**FCTUC DEPARTAMENTO DE ENGENHARIA CIVIL**
FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE DE COIMBRA

# Location Models in Humanitarian Logistics

**Dissertação apresentada para a obtenção do grau de Mestre em Engenharia Civil na Especialidade de Urbanismo, Transportes e Vias de Comunicação**

**Autor**

## Diogo Rafael dos Santos Forte

**Orientador**

## Miguel Gueifão Santos



**Coimbra, Junho, 2014**

## ACKNOWLEDGEMENTS

I would like to thank my advisor Prof. Dr. Miguel Gueifão Santos for making this opportunity possible. Furthermore, I would like to express my gratitude towards his guidance, corrections and knowledge provided not only in this project but throughout this last academic year of studies. I would like to state that without his contribution, the conclusion of this thesis would not have been possible.

I also would like to express my gratitude to the Civil Engineering Department of the University of Coimbra and to the Rzeszow University of Technology for the tools and knowledge provided during the last five years and for allowing me to feel ready to embrace new challenges. To my friends, a special thanks for the help during the long work hours and for the good moments.

I want to thank my parents for their unconditional support throughout my entire life, for believing in me, for accepting every choice I made and above all else, for their love. Without their encouragement and sacrifice, this project would not have been possible.

Last but not least, I want to thank my girlfriend, Natacha Rodrigues, for being my companion, for always supporting and understanding me, for your unconditional love and most importantly, for making me feel as happy and as fulfilled as anyone can be.

## RESUMO

Logística humanitária designa o processo de planeamento, gestão e controlo dos recursos necessários ao socorro de pessoas afetadas por desastres naturais ou causados pela ação humana. A literatura em logística humanitária é extensa, em particular a baseada em técnicas de otimização. Os primeiros modelos de otimização para a resolução de problemas de logística humanitária foram desenvolvidos nos anos 1960s e 1970s, e a partir dos anos 1980s o número de modelos desenvolvidos cresceu rapidamente. A literatura em logística humanitária desenvolve-se em dois ramos, dependendo de se considerarem as operações antes ou depois do impacto do desastre, designadas respetivamente por operações pré-desastre e pós-desastre. No âmbito desta dissertação, são estudados alguns modelos de otimização desenvolvidos para o planeamento de operações de logística humanitária em situações de pré-desastre. Os modelos estudados abordam problemas de evacuação (localização de abrigos) e de distribuição de recursos (localização de centros de distribuição) durante situações de desastre. A aplicabilidade dos modelos é analisada através da sua implementação para a resolução de problemas hipotéticos.

## ABSTRACT

Humanitarian Logistics consists in the process of planning, managing and controlling resources needed to help people affected by natural disasters or by disasters caused by human activity. The literature on humanitarian logistics is extensive, particularly the one based on optimization techniques. The first optimization models to solve humanitarian logistics problems of were developed in the 1960s and 1970s, and since the 1980s the number of models developed grew rapidly. The literature on humanitarian logistics is developed into two branches, depending on whether they consider emergency operations before or after the impact of a disaster, designated respectively by pre-disaster and post-disaster operations. Within the context of this dissertation, we studied some optimization models developed for the planning of humanitarian logistics operations in pre-disaster situations. Models which were studied within this dissertation address the issues of evacuation (locating shelters) and resource distribution (location of distribution centers) during disaster situations. The applicability of the models is analyzed through its implementation for solving hypothetical instances.

# TABLE OF CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

## LIST OF SYMBOLS

$\boldsymbol{J}$ – Set of centers, indexed by *j*

$\boldsymbol{K}$ – Set of sites, indexed by *k*

$\boldsymbol{R}$ – Set of serviceability levels, indexed from 0 to $p-1$

$\boldsymbol{A}$ – Set of network zones, indexed by *a*

$\boldsymbol{C}$ – Set of proximity levels, indexed by *c*;

$\boldsymbol{S}$ – Set of failure scenarios, indexed by *s*

$\boldsymbol{I}$ – Set of LSA locations, indexed by *i*

$\boldsymbol{N}$ – Set of types of distribution centers, indexed by *n*

$\boldsymbol{T}$ – Set of types of commodities, indexed by *t*

$\boldsymbol{L}$ – Set of facility types, indexed by *l*

$d_{jk}$ –Shortest path distance from center $j$ to site $k$;

$h_j$ – Number of residents in center $j$;

$q$ – Probability of failure occurring;

$p$ – Maximum number of shelters to be located;

$s_{ka}$ – Zone $a$ in which site $k$ is located;

$j_c$ – $c^{\text{th}}$ closest shelter to population in center *j*;

$d_j^c$ – Distance between population in center $j$ and its $c^{\text{th}}$ closest operational shelter;

$f$ – Maximum number of fortified shelters;

$q_s$ – Probability of scenario $s$ occurring;

$a_j$ – Demand of resources in location $j$;

$d_{ik}$ – Distance from distribution center $k$ to LSA $i$;

$p_n$ – Maximum number of distribution centers of type $n$;

$e_n$ – Maximum holding capacity in distribution centers of type $n$;

$v_{jts}$ – Demand of commodity $t$ in center $j$ during scenario $s$;

$m_{lt}$ – Maximum holding capacity of commodity $t$ in facility type $l$;

$g_{kt}$ – Amount of commodity $t$ pre-stocked on site $k$;

$p_l$ – Maximum number of facility type $l$;

$qr_s$ – Probability of scenario $s$ occurring;

$M$ – Penalty factor;

$L_{jra}$ – Zone $a$ to which residents of center $j$ are assigned to in level $r$;

$Y_k$ – Binary variable. Assumes the unitary value if a shelter is located in *k*, and 0 otherwise;

$X_{jkr}$ – Binary variable. Assumes the unitary value if population from *j* is served in site $k$ on level $r$, and 0 otherwise;

$Z_k$ – Binary variable. Assumes the unitary value if a shelter is fortified in site *k*, and 0 otherwise;

$W_{jc}$ – Binary variable. Assumes the unitary value if the $c - 1$ closest shelters to center *j* are not protected but the $c^{\text{th}}$ closest shelter is, and 0 otherwise;

$a_{ks}$ – Binary value. Assumes the unitary value of site *k* fails in scenario *s*, and 0 otherwise;

$X_{jks}$ – Binary variable. Assumes the unitary value if population from *j* is served in site $k$ on scenario *s*, and 0 otherwise;

$U$ - Variable which defines the distances travelled in the worst scenario;

$X_{ikn}$ – Variable. Resources transported from LSA $i$ to distribution center $k$ of type $n$;

$Z_{kn}$ – Binary variable. Assumes the unitary value if a distribution center of type *n* is located in site *k*, and 0 otherwise;

$Y_{jkn}$ – Binary variable. Assumes the unitary value if population from $j$ are assigned to distribution center *k* of type *n*, and 0 otherwise;

$X_{jkts}$ – Variable. Demand of commodity $t$ from center *j* that is served in facility located in site $k$ during scenario $s$;

$Y_{kl}$ – Binary variable. Assumes the unitary value if facility of type *l* is opened in site *k*, and 0 otherwise;

# 1 INTRODUCTION

A disaster is the result of a sudden disruption in the ecological balance between man and his environment, posing a significant, widespread threat to human life, health, property or the environment (United Nations, 1999). Regarding its origin, a disaster can either be categorized as natural or as man-made (Caunhye et al., 2012). Natural disasters are events such as earthquakes, floods or hurricanes, whereas man-made disasters events include leakage of chemical products and terrorist attacks, among others. Wassenhove (2006) states that disasters can be further categorized according to the quickness in which its effects take place: sudden-onset disasters have much quicker and uncontainable effects on the environment, and slow-onset disasters take time to develop into a full blown catastrophe. Events such as earthquakes, hurricanes, coup d'états and terrorist attacks are considered to be sudden-onset disasters, while famine, drought and political/refugee crisis are events which are considered to be slow-onset disasters.

The number of occurring disasters has consistently grown over the years, and every year, an average of more than 500 disasters are estimated to strike our planet, leaving a destruction path that kills around 75,000 people and severely affects 200 million others (Balcik and Beamon, 2008). In an annual statistical review, Guha-Sapir et al. (2013) show that the number of reported disasters has maintained significantly higher when compared to the previous decade (see Figure 1.1). Among the valuable statistical data presented, some figures come out as worrying. According to this study, in the year 2012, out of the top ten countries affected by natural disasters, three countries – India, Indonesia and Philippines – are middle-lower income economies, and another three countries – Afghanistan, Bangladesh and Haiti – are low income economies (according to the World Bank income classification).

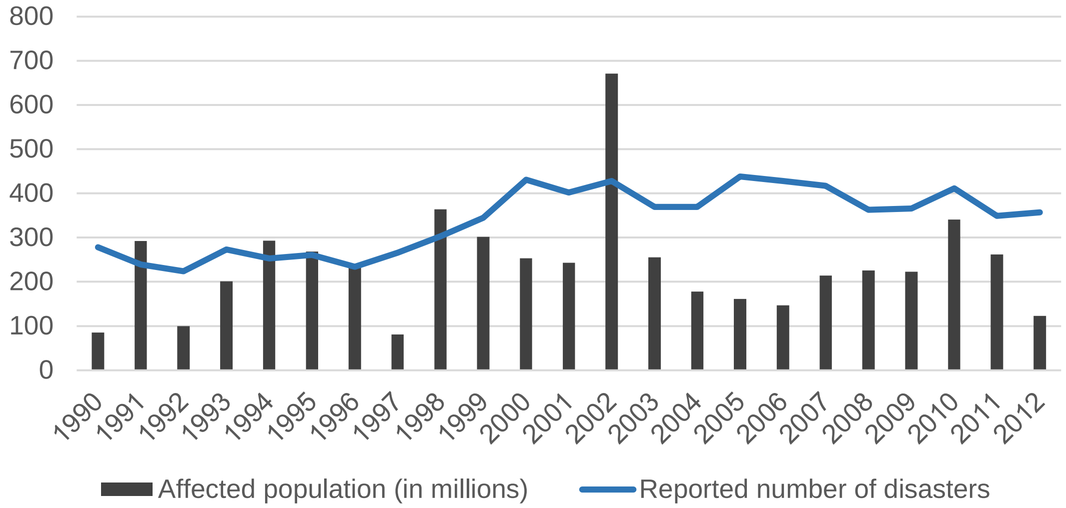


Figure 1.1 – Number of disaster occurrences and affected population between 1990 and 2012 (source: Guha-Sapir et al., 2013)

To add up to the increasing number of disaster occurrences, Fritz Institute (2005) indicates that during the 2004 Indian Ocean tsunami, one of the most devastating recent natural incidents, no pre-established plan of action was set up in case a disaster took place, thus, leading to a massive amount of aid that ended up not being delivered to people in need. Later reports (BBC News, 2010) provided by on field reporters during the 2010 Haiti earthquake indicate that the same neglect might have taken place as well. Furthermore, Haiti's case has become an example on how the lack of planning can result in catastrophic consequences.

Taking into account the impact that disasters create, especially when affecting under-developed areas, and the trends of occurrence in disasters, it is clear that there is a need for the development of efficient planning for responses to disasters. These disaster response operations are addressed in humanitarian logistics, which is defined as "the process of planning, implementing and controlling the efficient, cost-effective flow and storage of goods and materials as well as related information, from point of origin to point of consumption for the purpose of meeting the end beneficiary's requirements" (Wassenhove, 2006). Humanitarian logistics problems are those which look for ways to alleviate the damages caused to the population by disaster situations.

Humanitarian logistics diverge from traditional business-related logistics, which is the most developed area of research in logistic problems. Problems found within this area commonly have the main focus of locating a series of warehouses and defining transportation plans that guarantee the most cost effective way of supplying each warehouse with goods. The main differentiating factor between these areas is the non-emergency environment that traditional logistics problems are developed in. According to Balcik and Beamon (2008) and Kaynak and

Tuğer (2014), approaches that solve a traditional logistics problem may not do as well for a humanitarian logistics problem. Factors such as the uncertainty of safety conditions, on roads and buildings, and demand levels, for resources and people in need of assistance, present a serious problem to the creation of transportation plans and in determining the assistance needs. To cover issues related with uncertainties, humanitarian logistics problems may take into account stochastic data to define several possibilities of demand levels and conditions of the overall network of roads and surrounding buildings, according to the magnitude of the disaster. Furthermore, humanitarian logistics is a derivation of emergency logistics, a specific area of business-related logistics which is aimed at coping with uncertainties in emergency/disaster situations and which takes into account stochastic data. However similar, while humanitarian logistics aim at providing relief to affected people, emergency logistics aim at reducing company costs inherent to disaster occurrences (Snyder et al. 2005).

During recent years, humanitarian logistics have been the focus of a great number of research efforts. In the available literature regarding this subject, 45% of studies were published after the year 2000, giving a clear indication that the research on the topic is rapidly expanding (Altay and Green 2006). Research efforts in humanitarian logistics problems are divided according to the time phase when disaster operations take place, be it before or after a disaster strikes (Caunhye et al. 2012). Pre-disaster operations are all the efforts that take place before the impact of a disaster. During this phase, strategic planning and disaster mitigation take center stage, focusing on facility location, stock pre-positioning and evacuation efforts. Post-disaster operations, take place after a disaster occurs and have the sole purpose of attending to disaster impacts, be it by way of distributing resources or by providing transportation to disaster casualties (injured or deceased).

In this dissertation, we will focus on two of the most important problems addressed in pre-disaster operations: evacuation and relief distribution with stock pre-positioning. Studies performed to address both problems use facility location models, which are derived from classical models for the location of public facilities.

Evacuation models determine the best location of shelters, and define routes of access from the populated centers to the located shelters. These models present solutions which may consider events such as the failure of certain road links, which may alter the choice of routes to get to a shelter, and the failure of one/several located shelters, which may require to opt for a different and farther away point of assistance. These models present solutions that encompass the number of people served by each shelter and the routes used between their location of residency and closest shelter under operation.

Relief distribution and stock pre-positioning models look for ways to provide the population with access to resources (such as food, water and medication) which may be needed in disaster situations. Furthermore, these models determine the amount of goods that should be placed in each warehouse/distribution center and also the routes to be used. Some models in this context require that a set of suppliers be considered, in order to account for a first phase of planning that involves the transportation of resources from each supplier to the located distribution centers. This consideration allows us to determine the amount of resources that is stocked in each distribution center and the supplier from which it was originally transported. Solutions commonly presented for these problems consider the minimization of aggregated travelled distances between the suppliers and the distribution centers as well as the distances between the distribution centers and each populated center in need.

## 1.1 Objective Statement and Overall Structure

Regardless of the moment of action, humanitarian logistics problems focus essentially on the response time and user accessibility under chaotic situations. As facility location problems look to improve timeliness of demand response and functional cost in the best way possible, they present themselves to be one of the most important and efficient areas of study in order achieve an adequate emergency plan, especially with the usage of optimization models. Given that fact, in this dissertation, several facility location optimization models used for the planning of pre-disaster operations will be presented.

For the development of this dissertation we defined two main goals: providing a description of the main problems addressed in humanitarian logistics for pre-disaster operations; and performing a detailed study of optimization models used to solve those main problems.

The remainder of this dissertation is organized in four Chapters. In Chapter 2, a brief overview of the existing literature on evacuation and relief distribution models is presented. In Chapters 3 and 4, the matters of location-evacuation problems (planning the location of facilities with the objective of minimizing evacuation time) and location with relief distribution and stock pre-positioning (planning the location of facilities with the objective of positioning and delivering the necessary resources), are addressed, respectively. For both types of situations, optimization models are presented in order to achieve a solution that minimizes the distances that residents have to travel in order to receive the needed assistance. In Chapter 5 we present conclusions regarding the objectives that were established.

# 2 FACILITY LOCATION PROBLEMS IN HUMANITARIAN LOGISTICS

In humanitarian logistics, facilities considered for location are either shelters or distribution centers (also referred to as warehouses). Facility location models are designed to aid decision regarding the best possible location of shelters/distribution centers taking into account problems such as the timeliness of evacuation efforts, preparedness of relief distribution and effectiveness of stock pre-positioning. Regardless of the problem being analyzed, most facility location models focus on a single-period. Moreover, facility location models are mainly based on mixed integer programs (usually referred to as MIP) with binary location variables. In addition, these models can be differentiated according to the available data type, which can use deterministic parameters or a mixture of deterministic and stochastic parameters (be it probabilistically distributed or scenario-based).

The available literature on facility location models in humanitarian logistics can be classified into two different categories according to the main problems that are considered: Location-evacuation and location with relief distribution and stock pre-positioning. The following sub-chapters elaborate on optimization models used in each of the mentioned categories.

## 2.1 Location-Evacuation

Location-evacuation models determine the best set of locations for a set of shelters, in order to serve the affected population at the minimum travel distance. Furthermore, common outputs from these models are the number and location of the optimal set of shelters as well the assignment routes from affected areas to the located shelters.

In our overview of available models on the topic of evacuation related problems we found that emergency logistics models are a frequently cited topic on operational research. As explained in Chapter 1 of this dissertation, emergency logistics tend to deal with problems related to the economic performance and efficiency of networks, considering the effects of disasters. Studies like the one conducted by Snyder et al. (2005), lead us to believe that if the necessary adaptations are taken under consideration, emergency logistics models can be used as a basis for the analysis of humanitarian logistics problems, which in turn, leads to a wider spectrum of possibilities in terms of the research material that is available. Chapter 3 of this dissertation, explores the similarities between the problems and expands on the notions that the models developed in emergency logistics are applicable to humanitarian logistics, if only factors such as costs of locating facilities are changed or dismissed.

Among the research conducted by Snyder et al. (2005), we considered the analysis of five emergency logistics models which are easily relatable to the topic of evacuation models in humanitarian logistics: Reliability Fixed-Charge Location Model (RFLM), Capacitated Reliability Fixed-Charge Location Model (CRFLM), Minimax-Cost Reliability Fixed-Charge Location Model (MMRFLM), *p*-Median Fortification Model (PMFM) and Capacitated *p*-Median Fortification Model (CPMFM). The RFLM considers the risks connected to disaster events by using probabilistic data regarding the failure of facilities. As a consequence of using probabilistic data, the output from this model presents a serviceability plan for each level of proximity from center of population to the location of the facilities. For each level of proximity, assignments are made to guarantee that if a facility closer to the customer fails, he can be served by a different, and non-disrupted, facility that is located further. The CRFLM considers scenario-based failure events and decides the location of facilities and determines the assignments of residents, after the occurrence of a random disruption (failure). The MMRFLM is developed to analyze the worst-case scenario. It aims at minimizing costs only throughout failure scenarios with noticeable probability of occurrence (e.g. scenarios with probability higher than 0.01). Furthermore, it determines the location solution which best suits the worst-case scenario. The PMFM and CPMFM are derived from RFLM and CRFLM, respectively, and determine what facilities (which location is known *a priori*) should be fortified. The CRFLM and CPMFM may consider capacity limits to the facilities. Furthermore, these capacity limits are not addressed for the study described in this dissertation.

Location-evacuation models developed specifically within humanitarian logistics are associated with a variety of uncertainties: traffic flows, evacuee panic and demand quantities being the most commonly considered. We present the following studies as examples of some of the applications found within the available literature regarding each of the cited uncertainty topics.

Sherali et al. (1991) studied hurricane/flood situations, with consideration of the impact of shelter locations on evacuation time. Furthermore, a model was proposed to select a set of shelters from among a group of potential locations and prescribe an evacuation plan, which minimizes the total evacuation time of the affected population. This model covers the study of traffic flow’s influence on decision making, by considering the effects that congestion in available road links has on travelling time. Based on the premise that Wardrop (1952) proposed, this model, considered that the time of travel when using a certain link, varies according to the level of saturation (percentage of link’s capacity being used) presented by that link at the point of being chosen as a part of the evacuation route.

