Fuzzy Optimization of a Belt-conveyor Bridge

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1. Abstract

In the structural optimization of a ring-stiffened cylindrical shell the unknown variables are the shell thickness as well as the thickness and the number of flat rings. The shell diameter enables to realize a belt-conveyor structure inside of the shell. The uniformly distributed vertical load consists of dead and live load. The design constraints relate to the local shell buckling strength, to the panel ring buckling and to the deflection of the simply supported bridge. The cost function includes the material and fabrication costs. The fabrication cost function is formulated according to the fabrication sequence and includes also the cost of forming of shell elements into the cylindrical shape as well as the cost of cutting of the flat plate ring-stiffeners. As an alternative to safety factors one may try to describe the uncertain data via a non-probabilistic description of uncertainty, in particular the fuzzy-set based analysis. Several procedures are described and the optimum design level can be obtained either based on failure possibility or of membership value satisfaction. The fuzzy-based optimization becomes a sequential minimization of unconstrained convex scalar functions, from which a Pareto solution is obtained. A branch and bound procedure is associated with this algorithm to provide a discrete solution.

2. keywords: Structural Optimisation; Buckling; Welded joints; Manufacturing Costs; Fuzzy-based design

3. Introduction

Stiffened shells are widely used in offshore structures, bridges, towers, etc. Rings and/or stringers can be used to strengthen the shape of cylindrical shells. Shells can be loaded by axial compression, bending, external or internal pressure or by combined load. Design rules for the shell buckling strength have been worked out by ECCS [1], API [2] and DNV [3]. The optimum design of a stiffened shell belt-conveyor bridge has been treated in [4]. The buckling behaviour of stiffened cylindrical shells has been investigated by several 'authors, e.g. Harding [5], Dowling and Harding [6], Ellinas *et al* [7], Frieze *et al.* [8], Shen *et al.* [9], Tian *et al.* [10].

In the calculation of shell buckling strength the initial imperfections should be taken into account. These imperfections are caused by fabrication and by shrinkage of circumferential welds. A calculation method for the effect of welding has been worked out by the first author [11] and it is used in the calculation of the local shell buckling strength.

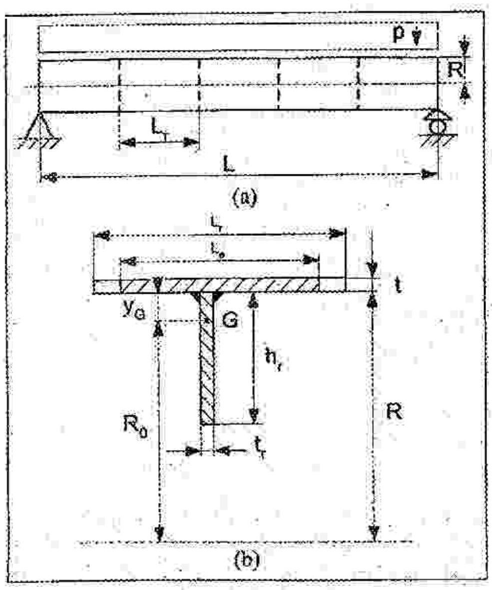

Fig. 1. (a) Simply supported belt conveyor bridge constructed as a ring stiffened cylindrical shell, (b) Cross-section of a ring stiffener including the effective width of the shell.

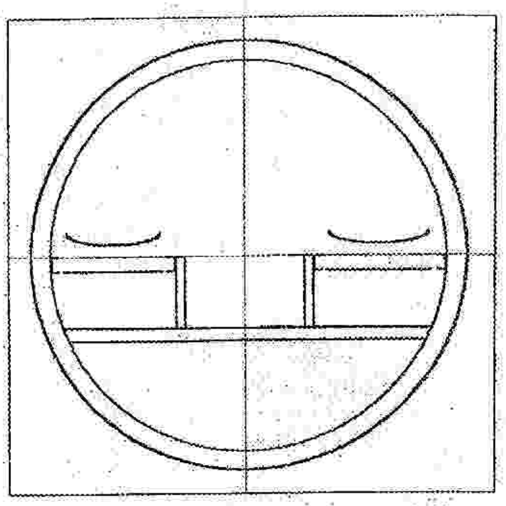

Fig. 2. Cross-section of a belt conveyor bridge with two belt conveyors and a service walkway in the middle.

