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Earthquake-resistant optimisation of Cable-Stayed Bridges

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ABSTRACT

A procedure for the application of structural optimisation to cable-stayed bridges undergoing seismic pseudo-static loads is discussed in this paper. These seismic loads are computed according to the Portuguese Safety and Actions Code of Practice (RSA) through a modal analysis and response spectra approach. The Complete Quadratic Combination Method is used to combine modes contributions. The seismic behaviour of a structure depends on its stiffness and mass properties and these change throughout the optimisation process together with the design variables (which are cross-section or geometry-related parameters). The sensitivity analysis is conducted by analytical means, the seismic forces set being evaluated for all updated intermediate structures, as well as their derivatives. An integrated analysis-optimisation computer program is briefly described and an application example is included.

INTRODUCTION

Large structures such as cable-stayed bridges are a natural domain for the application of optimisation techniques. Nevertheless, because of negative aspects such as the lack of proper software for this specific design area and the discouraging mathematical complexities of multi-purpose optimisation packages for the ordinary structural designer, optimisation is far from being watched as a current design resource.

The first author is developing an integrated analysis-optimisation program where most of repetitive and time-consuming tasks are automated so as to demand a minimum effort and specific knowledge from the user. The practicality of such program will depend strongly on the adequacy of the numeric/mathematical model to simulate real life conditions. Therefore, particular attention was paid to specific and conditioning aspects of cable-stayed bridges design, such as consideration of erection stages, non-linear behaviour of cables and automatic evaluation of seismic action. The subject of this paper concerns mostly this last feature.

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PC and DOS were the selected platform and environment. This option was considered because of the great hardware autonomy and code portability it provides and of the existence of a number of third-part applications for pre- and post-processing tasks. The severe memory restrictions of DOS alone were overcome by the use of PharLap DOS-Extender. The public domain program MODULEF (INRIA, 1991) was used as analysis core, being subjected to a filtering and downsizing process, in order to remove the unnecessary parts and reduce the code size. The equivalent Ernst modulus procedure was adopted to represent cables behaviour and a three dimensional linear elastic model was chosen to model the bridge. The element library contains 2- and 3-D bar and beam elements and plate-membrane 4- and 8-noded DKT elements, allowing for various types of bridge modelling.

DATA GENERATION

Throughout the optimisation process and depending on the pre-selected problem conditions, the element mesh, mechanical properties assignments and loadings may need to be updated. These tasks require a considerable amount of processing time if performed by hand or by any external resource, increasing the risk of human errors. Automatic updating routines are therefore provided.

The meshing routine is activated by the existence of geometric design variables. Because some geometry conditions must be satisfied whatever the geometry is (such as spacing and position of transverse beams or diaphragms), a remeshing has to be done to change, insert or remove a number of nodes for fitting the required condition.

Mechanical and cross-section parameters updating routine is invoked whenever design variables of this type are present.

Structure-independent (the case of live loads) and structure-dependent (the case of seismic loads) loadings require different procedures. For the first type no updating is required, since it remains constant through the whole optimisation process. Seismic loads, however, depend on the mass and stiffness structure characteristics and since these change continuously according to the values of the design variables, they need to be recomputed at each iteration, as well as their sensitivities. Self-weight, although a structure-dependent loading, is dealt with in the same way as live loads, because it is an explicit function of the mechanic and geometric properties of the structure, which are always known.

DESIGN VARIABLES

Several types of design variables (Simões, 1994) were considered and their sensitivities programmed at the element level, each type corresponding to a geometric or cross-section parameter. From the available set the user will select those types needed for his specific problem.

Explicit part of sensitivities are computed in an element basis, simultaneously with element stiffness, mass and right-hand side matrices. A *dependency matrix*, automatically generated at pre-processing time, establishes the correspondence between elements and design variables.