Kongsomsaksakul et al. (2005) presented a location-allocation model for flood evacuation planning that determines the number and locations of a set of capacitated shelters as well as the flow of movements within the road network. Contrary to most models, this proposed approach dismisses the idea that administrative authority has total control over evacuees' choices. Instead, this model factors in evacuee panic as a major decision factor on the destination of each affected person by defending that route decisions should be made by evacuees, taking into account the damage presented in the roads, traffic congestion of those same roads and the location and capacity of the destination shelters.

Song et al. (2009) take a different approach to most cases, by studying evacuation operations that are performed by a pre-assigned public bus pick-up service. This model aims at minimizing the total evacuation time and does it by identifying the optimal serving areas and vehicle routes to move evacuees from affected zones to their designated shelter location. Furthermore, shelter and population pick-up points' location must be determined by this model, as well as the transportation plan from each pick-up point to the located shelters. Furthermore, this model studies, with great focus, issues related to demand quantities, as they are critical to guarantee that all population is served (relation between bus capacity/evacuee number has to be carefully managed).

Although the majority of models that plan for location-evacuation consider a single-objective, Alçada-Almeida et al. (2009), consider a multi-objective analysis and aim at minimizing total travel distance, risk of primary path being impassable, risk of destruction of the shelters and on a complementary level, it aims at minimizing casualty transportation time from the shelters to local hospitals. This model diverges from previously referred ones as it takes into account the possibility of not only route failure, but also failure of located shelters. The authors develop a model for determining the number and location of shelters and identify the primary and secondary evacuation routes for affected residents to take to the operable shelters. For each residential zone, a back-up route and shelter are identified in order to avoid complications during the evacuation process.

## 2.2 Location with Relief Distribution and Stock Pre-Positioning

Facility location models with relief distribution and stock pre-positioning are used with the final objective of deciding upon the location of a set of warehouses/distribution centers and at the same time determining the amount and type of resources that those facilities should be stockpiled with. These models also create a plan for the distribution of stockpiled goods to each of the affected surrounding areas. The costs that are considered depend on the nature of the disaster, although, in general, costs related with building facilities, acquiring resources and distribution (transportation) of relief goods are commonly considered.

Early models such as the one presented in Psaraftis et al. (1986) were developed to help reduce damage caused by oil spill disasters and aimed at providing decision makers with a model that located the best set of resource storage facilities in order to achieve faster and less costly clean-up operations, by way of stocking the needed type of resources (each type of resource is associated with an acquisition cost and effectiveness of response) in the located facilities and creating a plan to allocate the resources to affected areas. Furthermore, the presented model takes into account stochastic data regarding the most likely points of disaster occurrence and the demand of resources that those disasters would generate. In addition, besides the more commonly used costs, this model considers unmet demand costs, that represent a penalization for escaping oil due to delay of distribution or by resource ineffectiveness. Wilhelm and Srinivasa (1996) reformulate oil spill models (due to the OPA – Oil Pollution Act) and consider a model that takes into account time phased requirements. Furthermore, the latter propose a different perspective of locating facilities by acknowledging that existent warehouses' expansion should be considered.

Chang et al. (2007), proposed a multi-level approach to flood situations that first formulates an organizational structure, by creating a grouping of regional distribution centers according to the likelihood of flood occurrences and secondly locates facilities with sufficient resources in each established region and determines the distribution flow of resources to be made from each distribution center to each disaster affected area. Moreover, this model provides decision makers with a logistics structure, organized by level of rescue importance, that considers the possibility of resource shortage in each region, but at the same time, allows for different lower level regional distribution centers to provide back-up in those cases. Mete and Zabinski (2010) additionally consider situations where route planning for the distribution of medical resources to local hospitals is affected by the magnitude of the occurring disaster (earthquakes were studied for this model), thus advocating the creation of scenarios where certain road links should be considered unusable/undesirable and creating alternative transportation routes that guarantee the timeliness of delivery of additionally stockpiled resources in a subset of existing hospital warehouses. Rawls and Turnquist (2010) expands on the uncertainty of conditions to be considered, by developing a model that aims at locating pre stockpiled warehouses and guarantees the distribution of resources to the affected population, even while considering scenarios where partial/total pre-positioned warehouse stock is destroyed due to the disaster's spectrum of action as well as damage caused to the network of roads. Furthermore, this model considers costs regarding the holding of excess resources (in case they are not used in certain scenarios) and a penalization for unmet demand (due to destruction of stock).

Horner and Downs (2010) propose a slightly different approach to relief distribution compared to the previous models, by developing an optimization model that considers the last phase of the distribution process to be handled by the affected population. It is done by assuming that population is in charge of travelling to the assistance point (much like the location-evacuation models), be it by vehicle or by foot, in case they are nearby or if the access road is destroyed; this assistance point is previously serviced by a central “logistics staging area”. The proposed model considers the location of additional relief facilities (called break of bulb points), which are smaller in cost as well as size and assistance capability, in order to guarantee that remote population can have better access to help. The objective is to locate normal and smaller size distribution centers in order to minimize costs and guarantee better accessibility to goods in all the affected region.

# 3 EVACUATION MODELS

Evacuation models are developed in order to determine the location of a network of shelters and define routes of evacuation from every populated center. These routes will lead people from every affected center to arrive as safely and as quickly as possible to one of the located shelters. Furthermore, these models look to achieve a solution that minimizes the overall distances travelled by all the affected population.

In this chapter we will consider the studies performed by Snyder et al. (2005) and apply them to a humanitarian logistics context. Models presented in that study were developed to cover emergency logistics problems and consider costs regarding the installation of facilities (fixed-charge). From a humanitarian logistics standpoint, it makes no sense to define an installation cost associated with each facility that is located and so, a limitation of the number of facilities is considered instead. By dropping the fixed location costs and replacing them with a limitation to the number of facilities, the models should be designated as $p$-median optimization models. This variable, $p$, is attributed a value, bearing in mind that every inhabitant of the affected region should be served in a way which allows for a good level of service across the entire region and at the same time guarantees that no unnecessary budget excess is made in terms of the solutions that are made available. Furthermore, we present plans that consider reliable facility locations. This type of planning efforts take in to account the occurrence of unexpected failures in facilities, each with a probability of occurrence associated with the site where the facility is to be located. Within the contents of our study, there are two types of models that are considered: design models and fortification models.

Design models represent situations where a solution has to be built from scratch and its aim is to determine the number and location of facilities. Furthermore, these models determine the assignments of population to be made to each located facility. Fortification models, consider a pre-defined set of facilities, and have the objective of determining which of the existing facilities should be fortified in order to cope with failure events and enhance the proximity of affected population to protected facilities. Additionally, most design models have a corresponding fortification model in terms of approach. It is possible to improve planning efforts by relating both phases and integrating the two models, in order to locate facilities and to identify a subset of those facilities to fortify against failure.

As an attempt to more easily understand the adaptation of emergency logistics models to humanitarian logistics requirements, in this dissertation, we devised the grouping of directly relatable design and fortification models, and created two different categories according the models' characteristics: proximity-based reliability models study the effects of disasters on a network of shelters by proposing hypothetical assignment solutions according to the level of proximity of the shelters to each of the populated centers; scenario-based reliability models study the occurrence of pre-defined specific failure combinations in certain areas of the territory and develop the best solution of assignments according to each disaster scenario.

For proximity-based reliability models, we considered two different models: a *p*-median location model (based on RFLM) and a *p*-median fortification model (based on PMFM). Both of these models consider that there are no limitations in terms of the capacity of each located shelter and always look to assign the population to their closest operable shelter. They present solutions that give indications on what might happen in case all populated centers are served by a shelter in a specific level of proximity. Furthermore, for each populated center, these models analyze the distances to all the located shelters and indicate which shelter is the closest, which is the furthest away and all other levels in between. At this point it should be noted that solutions presented in these models are merely hypothetical, as no specific disaster occurrences are considered (these are presented in scenario-based models).

The *p*-median location model presents the best possible set of shelter locations to minimize the distances travelled by the population. The solutions are presented according to the level of proximity that decision makers indicate. It basically generates solutions that illustrate the assignments that we should expect in case all the centers would be served by shelters in the same level of proximity.

The *p*-median fortification model determines a subset of shelters (which may be determined by the design model) that should be fortified against failures and that guarantee the least amount of distances travelled by the population. It indicates what would be the level of proximity for which a populated center might find the closest fortified shelter. Furthermore, for this model, it is assumed that all the population must be served by a facility that has been protected. Additionally, this hypothetical solution determines the worst case in which a center might be served during disaster occurrences, as it does not consider the possibility that a non-failing non-fortified shelter might serve population, which in reality will happen.

For scenario-based reliability models, we considered three different models: a scenario-based *p*-median location model (based on CRFLM), a scenario-based *p*-median fortification model (based on CPMFM) and a minimax *p*-median location model (based on MMRFLM). In this category, all models take into account specific disaster occurrences, represented by pre-defined scenarios associated with a probability of occurrence. Their objective is to grant a result of locations and assignments that presents a good level of functionality for the scenarios that are considered. No limitations in terms of capability are considered and for all models, a plan for the assignments in each scenario is provided. Furthermore, each scenario represents a different combination of located failures.

The scenario-based *p*-median location model presents a solution for the location of shelters that aims to minimize the amount of distances travelled across all the scenarios that are accounted for. It presents a solution that grants importance to all scenarios, according to their probability of occurrence: a scenario that is more likely, should have a higher level of importance, and scenarios that are unlikely to occur, are considered less important. In this model, for each scenario, we are presented with a plan of assignments that allocates the population in all centers to their closest operational shelter.

The scenario-based *p*-median fortification model takes the shelter location solutions provided by its corresponding design model and determines which shelters should be fortified. The decision on which subset of shelters to fortify is taken according to the same scenarios analyzed in the scenario-based *p*-median location model. The model develops a location solution that guarantees that throughout all the scenarios, the population travels the least amount of distances possible. Furthermore, a solution of assignments is also presented according to each scenario.

The minimax *p*-median location model presents a different approach from the other models in this category. Instead of considering a minimization of the overall distances travelled across all scenarios, this model looks for ways to optimize the results for the worst scenario of all. It takes into account the exact same premises as the scenario-based *p*-median location model, however in this case, only one scenario is viewed as being important. It develops a solution for the location of shelters that guarantees that the network has the best performance possible for the most devastating scenario. In this case, scenarios that are taken into consideration might be subjected to a reduction in order to look for a solution that does not compromise the functionally for scenarios that are not the worst case.

In this chapter, we will describe the approaches taken to reproduce the previously stated models, present the adaptations from the original models and compare results between each approach that is taken.

## 3.1 Description of Test Instances

A random test instance was used to demonstrate the applicability of the models. Taking into account the studies performed in Teixeira and Antunes (2008), the instance was built by considering a set of 20 centers and sites, with their coordinates (x,y) randomly generated in the range of 0-5 Km. Centers consist of the set of locations that serve as residence to the inhabitants of the region, while sites represent the set of locations where facilities might be placed. Furthermore, it was assumed that sites would be coincident with centers. After analyzing the coordinates for each center, a planar network was created by defining a Delaunay Triangulation and the corresponding Voronoi Diagram (Lee, 1980). The graphical representation of the resulting network is as follows:

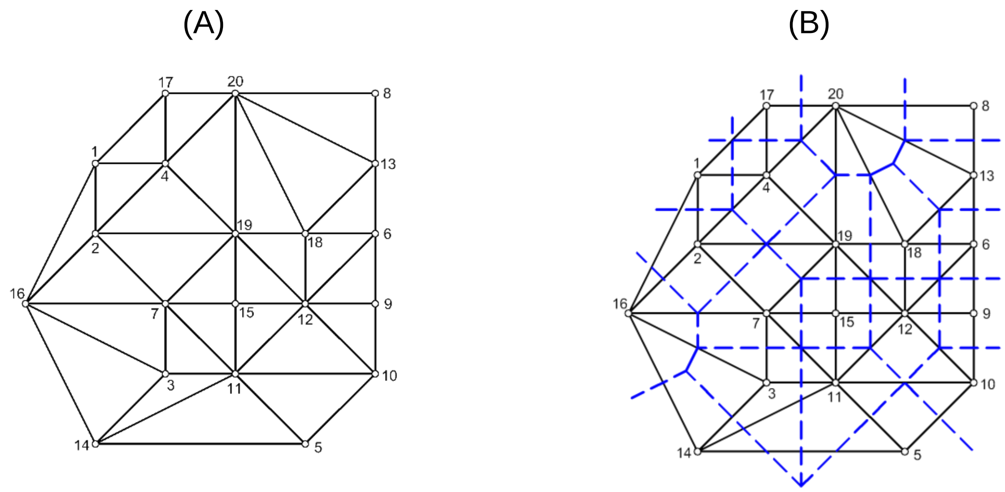


Figure 3.1 – Network design with (A) Delaunay Triangulation plus (B) Voronoi Diagram

This would generate a triangular mesh connecting all the centers and sites, simulating the existence of a series of roads that joins all the points in the network and creating a situation that is more closely related to a real network. Additionally, the Voronoi Diagram gives us a representation of the proximity between points in the network: a point located within a restricted area is more closely located to the center/site located in that restricted area than any other center/site in the whole region. Finally, with the network defined, the Euclidean distances, $d_{jk}$, were determined by finding the all shortest paths on the resulting network. Furthermore, demand levels were generated also considering the studies performed in Teixeira and Antunes (2008). For each center, we considered demand values randomly generated in the range of 5-100 units of demand. Data regarding the instances (coordinates, distances and demand) is presented in the Appendix: Table A.1 shows the network coordinates; Table A.6 illustrates the shortest path distance matrix; Table A.2 presents data regarding the generated demand levels.

A zoning of the network was developed and is presented in Figure 3.2. This zoning tries to reflect the division that is commonly present in cities nowadays. It generally aggregates areas that have similar topographical (terrain elevation) and structural characteristics (size of buildings), among others. As it pertains to the occurrence of disasters, this aggregation of areas in several zones is very useful when it comes to protecting the population. By separating each zone, we can prepare for the event of isolated or combined failures in the zones considered. Additionally, we can assume that when a disaster strikes a specific zone of the city, all shelters located within that zone are likely to fail and therefore the population that resides in that zone must be relocated to shelters on a different and safer zone of the city.

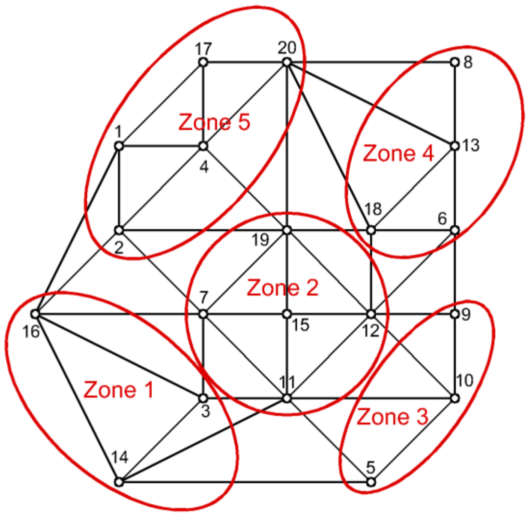


Figure 3.2 – Network of center/site locations divided into zones

## 3.2 Proximity-Based Reliability Models

Proximity-based reliability models are developed in this dissertation with the intent or creating or restructuring a network of shelters in order to guarantee that inhabitants of a certain area, which may be affected by a disaster occurrence, are able to find better conditions of safety in one of the located shelters. Furthermore, the models within this group generate plans for the routing of each inhabitant, from the center where they live until the site where their assigned shelter is to be located. Regarding the shelters, these types of models assume that there are no limitation in terms of their capacity, and therefore, the results that are presented, allocate people to the shelter that is more closely located to them.

Models developed within this category propose solutions for the hypothetical occurrence of disasters where all the population collectively would have to be assigned to receive assistance at a shelter that is at a certain level of proximity from their center of residence. We use the term “hypothetical” because, in fact, the assignments proposed by models within this category will never take place, at least not in the totality of assignments proposed. An analysis of Figure 3.3 allows for a better understanding of solutions proposed by the models within this category.

Figure 3.3 (A) makes clear that the assignments for the first proximity level will have all the population being served by its most closely located shelter. Furthermore, at a secondary level of proximity (B), the population from all centers will be reassigned to its respective second closest shelter. The population will only be served by its second closest shelter if the first one has failed, and so, it is impossible to assume that we will be able to reassign all the population to every level of proximity, as seen in Figure 3.3. This is due to the fact that for a certain center, its shelter in some proximity level might be at the same time a shelter in the previous proximity level for another center. Solutions proposed in the models within this category do not present definitive assignment results for a specific occurrence, but rather, shed a light on what would ensue if each center would have be served with a certain level of proximity. Furthermore, results provided by this type of models serve as a reference for the assignments that are expected to be made during more specific disaster occurrences.

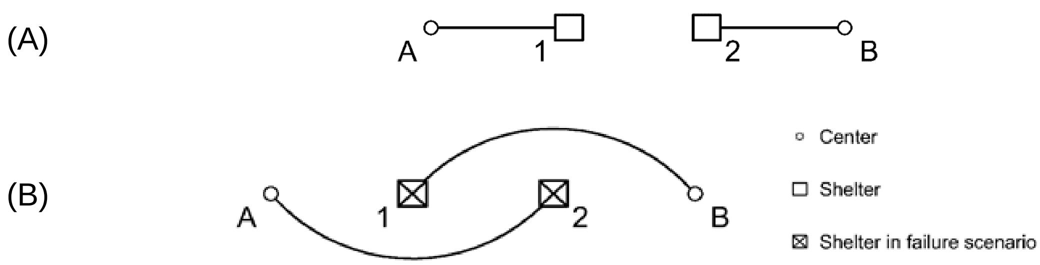


Figure 3.3 – Example of assignments for (A) proximity level 1 and (B) proximity level 2

Within this group, two different models can be found: a *p*-median location model (design) and a *p*-median fortification model (fortification). We will present the formulation for both models, using the location results from the design model as a basis for the application of the fortification model. Furthermore, a comparison between results will be provided.

### 3.2.1 *p*-median location model

For the purposes that are intended and taking into account the necessary changes of perspective from a decision standpoint, the developed *p*-median location model considers the following notation:

***Sets:***

$\boldsymbol{J}$ – Set of centers, indexed by $j$

$\boldsymbol{K}$ – Set of sites, indexed by $k$

$\boldsymbol{R}$ – Set of serviceability levels, indexed from 0 to $p-1$

$\boldsymbol{A}$ – Set of network zones, indexed by $a$

***Parameters:***

$h_j$ – Number of inhabitants in center $j \epsilon \boldsymbol{J}$

$d_{jk}$– Shortest path distance from center $j \epsilon \boldsymbol{J}$ to site $k \epsilon \boldsymbol{K}$

$q$ – Probability of failure occurring in an open shelter

$p$ – Maximum number of shelters to be located

$s_{ka}$ – Zone $a \epsilon \boldsymbol{A}$ in which site $k \epsilon \boldsymbol{K}$ is located

***Decision variables:***

$L_{jra}$ – Zone $a \epsilon \boldsymbol{A}$ to which inhabitants of center $j \epsilon \boldsymbol{J}$ are assigned to in level $r \epsilon \boldsymbol{R}$

$$Y_k = \begin{cases} 1, & \text{if shelter } k \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

$$X_{jkr} = \begin{cases} 1, & \text{if inhabitant } j \text{ is assigned to shelter } k \text{ at level } r \\ 0, & \text{otherwise} \end{cases}$$

Regarding the underlying methodology, in this model it was assumed that any open shelters may serve any resident and that there were no capacity restrictions to each open shelter. The model proposes the assignments that are to be made in case inhabitants from center $j$ are to be serviced at a certain level $r$. Furthermore, this models develops assignment solutions for each serviceability level that is considered. A "level-$r$" assignment is one for which there are $r$ closer open shelters. Taking that in to account, for each inhabitant's center of residence there should be made an analysis of the closest open shelters as well as the ordering of those shelters by level. A level-0 assignment represents a situation where the local resident is being assigned to the closest open shelter available, and so it is called a "primary" shelter. Subsequently an assignment in which the level $r$ is higher than 0, the shelter to which the resident is assigned is called a "backup" shelter for that particular resident, given the fact that the assignment will only take place if the $r$ closer shelters to the inhabitant have failed. Additionally, since we know in advance the maximum number of shelters that will open, the extent of the index $r$ should be considered from 0 through $p - 1$, as that is the maximum amount of serviceability levels there might be.