In the present study the design rules of Det Norske Veritas (DNV) are used for ring-stiffened cylindrical shells. The shape of rings is a simple flat plate, which is welded to the shell by double fillet welds. In the calculation of the fabrication cost, the cost of forming the shell elements into the cylindrical shape and the cutting of the flat ring-stiffeners is also taken into account.

In design and optimization problems material constants, loading, and structure geometry are usually considered as given data but in real world assumed values do not correspond with actual ones. All this is accounted by safety factors which amplify load magnitude or reduce material strength. As an alternative to safety factors one may try to describe the uncertain data via a non-probabilistic description of uncertainty, in particular the fuzzy-set based analysis. Wang [12] α-level cuts strategy has been recognized as the standard method for solving general optimum structural design problems with fuzzy constraints. Xu [13] proposed a 2^{nd} phase optimization to obtain a particular α^* by maximizing an established nonlinear fuzzy goal membership function. The former procedure obtains the optimum design level based on failure possibility instead of membership value satisfaction. Two alternative α-level cuts are also described in this work. The crisp solutions are obtained by a procedure described in [14]. The fuzzy-based optimization becomes a sequential minimization of unconstrained convex scalar functions, from which a Pareto solution is obtained. A branch and bound procedure is associated with this algorithm to provide a discrete solution.

The shell is a supporting bridge for a belt-conveyor, simply supported with a given span length of L = 60 m and radius of R = 1,800 mm (Figures 1 and 2). The intensity of the factored uniformly distributed vertical load is p = 16.5 N/mm + self-mass. Factored live load is 12 N/mm, dead load (belts, rollers, service-walkway) is 4.5 N/mm. For self-mass a safety factor of 1.35 is used, which is prescribed by Eurocode 3 (note that ECCS gives 1.3). The safety factor for variable load is 1.5. The flat plate rings are uniformly distributed along the shell. Note that the belt-conveyor supports are independent of the ring stiffeners, they can be realized by using local plate elements.
The unknown variables are as follows: shell thickness t, stiffener thickness t_r and number of stiffeners n. To ensure a stable cylindrical shape, a certain number of ring-stiffeners should be used. In the present study we consider a range of ring numbers n = 6 - 30. Those results for which the place of stiffeners coincides with the circumferential welds of the shell segments (n = 9, 19)are not applicable for fabrication reasons. The range of thicknesses t and t_r is taken as 4 – 20 mm, rounded to 1 mm.

4. Design Constraints

4.1 Local Buckling of the flat ring-stiffeners (Fig. 1)

According to DNV

$$\frac{h_r}{t_r} \leq 0.4\sqrt{\frac{E}{f_y}} \qquad (1)$$

Considering this constraint as active, for $E = 2.1x10^5$ MPa and yield stress $f_y = 355$ MPa, one obtains $h_r = 9t_r$.

4.2 Constraint on local shell buckling (as unstiffened) (Fig. 3)

$$p = 16.5 + 1.35\rho(2R\pi t + nA_r); \; \rho = 7.85x10^{-6} kg/mm^3; A_r = h_r t_r \qquad (2)$$

$$M_{\max} = \frac{pL^2}{8} \quad ; \quad \sigma_{\max} = \frac{M_{\max}}{\pi R^2 t} \leq \sigma_{cr} = \frac{f_y}{\sqrt{1+\lambda^4}} \tag{3}$$

