Accepted Manuscript

Title: External fixator configurations in tibia fractures: 1D optimization and 3D analysis comparison

Author: M. Luis Roseiro Augusta Neto Ana Amaro Rogerio P. leal Miguel C. Samarra

PII: S0169-2607(13)00325-8
DOI: http://dx.doi.org/doi:10.1016/j.cmpb.2013.09.018
Reference: COMM 3674

To appear in: *Computer Methods and Programs in Biomedicine*

Received date: 26-4-2012
Revised date: 11-9-2013
Accepted date: 23-9-2013

Please cite this article as: M.L. Roseiro, A. Neto, A. Amaro, R.P. leal, M.C. Samarra, External fixator configurations in tibia fractures: 1D optimization and 3D analysis comparison, *Computer Methods and Programs in Biomedicine* (2013), http://dx.doi.org/10.1016/j.cmpb.2013.09.018

External fixator configurations in tibia fractures: 1D optimization and 3D analysis comparison

Luis Roseiro [1], *M. Augusta Neto [2], Ana Amaro[3], Rogerio P. leal[4], Miguel C. Samarra[5]

[1] Departamento de Engenharia Mecânica - Instituto Superior de Engenharia de Coimbra, Rua Pedro Nunes- Quinta da Nora, 3030-199, Coimbra, Portugal.
e-mail: lroseiro@isec.pt

[2,3,4,5] Departamento de Engenharia Mecânica - Faculdade de Ciência e Tecnologia da Universidade de Coimbra (Polo II), Rua Luis Reis Santos, 3020-788 Coimbra, Portugal.
e-mail: augusta.neto@dem.uc.pt[2]; ana.amaro@dem.uc.pt; rogerio.leal@dem.uc.pt[4]; msc_samarra@hotmail.com[5]

Abstract *The use of external fixation devices in orthopedic surgery is very common in open tibial fractures. A properly applied fixator may improve the healing process while one improperly applied might delay the healing process. The several external fixator systems used in clinical today, can be categorized into uniplanar-unilateral, uniplanar-bilateral, biplanar and multiplanar. The stability on the fracture focus and, therefore, the fracture healing process, is related with the type of external fixator configuration that is selected. The aim of this study is to discuss the principles for the successful application of unilateral-uniplanar external fixation, the assembly of its components, for the case of a transverse fractures using computational models. In this context, the fixation stiffness characteristics are evaluated using a simplified 1D finite element model for the tibia and external fixator. The beams are modeled with realistic cross-sectional geometry and material properties instead of a simplified model. The VABS (the Variational Asymptotic Beam Section analysis) methodology is used to compute the cross-sectional model for the generalized Timoshenko model, which was embedded in the finite element solver FEAP. The use of Timoshenko beam theory allows accounting for several kinds of loads, including torsion moments. Optimal design is performed with respect to the assembly of fixator components using a genetic algorithm. The optimization procedure is based on the evaluation of an objective function, which is dependent on the displacement at the fracture focus. The initial and optimal results are compared by performing a 3D analysis, for which different three-dimensional finite element models are created. The geometrical model of a tibia is created on the basis of data acquired by CAT scan, made for a healthy tibia of a 22 year old male. The 3D comparison of the 1D optimal results show a clear improvement on the objective function for the several load cases and, therefore, it is shown that appropriate selection of the external fixator geometrical features can lead to an improvement on the stability of the external fixator. The results obtained show that the optimal position of the side beam and the first pin should be as close as possible to the bone interface and as close as possible to the fracture focus, respectively. Concerning the second pin, it should be placed away from the first pin in case of flexion loads, to axial and torsion loads the second pin should be placed near the first pin.*

* Corresponding author. augusta.neto@dem.uc.pt; Tel: +351 239 790 700; Fax +351 239 790 701

Keywords: Optimization; fracture tibia; Finite element method; external fixator; 3D analysis

1 Introduction

The bone fracture can be defined as a situation in which there is loss of bone continuity in addition to bone separation into several fragments. Some kind of fractures can be treated without surgery. In fact, on an everyday basis bones will support many kinds of forces naturally applied to them, specially the adolescent bones. Sometimes a jump or a blow on the adolescent bone may leads to an incomplete fracture (greenstick) or a separation of bone lamellae, which in turn can be responsible for a pain in the injured area. Naturally, fathers take their children to medical doctor and they will diagnose the problem. But, because healthy children heal quickly, many of these fractures can be treated with medical care only, and they heal in as little as one month. Younger children have greater potential for remodelling with growth. Fractures with angulation in the same direction as joint motion (bending and straightening) also have greater potential to remodel [1]. Nevertheless, due to the implications for citizens, fracture treatments have and will continue to have enormous socioeconomic consequences. In fact, although the curing process of a fracture is a phenomenon with biological features, it is directly related to the medical treatment carried out, namely how rapid the bone can heal and return to normality. Moreover, the pattern of healing of tibial fractures is profoundly influenced by the magnitude and distribution of mechanical stresses within the fracture bridging tissues, collectively referred as callus [2, 3]. It is the bone callus that re-establishes integrity, continuity and stiffness of the bone member, enabling a return to normality [4, 5]. Usually, fixation of the bone or fixation of bone fragments is the most suitable technique to ensure necessary stability for treatment of fractures, which can be done internally or through external fixation systems. The choice of stability methodology of fractures depends on the type of fracture and on the means available for treatment.

The external fixtures are systems that combine advantages of low cost and easy application, with the possibility to access soft tissues [6]. These systems consist of an assembly of several mechanical components so as to maintain stability and stiffness of the bone structure. The fixators are attached to the bone with linking pins. The structure and function of each external fixator depends, essentially, on the shape of its components. There are several types of external fixators and each one can be used in a certain type of fracture. The most complex kind of external fixator allows for a multiplanar fixation with any configuration, which allows it application in almost all clinical situations. However, these have high costs and limited access to soft tissues.

From the biomechanical point of view, it has been demonstrated that in the externally fixated tibial fracture, the axial load is shared by the fracture callus and the support device in proportion to the relative stiffness of the fixator and the callus [3, 7]. Therefore, the healing of bone is sensitive to the mechanical stability of fixators, which depends on the material and geometric characteristics of its components [8] and on the geometric configurations [9, 10]. A simple and versatile fixator that is well known, is the tubular system of type AO [11]. It is al-

also the widely used and it can be found in Ramos *et al.* [12]. This system is composed of pins, side beams and elements that assure the connection between pins and side beams. According the surgeons' criterion, the pins installation can vary in distance and inclination thus enabling the assembly of different configurations. There are various hypotheses about the configuration of these types of fixators: uniplanar and unilateral, uniplanar and bilateral, biplanar or multiplanar. The degree of stability of the focus of fracture is directly related to the type of configuration used.

In a study carried out on 45 patients with diaphysis fracture treated with unilateral external fixators, Kershaw *et al.* [13] related the weight discharge on the fractured tibia to the existence of micro-movements on the focus of the fracture. It was observed that the existence of micro-movements on the fracture focus lead to a significant reduction in the time to healing. Later, in a study done by Emami et al. [14] 68 patients were treated with unilateral external fixators fractures and, they conclude that the failure results were probably due to weight-bearing being too high in these patients relative to the mechanical stability provided by the external fixator system. The need of surgical revisions on 25 patients has disclosed the interest in the study of resistance and stiffness of this kind of systems. Epari *et al.* [15] observed the existence of a relation between the stability of fixation and the resistance and stiffness of the bone callus on the fracture focus that is formed after 9 weeks. Moreover, they conclude that moderate levels of axial stability were associated with the highest callus strength and stiffness and that optimizing axial stability and limiting shear instability appear to be important for creating conditions for timely fracture-healing. Therefore, the use of a software program enabling the selection of the optimum position of fixator components can be view as an additional tool of surgeons.

With regard to the growing potential of finite-element-analysis (FEA) in the field of orthopedic biomechanics [16-18], this study aims to contribute for the development of this kind of tools. Using a commercial model of an AO external fixator in the uniplanar-unilateral configuration and a side beam, the optimum position of the fixator mechanical components are obtained for the case of transverse fractures of the tibial diaphysis. The 3D tibia geometry is obtained from a human tibia using computerized axial tomography (CAT Scan). This 3D model is used to create a simplified 1D model of tibia, in which the natural frequencies and vibration modes of both geometrical models are compared [19, 20]. In fact, when a structure is excited its behavior is largely controlled by a set of preferable vibration modes, which are dependent on the spectral content of the excitation [21]. Moreover, assuming that the lower order modes have higher contribution to the global response of a system, often in structural dynamic analysis the structural components are described by a sum of selected modes of vibration [22]. Thus, the displacement field of a structural component can be spanned by a selected number of vibration modes, meaning that the structural global behavior of component is accounted with a smaller number of degrees of freedom [23]. Using this concept, is expected that if the lower order natural frequencies of both models are similar, the structural global behavior of both models also will be. Therefore, using this idea, the tibia-fixation stiffness characteristics are evaluated using a simplified 1D finite element model for the tibia and external fixator. The beams are modeled with realistic cross-sectional geometry and material properties instead of a simplified model. The VABS (the Variational Asymptotic Beam Section analysis) methodology is used to compute the cross-sectional model for a generalized Timoshenko model, which was embedded in the finite element solver FEAP [24, 25]. The use of Timoshenko beam theory allows accounting for several kinds of loads, including torsion moments. Optimal design is performed with respect to the assembly of fixator components using a genetic algorithm. The optimization procedure is based on the evaluation of an objective function, which is dependent on the displacement at the fracture focus. The initial and optimal results are compared by performing a 3D finite element analysis, for which different three-dimensional finite element models are created. The optimal 3D results show a clear improvement of the objective function for the several load cases and, therefore, it is shown that appropriate selection of the external fixator geometrical features can lead to an improvement on the stability of the external fixator.

2 Modal analyse: 3D and 1D models of tibia

The tibia used in this study was obtained from a CAT scan made for a healthy 22 year old male. From the CAT scan, the 3D geometrical model of the tibia was obtained, which is illustrated in figure 1(a). The tibia transfers loads between the foot and the femur when a human being stands or walks. Like other bones, its exterior is composed of cortical bone and its core is composed of trabecular bone. The cortical bones are made of hard, dense tissues and take charge of load transfers like bending and compression. The trabecular bones are made of sparse, foamlike tissue to reduce structural mass [26]. In the 3D geometrical model, the cortical and trabecular components of the tibia have been separated. Nevertheless, due to the complexity of the tibial bone and its differences from human to human, a methodology with a simplified model with an equivalent behavior was chosen. In fact, the development of a 3D FEM model of a human tibia is a relatively complex task that can only be achieved with specific skills and experience of using finite element analysis programs. Thus, an optimization methodology based on 1D simplified models for which the 1D mechanical behavior is equivalent to 3D models, can facilitate the implementation of this technique by surgeons. Moreover, the development of such tools based on simplified models of a human tibia should facility the creation of numerical models by specifying only a set of parametric variables.

The simplified model was updated such that the results from the 1D and the 3D finite element modal analysis were similar, i.e. the update scheme consists in selecting the 2D cross-section of the tibia, which is used to create the 1D numerical model, that allows to approximate better the first five natural frequencies that were obtained from a 3D modal analysis of the full tibia. In this context, the modal analysis of the 3D tibia was carried out in the NASTRAN/PATRAN [27] program, using a finite element model created with 10 node tetrahedral elements, for a total of 57674 elements and 90500 nodes. The convergence of the finite element results was confirmed by increasing mesh density. The 3D finite element mesh is illustrated in figure 1(b). The cortical bone has been considered with elasticity modulus E=17 GPa and Poisson coefficient ! =0.3. The trabecular bone has been considered with elasticity modulus E=7 GPa and Poisson coefficient ! =0.2. The specific mass of bone is considered with 1800 kg/m^3.

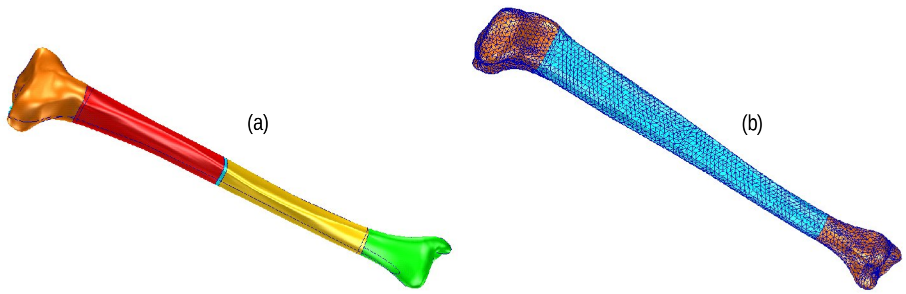

Fig. 1. 3D model of the tibia a) and tibia finite element mesh b)

The 1D finite element model of tibia is created assuming a constant cross-section defined from the middle level of the tibia diaphysis and is presented on figure 2. In this figure the blue area is occupied by trabecular bone while the gray one represents the area engaged by the cortical bone. The 1D finite element model is created with 55 quadratic finite elements following the Timoshenko beam theory [24]. This finite element has three nodes and six degrees of freedom per node, three displacements and three rotational degrees of freedom. The natural frequencies of both finite element models are obtained assuming a rigid fixation on the tibia surfaces that are in contact with the knee, according to Kim *et al.* [26].

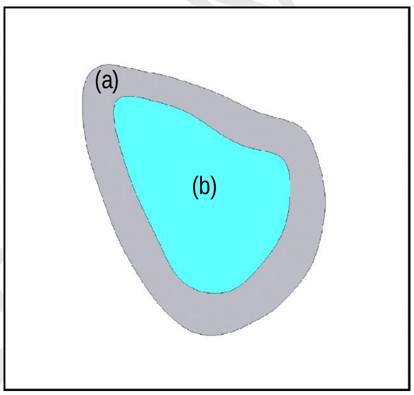

Fig. 2. Cross section geometry used at 1D finite element model of the human tibia: a) cortical bone; b) cancellous bone.

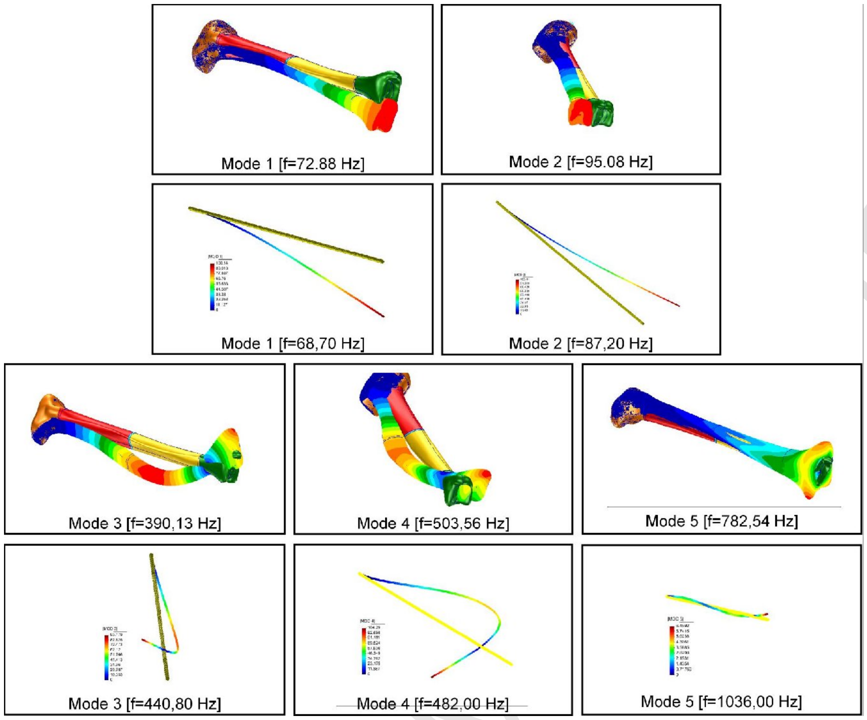

Fig. 3. Natural frequencies and associated vibration modes for 1D and 3D finite element models

Figure 3 shows the natural frequencies values and the vibration modes to the 3D model and to the simplified 1D model. The results obtained show a good agreement between models. Nevertheless, the 3D finite element model shows a stiff behavior in all natural frequencies except on the third and fifth frequencies, which are associated with the second flexion vibration mode in one flexion plan and the torsion vibration mode, respectively. Therefore, in static analysis is expected that the 3D finite element model will give smaller displacements than the 1D finite element model.

3 Optimization procedure

For the purpose of this study a transverse fracture (90 °) in the central tibia diaphysis is considered. According to Wong *et al.* [28], the model of the tibia can be considered with rigid fixation in the contact area with the knee, and free in the foot connection, where the loads are applied in a region of 20% equivalent to the bottom surface of the tibia.

The beam model of tibia is considered with the fracture gap simulated with a 4mm opening area and a symmetrical distribution of components in relation to the fracture focus. The tibia analyzed had a length of 366 mm and a constant section showed at figure 2. The fixator components are assumed to have a symmetrical distribution with respect to the fracture focus.

Figure 4 illustrates the simplified model utilized in the optimization procedure. The mechanical and geometrical properties of the external fixator components are also shown in figure 4.

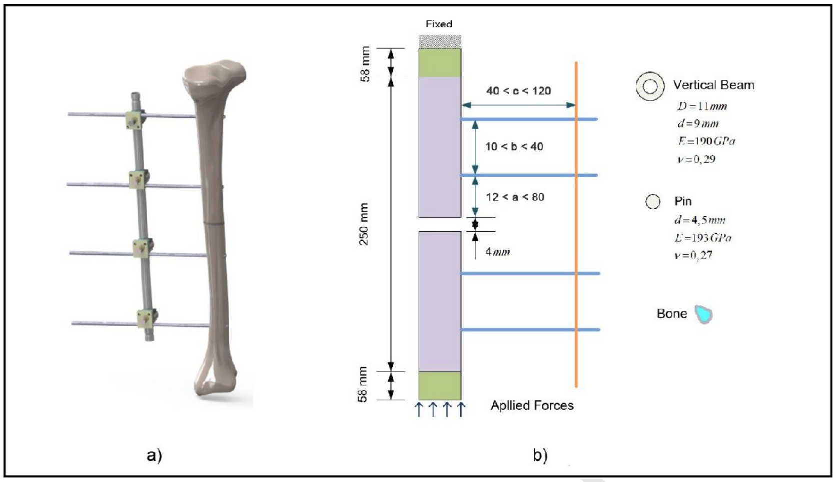

Fig. 4. Tibia – external fixator models: a) 3D model b) simplified model

The finite element model of the simplified model is defined using quadratic Timoshenko beam elements with 6 degrees of freedom per node [24] and a total number of 1101 finite elements. This number of finite elements enables a distance between nodes of 0.5 mm, which is important in the optimization procedure. This finite element is integrated in the FEAP library [29]. The finite element methodology developed has enabled us to obtain the properties associated with the numerical modeling of beams with arbitrary material and cross-section geometry, in which the element stiffness is computed on the bases of the variational asymptotic method presented by Yu *et al.* [30]. Therefore, the beam constitutive relation is computed by VABS routines library which were already integrated in the FEAP program by authors at previous work [12]. Thus, FEAP program is used to carry out static and modal analysis of 1D finite element models presented in this work. Moreover, during the optimization procedure, the objective function is evaluated at each genetic solution using FEAP program, see table 3. So, the initial configuration of tibia-external fixator needs to be changed in order to accounts the design variables a, b and c that were defined by the genetic algorithm, which is achieved by changing the connectivity of the finite elements.

The optimization procedure is repeated five times in order to obtain the optimal configuration for five situations (load cases). The design variables are defined on figure 4 and are denoted by letters *a*, *b* and *c*. The lower and upper limit constraints of each design variable are also presented on figure 4. The distance of the side bar from bone is determined by the depth of soft tissue [31]. Bringing the side bar closer to the bone improves stability and in general it should be kept as close as possible with enough space to facilitate pin site care, the distance of 40 to 50 mm from bone surface is advised [10]. For the case of design variable *a* special care must be taken. In fact, the proximity of any pin to the fracture itself is cautioned as the pin may be within the fracture hematoma and thereby carry the risk of a pin site infection spreading to within the fracture [31]. Giotakis and Narayan suggest that it should keep at least to 20 mm from the nearest fracture line. In this work the minimum value is set to 10 mm.

To all load cases the initial configuration is always the same and, the following positions of the pins and side beam were considered: a=15 mm, b=43 mm and c=50 mm. The value of the forces involved in each load case is presented in table 1. The first three cases consider the axial force together with a bending load on the fixator plane. The fourth case is the same as the third case but with an additional bending force on the perpendicular plane to the fixator. The fifth case only considers torsion.

Table 1. Load cases used within the optimization procedure

Load Type	Load Case				
	A	B	C	D	E
Axial [N]	100	-	100	100	-
Bending force in the fixator plane [N]	-	100	100	100	-
Bending force perpendicular to the fixator plane [N]	-	-	-	5	-
Torsion [N.mm]	-	-	-	-	100

The optimal position of the mechanical components on the external fixator is obtained from a real-coded genetic algorithm [32], in which the objective function is the minimization of the norm of the resulting displacement in the fracture focus according to

$$F_{obj} = \min \sqrt{u_x^2 + u_y^2 + u_z^2}$$

where u_x, u_y and u_z represents the displacement in the x, y and z directions, respectively.

Table 2. Genetic Parameters and Operators

Type	Value
Initial Population	100
Iterations	20
Elitism	10 %
Crossover Probability	75 %
Mutation Probability	2,5 %
Doping Roullet	20 %

The considered settings and genetic operators used within the genetic algorithm are presented in table 2. Genetic algorithms (Gas) are search and optimization techniques inspired by Darwin's theory of natural evolution [33] who’s generally deviate with better effectiveness from local optima, so tending to work better than the traditional optimization algorithms based on the sensitivity analysis. Thus, The genetic algorithm (GA) is a selective random search algorithm designed to improve the probability of achieving the global optimum within a large space of solutions, as proposed by Holland [34]. The initial solutions (population) of the GA algorithm are usually randomly generated. Then, for all the individuals called chromosomes and representing a solution to the problem, the objective function is calculated. This objective function measures the robustness of each solution. Therefore an objective-dependent technique is used to select the parents for the next population from the current one, setting them in the mating pool. This acts as a natural selection process where the strongest individuals have more probability of leaving their genetic information to the next generation.

Table 3. Diagram of the genetic algorithm used

1. Generate the initial population
2. Start GA iterations n = 1 to *nmax*:
2.1. Evaluate the objective function for all solutions, using the FEAP program
2.2. Select solutions to the mating pool
2.3. Combine solutions by crossover technique
2.4. Apply the mutation operator
2.5. Apply elitism strategy
3. Stop iteration process if *nmax* is reached or return to step 2

According to the fitness values, the next generation of solutions is obtained by a crossover technique from the individuals in the mating pool. In this study a single point crossover is used to perform this task. In order to ensure genetic diversity, a mutation operator is used. This is related to the exploration and exploitation balance that should be present in the convergence of the GA algorithm [35]. A way to increase this ‘‘converging pressure” is to maintain part of the population; this procedure is referred to as an elitism strategy. This algorithm is repeated until the end condition is reached. The diagram of the optimization problem, using the genetic algorithm is shown in table 3.

4 Optimization results

Table 4 summaries the numerical results obtained with the optimization procedure for the five load cases studied. Figure 5 illustrates the distribution of the norm of the resulting displacement for load case A in the initial and optimum positions.

In table 4 is possible to verify that for all load cases the optimum values of design variables *a* and *c* are always the lower limit of their design constraints. These results are in agreement with those presented in the literature. Nevertheless, the optimum value of design variable *b*, which is associated with the position of the second pin, is dependent on the load case. It is interesting to point out that, if the optimum position of the first pin is assumed, the available length in which the second pin may be placed is about 123 mm and, the results presented on table 4 show that in the presence of an axial load or a torsion moment, the optimum value of design variable *b* is about 1/6 of the available length while in the presence of axial and bending loads the optimum value is about 1/4 of the available length. Thus, the near and far rule for which the pins should be spread along a segment of bone such that the segment is spanned [9, 10, 36] is not the optimum solution presented on table 4.

Table 4. Results obtained in the optimization process

Design Variable	Initial De [mm]	Optimum Design [mm]				
		A	B	C	D	E
a	15	11	10	10	10	11
b	43	22	36	31	32	20
c	50	40	41	41	40	40
Displacement decrease at the facture focus [%]		*98.1*	*4.7*	*12.5*	*13.6*	*57.5*

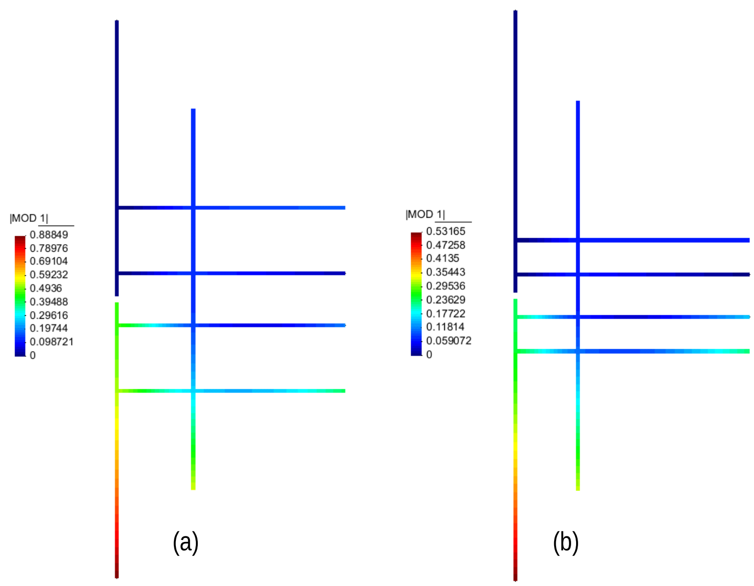

Fig. 5. Magnitude distribution of the resulting displacement for load case A: a) in the initial configuration; b) in the optimum configuration. The maximum magnitude: a) 0.88849; b) 0.53165

The higher improvements on the displacement at the fracture focus are verified to the optimum designs of A and E load cases. Therefore, in order to verify if these improvements are masked by the simplified model of tibia-fixator system used during the optimization procedure, in the next section both load cases will be analyzed using a 3D geometry of the tibia-fixator system at the initial and optimum configurations.

5 Comparison of the 3D Optimum Results

Because the optimization results are obtained using a 1D simplified model of tibia and fixator system, the main improvements obtained with the optimization procedure may need to be confirmed with a more realistic finite element model of the tibia-fixator system. Therefore, a transverse fracture is geometrically created on the 3D geometrical model of the tibia by generating a 4mm gap through the middle section of tibia. Next, the external fixator system composed with four pins and one side bar are assembled to the fractured tibia. The position of each fixator component is defined from the initial design presented at table 4. The pins are placed in the tibia and in the side bar assuming to be completely bonded without any loosening. The plane of the external OA fixator application is showed in figure 6 and is the transversal plane. It is worth highlighting that despite the optimum plan to apply the fixator is the sagittal plane [31], the optimization and analysis results from this study should not change significantly with the adjustment of fixator application plane.

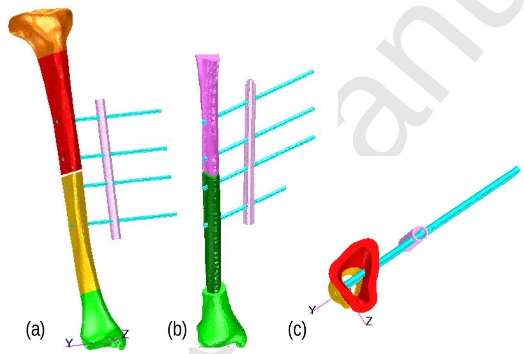

Fig. 6. Tridimensional configuration of the initial tibia-fixator assembling: a) full assembly; b) assembly without knee region and diaphysis without cortical bone; c) plant view of the diaphysis cortical bone.

Boundary conditions for the several static analyses are prescribed at the nodes placed on the colored green surfaces depicted at figure 7 a). Models are constrained in such away that the three nodal degrees of freedom of each node placed on the fixed surfaces are restrained.

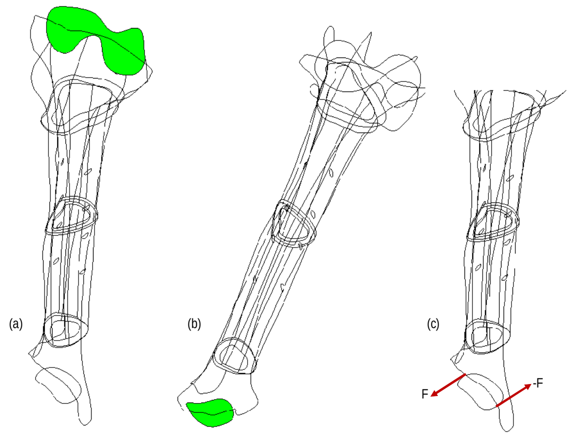

Fig. 7. Geometrical representation: a) the fixed surfaces; b) the axial loaded surfaces; c) points used to create torsion binary.

For the axial load case, the surfaces in which the force is applied are the colored green surfaces depicted at figure 7 b) and, to load case *E*, the torsional moment is created by applying the two opposite forces presented at figure 7 c).

Figure 8 shows the finite element mesh of the assembly. The commercial finite element code PATRAN/NASTRAN was used for the analysis and a 4-node 3D solid element was used for mesh generation with a total number of about 188300 finite elements.

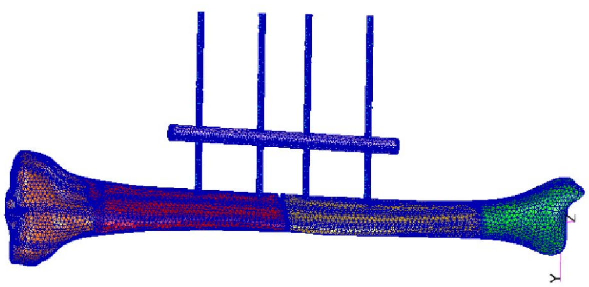

Fig. 8. Mesh generation of the tibia-fixator system for the initial configuration

Figure 9 shows the von Mises stress distribution in compact and trabecular bone of the tibia under axial loading for the initial and optimum configurations. The apparent stress concentration, on both models, is observed in the compact bone that is in contact with the first pin. The maximum von Mises stress in the initial model is of 44.4 MPa and in the optimum model is of 32.9 MPa.

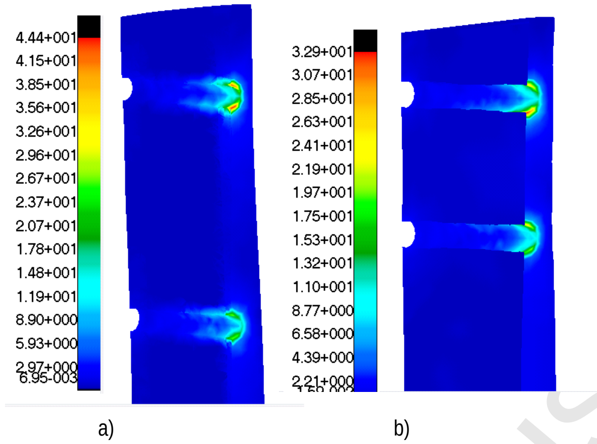

Fig. 9. Von Mises stress distribution in cancellous and trabecular bone of the fragmented tibia that covers the ankle, under the axial load and for: a) initial tibia-fixator configuration; b) optimum tibia-fixator configuration. Maximum von Mises stresses of 44.4 MPa is seen on the initial configuration and 32.9 MPa is seen on the optimum one.

All the data related with the numerical principal stresses evaluated for the initial and optimum configurations at load cases A and E are presented on table 5. A stress with a negative value means that is a compression stress and a positive value means that is a tension stress. In table 4 is possible to see that initial and optimum configuration models have similar levels for the compression stress, with the optimum configuration geometry showing a maximum increase of 30% for the minimum value of first principal stress for the axial load. Nevertheless, the initial configuration shows a clear increase on principal stress levels and on the value of maximum shear stress for both load cases.

Table 5. Peak values of principal stresses at tibia in axial load and torsinal moment for initial and optimum designs

	Initial [MPa]	Optimum [MPa]	Variation [%]*
	Axial load (load case A)		
First principal			
Maximum	+47.5	+29.7	60
Minimum	-2.47	-3.54	-30
Second principal			
Maximum	+8.62	+5.85	47
Minimum	-8.52	-7.16	19
Third principal			
Maximum	+2.31	+2.44	-5
Minimum	-49.1	-38.1	29

Maximum shear	23.6	17.2	37
Von Mises	44.4	32.9	35
Displacement fracture focus [mm]	0.319	0.161	98
Torsional moment (load case E)			
Maximum shear	2.02	1.75	15
Von Mises	1.09	0.941	16
Displacement fracture focus [mm]	0.034	0.0219	55

* Variation is evaluated by the formula: (initial- optimum)/ optimum × 100

Figure 10 shows the distribution of the displacement magnitude at the tibia-fixator system under axial loading for the initial and optimum configurations. It is interesting point out that the variation of the maximum displacement magnitude between both configurations is about 51% while the variation of the displacement magnitude at the fracture focus is about 98 %, which is quite similar to the variation obtained with the 1D simplified model. The higher variation verified on the fracture focus is a consequence of changing the distribution of the displacement magnitude at the optimum configuration.

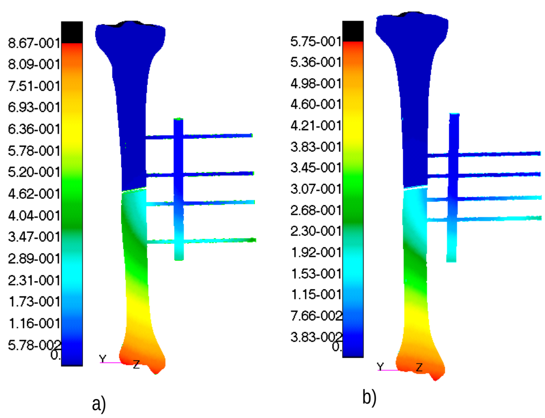

Fig. 10. Distribution of the displacement magnitude at the tibia-fixator system under axial loading for: a) the initial configuration; b) the optimum configuration.

Comparing figures 5 and 10 is possible to observe that in both finite element models, 1D and 3D finite element models, the values of the maximum displacement magnitude have a remarkable similarity.

6 Conclusions

The positions of fixators are usually defined by surgeons based on his/her own experience. Although from a clinical point of view there are several factors that can determine the pin and side bar positions, an optimization model can contribute to the best clinical decision. In this of work a simplified model of an AO external fixator in the uniplanar-unilateral configuration is used to search for the optimum position of its mechanical components for the case of tranverse fractures of the tibial diaphysis. The 3D tibia geometry is obtained from a human tibia using computerized axial tomography (CAT Scan). This 3D model is used to create a simplified 1D model of tibia, in which the natural frequencies and vibration modes of both models are compared. Therefore, the fixation stiffness characteristics are evaluated using a simplified 1D finite element model for the tibia and external fixator. The beams are modeled with realistic cross-sectional geometry and material properties instead of a simplified model. The VABS (the Variational Asymptotic Beam Section analysis) methodology is used to compute the cross-sectional model for a generalized Timoshenko model, which is embedded in the finite element solver FEAP [24]. The use of Timoshenko beam theory allows accounting for several kinds of loads, including torsion moments.

Optimal design is performed with respect to the assembly of fixator components using a genetic algorithm. The optimization procedure is based on the evaluation of an objective function, which is dependent on the displacement at the fracture focus. The initial and optimal results are compared by performing a 3D analysis, for which different three-dimensional finite element models are created, assuming that the pins-bone and pins-bar interfaces are completely bonded. This simplification may lead to an over-estimation of the structural stiffness of the biomechanical system. However, because the same simplification conditions were assumed for all numerical models, this study can be considered as a comparative study. Moreover, MacLeod and co-workers [37] showed that the global load-deformation response is not influenced by the interface modelling approach employed; the deformation varied by less than 1% between different interaction models. However, interface modelling is found to have a considerable impact on the local stress-strain environment within the bone in the vicinity of the screws.

The 1D optimization results show that the optimal position of the side beam it should be as close as possible to the tibia bone and the first pin must be placed as close as possible at the fracture focus. Concerning the second pin, in the presence of an axial load or a torsion moment, the optimum position is about 1/6 of the available length while in the presence of axial and bending loads the optimum position is about 1/4 of the available length. Thus, the near and far rule for which the second pins should be spread along a segment of bone such that the segment is spanned [9, 10] is not the optimum solution presented on this work. Moreover, the 3D results obtained allow us to conclude that it is possible to move in the direction of the development of tools capable of determine the optimum position of fixators, thus serving as a useful aid to the surgeon.

References

[1] ASSH, *Fractures in children*. American Society for Surgery of the Hand, 2012, http://www.assh.org/Public/HandConditions/Pages/Fractures-in-Children.aspx, 2013.

[2] L. E. Claes and C. A. Heigele, Magnitudes of local stress and strain along bony surfaces predict the course and type of fracture healing. *Journal of Biomechanics*, **32** (3), 255-266, 1999.

[3] V. Vijayakumar, L. Marks, A. Bremmer-Smith, J. Hardy and T. Gardner, Load transmission through a healing tibial fracture. *Clinical Biomechanics*, **21** (1), 49-53, 2006.

[4] M. Kunze, A. Schaller, H. Steinke, R. Scholz and C. Voigt, Combined multi-body and finite element investigation of the effect of the seat height on acetabular implant stability during the activity of getting up. *Computer methods and programs in biomedicine*, **105** (2), 175-182, 2012.

[5] J.-T. Hsu, K.-A. Lai, Q. Chen, M. E. Zobitz, H.-L. Huang, K.-N. An and C.-H. Chang, The relation between micromotion and screw fixation in acetabular cup. *Computer methods and programs in biomedicine*, **84** (1), 34-41, 2006.

[6] D. P. Moss and N. C. Tejwani, Biomechanics of external fixation: a review of the literature. *Bulletin of the NYU Hospital for Joint Diseases*, **65** (4), 294-299, 2007.

[7] T. N. Gardner and M. Evans, Relative stiffness, transverse displacement and dynamization in comparable external fixators. *Clinical Biomechanics*, **7** (4), 231-239, 1992.

[8] V. L. Caja, W. Kim, S. Larsson and E. Y. S. Chao, Comparison of the mechanical performance of three types of external fixators: linear, circular and hybrid. *Clinical Biomechanics*, **10** (8), 401-406, 1995.

[9] B. T. Briggs and E. Y. S. Chao, The mechanical performance of the standard Hoffmann-Vidal external fixation apparatus. *Journal of Bone and Joint Surgery*, **64** (4), 566-573, 1982.

[10] R. Huiskes and E. Y. S. Chao, Guidelines for external fixation frame rigidity and stresses. *journal of Orthopaedic Research*, **4** (1), 68-75, 1986.

[11] A. T. Fragomen and S. R. Rozbruch, The mechanics of external fixation. *HSS J.*, **3** (1), 2007.

[12] L. Ramos, I. Rotbande, I. Shehata and I. Knackfuss, Contribuição ao estudo mecânico do fixador externo tubular AO. *Rev Bras Ortop*, **34** (2), 134-138, 1999.

[13] C. J. Kershaw, J. L. Cunningham and J. Kenwright, Tibial external fixation, weight bearing, and fracture movement. *clinical Orthopedic Related Research*, **293** 28-36, 1993.

[14] A. Emami, B. Mjoberg, G. Karlstrom and S. Larsson, Treatment of closed tibial shaft fractures with unilateral external fixation. *Injury*, **26** (5), 299-303, 1995.

[15] D. Epari, J. Kassi, H. Schell and G. Duda, Timely fracture-healing requires optimization of axial fixation stability. *The Journal of Bone and Joint Surgery*, **89** (7), 1575-1585, 2007.

[16] S. K. Easley, S. Pal, P. R. Tomaszewski, A. J. Petrella, P. J. Rullkoetter and P. J. Laz, Finite element-based probabilistic analysis tool for orthopaedic applications. *Computer methods and programs in biomedicine*, **85** (1), 32-40, 2007.

[17] C.-C. Hsu, J. Lin and C.-K. Chao, Comparison of multiple linear regression and artificial neural network in developing the objective functions of the orthopaedic screws. *Computer methods and programs in biomedicine*, **104** (3), 341-348, 2011.

[18] D. Kluess, R. Souffrant, W. Mittelmeier, A. Wree, K.-P. Schmitz and R. Bader, A convenient approach for finite-element-analyses of orthopaedic implants in bone contact: Modeling and experimental validation. *Computer methods and programs in biomedicine*, **95** (1), 23-30, 2009.

[19] J. A. Loya, L. Rubio and J. Fernández-Sáez, Natural frequencies for bending vibrations of Timoshenko cracked beams. *Journal of Sound and Vibration*, **290** (3â€“5), 640-653, 2006.

[20] M. R. Mitchell, R. E. Link, F. Antunes, A. Ramalho, J. A. M. Ferreira, C. Capela and P. Reis, Determination of Elastic Properties by Resonant Technique: A Sensitivity Analysis. *Journal of Testing and Evaluation*, **36** (1), 2008.

[21] M. Neto, J. C. Ambrósio, L. M. Roseiro, A. Amaro and C. M. A. Vasques, Active vibration control of spatial flexible multibody systems. *Multibody System Dynamics*, **30** (1), 13-35, 2013.

[22] M. A. Neto, J. A. C. Ambrósio and R. P. Leal, Composite materials in flexible multibody systems. *Computer Methods in Applied Mechanics and Engineering*, **195** (50â€“51), 6860-6873, 2006.

[23] J. Cuadrado, J. Cardenal and E. Bayo, Modeling and Solution Methods for Efficient Real-Time Simulation of Multibody Dynamics. *Multibody System Dynamics*, **1** (3), 259-280, 1997.

[24] M. A. Neto, W. Yu and R. P. Leal, Generalized Timoshenko modelling of composite beam structures: sensitivity analysis and optimal design. *Engineering Optimization*, **40** (10), 891-906, 2008.
[25] M. A. Neto, W. Yu and S. Roy, Two finite elements for general composite beams with piezoelectric actuators and sensors. *Finite Elements in Analysis and Design*, **45** (5), 295-304, 2009.
[26] S.-H. Kim, S.-H. Chang and H.-J. Jung, The finite element analysis of a fractured tibia applied by composite bone plates considering contact conditions and time-varying properties of curing tissues. *Composite Structures*, **92** (9), 2109-2118, 2010.
[28] C. Wong, P. Mikkelsen, L. B. Hansen, T. Darvann and P. Gebuhr, Finite element analysis of tibial fractures. *Dan Med Bull*, **57** (5), 2010.
[29] R. Taylor, *FEAP, A finite element analysis program*. California, USA, 2000.
[30] W. Yu, D. H. Hodges, V. Volovoi and C. E. S. Cesnik, On Timoshenko-like modeling of initially curved and twisted composite beams. *International Journal of Solids and Structures*, **39** (19), 5101-5121, 2002.
[31] N. Giotakis and B. Narayan, Stability with unilateral external fixation in the tibia. *Strat Traum Limb Recon*, **2** 13-20, 2007.
[32] B. Chaparro, S. Thuillier, L. Menezes, P. Manach and J. Fernandes, Material parameters identifica-tion: Gradient-based, genetic and hybrid optimization algorithms. *Computational Material Sci-ence*, **44** (2), 339-346, 2008.
[33] D. Goldberg, *Genetic Algorithms in search, optimization and machine learning*. Addison-Wesley, Reading, M.A., 1989.
[36] O. O. A. Oni, M. Capper and C. Soutis, A finite element analysis of the effect of pin distribution on the rigidity of a unilateral external fixation system. *Injury*, **24** (8), 525-527, 1993.
[37] A. R. MacLeod, P. Pankaj and A. H. R. W. Simpson, Does screw-bone interface modelling matter in finite element analyses? *Journal of Biomechanics*, **45** (9), 1712-1716, 2012.