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Reliability-based optimum design of welded steel cellular plates

Luis M. C. Simões[1], József Farkas[2], Károly Jármai [3]

[1] Dep. Civil Eng., University of Coimbra, Portugal, lcsimoes@dec.uc.pt
[2] University of Miskolc, Miskolc, Hungary , altfar@uni-miskolc.hu
[3] University of Miskolc, Miskolc, Hungary , altjar@uni-miskolc.hu

1. Abstract
Cellular plates can be calculated as isotropic ones, bending moments and deflections being determined by using the classic results of isotropic plates for various loads and support types. A cellular plate subject to a uniformly distributed vertical load supported at four corners is optimized here. Half rolled I section (UB) stiffeners are used, their web being welded to the upper base plate by double fillet welds and the bottom base parts are welded to the stiffener flanges by fillet welds. The cost function to be minimised includes the costs of the materials, assembly, welding and painting. The design variables are the number and profile of stiffeners and the thicknesses of the upper and bottom base plates.
A cantilever stub column of a square box section composed of welded cellular plates is also optimized. The column is subject to compression and bending and is constructed from four equal cellular side plates. The constraints on overall buckling are formulated according to the Det Norske Veritas design rules. The horizontal displacement of the column top is limited. The minimum distance between stiffeners is prescribed to ease the welding of stiffeners to the base plate. Halved rolled I section (UB) stiffeners are used. The cost function is formulated according to the fabrication sequence.
Randomness is considered both in loading and material properties. A level II reliability method (FORM) is employed. Individual reliability constraints related with normal stress due to local bending of an upper base plate part with built-in edges, shear stress at the corners and maximum deflection are considered. The overall structural reliability is obtained by using Ditlevsen method of conditional bounding.
A branch and bound strategy coupled with a entropy-based algorithm is used to solve the reliability-based optimization. The entropy-based procedure is employed to find optimum continuous design variables giving lower bounds on the decision tree and the discrete solutions are found by implicit enumeration. Results are given illustrating the influence of the coefficient of variation of the loading and the probability of failure requirements.
2. Keywords: reliability, optimization, cellular plates, box column

3. Introduction
Cellular plates can be employed in various types of structures such as bridges, ships and buildings floors and roofs. Cellular plates have the following advantages over the plates stiffened on one side: (a) because of their large torsional stiffness the plate thickness can be decreased, which results in decrease of welding cost, (b) their planar surface is more suitable to corrosion protection, (c) their symmetric welds do not cause residual distortion. In the case of square symmetry the torsional stiffness of cellular plates equal the bending stiffness. Thus they can be calculated as isotropic and the bending moment and deflection for a square plate can be obtained by using the formulae for isotropic plates [1]. Welded cellular plates for ships investigated in [2] consist of two face sheets and some longitudinal ribs of square hollow section welded by using arc-spot technology. Later a cellular plate supported at four corners and subject to a uniformly distributed load was designed in [3]. Halved rolled I-section stiffeners were used, their web being welded to the upper base plate by double fillet welds and the bottom base parts are welded to the stiffener flanges also by fillet welds. The cost function minimized includes the costs of the materials, assembly, welding and painting. The design variables are the number and profile of stiffeners and the thicknesses of the upper and bottom base plates.
This work also covers a square box column composed of welded cellular plates. Box beams and columns of large load-carrying capacity are widely applied in bridges, buildings, highway piers and pylons. Since the thickness required for an unstiffened box column can be too large, stiffened plate elements or cellular plates should be used. It has been shown that in the case of uniaxial compression, cellular plates are more economic than longitudinally stiffened ones [4]. A cantilever column loaded by a compression force and a horizontal load is subject to compression and bending. From this loading a compression force is calculated for two opposite plate elements, while the remaining plate elements are subject to compression and bending. Since this loading is not as dangerous for the buckling of the remaining side plate elements, it is sufficient to design only the two main plate elements. The thickness and width of side plates as well as the dimensions and number of longitudinal stiffeners are calculated to fulfil the constraints and minimize the cost function which includes material and fabrication costs.

