

 α_{method} was defined to indicate the error in the result obtained by the method used. It is defined as

$$\alpha_{method} = \frac{\chi_{method}}{\chi_{GMNIA}} = R_{Method} R_{GMNIA}$$
(4.31)

where R_{Method} , and R_{GMNIA} are resistance obtained from the method and by GMNIA.

4.8 Summary

The chapter explains the methodology used for the calculation of resistances using different methods.

Resistances were calculated from clauses 6.3.1 to 6.3.3 of EC3-1-1, general method of EC3-1-1 and from GMNIA. Table 4.3 summarizes the methods and the set of results in them.

I

Abbreviation

6.3.1

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Method			ϕ range	Remarks					
Using 6.3.1	Eurocode	clause	ω	-SEMICOMP+ used for class of members and resistance of class 3 cross sections.					
Using 6.3.2	Eurocode	clause	0	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. $-\chi_{LT}$ from general case.					
Using 6.3.2	Eurocode	clause	0	-SEMICOMP+ used for class of members and resistance of class 3 cross sections.					

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6.3.2-GC	6.3.2	0	resistance of class 3 cross sections. - χ_{LT} from general case.
6.3.2-SC	Using Eurocode clause 6.3.2	0	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. $-\chi_{LT}$ from special case.
6.3.2-Taras	Using Eurocode clause 6.3.2	0	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. - χ_{LT} from recent proposals in Taras A. (2010).
6.3.3-GC	Using Eurocode clause 6.3.3	(0,∞)	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. $-\chi_{LT}$ from general case.
6.3.3-SC	Using Eurocode clause 6.3.3	(0,∞)	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. $-\chi_{LT}$ from special case.
6.3.3-Taras	Using Eurocode clause 6.3.3	(0,∞)	-SEMICOMP+ used for class of members and resistance of class 3 cross sections. - χ_{LT} from recent proposals in Taras A. (2010).
GM	General Method	8	$-\chi_{op}$ calculated from expression for χ in EC3-1-1.
GM-GC	General Method	0	$-\chi_{op}$ calculatedfromexpressionfor χ_{LT} in EC3-1-1. $-\chi_{LT}$ calculated from general case.
GM-SC	General Method	0	$-\chi_{op}$ calculatedfromexpressionfor χ_{LT} in EC3-1-1. $-\chi_{LT}$ calculated from special case.
GM-Min, GC	General Method	(0,∞)	$-\chi_{op}$ calculated from minimum b/w χ and χ_{LT} . $-\chi_{LT}$ calculated from general case.
GM-Min, SC	General Method	(0,∞)	$-\chi_{op}$ calculated from minimum b/w χ and χ_{LT} . $-\chi_{LT}$ calculated from special case.
GM-Int, GC	General Method	(0,∞)	$-\chi_{op}$ calculated by interpolation b/w χ and χ_{LT} . $-\chi_{LT}$ calculated from general case.
GM-Int-SC	General Method	(0,∞)	$-\chi_{op}$ calculated by interpolation b/w χ and χ_{LT} . $-\chi_{LT}$ calculated from special case.
GMNIA	Geometrically and materially non-linear analysis	All ϕ	-Imperfections from LBA.

Table 4.3: Summary of methods with abbreviations.



5 Results and Analysis

5.1 Introduction

Resistances for a total of 4144 cases were obtained from GMNIA, general method and from clauses 6.3.1 to 6.3.3 of EC3-1-1 using the procedure outlined in chapter 4.

This chapter presents the results and makes comparisons for assessment and validation of these methods. Results were found to be scattered on both sides of GMNIA results (both greater than and less than GMNIA results).

The results are described in the sections below.

5.2 Overview of the results

The scatter of the results is shown in figure 5.1. In figure 5.1, χ_{method} for different methods is plotted against χ_{GMNIA} .





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Figure 5.1: Scatter plot of self defined reduction factor χ for different methods against χ GMNIA.

In Figure 5.1, the points below the red dashed line represent conservative results.

In comparison to results from clauses 6.3.1 to 6.3.3 of EC3-1-1, the scatter for general method is generally higher in all of its cases in comparison to the corresponding special or general case from 6.3.1 to 6.3.3 with more number of un-conservative results. Results from clauses 6.3.1 to

6.3.3 when χ_{LT} is calculated from recent proposals of Taras (2010) are more closer to GMNIA results than when χ_{LT} is calculated from general case and special case of EC3-1-1. The number of unsafe results are highest when special case is used in comparison with other methods for calculation of χ_{LT} in general method (using interpolation or minimum) and 6.3.3. The effect of special case is much more noticeable when interpolation is used in general method in comparison with the minimum criteria.