Once all the factors mentioned above have been taken into account, the model is formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad \sum_{j\in \boldsymbol{J}}\sum_{r\in \boldsymbol{R}}\sum_{k\in \boldsymbol{K}} h_j d_{jk} q^r (1-q) X_{jkr} \tag{1}$$

(s.t):

$$\sum_{k\in \boldsymbol{K}} Y_k \leq p \tag{2}$$

$$\sum_{k\in \boldsymbol{K}} X_{jkr} = 1, \qquad \forall j \in \boldsymbol{J}, r \in \boldsymbol{R} \tag{3}$$

$$X_{jkr} \leq Y_k, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K}, r \in \boldsymbol{R} \tag{4}$$

$$\sum_{r\in \boldsymbol{R}} X_{jkr} \leq 1, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K} \tag{5}$$

$$Y_k \in \{0,1\}, \qquad \forall k \in \boldsymbol{K} \tag{6}$$

$$X_{jkr} \in \{0,1\}, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K}, r \in \boldsymbol{R} \tag{7}$$

Each of the equations that are listed above represent the mathematical expressions of all the considerations that were taken into account to develop the *p*-median location model.

The objective function (1), minimizes the sum of travelled distances by all the inhabitants in all levels of service. It reflects the fact that if a resident $j$ is assigned to shelter $k$ at level $r$, it will only, in fact, be served by $k$ if all the $r$ closer facilities have failed (reflected in the equation by the probability $q^r$) and if $k$ itself has not failed (with probability $1-q$).[1]

Constraints (2) limit the number of open shelters to a maximum of $p$. Constraints (3) ensure that each inhabitant $j$ is to be assigned to some shelter at level $r$. Constraints (4) guarantee that assignments can only be made to a shelter that is open, while constraints (5) impede a resident from being assigned to the same shelter at more than one level, since if a shelter is the $r$ closest

[1] For a numerical example let's say that all shelters have a 30% percent change of failure once they are open, and that for a center j=7, the order of proximity to an open shelter, is: 3, 12 and finally 9 (with level-0 = shelter 3; level-1: shelter 12; level-2: shelter 9). The occurrence of a hypothetical assignment from j=7 to, for example, shelter k=9, which represents a level-2 assignment, will only happen, if the *r* closer shelters (in this case 2) have failed and at the same time if k=9 has not failed. This situation happens with probability: $q^r \times (1-q) = 0.3^2 \times (1-0.3) = 0.063 \equiv 6.3\%$.

to a certain center, it cannot, at the same time, be the $r+1$ closest to that same center. Finally, constraints (6) and (7), require the decision variables to be binary.

## **Application**

Due to the zoning approach that was taken to apply this model, the following additional constraints had to be considered:

$$L_{jra} = \sum_{k \in \boldsymbol{K}} X_{jkr} s_{ka}, \qquad \forall j \in \boldsymbol{J}, a \in \boldsymbol{A}, r \in \boldsymbol{R} \tag{8}$$

$$\sum_{r \in \boldsymbol{R}} L_{jra_1} - \sum_{r \in \boldsymbol{R}} L_{jra_2} \leq 1, \qquad \forall j \in \boldsymbol{J}, r \in \boldsymbol{R}, a_1 \neq a_2 \in \boldsymbol{A} \tag{9}$$

Given the fact that we associate failure occurrences with specific zones of the network, this will mean that once a zone fails, then, shelters which are located within the confinements of that zone will all fail, since they are all subjected to the same conditions. To prevent situations of this nature, which may prove to be costly, we decided that each zone must only have a single located shelter within its limits. Furthermore, since assignments are performed according to serviceability levels, we must guarantee that if the population from a center has been assigned to shelter in a certain zone at a specific level of service then, population from that same center may not be served by a shelter from that same zone on a different level of service. Constraints (8) and (9) reflect our proposed approach of diving the networks into different zones. Constraints (8) will determine the zone to which every assignment is made on all levels and constraints (9) ensure that residents which were served in a specific zone at a certain level can no longer be assigned to that same zone on a different level.

This model is applied taking into account: a maximum of five shelters ($p = 5$) and a 30% probability ($q = 0.3$), of each shelter failing. For these considerations, the model presents the results shown in Table 3.1.

Table 3.1 – Assignment results for *p*-median location model

| | | Centers | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Level of Service*** | ***0*** | 4 | 4 | 3 | 4 | 3 | 13 | 3 | 13 | 9 | 9 | 3 | 9 | 13 | 3 | 19 | 3 | 4 | 19 | 19 | 4 |
| | ***1*** | 19 | 19 | 19 | 19 | 9 | 9 | 19 | 9 | 13 | 19 | 19 | 19 | 9 | 19 | 3 | 4 | 19 | 13 | 4 | 19 |
| | ***2*** | 3 | 3 | 9 | 13 | 19 | 19 | 4 | 4 | 19 | 13 | 9 | 13 | 19 | 9 | 9 | 19 | 13 | 9 | 13 | 13 |
| | ***3*** | 13 | 13 | 4 | 9 | 13 | 4 | 9 | 19 | 3 | 3 | 4 | 3 | 4 | 4 | 4 | 9 | 9 | 4 | 9 | 9 |
| | ***4*** | 9 | 9 | 13 | 3 | 4 | 3 | 13 | 3 | 4 | 4 | 13 | 4 | 3 | 13 | 13 | 13 | 3 | 3 | 3 | 3 |

By analyzing Table 3.1 and the illustrations in Figure 3.4, we can observe that, for the conditions that were taken into account, the network will have shelters located in centers 3, 4, 9, 13 and 19. This solution allows us to minimize the overall amount of distances travelled by the region's inhabitants in all the levels of service ($14251.5\ Km$).

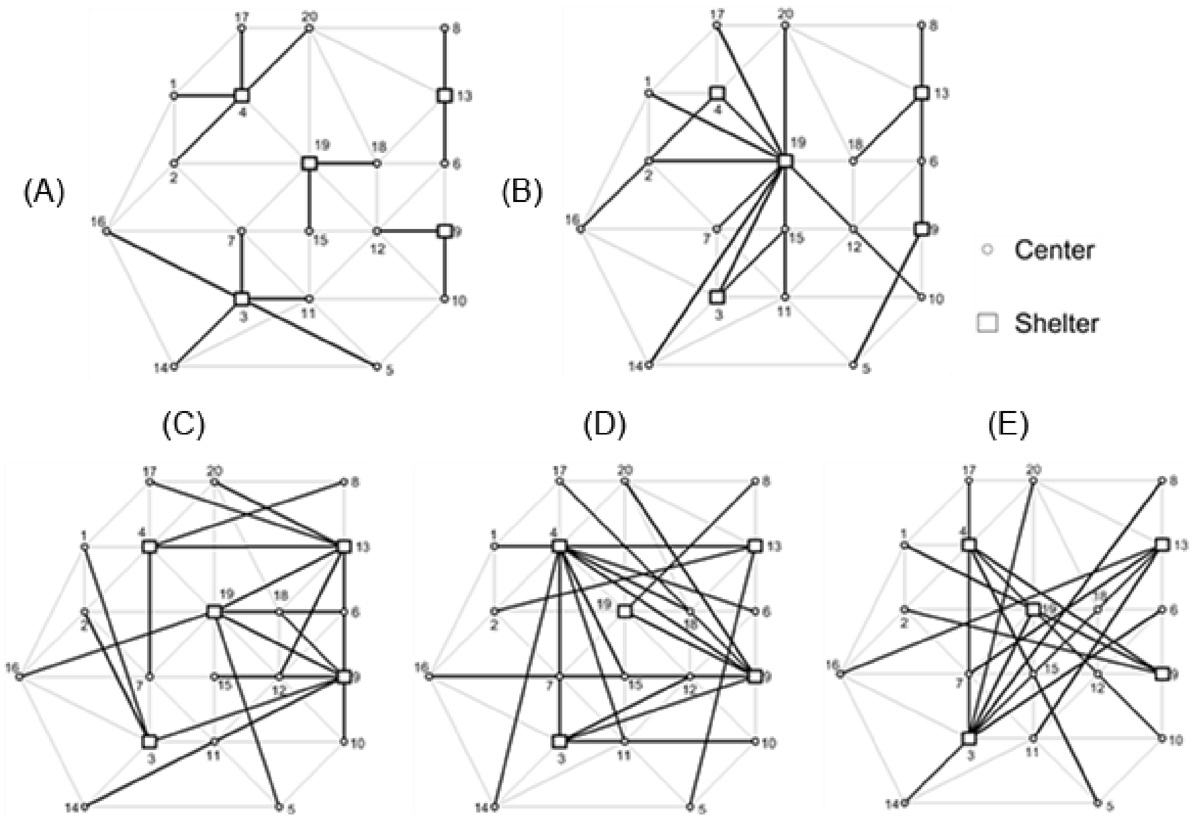


Figure 3.4 – $p$-median location model for level of service: (A) 0 (B) 1 (C) 2 (D) 3 and (E) 4

For a better understanding of the resulting assignments, in Table 3.2 we summarize the amount of distances travelled according to each level of service:

Table 3.2 – $p$-median location model results in travelled distances

| ***Level of service*** | **0** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|---|
| ***Overall travelled distance (Km)*** | 928.9 | 2314.8 | 3227.5 | 4039.5 | 4740.5 |
| ***Average travelled distance per resident (Km/resident)*** | 0.8 | 2.0 | 2.9 | 3.6 | 4.2 |

Bearing in mind the objective, this model prioritizes locating a number of shelters (for this instance 5) in the best set of places possible, in order to minimize the $h_j d_{jk} X_{jkr}$ component of the problem in the totality of all levels of service, and not necessarily the best for each particular group of residents. The solution presented in Figure 3.4 (A) shows the level-0 assignments ($r = 0$) for the instance that was described, representing what would ensue in the case of no failures in any of the shelters. As explained earlier, solutions developed by this model are simply hypothetical representations of what can follow a disaster. By analyzing the remaining illustrations in Figure 3.4 (B; C; D and E), we can clearly understand that all represented assignment solutions are impossible to take place. This is explained by the fact that a situation where all population is assigned to their second, third, fourth of fifth closest shelter at the same time is impossible. However, what is possible is situations where some populated centers are assigned to their closest shelter, others are assigned to their second closest shelter and so on for the remaining levels. So, in order to understand the solutions proposed, we have to consider that assignments presented for each level are merely hypothetical situations of what would occur if all population was assigned to shelters at a specific level of proximity. Furthermore, solutions presented by this model are useful to understand assignments proposed by models where specific failure occurrences are studied, since we already know in advance the assignments that will be proposed by those models in case a certain populated center has to be served by its first, second, third, fourth of fifth closest located shelter.

### 3.2.2 *p*-median fortification model

The developed *p*-median fortification model considers, for the most part, the same notations as the *p*-median location model. However, seeing as it is developed with the intent of fortifying an existent network of shelters, this model considers the following additional notation:

- $\boldsymbol{K}$ denotes the set of existent rather than potential shelter locations – indexed with the resulting shelter locations provided by the *p*-median location model;
- $f$ reflects the number of shelters to fortify;
- $j_c$ denotes the $c^{\text{th}}$ closest shelter to population in center $j$;
- $d_j^c$ represents the distance between population in center $j$ and its closest operational shelter, given the fact that $c - 1$ closest shelters to center $j$ are not protected and the $c^{\text{th}}$ closest shelter to $j$ is.

The distance, $d_j^c$, is calculated as follows:

$$d_j^c = \sum_{k=1}^{c-1} q^{k-1}(1-q)d_{jj_k} + q^{c-1}d_{jj_c} \tag{10}$$

The latter differs from $d_{jk}$ in the sense that it does not represent the distance from the population in center $j$ to a shelter in site $k$, but rather that same distance influenced by the probability of failure in each shelter, giving us information regarding the expected travelled distances.

The models determines the subset of existing shelters which are most suitable to be fortified. In disaster situations, residents may be assigned to all the existing shelters but on this model, it analyzes what would occur if the population was to be served exclusively by fortified shelters. Regarding the notations, as no decision regarding the location of shelters has to me made in this $p$-median fortification model, it does not take into account the same decision variables as the model shown in previous chapter. The following sets of decision variables are taken into consideration:

$$Z_k = \begin{cases} 1, & if\ shelter\ k\ is\ fortified \\ 0, & otherwise \end{cases}$$

$$W_{jc} = \begin{cases} 1, & if\ the\ c-1\ closest\ shelters\ to\ population\ in\ j\ are \\ & not\ protected\ but\ the\ c^{\text{th}}\ closest\ shelter\ is, \\ 0, & otherwise \end{cases}$$

Once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad \sum_{j \in \boldsymbol{J}} \sum_{c=1}^{p-f+1} h_j d_j^c W_{jc} \tag{11}$$

(s.t):

$$\sum_{c=1}^{p-f+1} W_{jc} = 1, \qquad \forall j \in \boldsymbol{J} \tag{12}$$

$$W_{jc} \leq Z_{j_c}, \qquad \forall j \in \boldsymbol{J}, c = 1, \dots, p-f+1 \tag{13}$$

$$W_{jc} \leq 1 - Z_{j_{c-1}}, \qquad \forall j \in \boldsymbol{J}, c = 2, \dots, p-f+1 \tag{14}$$

$$\sum_{k \in \boldsymbol{K}} Z_k = f \tag{15}$$

$$W_{jc} \in \{0,1\}, \qquad \forall j \in \boldsymbol{J}, c = 1, \dots, p-f+1 \tag{16}$$

$$Z_k \in \{0,1\}, \qquad \forall k \in \boldsymbol{K} \tag{17}$$

The objective function (11) minimizes the overall expected travelled distances. It should be noted at this point that the variable, $W_{jc}$, and the travelled distances, $d_j^c$, only need to be defined for values of $c$ between 1 and $p-f+1$, as in the worst case possible, the closest protected shelter to a resident living in center $j$ is $p-f+1$. This situation occurs if the $f$ fortified shelters are the furthest shelters from center $j$. So, if the $p - f$ closest shelters to center $j$ fail, a resident living in $j$ will be assigned to its $p - f + 1$ closest shelter.[2]

Constraints (12) guarantee that one of the $p - f + 1$ nearest shelters to center $j$ will be its closest fortified shelter. The joint effect of constraints (13) and (14) ensure that an assignment can only be made to a shelter that is protected, plus the assignment variable $W_{jc}$ that equals 1 is the one linked with the smallest value of $c$, such that the $c^{\text{th}}$ closest shelter to $j$ is protected. Summarizing, constraints (13) and (14) guarantee that once a shelter in whichever level of proximity from a specific populated center is protected, then no further assignments are made to a shelter on a higher level of proximity from that same center, even if it is protected. We take this into account because once a protected shelter is found, on a certain level of proximity, then, there will be no need for the population to considered further reallocation, since the shelter to which they are assigned will already never fail. Constraints (15) limit the fortification resources to a maximum of $f$ shelters that can be protected. Finally, constraints (16) and (17), require the decision variables to be binary.

**<u>Application</u>**

Data presented in Table 3.3, illustrates the level of proximity of each populated center to all of the shelters considered is our test instance. As an example, for center $j = 15$, the order of closest opened shelters is: 19, 3, 9, 4 and 13, which means that for $j = 15$, its closest shelter ($c = 1$) is 19, its second closest shelter ($c = 2$) is 3 and so on for the remaining levels of proximity.

Table 3.3 – Shelters per level of proximity to populated centers ( $j_c$) in the *p*-median fortification model

| | | **Centers** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Proximity (c)*** | ***1*** | 4 | 4 | 3 | 4 | 3 | 13 | 3 | 13 | 9 | 9 | 3 | 9 | 13 | 3 | 19 | 3 | 4 | 19 | 19 | 4 |
| | ***2*** | 19 | 19 | 19 | 19 | 9 | 9 | 19 | 9 | 13 | 19 | 19 | 19 | 9 | 19 | 3 | 4 | 19 | 13 | 4 | 19 |
| | ***3*** | 3 | 3 | 9 | 13 | 19 | 19 | 4 | 4 | 19 | 13 | 9 | 13 | 19 | 9 | 9 | 19 | 13 | 9 | 13 | 13 |
| | ***4*** | 13 | 13 | 4 | 9 | 13 | 4 | 9 | 19 | 3 | 3 | 4 | 3 | 4 | 4 | 4 | 9 | 9 | 4 | 9 | 9 |
| | ***5*** | 9 | 9 | 13 | 3 | 4 | 3 | 13 | 3 | 4 | 4 | 13 | 4 | 3 | 13 | 13 | 13 | 3 | 3 | 3 | 3 |

[2] For a numerical example, the closest shelters to a resident in center j=1 are k={3;5;12;18;6}, by order of proximity. Let's consider that of those five shelters (*p*=5), only two are protected (*f*=2), those being shelters k=18 and k=6. The previous examples, lead us to conclude, that at most, the population in center j=1, will be served by shelter k=18, which is its 5-2+1=4$^{\text{th}}$ nearest shelter.

As the new distance matrix, $d_j^c$, is formulated according to the level of proximity of populated centers to available shelters, the shelter corresponding to a certain value of proximity level, $c$, changes from center to center, as each center has its own order of proximity to the located shelters. The new distance matrix is presented Table 3.4 (See Appendix - Table A.7 for a clearer and higher scale representation of the new distance matrix).

Table 3.4 – Distance matrix $d_j^c$ (*in Km*) for the *p*-median fortification model

| | | ***Centers*** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Proximity (c)*** | ***1*** | 1 | 1.4 | 0 | 0 | 2.4 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1.4 | 1 | 2.2 | 1 | 1 | 0 | 1.4 |
| | ***2*** | 1.4 | 1.6 | 0.7 | 0.4 | 2.4 | 1 | 1.1 | 1.6 | 0.6 | 1.6 | 1.3 | 1.1 | 0.6 | 2.1 | 1.3 | 2.4 | 1.4 | 1.1 | 0.4 | 1.6 |
| | ***3*** | 1.5 | 1.6 | 0.8 | 0.6 | 2.5 | 1.1 | 1.3 | 1.6 | 0.6 | 1.6 | 1.3 | 1.2 | 0.6 | 2.2 | 1.3 | 2.5 | 1.5 | 1.2 | 0.5 | 1.6 |
| | ***4*** | 1.6 | 1.7 | 0.8 | 0.6 | 2.5 | 1.1 | 1.3 | 1.6 | 0.7 | 1.6 | 1.4 | 1.2 | 0.7 | 2.3 | 1.3 | 2.5 | 1.5 | 1.2 | 0.5 | 1.7 |
| | ***5*** | 1.6 | 1.7 | 0.8 | 0.6 | 2.5 | 1.1 | 1.3 | 1.7 | 0.7 | 1.6 | 1.4 | 1.2 | 0.7 | 2.3 | 1.3 | 2.5 | 1.5 | 1.2 | 0.5 | 1.7 |

The *p*-median fortification model, previously formulated in this chapter, is applied taking into account: a maximum of amount two shelters which might be fortified ($f = 2$) and a 30% probability ($q = 0.3$), of each shelter failing. Furthermore, it was assumed that the existing network of shelters is a result of the locations determined by the *p*-median location model. For these considerations, the model develops the results presented in Figure 3.5.