Halved rolled I-section stiffeners are used.
Stresses and displacements can be computed given the deterministic parameters of loads, geometry and material behaviour. Some structural codes specify a maximum probability of failure within a given reference period (lifetime of the structure). This probability of failure is ideally translated into partial safety factors and combination factors by which variables like strength and load have to be divided or multiplied to find the so called design values. The structure is supposed to have met the reliability requirements when the limit states are not exceeded. The advantage of code type level I method (using partial safety factors out of codes) is that the limit states are to be checked for only a small number of combinations of variables. The safety factors are often derived for components of the structure disregarding the system behaviour. The disadvantage is lack of accuracy. This problem can be overcome by using more sophisticated reliability methods such as level II (first order second order reliability method, FOSM [5] and level III (Monte Carlo) reliability methods. In this work FOSM was used and the sensitivity information was obtained analytically. Besides stipulating maximum probabilities of failure for the individual modes, the overall probability of failure which account for the interaction by correlating the modes of failure is considered.
A branch and bound strategy coupled with a entropy-based algorithm [6] is used to solve the reliability-based optimization. The entropy-based procedure is employed to find optimum continuous design variables giving lower bounds on the decision tree and the discrete solutions are found by implicit enumeration. Results are given illustrating the influence of the coefficient of variation of the loading and the probability of failure requirements.

4. Cellular plate supported at four corners

4.1. Design variables
The variables to be optimized are the number of stiffeners in one direction (square symmetry) *n*, the height of the rolled I-section stiffener *h*, the thickness of the upper and bottom base plates t_1 and t_2. Available series of rolled I sections UB profiles according to the Arcelor catalogue (sales program 2007) in the range of *h* from 607.6 to 1008.1

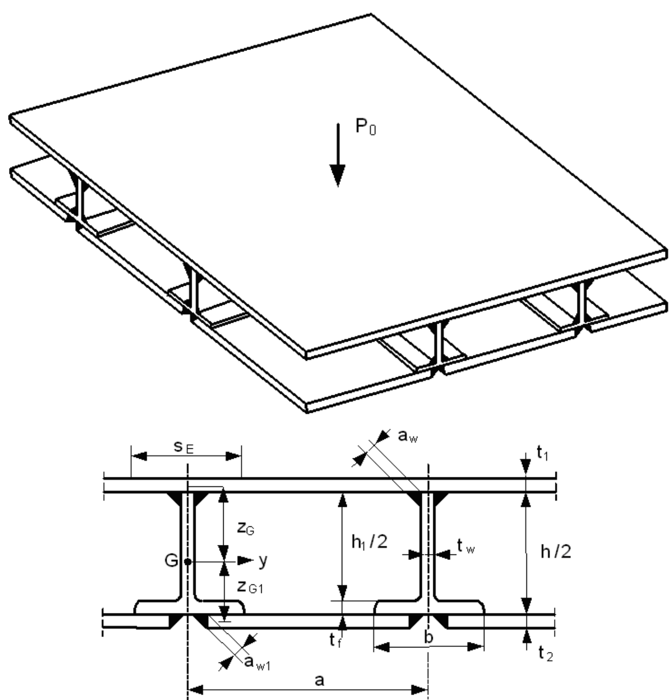

Figure 1 Cellular plate and dimensions of halved rolled I-section stiffeners

4.2. Geometric characteristics, bending moments and deflections
Effective width of the compressed upper base plate according to ECCS(1998)

$$s_E = \sqrt{\frac{E}{f_y}} \tag{1}$$

Cross sectional area of a halved rolled I-section stiffener,

$$A_s = \frac{h_1 t_w}{2} + bt_f \quad , \quad h_1 = h - 2t_f \tag{2}$$

Cross sectional area of a stiffener with upper and bottom base plate parts,

$$A = s_E t_1 + at_2 + A_S \quad , \quad a = \frac{L}{n+1} \tag{3}$$

Distances of centroid,

$$z_G = \frac{1}{A}\left[at_2\left(\frac{h}{2}+\frac{t_1}{2}+\frac{t_2}{2}\right) + bt_f\left(\frac{h_1+t_1+t_f}{2}\right) + \frac{h_1 t_w}{2}\left(\frac{h_1}{4}+\frac{t1}{2}\right)\right] \tag{4}$$

$$z_{G1} = \frac{h+t_1+t_2}{2} - z_G \tag{5}$$

Moment of inertia,

$$I_y = s_E t_1 z_G{}^2 + at_2 z_{G1}{}^2 + bt_f\left(\frac{h_1+t_1+t_f}{2} - z_G\right)^2 + I_{y1} \tag{6}$$

$$I_{y1} = \frac{h_1^3 t_w}{96} + \frac{h_1 t_w}{2}\left(\frac{h_1}{4}+\frac{t_1}{2} - z_G\right)^2 \tag{7}$$

The bending and torsional stiffness used in the Huber equation for orthotropic plates in the case of uniform transverse load are:

$$B_x = \frac{E_1 I_y}{a_y} \quad ; \quad B_y = \frac{E_1 I_x}{a_x} \quad ; \quad E_1 = \frac{E}{1-\upsilon^2} \tag{8}$$

Formulae are given for bending moments and deflections as a function of bending and torsional stiffness. In the case of a square cellular plate the bending stiffness are equal to the torsional stiffness (B_x=B_y=H) and the maximum bending moment is,

$$M_{max} = 0.15\, p\, L^2 \tag{9}$$

and the maximum deflection is expressed by

$$w_{max} = 0.025\, p_o\, L^4 / B_x \tag{10}$$

where L is the plate edge length, p_o is the factored intensity of the uniformly distributed normal load and p is the load intensity including the self mass of the plate.