	6.3.1 to	6.3.1 to	6.3.1 to	GM-	GM-	GM-	GM-
	6.3.3-Taras	6.3.3-GC	6.3.3-SC	Min,GC	Min,SC	Int,GC	Int,SC
Mean	0.91	0.89	0.91	0.89	0.90	0.93	0.98
SD	0.06	0.06	0.06	0.08	0.09	0.08	0.07
CoV	0.06	0.07	0.07	0.09	0.10	0.09	0.08
Mean+SD	0.97	0.95	0.97	0.98	0.99	1.01	1.05
Mean-SD	0.85	0.82	0.85	0.81	0.81	0.84	0.90

Table 5.1 show some statistical parameters for α for all the cases analyzed.

Table 5.1: Statistical parameters for α for all cases

5.2 Variation of Results with h/b Ratio

Different buckling curves for different h/b ratio and thickness limits are used for calculation of reduction factors. Based upon these ratios and limits, different statistical parameters were calculated for comparison of results in these ranges.

Table 5.2 gives different statistical parameters calculated for α for different groups of h/b ratios and thickness limits. Mean, standard deviation, co-efficient of variation, minimum and maximum of α is calculated along with percentage of cases when α is less than 0.9 and when it is greater than 1.03 is also calculated.

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	tf	Method	n	Mean (α)	SD	CoV (%)	Min (α)	Max (α)	%cases	%cases
h/b Limit	Limit (cm)								α<0.9	α>1.03
		GM-Min,GC	1454	0.86	0.07	7.80	0.69	1.04	74.07	0.21
		GM-Min,SC	1454	0.86	0.08	8.97	0.69	1.05	73.11	1.99
h/h<1.2	-	GM-Int,GC	1414	0.91	0.04	4.78	0.79	1.05	46.68	0.35
11/0<1.2		GM-Int,SC	1414	0.95	0.05	5.18	0.84	1.06	18.53	3.11
		6.3.2-Taras	1461	0.91	0.05	5.57	0.75	1.05	44.49	0.48
		6.3.2-GC	1461	0.89	0.06	6.34	0.71	1.05	54.76	0.34
		6.3.2-SC	1461	0.91	0.05	5.98	0.75	1.06	43.60	2.05
		GM-Min,GC	1477	0.93	0.05	5.11	0.78	1.13	23.70	0.88
		GM-Min,SC	1477	0.94	0.06	5.90	0.78	1.18	22.07	4.60
	_	GM-Int,GC	1429	0.96	0.04	4.32	0.84	1.13	7.63	4.55
1.2 <h b<2<="" td=""><td></td><td>GM-Int,SC</td><td>1437</td><td>1.01</td><td>0.04</td><td>3.63</td><td>0.91</td><td>1.18</td><td>0.00</td><td>26.37</td></h>		GM-Int,SC	1437	1.01	0.04	3.63	0.91	1.18	0.00	26.37
		6.3.2-Taras	1477	0.92	0.05	5.37	0.78	1.17	37.24	0.41
		6.3.2-GC	1477	0.90	0.06	6.62	0.71	1.14	47.19	0.81
		6.3.2-SC	1477	0.93	0.06	6.04	0.78	1.17	35.00	4.60
	+f<10	GM-Min,GC	570	0.94	0.05	5.27	0.79	1.06	18.07	1.93
		GM-Min,SC	570	0.95	0.05	5.24	0.81	1.06	13.86	4.21
		GM-Int,GC	556	0.94	0.05	5.27	0.79	1.06	18.53	1.98
	11\40	GM-Int,SC	556	1.01	0.03	3.18	0.94	1.11	0.00	19.24
		6.3.2-Taras	574	0.92	0.06	6.08	0.71	1.04	37.80	0.70
h/b>2		6.3.2-GC	574	0.88	0.07	7.80	0.64	1.02	55.05	0.00
		6.3.2-SC	574	0.92	0.06	6.45	0.72	1.06	39.37	2.26
		GM-Min,GC	592	0.87	0.06	6.98	0.72	1.07	75.00	0.17
		GM-Min,SC	592	0.88	0.07	8.23	0.72	1.13	69.93	1.52
	tf>40	GM-Int,GC	576	0.89	0.05	5.68	0.78	1.07	67.88	0.35
		GM-Int,SC	576	0.95	0.05	5.21	0.86	1.13	15.10	7.81
		6.3.2-Taras	592	0.87	0.06	6.80	0.75	1.07	76.52	0.34
		6.3.2-GC	592	0.83	0.06	7.10	0.67	1.03	87.67	0.00
		6.3.2-SC	592	0.86	0.06	6.88	0.75	1.07	76.69	1.35

Table 5.2: Statistical parameters for values of α calculated from all results.



Figure 5.2 to 5.8 show the position of mean and standard deviation of α calculated from different methods.

