SHAPE OPTIMIZATION OF GRAVITY DAMS SUBJECT TO SEISMIC LOADING

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Summary

The purpose of this paper is to show an application of the maximum entropy formalism to the shape optimization of dams subject to seismic loading. The stresses in gravity dams due to static loads have little resemblace to the dynamic response to earthquake ground motion.The standard requirements do not predict large tensile stresses associated with cracking that may occur during earthquakes. The structural system is discretized by the finite element method and the Newmark-β time history procedure is used for the nonlinear dynamic response analysis. The hydrodynamic pressure is evaluated by using an approximation of the expressions given by Yang [1], which are a generalization of Westergaard's theory. The shape optimization problem is set in a multiobjective optimization context with goals of minimum volume of concrete, stresses and maximum safety against sliding and overturning. By using the maximum entropy formalism a Pareto solution is found indirectly by the unconstrained optimization of a scalar function. Emphasis is placed on the derivation of the sensitivity analysis equations done by the semi-analytic method and based on the step-by-step integration procedure.

Shape Representation

The boundary shape is described by piecewise polynomials such that the finite element mesh needs not to be changed during the optimization process to give accurate results. The initial geometry of a concrete gravity dam described by 3, or 5 design (shape) variables is shown in Figure 1.

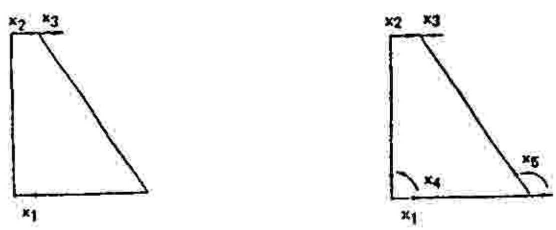

Figure 1

Structural Analysis

The gravity dam is subject to static (dead weight; self weight, water pressure, uplift) and dynamic (dead weight, water pressure, uplift, earthquake; dead weight, earthquake) loading conditions. The static structural analysis consists of solving the equilibrium equations,

$$K\,u = P \tag{1}$$

controls the overturning moment is met by imposing limits on the eccentricity of the global forces at the base of the dam. The crest of a dam must have substantial thickness to resist the shock of floating objects, to afford a roadway and to support auxiliary structures. This requirement is met by penalizing the design if the crest thickness is lower than a specified value. The objective is to minimize all of these goals over shape variables x. This is achieved by solving the minimax optimization problem:

$$\text{Min}_x \ \text{Max}_j \,(g_1, \ldots, g_j \ldots g_J) = \text{Min}_x \ \text{Max}_{j=1,J} \ < g_j(x) > \tag{6}$$

The minimax problem (6) is discontinuous and non-differentiable, both of which attributes make its numerical solution by direct means difficult. Ref.[2] explores the role of the maximum entropy formalism in minimax optimization. It is shown that a Pareto solution may be found indirectly by the unconstrained optimization of a scalar function which is both continuous and differentiable and thus considerably easier to solve.

$$\text{Min}_x \ (1/\rho) \ \log\{\Sigma_{j=1,J} \exp[\rho \ g_j(x)]\} \tag{7}$$

with a sequence of values of increasingly large positive $\rho \geq 1$. Since the goal functions $g_j(x)$ do not have explicit algebraic form in most cases, the strategy adopted was to solve (7) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(x)$ truncated after the linear term.

Sensitivity Analysis

In the analytic method of sensitivity calculation Q, u and K are all functions of x and the obtaining of the expresssions for $\partial K/\partial x_i$ and $\partial Q/\partial x_i$ involves lengthy derivation. The semi-analytic method of sensitivity analysis is used to compute the responses due to static loading and consists of the following steps:

1 - Given a proper step length vector $\Delta x_i = (0, 0,\ldots, \Delta x_i,\ldots,0)$, the difference approximation of pseudo-load vector Q_p is:

$$Q_p = \Sigma_{e\in E} \ (-K_e(x+\Delta x_i)\, u + Ke(x)\, u + P_e(x+\Delta x_i) - P_e(x)) \,/\, \Delta x_i \tag{8}$$

where subscript e denotes the eth element and E is the set of elements related to the design variable x_i.

2. - Solve $\partial u/\partial x_i$ from,

$$\partial u/\partial x_i = K^{-1} Q_p \tag{9}$$

3. - Determine the first-order approximation of displacement at design $x+\Delta x_i$,

$$u(x+\Delta x_i) \cong u(x) + \partial u/\partial x_i \ \Delta x_i \tag{10}$$

4.- Obtain the sensitivity of the response by local differences:

$$\partial R/\partial x_i \cong [\, R(x+\Delta x_i, u+\Delta u) - R(x,u)\,] \,/\, \Delta x_i \tag{11}$$

When the structure is subjected to seismic loading, the gradients of the accelerations can be computed by differentiation of equation (5) with respect to each design variable x_i. Rearranging the resulting expression, one obtains:

$$\partial \ddot{u}(t+\delta t)/\partial x_i = [M + \delta t/2\, C + \beta \ \delta t^2 K]^{-1}$$

$$\{A_1\, u(t) + A_2\, \dot{u}(t) + A_3\, \ddot{u}(t) + K \frac{\partial u(t)}{\partial x_i} + A_4 \frac{\partial \dot{u}(t)}{\partial x_i} + A_5 \frac{\partial \ddot{u}(t)}{\partial x_i} + A_6\, \ddot{u}(t+\delta t)\} \tag{12}$$

where K is the structural stiffness matrix, u is the nodal displacement vector and P the generalized load vector. Both K and P are functions of the shape variables x_i. Given the nodal displacements, a structural response R can be related to them through matrix Q:

$$R = Q\,u \qquad (2)$$

The equations of motion at time t for the dynamic structural analysis are:

$$M\,\ddot{u}(t) + C\,\dot{u}(t) + K\,u(t) = P(t) \qquad (3)$$

where M, C, and K are functions of time or constants in linear systems, while they are also functions of displacements, velocities, etc. in non-linear systems. Among step-by-step integration techniques, Newmark's β method is widely used. Let all quantities in equation (3) be known at time t. The displacements and velocities at time $t+\delta t$ are assumed as follows:

$$u(t+\delta t) = u(t)+\delta t\,\dot{u}(t) + (1/2-\beta)\,\delta t^2\,\ddot{u}(t) + \beta\,\delta t^2\,\ddot{u}(t+\delta t) \qquad (4a)$$

$$\dot{u}(t+\delta t) = \dot{u}(t)+\delta t/2\,[\ddot{u}(t) + \ddot{u}(t+\delta t)] \qquad (4b)$$

where $0 \leq \beta \leq 1/6$. The time interval δt is generally selected to be about one-sixth of the shortest natural period of the system. Substitution of equations (4a-b) into equation (3) at the time $(t+\delta t)$will give:

$$\ddot{u}(t+\delta t) = [M + \delta t/2\,C + \beta\,\delta t^2 K]^{-1}\,\{P(t+\delta t) - C[\dot{u}(t)+\delta t/2\,\ddot{u}(t)]$$
$$- K\,[u(t) + \delta t\,\dot{u}(t) + (1/2-\beta)\,\delta t^2\,\ddot{u}(t)]\} \qquad (5)$$

The finite element idealization for the initial configuration (and linear behavior) is shown in Figure 2 and consists of 18 quadrilateral elements and 77 nodes, which provide 154 degrees of freedom (90 in the dam and 64 on the foundation).

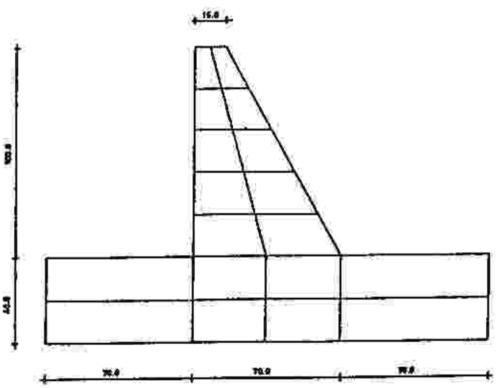

Figure 2

<u>Optimization</u>

The overall objective of concrete dam design is to achieve an economic and safe solution. The optimization method requires that all the goals should be casted in a normalized form. One of the factors influencing the economics of a design is the concrete volume. If some reference volume $\underline{V}$ is specified, it is necessary in the next iteration to reduce $\underline{V}$ further. A second set of goals arises from the requirement that the stresses should be as small as possible. A further goal comes from the imposition of stability against sliding. The global moment requirement that where, $A_1 = \partial K/\partial x_i$; $A_2 = \partial C/\partial x_i + \delta t\ \partial K/\partial x_i$

$A_3 = \delta t/2\ \partial C/\partial x_i + (1/2 - \beta)\ \delta t^2\ \partial K/\partial x_i$; $A_4 = C + \delta t\, K$

$A_5 = \delta t/2\, C + (1/2 - \beta)\ \delta t^2 K$; $A_6 = \partial M/\partial x_i + \delta t/2\ \partial C/\partial x_i + \beta\ \delta t^2\ \partial K/\partial x_i$

and $\partial K/\partial x_i, \partial M/\partial x_i$ and $\partial C/\partial x_i$ are given by finite differences. Equation (5) can be directly used for computation of the sensitivities. In fact, since the displacements and velocities can be found by substitution of equation (5) into equations (4a) and (4b), respectively, the gradients for these quantities can also be obtained by direct differentiation with respect to each design variable x_i. Thus, for the velocities and displacements, one will obtain:

$$\partial u(t+\delta t)/\partial x_i = \partial/\partial x_i\ \{u(t) + \delta t/2\ [u(t) + u(t+\delta t)]\} \qquad (15c)$$

$$\partial u(t+\delta t)/\partial x_i = \partial/\partial x_i\ \{u(t) + \delta t\, u(t) + (1/2-\beta)\ \delta t^2\, u(t) + \beta\ \delta t^2\, u(t+\delta t)\} \qquad (15b)$$

The derivatives of the accelerations and velocities are necessary to find the gradients for the displacements. Once $\partial u(t+\delta t)/\partial x_i$ are known, the structural responses R(t) can be obtained by using eq.(10) and (11).

Example

A 100 m height concrete dam having 6 m minimum thickness chosen to illustrate the optimization procedure. The optimal geometries are shown in Figure 3. The seismic effects are computed by standard static forces and dynamic analysis. Sliding, overturning and minimum thickness at the top are the critical goals in the static approach. This design fail to antecipate the large tensile stresses and associated cracking of concrete that occur due to earthquake ground motion. Maximum tensile stresses in the air face are dominant when the linear dynamic response is considered. These tensile stresses cause a substantial increase in concrete volume. Nevertheless the concrete volume can be reduced by modelling the effects of stress concentration by nonlinear yield or fracture criteria.

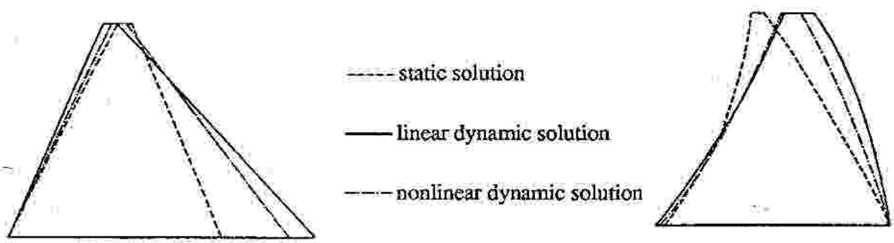

Figure 3

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