Bending stiffness,

$$B_x = \frac{E_1 I_y}{a} \quad , \quad E_1 = \frac{E}{1-\upsilon^2} \tag{11}$$

Structural volumes corresponding to each fabrication phase are as follows,

$$V_1 = L^2 t_1 \quad , \quad V_2 = V_1 + (n+2)A_s \quad , \quad V_3 = V_2 + (n+2)A_s \quad , \quad V_4 = V_3 + L^2 t_2 \tag{12}$$

Load intensity including the self mass,

$$p = p_o + \frac{\rho_o V_4}{L^2} \tag{13}$$

4.3. Constraints

Stress constraint including normal stress due to local bending of an upper base plate part with built-in edges,

$$\sigma_P = 0.3078 \frac{p_o a^2}{t_1^2} \tag{14}$$

$$\sigma_2 = \frac{0.15 p L^2 z_G}{I_y} + \sigma_P \le \frac{f_y}{1.1} \tag{15}$$

$$\sigma_1 = \frac{0.15 p L^2 z_{G1}}{I_y} \le \frac{f_y}{1.1} \tag{16}$$

Shear constraint at the corners,

$$\tau = \frac{pL^2}{4 h_1 t_w} \le \frac{f_y}{1.1\sqrt{3}} \tag{17}$$

Deflection constraint,

$$w_{\max} \le w_{allow} = \frac{L}{\phi} \tag{18}$$

where the assumed ϕ is 300.

Thickness limitation,

$$t_{min} = 4 \text{ mm} \tag{19}$$

Limitation of the distance between stiffener flanges to allow the welding of the stiffener web to the upper base plate

$$a - b \geq 300mm \tag{20}$$

4.4. Cost Function

The cost function is formulated according to the fabrication sequence. Welding of the upper base plate (18x18m2) from 36 pieces of size 6 m x 1.5 m using single or double bevel welds with complete joint penetration (GMAW-C gas metal arc welding with CO_2)

$$K_{w1} = k_w\left[\Theta\sqrt{36\rho V_1} + 1.3C_1 t_1^{n_1} 13L\right] \tag{21}$$

Welding cost factor k_w = 1$7kg, factor for the complexity of the assembly $\Theta = 3$

For $t < 15$, $C_1 = 0.1939\ 10^{-3}$; $n_1 = 2$ (22a)

For $t \geq 15$, $C_1 = 0.1496\ 10^{-3}$; $n_1 = 1.9029$ (22b)

Welding of n+2 continuous stiffeners to the upper base plate by double fillet welds (GMAW-C)

$$K_{w2} = k_w\left[\Theta\sqrt{(n+3)\rho V_2} + 1.3\ 0.3394\ 10^{-3} a_w^{\ 2} 2(n+2)L\right] \tag{23}$$

$a_w = 0.4\ t_w$ but $a_{w\min} = 4mm$

Welding of n+2 intermittent stiffeners to the upper base plate and to the continuous stiffeners (webs with fillet welds, flanges with butt welds GMAW-C)

$$K_{w3} = k_w\left[\Theta\sqrt{(n^2+3n+3)\rho V_3} + T_1 + T_2\right] \tag{24}$$

$$T_1 = 1.3\ 0.3394\ 10^{-3} a_w^{\ 2}(h_1 + b)\ 2\ (n+1)\ (n+2) \tag{25}$$

$$T_2 = 1.3\ C_1 t_f^{\ n_1}\ 2\ b\,(n+1)\ (n+2) \tag{26}$$

Welding of the bottom plate parts to the flanges by fillet welds (GMAW-C)

$$K_{w4} = k_w\left[\Theta\sqrt{(n^2+2n+2)\rho V_4} + 1.3\ 0.3394\ 10^{-3} a_{w1}^{\ 2} 4L(n+1)\right] \tag{27}$$

$a_{w1} = 0.4\ t_2$ but $a_{w1\min} = 3mm$

Cost of material

$$K_M = k_M \rho V_4 \quad , \quad k_M = 1.0\$/kg \tag{28}$$

Painting cost is calculated as,

$$K_P = k_P \Theta_P S_P \quad \Theta_P = 3 \quad ; \quad k_P = 2\text{x}14.4\text{x}10^{-6}\ \$/\text{mm}^2 \tag{29}$$