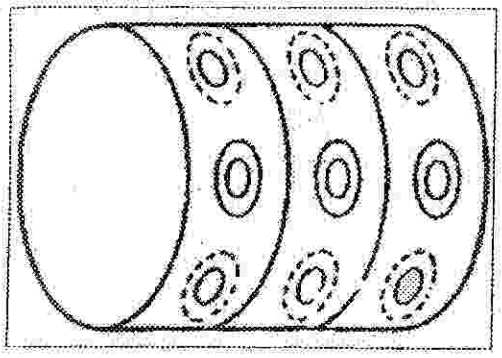

Fig. 3. Top-view of the shell with local buckling.

$$\lambda^2 = \frac{f_y}{\sigma_E}, \sigma_E = (1.5 - 50\beta)C\frac{\pi^2 E}{10.92}\left(\frac{t}{L_r}\right)^2 ; \; L_r = \frac{L}{n+1} \tag{4}$$

The factor *(1.5-50β)* expresses the effect of the initial radial shell deformation caused by the shrinkage of circumferential welds [11]. Introducing the reduction factor of β for which

$$0.01 \leq \beta = \frac{8.197 A_w 10^{-3}}{t^2} \leq 0.02 \tag{5}$$

When $t \leq 10mm, A_w = 10t$

When $t > 10mm, A_w \cong 3.05t^{1.45}$

For $\beta \leq 0.01 \quad \beta = 0.01$, for $\beta \geq 0.02 \quad \beta = 0.02$

The imperfection factor for shell buckling strength should be multiplied by (1.5-50β).
Furthermore

$$C = \psi\sqrt{1+\left(\frac{\rho_0\xi}{\psi}\right)^2}, Z = 0.9539\frac{L_r^2}{Rt}; \psi = 1, \xi = 0.702 \; Z, \rho_0 = 0.5\left(1+\frac{R}{300t}\right)^{-0.5} \tag{6}$$

It can be seen that σ_E does not depend on L_r, since in Eq. (4) L_r^2 is the denominator and in C (Eq. 6) it is in the numerator. The fact that the buckling strength does not depend on the shell length is first derived by Timoshenko and Gere [15]. Note that API design rules [2] give another formula. On the contrary, in the case of external pressure the distance between ring-stiffeners plays an important role [4,6].

4.3 Constraint on panel ring buckling (Fig. 4)

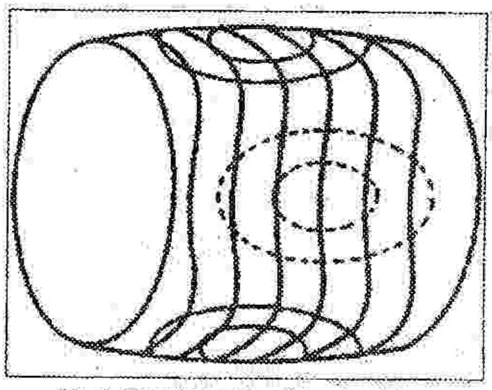

Fig. 4. Top-view of panel ring buckling.

Requirements for a ring stiffener are as follows:

$$A_r = h_r t_r \geq \left(\frac{2}{Z^2} + 0.06\right) L_r t \tag{7}$$

$$I_r = \frac{h_r^3 t_r}{12} \cdot \frac{1+4\omega}{1+\omega} \geq \frac{\sigma_{max} t R_0^4}{500 E L_r} \tag{8}$$

where,

$$R_0 = R - Y_G; Y_G = \frac{h_r}{2(1+\omega)}; \omega = \frac{L_e t}{h_r t_r}; L_e = \min\left(L_r, L_{e0} = 1.5\sqrt{Rt}\right) \tag{9}$$

4.4 Deflection constraint

$$W_{max} = \frac{5 p_0 L^4}{384 E I_x} \leq \frac{L}{500} \tag{10}$$

where I_x and the unfactored load p are, respectively:

$$I_x = \pi R^3 t;\ p_0 = 12/1.5 + 4.5/1.35 + \rho(2R\pi t + nA_r) = 11.33 + \rho(2R\pi t + nA_r) \tag{11}$$