SENSITIVITY ANALYSIS

For sensitivity analysis the analytical procedure was carried out by one, in view of more accurate results and reduction in processing time. Differentiation of the equilibrium equations leads to

$$\frac{\partial \underline{\mathbf{K}}}{\partial x}\underline{\mathbf{u}} + \underline{\mathbf{K}}\frac{\partial \underline{\mathbf{u}}}{\partial x} = -\frac{\partial \underline{\mathbf{P}}}{\partial x} \qquad \frac{\partial \underline{\mathbf{u}}}{\partial x} = \underline{\mathbf{K}}^{-1}\underline{\mathbf{Q}}_P \qquad \text{with} \qquad \underline{\mathbf{Q}}_P = -\frac{\partial \underline{\mathbf{P}}}{\partial x} - \frac{\partial \underline{\mathbf{K}}}{\partial x}\underline{\mathbf{u}} \tag{1}$$

The second term of the virtual pseudo-load expression is not explicit and must be computed after solving the system equation for the real load cases. The first term may be either explicit or not, depending on the kind of loadings. So, where dead and live loads only are considered, it may be directly evaluated at element level, simultaneously with mass, stiffness and real right-hand sides. If seismic action is to be considered, we now have

$$\underline{\mathbf{P}} = \underline{\mathbf{P}}(\underline{\mathbf{X}},\underline{\mathbf{W}}) \qquad \text{and the first term becomes} \qquad \frac{\partial \underline{\mathbf{P}}}{\partial x} + \sum_{L}\frac{\partial \underline{\mathbf{P}}}{\partial \omega}\frac{\partial \omega}{\partial x} \tag{2}$$

expressions in which $\underline{\mathbf{X}}=\{x_1, x_2, \ldots, x_{NV-1}, x_{NV}\}^T$ and $\underline{\mathbf{W}}=\{\omega_1, \omega_2, \ldots, \omega_{L-1}, \omega_L\}^T$ are the design variable set and the vector of L eigenfrequencies inside the desired range. It is then necessary, in this case, to make an intermediate resolution of the eigenproblem and its sensitivities, which is much more expensive then the ordinary analysis-optimisation procedure.

Objectives (stresses) sensitivities are finally computed from the addition of both explicit and implicit contributions of the F.E. stress vector sensitivities

$$\frac{\partial \sigma}{\partial x} = \frac{\partial}{\partial x}\left(\underline{\mathbf{DB}_e\mathbf{u}_e}\right) \Rightarrow \frac{\partial}{\partial x}(\mathbf{DB}_e)\mathbf{u}_e + \mathbf{DB}_e\frac{\partial \mathbf{u}_e}{\partial x} \tag{3}$$

SEISMIC ANALYSIS BY RESPONSE SPECTRA AND MODE SUPERPOSITION

When seismic analysis is required, both step-by-step methods or modal analysis may be used.

Spectral analysis associated with modal analysis is accepted in most codes of practice and has some comparative advantages with respect to step-by-step method: It generates a smaller amount of data for the analysis-optimisation module (corresponding to a load case for each seismic load case and each design variable); It may be used recursively as a black box where input data and results are exchanged with the main analysis and optimisation core, with no need of significant changes of this one. Therefore, all the main memory may be set free at the end of a task to be used by the other, and memory is critical in the used environment. However, its use is limited to linear elastic behaviour.

Complete Quadratic Combination Method (CQC) was used for seismic forces evaluation.

PSEUDO-STATIC SEISMIC FORCES EVALUATION

The evaluation of seismic loads follows the usual procedure of mode superposition. It is supposed that eigenvalues/vectors in the required range are available, the subspace or Bathe method (Bathe, 1976) having been used for that purpose. Using the response spectra approach, and denoting by $\underline{\mathbf{M}}$ and $\underline{\mathbf{f}}_n$ the mass matrix and the eigenvector related to mode n, L_n the modal participation factor, M_n the generalised mass and $S_{pa,n}$ the spectral acceleration (extracted from the response spectra) for the same mode, the maximum seismic force contribution of mode n is known to be

$$\underline{\mathbf{f}}_{n(max)} = \underline{\mathbf{M}}\,\underline{\phi}_n \frac{L_n}{M_n} S_{pa,n} \tag{4}$$

CQC (Wilson, 1981) method is then used to obtain the maximum design forces

$$f_k = \sqrt{\sum_i \sum_j f_{ki}\,\rho_{ij}\,f_{kj}} \qquad \text{with} \qquad \rho_{ij} = \frac{8\xi^2 (1+r)\,r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} \tag{5}$$

k denoting component k of the maximum forces vector and ξ the damping coefficient, assumed to be constant for all modes. r is the frequencies ratio $r=\min(\omega_i,\omega_j)/\max(\omega_i,\omega_j)$.