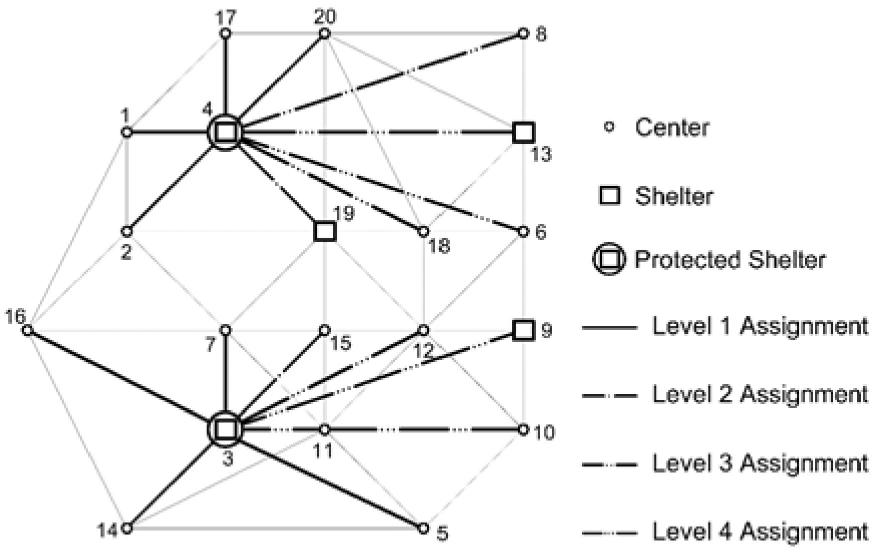


Figure 3.5 – *p*-median fortification model considering $f = 2$

As expected, this model presents a significantly different solution than the one provided by the *p*-median location model. The *p*-median location model produces assignments according to the level of service that is considered, while the $p$-median fortification model takes into account the locations of shelters determined by the design model and delivers a single solution, which reassigns the population to a smaller subset of shelters (fortified) than the ones originally determined by the design model.

By analyzing Figure 3.5 , we can observe that the model develops a solution that will have the fortification of shelters located in sites 3 and 4. In this model, previously located shelters which are not protected, are presumed to end up not being available, and therefore, all population that is served by them has to be reassigned to a shelter that is protected. Once again, this is a hypothetical solution, since we cannot presume that all non-fortified shelters will be subject to failure. There will be times when a disaster strikes the region and it might still leave up to three operable non-fortified shelters. Due to that fact, assignment solutions presented within the context of this model, reflect the worst case of when a populated center might be served. For a clear example of this fact, let's analyze the situation of center $j = 8$ being assigned to the fortified shelter in site $k = 4$, with level of proximity $c = 3$. This assignment means that at most, the population in center 8 will be serviced by shelter 4, which is its third closest shelter and which is fortified. However, this will only happen if both shelters on levels of proximity $c = 1$ and $c = 2$ from center 8 have both failed. Furthermore, in case none have failed, center 8 is assigned to its shelter on level of proximity $c = 1$ and if its shelter on level of proximity $c = 1$ fails, then center 8 is assigned to its shelter on level of proximity $c = 2$. Regarding the models' objective, it determines that a total of $2173.5$ $Km$, is travelled by the entire population (each resident has to travel an average of $1.9$ $Km$ in order to reach its designated shelter).

However different, the solutions for the previous two models may be comparable, as the reassignments performed in the fortification model resemble the levels of service analyzed in the design model. This becomes comprehensible, by performing a comparison between the assignments presented in Figure 3.4 (A) and in Figure 3.5. The difference from one solution to another resides on the fact that residents from centers 6, 8, 9, 10, 12, 13, 15, 18 and 19 are no longer assigned to their closest shelter ($c = 1$ or $r = 0$) . Furthermore, as a way to compare data, we will analyze the level of service of the reassignments that are made in this $p$-median fortification model.

As presented in Table 3.5, the average level of service of all centers, when considering the reassignments made by the fortification model, is approximately $r = 1.10$, which in practical terms is acceptable to consider $r = 1$. This value indicates that in average, for the fortification model, centers are assigned to their second closest shelter ($c = 2$), which is same as saying that the shelter that is closest to them is failing ($r = 1$). That said, and taking into account result of $2.0$ $Km$ per resident provided by the design model for a value of $r = 1$, we are able to see that the result of $1.9$ $Km$ per resident provided by the fortification model is very similar.

Table 3.5 – Levels of service in fortification reassignments

| ***Center*** | ***6*** | ***8*** | ***9*** | ***10*** | ***12*** | ***13*** | ***15*** | ***18*** | ***19*** |
|---|---|---|---|---|---|---|---|---|---|
| ***New level of service (r)*** | 3 | 2 | 3 | 3 | 3 | 3 | 1 | 3 | 1 |
| ***Average reassignment level of service*** | 2.44 | | | | | | | | |
| ***Average level of service (all centers)*** | 1.1 | | | | | | | | |

## 3.3 Scenario-Based Reliability Models

Scenario-based reliability models are designed with the intent of defining plans of evacuation for specific disaster situations. For all scenarios, the model considers combinations of failure events in the available site locations with the intent of preparing the network for as much different situations as possible. Each scenario is associated with a probability of occurrence. Furthermore, depending on the models' objective, the number of scenarios which are considered is subject to changes.

These models produce results that indicate the best course of action according to the disaster occurrence that is being considered and in general, look to achieve a solution of shelter locations and assignments that guarantees the minimum amount of travelled distances across all the scenarios that are considered. Moreover, these models develop assignment solutions that always allocate people to a shelter which is in a safe area and is operable. Like the models in the previous chapter, the models within this group generate plans for the routing of each inhabitant, from the center where they live until the site where their assigned shelter is to be located. Furthermore, these models ensure that if any of the located shelters fails due to the occurrence of a disaster, people are reassigned to the shelter that is immediately closer to them.

Within this group, three different models can be found: a scenario-based *p*-median location model (design), a scenario-based *p*-median fortification model (fortification) and a minimax *p*-median location model (design). We will present the formulation for all models, using the location results from the design model as a basis for the application of the fortification model. Furthermore, a comparison between results will be provided.

### 3.3.1 Scenario-based *p*-median location model

The scenario-based *p*-median location model takes into consideration most of the notations of its proximity-based version, presented in Chapter 3.2.1. However, seeing as it is developed with the intent of providing a solution that considers specific scenarios rather than general failure occurrences, this model is subject to the following changes in notation, when compared to the *p*-median location model:

***Sets:***

$\boldsymbol{S}$ – set of failure scenarios, indexed by $s$

***Parameters:***

$q_s$ – Probability of scenario $s$ occurring

$$a_{ks} = \begin{cases} 1, & if\ shelter\ k\ fails\ in\ scenario\ s \\ 0, & otherwise \end{cases}$$

***Decision variables:***

$$X_{jks} = \begin{cases} 1, & if\ inhabitant\ j\ is\ assigned\ to\ shelter\ k\ in\ scenario\ s \\ 0, & otherwise \end{cases}$$

Once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad \sum_{s \in \boldsymbol{S}} q_s \sum_{j \in \boldsymbol{J}} \sum_{k \in \boldsymbol{K}} h_j d_{jk} X_{jks} \tag{18}$$

(s.t): (2); (6)

$$\sum_{k \in \boldsymbol{K}} X_{jks} = 1, \qquad \forall j \in \boldsymbol{J}, s \in \boldsymbol{S} \tag{19}$$

$$X_{jks} \leq (1 - a_{ks}) Y_k, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K}, s \in \boldsymbol{S} \tag{20}$$

$$X_{jks} \in \{0,1\}, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K}, s \in \boldsymbol{S} \tag{21}$$

The objective function (18), minimizes the sum of travelled distances by all the inhabitants for all the scenarios weighted by the probability of each scenario occurrence. It determines which shelter locations are best in order for the travelled distances, corresponding the resulting assignments, to be minimized across all scenarios. Constraints (19) state that each inhabitant $j$ is to be assigned to some shelter in each scenario. Constraints (20) guarantee that assignments for each scenario can only be made to a shelter that is open and which as not failed. Finally, constraints (6) and (21), require the decision variables to be binary.

**Application**

To ensure that a level of continuity is kept throughout all the models that are developed in this dissertation, we continued to assume that the maximum number of located shelters would be five ($p = 5$) and that a probability of failure of 30% would still be considered. However, this time, that failure probability reflects the possibility of a certain zone being affected by a disaster.

We considered that failure occurrences were intimately connected to the zones that define the network. Furthermore, as mentioned in Chapter 3.1, the association of failure occurrences to specific zones enables us to present solutions that more easily reflect disaster situations. Looking at the fact that we divided our network into five different zones, we propose only taking into account situations where up to four out of those five zones are affected by disasters.

Table 3.6 shows the failure events that take place according to each scenarios that is considered and presents their probability of occurrence. These occurrences of disasters in different zones are independent events, and by dismissing the scenarios where five zones are affected simultaneously, we will only be dismissing situations that occur with a probability of 0.24% ($q_s = 0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3 = 0.0024$), which lets us plan for events of failure with a good level of reliability. Furthermore, taking into account the notations that were used, the single scenario that considers that no failures occur in any of the zones ($s = 1$), has a 16.8% chance of taking place ($q_s = (1 - 0.3)^5 = 0.168$). Furthermore, each scenario that reflects the situation of a single zone being struck (scenarios $2 \leq s \leq 6$) has a 7.2% probability of occurring ($q_s = 0.3 \times (1 - 0.3)^4 = 0.072$), scenarios which consider two simultaneous disasters (scenarios $7 \leq s \leq 16$) have a 3.1% probability of occurring ($q_s = 0.3 \times 0.3 \times (1 - 0.3)^3 = 0.031$), scenarios which consider three disasters occurrences (scenarios $17 \leq s \leq 22$) have a 1.3% probability of occurring ($q_s = 0.3 \times 0.3 \times 0.3 \times (1 - 0.3)^2 = 0.013$) and finally, scenarios which consider four simultaneous disasters (scenarios $23 \leq s \leq 25$) have a 0.6% probability of taking place ($q_s = 0.3 \times 0.3 \times 0.3 \times 0.3 \times (1 - 0.3) = 0.006$).The previously described failure scenarios are illustrated as a binary matrix presented in the Appendix – Table B.1.

Table 3.6 – Combination of failing zones according to scenarios

| ***Total number of failing zones*** | ***Scenarios*** | ***Combination of zones that fail*** | ***Probability of each scenario*** |
|---|---|---|---|
| 0 | 1 | – | 0.168 |
| 1 | 2 | 1 | 0.072 |
| | 3 | 2 | |
| | 4 | 3 | |
| | 5 | 4 | |
| | 6 | 5 | |
| 2 | 7 | 1+2 | 0.031 |
| | 8 | 1+3 | |
| | 9 | 1+4 | |
| | 10 | 1+5 | |
| | 11 | 2+3 | |
| | 12 | 2+4 | |
| | 13 | 2+5 | |
| | 14 | 3+4 | |
| | 15 | 3+5 | |
| | 16 | 4+5 | |
| 3 | 17 | 1+2+3 | 0.013 |
| | 18 | 1+2+4 | |
| | 19 | 1+2+5 | |
| | 20 | 2+3+4 | |
| | 21 | 2+3+5 | |
| | 22 | 3+4+5 | |
| 4 | 23 | 1+2+3+4 | 0.006 |
| | 24 | 1+2+3+5 | |
| | 25 | 2+3+4+5 | |

Furthermore, we assumed that each zone should only have located a shelter within its confinements. This approach ensures that if a zone is affected by a disaster, no more than one shelter will be left inoperable. To ensure this, we consider the following additional constraints to the formulation:

$$\sum_{k=\{3,14,16\}} Y_k = 1 \tag{22}$$

$$\sum_{k=\{7,11,12,15,19\}} Y_k = 1 \tag{23}$$

$$\sum_{k=\{5,9,10\}} Y_k = 1 \tag{24}$$

$$\sum_{k=\{6,8,13,18\}} Y_k = 1 \tag{25}$$

$$\sum_{k=\{1,2,4,17,20\}} Y_k = 1 \quad (26)$$

As the model develops different assignments for each scenario that is considered, we will present an illustration of the resulting network assignments for the worst scenario in each of the five situations that were considered. Furthermore as the assignments for the situation of no failures are the exact same as in the *p*-median location model, presented in Figure 3.4 (A), there will be no need to illustrate that situation.

In Figure 3.6, we can observe the resulting assignment solutions for each of the worst scenarios in the remaining situations that were considered. As the considerations regarding the choice of sites in which to locate shelters remain the same from previous models and given the fact that no capacity limitations where considered, as expected, this model determines that shelters are to be located in sites 3, 4, 9, 13 and 19. Furthermore, these are the five locations (one in each zone) which offer the best conditions to guarantee that travelled distances amongst all the population remains at a minimum.

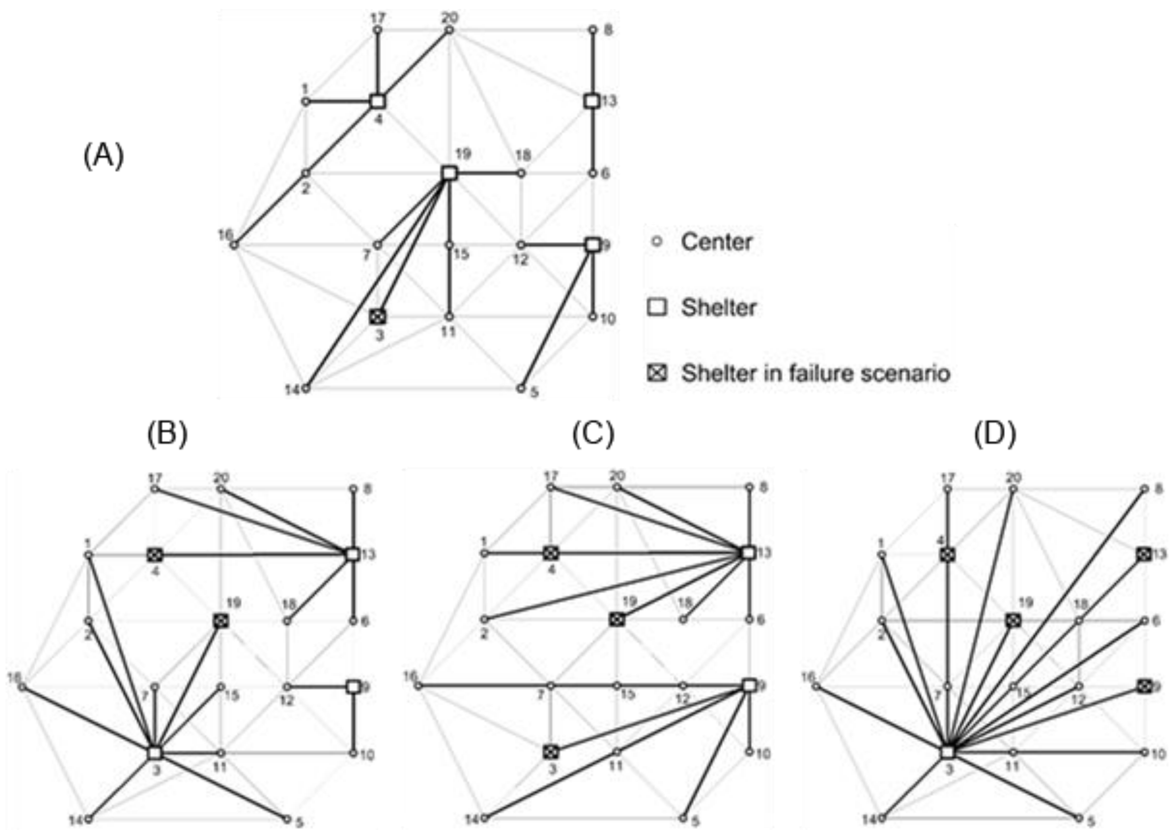


Figure 3.6 – Scenario-based *p*-median location model for scenarios: (A) 2 (B) 13 (C) 19 and (D) 25

As it pertains to the resulting assignments, it is now clear that this model presents solutions at each scenario that always ensure that people from every center are being assigned to their closest operable shelter. The data presented from Table 3.7 shows us all the results in terms of average distances travelled by each resident. It now clear that the illustrated scenarios in Figure 3.6 are, in fact, the worst case for each of the disaster situations. From these illustrations we can conclude an interesting fact: shelter 3 is the shelter that is located the furthest away from most of the population, thus, when analyzing the situation where only one shelter remains functional, the worst solution is the one where shelter 3 is operable (scenario 25). Another fact that is easily verified is that the results get worse as the number of failing zones increases. Furthermore, for the remaining scenarios that were not illustrated, situations would occur where less reassignments due to shelter failures would take place. Additionally, the developed solution allows for a total of $44955.4\ Km$ travelled across all the scenarios (absolute value, unaffected by probabilities), which is the minimum possible, taking into account that each zone should only have one shelter located within its confinements.

Table 3.7 – Results of scenario-based $p$-median location model

| ***Scenarios*** | ***Combination of zones that fail*** | ***Average travelled distance per resident (Km/resident)*** |
|---|---|---|
| 1 | – | 0.82 |
| 2 | 1 | 1.16 |
| 3 | 2 | 0.99 |
| 4 | 3 | 1.04 |
| 5 | 4 | 1.05 |
| 6 | 5 | 1.10 |
| 7 | 1+2 | 1.46 |
| 8 | 1+3 | 1.41 |
| 9 | 1+4 | 1.38 |
| 10 | 1+5 | 1.45 |
| 11 | 2+3 | 1.29 |
| 12 | 2+4 | 1.25 |
| 13 | 2+5 | 1.62 |
| 14 | 3+4 | 1.40 |
| 15 | 3+5 | 1.32 |
| 16 | 4+5 | 1.32 |
| 17 | 1+2+3 | 1.41 |
| 18 | 1+2+4 | 1.72 |
| 19 | 1+2+5 | 2.31 |
| 20 | 2+3+4 | 1.92 |
| 21 | 2+3+5 | 1.93 |
| 22 | 3+4+5 | 1.67 |
| 23 | 1+2+3+4 | 2.69 |
| 24 | 1+2+3+5 | 2.87 |
| 25 | 2+3+4+5 | 3.01 |

#### 3.3.2 Scenario-based *p*-median fortification model

Taking into account the necessary changes of perspective from a decision standpoint, in order to develop this scenario-based $p$-median fortification model, we had to apply some changes to notions used in the scenario-based $p$-median location: $\boldsymbol{K}$ denotes the set of existent rather than potential shelter locations – indexed with the resulting shelter locations provided by the the scenario-based $p$-median location model; $f$ reflects the limitations in terms of the number of shelters that can be fortified; as in the $p$-median fortification model, $Z_k$ is now the main decision variable, deciding upon which shelters will be fortified. Furthermore, once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

Objective function: (18)

(s.t): (15); (17); (19); (21)

$$X_{jks} \leq (1 - a_{ks}) + a_{ks}Z_k, \qquad \forall j \in \boldsymbol{J}, k \in \boldsymbol{K}, s \in \boldsymbol{S} \tag{27}$$

Sharing every other restriction with the $p$-median fortification model and the scenario-based $p$-median location, only constraints (27) illustrate a different reality, by imposing that a shelter that has failed ($a_{ks} = 1$) cannot accommodate any resident, unless it has been fortified. Furthermore, the previous constraints allow a non-failing shelter ($a_{ks} = 0$), be it fortified or not, to accommodate residents. Because of this, it is now clear how considering pre-defined disaster scenarios affects the assignment results. More specifically, when compared to the $p$-median fortification model, this model produces assignments which resemble disaster situations: if a shelter is failing, then the population assigned to that shelter is immediately reassigned to their second closest operational shelter, which isn't necessarily a fortified shelter, as it might simply be another non-fortified shelter that has not failed in the scenario that is being considered. As it pertains to the results, this model presents solutions that guarantee that each resident is always being assigned to their closest operational shelter and also ensure that the location of fortified shelters are determined in order have the least amount of travelled distances across all scenarios.