Surface to be painted

$$S_P = 3L^2 + 2L(h_1 + b)(n+2) \tag{30}$$

Total Cost

$$K = K_M + K_{w1} + K_{w2} + K_{w3} + K_{w4} + K_P \tag{31}$$

4.5. Numerical data

Plate edge length L=18 m, Young modulus E =2.1 10^5 MPa, Poisson Ratio ν =0.3, steel density ρ = 7.85 10^{-5} N/mm3

5. Square box column composed of cellular plates

5.1. Design variables

The following variables should be optimized: the column width b_o, the outer and inner plate thickness t, dimensions and number of stiffners h, n. The remaining profile dimensions are determined for a series of UB sections according to the Arcelor catalogue in the range of h from 152.4 to 607.6.

5.2. Geometric characteristics, bending moments and deflections

Effective cross-sectional area

$$A_e = \frac{h_1 t_w}{2} + bt_f + 2s_{ey}t \quad , \quad s_y = \frac{b_o}{n} \tag{32}$$

Effective plate width

$$s_{ey} = s_y C \quad \text{where} \quad C = \frac{1.8}{\beta} - \frac{0.8}{\beta^2} \tag{33}$$

$$\beta = \frac{s_y}{t}\sqrt{\frac{f_y}{E}} \ \text{ if } \ \beta \geq 1 \tag{33}$$

but $\beta = 1$ if $\beta < 1$.

Distance of the centroid G

$$z_G = \frac{1}{A_e}\left[\frac{h_1 t_w}{2}\left(\frac{h_1}{4}+\frac{t}{2}\right)\right] + bt_f\left(\frac{h+t-t_f}{2}\right) + s_{ey}t\left(\frac{h}{2}+t\right) \tag{34}$$

Moment of Inertia

$$I_y = s_{ey}tz_G^{\,2} + s_{ey}t\left(\frac{h}{2}+t-z_G\right)^2 + \frac{h_1^3 t_w}{96} + \frac{h_1 t_w}{2}\left(\frac{h_1}{4}+\frac{t}{2}-z_G\right)^2 + bt_f\left(\frac{h+t-t_f}{2}-z_G\right)^2 \tag{35}$$

$$W_\xi = \frac{I_\xi}{\frac{b_o}{2}-z_G} \quad ; \quad b_1 = b_o - h - t \tag{36}$$

$$I_\xi = 2\left\langle\left[I_y + A_e\left(\frac{b_o}{2}-z_G\right)^2\right](n-1) + I_{\xi S} + \frac{b_o^{\,3}t}{12} + \frac{b_1^{\,3}t}{12}\right\rangle \tag{37}$$

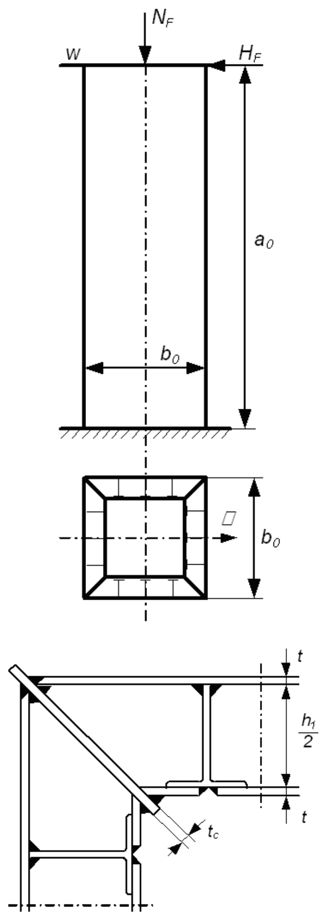

Figure 2. A cantilever stub-column of square box section with cellular side plates and the welded corner

If n is even

$$I_{\xi S} = \frac{b^3 t_f}{12}(n-1) + 2\left(bt_f + \frac{h_1 t_w}{2}\right)\sum_{i=1}^{\frac{n_y}{2}-1}\left(s_y^{\,2} i^2\right) \tag{38a}$$

If n is odd

$$I_{\xi S} = \frac{b^3 t_f}{12}(n-1) + 2\left(bt_f + \frac{h_1 t_w}{2}\right)\sum_{i=1,3,5}^{n_y-2}\left[\left(\frac{s_y}{2}\right)^2 i^2\right] \tag{38b}$$

The classic buckling force is derived from the Huber´s differential equation for orthotropic plates,

$$N_E = \frac{\pi^2}{b_o^2}\left(B_x \frac{b_o^2}{a_o^2} + 2H + B_y \frac{a_o^2}{b_o^2} \right) \quad (39)$$

