

Sadi Carnot on Carnot's theorem

J. Güémez^{a)}

Departamento de Física Aplicada, Universidad de Cantabria, E-39005 Santander, Spain

C. Fiolhais^{b)} and M. Fiolhais^{c)}

Departamento de Física and Centro de Física Computacional, Universidade de Coimbra, P-3004-516 Coimbra, Portugal

(Received 26 February 2001; accepted 28 August 2001)

We reproduce the original calculations of Sadi Carnot that led him to formulate the so-called Carnot's theorem. We rephrase Carnot's calculations in SI units and compare his results with those obtained using modern data. © 2002 American Association of Physics Teachers.
[DOI: 10.1119/1.1412642]

I. INTRODUCTION

In his book *Reflections on the Motive Power of Fire* (1824),¹ Sadi Carnot presented calculations on the “motive power” defined as

*[...] the useful effect that an engine is capable of producing. The effect can always be expressed in terms of a weight being raised to a certain height. It is measured, of course, by the product of the weight and the height to which the weight is considered to have to be raised.*²

He considered thermal cycles operating with different substances and, based on particular cases, formulated the following general propositions:

*Wherever there is a difference in temperature, motive power can be produced.*³

*[...] the maximum amount of motive power gained by the use of steam [in a Carnot cycle] is also the maximum that can be obtained by any means whatsoever.*⁴

The motive power of heat is independent of the working substances that are used to develop it. The quantity is determined exclusively by the temperature of the bodies between which, at the end of the process, the passage of caloric has taken place.^{5,6}

According to this first statement, both a cold and a hot body are required to produce work. The second statement, equivalent to the second law of thermodynamics, says that a heat engine more efficient than the Carnot machine cannot exist. The third statement evolved to what is now called Carnot's theorem.^{7,8}

In spite of the recognition of Carnot's contribution to thermodynamics, pedagogical presentations of his original work are scarce. Several authors discuss this work, but very few consider its quantitative aspects. Truesdell should be cited⁹ for his effort to put Carnot's results in a modern perspective. On the other hand, Carnot's booklet is difficult to read because of its terminology and units. This difficulty motivated us to present some of his original calculations, using modern concepts and SI units.

Carnot based his calculations on a cycle that differs from what became known as the Carnot cycle (two adiabatic and two isothermic processes, all reversible⁸). The cycle used by Carnot was composed of two isobarics and two isochorics: we call it the “modified Carnot cycle.” We present his reasonings and calculations and discuss their validity. We also show how the results change if we use current data.

In Sec. II we describe the processes that Carnot conceived to obtain “motive power.” In Sec. III we reproduce his calculations for five cycles, using air, water, and alcohol. In Sec. IV we discuss the results, comparing them with modern ones, and summarize the pedagogical relevance of Carnot's work. In the Appendix we give some technical details.

II. MODIFIED CARNOT CYCLE

In his memoir, Carnot described the *reversible cycle*¹⁰ undergone by a fluid, with seven states,¹¹ but equivalent to what is known today as the *Carnot cycle*. Two bodies were needed: body A (the hot source) at temperature T_A , and body B (the cold source) at temperature $T_B < T_A$. The original description of the cycle is intricate,¹² but Carnot noted that such a cycle could be simplified if $T_A \approx T_B$.⁴ In all his calculations, he considered a cycle with the following stages^{13,14} (Fig. 1):

Contact between the gas abcd and the body A; movement of the piston from cd to ef.

Removal of the body A; contact between the gas in abef with the body B; return of the piston from ef to cd.

Removal of the body B; contact between the gas and the body A; movement of the piston from cd to ef. In other words, the first stage is repeated, and so on.

Figure 1 shows the *modified Carnot cycle* in a (P,V) diagram. The process 1→2 is a quasi-static isobaric expansion at pressure P_i in contact with the hot reservoir; in 2→3, the temperature decreases at constant volume V_f (irreversible isochoric); the process 3→4 is a quasi-static isobaric compression at a lower pressure P_f in contact with the cold reservoir until the initial volume V_i is recovered; finally, in 4→1, the temperature increases at constant volume (irreversible isochoric) until the initial temperature is reached.