**Application**

Once again, to ensure that a level of continuity is kept throughout the all the models that are developed in this dissertation, we continued to assume that the maximum number of fortified shelters would be two ($f = 2$) and that a probability of a disaster occurrence in each zone of the network would still be 30%. Furthermore, we consider the same exact situations and scenario probabilities as in the scenario-based $p$-median location model. Furthermore, as this model also develops different assignments for each scenario that is considered, we will present an illustration of the resulting network assignments for the worst scenario in each of the five situations that were considered. Additionally, as the assignments and results for the situation of no failures are the exact same as in the *p*-median location model, presented in Figure 3.4 (A), with the exception of shelters 3 and 4 being fortified, there will be no need to illustrate that situation.

In Figure 3.7, we can observe the resulting assignment solutions for each of the worst scenarios from situations of one, two, three and four simultaneous disasters. For each scenario, the model determines the assignments that should be made in order for every person to be served as closely as possible. This model takes into account the set of shelters located in the previous model and determines which of those are the most suited to be fortified. As the considerations regarding the choice of shelters which to fortify remain the same as in the *p*-median fortification model and given the fact that no capacity limitations where considered, as expected, the scenario-based *p*-median fortification model determines that the shelters located in sites 3 and 4 should be fortified. Furthermore, these are the two locations, which offer the best conditions to guarantee that travelled distances amongst all the population remains at a minimum.

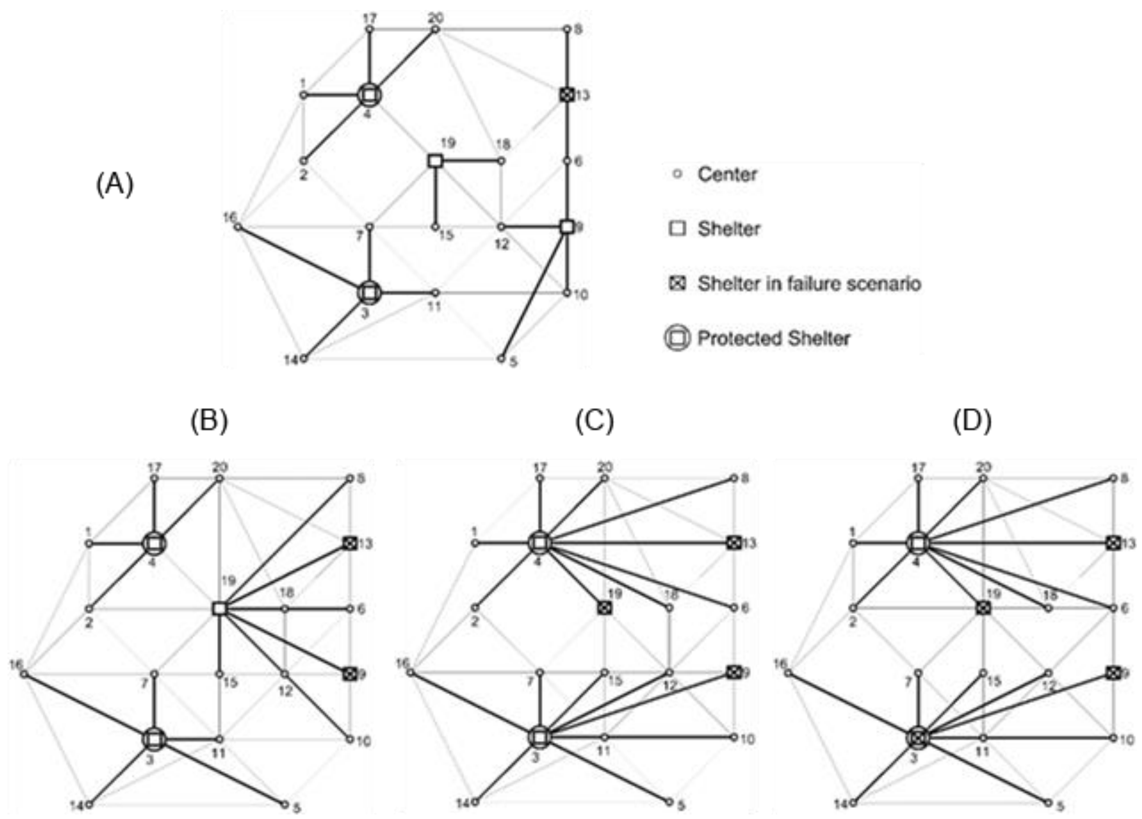


Figure 3.7 – Scenario-based *p*-median fortification model for scenarios: (A) 5 (B) 14 (C) 20 and (D) 23

Regarding the resulting assignments, it is now clear that this model presents solutions at each scenario that always ensure that people from every center are being assigned to their closest operable shelter. Furthermore, as in the previous chapter, we will summarize the resulting assignments according to the disaster situation that is being considered. The data presented on Table 3.8 shows us all the results in terms of average distances travelled by each resident. It is now clear that the illustrated scenarios in Figure 3.7 are, in fact, the worst case for each of the disaster situations. Additionally, regarding the models' objective, the developed solution allows for a total of $33358.4\ Km$ travelled across all the scenarios (absolute value, unaffected by probabilities), which is the minimum possible.

Table 3.8 – Results of scenario-based $p$-median fortification model

| ***Scenarios*** | ***Combination of zones that fail*** | ***Average travelled distance per resident (Km/resident)*** |
|---|---|---|
| 1 | – | 0.82 |
| 2 | 1 | 0.82 |
| 3 | 2 | 0.99 |
| 4 | 3 | 1.04 |
| 5 | 4 | 1.05 |
| 6 | 5 | 0.82 |
| 7 | 1+2 | 0.98 |
| 8 | 1+3 | 1.04 |
| 9 | 1+4 | 1.05 |
| 10 | 1+5 | 0.82 |
| 11 | 2+3 | 1.29 |
| 12 | 2+4 | 1.25 |
| 13 | 2+5 | 0.99 |
| 14 | 3+4 | 1.40 |
| 15 | 3+5 | 1.04 |
| 16 | 4+5 | 1.05 |
| 17 | 1+2+3 | 1.04 |
| 18 | 1+2+4 | 1.25 |
| 19 | 1+2+5 | 0.99 |
| 20 | 2+3+4 | 0.92 |
| 21 | 2+3+5 | 1.29 |
| 22 | 3+4+5 | 1.40 |
| 23 | 1+2+3+4 | 1.92 |
| 24 | 1+2+3+5 | 1.29 |
| 25 | 2+3+4+5 | 1.92 |

At this point, a comparison between the results proposed by the scenario-based $p$-median location and fortification models, respectively, would contribute towards a better understanding of the assignments that are proposed. Analyzing the data presented in Table 3.7 and Table 3.8, it is clear that the fortification model develops solutions that are much more advantageous than the ones presented in the design model. Furthermore, this is always to be expected (no matter the failure conditions), since the fortification model selects the best subset of two shelters located in the design model and considers that they are immune to disaster effects. By taking this approach, we always guarantee, like it was expressed in the *p*-median fortification model, that a populated center is never assigned to the shelter that is its fifth closest (in terms of proximity: $c = 5$), which consequently means that the hypothetical assignments for the last level of service ($r = 4$), produced in the *p*-median location model, will never take place. In the same line of thought, this model tell us, that at most, in the worst case possible, a center will always be served by a shelter that is its fourth closest and which is fortified. All of this facts are reflected in the difference of results in terms of assignments between both of the previous two scenario-based models: we register an average reduction of travelled distances per scenario of about 22%, if we take into account the solutions presented by the fortification model; the objective function value of the fortification model ($33358.4\ Km$) represents a 35% reduction of the value determined by its corresponding location model ($44955.4\ Km$).

#### 3.3.3 Minimax *p*-median location model

The minimax *p*-median location model developed in this dissertation, aims at optimizing the assignments for the worst possible scenario of shelter locations. More specifically, this model develops a solution that guarantees that the scenario with the most travelled distances, will be the one to which the optimal solution is found. In disaster situations, this model reflects the need to consider the effects of catastrophic and unlikely disaster events, and ensure that the planning for those situations are conducted with the utmost level of importance. Furthermore, in regards to its notations and considerations, this model differs only on a single constraint when compared to the scenario-based *p*-median location model. Additionally, a new decision variable $U$, which equals the distances travelled in the worst scenario of all that are considered.

Once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad U \tag{28}$$

(s.t): (2); (6); (19); (20); (21)

$$\sum_{j \in \boldsymbol{J}} \sum_{k \in \boldsymbol{K}} h_j d_{jk} X_{jks} \leq U, \qquad \forall s \in \boldsymbol{S} \tag{29}$$

The objective function (28), determines the location of shelters, which minimizes the distances travelled in the worst case scenario, exclusively. Furthermore, constraints (29) determine the distances that are travelled in the worst scenario, $U$, and identifies in which scenario that worst-case result in taking place.

**Application**

To ensure a good level of cohesion between all the presented solutions, for this model, we continued to consider that the maximum number of located shelters would be five ($p = 5$) and that a probability of a disaster occurrence in each zone would be 30%. In order to perform a thorough study of the solutions developed within the context of prioritizing the planning for the worst case scenario, we should analyze the compromises that we would be making in order to guarantee the upmost level of safety for the worst case scenario. For that, we start by analyzing what would occur if we remained considering the possibility of up to four simultaneous disasters. For these conditions, it is expected that the model prioritizes the optimization of assignments for one of the scenarios which considers four simultaneous failures, since the solution with the worst assignments is clearly represented within this group of scenarios (scenarios $23 \leq s \leq 25$). Table 3.9 illustrates the solutions achieved with this approach.

Table 3.9 - Results of minimax *p*-median location model

| ***Scenarios*** | ***Combination of zones that fail*** | ***p-median location model result (Km/resident)*** | ***Minimax p-median model result (Km/resident*** | ***Increase in percentage (%)*** |
|---|---|---|---|---|
| 1 | – | 0.80 | 2.79 | 240 |
| 2 | 1 | 1.16 | 2.61 | 125 |
| 3 | 2 | 0.99 | 2.69 | 173 |
| 4 | 3 | 1.04 | 2.74 | 164 |
| 5 | 4 | 1.05 | 2.33 | 123 |
| 6 | 5 | 1.10 | 3.01 | 175 |
| 7 | 1+2 | 1.46 | 2.78 | 91 |
| 8 | 1+3 | 1.41 | 2.75 | 95 |
| 9 | 1+4 | 1.38 | 2.34 | 70 |
| 10 | 1+5 | 1.45 | 2.75 | 90 |
| 11 | 2+3 | 1.29 | 2.83 | 119 |
| 12 | 2+4 | 1.25 | 2.88 | 129 |
| 13 | 2+5 | 1.62 | 3.01 | 85 |
| 14 | 3+4 | 1.40 | 2.82 | 102 |
| 15 | 3+5 | 1.32 | 3.01 | 129 |
| 16 | 4+5 | 1.32 | 3.01 | 128 |
| 17 | 1+2+3 | 1.41 | 2.71 | 92 |
| 18 | 1+2+4 | 1.72 | 2.96 | 72 |
| 19 | 1+2+5 | 2.31 | 2.91 | 26 |
| 20 | 2+3+4 | 1.92 | 2.75 | 43 |
| 21 | 2+3+5 | 1.93 | 3.01 | 56 |
| 22 | 3+4+5 | 1.67 | 3.01 | 80 |
| 23 | 1+2+3+4 | 2.69 | 2.69 | 0 |
| 24 | 1+2+3+5 | 2.87 | 2.98 | 0 |
| 25 | 2+3+4+5 | 3.01 | 3.01 | 0 |

The results presented in Table 3.9 enable us to understand just how much effect the optimization of a statistically insignificant has on an entire array of more likely scenarios. The results presented by this model, while still considering occurrences of up to four failures (which occur with 0.6% probability), represent an average increase of 109% for the scenarios that were subject to changes. Furthermore, these results allow us to say, without a shadow of a doubt, that determining the location of shelters in order to plan specifically for the occurrence of scenario 25 (which is the one prioritized) is the wrong course of action to take in this situation. As we can see, although the model aims at optimizing the worst case scenario, it fails to do so, because for the situations of four failure occurrences, the location of shelters and assignments are already optimized, thus the 0% difference in the results from scenarios 23 until 25.

In an effort to find a compromise that seems admissible, we propose analyzing the results of shelter locations and assignments only for situations which are more likely to occur. In that sense, situations of three and four simultaneous disasters should be dismissed, since they present low and almost insignificant probabilities of occurring: 0.6 % for situations of four simultaneous failures and 1.3% for situations of three disasters. With this approach, the array of scenarios that are considered is reduced to a set of 16 different scenarios, covering situations of no failures, one failure and two simultaneous failures (Table 3.6). For this approach, we will once again compare the results provided by this model with the ones provided by the scenario-based *p*-median location model, in an attempt to determine whether or not a compromise should be made to achieve better results for significantly more dangerous disaster occurrences. This comparison is represented in Table 3.10.

Table 3.10 – Results for the second approach of the minimax *p*-median location model

| ***Scenarios*** | ***Combination of zones that fail*** | ***p-median location model result (Km/resident)*** | ***Minimax p-median model result (Km/resident*** | ***Increase in percentage (%)*** |
|---|---|---|---|---|
| 1 | – | 0.80 | 1.35 | 64 |
| 2 | 1 | 1.16 | 1.37 | 18 |
| 3 | 2 | 0.99 | 1.50 | 52 |
| 4 | 3 | 1.04 | 1.40 | 35 |
| 5 | 4 | 1.05 | 1.29 | 23 |
| 6 | 5 | 1.10 | 1.44 | 32 |
| 7 | 1+2 | 1.46 | 1.37 | -6 |
| 8 | 1+3 | 1.41 | 1.50 | 6 |
| 9 | 1+4 | 1.38 | 1.51 | 9 |
| 10 | 1+5 | 1.45 | 1.52 | 5 |
| 11 | 2+3 | 1.29 | 1.49 | 15 |
| 12 | 2+4 | 1.25 | 1.44 | 15 |
| 13 | 2+5 | 1.62 | 1.46 | -10 |
| 14 | 3+4 | 1.40 | 1.40 | 0 |
| 15 | 3+5 | 1.32 | 1.41 | 7 |
| 16 | 4+5 | 1.32 | 1.45 | 10 |

Results for the consideration of only up to two simultaneous disasters, illustrated in Table 3.10, as expected, present less of a compromise in order to guarantee better results for the worst case scenario. With this approach, the increase in terms of travelled distances per resident is a mere 21%, when compared to the massive 108% proposed for the first approach. The solution that is proposed by this second approach might be preferred by risk averse decision makers, as it presents a more realistic representation of what happens in case a disaster strikes. To comprehend what sort of assignments this model produces, we present an illustration of the assignment solutions, in the approach of only up to two failures occuring simultaneously:

Figure 3.8 shows us the assignments developed by this model for scenario 13, taking into account only situations of up to two simultaneous failures. Furthermore, we illustrate this scenario because it represents the worst case scenario for the situation of two disasters occurring at the same time. Additionally, comparing Figure 3.6 (B) and Figure 3.8 we can see the effects of optimizing the assignments for a single scenario, instead of optimizing for the entire array of possibilities. On this approach, the model determines that shelters should be placed in sites 3, 4, 10, 18 and 19, instead of the ones which were initially proposed in our first approach. Furthermore, this alteration in the location of shelters indicates that this is the best set of shelters to comply with the model's primary objective of optimizing the functionality for the worst case scenario. Finally, as expressed in Table 3.10, this approach, when compared to the one taken in Figure 3.6 (B), allows for a decrease of 10% in terms of distances travelled amongst all the population, which in real life disaster situations, might prove to be a determining factor on whether or not the residents reach its assigned shelter safely.

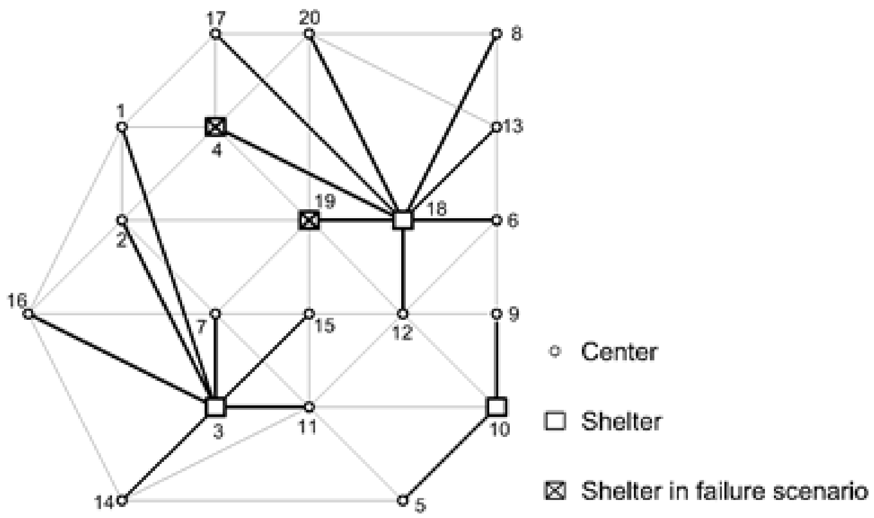


Figure 3.8 – Minimax $p$-median location model (2nd approach) for scenario 13 ($s = 13$)

# 4 RELIEF DISTRIBUTION / STOCK PRE-POSITIONING MODELS

Within the available literature, as it was expressed in Chapter 2.2, there are a variety of models that consider planning for relief distribution. From all the analyzed models, we selected two for their simplicity and applicability: Horner and Downs (2010) and Rawls and Turnquist (2010). Both models look to provide solutions that allow for a better understanding of how to overcome uncertainties related to disaster situations. Furthermore, they both have the same basic underlying problem: locating a set of distribution centers and determining the amount of commodities that they should hold in order to serve as relief sources for all the needs that the affected population has.

However similar, these models present a fundamental difference: Horner and Downs (2010) consider a single type of general commodity that is demanded while Rawls and Turnquist (2010) take into account different types of commodities, each one needing a different level of demand to be serviced. Although they consider different types of conditions, they both take an approach to locate a single type of general relief distribution facility, divided into categories/level according to the stock holding capacity that each one has. This disparity, lead us to analyze a different approach for the model proposed by Rawls and Turnquist (2010) and develop a model that would allow for the location of different types of facilities, which would be assigned to serve only demand from a specific type of commodity. In sum, we will present one model single commodity/single that locates a single type of facility (with different capacity levels) which serves a single type of commodity demand and develop another model that considers a variety of demand types and locates different types of facilities, according to their capability of holding stock from the various types of commodity demand. In this chapter, we will describe the approaches taken to reproduce the previously stated models, present the adaptations from the original models and compare results between each approach that is taken.