PSEUDO-STATIC FORCES SENSITIVITY ANALYSIS

Sensitivities for the seismic maximum design forces (5) are obtained by chain derivation of that equation, in which all the parameters involved are explicit or implicit functions of $\underline{\mathbf{X}}$:

$$\frac{\partial f_k}{\partial x} = \frac{1}{2f_k} \sum_i \sum_j \left(\frac{\partial f_{ki}}{\partial x} \rho_{ij}\, f_{kj} + f_{ki} \frac{\partial \rho_{ij}}{\partial x} f_{kj} + f_{ki}\, \rho_{ij} \frac{\partial f_{kj}}{\partial x} \right) \tag{6}$$

where a non zero value of f_i will be ensured either because of the processor idiosyncrasies or by imposing some twilight minimum. Maximum modal forces derivatives are obtained from derivation of equation (4), leading to

$$\frac{\partial \underline{\mathbf{f}}_n}{\partial x} = \underline{\mathbf{M}} \left[\frac{\partial \underline{\phi}_n}{\partial x} \frac{L_n}{M_n} S_{pa,n} + \underline{\phi}_n \left(\left(\frac{1}{M_n} \frac{\partial L_n}{\partial x} - \frac{L_n}{M_n^2} \frac{\partial M_n}{\partial x} \right) S_{pa,n} + \frac{L_n}{M_n} \frac{\partial S_{pa,n}}{\partial x} \right) \right] + \frac{\partial \underline{\mathbf{M}}}{\partial x} \underline{\phi}_n \frac{L_n}{M_n} S_{pa,n} \tag{7}$$

This expression requires finding modal participation factors, generalised modal mass and spectral acceleration derivatives. The former two are obtained by using

$$\frac{\partial L_n}{\partial x} = \left[\frac{\partial \underline{\phi}_n^T}{\partial x} \underline{\mathbf{M}} + \underline{\phi}_n^T \frac{\partial \underline{\mathbf{M}}}{\partial x} \right] \underline{\mathbf{r}} \qquad \frac{\partial \underline{\mathbf{M}}_n}{\partial x} = \left[\frac{\partial \underline{\phi}_n^T}{\partial x} \underline{\mathbf{M}}\, \underline{\phi}_n + \underline{\phi}_n^T \frac{\partial \underline{\mathbf{M}}}{\partial x} \underline{\phi}_n + \underline{\phi}_n^T \underline{\mathbf{M}} \frac{\partial \underline{\phi}_n}{\partial x} \right] \tag{8}$$

If an orthonormal set of eigenvectors is used, then the generalised mass always equals 1 and its derivative vanishes.

Concerning spectral acceleration derivative and since the response spectra is built from empirical and experimental data, no explicit analytic expression is available. To overcome this difficulty, an analytical expression was fitted to the discrete available data which is composed by a non-linear followed by a constant value curve. The approximation made to the response spectra is such that no discontinuity occurs both in spectral acceleration and its derivative. With a 8th degree polynomial the error bounds were less then 1%.

As to CQC expression derivatives, we have

$$\frac{\partial}{\partial x}\rho_{ij} = \frac{\partial}{\partial x} \frac{8\xi^2 (1+r)\,r^{3/2}}{(1-r^2)^2 + 4\xi^2 r(1+r)^2} = \frac{\partial}{\partial x}\frac{A}{B} = \frac{\frac{\partial A}{\partial x} B - A \frac{\partial B}{\partial x}}{B^2} \tag{9}$$

$$\frac{\partial A}{\partial x} = 8\xi^2 \left(r^{3/2} + \tfrac{3}{2}(1+r) r^{1/2} \right) \frac{\partial r}{\partial x} \qquad \frac{\partial B}{\partial x} = 4(1+r)\left[r(1+3\xi^2) + \xi^2 - 1 \right] \frac{\partial r}{\partial x} \tag{10}$$

For the derivative of the frequencies ratio, one has

$$\frac{\partial r}{\partial x} = \frac{\frac{\partial \omega_j}{\partial x}\omega_i - \frac{\partial \omega_i}{\partial x}\omega_j}{\omega_i^2} \qquad \omega_j =< \omega_i \tag{11}$$

The eigenfrequencies are related with the eigenvalues through $\lambda=\omega^2$

$$\frac{\partial \omega}{\partial x} = \frac{1}{2\sqrt{\lambda}}\frac{\partial \lambda}{\partial x} \tag{12}$$

the eigenvalues sensitivities being, finally, evaluated through a numeric process.
For this purpose the method described by (Haftka, 1992) is used:

$$\frac{d\lambda_n}{dx} = \frac{\underline{\phi}_n^T\left[\frac{d\mathbf{K}}{dx} - \lambda\frac{d\mathbf{M}}{dx}\right]\underline{\phi}_n}{\underline{\phi}_n^T\mathbf{M}\underline{\phi}_n} \tag{13}$$