5. Cost Function

The cost function is formulated according to the fabrication sequence. A possible fabrication sequence is as follows:

(1) Fabricate 20 shell elements of length 3 m without rings (using 2 end ring stiffeners to assure the cylindrical shape). For one shell element 2 axial butt welds are needed (GMAW-C). The welding of end ring stiffeners is not calculated, since it does not influence the variables. The cost of the forming of the shell element to a cylindrical shape is also included (K_{F0}). According to the time data obtained from a Hungarian production company (Jászberényi Aprítógépgyar, Crushing Machine Factory, Jászberény) for plate elements of 3m width, the times $(T_a + T_b)$ can be approximated by the following function of the plate thickness (Eq. 12).

$$K_{F0} = k_F \Theta \left(212.18 + 42.824t - 0.2483\ t^2 \right) \tag{12}$$

The cost of welding of a shell element is

$$K_{F1} = k_F \left[\Theta\sqrt{\kappa \rho V_1} + 1.3x0.2245x10^{-3} t^2 (2x3{,}000) \right] \tag{13}$$

where Θ is a difficulty factor expressing the complexity of the assembly and κ is the number of elements to be assembled

$$\kappa = 2; V_1 = 2R\pi t x 3{,}000; \Theta = 2 \tag{14}$$

The first term of Eq.(13)expresses the time of assembly and the second calculates the time of welding and additional works [16].

(2) Welding the whole unstiffened shell from 20 elements with 19 circumferencial butt welds

$$K_{F2} = k_F \left(\Theta\sqrt{20 \rho V_1} + 1.3x0.2245x10^{-3} t^2 x19x2R\pi \right) \tag{15}$$

(3) Cutting of n flat plate rings with acetylene gas

$$K_{F3} = k_F \Theta_c C_c t_r^{0.25} L_c \tag{16}$$

Where Θ_c, C_c and L_c are the difficulty factors for cutting, cutting parameter and length respectively,

$$\Theta_c = 3, C_c = 1.1388, L_c \approx 2R\pi n + 2(R - h_r)\pi n. \tag{17}$$

(4) Welding n rings into the shell with double-sided GMAW-C fillet welds. The number of fillet welds is 2n

$$K_{F4} = k_F \left(\Theta\sqrt{(n+1)\rho V_2} + 1.3x0.3394x10^{-3} a_w^2 x4R\pi n \right) \tag{18}$$

$a_w = 0.5 t_r$, but $a_{w\min} = 3mm$.

$$V_2 = 20V_1 + 2\left(R - \frac{h_r}{2}\right)\pi h_r t_r n \tag{19}$$

a_w is taken so that the double fillet welded joint be equivalent to the stiffener thickness.

The total material cost is

$$K_M = k_M \rho V_2 \tag{20}$$

The total cost is

$$K = K_M + 20(K_{F0} + K_{F1}) + K_{F2} + K_{F3} + K_{F4} \tag{21}$$

where $k_M = 1\$/kg; k_F = 1\$/\min$.

6. Nonlinear Programming with Fuzzy Resources

The available general model of a nonlinear programming with fuzzy resources can be formulated as:

$$\operatorname{Min} f(\mathbf{X}) \tag{22}$$

$$s.t.\ g_i(X) \leq \tilde{b}_i, i = 1,2,\ldots,m$$

$$X^L \leq X \leq X^U$$

where the objective function and the *i*th in-equality constrained function are indicated as f (**X**) and $g_i(X)$, respectively. The fuzzy number $\tilde{b}_i, \forall i$, are in the fuzzy region of $[b_i, b_i + p_i]$ with given fuzzy tolerance p_i. Assume the fuzzy tolerance p_i for each fuzzy constraint is known, then, $\tilde{b}_i$ will be equivalent to $(b_i + \theta p_i), \forall i$, where θ is in [0,1]. Several α-level cuts methods for Fuzzy Linear Programming applied to Fuzzy Nonlinear Programming are described in the following sections.

6.1 Verdegay's approach: α-level cuts method

Verdegay [17] considered that the membership function of the fuzzy constraint representing in Figure 5(a) has the following form:

$$\mu_{gi}(X) = \begin{cases} 1 & \text{if } g_i(X) < b_i \\ 1 - \dfrac{g_i(X) - b_i}{p_i} & \text{if } b_i \le g_i(X) \le b_i + p_i \\ 0 & \text{if } g_i(X) > b_i + p_i \end{cases} \quad (23)$$

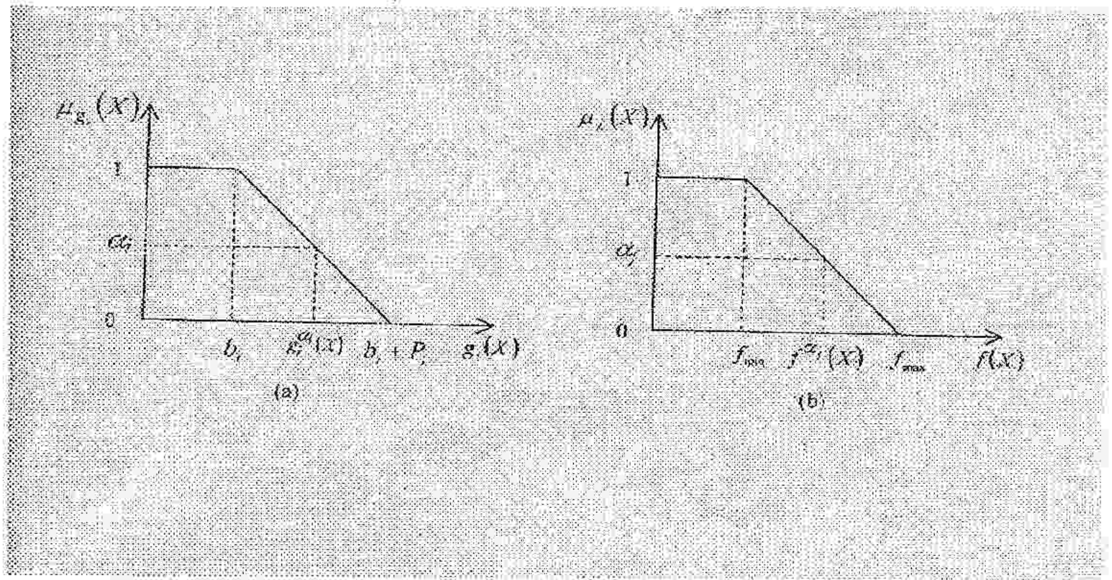

Fig.5 Membership functions $\mu_{gi}(X)$ (a) of and $\mu_f(X)$ (b) with level cut of α_i and α_f, respectively.

Simultaneously, the membership functions of $\mu_{gi}(X)$, $\forall i$, are continuous and monotonic functions, and trade-off between those constraints are allowed; then program (22) is equivalent to the following formulation:

$$\text{Min} f(\mathbf{X}) \quad (24)$$
$$\text{s.t. } X \in X_\alpha$$

where $X_\alpha = \{x | \mu_{gi}(X) \ge \alpha, \forall_i, X \ge 0$, for each $\alpha \in [0,1]$. This is the fundamental concepts of α-level cuts method of fuzzy mathematical programming. Furthermore, the membership function in Eq. (23) indicates that the more resource consumed, the less satisfaction the decision makes feel. One can replace Eq. (23) into program (24) and obtain the following formulation:

$$\text{Min} f(\mathbf{X}) \quad (25)$$
$$\text{s.t. } g_i(X) \le b_i + (1-\alpha)p_i, \forall_i$$
$$X^L \le X \le X^U \text{ and } \alpha \in [0,1]$$

Thus, program (25) is equivalent to a parametric programming formulation while $\alpha = 1 - \theta$. For each α, one will have an optimal solution; therefore, the solution with α grade of membership function is fuzzy.

6.2 Werner's approach: max- α method

Werner's [12] proposed the objective function should be fuzzy because of fuzzy total resources (or fuzzy inequality constraints). For solving program (22), one needs to define $f_{\max}$ and $f_{\min}$ follows:

$$f_{\max} = \text{Min } f(X), \text{ s.t. } g_i(X) \le b_i, \forall_i, \text{ and } X^L \le X \le X^U \quad (26)$$

$$f_{\min} = \text{Min } f(X), \text{ s.t. } g_i(X) \le b_i + p_i, \forall_i, \text{ and } X^L \le X \le X^U \quad (27)$$

The membership function $\mu_f(X)$, shown on Figure 5(b), of the objective function is stated as:

$$\mu_f(X)=\begin{cases} 1 & if \quad f(X)<f_{\min} \\ 1-\dfrac{f(X)-f_{\min}}{f_{\max}-f_{\min}} & if \quad f_{\min}\le f(X)\le f_{\max} \\ 0 & if \quad f(X)>f_{\max} \end{cases} \tag{28}$$

One can consequently apply the max-min operator to obtain optimal decision. Then, program (22) can be solved by the strategy of max- α, where α= min $[\mu_f(X), \mu_{gi}(X), \mu_{g2}(X),..., \mu_{gm}(X)]$. That is:

$$\begin{aligned} &\text{Maximize } \alpha \\ &\text{s.t. } \alpha\le \mu_f(X) \\ &\alpha\le \mu_{gi}(X), \forall_i \\ &\alpha\in[0,1] \text{ and } X^L\le X\le X^U \end{aligned} \tag{29}$$

By defining $f_{\max}-f_{\min}=p_f$, and $f_{\min}=b_f$, Eq. (28) can be written as:

$$\begin{aligned} &\text{Maximize } \alpha \\ &\text{s.t. } f(X)\le b_f+(1-\alpha)p_f \\ &g_i(X)\le b_i+(1-\alpha)p_i, \forall_i \\ &\alpha\in[0,1] \text{ and } X^L\le X\le X^U \end{aligned} \tag{30}$$

Program (30) construct a crisp NLP problem and an optimum solution can be obtained. This technique is considered as a practical method for engineering design.

6.3 Xu's approach: bound search method

Suppose there are a fuzzy goal function f and a fuzzy constraint C in a decision space **X**, which are characterized by their membership functions $\mu_f(X)$ and $\mu_C(X)$, respectively. The combined effect of those two can be represented by the intersection of the membership functions, as shown in Figure 6 and the following formulation.

$$\mu_D(X)=\mu_{f\cap C}(X)=\mu_f(X)\wedge\mu_C(X)=\min\{\mu_f(X),\mu_C(X)\} \tag{31}$$

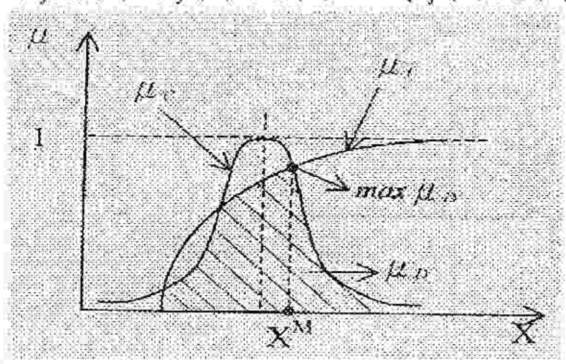

Fig. 6 Relationships of μ_f, μ_c and μ_D in fuzzy decision making

Bellman and Zadeh [18] proposed that a maximum decision could be defined as:

$$\mu_D(X^M)=\max\mu_D(X) \tag{32}$$

If $\mu_D(X)$ has a unique maximum at X^M, then the maximizing decision is a uniquely defined crisp decision. From Eq. (32)], one can obtain the particular optimum level α * corresponding to the optimum point X^M such that:

$$\mu_f(X^M)=\max_{X\in C_\alpha *}\mu_f(X) \tag{33}$$

Where C_α * is the α *-level cut of the fuzzy constraint set C. Xu [13] used a goal membership function of f(**X**) as following:

$$\mu_f(X)=\frac{f_{\min}}{f(X)} \tag{34}$$

Where $f_{\min}$ is defined by Eq. (27). It is clearly to see upper and lower bound of this goal membership function is between 1 and $f_{\min} / f_{\max}$. The optimum α * can be achieved through a simple iteration computation. This method has been called the 2nd phase of α-cut method.

6.4 Alternative Level Cut Methods for NLP Problems with Fuzzy Resources

As observed the original α-cut method (Verdegay's approach) in which each α value can yield to a set of optimum solution X_{α}. Every constrained function in program (25) has the same α value; thus, it is single α-cut approach. In Werner's approach program (29) and Eq. (30) and Figure 5, lead to the value of final α * as the minimum among $\alpha_f, \alpha_1, \alpha_2, \ldots$ and α_m. The actual value of α-level corresponding to α_f and each α_i (i=1,2,...,m) is different to final design X*. This phenomenon means that each level cut among fuzzy domain is not necessarily identical. In Xu's approach in Eq. (33) and Figure 6 to maximize $\mu_f(X)$ is equivalent to maximize α in program (29) of Werner's approach; therefore, the final results should be very close to each other. However, a nonlinear function defined in Eq. (34) of design goal that is different from the linear one in Eq. (28).

For obtaining a single solution of the original α-level cut approach in NLP problem with fuzzy resources, two alternative strategies are presented next: single α-cut approach and double α-cuts approach [19]. Each approach contains both linear membership function shown in Figure 5(b) and nonlinear membership function (Eq.35) of objective function shown in Figure 7.

$$\mu_f(X) = \frac{f_{\max} - f(X)}{f_{\max} - f_{\min}} = 1 - \frac{f(X) - f_{\min}}{f_{\max} - f_{\min}} \qquad (35)$$

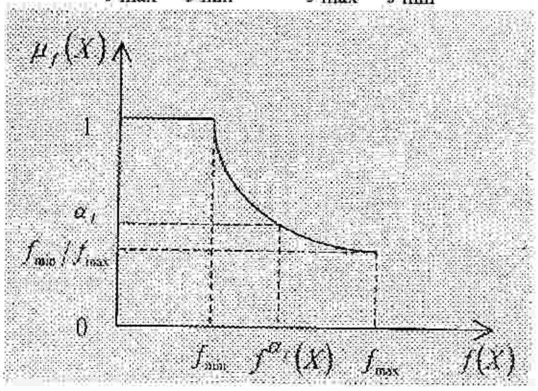

Fig.7 Nonlinear membership function μ_f of goal functions f(X)

6.4.1 Single α-cut approach

To obtain the unique solution for fuzzy problem en program (22), the mathematical formulation with directly single α-cut approach can be written as following:

$$\begin{aligned} & \text{Min } f(X) \qquad (36) \\ \text{s.t. } & f(X) - [f_{\max} - \alpha(f_{\max} - f_{\min})] = 0 \text{ (for linear } \mu_f(X)) \\ & f(X) - (f_{\min}/\alpha) = 0 \text{ (for nonlinear } \mu_f(X)) \\ & g_i(X) \le b_i + (1-\alpha)p_i, \forall_i \\ & X^L \le X \le X^U \\ & \alpha \in [0,1] \text{ (for linear } \mu_f(X)) \\ & \alpha \in [f_{\min}/f_{\max}, 1] \text{ (for nonlinear } \mu_f(X)) \end{aligned}$$

The above formulation can compare with max-α method in Eq. (30) where $\mu_f(X)$ can be nonlinear (Eq. 34) or linear (Eq. 35) membership functions.