## 4.1 Single Commodity/Single Facility Model

Horner and Downs (2010) formulated a model to optimize the distribution of disaster relief goods during the occurrence of events like hurricanes. The model presented in this chapter is an adaptation of the model developed by those authors and studies the problem of locating two different classes of emergency facilities: primary and “break of bulb” distribution centers (BOB). Furthermore, the model assures that each facility is stocked with the necessary amount of relief goods in order to serve the assigned population with the demanded level of quantity. Additionally, the two classes of distribution centers considered vary according to the level of stock that they are able to store and distribute: primary distribution centers have bigger capacity while BOB's have a reduced level of capacity. As a result, this will likely lead BOB's to be more closely located to isolated populations, therefore allowing for a better overall solution in terms of accessibility amongst residents. Finally, a set of "logistical staging areas" (LSA) are considered in order to represent the primary source of relief goods, which is responsible for the transportation and stocking of relief goods to each located distribution center. In this chapter, we will present the changes that were applied to the original network of centers (presented in Chapter 3.1) as well as the factors and notations that were taken into account to develop a similar method to that of Horner and Downs (2010).

For the purposes that are intended and taking into account the necessary changes to fit the available data, the single commodity/single facility model, developed in this dissertation, considers the following notations:

***Sets:***

$\boldsymbol{J}$ – set of centers, indexed by $j$

$\boldsymbol{K}$ – set of sites, indexed by $k$

$\boldsymbol{I}$ – set of LSA locations, indexed by $i$

$\boldsymbol{N}$ – set of types of distribution centers, indexed by $n$

***Parameters:***

$a_j$ – Demand of resources in location $j \in \boldsymbol{J}$

$d_{ik}$ – Distance from distribution center $k \in \boldsymbol{K}$ to LSA location $i \in \boldsymbol{I}$

$d_{jk}$ – Shortest path distance from center $j \in \boldsymbol{J}$ to site $k \in \boldsymbol{K}$

$p_n$ – Maximum number of distribution centers of type $n \in \boldsymbol{N}$

$e_n$ – Maximum holding capacity of distribution centers of type $n \in \boldsymbol{N}$

***Decision variables:***

$X_{ikn}$ – Resources transported from LSA $i \in \boldsymbol{I}$ and stocked on facility $k \in \boldsymbol{K}$ of type $n \in \boldsymbol{N}$

$$Z_{kn} = \begin{cases} 1, & if\ distribution\ center\ k\ of\ type\ n\ is\ opened \\ 0, & otherwise \end{cases}$$

$$Y_{jkn} = \begin{cases} 1, & if\ inhabitants\ from\ center\ j\ are\ assigned\ to\ facility\ k\ of\ type\ n \\ 0, & otherwise \end{cases}$$

Once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad \sum_{i\in \boldsymbol{I}}\sum_{k\in \boldsymbol{K}}\sum_{n\in \boldsymbol{N}} d_{ik}X_{ikn} + \sum_{j\in \boldsymbol{J}}\sum_{k\in \boldsymbol{K}}\sum_{n\in \boldsymbol{N}} a_j d_{jk} Y_{jkn} \tag{30}$$

(s.t):

$$\sum_{k\in \boldsymbol{K}} Z_{kn} \leq p_n, \qquad \forall n \in \boldsymbol{N} \tag{31}$$

$$\sum_{i\in \boldsymbol{I}} X_{ikn} = \sum_{j\in \boldsymbol{J}} a_j Y_{jkn}\,, \qquad \forall k \in \boldsymbol{K}, n \in \boldsymbol{N} \tag{32}$$

$$\sum_{k\in \boldsymbol{K}}\sum_{n\in \boldsymbol{N}} Y_{jkn} = 1, \qquad \forall j \in \boldsymbol{J} \tag{33}$$

$$Z_{kn} \geq Y_{jkn}, \qquad \forall k \in \boldsymbol{K}, j \in \boldsymbol{J}, n \in \boldsymbol{N} \tag{34}$$

$$\sum_{j\in \boldsymbol{J}} a_j Y_{jkn} \leq e_n, \qquad \forall k \in \boldsymbol{K}, n \in \boldsymbol{N} \tag{35}$$

$$Z_{kn} = 0, \qquad \forall n \in \boldsymbol{N}, k = 21, \dots, 24 \tag{36}$$

$$Y_{jkn} \in \{0,1\}, \qquad \forall\, j \in \boldsymbol{J}, k \in \boldsymbol{K}, n \in \boldsymbol{N} \tag{37}$$

$$Z_{kn} \in \{0,1\}, \qquad \forall k \in \boldsymbol{K}, n \in \boldsymbol{N} \tag{38}$$

The objective function (30), minimizes the overall travelled distances while distributing relief goods from LSA's to population centers through $n$ types of intermediate distribution centers. The first term represents the distances that are travelled in order to pre-position stock of relief goods in both types of distribution center. The second term represents the distances travelled by the population in order to have access to relief goods in both types of distribution centers. The inclusion of (30) aims at creating a balance between the two phases of transportation involved, in order to guarantee the quickest stocking of facilities possible while minimizing the travelled distances by the population.

Constraints (31) reflect the budgetary and operational limitations by way of imposing that the maximum number of each $n$-type distribution center is $p_n$. Constraints (32) mandate that demand served by distribution centers is to be provided by LSA's. Constraints (33) regulate that demand from populated areas may only be served by a single distribution center, be it a primary distribution center or a BOB. Constraints (34) assure that population is only assigned to distribution centers that have been sited and are operational. Constraints (35) guarantee that distribution centers do not distribute more goods than their stipulated maximum holding capacity. Given the notations that were used, constraints (36), ensure that LSA locations are not treated as possible distribution centers by impeding location of facilities in said areas. Constraints (37) and (38) require the decision variables to be binary.

**Application**

This single commodity/single facility model requires the existence of a set of "logistical staging areas". To take that into account the original network design had to be altered consider this part of the organizational structure underlying relief distribution efforts. To the originally proposed network design, presented in Chapter 3.1, we assumed that there would be an additional set of four "logistical staging areas" (LSA). Furthermore, the original design maintained untouched in terms of the location of each center and we simply looked at ways of locating the four missing LSA's. We assumed that each of these administrative facilities would be located at the corners of a square unit with coordinates (x,y) now scaled from -1 to 6. Figure 4.1 illustrates the graphical representation of the resulting network.

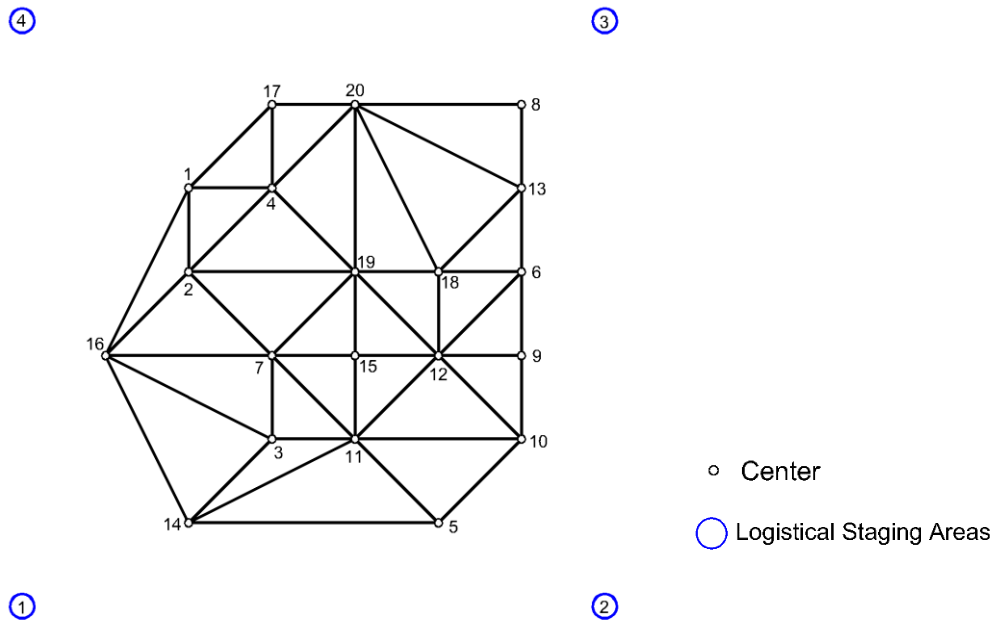


Figure 4.1 – Network design with location of "logistical staging areas"

This model is applied taking into account that there would be a maximum of three primary distribution centers ($p_1 = 3$) and a maximum of two BOB's ($p_2 = 2$). Furthermore, we assumed that every primary distribution center would have a maximum holding capacity of 300 units ($e_1 = 300$) and that BOB's would a maximum of 150 units ($e_2 = 150$). Regarding the demand data, we assumed that the number of people in need of emergency commodities would be the same as the number of people in need of evacuation (see Appendix – Table A.2).

Figure 4.2 shows the results of this model, for the conditions that were mentioned above. It shows that, for the conditions that were taken into account, a set of five distribution centers should be located in order to achieve the desired objectives. More specifically, three primary distribution centers, located in sites 1, 8 and 10, and a complementary set of two BOB's, located in sites 5 and 14. Furthermore, this solution represents an overall amount of $3926.6\ Km$ in travelled distances, more specifically, $2469\ Km$ in the transportation of stock and $1457.6\ Km$ in the distribution of resources to the population. Additionally, these values represent an average of $2.2\ Km$ and $1.3\ Km$ of travelled distances per demand unit in the transportation of stock and in the distribution of resources, respectively.

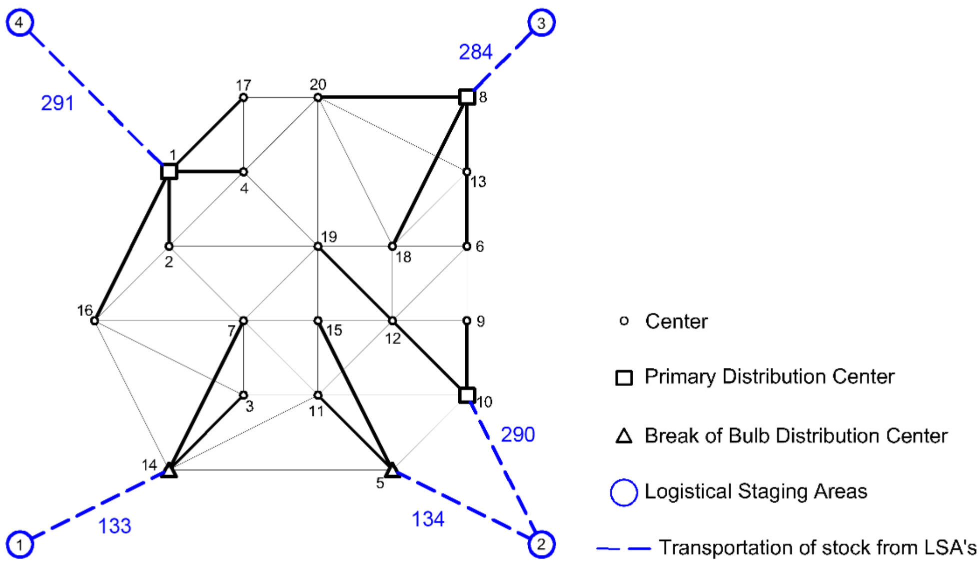


Figure 4.2 – Solution for the single commodity/single facility model

Regarding the locations and assignments that were determined, it is clear that by assuming the same level of importance on both terms of the objective function (30), the model always looks to minimize the component of positioning stock in distribution centers first and only then looks for the best combination of assignments to minimize the distances travelled by the population. Although the population does not travel the ideally smallest amount of distances possible, the approach that is taken in this model allows for the fastest pre-positioning of stock in located distribution centers, which is an indirect benefit to the population that offsets the disadvantages of them having to travel a slightly increased distance to receive the necessary aid.

This model is mainly influenced by the number of each type of distribution centers. Furthermore, if we were to consider a limited number of primary distributions centers ($p_1 = 2$), an amount of two additional BOB's ($p_2 = 4$) would have to be created to provide the necessary number of relief goods. This would result in an overall solution of $2613\,Km$ ($2.3\,Km$ per demand unit) travelled distances in transportation of stock and $1313.6\,Km$ ($1.2\,Km$ per demand unit) in travelled distances in the distribution of resources to the population. The sum of both parts ($3926.6\,Km$) matches the total result presented in Figure 4.2. Although the results are identical on both situations in terms of combined distances (stocking + distribution), in this last case, the distances travelled by the population would be smaller since the total number of distribution centers would be six instead of the original five shown in Figure 4.2. All told, an increase in the overall number of facilities is always accompanied by a decrease in the distances travelled by the population and not necessarily in the distances travelled to transport supplies from LSA's to distribution centers.

## 4.2 Multiple Commodities/Multiple Facilities Model

Rawls and Turnquist (2010) formulate a model for the location of facilities that perform the distribution of various types of pre-stocked commodities according to demands provided by scenario-based disaster situations. This multiple commodities/multiple facilities model developed within this dissertation consists of a variation of the latter. It considers that each type of facility as a specific kind of demand that it might serve and that the levels of demand for each type of resource vary according to the gravity of scenarios that are considered. Each scenario is associated with a probability of occurrence. The problem is formulated as an optimization model, in which the objective is to minimize the distances travelled by the population in order to receive assistance for all the commodities that might be needed. The model looks to minimize the distance results over all scenarios and presents assignment solutions according to each commodity and each scenario of demand levels.

In order to adapt the original model presented by Rawls and Turnquist (2010) to the changes of perspective that were taken in this dissertation for this particular problem, the following notations were considered:

***Sets:***

***J*** – set of centers, indexed by *j*

***K*** – set of sites, indexed by *k*

***T*** – set of types of commodities, indexed by *t*

***L*** – set of facility types, indexed by *l*

$\boldsymbol{S}$ – set of scenarios, indexed by $s$

***Factors:***

$v_{jts}$ – Demand of commodity $t \in \boldsymbol{T}$ in center $j \in \boldsymbol{J}$ during scenario $s \in \boldsymbol{S}$

$m_{lt}$ – Maximum holding capacity of commodity $t \in \boldsymbol{T}$ in facility type $l \in \boldsymbol{L}$

$g_{kt}$ – Amount of commodity $t \in \boldsymbol{T}$ pre-stocked on site $k \in \boldsymbol{K}$

$d_{jk}$– Shortest path distance from center $j \in \boldsymbol{J}$ to site $k \in \boldsymbol{K}$

$p_l$ – Maximum number of facility type $l \in \boldsymbol{L}$

$qr_s$ – Probability of scenario $s \in \boldsymbol{S}$ occurring

$M$ – Penalty factor

***Decision variables:***

$X_{jkts}$ – Demand of commodity $t \in \boldsymbol{T}$ from center $j \in \boldsymbol{J}$ that is served in facility located in site $k \in \boldsymbol{K}$ during scenario $s \in \boldsymbol{S}$

$$Y_{kl} = \begin{cases} 1, & \textit{if facility of type } l \in \boldsymbol{L} \textit{ is opened in site } k \in \boldsymbol{K} \\ 0, & \textit{otherwise} \end{cases}$$

Once all the factors mentioned above have been taken into account, the model is then formulated as an integer programming problem, with the objective and constraint functions as follows:

$$minimize \quad \sum_{s \in \boldsymbol{S}} qr_s \sum_{j \in \boldsymbol{J}} \sum_{k \in \boldsymbol{K}} \sum_{t \in \boldsymbol{T}} d_{jk} X_{jkts} + M \times \sum_{k \in \boldsymbol{K}} \sum_{t \in \boldsymbol{T}} g_{kt} \tag{39}$$

*(s.t):*

$$\sum_{k \in \boldsymbol{K}} X_{jkts} = v_{jts}, \qquad \forall j \in \boldsymbol{J}, t \in \boldsymbol{T}, s \in \boldsymbol{S} \tag{40}$$

$$g_{kt} \leq \sum_{l \in \boldsymbol{L}} m_{lt} Y_{kl}, \qquad \forall k \in \boldsymbol{K}, t \in \boldsymbol{T} \tag{41}$$

$$g_{kt} \geq \sum_{j \in \boldsymbol{J}} X_{jkts}, \qquad \forall k \in \boldsymbol{K}, t \in \boldsymbol{T}, s \in \boldsymbol{S} \tag{42}$$

$$\sum_{l \in \boldsymbol{L}} Y_{kl} \leq 1, \qquad \forall k \in \boldsymbol{K} \tag{43}$$

$$\sum_{k \in \boldsymbol{K}} Y_{kl} \leq p_l, \qquad \forall l \in \boldsymbol{L} \tag{44}$$

$$Y_{kl} \in \{0,1\}, \qquad \forall k \in \boldsymbol{K}, l \in \boldsymbol{L} \tag{45}$$

The first term of the objective function (39) minimizes the distances travelled by the population over all scenarios. The second term represents a penalty associated with the acquisition of commodities to pre-position in the located facilities and leads to a solution that stocks only the minimum amount of commodities possible to be able to serve the population. Furthermore, this second term assembles, in a way, an objective to minimize costs associated with the holding of excess commodities. In regards to effective results, the model provides a solution that is able to cope with the most challenging set of commodity demands and presents a solution to the location of facilities that is maintained throughout all disaster scenarios, with assignments varying according to the commodities and scenarios that are considered. This means that even if a smaller scale disaster occurs, the amount of stock that is pre-positioned in all the facilities must be able to cope with a larger scale disaster, which consequently results in a certain amount of commodities that end up not being used if a scenario with fewer demand occurs. Although, the model presented does not take it into account, a cost of unmet demand may be considered in order to generate a solution that performs well under catastrophic events (with lower probability of occurring). Finally, this model provides a result which consists of three different schemes of network assignments: one for the distribution of water, another for the distribution of food and lastly one for the distribution of medication.

Constraints (40) guarantee that all demand for each commodity must be served. Constraints (41) ensure that pre-positioning of stock may only occur if the facility has been located and that the amount of stock must not exceed the facility's capacity for each commodity. Constraints (42) determine that the amount of pre-positioning of stock of each commodity in a certain facility must be equal or exceed the overall number of demand from all the population that is assigned to that same facility. Constraints (43) certify that only one type of facility may be located at each site. Constraints (44) reflect the budgetary and operational limitations by way of commanding that the maximum number of each l-type facility is $p_l$. Finally, constraints (45) require the location decision variable to be binary.

In an attempt to achieve the best set of results possible, the previous formulation (**1st approach**), which determines a solution in which the demand for a certain type of commodity may be serviced by a multitude of facilities, will be compared to a second approach that mandates that demand for a certain type of commodity may only be served by a single facility. This second approach (**2nd approach**) may provide solutions that are easier to implement in real-life situations, due to the clear indications that are provided to citizens on where to go to receive assistance, which will likely not happen in the first approach.

**Application**

We consider a set of three different types of commodities (1 – Water; 2 – Food; 3 – Medication), for which there is likely to be demand during a disaster event, and consider their pre-positioning in three different types of storage/distribution facilities, according to the suitability of said facilities to hold a certain type of commodity (1 – Water Silo; 2 – Food Distribution Center; 3 – Medical Center).

Both approaches of the multiple commodities/multiple facilities model will be tested and compared in this chapter, according to a certain group of factors. We took into account that there would be a maximum of two water silos ($p_1 = 2$), two food distribution centers ($p_2 = 2$) and also two medical centers ($p_3 = 2$). Additionally, we generated random values for commodity demand, varying from 0 – 150 units, depending on the gravity of the disaster that the scenario portraits: 0-50 small scale; 50-100 mid-scale; 100-150 high scale (See Appendix – Tables A.3; A.4 and A.5 for the detailed demand data). Furthermore, demand scenarios were associated with the following random probabilities: $qr_1 = 0.15$ (small scale); $qr_2 = 0.10$ (mid-scale); $qr_3 = 0.05$ (high scale). Finally, we assumed that water silos would only be able to serve demand for water ($m_{11} = 1500$ ; $m_{12} = 0$ ; $m_{13} = 0$), that food distribution centers would have capacity to primarily serve demand for food and have a reserve for medication ($m_{21} = 0$ ; $m_{22} = 2000$ ; $m_{23} = 250$), and that medical centers have the primary objective of serving medication demand but also contain a reserve for food supplies ($m_{31} = 0$ ; $m_{32} = 250$ ; $m_{33} = 2000$).