Eigenvector derivatives are also required, and the method based on eigenvector linear combination was adopted, because a limited number of terms provide enough accuracy for our purpose. The eigenvector derivative expression is then obtained from the expansion

$$\frac{\partial \underline{\phi}_n}{\partial x} = \sum_{j=1}^{P} c_{nj}\,\underline{\phi}_j \qquad (P =< L) \tag{14}$$

In this expression, P is the number of eigenvectors to be considered and a few terms only may be used, depending on the desired accuracy level. Note than an higher number of modes is needed for evaluating (14) than the combined CQC forces, because there is a loss of accuracy of numeric evaluation of derivatives and its values strongly condition the whole process. c_{nj} evaluates by

$$c_{nj} = \frac{\underline{\phi}_j^T\left(\frac{d\mathbf{K}}{dx} - \lambda_n\frac{d\mathbf{M}}{dx}\right)}{(\lambda_n - \lambda_j)\,\underline{\phi}_j^T\mathbf{M}\underline{\phi}_j} \qquad j \neq n \qquad c_{nn} = -\frac{1}{2}\underline{\phi}_n^T\frac{d\mathbf{M}}{dx}\underline{\phi}_n \tag{15}$$

OPTIMISATION METHOD

The cable-stayed design problem is posed in a multi-objective optimisation format with goals of minimum cost and stress and a Pareto solution is sought. This problem is equivalent to a minimax formulation which is discontinuous and non-differentiable, both of which attributes make its numerical solution by direct means difficult. An entropy-based technique (Templeman, 1989) is used to determine the minimax solution via the minimisation of a convex scalar function. Although the scalar function is unconstrained and differentiable, the stress goal functions do not have an explicit algebraic form. Explicit approximations were made by taking Taylor series expansions truncated after the linear term. Assigning the normalised volume (or cost) to objective g_1 and normalised stresses and/or displacements all over the structures to objectives $g_2, g_3, \ldots, g_J$, one can represent this scalar function and its linearized approximation by

$$F(\underline{X}) = \frac{1}{\rho}\ln\left(\sum_{j=1,J} e^{\rho g_j}\right) \qquad\qquad F(\underline{X}) = \frac{1}{\rho}\ln\left\{\sum_{j=1,J} e^{\rho\left[g_j(\underline{X}_o) + \sum_{i=1,NV}\frac{\partial g_j}{\partial x_i}(x_i - x_{oi})\right]}\right\} \qquad (16)$$

ρ is a positive control parameter initially set by the user at values usually in the range 10-100. During the iterative process it must be increased so as to ensure that a minimax solution is found for function (16). Index 0 in the approximation function denotes starting values, computed at the analysis step. The solution of this equation constitutes the starting design for the next iteration and convergence is attained when the objective function $F(\underline{X})$ decrease from one iteration to the following is smaller then some pre-defined value.

APPLICATION EXAMPLE

With the purpose of testing the global behaviour of this method, an example with simplified conditions was modelled. It consists of a 250m long steel cable-stayed bridge, the central span being 130m. Its geometry is represented in Figure 1. The initial mechanical characteristics were set up by full stress design, considering two asymmetric I-typed cross-sections for main girders supporting transverse symmetric I beams which in turn support the deck. The effect of transversal stiffening of the deck was modelled by placing weightless diagonal bars between the transverse beams throughout the span. Pylons cross section are of hollow rectangular type, different sections being considered below and above the deck level. An hinged deck-to-pylon connection was considered in left pylons, while a single vertical connection was considered for the right-hand side. No erection stages were considered for the seismic loading. A bracing beam was introduced at the top of pylons to increase their transverse stiffness. A semi-harp arrangement was adopted for the cables, anchorages being spaced 10m on side spans and 15m on central span, while spacing is 5m in the pylons. The transverse deck beams are also spaced 5m. Allowable stress for deck and pylons steel was assumed to be 200MPa, while for the stays a value of 500MPa was considered. All the types of structural elements were given the same unit volume cost.

