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Shape and sizing optimization of box-girder decks of cable-stayed bridges

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Abstract

Box-girder cross-sections are characterized by large torsional stiffness when compared with alternative solutions. This is a fundamental issue as the bridge span increases. Transverse winds may lead to critical stress conditions or cause a number of aerodynamic instability phenomena, which are related to the torsional stiffness of the deck. Moreover, in single-plane stayed bridges, the use of box-girder decks is dictated by equilibrium requirements.
The definition of the cross-section may be rather complex. There are a large number of parameters involved, load combinations, critical stress locations and the dependence of the wind loading or seismic response on the geometric and stiffness characteristics of the deck.
Mathematical programming techniques may thus be valuable tools to assess designer expertise in the preliminary design stage. In this paper, one such study is undertaken. Plate thickness and size of the box-girder, cross-sectional area and prestress in stays, anchorage positions and cross-sectional parameters of the pylons are the continuous design variables considered. The optimization problem is stated as the minimization of stresses, displacements and the bridge cost. A finite-element approach is used for structural analysis. It includes a direct analytic sensitivity analysis module that provides the structural behaviour responses to changes in the values of the design variables.
An equivalent multicriteria approach is used to solve the non-differentiable non-linear optimization problem, turning the original problem into the sequential minimization of unconstrained convex scalar functions, from which a Pareto optimum is obtained. An illustrative example is presented.

1 Introduction

Preliminary design of cable-stayed bridges is usually undertaken by the so-called *pendulum method*, e.g.Virlogeux[1]. The bridge deck is assumed to behave like a multi-span continuous beam laying on sloping supports which represent the suspending effect of the stays. These are assumed to be rigid, in order to match the undeflected shape required for dead load condition. This can be ensured by properly prestressing the stays during the erection stages. The values of the actual installing forces are determined by dismantling and back-analysing the resulting temporary structures, in an order opposite to that of the erection.
From a mechanical point of view this procedure leads to a nearly optimum design under dead load condition. In fact, bending on deck is reduced to the unavoidable local effect related to cable spacing. Thus, load is transferred to foundation mostly by axial force (tension in stays and compression in deck), which is the most effective way for using the available material. Besides, if a symmetric cable arrangement is used, forces in cables of opposite sides of the pylon are balanced and bending stresses in pylons also vanish.
However, this almost ideal behaviour no longer applies under the action of live, seismic and wind loads. These lead to a much less predictable stress envelope. Therefore, one can not ensure that the preliminary design based on the dead load effects will still be the best suited to those stresses, which actually determine the design.
Another important issue is that the traditional preliminary design provides no information on the best distribution of material for deck, where most of the structural mass is concentrated. This problem is especially complex with box-girder decks, due to its elaborated geometry and three-dimensional nature. The use of such structural solution becomes more and more frequent, as the average span of cable-stayed bridges increases or simply because of fundamental equilibrium requirements, as is the case with bridges with a single plane of stays.
The purpose of this paper is to show the potential of mathematical programming tools for providing suitable solutions for this problem. The optimization problem is posed as that of the minimization of structural cost, subjected to constraints on maximum stresses throughout the structure, non-slackness conditions for the stays and dead load geometry matching for the deck. Although the aspects concerning the optimal size and shape of the deck are emphasised, issues concerning other structural parts are also referred to, because neither part of the structure can be optimized on its own.

2 Design variables

There are a wide variety of parameters that play an important role in the structural behaviour of a cable-stayed bridge, and a virtually endless number of possible combinations of their values. Each of those requires specific numeric procedures for sensitivity analysis. The least time-consuming way to specify a starting trial design and the set of associated design variables is by referring to a *design variable library*, e.g. Negrão and Simões[2,4], Simões and Negrão[3,5]. From this the appropriate procedures are selected by simply defining links between the element mesh and the desired design variable.

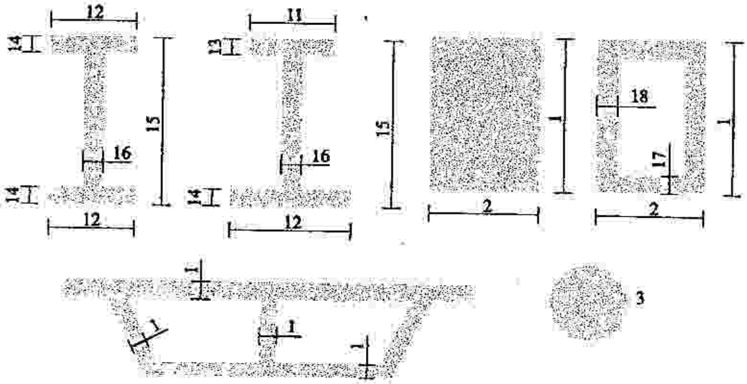

Figure 1: Sizing design variables

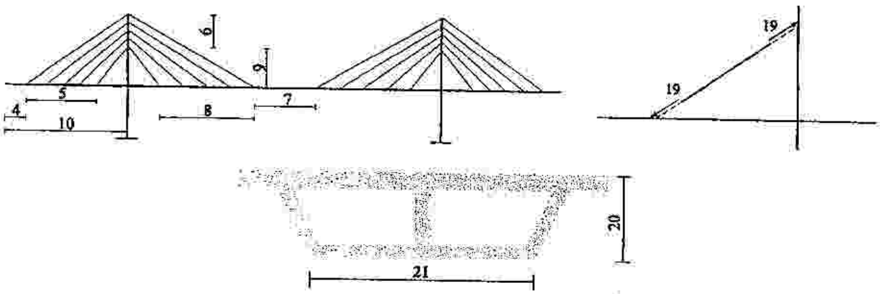

Figure 2: Shape and mechanical design variables

Figures 1-2 show the current contents of this library. Sizing design variables directly lead to weight (or cost) reduction. Its presence in the design variable set is therefore unavoidable for that purpose. Mechanical and geometric design variables are almost cost insensitive but allow for better stress distributions, which in turn lead to additional reductions in sizing variables. Other important feature of the latter is that they require remeshing after each analysis-optimization cycle, because they produce changes on nodal co-ordinates or even on element connections. Design variable types 20 and 21 are hybrid, because its use results in nodal co-ordinates updating but produces no changes in element connections and numbers.

Among the several descriptive parameters of the structure, cross section area and initial prestress of the stays play a fundamental role on the stress distribution throughout the structure, because they determine the extent of the beam-like behaviour of the deck. Besides, they are essential for a successful deflection control, which otherwise could only be achieved by a severe stiffening of the deck, in opposition to the expected reduction of material. Therefore, its use as design variables is essential in any optimization procedure.

3 Sensitivity analysis

Given the availability of the F.E. analysis code and the better accuracy, an analytical procedure was chosen for sensitivity analysis. The discrete nature of the structural topology and the large number of constraints under control dictated the choice of the discrete direct method. In this, the sensitivities of displacements are obtained from the differentiation at the element level of the stiffness method equilibrium equations:

$$\underline{K}\frac{d\underline{u}}{dx}=\frac{d\underline{P}}{dx}-\frac{d\underline{K}}{dx}\underline{u} \tag{1}$$

Derivatives of stiffness matrix, right-hand sides and strain-displacements matrices and vectors are explicitly coded at the element routines and are computed simultaneously with the quantities required for analysis. Design variables of types 20 and 21 (see Figure 2) show the particular issue of affecting the local axes cosines of some deck elements, as shown in Figure 3. This implies the use of an extended expression for the derivative of the stiffness matrix, in which the variation of the transformation matrix is accounted for:

$$\frac{d\underline{K}_G}{dx}=\frac{d\underline{A}^T}{dx}\underline{K}_L\,\underline{A}\;+\;\underline{A}^T\frac{d\underline{K}_L}{dx}\underline{A}\;+\;\underline{A}^T\,\underline{K}_L\frac{d\underline{A}}{dx} \tag{2}$$

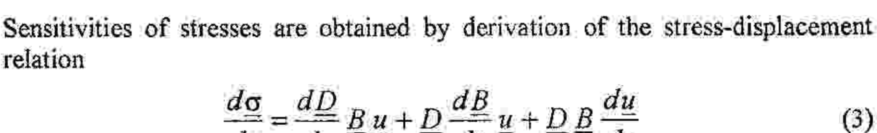

Figure 3: Dependancy of local axes cosines on design variables of types 20-21

Sensitivities of stresses are obtained by derivation of the stress-displacement relation

$$\frac{d\underline{\sigma}}{dx}=\frac{d\underline{D}}{dx}\underline{B}\,\underline{u}+\underline{D}\frac{d\underline{B}}{dx}\underline{u}+\underline{D}\,\underline{B}\frac{d\underline{u}}{dx} \tag{3}$$

The first two contributions in equation (3) become readily available from the solution of the ordinary analysis problem and are therefore called *explicit*. The last term further requires the solution of the pseudo-loading system (1) and is referred to as *implicit*. Explicit terms reflect mostly the local effect of changes of sizing variables, while implicit term represents the overall effect.

4 Optimization

The optimization problem is stated as that of the minimization of structural cost (or volume) and the maximum stresses throughout the structure. Additional objectives are aimed at the deflections at selected points, namely the anchorage nodes of the stays on deck and horizontal displacements at the top of the pylons. These criteria are essential to ensure that the optimized design will match the dead load condition. Another important set of objectives concerns the non-slackness condition for the stays. They intend to prevent the loss of structural effectiveness that may result from a low tensile strength under certain load cases, or even meaningless situations such as compression in some stays.
This is a minimax problem, which is discontinuous and non-differentiable and thus difficult to solve. However, by using the concept of entropy, Templeman[6] has shown that the solution of this problem is the same of that of the minimization of a non-linear convex scalar unconstrained function, which may be much easily solved by conventional NLP methods. This function is:

$$F(\underline{x}) = \frac{1}{\rho} \ln\left(\sum_{j=1,M} e^{\rho\, g(x)} \right) \tag{4}$$

With these conflicting objectives, the algorithm supplies a minimum in the Pareto sense. Thus, a few starting trial designs may be used for the sake of comparison of alternative solutions. M is the number of constraints. ρ is a control parameter that must be steadily increased as the current function value approaches the optimum.
For numerical purposes, equation (4) is replaced by the following approximation:

$$F(\underline{x}) = \frac{1}{\rho} \ln\left(\sum_{j=1,M} e^{\rho\left[g_j(x_0) + \sum_{i=1,N} \frac{\partial g_j}{dx_i}(x_i - x_{0i}) \right]} \right) \tag{5}$$

In (5), first order estimates of the current values of the constraints are computed by using the sensitivities provided by the solution of equations (1) and (3).

5 Numerical example

In order to illustrate the potential of this method, the symmetrical three-span cable-stayed bridge represented in Figure 4 was studied. The large span and multi-cable semi-harp pattern is representative of the present trend on such structures. There were no exhaustive studies for the selection of the preliminary design, only reasonable conditions concerning aesthetics and element sizing being accounted for. This is because the algorithm itself will naturally lead to a feasible design with respect to stresses and selected deflections.

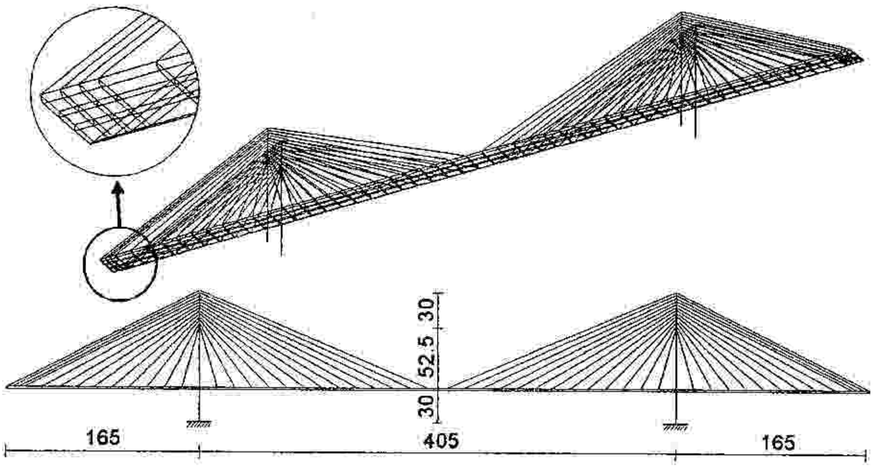

Figure 4: Geometry of the model

Torsional stiffness and aerodynamic stability requirements dictated the choice of a box-girder solution for the deck. A 24m width was considered in order to accommodate 6 lanes of traffic. Two intermediate septs divide the bottom face into three equal panels. Diaphragms are placed every 15m, to coincide with the positions of cable anchorages. Assuming a steel deck, an equivalent thickness had to be used to match the true cross-sectional area and inertia of the ribbed panels. By using fictitious values of uniform thickness, Young's modulus and mass, the actual bending and axial stiffness and self-weight values were preserved. The unit cost for the deck is adapted accordingly. Yield stresses of 200MPa were used for the deck and pylons and 500MPa for the stays. All element types were assigned the unit volume cost of 1, which is, in some degree, an arbitrary choice.

Three load cases were accounted for the sake of ultimate limit state design: live load throughout the deck, on side spans and on central span only. A uniformly distributed load of 6KN/m^2 was assigned to non-structural dead load such as the bituminous layer and fixed equipment. As to the live traffic load, a value of 4KN/m^2 was prescribed. This is obviously a minimalist approach, because wind and/or earthquake may in some instances be dominating in the design of such long bridges. However, the essential features and purposes of this paper would not significantly be emphasised by its inclusion and thus they were discarded, though the software is prepared to handle them. An additional case of dead load was also included to provide the kinematic constraints concerning the sub-problem of geometry matching.

Initial tensile fixed-end forces were prescribed for the stays. These have not a practical meaning, but they affect the erection initial stress to put the cable in place. They play a fundamental role in both controlling the deflections and providing for optimal stress distributions on deck and pylons. Differently from other structural parameters, their direct impact on the overall structural cost is not a function of a unit material cost but, instead, that concerning to the prestres-

sing equipment and staff.
The design variable set consists of 78 parameters:

- upper, bottom and side panel thicknesses in side span, central span and neighbourhood of pylons (#1-9)
- diaphragm thickness (#10)
- deck height and bottom width for the whole span (#11-12)
- sizes and thicknesses of the pylon panels for the lower, intermediate and upper stages (#13-24)
- cables cross-sectional areas (#25-50)
- cables fixed-end prestressing force (#51-76)
- distance between the deck and the lower cable anchorage in pylons (#77)
- length of the anchorage region of cables in the pylons (#78)

The values of the design variables are as indicated in Table 1.

Table 1 – Starting and optimal values of design variables

VD	*X* ini	*X* opt	VD	*X* ini	*X* opt	VD	*X* ini	*X* opt
1	10.0	5.8	27	200	206	53	2000	4523
2	10.0	5.8	28	200	189	54	2000	4157
3	10.0	19.3	29	200	114	55	2000	2520
4	10.0	6.6	30	200	109	56	2000	2381
5	10.0	7.1	31	200	91	57	2000	2006
6	10.0	7.1	32	200	108	58	2000	2480
7	10.0	5.0	33	200	173	59	2000	4274
8	10.0	5.0	34	200	174	60	2000	4671
9	10.0	11.5	35	200	137	61	2000	3964
10	10.0	5.0	36	200	122	62	2000	3713
11	1.50	1.41	37	200	76	63	2000	2332
12	21.00	22.50	38	200	97	64	2000	2610
13	5.00	7.00	39	200	100	65	2000	2465
14	5.00	7.00	40	200	107	66	2000	2437
15	5.0	8.0	41	200	125	67	2000	2633
16	5.0	8.0	42	200	132	68	2000	2356
17	4.50	6.00	43	200	165	69	2000	3102
18	4.50	6.00	44	200	126	70	2000	2299
19	4.0	6.0	45	200	126	71	2000	2267
20	4.0	6.0	46	200	195	72	2000	3521
21	4.00	5.00	47	200	217	73	2000	4000
22	4.00	5.00	48	200	212	74	2000	4000
23	4.0	6.0	49	200	199	75	2000	3873
24	4.0	6.0	50	200	166	76	2000	3351
25	200	251	51	2000	5332	77	52.50	64.50
26	200	257	52	2000	5519	78	30.00	25.82

The overall cost reduction achieved was about 16% of the initial cost. This value strongly depends on how much the minimum feasibility level was exceeded by the trial starting design. Thus, the important issue to highlight is the high effectiveness of the solution provided by the algorithm. The optimization of the deck shape and size, which is one of the main goals of this paper, resulted in only slight changes on shape but in large variations of plate thicknesses. The relatively sharp profile is a consequence of neglecting the quasi-static or aerodynamic wind effects. In such conditions, the nearly vertical slope of the lateral septs enhances the capability to carry the high cable anchorage forces and maximizes bending stiffness at the cost of almost no extra material. Figure 5 shows the starting and optimized shapes of the deck for the three regions considered.

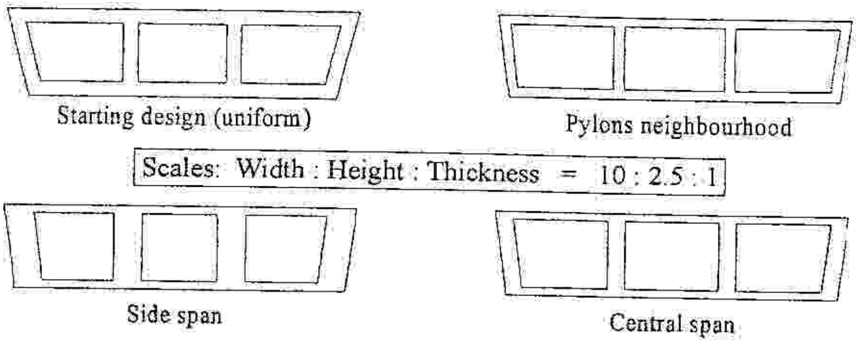

Figure 5: Comparative shapes and sizes for the deck

As to the height of the pylons, a steady trend towards the fan pattern was noticed throughout the optimization process, with anchorages getting closer and higher from iteration to iteration, which enhances the effectiveness of the cables as elastic supports for the deck. In Figure 6 one may compare the starting and optimized profile of the bridge.

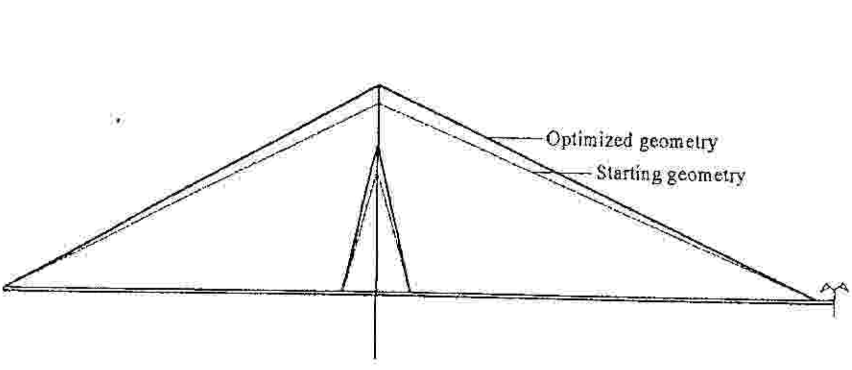

Figure 6: Comparison between starting and optimized geometry

An allowable deflection tolerance of 5cm is initially prescribed. Kinematic objectives are obtained by normalisation of the deflections throughout the deck to this value. Figure 7 compares the starting and final deflected shape of the deck. Displacements scale is 100 times that of the geometry.

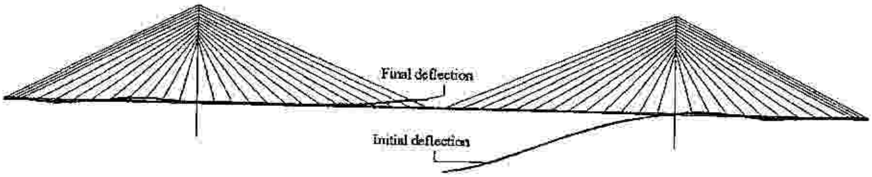

Figure 7: Deflection of the deck in starting and optimized design

6 Conclusions

Structures such as cable-stayed bridges are becoming too complex and expensive to rely only on human expertise for the stage of preliminary design. Mathematical programming tools may be very effectively used for such purpose. The large number of intervening parameters, load cases and constraints make difficult an objective choice of the proper trial design. The deck concentrates most of the structural mass of the structure and is thus obvious that design variables directly related to its weight must be considered for the sake of cost reduction. However, other parameters such as cable cross-section areas and erection prestressing forces play a fundamental role in achieving a rational solution, besides providing for an effective geometry control.

A large number of design variables enlarges the feasible search domain of the problem and is therefore recommendable. However, the choice of these parameters will strongly depend on which extent the designer accepts trading savings for uniformity. The greater the number of design variable, the greater the cost reduction but also the loss of structural homogeneity. The designer still must take this decision, but the answer is certainly easier than that of selecting all the relevant parameters out of nothing. We think it's worthy.

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