We obtain the solutions presented in Figure 4.3, Figure 4.4 and Figure 4.5, which represent the resulting assignments and facility locations associated with the distribution of water, food and medication, respectively.

The results presented in Figure 4.3 show that the location of water silos is not affected by the approach that is taken into account. For both cases, the resulting number and location of facilities for the distribution of water is the same, with a total of two water silos located in centers 7 and 18. Moreover, due to the fact that the assignments remain untouched from one approach to another, on both situations the amount of pre-positioned water resources is 1094 units in center 7 and 1338 units in center 18.

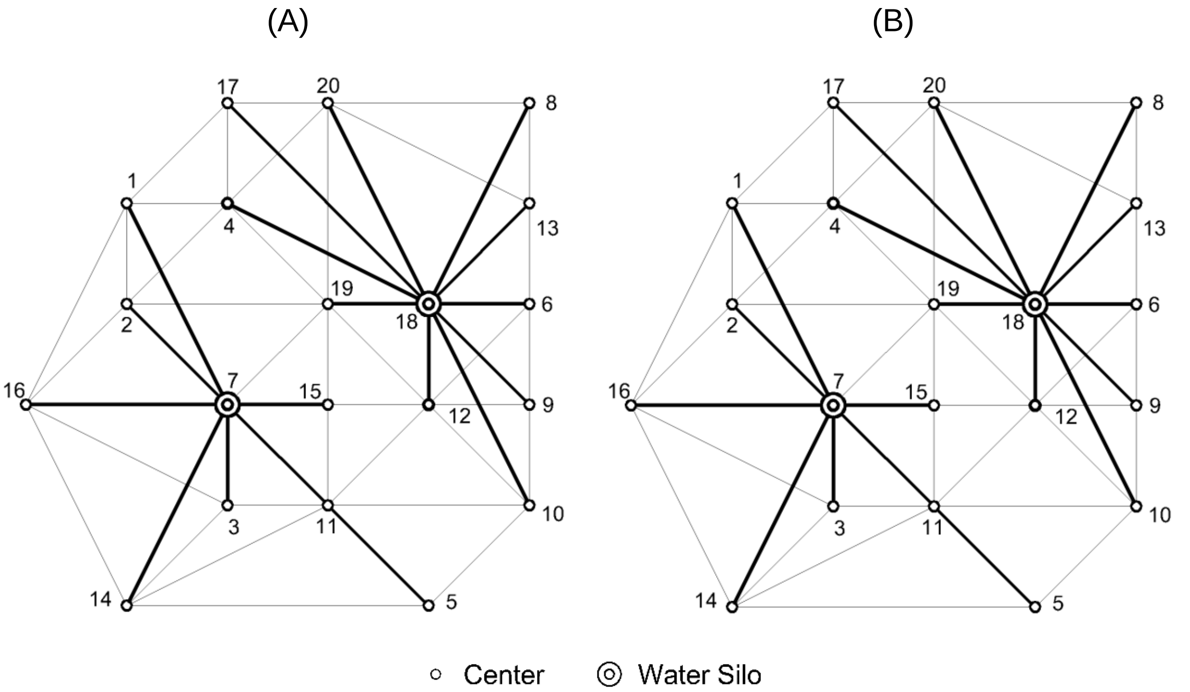


Figure 4.3 – Multiple Commodities/Multiple facilities model – results for the distribution of water in (A) 1st approach and (B) 2nd approach

Regarding the distribution of food, it is clear by the results illustrated in Figure 4.4 that the decision regarding the location of facilities is subjected to changes, depending on the approach that is taken, which consequently leads to a change in terms of the resulting assignments. For the 1st approach, there will be two food distribution centers, located in centers 3 and 13, each one with 1109 and 863 of pre-positioned commodity units, respectively. Furthermore, with each medical center being capable of distributing up to 250 units of food, the results that are achieved provide a solution with a better overall distribution of travelled distances. For the 1st approach, in specific, the medical centers in 4 and 12, provide the population with a total of 500 food units, as they are both stocked to their maximum capacity. The latter enables population from centers such as 1, 4, 10 and 12 to receive quicker assistance. For the 2nd approach, food distribution centers are to be located in centers 4 and 11, with each distributing 874 and 1106 food units, respectively. Regarding the backup provided by medical centers in the distribution of food resources, both centers in 2 and 6 provide the distribution of 246 units of food.

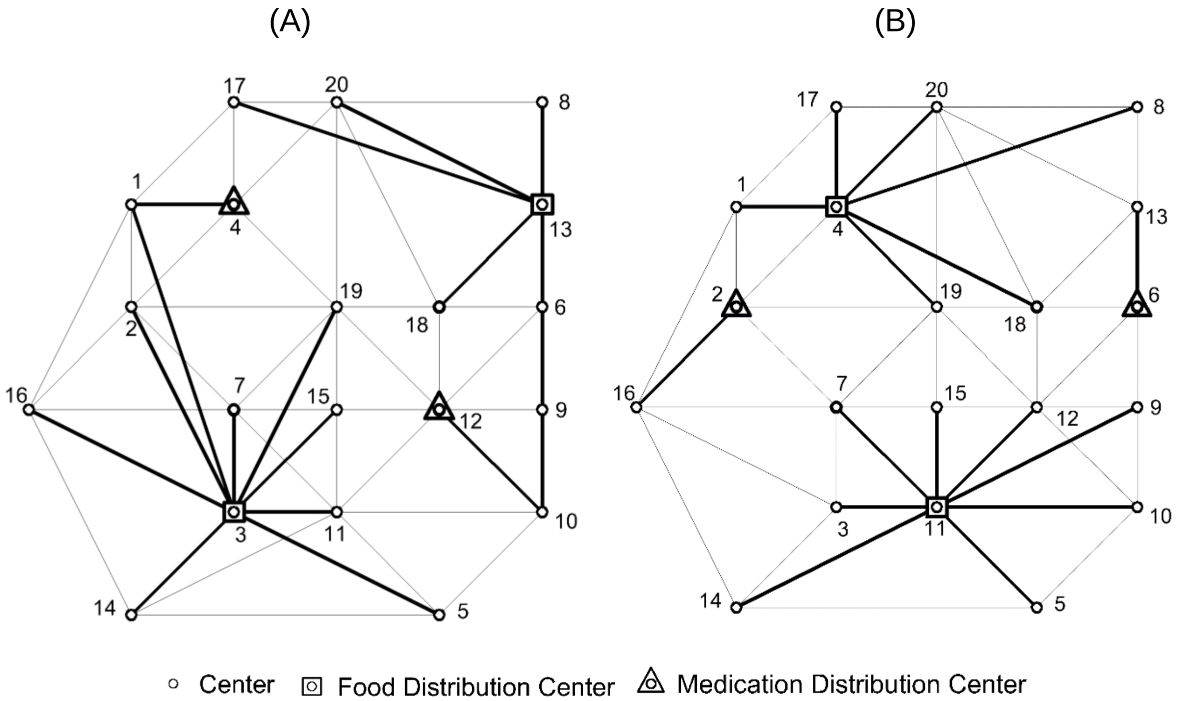


Figure 4.4 – Multiple Commodities/Multiple facilities model – results for the distribution of food in (A) 1st approach and (B) 2nd approach

As for the distribution of medical supplies, the results presented in Figure 4.5, again show that there is a difference of results according to the approach that is taken into consideration. For the 1st approach, the model indicates that there should be located medical centers in centers 4 and 12. Furthermore, each of these centers distributes a total of 895 and 1139 medical supplies. As for the backup provided by food distribution centers, centers 3 and 13 both distribute the maximum amount of 250 medical supplies each. Regarding the 2nd approach, the results indicate that there should be located medical centers in centers 2 and 6 with each distributing a total of 1023 and 1018 medical supplies, respectively. As for the backup provided by food distribution centers, a total of 243 medical supplies is stocked in center 4, while center 11 is used in its' maximum capacity, providing relief with the distribution of 250 units of medical supplies.

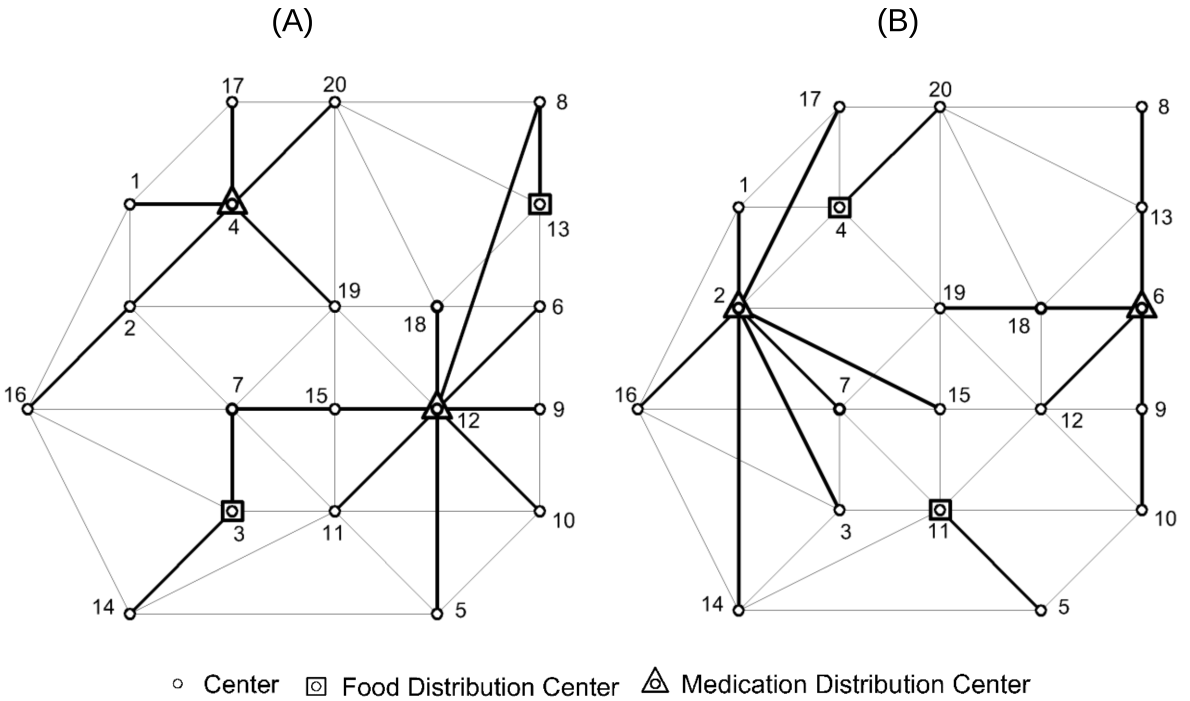


Figure 4.5 – Multiple Commodities/Multiple facilities model – results for the distribution of medication in (A) 1st approach and (B) 2nd approach

In regards to the overall solution that each approach develops, it is expected that the 1st approach develops results with lower travelled distances amongst all the population, as it considers that population from the same center may be served in different distribution centers. In fact, this expectation is met, as the assignments and locations proposed by the 1st approach result in an overall amount of $18382.6\ Km$ travelled by all the population ($2.5\ Km$ per demand unit), as contrasting to the solution proposed by the 2nd approach, which reproduces a total of $18947.4\ Km$ in travelled distances ($2.55\ Km$ per demand unit).

As it pertains to the disparity between the approaches in terms of travelled distances, the registered difference of $564.8\ Km$, represents an increase of approximately $500\ m$ per unit of commodity demand for the 2nd approach. Taking into account the added level of safety (less confusion in assignments) that the 2nd approach offers when compared to the 1st approach, the small increase in distances travelled looks to be a logical compromise for the population to accept the 2nd approach as the best solution.

# 5 CONCLUSION

In this dissertation, we aimed at studying planning efforts for the prevention of damages associated with the occurrence of catastrophic events. Within this context, humanitarian logistics studies take center stage and present themselves as the most important tool in constructing a coordinated and effective response plan for the eventuality of a disaster. These studies address the topics of what are the necessities during a disaster crisis and plan for the best way of implementing an efficient plan of action which guarantees that all those necessities are met promptly. Humanitarian logistics, study relief efforts motivated by disaster events and appoint specific solutions depending on the need that is presented (e.g. evacuation plans, distributions of materials in need). Additionally, humanitarian logistics is a derivation of emergency logistics, an area of business-related problems which studies the effects of disasters on the functionality of a specific business.

Necessities and obstacles for the fulfilment of those necessities during disaster situations may fluctuate according to the phase of actions that is considered, and so, humanitarian logistics is divided into two specific planning stages: pre-disaster and post-disaster operations. Pre-disaster operations are all the efforts that take place before the impact of a disaster and which look for ways to alleviate or prevent the damages that are caused. Post-disaster operations, take place after a disaster occurs and have the purpose of attending to disaster impacts. In this project, we analyzed pre-disaster operations and found that they are mainly performed by way of developing optimization models. Furthermore, in this area of studies, these models focus primarily on facility location problems. They study the creation of a network of facilities and have the objective of locating those facilities as closely as possible to areas which are likely to be affected during a disaster.

Facility location problems, in humanitarian logistics, study the implementation of two different types of network of facilities: a network of shelters or a network of distribution centers. Problems which locate shelters aim at providing the population from all inhabited centers with quick and safe access to protected zones. Problems that focus on locating distribution centers aim at providing the population with access to emergency resources (e.g. food, water and medicine), which are stocked in each distribution center. Furthermore, each of these different types of networks is associated with a specific issue: a network of shelters provides solutions that enable to solve problems related to the evacuation of people from affected areas – these are called evacuation models; a network of distribution centers allows to solve problems related with the stocking and transportation of resources to people in need – these are referred to as relief distributions and stock pre-positioning models.

The first type of models which we present, are evacuation models. They determine the location of a network of shelters and define the routes of evacuation that should be taken from each populated center in order to reach its assigned safe shelter. As an objective, these models look for the best set of locations of shelters in order to minimize travelled distances. They take into account probabilities of failure in shelters and specific zones of the city in order to present a solution that guarantees the best level of preparedness.

We identified and studied two different types of approaches for evacuation models: proximity-based models and scenario-based models. Proximity-based models provide different assignment solutions according the levels of proximity of centers and sites. Furthermore, models with this type of approach, study the hypothetical occurrence of every resident being assigned to shelters at a specific level of proximity. Scenario-based models provide routing assignments by taking into account scenarios that define a specific occurrence in terms of the number of failures that occur due to a disaster. On this approach, models present a solution for each pre-defined scenario of occurrences and give solid insight on the course of action for every eventuality. We observed that for whichever approach, proximity-based or scenario-based, there were two types of situations that were considered: design of networks and fortification of networks. Network design models aim at building a network of shelters from scratch. Network fortification models select a subset of shelters from the solutions presented by the design model and then apply measures to fortify their structure, which turns them immune to the occurrence of a disaster.

For proximity-based evacuation models, we presented the formulation and application of two optimization models: a $p$-median location model (design) and a $p$-median fortification model (fortification). The $p$-median location model presented solutions which allowed us to have a better understanding of what would incur in terms of assignments, when a certain populated center is assigned to a shelter at a certain level of proximity. From this model, we were able to conclude upon what assignments to expect in a case a populated had its one, two, three or four closest shelters failing. The $p$-median fortification model determined the subset of previously existing shelters that should be fortified against failures. It indicated what would be the level of proximity for which a populated center might find the closest fortified shelter.

For scenario-based evacuation models, we presented the formulation and application of three optimization models: a scenario-based $p$-median location model (design), a scenario-based $p$-median fortification model (fortification) and a minimax $p$-median location model (design). The scenario-based $p$-median location model enabled us to determine a solution for the location of shelters and evacuation of people that performed well for the majority of the scenarios that were considered, regardless of the probability of occurrence associated with a specific scenario. We concluded that these types of solutions were useful for decision makers which concerned themselves with presenting a solution that worked well overall and which devoted less time to the specific planning of more catastrophic situations. The scenario-based *p*-median location model allowed us to present the ideal solution for assignments and shelter locations, however, we concluded that this solution would always require a bigger budget, and therefore, not be at the reach of every decision maker. At last, the minimax *p*-median location model determined solutions that looked for a compromise of the overall functionality of the network in order to work in the best way possible for the most catastrophic event. We came to the conclusion that this would be indicated for risk averse decision makers, which do not mind losing a significant amount of responsiveness in most scenarios in order to avoid complications in the worst scenario.

The second and also last type of models which we present are relief distribution and stock pre-positioning models. These models have the objective of locating a set of distribution centers and determining the amount of resources (like food, water and medication) that they should hold in order to serve as relief sources for all the needs that the population has. Within relief distribution and stock pre-positioning models, we analyzed and applied two different variations: a single commodity/single facility model and a multiple commodities/multiple facilities model.

The single commodity/single facility model studies situations where there is a demand for a single type of resource. This model assumes that to serve all the demand, there will only be a single type of facility that is located. However, that single type of facility is divided into two categories, according to the capacity that it has to hold and distribute stock: a primary distribution center which holds the most amount of stock and a "break of bulb" facility which has the capacity to serve a much smaller amount of people. The model aims at determining the combined locations of each category of distribution centers that guarantees that people have the quickest access to the stocked resources. Furthermore, the model considers the existence of "logistical staging areas" which are in charge of delivering and stocking resources in each of the located distribution centers. As a secondary objective, this model looks to minimize the distances travelled to perform this transportation of stock from the "logistical staging areas" to distribution centers, and so, in the end, the model presents a solution of distribution center locations that guarantees the quickest combined process of stocking the located distribution centers and reaching the population with the stocked resources. We were able to conclude that this model presents solutions that are very balanced in terms of each stage that is considered and also, that the inclusion of a smaller category of the same type of facility allows population from remote zones of the city to have better access to the resources.

The multiple commodities/multiple facilities model studies situations where is demand for different types of resources and that the demand for each must be serviced by a specialized distribution center. We solved a problem that had differentiated demands for food, water and medication. Furthermore, we assumed that there would be located three types of facilities: water silos, food distribution centers and medical centers. Water silos were exclusively serving water demand, food distribution centers served food demand and had a small reserve of medicine also stocked and medical centers served primarily demand for medication and had a backup for food demand. For this model we took two different approaches regarding the possibility of assignments. The first approach considered that a center might be served by more than one facility for each type of resources. The second approach assumed that, to avoid confusion, residents of each center had to be served exclusively by one facility for each type of demand. This allowed us to conclude that the solutions presented for the second approach represented a very good compromise and should always be taken in order to avoid misunderstandings in the assignments that are proposed to the residents.

# REFERENCES

Alçada-Almeida, L., Tralhão, L., Santos, L. and Coutinho-Rodrigues, J. (2009). "A multiobjective approach to locate emergency shelters and identify evacuation routes in urban areas". Geographical Analysis, 41: 9 – 29.

Altay, N. and Green III, WG. (2006). "OR/MS research in disaster operations management". European Journal of Operational Research, 175(1):475 – 493.

Balcik, B. and Beamon, B.M. (2008). "Facility location in humanitarian relief". International Journal of Logistics: Research and Applications, 11(2):101 – 21.

BBC News (2010). "Logistical nightmare hampers Haiti aid effort". Report. January 22, 2010. Available from: http://news.bbc.co.uk/2/hi/americas/8460787.stm - accessed in 21/03/2014.