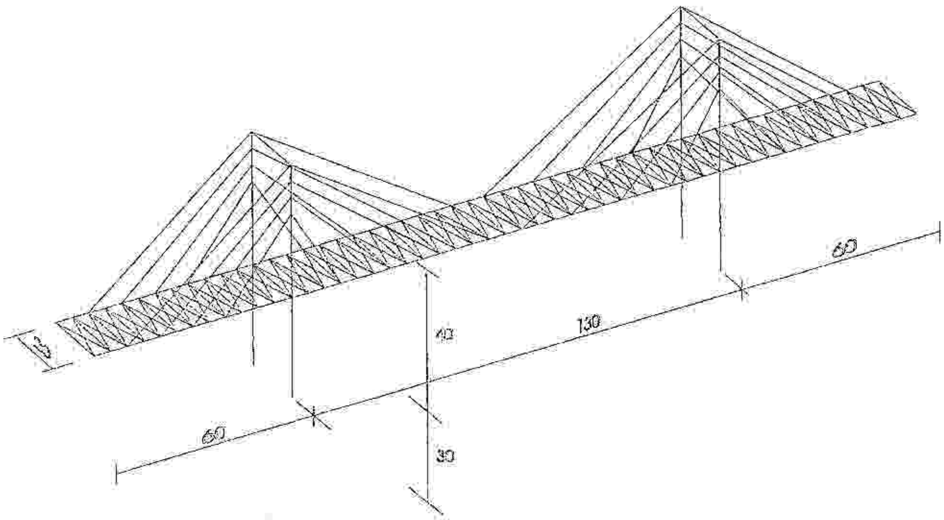

Figure 1

A uniformly distributed live load on deck with a value of $4KN/m^2$ and seismic action acting both in longitudinal, transversal and vertical directions were the considered load cases, for the latter, according to the RSA prescriptions, the response spectra ordinates being reduced in 1/3. Dead loads associated with current cross sectional area values and a constant uniformly distributed load of $6KN/m^2$ for the deck slab self-weight were considered. Seismic actions of types 1 (moderate seismic magnitude and a small focal distance) and 2 (high magnitude and a distant focus) were considered. The erection site was assumed to be located at zone A (with strongest seismic intensity) and a moderately stiff foundation was assumed. For reasons of simplicity, no spatial variability of seismic action was considered, although the span value recommends its evaluation.

22 design variables were used, their meanings, initial and final values being listed in Table 1. All design variables are of mechanical types. Italic typeface stands for lower bound of design variables. No upper bounds were active at the optimum.

Table 1 - Design variables set

x_i	Meaning	Initial	Final
1	Upper flange width in side spans girders	1.2000	0.6380
2	Bottom flange width in side spans girders	1.2000	1.4390
3	Upper flange thickness in side spans girders	*0.0200*	*0.0200*
4	Bottom flange thickness in side spans girders	*0.0200*	*0.0200*
5	Height of side spans cross section	2.0000	2.1320
6	Web thickness of side spans cross section	*0.0200*	*0.0200*
7	Upper flange width in central span girders	1.2000	0.6380
8	Bottom flange width in central span girders	1.2000	1.6310
9	Upper flange thickness in central span girders	*0.0200*	*0.0200*
10	Bottom flange thickness in central span girders	*0.0200*	*0.0200*
11	Height of central span cross section	2.0000	1.0630
12	Web thickness of central span cross section	*0.0200*	*0.0200*
13	Height of hollow section of bottom pylons	2.5000	*1.5000*
14	Width of hollow section of bottom pylons	2.5000	3.7360
15	Top/bottom walls thickness of bottom pylons	0.0300	*0.0200*
16	Side walls thickness of bottom pylons	0.0300	0.0300
17	Height of hollow section of top pylons	2.5000	*1.5000*
18	Width of hollow section of top pylons	2.5000	*1.5000*
19	Top/bottom walls thickness of top pylons	*0.0200*	*0.0200*
20	Side walls thickness of top pylons	*0.0200*	*0.0200*
21	Side spans stays cross section	0.0150	*0.0080*
22	Central span stays cross section	0.0150	*0.0080*

The shape modes 1 to 5 for the initial design are represented in Figure 2. It may be seen that the design variables which affect most strongly the longitudinal stiffness (and therefore mode 1) were greatly reduced, resulting in a significant reduction in frequency and, consequently, in the associated pseudo-static forces. Of course, this leads to a flexible solution in longitudinal direction, because no constraints were formulated for the mid-span and top of towers displacements.

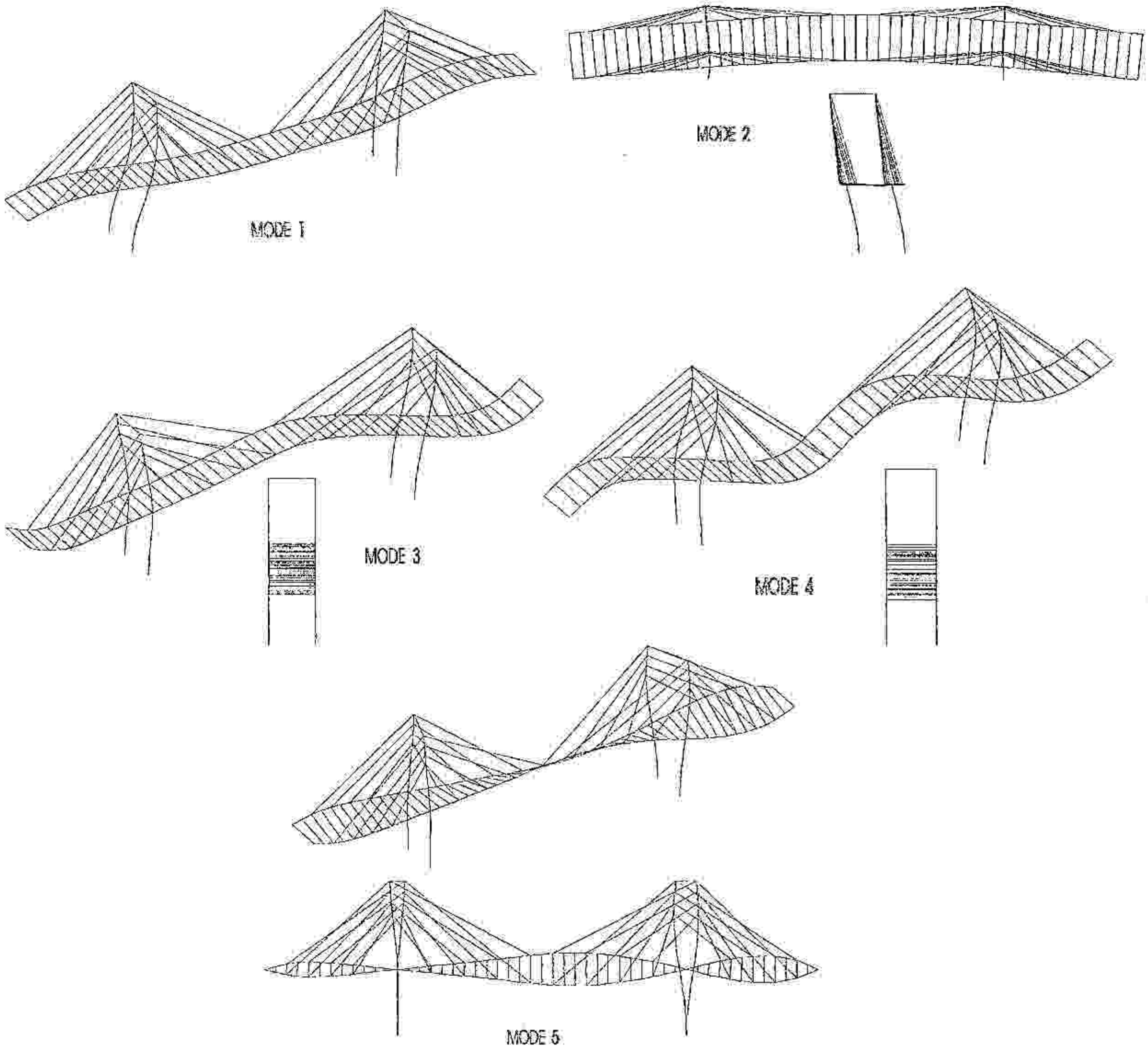

Figure 2

The main 5 modes remain the same, but modes 2 and 3 change their relative position (optimised structure is relatively stiffer in the transversal direction). Initial and final values of first 5 eigenfrequencies are listed in Table 2. Base shear is reduced from the initial value of 2805KN to 1720KN after optimisation. A global cost reduction of 22.5% with respect to the starting value is achieved. It may be seen that the average reduction in seismic forces is greater then that of gravity loads, which indicate a more favourable distribution of mass and stiffness. Initial and final maximum stress distribution in girders and pylons is shown in Figure 3. It can be seen that a further increase in the solution could result if the girders cross-sections were allowed to vary through central span. The same conclusion can be drawn to the pylons.