Fig 5.2: Position of mean and standard deviation for results calculated from 6.3.1 to 6.3.3-GC.



Fig 5.3: Position of mean and standard deviation for results calculated from 6.3.1 to 6.3.3-SC.



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Fig 5.4: Position of mean and standard deviation for results calculated from 6.3.1 to 6.3.3-Taras.



Fig 5.5: Position of mean and standard deviation for results calculated from GM-Int,GC



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Fig 5.6: Position of mean and standard deviation for results calculated from GM-Int,SC



Fig 5.7: Position of mean and standard deviation for results calculated from GM-Min,GC



Fig 5.8: Position of mean and standard deviation for results calculated from GM-Min,SC

The general method gives more unsafe results in groups h/b>2 and 1.2<h/b<2 except for the case h/b>2 and tf>40 where it gives results comparatively better than the method's own results in other groups with higher h/b. GM-Int, SC gives more unsafe results when compared with other alternatives in general method whereas GM-Min,GC appears to give safer results in comparison with other alternatives in general method.

5.3 Variation of Results by ϕ

Fig 5.9 below shows the mean of the ratio α for different methods and its variation with increasing ϕ for sections with h/b<1.2.





Fig: 5.9: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b<1.2



Fig 5.10 below shows the mean of α and its variation with increasing ϕ for sections with 1.2<h/b<2.

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Fig: 5.10: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with 1.2< h/b<2.

Fig 5.11 below shows the mean of the ratio α for different methods and its variation with increasing ϕ for sections with h/b>2, tf<40mm.



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Fig: 5.11: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2,tf<40mm.



Fig 5.12 below shows the mean of α and its variation with increasing ϕ for sections with h/b>2, tf>40mm.



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Fig: 5.12: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2,tf>40mm.

Generally, the standard deviation reduces with increasing ϕ for all methods. This does not happen for general method when calculated using interpolation with special case. Results from

general method calculated like this have more unsafe results in all h/b groups. Also the results become less unsafe with increasing ϕ . Results calculated from clauses 6.3.1 to 6.3.3 have mean ratio lower than mean α from general method and is always positive, no matter how the reduction factor for lateral torsional buckling is calculated.

5.4 Variation of Results by $\overline{\lambda}$

Fig 5.13 below shows the mean α and its variation with increasing $\overline{\lambda}_{z}$ for sections with h/b<1.2.





Fig: 5.13: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b<1.2.

Fig 5.14 below shows the mean α and its variation with increasing λ_z for sections with 1.2<h/b<2.



European Erasmus Mundus Master Sustainable Constructions under natural hazards and catastrophic events 520121-1-2011-1-CZ-ERA MUNDUS-EMMC



Fig: 5.14: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with 1.2<h/b<2.

Fig 5.15 below shows the mean of the ratio of results obtained from different methods and GMNIA and its variation with increasing $\overline{\lambda}_{z}$ for sections with h/b>2, tf<40mm.





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Fig: 5.15: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2,tf<40mm.

Fig 5.16 below shows the mean of the ratio α for different methods and its variation with increasing $\overline{\lambda}_{z}$ for sections with h/b>2, tf>40mm.







Fig: 5.16: Mean and standard deviations for the ratio of results from different methods with GMNIA for sections with h/b>2,tf>40mm.

The results from clauses 6.3.1 to 6.3.3 appear to have the same level of difference from GMNIA results at both high slenderness and low slenderness except for sections with h/b>2,tf>40mm group where they slightly decrease in comparison with GMNIA results. The standard deviation also remains constant for these results. The mean of results from these clauses is lower than GMNIA results in all h/b groups. The results obtained from general method vary along slenderness. This was also noted in Simões da Silva et al. (2010). The mean of the ratio between results obtained from general method and results from GMNIA generally decrease with increasing slenderness. The results obtained by general method are more conservative results at higher slenderness than at lower slenderness. More unsafe results are obtained when interpolation is used in comparison with when minimum criteria is used for reduction factor in general method. Also results for general method are greater when special case is used than when general case is used for calculation of lateral torsional buckling reduction factor. General method gives more un-conservative results for sections in group h/b>2, tf<40mm and group 1.2<h/b<2 than for groups h/b<1.2 and h/b>2,tf<40mm. This is consistent with results in Simões da Silva et al. (2010) where more un-conservative results are obtained for more slender cross sections.

5.5 Variation of Results with Different Loading Cases

Figure 5.17 describes the mean and range between mean ± 1 standard deviation of α when only axial force is present (results are calculated from general method for different h/b groups).



Figure 5.17: Mean along with range between mean ± 1 standard deviation of α for general method when only axial force is present.

Figure 5.18 describes the mean and range between mean ± 1 standard deviation of α when only axial force is present (results are calculated from 6.3.1 for different h/b groups).