Bending and torsional stiffeness,

$$B_x = \frac{E_1 I_y}{s_y} \quad ; \quad B_y = \frac{E_1 I_x}{s_x} = \frac{E_1 t(h+2t)^2}{8} \quad ; \quad H = \frac{B_x + B_y}{2} \quad (40)$$

5.3. Constraints

The buckling constraints are formulated according to the Det Norske Veritas rules [7]. Overall buckling constraint,

$$\sigma = \frac{N_F}{4A_e(n-1)} + \frac{0.1 N_F a_o}{W_\xi} \le \sigma_{cr} = \frac{f_{y1}}{\sqrt{1+\lambda^4}} \quad (41)$$

where,

$$\sigma_E = \frac{N_E s_y}{A_e} \quad ; \quad \lambda_i = \sqrt{\frac{f_{y1}}{\sigma_E}} \quad (42)$$

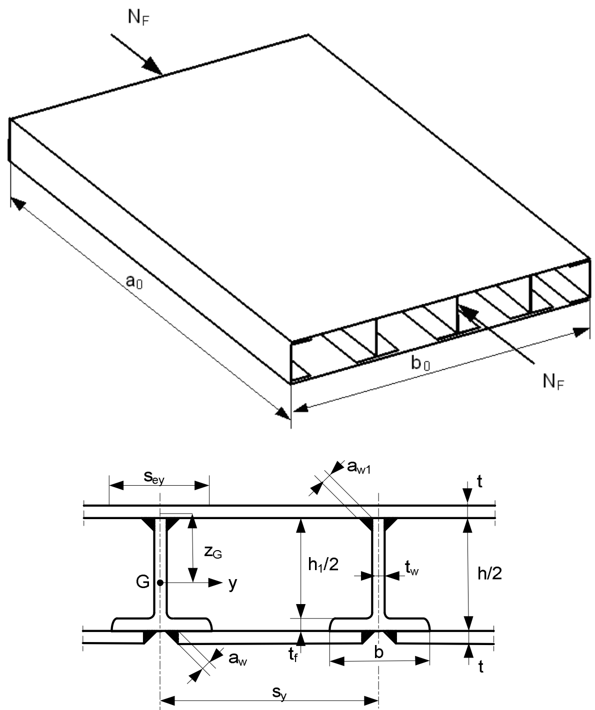

Figure 3 Cellular plate with longitudinal stiffeners

Constraint on horizontal displacement of the column top,

$$\omega_{max} = \frac{H_F}{\gamma_M} \frac{L^3}{3EI_\xi} \le \frac{L}{\phi} \quad (43)$$

ϕ is in the range 300-1000.

Constraint on local buckling of face plates connecting the transverse stiffeners,

$$t_c = \frac{\sqrt{2}h}{28\varepsilon} \quad ; \quad \varepsilon = \sqrt{\frac{235}{\sigma}} \quad (44)$$

Ranges of unknowns $4 < t < 20$ mm, $152 < h < 1016$ mm, $4 < n < n_{max}$. n_{max} is determined by fabrication constraints.

$$\frac{b_o}{n} - b_y \ge 300 \text{ mm} \quad (45)$$

5.4. Cost function

Welding of the base plate with butt welds (submerged arc welding): A fabricated plate element has sizes of 6000x1500 mm or less.

The fabrication cost factor is taken as $k_F = 1.0\$/\min$, the complexity factor of the assembly $\Theta_w = 2$

$$K_{Fo} = k_F \left[\Theta_w \sqrt{3n\rho V_o} + 1.3 C_w t^{n_1} L_{w1} \right] \tag{46}$$

$$V_o = a_o b_o t \quad ; \quad L_{w1} = 2b_o + a_o(n-1) \tag{47}$$

For $t < 11$, $C_w = 0.1346\ 10^{-3}$; $n_1 = 2$ (48a)

For $t \geq 11$, $C_w = 0.1033\ 10^{-3}$; $n_1 = 1.904$ (48b)

Welding (n-1) stiffener webs to the base plate with double fillet welds (GMAW-C gas metal arc welding with CO_2):

$$K_{F1} = k_F \left[\Theta \sqrt{n\rho V_1} + 1.3\ 0.3394\ 10^{-3} a_{w1}^{\ 2} 2a_o(n-1) \right] \tag{49}$$

a_{w1}=0.4 t_{wx} but $a_{w1\min}$ =3 mm

$$V_1 = a_o b_o t + \left(\frac{h_1 t_w}{2} + b t_f \right) a_o(n-1) \tag{50}$$

Welding (n-2) inner plate strips with butt welds,

$$K_{F2} = k_F \left[\Theta \sqrt{3\rho V_2} + 1.3\ C_W t^{n1} 2s_y \right](n-2) \tag{51}$$