Table 2 - Frequencies (Hz) for first five modes

	f_1	f_2	f_3	f_4	f_5
Initial	0.370	0.552	0.620	0.968	1.103
Optimised	0.214	0.553	0.470	0.664	0.785

Contributions from 5 first modes were considered for the evaluation of seismic forces. Corresponding eigenvectors sensitivities were computed according to expression (14), all the available eigenvectors (59) in the specified range ($\lambda = < 500$ or $f = < 3.55$Hz) being considered for the expansion.

Execution took nearly 6 hours of processing time until convergence occurred at iteration number 6. With non-seismic loadings only, about 20% of this time is needed, showing that the solution of eigenproblem and modal analysis (including sensitivities evaluation) are the most expensive tasks in the process.

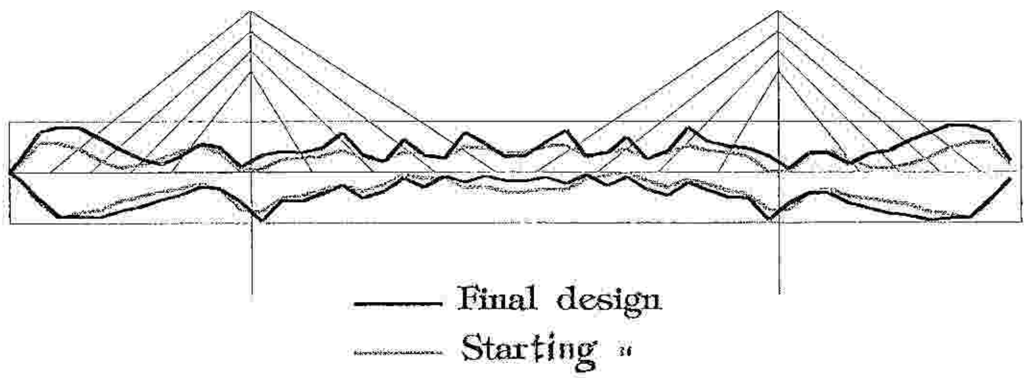

Figure 3 - Maximum stresses on deck girders

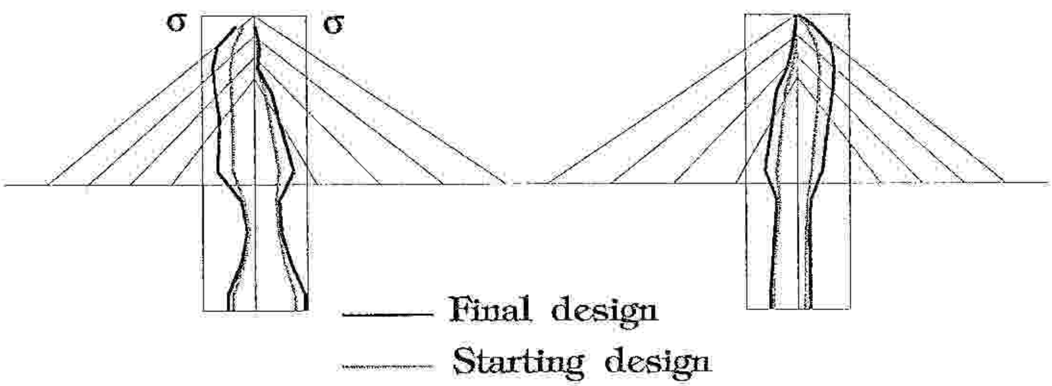

Figure 4 - Maximum stresses on pylons

CONCLUDING REMARKS

Although the evaluation of seismic loadings is a major and probably the most time-consuming task in cable-stayed bridges structural analysis, the use of an integrated analysis-optimisation program can greatly enhance the efficiency of the process, allowing for improved solutions to be obtained from a starting coarse design. Considering that the Pareto solution supplied by the used optimisation method is not necessarily unique, several design alternatives can be tested just by modifying the initial design or design variable set. Some features not considered in the present example, such as consideration of erection stages, spatial variability of seismic excitation or simultaneous consideration of various seismic actions are available or currently being developed. The validity of the describe method is limited to the assumptions of the CQC approach and the procedure presented here results from direct differentiation of its expressions.

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