Figure 5.18: Mean along with range between mean ± 1 standard deviation of α calculated for 6.3.1 when only axial force is present.

Figure 5.19 show the mean and range between mean ± 1 standard deviation of α for different methods when the loading case is axial force and different bending moments, for sections with h/b<1.2.



a)



d)



Fig: 5.19: Mean and standard deviations for the ratio α from different methods with GMNIA for sections with h/b<1.2.

Figure 5.20 show the mean and range between mean ± 1 standard deviation of α for different methods when the loading case is axial force and different bending moments for sections with 1.2<h/b<2.



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Fig: 5.20: Mean and standard deviations for the ratio α for sections with h/b<1.2.

Figure 5.21 show the mean and range between mean ± 1 standard deviation of α for different methods when the loading case is axial force and different bending moments for sections with h/b>2,tf<40mm.



a)



d)

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g)

Fig: 5.21: Mean and standard deviations for the ratio α from different methods for sections with h/b>2,tf<40mm.



Figure 5.22 show the mean and range between mean ± 1 standard deviation of α for sections with h/b>2,tf>40mm.

b)



e)

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Fig: 5.22: Mean and standard deviations for the ratio α for sections with h/b>2,tf>40mm.

The mean of ratio α is higher for uniform bending moment and parabolic moment for results calculated from 6.3.3 in all h/b groups. Similar behavior is noted for GM-Min,GC, GM-Min,SC, GM-Int,GC for sections with 1.2<h/b<2 and h/b>2,tf<40mm.

5.6 Summary

Results obtained from clauses 6.3.1 to 6.3.3 and general method are compared with results from GMNIA in this chapter.

The results from clauses 6.3.1 to 6.3.3 are more conservative in comparison with results from general method. It was also seen that the number of un-conservative results from general method are more than from clauses 6.3.1 to 6.3.3.

The scatter of results was found to be higher when bending moment was greater than axial force. The results of general method vary in different groups of cross sectional slenderness defined by using h/b ratios and tf. The results from general method also vary with the length of the member. These conclusions were found to be consistent with conclusion from Simões da Silva et al (2010).

6 Conclusions

Stability verification according to the Eurocode could be made by either clauses 6.3.1 to 6.3.3 or from 6.3.4-general method. Results from these two methods of the Eurocode were compared with GMNIA results for assessment of these methods. Results from clauses 6.3.1 to 6.3.3 were calculated using general case, special case and recent proposals from Taras A. (2010) for lateral torsional buckling reduction factor. Similarly for general method, results were calculated using the interpolation and minimum criteria for calculation of reduction using both the general case and the special case.

Out of the different ways for calculation of lateral torsional buckling reduction factor when results are calculated from clauses 6.3.2 and 6.3.3, results from proposals from Taras A. were found to be much more closer to GMNIA results than from the general case. The results from special case were also much more closer to GMNIA results than general case but they also had more unsafe results. General case was found to be most conservative of the three different options.

In the general method, the results were more unsafe when interpolation criteria was used than when minimum criteria was used. More scatter was present on the conservative side when general case was used than when special case was used. Results from general method when interpolation and special case was used had the most number of un-conservative results. The unconservative results from the general method could reach up to 10% difference from GMNIA results.

Most of the results obtained from the clauses 6.3.1 to 6.3.3 were lower in comparison with GMNIA results. General method gave more un-conservative results.

The scatter of the results shown by the standard deviation of difference in results from GMNIA was found to more larger when ϕ was closer to zero (bending moment greater than axial force) for all methods.

In the general method, difference in results from GMNIA varied by cross sectional slenderness. More slender cross sections were found to have more un-conservative results. This is consistent with the finding in Simões da Silva et al (2010). Also variability of difference in results from GMNIA was noted with increasing length of the cross sections. Generally in the general method, results became more conservative with increase in length. This is also consistent with the finding in Simões da Silva et al (2010).

The mean+standard deviation calculated from all results was always greater for general method than the corresponding case in 6.3.3.

In conclusion, the general method should be avoided because of its un-conservative nature. General method is proposed as an alternative for cases where clauses 6.3.1 to 6.3.3 do not apply. Additional problem of applicability in these cases might arise due to non-availability of proper guidelines/recommendations for application to these cases. This may further aggravate the un-conservative nature of general method.

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The number of cases with only axial force were limited in this thesis. Also the numbers of cases with h/b>2,tf<40mm and tf>40mm were less. Future work should include similar investigation for the column case and for sections in these h/b and thickness limits.

The range of applicability of the general method includes non-prismatic members, members with complex supports and plane members. The general method is provided in Eurocode as an alternative to clauses 6.3.1 to 6.3.3 for cases when these would not apply. A study could be carried out to investigate how safe this method is for such cases for which this method is intended.

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