$$V_2 = a_o s_y t \tag{52}$$

Welding of inner plate strips to the stiffener flanges with 2 fillet welds (excluding 2 side strips)

$$K_{F3} = k_F \left[\Theta \sqrt{(n-1)\rho V_3} + 1.3\ 0.3394\ 10^{-3} a_{w2}^{\ 2} 2a_o(n-2) \right] \tag{53}$$

a_{w2}=0.7 t but $a_{w2\min}$ =3 mm

$$V_3 = V_1 + V_2(n-2) \tag{54}$$

Welding of 4 outer plates of cellular pltes to the corner plates with 4 fillet welds,

$$K_{F4} = k_F \left[\Theta \sqrt{8\rho V_4} + 1.3\ 0.3394\ 10^{-3} a_{w2}^{\ 2} 16a_o \right] \tag{55}$$

$$V_4 = V_3 + 4t_c a_o \left(\frac{h\sqrt{2}}{2} + 3t_c \right) \tag{56}$$

Welding of 8 inner side plate strips with 3 butt welds,

$$K_{F5} = k_F \left[\Theta \sqrt{3\rho V_5} + 1.3\ C_w t^{n_1} 2\left(s_y - \frac{h}{2} \right) \right] 8 \tag{57}$$

$$V_5 = a_0 t \left(s_y - \frac{h}{2} \right) \tag{58}$$

Welding of 8 inner side plate strips to the corner plates and side stiffener flanges with fillet welds,

$$K_{F6} = k_F \left[\Theta \sqrt{9\rho V_6} + 1.3\ 0.3394\ 10^{-3} a_{w2}^{\ 2} 16a_o \right] 8 \tag{59}$$

$$V_6 = V_4 + 8V_5 \tag{60}$$

Painting cost is calculated as

$$K_P = k_P \Theta_P S_P \quad ; \quad \Theta_P = 2,\ k_P = 2\text{x}14.4\text{x}10^{-6}\ \$/\text{mm}^2 \tag{61}$$

The total cost is formulated according to the fabrication sequence and includes the cost of material, assembly, welding and painting.

$$K = K_M + 4\left(K_{F0} + K_{F1} + K_{F2} + K_{F3} + K_{F4}\right) K_{F5} + K_{F6} + K_P \tag{62}$$

The cost of material

$$K_M = k_M \rho V_6 \quad , \quad k_M = 1.0\$/kg \tag{63}$$

5.5. Numerical data

a_o=15000, Young modulus E=2.1 10^5 MPa, Shear modulus G=0.81 10^5, density ρ=7.85 10^{-6} kg/mm3, Poisson ratio ν=0.3, selected rolled I-sections UB profiles

6. Reliability-based optimization

The following assumptionsare considered: (1) the general configuration including the length of all members is specified in a fixed (deterministic) manner; (2) the failure modes are overall buckling, local buckling and maximum deflections; (3) the magnitudes of the static loads that form the load vector are random, but their locations deterministic; (4) the allowable stresses and displacements are random, but their position is deterministic. In the case the limit state function g (x) is a linear function of the normally distributed basic random variables x the probability of failure can be written in terms of the linear safety margin M as:

$$P_F = P\{g(x) \leq 0\} = P(M \leq 0) \quad (64)$$

which reduces to the evaluation of the standard normal distribution function

$$P_F = \Phi(-\beta) \quad (65)$$

where β is the reliability index given as

$$\beta = \mu_M \,/\, \sigma_M \quad (66)$$

The reliability index has the geometrical interpretation as the smallest distance from the line (or the hyperplane) forming the boundary between the safe domain and the failure domain. The evaluation of the probability of failure reduces to simple evaluations in terms of mean values and standard deviations of the basic random variables.
When the limit state function is not linear in the random variables x, the linearization of the limit state function in the design point of the failure surface represented in normalised space u must be performed,

$$u_i = \left(x_i - \mu_{x_i}\right) / \sigma_{x_i} \quad (67)$$

As one does not know the design point in advance, this has to be found iteratively. Provided that the limit state function is differentiable, the following simple iteration scheme may be followed:

$$\alpha_i = -\partial g(\beta\alpha) / \partial u_i \left[\sum_{j=1}^{n} \partial g(\beta\alpha)^2 \,/\, \partial u_i \right] \quad (68)$$

$$G(\beta\alpha_1, \beta\alpha_2, \ldots \beta\alpha_v) \quad (69)$$

which will provide the design point u* as well as the reliability index β.
In general, the admissible failure probability for structural design is very low. Commonly used approximations of the overall probability of failure are based either on the assumption of perfect statistical dependence (Cornell´s lower bound) or on that of their statistical independence (Cornell´s upper bound). These bounds may be widely different because correlation between failure modes is not included in the formulation. Ditlevsen´s method [8] which incorporates the effects of statistical dependence between any two failure modes, narrowed considerably the bounds on the system failure probability. An approximate method, which avoids calculating conditional probabilities resulting from conditions leading to failure via pais of failure modes is the PNET[9]. This method requires the determination of the coefficients of correlation between any two failure modes I and j and is based on the notion of demarcating correlation coefficient, assuming those failure modes with high correlation to be perfectly correlated and those with low correlation to be statistically independent.