6.4.2 Double α-cut approach

When one observe membership functions $\mu_f(X)$ and $\mu_i(X)$ (i=1,2,...,m) corresponding to objective function and constraints expressed in Figure 5, the value of $f(X)$ goes to $f_{\min}$ direction as α_f approaches to one, while the value of $g_i(X)$ goes to $b_i + p_i$ as α_i approaches to zero. This tendency in the optimization process pulls α_f back to an allowable lower limit and pushes α_i up to an allowable upper limit. This consideration guiding us develops the solution finding algorithm. Consequently, the fuzzy problem in program (22) can be solved by the following formulation:

$$\text{Min } f(X)x\frac{\alpha_f}{\alpha} \quad (37)$$

$$\text{s.t. } f(X)-\left[f_{\max}-\alpha_f\left(f_{\max}-f_{\min}\right)\right]=0 \text{ (for linear } \mu_f(X))$$

$$f(X)-\left(f_{\min}/\alpha_f\right)=0 \text{ (for nonlinear } \mu_f(X))$$

$$g_i(X)\leq b_i+(1-\alpha)p_i, \forall_i$$

$$X^L \leq X \leq X^U$$

$$\alpha_f \in [0,1] \text{ (for linear } \mu_f(X))$$

$$\alpha_f \in \left[f_{\min}/f_{\max},1\right] \text{ (for nonlinear } \mu_f(X))$$

$$0.01 \leq \alpha \leq 1$$

The program (37) for nonlinear $\mu_f(X)$ can be viewed as minimizing $f(X)$ and minimizing α_f/α simultaneously.

7. Optimum Design

The shell thickness is determined in this example by the constraints on local shell buckling. Since the number of ring-stiffeners does not influence these constraints, in order to assure a stable circular shell shape, a certain number of rings should be used. The minimum area requirement to avoid panel ring buckling is also dependent on the shell thickness and fixes the stiffener thickness. Since the design rules do not give any prescriptions for the minimum number of ring stiffeners, for the investigated case a ring number domain of n = 6 - 30 was selected.

The optimisation results show that, due to the cutting and welding costs of stiffeners, the smaller number of stiffeners is more economic. By fixing n and using the entropy-based algorithm described in [14] the optimum continuous solution is:

Table 1. Number of stiffeners, thickness of stiffeners, shell thickness and total cost

n	t_r	t	Cost
6	18.9	6.23	67300

Fuzzy goals were generated involving stress limits and loading. An increased tolerance of 25% is employed to fuzzify the loading. A membership function of inclined straight lines is adopted and (25) will become a parametric program. In the first phase, by solving this program with different α values yield deterministic designs together with a level of acceptability with respect to the stress limits fuzziness. In the second phase, the crisp solution is obtained by maximazing the membership value.

n	t_r	t	Cost
6	19.7	6.78	67300

M = W(1) = 72570. m = W (0) = 67300. $0.927 \leq \alpha \leq 1$

$\alpha = 0.932$ W(α) = 72210. and μ_G = m/W(α) = 0.932

Once the optimum α is known the sizing variables are obtained. The fuzzy optimum is very close to the original deterministic solution.
The influence of 10% tolerance in the stress limits is even less significant.

n	t_r	t	Cost
6	18.9	6.21	67140

M = W(1) = 67300. m = W (0) = 67140. $0.998 \leq \alpha \leq 1$

However shell and stiffener thicknesses should be rounded to 1 mm. A branch and bound strategy is then used to find the optimum discrete solution.

n	t_r	t	Cost
7	19	7	75320

Given it is not feasible to choose a shell thickness of 6 mm, the increase in t leads to more rings being needed to ovserve the design requirements. Both to increase the number of rings (and reduce t_r) and the opposite are associated with more expensive solutions. Material cost is about half of total cost and is insensitive to the variation of the rings. The forming cost of the shell elements is significant.

11. Acknowledgement

The research work was supported by the Hungarian-Portuguese Intergovernmental S&T Co-operation programme. The Hungarian partner is the Ministry of Education, R&D Deputy Undersecretary of State, the Portuguese partner being the Ministry of Universities and Technology, GRICES.

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