Berman, O., Krass, D. and Menezes M (2005). "MiniSum with imperfect information: Trading off quantity for reliability of locations". Working paper, Rotman School of Management, University of Toronto.

Berman, O., Krass, D. and Menezes M (2007). "Facility reliability issues in network p-median problems: Strategic centralization and co-location effects". Operations Research, 55(2): 332 – 350.

Caunhye, A.M., Nie, X. and Pokharel, S. (2012). "Optimization models in emergency logistics: A literature review". Socio-Economic Planning Sciences, 46: 4 – 13.

Chang, M.S., Tseng, Y.L. and Chen, J.W. (2007). "A scenario planning approach for the flood emergency logistics preparation problem under uncertainty". Transportation Research Part E: Logistics and Transportation Review, 43(6):737 – 754.

Church, R.L. and Scaparra M.P. (2005). "Protecting critical assets: The r-interdiction median problem with fortification". Working paper 79, Kent Business School, Canterbury, England, February.

Church, R.L., Scaparra, M.P. and Middleton R.S. (2004). "Identifying critical infrastructure: The median and covering facility interdiction problems". Annuals of the Association of American Geographers, 94(3):491–502.

Fritz Institute (2005). "Logistics and the effective delivery of humanitarian relief". Report. Available from: http://www.fritzinstitute.org/PDFs/Programs/TsunamiLogistics0605.pdf – accessed in 23/03/2014.

Guha-Sapir, D., Hoyois, Ph. and Below, R. (2013). Annual Disaster Statistical Review 2012: The Numbers and Trends. Brussels: CRED.

Haghani, A., Oh, S.C. (1996). “Formulation and solution of a multi-commodity, multimodal network flow model for disaster relief operations”. Transportation Research Part A: Policy and Practice, 30(3):231 – 250.

Horner, M.W. and Downs, J.A. (2010). “Optimizing hurricane disaster relief goods distribution: model development and application with respect to planning strategies”. Disasters, 34(3):821 – 844.

Kaynak, R. and Tuğer, A.T. (2014). “Coordination and collaboration functions of disaster coordination centers for humanitarian logistics”. Procedia – Social and Behavioral Sciences 109, 432 – 437.

Kongsomsaksakul, S., Yang, C. and Chen, A. (2005). “Shelter location-allocation model for flood evacuation planning”. Journal of the Eastern Asia Society for Transportation Studies, 6: 4237 – 425

Mete, H.O. and Zabinsky, Z.B. (2010). “Stochastic optimization of medical supply location and distribution in disaster management”. International Journal of Production Economics, 126(1):76 – 84.

Psaraftis, H.N., Tharakan, G.G. and Ceder, A. (1986). “Optimal response to oil spills: the strategic decision case”. Operations Research, 34(2):203 – 217.

Rawls, C.G. and Turnquist, M.A. (2010). “Pre-positioning of emergency supplies for disaster response”. Transportation Research Part B: Methodological, 44(4): 521 – 534.

Scaparra, M.P. and Church, R.L. (2005). “An optimal approach for the interdiction median problem with fortification”. Working paper 78, Kent Business School, Canterbury, England, February.

Sherali, H.D. and Carter, T.B and Hobeika, A.G. (1991). “A location-allocation model and algorithm for evacuation planning under hurricane/flood conditions”. Transportation Research Part B: Methodological, 25(6):439 – 452.

Snyder, L.V. and Daskin, M.S. (2006). “Models for reliable supply chain network design”. In Alan T. Murray and Tony H. Grubesic, editors, Reliability and Vulnerability in Critical Infrastructure: A Quantitative Geographic Perspective. Springer, forthcoming.

Snyder, L.V., Scaparra, M.P., Daskin, M.S. and Church, R.L. (2005). “Planning for disruptions in supply chain networks”. Tutorials in Operations Research INFORMS, 234–257.

Song, R., He, S. and Zhang, L. (2009). “Optimum transit operations during the emergency evacuations”. Journal of Transportation Systems Engineering and Information Technology, 9(6):154 – 160.

United Nations International Strategy for Disaster Reduction (1999). Available from: http://www.unisdr.org/who-we-are/what-is-drr - accessed in 20/03/2014.

Van Wassenhove, L.N. (2006). “Blackett Memorial Lecture Humanitarian aid logistics: supply chain management in high gear”. Journal of the Operational Research Society, 57,475-489.

Wardrop, J.G. (1952). “Road paper: some theoretical aspects of road traffic research”. ICE Proceedings: Engineering Divisions, 1(3): 325 – 362.

Wilhelm, W.E.and Srinivasa, A.V. (1996). “A strategic, area-wide contingency planning model for oil spill cleanup operations with application demonstrated to the Galveston Bay area”. Decision Sciences, 27(4):767 – 799.

Yi, W. and Özdamar, L. (2007). “A dynamic logistics coordination model for evacuation and support in disaster response activities”. European Journal of Operational Research, 179(3):1177 – 1193.

# APPENDIX

## A. NETWORK GEOMETRY AND CHARACTHERISTICS

## B. SCENARIOS

# A. NETWORK GEOMETRY AND CHARACTERISTICS

Table A.1 – Network coordinates

| | ***Centers/Sites*** | | | | | | | | | | | | | | | | | | | | ***LSA's*** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** | ***1*** | ***2*** | ***3*** | ***4*** |
| **X** | 1 | 1 | 2 | 2 | 4 | 5 | 2 | 5 | 5 | 5 | 3 | 4 | 5 | 1 | 3 | 0 | 2 | 4 | 3 | 3 | -1 | 6 | 6 | -1 |
| **Y** | 4 | 3 | 1 | 4 | 0 | 3 | 2 | 5 | 2 | 1 | 1 | 2 | 4 | 0 | 2 | 2 | 5 | 3 | 3 | 5 | -1 | -1 | 6 | 6 |

Table A.2 – Demand of people in need of evacuation and supplies (for models with a single type of demand)

| ***Center*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Demand*** | 14 | 77 | 55 | 93 | 41 | 67 | 11 | 74 | 69 | 41 | 65 | 89 | 54 | 67 | 28 | 30 | 77 | 77 | 91 | 12 |

Table A.3 – Demand of water supplies in Rawls and Turnquist

| | ***Centers*** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Scenario 1*** | 5 | 47 | 3 | 19 | 26 | 13 | 45 | 45 | 18 | 42 | 50 | 3 | 47 | 16 | 18 | 14 | 37 | 37 | 25 | 23 |
| ***Scenario 2*** | 65 | 76 | 84 | 94 | 79 | 86 | 87 | 92 | 53 | 51 | 81 | 71 | 51 | 66 | 92 | 80 | 68 | 74 | 69 | 58 |
| ***Scenario 3*** | 135 | 133 | 110 | 131 | 112 | 107 | 107 | 106 | 123 | 101 | 124 | 125 | 147 | 148 | 107 | 118 | 114 | 135 | 124 | 125 |

Table A.4 – Demand of food supplies in Rawls and Turnquist

| | ***Centers*** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Scenario 1*** | 38 | 19 | 36 | 32 | 33 | 38 | 4 | 14 | 12 | 42 | 34 | 19 | 47 | 35 | 13 | 31 | 22 | 12 | 25 | 26 |
| ***Scenario 2*** | 96 | 77 | 96 | 91 | 52 | 76 | 61 | 78 | 50 | 62 | 71 | 84 | 66 | 92 | 78 | 83 | 93 | 92 | 97 | 57 |
| ***Scenario 3*** | 109 | 113 | 114 | 145 | 144 | 108 | 123 | 117 | 111 | 129 | 110 | 130 | 138 | 113 | 132 | 133 | 121 | 119 | 123 | 140 |

Table A.5 – Demand of medication in Rawls and Turnquist

| | ***Centers*** | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** |
| ***Scenario 1*** | 26 | 33 | 34 | 44 | 12 | 38 | 14 | 26 | 24 | 2 | 38 | 24 | 45 | 0 | 49 | 8 | 49 | 5 | 3 | 21 |
| ***Scenario 2*** | 89 | 79 | 59 | 75 | 63 | 95 | 52 | 90 | 84 | 80 | 73 | 77 | 56 | 54 | 57 | 100 | 99 | 71 | 88 | 90 |
| ***Scenario 3*** | 148 | 108 | 121 | 107 | 105 | 107 | 119 | 123 | 147 | 128 | 145 | 108 | 131 | 121 | 144 | 114 | 148 | 140 | 134 | 136 |

Table A.6 – Shortest path distance between nodes $d_{jk}$ $(in\ Km)$

| | | ***Centers and Sites*** | | | | | | | | | | | | | | | | | | | | ***LSA's*** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** | ***9*** | ***10*** | ***11*** | ***12*** | ***13*** | ***14*** | ***15*** | ***16*** | ***17*** | ***18*** | ***19*** | ***20*** | ***1*** | ***2*** | ***3*** | ***4*** |
| ***Centers and Sites*** | ***1*** | 0 | 1.0 | 3.4 | 1.0 | 5.2 | 4.4 | 2.4 | 4.4 | 5.4 | 5.2 | 3.8 | 3.8 | 4.7 | 4.5 | 3.4 | 2.2 | 1.4 | 3.4 | 2.4 | 2.4 | 5.4 | 7.5 | 5.8 | 2.8 |
| | ***2*** | 1.0 | 0 | 2.4 | 1.4 | 4.2 | 4.0 | 1.4 | 5.1 | 4.4 | 4.8 | 2.8 | 3.4 | 4.4 | 3.7 | 2.4 | 1.4 | 2.4 | 3.0 | 2.0 | 2.8 | 4.6 | 6.5 | 6.5 | 3.8 |
| | ***3*** | 3.4 | 2.4 | 0 | 3.8 | 2.4 | 3.8 | 1.0 | 5.8 | 3.4 | 3.0 | 1.0 | 2.4 | 4.8 | 1.4 | 2.0 | 2.2 | 4.8 | 3.4 | 2.4 | 4.4 | 3.6 | 4.6 | 7.2 | 6.2 |
| | ***4*** | 1.0 | 1.4 | 3.8 | 0 | 4.8 | 3.4 | 2.8 | 3.4 | 3.8 | 4.2 | 3.4 | 2.8 | 3.7 | 5.1 | 2.4 | 2.8 | 1.0 | 2.4 | 1.4 | 1.4 | 6.0 | 6.5 | 4.8 | 3.8 |
| | ***5*** | 5.2 | 4.2 | 2.4 | 4.8 | 0 | 3.4 | 2.8 | 5.4 | 2.4 | 1.4 | 1.4 | 2.8 | 4.4 | 3.0 | 2.4 | 4.7 | 5.8 | 3.8 | 3.4 | 5.4 | 5.2 | 2.2 | 6.8 | 8.1 |
| | ***6*** | 4.4 | 4.0 | 3.8 | 3.4 | 3.4 | 0 | 3.4 | 2.0 | 1.0 | 2.0 | 2.8 | 1.4 | 1.0 | 5.1 | 2.4 | 5.4 | 4.2 | 1.0 | 2.0 | 3.2 | 7.3 | 4.2 | 3.4 | 7.2 |
| | ***7*** | 2.4 | 1.4 | 1.0 | 2.8 | 2.8 | 3.4 | 0 | 4.4 | 3.0 | 3.4 | 1.4 | 2.0 | 3.8 | 2.4 | 1.0 | 2.0 | 3.8 | 2.4 | 1.4 | 3.4 | 4.6 | 5.1 | 5.8 | 5.2 |
| | ***8*** | 4.4 | 5.1 | 5.8 | 3.4 | 5.4 | 2.0 | 4.4 | 0 | 3.0 | 4.0 | 4.8 | 3.4 | 1.0 | 7.1 | 4.4 | 6.2 | 3.0 | 2.4 | 3.4 | 2.0 | 9.3 | 6.2 | 1.4 | 6.2 |
| | ***9*** | 5.4 | 4.4 | 3.4 | 3.8 | 2.4 | 1.0 | 3.0 | 3.0 | 0 | 1.0 | 2.4 | 1.0 | 2.0 | 4.7 | 2.0 | 5.0 | 4.8 | 2.0 | 2.4 | 4.2 | 6.9 | 3.2 | 4.4 | 8.0 |
| | ***10*** | 5.2 | 4.8 | 3.0 | 4.2 | 1.4 | 2.0 | 3.4 | 4.0 | 1.0 | 0 | 2.0 | 1.4 | 3.0 | 4.2 | 2.4 | 5.2 | 5.2 | 2.4 | 2.8 | 4.7 | 6.5 | 2.2 | 5.4 | 8.1 |
| | ***11*** | 3.8 | 2.8 | 1.0 | 3.4 | 1.4 | 2.8 | 1.4 | 4.8 | 2.4 | 2.0 | 0 | 1.4 | 3.8 | 2.2 | 1.0 | 3.2 | 4.4 | 2.4 | 2.0 | 4.0 | 4.5 | 3.6 | 6.2 | 6.6 |
| | ***12*** | 3.8 | 3.4 | 2.4 | 2.8 | 2.8 | 1.4 | 2.0 | 3.4 | 1.0 | 1.4 | 1.4 | 0 | 2.4 | 3.7 | 1.0 | 4.0 | 3.8 | 1.0 | 1.4 | 3.2 | 5.9 | 3.6 | 4.8 | 6.6 |
| | ***13*** | 4.7 | 4.4 | 4.8 | 3.7 | 4.4 | 1.0 | 3.8 | 1.0 | 2.0 | 3.0 | 3.8 | 2.4 | 0 | 6.1 | 3.4 | 5.8 | 3.2 | 1.4 | 2.4 | 2.2 | 8.3 | 5.2 | 2.4 | 6.4 |
| | ***14*** | 4.5 | 3.7 | 1.4 | 5.1 | 3.0 | 5.1 | 2.4 | 7.1 | 4.7 | 4.2 | 2.2 | 3.7 | 6.1 | 0 | 3.8 | 3.2 | 5.9 | 3.7 | 3.8 | 5.8 | 2.2 | 5.2 | 8.5 | 7.3 |
| | ***15*** | 3.4 | 2.4 | 2.0 | 2.4 | 2.4 | 2.4 | 1.0 | 4.4 | 2.0 | 2.4 | 1.0 | 1.0 | 3.4 | 3.8 | 0 | 3.0 | 3.4 | 2.0 | 1.0 | 3.0 | 6.1 | 4.6 | 5.8 | 6.2 |
| | ***16*** | 2.2 | 1.4 | 2.2 | 2.8 | 4.7 | 5.4 | 2.0 | 6.2 | 5.0 | 5.2 | 3.2 | 4.0 | 5.8 | 3.2 | 3.0 | 0 | 3.7 | 4.4 | 3.4 | 4.2 | 3.2 | 6.9 | 7.6 | 4.1 |
| | ***17*** | 1.4 | 2.4 | 4.8 | 1.0 | 5.8 | 4.2 | 3.8 | 3.0 | 4.8 | 5.2 | 4.4 | 3.8 | 3.2 | 5.9 | 3.4 | 3.7 | 0 | 3.2 | 2.4 | 1.0 | 6.8 | 7.5 | 4.4 | 3.2 |
| | ***18*** | 3.4 | 3.0 | 3.4 | 2.4 | 3.8 | 1.0 | 2.4 | 2.4 | 2.0 | 2.4 | 2.4 | 1.0 | 1.4 | 3.7 | 2.0 | 4.4 | 3.2 | 0 | 1.0 | 2.2 | 5.9 | 4.6 | 3.8 | 6.2 |
| | ***19*** | 2.4 | 2.0 | 2.4 | 1.4 | 3.4 | 2.0 | 1.4 | 3.4 | 2.4 | 2.8 | 2.0 | 1.4 | 2.4 | 3.8 | 1.0 | 3.4 | 2.4 | 1.0 | 0 | 2.0 | 6.1 | 5.1 | 4.8 | 5.2 |
| | ***20*** | 2.4 | 2.8 | 4.4 | 1.4 | 5.4 | 3.2 | 3.4 | 2.0 | 4.2 | 4.7 | 4.0 | 3.2 | 2.2 | 5.8 | 3.0 | 4.2 | 1.0 | 2.2 | 2.0 | 0 | 7.4 | 6.9 | 3.4 | 4.2 |
| ***LSA's*** | ***1*** | 5.4 | 4.6 | 3.6 | 6.0 | 5.2 | 7.3 | 4.6 | 9.3 | 6.9 | 6.5 | 4.5 | 5.9 | 8.3 | 2.2 | 6.1 | 3.2 | 6.8 | 5.9 | 6.1 | 7.4 | 0 | 8.1 | 10.7 | 7.3 |
| | ***2*** | 7.5 | 6.5 | 4.6 | 6.5 | 2.2 | 4.2 | 5.1 | 6.2 | 3.2 | 2.2 | 3.6 | 3.6 | 5.2 | 5.2 | 4.6 | 6.9 | 7.5 | 4.6 | 5.1 | 6.9 | 8.1 | 0 | 9.1 | 10.9 |
| | ***3*** | 5.8 | 6.5 | 7.2 | 4.8 | 6.8 | 3.4 | 5.8 | 1.4 | 4.4 | 5.4 | 6.2 | 4.8 | 2.4 | 8.5 | 5.8 | 7.6 | 4.4 | 3.8 | 4.8 | 3.4 | 10.7 | 9.1 | 0 | 9.0 |
| | ***4*** | 2.8 | 3.8 | 6.2 | 3.8 | 8.1 | 7.2 | 5.2 | 6.2 | 8.0 | 8.1 | 6.6 | 6.6 | 6.4 | 7.3 | 6.2 | 4.1 | 3.2 | 6.2 | 5.2 | 4.2 | 7.3 | 10.9 | 9.0 | 0 |

Table A.7 – Distance matrix $d_j^c$ for $p$-median fortification model (*in Km*)

| | | Centers | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| **Proximity (c)** | **1** | 1 | 1.41 | 0 | 0 | 2.41 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1.41 | 1 | 2.24 | 1 | 1 | 0 | 1.41 |
| | **2** | 1.42 | 1.59 | 0.72 | 0.42 | 2.41 | 1 | 1.12 | 1.60 | 0.60 | 1.55 | 1.30 | 1.12 | 0.60 | 2.13 | 1.30 | 2.41 | 1.42 | 1.12 | 0.42 | 1.59 |
| | **3** | 1.51 | 1.62 | 0.81 | 0.62 | 2.50 | 1.09 | 1.25 | 1.64 | 0.64 | 1.56 | 1.34 | 1.21 | 0.64 | 2.21 | 1.30 | 2.47 | 1.50 | 1.18 | 0.51 | 1.61 |
| | **4** | 1.55 | 1.68 | 0.82 | 0.63 | 2.53 | 1.13 | 1.25 | 1.64 | 0.66 | 1.56 | 1.36 | 1.21 | 0.67 | 2.25 | 1.31 | 2.51 | 1.54 | 1.19 | 0.51 | 1.66 |
| | **5** | 1.55 | 1.68 | 0.83 | 0.63 | 2.53 | 1.13 | 1.26 | 1.66 | 0.67 | 1.57 | 1.37 | 1.22 | 0.68 | 2.26 | 1.32 | 2.52 | 1.54 | 1.20 | 0.51 | 1.66 |

# B. SCENARIOS

Table B.1 – Binary matrix of failure scenarios ($a_{ks}$)

| | | Numbers of simultaneous failures | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Zero | One | | | | | Two | | | | | | | | | | Three | | | | | | Four | | |
| Scenario number | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
| Sites | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| | 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| | 5 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 6 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | 7 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| | 8 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | 9 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 10 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 11 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| | 12 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| | 13 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | 14 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| | 15 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| | 16 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| | 17 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| | 18 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| | 19 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| | 20 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |