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A Bi-objective Modeling Approach Applied to an Urban Semi-Desirable Facility Location Problem

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Abstract

This paper introduces a mixed-integer, bi-objective programming approach to identify the locations and capacities of semi-desirable (or semi-obnoxious) facilities. The first objective minimizes the total investment cost; the second one minimizes the dissatisfaction by incorporating together in the same function “pull” and “push” characteristics of the decision problem (individuals do not want to live too close, but they do not want to be too far, from facilities). The model determines the number of facilities to be opened, the respective capacities, their locations, their respective shares of the total demand, and the population that is assigned to each candidate site opened. The proposed approach was tested with a case study for a particular urban planning problem: the location of sorted waste containers. The complete set of (supported or unsupported) non-inferior solutions, consisting of combinations of multi-compartment containers for the disposal of four types of sorted waste in nineteen candidate sites, and population assignments, was generated. The results obtained for part of the historical center of an old European city (Coimbra, Portugal) show that this approach can be applied to a real-world planning scenario.

Keywords:

Combinatorial optimization; multiple objective modeling; facility location; semi-obnoxious facility; urban facilities planning; waste management.

1. Introduction

Researchers have been interested in multiobjective location problems for over three decades (Alves and Clímaco, 2007; Cohon et al., 1980; Current et al., 1990; Current et al., 2001; Erkut and Neuman, 1989; Ross and Soland, 1980). The inclusion of multiple, conflicting objectives, enhances the analysis and leads to model formulations where the concept of an optimal solution is replaced with that of an efficient solution also referred to as non-dominated, non-inferior, or Pareto-optimal solution (Cohon, 1978).

The development of the modeling approach introduced in this paper has been fostered by an urban waste management problem that includes the location of facilities. The advantages of multiobjective approaches rather than optimizing a single dimensional objective function (such as cost-benefit analysis) have already been recognized in waste management analysis. Some of the advantages referred to are the enhancement of the cognitive capabilities of the decision-maker by considering several dimensions of the problems, and the additional flexibility relative to purely economic based models (Morrissey and Browne, 2004). Multiobjective approaches to waste management problems include Current and Ratick (1995), Melachrinoudis et al. (1995), Wyman and Kuby (1995), Coutinho-Rodrigues et al. (1997) and Alçada-Almeida et al. (2009a), who included in their approaches objectives related to risk, equity, and economic costs. In turn, Boffey et al. (2008) considered in the analysis aspects such as travel cost, route nuisance, facility cost and equity. In addition, Erkut et al. (2008) included objectives related to greenhouse effects, final disposal to the landfill, total cost, energy recovery and material recovery. Furthermore, Minciardi et al. (2008) considered objectives related to economic costs, unrecycled waste, sanitary landfill disposal and environmental impact. Finally, Tralhão et al. (2010) introduced a four objective approach to locate urban waste containers where the investment cost, travel distance and other two objectives related to the semi-obnoxious nature of the problem were considered.

This research introduces a bi-objective modeling approach developed to determine the most appropriate locations and capacities for semi-desirable facilities and was tested in an urban planning problem: the location of multi-compartment sorted waste containers. This is a complex and important urban waste management problem as such facilities impose environmental costs (e.g., noise, smell and/or visual pollution) on individuals who live too close to them, and travel costs for those who live too far away from them. Due to these “push” and “pull” factors, they fall into the class of semi-desirable (or semi-obnoxious) facilities (Brimberg and Juel, 1998; Carrizosa and Conde, 2002; Melachrinoudis and Xanthopulos, 2003; Skriver and Andersen, 2003; Romero-Morales et al., 1997; Tralhão et al., 2010; Zhou et al., 2005; Yapicioglu et al., 2007). As recognized by Revelle and Eiselt (2005), in location problems, the fact that "push'' objectives will attempt to locate towards infinity, make them to be often coupled with other predominantly ''pull'' objectives. Furthermore, Berman and Wang (2008) stated that the traditional *minimax* criterion for desirable facilities is not appropriate for semi-obnoxious facilities because customers too close to the facilities are ignored, and the traditional *maximin* criterion for obnoxious facilities is not appropriate because the resulting location might be too far away from some customers.

Tralhão et al. (2010) proposed a multiobjective approach to locate semi-desirable urban facilities considering four objectives: the first minimizes the total investment cost; the second minimizes the average distance from customers to facilities; the last two objectives address the "pull" and "push" characteristics of the decision problem, one by minimizing the number of individuals too close to any facility, and the other by minimizing the number of customers too far from the respective facility. Conceived to deal with this particular kind of semi-obnoxious location problems, considering separately "push" and "pull" objectives, that approach was developed to analyze problems with heterogeneous demand stakeholders (i.e., assuming distinct stakeholders and preferences for the "pull" and "push" factors of the problem, such as demands from both residents and commerce/services in an urban planning problem).

With homogenous demand (e.g., mostly related to residents), a different approach may be adopted. Instead of four objectives as considered by Tralhão et al. (2010), we adopted two objectives in this research, addressing basically the investment cost and a measure of "acceptability" by the residents (or its opposite, the "dissatisfaction", which is more useful in our case due to canonical reasons given that the minimization of both objectives was adopted). Comparing solutions in a two-dimensional space also requires less effort from a decision-maker than making comparisons, for example, in a four-dimensional space. In fact, an advantage of a bi-objective approach is to make the output analysis easier: as the number of objectives increases, the analysis of the trade-offs among the various objectives and among the various efficient alternatives becomes more difficult (Cohon, 1978; Teghem and Kunsch, 1986). According to Cohon (1978, pg. 100), several objectives (usually more than three) cause two problems: high computational burden and complexity of results display. Other authors have adopted bi-objective approaches in semi-obnoxious facility location problems (Brimberg and Juel, 1998; Melachrinoudis and Xanthopulos, 2003; Romero-Morales et al., 1997; Skriver and Andersen, 2003; Yapicioglu et al., 2007); Ohsawa et al. (2006) advocate that such type of approach facilitates the trade-off evaluations.

A methodology is proposed in this research in order to address the “push” and “pull” factors together in an objective function that represents the dissatisfaction level assumed by people, taking into consideration the required walking effort and the undesirable convenience of living too close to such facilities. Thus, in the proposed two objectives modeling approach, one minimizes the dissatisfaction level, and the other minimizes the total investment cost.

The modeling approach presented in this research was applied to a case study. All non-dominated solutions in the objective space (also designated by outcomes), were iteratively generated by imposing a constraint on the value of one objective, which eliminates solutions already known, and minimizing the other objective (i.e., applying the constraint method). Changing conveniently the value of the constraint on one objective, a series of single-objective problems were solved generating non-inferior solutions. Thus, both non-dominated supported and unsupported solutions (Erghott, 2005, pp 204) were generated.

The remainder of this paper is organized as follows. The construction of a dissatisfaction function and the new bi-objective model are introduced in section 2. Computational results and comparisons among generated solutions for a test case study (the location of multi-compartment sorted waste containers in an urban area) are given in section 3. A summary and conclusions are presented in the last section.

2. A bi-objective approach to locate urban semi-desirable facilities

2.1. The construction of acceptance and dissatisfaction functions

As the bi-objective approach developed in this research relies on the use of a function aggregating the push and pull characteristics of the semi-desirable problem, this subsection presents the respective definition.

Let’s consider a pair of nodes, i and j, of the (oriented) graph representing the problem street network, *N*, such as a household is assigned to i and a waste facility is located in j.

Pull characteristics of this problem addresses people desire to have their assigned waste containers not far away from their homes, in order to reduce the walking effort required to deposit waste. This may be addressed by a bounded, non increasing non negative function, $s^{(a)}$, decreasing with "*attraction* distance", $d_{i,j}$. It is important to note here that $d_{i,j}$ is not the distance measured in the network (ex. shortest path length from i to j) but rather an "equivalent walking distance". This is evaluated considering the actual lengths affected by slopes and/or other impedances. In fact, it is well known that walking speed depends on the path slope and also on the type and quality of the footpath pavement (TRB, 2003). Thus, considering the minimum time, $t'_{i,j}$, spent to walk from i to j, let $d'_{i,j}$ be the length of a horizontal path (slope zero) requiring the same time, $t'_{i,j}$, to be traversed. $d'_{i,j}$ is calculated as follows, considering only the influence of footpath slopes (no additional impedances). Consider the set, $\mathrm{P}_{i,j}$, of (oriented) paths from i to j; for each path $p \in \mathrm{P}_{i,j}$, let $d^{(m)}_{i,j,p}$ be the length of p, $v^{(w)}_{i,j,p}$ the average (walking) speed associated to p, and $v^{(0)}_{i,j,p}$ the walking speed in a horizontal path with the same length (having no slopes or other impedances); then:

$$d'_{i,j} = \min_{p \in \mathrm{P}_{i,j}} \left\{ \frac{v^{(0)}_{i,j,p}}{v^{(w)}_{i,j,p}} \, d^{(m)}_{i,j,p} \right\} \quad (1)$$

It should be noticed that, as no linear relation exists between walking speeds and slopes, it may happen that the path corresponding to $d'_{i,j}$ may not coincide with the path in the actual network with the shortest length. Thus, the minimization presented in (1). Besides, usually, the waste deposition in urban containers implies a path from home to the facility and a return home path. Thus, in our approach we consider, for $d_{i,j}$, the average of such path lengths:

$$d_{i,j} = \frac{d'_{i,j} + d'_{j,i}}{2} \quad (2)$$

However, push characteristics of the problem (related to visual, aesthetics and smell) addresses people desire to have facilities as far away as possible from their homes. This may be modeled by a bounded non decreasing non positive "*repulse* function", $s^{(r)}$, approaching zero with "repulsion distance", $d''_{i,j}$. For this distance, slopes (or other impedances) are not relevant. Thus, Euclidean distance from i to j could be used. However, due to the "barrier effect" of buildings, walls, etc., we preferred to use the actual network shortest length path, $d^{(s)}_{i,j}$.

At this point we can aggregate both push and pull characteristics in one function, $f(i,j)$, simply by adding both attraction and repulsion functions:

$$f(i,j) = s^{(r)}\left(d_{i,j}^{(s)}\right) + s^{(a)}\left(d_{i,j}\right) \tag{3}$$

If *N* is a disconnected network, oriented arcs may be added to *N*, connecting the sub-networks; for those additional arcs, d'' and d' can also be adopted with convenient very high values.

Due to canonical reasons (minimizing instead of maximizing objective 2), the acceptance function is not used in the model. Instead, with no lack of generality, a complement, *dissatisfaction objective function* $u(i,j)$ (to minimize), is used:

$$u(i,j) = \kappa''' - f(i,j), \qquad \kappa''' \geq M^{(f)} \tag{4}$$

Where $M^{(f)}$ is the supremum of the image of f.

A possible interpretation for f is considering it as the probability of an inhabitant to be satisfied with its assigned facility at attraction distance $d_{i,j}$ and repulse distance $d_{i,j}^{(s)}$. In this case u could be seen as the complementary probability (i. e, $\kappa''' = 1$).

2.2. Model formulation

The terms defined below are pertinent in the mixed integer linear programming (MILP) formulation and discussion:

- Facility - A physical entity for the disposal of sorted waste in distinct receptacles; it may support |K| types of waste; in our case study, K = {paper, plastic, glass, other}.
- Candidate location (site) - A site where facilities may be located.
- Open candidate - A site where a facility has been located by the MILP.
- Container - Consists of a set of receptacles, in each facility, for the disposal of a given type of waste; its capacity (in liters) is determined by the MILP model.
- Arrangement (of containers) - A sequence of |K| capacities (one for each type of waste) and a vendor brand. The K capacities corresponding to physical containers, of a given vendor brand, that store different kinds of waste – in the case study K=4. Each arrangement may be associated to several facilities, but each facility is associated with just one arrangement. Therefore, a facility is characterized by a given arrangement. The MILP determines the capacity for each type of waste collected, and the vendor brand. The concept of arrangement is important given that arrangements

can be organized in classes (sets). An "additional" cost is assigned to each class of arrangements due to the requirements of a special vehicle acquisition or the adaptation of existing vehicles used in waste collection. The additional cost corresponding to a given arrangement class is accounted whenever one or more facilities with arrangements in that class are located.

- Pull Distance Threshold - Maximum desired equivalent walking distance to an open facility.
- Sector - Represents the basic (aggregated) demand unit in the MILP; each consists of one or more dwellings. The distances between a sector and a candidate location are measured from the midpoint of the sector.

The MILP formulated includes two objectives. The first of these minimizes total facility investment costs. These include the containers themselves and any costs required for new or modified vehicles to service them. The second objective minimizes total dissatisfaction.

The main purpose of the bi-objective model is to identify non-dominated (Cohon, 1978; Ehrgott, 2005) siting schemes for the facilities. The model determines the number of facilities to be opened, the respective facility composition (in terms of containers), their locations, and their respective shares of the total waste of each type to be collected.

Due to the computational complexity of location problems, demand aggregation is frequently used to reduce dimensionality and solution time (e.g., Current and Schilling (1987; 1990), Francis et al. (1999; 2004a; 2004b), Horner and O'Kelly (2005), Alçada-Almeida et al. (2009b), Tralhão et al. (2010)). Following what was considered in a previous research applied to another part of the historical center of Coimbra (the "Baixa") (Tralhão et al., 2010), the dwellings of a different part of the historical center (the "Alta") were aggregated into linear sectors, generally with a given length (20 meters long in our case study), along the streets, and distances were calculated from sector midpoints. Demand (dwellings and their residents) within a given sector was assumed to be located at the sector's midpoint. Street intersections are represented by nodes, and the street segments forming "node sectors" generally radiate 10 meters from the node. For equity of service considerations, a constraint in the model ensures that no sector may be more than a given "distance" (defined by Eq. 2), D, from an open candidate site (pull distance threshold).

The relevant sets, parameters, and variables used in the bi-objective approach of this research follow overall the definitions adopted in that previous research, as follows:

- C: Set of candidate sites where facilities can be located.

- K: Set of the types of waste. In the case study K = {paper, plastic, glass, other}.
- M: Set of arrangements to be considered in each instance of the problem.
- M_j: is the set of arrangements defined specifically for the candidate site $j \in C$; in a particular problem instance, $\bigcup M_j = M$.
- S: Set of the sectors.
- T: Set of the classes of arrangements (related to additional costs), to be considered in each instance of the problem.
- α_j: Maximum number of facilities allowed in the candidate site $j \in C$. In the case study $\alpha_j \in \{1, 2, 3\}$ (integer parameter).
- c_m: Cost of a facility of arrangement $m \in M$ (real parameter).
- c'_t: Additional cost of waste collection associated to the type of facilities $t \in T$ (real parameter).
- $d_{i,j}$: "Distance" (defined by Eq. (2)) between the sector $i \in S$ and the candidate site $j \in C$ (real parameter).
- δ_i: Number of inhabitants in sector $i \in S$ (integer parameter).
- $g_{m,k}$: Capacity (liters) of the waste container of type $k \in K$ in the arrangement $m \in M$ (real parameter).
- $v'_{m,t}$: 1 if arrangement $m \in M$ of facilities belongs to the additional collection costs class $t \in T$, 0 otherwise (binary parameter).
- $w_{i,k}$: Amount (liters) of waste of type $k \in K$ produced in the sector $i \in S$ (real parameter).
- r_t: 1 if any arrangement of additional cost class $t \in T$ is chosen, 0 otherwise (binary variable).
- v_m: 1 if one or more arrangement $m \subset M$ is chosen to be located at any site, 0 otherwise (binary variable).

- $x_{i,j}$: 1 if the sector $i \in S$ is assigned to the candidate site $j \in C$ for waste deposition, 0 otherwise (binary variable).
- $y_{j,m}$: Number of facilities of arrangement $m \in M$ to be installed in candidate site $j \in C$. These variables must be equal to zero for $m \notin M_j$, in those cases they become parameters (integer variable).
- y'_j: 1 if the candidate $j \in C$ is opened, 0 otherwise (binary variable).

The underlying bi-objective mathematical model is formulated below. The two objectives are:

$$Min \sum_{j \in C} \sum_{m \in M} c_m y_{j,m} + \sum_{t \in T} c'_t r_t$$ minimizes total investment cost; ((5)

$$Min \sum_{i \in S} \delta_i \sum_{j \in C} u(i,j) x_{i,j}$$ minimizes total (and, consequently, weighted average) dissatisfaction; $u(i,j)$ is given by (4); ((6)

The constraints considered in the model, described below, are similar to those presented by Tralhão et al. (2010):

$$\sum_{j \in C} x_{i,j} = 1 \quad , \quad i \in S \tag{(7}$$

ensures that each sector will be assigned to one and only one open candidate site; $|S|$ = number of sectors = number of constraints;

$$\sum_{m \in M} y_{j,m} \leq \alpha_j \quad , \quad j \in C \tag{(8}$$

ensures that the number of facilities to be installed in location j is not greater than the maximum number of facilities that is physically possible to be installed in that location; $|C|$ = total number of candidate sites = number of constraints;

$$\kappa \sum_{m \in M} y_{j,m} \geq \sum_{i \in S} x_{i,j} \quad , \quad j \in C \quad , \quad \kappa \geq |S| \tag{(9}$$

ensures that, at least, one facility is located at the candidate site j, if at least one sector is assigned to this site; κ is a integer constant with arbitrary value not less than the total number of sectors;

$|C|$ = total number of candidate sites = number of constraints;

$$\sum_{m\in M} g_{m,k} y_{j,m} \geq \sum_{i\in S} w_{i,k} x_{i,j} \quad , \quad j \in C \quad , \quad k \in K \tag{(10)}$$

ensures that the capacity installed for each waste type $k \in K$ and in each (open) candidate site $j \in C$ satisfies the total demand of the sectors assigned to *j*;

$|C| \times |K|$ constraints;

$$\kappa' y'_j \geq \sum_{m\in M} y_{j,m} \quad , \quad j \in C \quad , \quad \kappa' \geq \sum_{j\in C} \alpha_j \tag{(11)}$$

ensures that the candidate site *j* is opened if facilities are located there; κ' is an integer constant with arbitrary value greater than the maximum total number of facilities;

$|C|$ = total number of candidate sites = number of constraints;

$$\kappa' v_m \geq \sum_{j\in C} y_{j,m} \quad , \quad m \in M \quad , \quad \kappa' \geq \sum_{j\in C} \alpha_j \tag{(12)}$$

sets $v_m = 1$ if a facility of arrangement *m* is located somewhere;

$|M|$ = number of arrangements = number of constraints;

$$\kappa'' r_t \geq \sum_{m\in M} v'_{m,t} v_m \quad , \quad t \in T \quad , \quad \kappa'' \geq |M| \tag{(13)}$$

ensures that additional costs c'_t, $t \in T$ should be considered in the total costs if an

arrangement of the corresponding class *t* is located somewhere; κ'' is an integer constant with arbitrary value greater than the maximum total number of arrangements;

$|T|$ = total number of classes of arrangements = number of constraints;

$$d_{i,j} x_{i,j} \leq D \quad , \quad i \in S \quad , \quad j \in C \tag{(14)}$$

ensures that the "distances" (Eq. (2)) between sectors and respective open candidate sites are not greater than a certain maximum value ($D = 250m$ was considered in our particular application); however, these constraints may be eliminated if $x_{i,j}$ is not defined for the sector-candidate site pairs for which $d_{i,j} > D$;

It should be noted that constraint (8) allows for the possibility of two or more facilities with different arrangements to be installed at the same open candidate site. If this is not desired for aesthetic or other reasons it may be prohibited at one or more candidate locations via the definition of the pertinent $\{M_j\}$ sets.

It should also be noted that, repulse distances measured from sector extreme points could have been adopted (e.g., subtracting half of the sector length except when that could produce negative repulse distances). But, in our case study only demand located in a radius of 30m of candidate locations would be affected. Besides, in our case study, repulse function is linear in the interval [0, 20[and null otherwise, which almost eliminates this effect.

3. Case study analysis

3.1. Area studied

The model was tested in the part of the historical center of Coimbra (known as "Alta"), which surrounds the old university, one of the oldest Universities in the world (actually, a candidate for the classification as UNESCO's World Heritage). Tralhão et al. (2010) presented a model applied to another area of the center of this city, with distinct topographic and demand characteristics (not so hilly, with heterogeneous demand originated from residents and from commerce/services). Coimbra is a city with more than 2000 years of history and home for one of the oldest universities in the world (720 years old), with a population of about 150000 inhabitants. The particular test area, located in the medieval city-center, is a hilly section of the city with many narrow streets and old buildings (some of them dating back to medieval times). It covers about 15 hectares that include: 420 buildings, 138 sectors, and 1558 inhabitants. This part of the city is being renovated. The project includes rehabilitation of buildings as well as new plans for urban infrastructures - the urban waste collection system is one of the project components to be studied. In recent years there has been an increasing growth of environmental concern leading to innovations in urban waste management. Directives from the European Union originated regulations in the different countries enforcing municipalities to ensure proper separation for selective collection of waste (Bautista et al., 2008). New and interesting management problems arise in the affected municipalities: what collection system should be applied, where to locate the collection points, how many containers and of what type should be assigned to each area, which are the most appropriate collection routes, how big the fleet of vehicles should be, etc. (Bautista et al., 2008; Coutinho-Rodrigues et al., 1993; Santos et al., 2008; Santos et al., 2010; Santos et al., 2011).

In the case study, as usual, four types of urban waste are considered (Tralhão et al., 2010): glass, plastic, paper and other (mainly organic waste). The facilities must have containers with adequate sizes for the above four waste types. However, it should be noted that the bi-objective model presented is flexible enough to support any number of waste types. The buildings were aggregated into linear sectors of 20 meters long.

3.2. The dissatisfaction function in the case study

In what concerns the repulse function used in this case study, as it is related to visual, aesthetics and smell, it seems reasonable to admit that it is effective and linear only for relatively short distances (until 20m). This may be expressed as follows:

$$s^{(r)}\left(d_{i,j}^{(s)}\right) = \begin{cases} \frac{24}{20} d_{i,j}^{(s)} - 24 & if \quad d_{i,j}^{(s)} \in [0, 20[\\ 0 & otherwise \end{cases} \tag{15}$$

For the attraction function we considered:

- $M^{(f)} = 24$ (16)
- A stepwise attraction part, usually with 50m wide steps, followed by a linear (smooth) descent part until 400m; in fact this is effective until 250m due to the pull distance threshold of 250m (equivalent distances) that was adopted in this research:

$$s^{(a)}(d_{i,j}) = \begin{cases} 24 & if \quad d_{i,j} \in [0, 50[\\ 17.5 & if \quad d_{i,j} \in [50, 100[\\ 10 & if \quad d_{i,j} \in [100, 150[\\ 6 & if \quad d_{i,j} \in [150, 200[\\ 12 - \frac{3 d_{i,j}}{100} & if \quad d_{i,j} \in [200, 400[\\ 0 & if \quad d_{i,j} \geq 400 \end{cases} \tag{17}$$

Repulsion and attraction functions in the case study were constructed in order to fit Zhou et al. (2005) histogram (not considering slopes and other impedances), where $M^{(f)}$ corresponds approximately to the maximum of the "acceptance rate" defined by those authors. In what concerns the pull effect, (17) is a "step-wise" version of the model presented by Yapicioglu et al. (2007).

Thus, from (3) and (4), considering $k''' = 24$, the dissatisfaction function, $u(i,j)$, as defined by (7), becomes:

$$u(i,j) = 24 - \left(s^{(r)}\left(d_{i,j}^{(s)}\right) + s^{(a)}\left(d_{i,j}\right)\right) \quad (18)$$

In what concerns the metrics $d^{(s)}$ and d, the values given in the Transit Capacity and Quality of Service Manual (TRB, 2003) for walking speeds were adopted:

- only slopes were considered, no other impedances were accounted;
- walking down speeds are equal to walking speeds on horizontal surfaces, except for stairs (slope $\pm$45);
- for positive slopes speeds were calculated according to the Transit Capacity and Quality of Service Manual (TRB, 2003, pg 3-93);
- equivalent distances were calculated for each arc in order to assign them the respective value.

A graphical representation of the dissatisfaction function adopted in the case study, applied to a stairway (slope $\pm$45), is shown in Fig. 1.

Insert Figure 1 about here

3.3. Facilities and candidate locations considered

Sixteen arrangements of containers (labeled T1 to T16) were considered as shown in Table 1. The different truck requirements correspond to different additional costs to be added to the price of acquisition and installation of facilities. The costs of containers and the costs of new trucks to operate with them (required by T13 – T16), or alternatively the costs of adapting existing trucks (required by T11 and T12), were obtained from commercial suppliers.

Insert Table 1 about here

The number, capacity and arrangements at candidate locations may vary by location. Consequently, the maximum number of feasible facilities and a feasible subset of arrangements are associated with each candidate site.

After the elimination of other potential locations, taking into account space availability, historical/aesthetic reasons, and structural issues, nineteen candidate locations remained in the study area.

3.4. Procedure for generating non-dominated solutions

The mixed-integer linear programming model presented in the previous section is used to generate non-dominated solutions to the problem. A solution consists of a set of open facility locations and the sectors (and corresponding people and waste production) assigned to each open candidate site. In our case study, the model consists of 2 objective functions and approximately 280 constraints with over 9300 entries (parameters, variables, etc.), including 758 variables (all variables are integer in the model).

In the tests, non-inferior solutions were generated to provide a general understanding of the underlying tradeoffs between the two objectives, helping the ultimate identification of the most preferred solution. The first two of these solutions optimize the two objectives individually and form the payoff table, defining the “ideal solution” (Zeleny, 1982). These were used as a benchmark to compare generated solutions.

The particular structure of some bi-objective problems has been explored in some researches in order to produce efficient procedures aimed at generating supported and unsupported non-dominated solutions (Coutinho-Rodrigues et al., 1999). In our research we could generate the complete non-inferior set of outcomes (i.e., including the unsupported non-dominated solutions) testing the proposed model with a real case study. The constraint method was repeatedly applied. As considered by other authors dealing with bi-objective approaches (e. g., Steiner and Radzik (2008), this procedure presents to the decision maker (DM) all possible non-inferior outcomes, letting him/her make the final selection. As also recognized by Melachrinoudis and Xanthopulos (2003), the generation of many “good” choices including those that optimize each individual objective is indeed an added value of the bi-objective model over the single objective ones. The CPU time for generating a solution in a PowerBook Macintosh 17” laptop computer with a 2.8 GHz Intel CPU ranged from 2 seconds to 5 minutes. As in previous researches (Alçada-Almeida et al., 2009a; Coutinho-Rodrigues et al., 2012; Tralhão et al., 2010), a MILP solver implemented by the authors, compiled to run either in Windows or in UNIX-OSX, has been used.

In each step, the total dissatisfaction (objective 2, defined by Eq. (6)) was minimized imposing an additional constraint that limits the maximum value of the total investment cost (objective 1, defined by Eq. (5), whose value must be less than its value obtained in the previous iteration, as presented in column 1 of Table 2 for solutions #02 - #64). That is, after identifying the first two solutions that optimize the objectives individually (the individual optimum values are opt_1, opt_2), the best solution for objective 2 is repeatedly identified imposing an updated constraint on objective 1 (investment cost).

3.5. Results obtained

The complete set of non-inferior outcomes obtained is shown in Table 2 (sixty-four solutions labeled #01 - #64); these are plotted as circles in Fig. 2. The larger circles represent supported non-dominated solutions and are labeled in roman numerals (I - XVI). These sixteen solutions are fairly spaced in the objectives space, except for solutions XII and XIII that are almost coincident.

Insert Figure 2 about here

Insert Table 2 about here

The objective function values for these sixty-four solutions are presented in columns 4 and 6 of Table 2 (where individual optimum values: opt_1, opt_2, are highlighted in bold). Objective 1 (investment cost) is expressed in currency units. Objective 2 represents the total dissatisfaction assumed by the producers of waste, as defined by Eq. (6). For each solution, the distances of each objective value from their respective optimum are given in columns "ΔOpt_1" and "ΔOpt_2" – the minimum dissatisfaction (solution #01) would be attained with an increase of 240.2% on the optimum value of the cost; vice-versa, the solution with the best cost (solution #64) would imply a degradation of 87.6% relatively to the minimum possible value of the dissatisfaction.

The objective function values for the "ideal solution" are given in the row labeled "Ideal" in columns 4 and 6. The two objective function values for the "anti-ideal solution" are the "worst" (i.e., maximum) values attained for each objective (given in the row labeled "Anti-ideal" in columns 4 and 6).

The distance of each solution from the "ideal solution" (considering the objectives' space) is evaluated using three frequently adopted metrics (Bowman Jr., 1976; Steuer, 1986): the Rectilinear or Manhattan (L_1), the Euclidean (L_2) and the Chebyshev (L_∞)

metrics. L_1, L_2 and L_∞ were used to compare the solutions - the distances from each solution to the "ideal solution", are presented as a percentage above the ideal in columns "ΔL_1", "ΔL_2" and "ΔL_∞" of Table 2. The three indicators are used to give the DM a better idea of the options available. Three non-dominated solutions, #59 (XIV), #53 (XIII), #52 (XII), perform the best of the sixty-four solutions generated in regard to minimizing the distance to the "ideal solution" as measured respectively by the L_1, L_2 and L_∞ metrics ("*Min* L_1", "*Min* L_2" and "*Min* L_∞" also designate those solutions respectively).

As a pair of objective weights can be calculated from the slope of the line segment between any two contiguous supported non-inferior solutions (Cohon, 1978), the pairs of weights corresponding to each of the sixteen supported solutions, as well as the respective range of variation for each solution (Fig. 3), were obtained *a posteriori*. A methodology as that proposed by Duin and Volgenant (2012, in press) could be used for that purpose. This illustrates known limitations of the weighting method: as can be seen in Fig. 3, relatively large range of weight values lead to the generation of the same outcome (e.g., solution XV), while some outcomes would be generated just for a very narrow range of weight values (e.g., solutions X and XII). Fig. 3 also shows that, in our case study, XII (Opt L_∞) and XIII (Opt L_2) are supported solutions.

Insert Figure 3 about here

Considering the sixteen supported non-dominated solutions, Table 3 displays how many candidate sites are opened (column 3) and the number of inhabitants assigned to each one (columns C01-C19, whose labels refer to the nineteen candidates). Such information can give the DM insight into the desirability of the various candidate sites. The minimum number of candidate sites opened is five, (solutions #59, #63 and #64 - #64 minimizes the investment cost). The candidate sites C04 and C11 are opened in all sixteen of the supported non-dominated solutions; C11 is opened in all sixty-four of the non-dominated solutions. The largest average number of inhabitants (238.8) of the sixteen solutions was assigned to site C17, while site C12 is assigned the minimum average (0.8).

Insert Table 3 about here

Analyzing the arrangements that were used across solutions, only T1, T2, T3, T4 were selected by the model in some of the sixty-four solutions. These arrangements have the lower capacities for three types of waste (glass, plastic and paper), and do not require additional investments in vehicles. As Table 3 shows, T1 (4 x 800 liters) and T2 (3 x 800 + 1 x 1000 liters) were used in thirteen out of the sixteen supported solutions. T1 was the most used in the sixty-four solutions, and has the highest number of units used both in the sixteen and in the sixty-four solutions.

The solutions obtained can be exported into a geographical information system (GIS) in order to produce color-coded maps to facilitate the comparison in the “decision” (i.e., “geographic”) space (Cohon, 1978). Solution #63 is shown in Fig. 4 - the locations of the five opened sites (C04, C07, C08, C11, C17) are identified with colored circles, and the buildings assigned to a given site are mapped with the respective site’s color. For this purpose, each building is assigned a GIS code that facilitates the communication between the MILP solver and the GIS.

Insert Figure 4 about here

4. Conclusions

The motivation for this research was the analysis of urban sorted waste containers location in part of the historical center of Coimbra, Portugal, surrounding the old University. This is a semi-desirable (or semi-obnoxious) location problem. Due to the dimensions of the problem, the demand for the facilities was aggregated into 138 demand sectors.

As is the case with many public sector location problems, the analysis involved multiple objectives (Current et al., 1990). A two objective MILP was developed. In addition to an investment cost objective, the model included “push” and “pull” dimensions of the problem in a second objective to minimize the dissatisfaction of residents related to the facilities location. Analysis of multiobjective location problems is a complex task due to the potentially large number of non-dominated solutions and the difficulty of analyzing the tradeoffs among the objectives. The bi-objective approach facilitates comparisons. The constraint method (imposing a constraint on the maximum value of the investment cost objective, and minimizing the dissatisfaction objective) was used repeatedly in order to obtain the complete set of non-inferior outcomes (e.g. including unsupported non-dominated solutions). A graphical representation of the complete set of non-inferior solutions, is provided to help the decision makers in comparing solutions. As the complete set of the non-inferior solutions (sixty-four) for the case study was generated, a more complete analysis of the trade-offs is allowed.

The present results illustrate how a generation method may be used to obtain solutions in a bi-objective problem. The quality of the solutions generated was evaluated through the respective distances to the "ideal solution" using three metrics commonly used in the literature.

Complex decision problems are frequently encountered in urban and environmental planning, typically involving the consideration of a wide range of incommensurable and conflicting objectives. The study carried out produced relevant information shedding light for decision-making in a complex multidimensional decision problem.

We believe that this research is a relevant contribution to the field given the following reasons. First, it provides a new bi-objective MILP formulation to a practical urban location problem, aggregating within the same objective function the "push" and "pull" factors of the semi-desirable problem addressed. Second, this problem is important, timely, and pertinent in many urban areas, especially in the center of old European towns and cities. Third, the application to a case study demonstrates that the approach can be applied to a real-world planning scenario.

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Table_1

Arrangements		Container Volumes by Type of Waste (liters)				Additional Truck Requirement	
ID	Brand	Glass	Plastic	Paper	Organic	Description	Cost
T1	Brand_1	800	800	800	800	None	0
T2		800	800	800	1000	None	0
T3		800	800	800	1600	None	0
T4		800	800	800	2000	None	0
T5		800	800	1000	1000	None	0
T6		800	800	1000	1600	None	0
T7		800	800	1000	2000	None	0
T8		1000	1000	1000	1000	None	0
T9		1000	1000	1000	1600	None	0
T10		1000	1000	1000	2000	None	0
T11	Brand_2	3000	3000	3000	3000	Adaptation	2300
T12		3000	3000	3000	5000	Adaptation	2300
T13	Brand_3	3000	3000	3000	12000	New Truck	144000
T14		3000	3000	4000	9000	New Truck	144000
T15		3000	3000	4000	12000	New Truck	144000
T16		3000	3000	4000	15000	New Truck	144000

Table_2

Solutions (#01-#64) \ Objectives	Supported Solutions (I-XVI)		Objective 1		Objective 2		Δ (%)		
	Designation	Obj1 Weights Interval	Cost	ΔOpt_1	Dissatisfaction	ΔOpt_2	ΔL_1	ΔL_2	ΔL_∞
(#01) Min Obj 2	Sol. I; **Opt Obj2**	[0.0; 0.8]	44 260	+240.2%	**12 331**	**+0.0%**	240.2%	240.2%	240.2%
(#02) Cost < 44 260	Sol. II	]0.8; 3.4]	41 730	+220.8%	12 351	+0.2%	220.9%	220.8%	220.8%
(#03) Cost < 41 730	Sol. III	]3.4; 8.4]	39 280	+201.9%	12 437	+0.9%	202.8%	201.9%	201.9%
(#04) Cost < 39 280			39 200	+201.3%	12 609	+2.3%	203.6%	201.3%	201.3%
(#05) Cost < 39 200	Sol. IV	]8.4; 11.6]	36 830	+183.1%	12 661	+2.7%	185.8%	183.1%	183.1%
(#06) Cost < 36 830			36 750	+182.5%	12 833	+4.1%	186.5%	182.5%	182.5%
(#07) Cost < 36 750	Sol. V	]11.6; 13.2]	34 380	+164.3%	12 983	+5.3%	169.5%	164.3%	164.3%
(#08) Cost < 34 380			34 300	+163.6%	13 155	+6.7%	170.3%	163.8%	163.6%
(#09) Cost < 34 300	Sol. VI	]13.2; 14.4]	31 930	+145.4%	13 354	+8.3%	153.7%	145.7%	145.4%
(#10) Cost < 31 930			31 850	+144.8%	13 526	+9.7%	154.5%	145.1%	144.8%
(#11) Cost < 31 850	Sol. VII	]14.4; 21.4]	29 630	+127.7%	13 742	+11.4%	139.2%	128.3%	127.7%
(#12) Cost < 29 630			29 560	+127.2%	13 834	+12.2%	139.4%	127.8%	127.2%
(#13) Cost < 29 560			29 550	+127.1%	13 882	+12.6%	139.7%	127.8%	127.1%
(#14) Cost < 29 550			29 480	+126.6%	14 006	+13.6%	140.2%	127.3%	126.6%
(#15) Cost < 29 480			29 400	+126.0%	14 192	+15.1%	141.1%	126.9%	126.0%
(#16) Cost < 29 400	Sol. VIII	]21.4; 25.1]	27 180	+108.9%	14 408	+16.8%	125.8%	110.2%	108.9%
(#17) Cost < 27 180			27 110	+108.4%	14 502	+17.6%	126.0%	109.8%	108.4%
(#18) Cost < 27 110			27 100	+108.3%	14 548	+18.0%	126.3%	109.8%	108.3%
(#19) Cost < 27 100			27 030	+107.8%	14 674	+19.0%	126.8%	109.4%	107.8%
(#20) Cost < 27 030			26 950	+107.1%	14 889	+20.7%	127.9%	109.1%	107.1%
(#21) Cost < 26 950	Sol. IX	]25.1; 28.3]	24 960	+91.9%	15 150	+22.9%	114.7%	94.7%	91.9%
(#22) Cost < 24 960			24 890	+91.3%	15 181	+23.1%	114.4%	94.2%	91.3%
(#23) Cost < 24 890			24 820	+90.8%	15 276	+23.9%	114.7%	93.9%	90.8%
(#24) Cost < 24 820			24 810	+90.7%	15 321	+24.3%	114.9%	93.9%	90.7%
(#25) Cost < 24 810			24 800	+90.6%	15 347	+24.5%	115.1%	93.9%	90.6%
(#26) Cost < 24 800			24 730	+90.1%	15 407	+24.9%	115.0%	93.5%	90.1%
(#27) Cost < 24 730			24 660	+89.5%	15 510	+25.8%	115.3%	93.2%	89.5%
(#28) Cost < 24 660			24 650	+89.5%	15 552	+26.1%	115.6%	93.2%	89.5%
(#29) Cost < 24 650			24 580	+88.9%	15 682	+27.2%	116.1%	93.0%	88.9%
(#30) Cost < 24 580			24 500	+88.3%	15 978	+29.6%	117.9%	93.1%	88.3%
(#31) Cost < 24 500			22 660	+74.2%	16 071	+30.3%	104.5%	80.1%	74.2%
(#32) Cost < 22 660	Sol. X	]28.3; 28.6]	22 580	+73.6%	16 090	+30.5%	104.0%	79.6%	73.6%
(#33) Cost < 22 580			22 510	+73.0%	16 121	+30.7%	103.8%	79.2%	73.0%
(#34) Cost < 22 510			22 440	+72.5%	16 180	+31.2%	103.7%	78.9%	72.5%
(#35) Cost < 22 440			22 430	+72.4%	16 284	+32.1%	104.5%	79.2%	72.4%
(#36) Cost < 22 430			22 370	+71.9%	16 330	+32.4%	104.4%	78.9%	71.9%
(#37) Cost < 22 370			22 360	+71.9%	16 335	+32.5%	104.3%	78.9%	71.9%
(#38) Cost < 22 360			22 350	+71.8%	16 437	+33.3%	105.1%	79.1%	71.8%
(#39) Cost < 22 350			22 290	+71.3%	16 502	+33.8%	105.2%	78.9%	71.3%
(#40) Cost < 22 290			22 280	+71.3%	16 511	+33.9%	105.2%	78.9%	71.3%
(#41) Cost < 22 280			22 200	+70.6%	16 723	+35.6%	106.3%	79.1%	70.6%
(#42) Cost < 22 200	Sol. XI	]28.6; 30.6]	20 280	+55.9%	17 011	+38.0%	93.8%	67.5%	55.9%
(#43) Cost < 20 280			20 210	+55.3%	17 042	+38.2%	93.5%	67.2%	55.3%
(#44) Cost < 20 210			20 140	+54.8%	17 096	+38.6%	93.4%	67.1%	54.8%
(#45) Cost < 20 140			20 130	+54.7%	17 169	+39.2%	94.0%	67.3%	54.7%
(#46) Cost < 20 130			20 070	+54.3%	17 192	+39.4%	93.7%	67.1%	54.3%
(#47) Cost < 20 070			20 060	+54.2%	17 223	+39.7%	93.9%	67.2%	54.2%
(#48) Cost < 20 060			19 990	+53.7%	17 335	+40.6%	94.2%	67.3%	53.7%
(#49) Cost < 19 990			19 980	+53.6%	17 404	+41.1%	94.7%	67.5%	53.6%
(#50) Cost < 19 980			19 910	+53.0%	17 532	+42.2%	95.2%	67.8%	53.0%
(#51) Cost < 19 910			19 900	+53.0%	17 662	+43.2%	96.2%	68.4%	53.0%
(#52) Cost < 19 900	Sol. XII; Min L_∞	]30.6; 30.7]	17 830	+37.0%	18 090	+46.7%	83.8%	59.6%	**46.7%**
(#53) Cost < 17 830	Sol. XIII; Min L_2	]30.7; 39.2]	17 760	+36.5%	18 121	+47.0%	83.5%	**59.5%**	47.0%
(#54) Cost < 17 760			17 690	+36.0%	18 217	+47.7%	83.7%	59.8%	47.7%
(#55) Cost < 17 690			17 680	+35.9%	18 289	+48.3%	84.2%	60.2%	48.3%
(#56) Cost < 17 680			17 610	+35.4%	18 405	+49.3%	84.6%	60.6%	49.3%
(#57) Cost < 17 610			17 600	+35.3%	18 727	+51.9%	87.1%	62.7%	51.9%
(#58) Cost < 17 600			15 460	+18.8%	19 646	+59.3%	78.2%	62.2%	59.3%
(#59) Cost < 15 460	Sol. XIV; Eq. Weights & Min L_1	]39.2; 50.5]	15 380	+18.2%	19 654	+59.4%	**77.6%**	62.1%	59.4%
(#60) Cost < 15 380			15 310	+17.7%	19 785	+60.4%	78.1%	63.0%	60.4%
(#61) Cost < 15 310			15 300	+17.6%	20 511	+66.3%	83.9%	68.6%	66.3%
(#62) Cost < 15 300			13 250	+1.8%	21 962	+78.1%	79.9%	78.1%	78.1%
(#63) Cost < 13 250	Sol. XV	]50.5; 93.5]	13 090	+0.6%	21 994	+78.4%	79.0%	78.4%	78.4%
(#64) Cost < 13 090	Sol. XVI; **Opt Obj1**	]93.5; 100.0]	**13 010**	**+0.0%**	23 135	+87.6%	87.6%	87.6%	87.6%
Ideal			**13 010**	**+0.0%**	**12 331**	**+0.0%**	0.0%	0.0%	0.0%
Anti Ideal			44 260	+240.2%	23 135	+87.6%	327.8%	255.7%	240.2%

Table_3

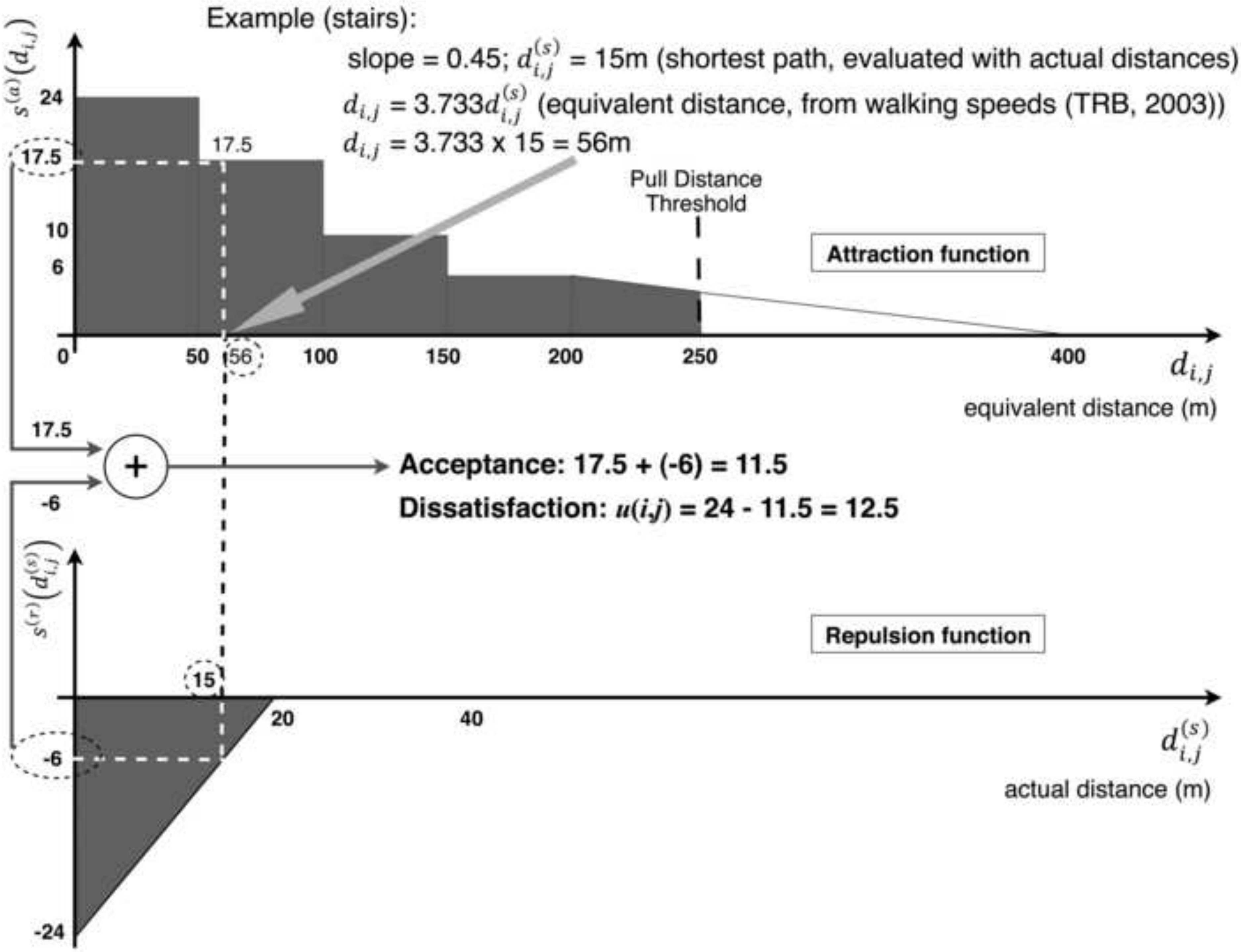
Supported Solutions (I - XVI)		# Open Candidate Sites	C01	C02	C03	C04	C05	C06	C07	C08	C09	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	T1	T2	T3	T4
			Candidate Sites (C01 - C19) and # Inhabitants Assigned																			Arrangements (T1 - T4) # Units Used			
I	(#01) Min Obj 2 (Opt Obj2)	16	55	8	91	159	117	64	88	204	41	50	73	13	42	29		48	167	105	204	16	2		
II	(#02) Cost < 44 260	17	55	8	91	159	117	64	88	204	86	50	73		55	29		48	167	105	159	16	1		
III	(#03) Cost < 41 730	15	55		99	158	140	64	88	161	86	89	73		55	29			153	140	167	15			
IV	(#05) Cost < 39 200	12			99	159	155	64	88	209		109	128		66				167	105	209	10	2		
V	(#07) Cost < 36 750	12			99	159	155	64	88	149		90	128		61				161	105	299	11		1	
VI	(#09) Cost < 34 300	10				204	163	88	106	209		102	209						163	105	209	6	4		
VII	(#11) Cost < 31 850	9				204	163	88	111	286		90	209						163		244	5	2	2	
VIII	(#16) Cost < 29 400	8				204	144	107		204			209						332	105	253	3	3	2	
IX	(#21) Cost < 26 950	8				165	164	122		149			209						336	105	308	5	1	2	
X	(#32) Cost < 22 660	7				204	167	143		293			209						333		209	2	3	2	
XI	(#42) Cost < 22 200	7				165	164	168		306			209						336		210	3	2	2	
XII	(#52) Cost < 19 900 (Min L∞)	6				326		179		288			175						336		254		2	4	
XIII	(#53) Cost < 17 830 (Min L2)	6				335		166		324			189						335		209	1	2	3	
XIV	(#59) Cost < 15 460 (Equal Weights & Min L1)	5				371			309	367			175						336				1	2	2
XV	(#63) Cost < 13 250	5				335			331	367			189						336				1	3	1
XVI	(#64) Cost < 13 090 (Opt Obj1)	5				331			308				163				336				420	1		3	1
16 sol.	Times Opened (Sites) or Used (Arrangements)		3	2	5	**16**	11	13	10	15	3	7	**16**	1	5	3	1	2	15	8	14	13	13	11	3
64 sol.	Times Opened (Sites) or Used (Arrangements)		6	2	31	63	57	60	39	63	11	40	**64**	1	14	8	2	4	62	37	62	61	47	42	3
16 Supported solutions	Column Grand Total		165	16	479	3639	1649	1381	1605	3720	213	580	2620	13	279	87	336	96	3821	875	3354	94	26	26	4
	Column Average		10.3	1.0	29.9	227.4	103.1	86.3	100.3	232.5	13.3	36.3	163.8	0.8	17.4	5.4	21.0	6.0	238.8	54.7	209.6	5.9	1.6	1.6	0.3
	Column Minimum		55	8	91	159	117	64	88	149	41	50	73	13	42	29	336	48	153	105	159	1	1	1	1
	Column Maximum		55	8	99	371	167	179	331	367	86	109	209	13	66	29	336	48	336	140	420	16	4	4	2
64 solutions	Grand Total for the complete set of outcomes		330	16	3262	12888	8592	6533	4812	14024	901	3788	10581	13	803	232	672	192	13278	4338	14457	449	84	89	4
	Average for the complete set of outcomes		5.2	0.3	51.0	201.4	134.3	102.1	75.2	219.1	14.1	59.2	165.3	0.2	12.5	3.6	10.5	3.0	207.5	67.8	225.9	7.0	1.3	1.4	0.1

Table 1. Arrangements of containers considered and respective attributes

Table 2. Summary of the complete set of nondominated solutions generated

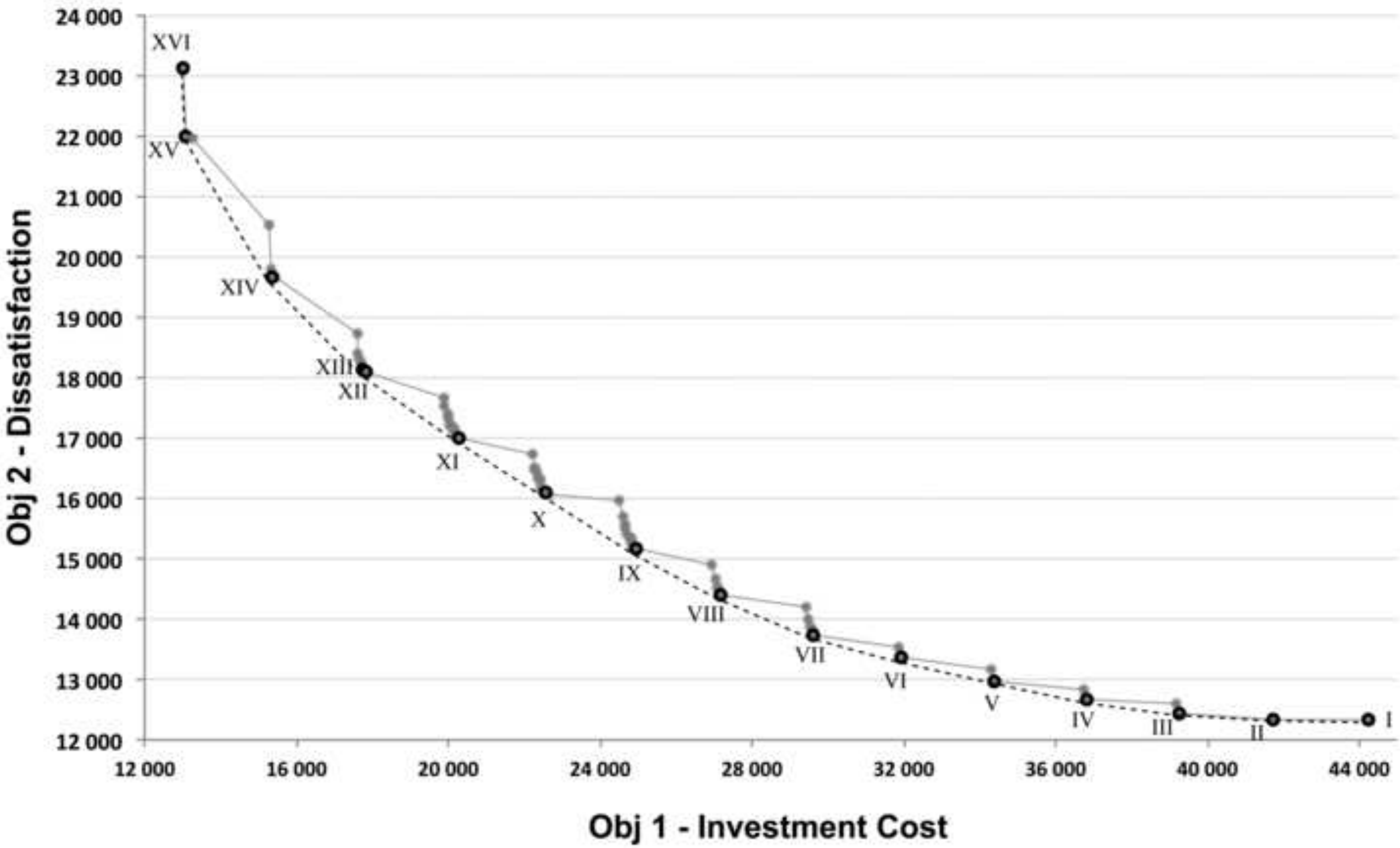
Table 3. Number of inhabitants assigned to each candidate site opened and arrangements selected for each solution generated

Fig_1



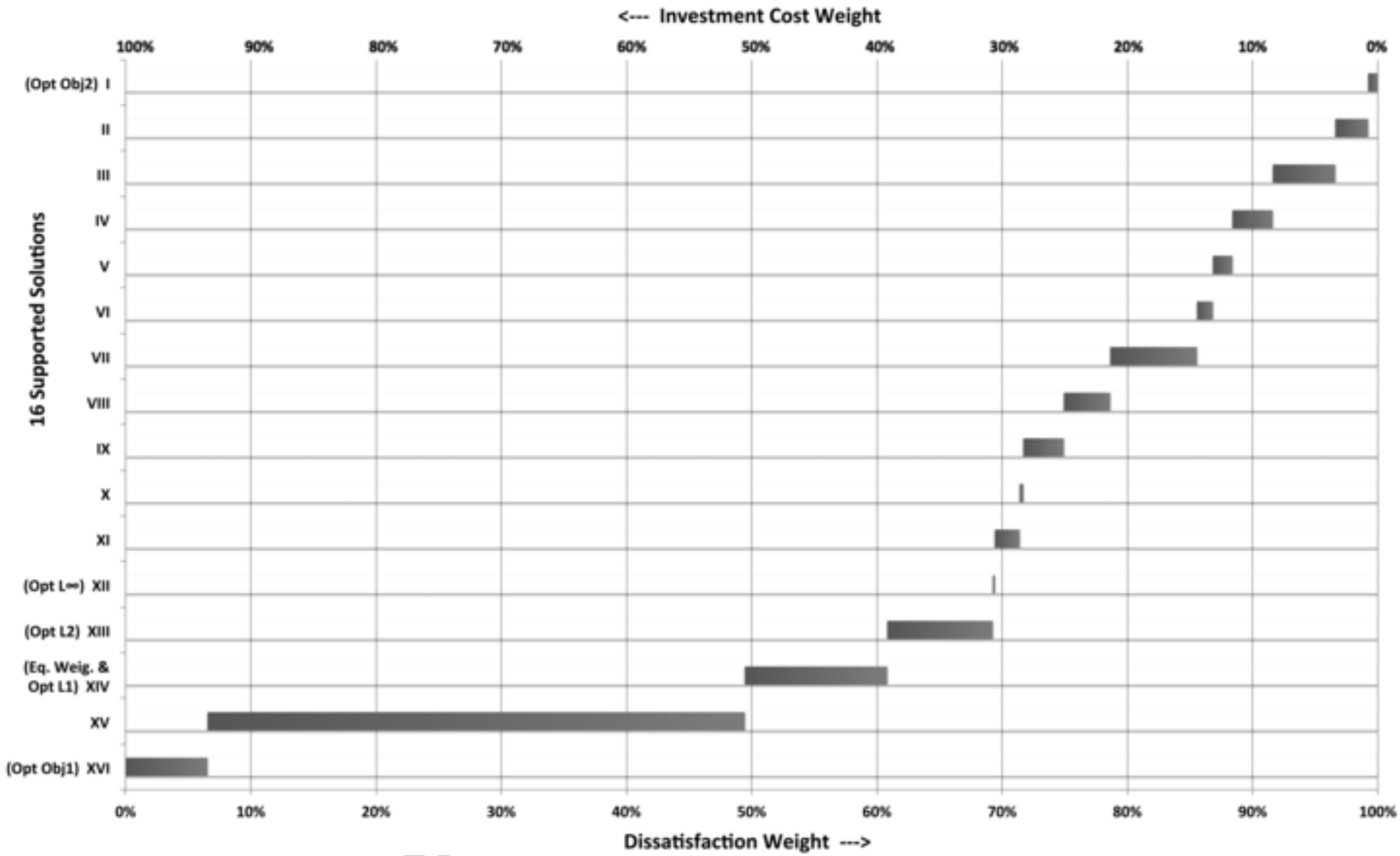
Example (stairs):
slope = 0.45; $d_{i,j}^{(s)}$ = 15m (shortest path, evaluated with actual distances)
$d_{i,j} = 3.733 d_{i,j}^{(s)}$ (equivalent distance, from walking speeds (TRB, 2003))
$d_{i,j}$ = 3.733 x 15 = 56m
Pull Distance Threshold
Attraction function
$s^{(a)}(d_{i,j})$
24
17.5
17.5
10
6
0
50
56
100
150
200
250
400
$d_{i,j}$
equivalent distance (m)
17.5
+
-6
Acceptance: 17.5 + (-6) = 11.5
Dissatisfaction: $u(i,j)$ = 24 - 11.5 = 12.5
$s^{(r)}(d_{i,j}^{(s)})$
Repulsion function
15
20
40
$d_{i,j}^{(s)}$
actual distance (m)
-6
-24

Fig_2



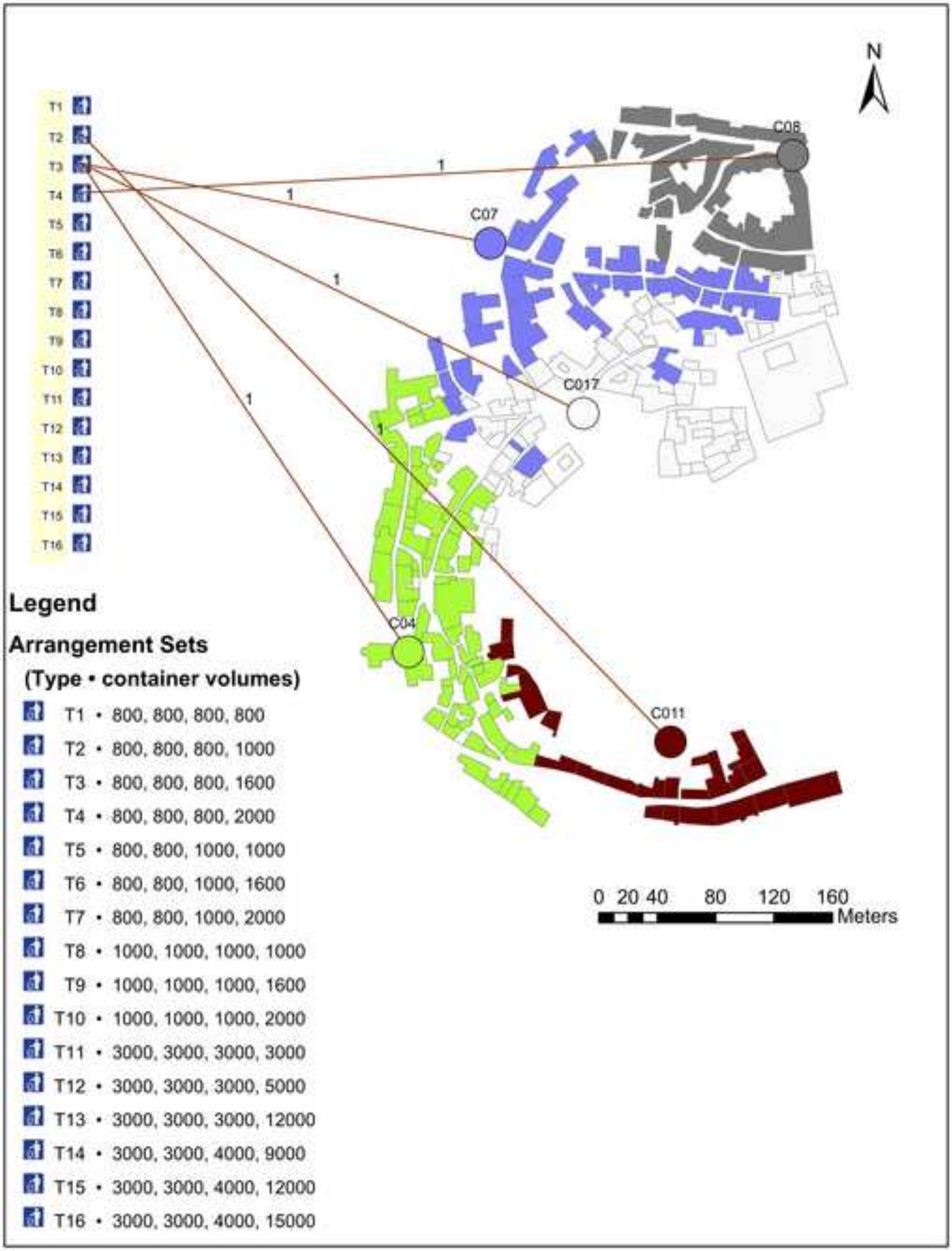
Obj 2 - Dissatisfaction
24 000
23 000
22 000
21 000
20 000
19 000
18 000
17 000
16 000
15 000
14 000
13 000
12 000
12 000
16 000
20 000
24 000
28 000
32 000
36 000
40 000
44 000
Obj 1 - Investment Cost
XVI
XV
XIV
XIII
XII
XI
X
IX
VIII
VII
VI
V
IV
III
II
I

Fig_3



<--- Investment Cost Weight
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
16 Supported Solutions
(Opt Obj2) I
II
III
IV
V
VI
VII
VIII
IX
X
XI
(Opt L∞) XII
(Opt L2) XIII
(Eq. Weig. & Opt L1) XIV
XV
(Opt Obj1) XVI
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Dissatisfaction Weight --->

Fig_4



N
T1
T2
T3
T4
T5
T6
T7
T8
T9
T10
T11
T12
T13
T14
T15
T16
C08
C07
C017
C04
C011
1
1
1
1
1
0 20 40 80 120 160
Meters
Legend
Arrangement Sets
(Type • container volumes)
T1 • 800, 800, 800, 800
T2 • 800, 800, 800, 1000
T3 • 800, 800, 800, 1600
T4 • 800, 800, 800, 2000
T5 • 800, 800, 1000, 1000
T6 • 800, 800, 1000, 1600
T7 • 800, 800, 1000, 2000
T8 • 1000, 1000, 1000, 1000
T9 • 1000, 1000, 1000, 1600
T10 • 1000, 1000, 1000, 2000
T11 • 3000, 3000, 3000, 3000
T12 • 3000, 3000, 3000, 5000
T13 • 3000, 3000, 3000, 12000
T14 • 3000, 3000, 4000, 9000
T15 • 3000, 3000, 4000, 12000
T16 • 3000, 3000, 4000, 15000

Fig. 1. Dissatisfaction function in the case study– graphical representation

Fig. 2. Scattergram with the complete set of nondominated outcomes

Fig. 3. Ranges of variation of objective weights for supported solutions

Fig. 4. Solution #63 represented in the map (GIS) - locations of the five opened sites, respective arrangement sets and building assignments

Highlights

- We introduce a mixed-integer bi-objective programming approach in a location problem
- Investment costs and dissatisfaction related to semi-desirable facilities are modeled
- “Pull” and “push” factors are considered together in a dissatisfaction function
- The location of multi-compartment containers for sorted urban waste was tested
- Results were obtained for part of the historical center of an old European city