7. Optimization Strategy

7.1 Branch and Bound

The problem is non-linear and the design variables are discrete. Given the small number of discrete design variables an implicit branch and bound strategy was adopted to find the least cost solution. The two main ingredients are a combinatorial tree with appropriately defined nodes and some upper and lower bounds to the optimum solution associated the nodes of the tree. It is then possible to eliminate a large number of potential solutions without evaluating them.
Three levels were considered in the combinatorial tree. The number of stiffeners *n* is fixed at the top of the tree, the remaining levels corresponding to the UB profile *h* and the thickness *t* for the square box column. b_o is obtained, once the remaining variables are known. The same levels were adopted in the cellular supported plate, t_1 and t_2 being found at the third level. A strong branching rule was employed. Each node can be branched into n_s new nodes, each of these being associated with the number of stiffeners needed in the x direction. This requires using continuous values close to the geometric characteristics of an UB section, (b, t_f, t_w), which are approximated by curve-fitting functions written as a function of *h*. The stiffener height is also obtained from a curve fitting of the heights *h*. Care has to be taken to find geometrical properties leading to convex underestimates of the actual UB section, so that the solution obtained by using the real UB geometric characteristics is more costly than the solution given by using continuous approximations. In the second level of the tree the branches correspond to different stiffener UB profiles. At the third level the resulting minimum discrete solution becomes the incumbent solution (upper bound). Any leaf of the tree whose bound is strictly less than the incumbent is active. Otherwise it is designated as terminated and need not to be considered further. The B&B tree is developed until every leaf is terminated. The branching strategy adopted was breadth first, consisting of choosing the node with the lower bound.

7.2 Optimum design with continuous design variables

For solving each relaxed problem with continuous design variables the simultaneous minimization of the cost and constraints is sought. All these goals are cast in a normalized form. For the sake of simplicity, the goals and variables described in the following deal with stiffened shells. If a reference cost K_0 is specified, this goal can be written in the form,

$$g_1(t,n,h) = K(t,n,h)/K_0 - 1 \leq 0 \tag{70}$$

Another goal arise from the reliability constraint on overall buckling for the square box column and the stress constraint due to local bending in the case of the supported plate,

$$g_3(t,n,h) = \sigma_c / \sigma_{cr} - 1 \leq 0 \tag{71}$$

The third goal deals with the reliability constraint arising from the horizontal displacement at the top column and the probability of the allowable maximum plate deflection to be exceeded,

$$g_3(t,h) = w/w_{\max} - 1 \leq 0 \tag{72}$$

The remaining goals deal with the local buckling in the square box column which can be solved by deterministic means and the probability that the shear stress at the corners exceeeds the allowable values.
The objective of this Pareto optimization is to obtain an unbiased improvement of the current design, which can be found by the unconstrained minimization of the convex scalar function [10]:

$$F(t,h) = \frac{1}{\rho}.1n\left[\sum_{j=l}^{3}\exp\rho\left(g(t,h)\right)\right] \tag{73}$$

This form leads to a convex conservative approximation of the objective and constraint boundaries. Accuracy increases with ρ.
The strategy adopted was an iterative sequence of explicit approximation models, formulated by taking Taylor series approximations of all the goals truncated after the linear term. This gives:

$$\text{Min } F(t,h) = \frac{1}{\rho}.1n\left[\sum_{j=l}^{3}\exp\rho\left(g_0(t,h) + \frac{\partial g_{0j}(t,h)}{\partial t}dt + \frac{\partial g_{0j}(t,h)}{\partial h}dh\right)\right] \tag{74}$$

This problem has an analytic solution giving the design variables changes dt and dh. Solving for a particular numerical value of g_{oj} forms an iteration of the solution to problem (74). Move limits must be imposed on the design variable changes to guarantee the accuracy of the approximations. Given the small number of design variables an analytic solution is available. During the iterations the control parameter ρ, which should not be decreased to produce an improved solution, is increased.

8. Numerical results and discussion

Consistent with the traditional limit state design (level 1 approach), yield stress of steel f_y=355 MPa were considered. With a safety factor for structural steel of 1.10 and an assumed coefficient of variation of 0.10 this corresponds to mean values of 440 MPa. Design factored load intensity p_o= 0.0015 N/mm2 for the cellular plate. Gaussian distribution was adopted for the random variables. The randomness of the Young modulus was not considered for the sake of simplicity. In the square box example the design axial load is N_x=3 10^7 N and the safety factor $\gamma_M=1.5$ must be considered to define the average loading.
In the cellular plate supported at the four corners case calculations were carried out for increasing coefficients of variation for the loading and p_F requirements. The reliability constraint associated with the maximum displacement is always mobilized, even for the lowest ϕ=300, the UB profile with higher UB being always chosen. The limit state equations governing both stress constraints and shear constraints at the corners lead to highly unlikely failure modes. The total costs are summarized in Table 1.

Table 1 Cost x 10^3. Optima are marked by bold letters

	$p_F=10^{-3}$	$p_F=10^{-4}$	$p_F=10^{-5}$
Cov=0.15	102.3	105.1	106.1
Cov=0.20	102.3	106.1	109.4
Cov=0.25	105.1	109.4	111.0

The data corresponding to these optima is represented in Table 2. There is no general conclusion concerning the optimum number of stiffeners or the trend of cost vs fabrication as the requirements become more demanding. There are close discrete solutions the best of these being selected by the implicit enumeration procedure.
In the square box composed of cellular plates calculations were carried out for increasing coefficients of variation for the loading and p_F requirements. The reliability constraint associated with the maximum displacement is always mobilized with ϕ=300 and not needed when ϕ=1000. The limit state equation governing stresses are usually unlikely in the latter case. The total costs are summarized in Table 3.

Table 2. Main dimensions (in mm) of the optimum designs

Cost x10^3	n	t_1	t_2	h
102.3	5	4	4	1008
105.1	5	4	5	1008
106.1	5	5	4	1008
102.3	5	4	4	1008
106.1	5	5	4	1008
109.4	6	4	4	1008
105.1	5	4	5	1008
109.4	6	4	4	1008
111.0	4	8	4	1008

Table 3 Cost x 10^3. Optima are marked by bold letters

	ϕ=300	ϕ=1000
$p_F=10^{-3}$ Cov=0.15	52.0	55.5
$p_F=10^{-4}$ Cov=0.20	58.1	60.2
$p_F=10^{-5}$ Cov=0.25	63.0	63.9

Table 4. Main dimensions (in mm) of the optimum designs

Cost x10^3	n	t	b_o	h
52.0	8	5	3770	454.6
55.5	8	5	4800	403.2
58.1	9	5	4400	454.6
60.2	10	5	4875	403.2
63.0	8	5	4570	533.1
63.9	10	5	4985	454.6

The minimum thickness is chosen in all designs. The most demanding displacement requirements lead to larger b_o, smaller UB profile and usually more stiffeners. Increasing p_F and Cov are associated with larger b_o, n and/or h. the choice being made by the algorithm.

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10. References

[1] J. Farkas, Discussion to the paper of Evas,R. & Shanmugam,N.E.:Simplified analysis for cellular structures, *J.Struct. Eng. ASCE*, 11 (10), 2269-2271, 1985.

[2] K. Jarmai and J. Farkas, Cost calculation and optimization of welded steel structures, *J.Const. Steel Research*, 50, 115-135, 1999.

[3] J. Farkas and K. Jármai, *Design and optimization of metal structures*. Chichester, Horwood, 2008.

[4] J. Farkas and K. Jarmai, Optimum design and cost comparison of a welded plate stiffened on one side and a cellular plate both loaded by uniaxial compression, *Welding in the World* 50 (7-8), 74-78, 2007.

[5]A.M. Hasofer and N.C. Lind, Exact and invariant second moment code format, *J.Eng. Mech Div.* 100 (1), 111-121, 1974.

[6]L.M.C.Simoes, J.Farkas and K. Jarmai, Reliability-based optimum design of a welded stringer-stiffened steel cylindrical shell subject to axial compression and bending, *Struct. and Multidis. Optim.*,31 (2), 147-155,2006.

[7] Det Norske Veritas (DNV) *Buckling strength analysis.* Classification Notes No.30.1. Høvik, Norway, 1995.

[8]O. Ditlevsen, Narrow reliability bounds for structural systems, *J.Struct. Mech*, 7 (4), 453-472, 1979.

[9] A.H.S. Ang and H. Ma, On the reliability of structural systems, Proc. *3rd Int. Conference on Structural Safety*, Trondheim, Norway, 1982.

[10] L.M.C. Simões and A.B. Templeman, Entropy-based synthesis of pretensioned cable net structures, *Engineering Optimization*, , 15, 121-140, 1989