Carnot considered cycles for air, water, and alcohol with $T_A - T_B = 1^\circ\text{C}$, which is rather small in comparison with T_A and T_B . As we shall see, replacing the Carnot cycle by the modified Carnot cycle is quite reasonable for water and alcohol, where liquid–vapor equilibrium phase transitions are present. On the other hand, the modified cycle is impossible for air if it is in contact with bodies A and B only.

Carnot implicitly defined¹⁵ the efficiency of the cycle as the ratio of the *net* (or useful) work delivered by the cycle to the heat taken from the hot reservoir:

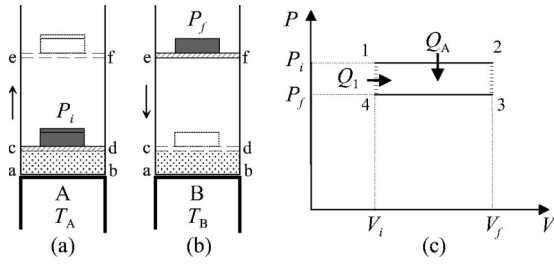


Fig. 1. Cycle described in Carnot's memoir (modified Carnot cycle). (a) Isobaric expansion in diathermal contact with hot reservoir A at temperature T_A ; (b) isobaric compression in contact with cold reservoir B at temperature T_B ; (c) Clapeyron's diagram for the cycle.

$$\eta = \frac{W_N}{Q_1 + Q_A}. \quad (1)$$

The substance absorbs heat in one of the isochoric processes (Q_1) and in the expansion in contact with body A (Q_A), but in many cases, Carnot assumed that Q_1 could be neglected in comparison with Q_A .¹⁵

For W_N he took the difference between the work in processes $1 \rightarrow 2$ and $3 \rightarrow 4$.¹⁶ He knew that the work could be calculated from $W = P_{\text{ext}} \Delta V$, where P_{ext} is the external pressure and ΔV the volume variation. Hence,

$$W_N = \Delta P \Delta V, \quad (2)$$

where $\Delta P = P_i - P_f$ and $\Delta V = V_f - V_i$. This work is given by the area 1234 in Fig. 1(c).

III. CARNOT'S CALCULATIONS

Carnot studied cycles with air between 0 and 1 °C, with water (liquid-steam) in three cycles between 99 and 100 °C, between 0 and 1 °C, and between 77 and 78 °C, and with alcohol (liquid-vapor) between 77.7 and 78.7 °C.¹⁷

A. Cycle with air between 0 and 1 °C

Carnot considered 1 kg of air at 0 °C and volume $V_i(0^\circ\text{C}) = 77 \text{ m}^3$ —Ref. 17 [state 4 in Fig. 1(c)], which is heated at constant volume, until it reaches 1 °C, absorbing heat Q_1 (which he neglected for the efficiency). Then, at constant pressure ($P_i = 1 \text{ atm} = 760 \text{ mm Hg} = 1.013 \times 10^5 \text{ Pa}$), air expands in contact with body A absorbing heat Q_A . Carnot assumed that this expansion involved two contributions, one isothermal, with $\Delta V = V_i/267$, and another, adiabatic, with $\Delta V = V_i/116$ (see Appendix A 1), estimating the total volume increase as

$$\frac{\Delta V}{V} = \frac{1}{267} + \frac{1}{116} = 1.24 \times 10^{-2} \Rightarrow \Delta V = 9.52 \times 10^{-3} \text{ m}^3. \quad (3)$$

To calculate ΔP , Carnot used Gay-Lussac's rule (see again Appendix A 1). At constant volume, the relative pressure increment corresponding to the temperature variation from 0 to 1 °C was taken to be $\Delta P/P = 1/267$. Hence

$$\Delta P = \frac{1}{267} 10.4 = 3.89 \times 10^{-2} \text{ m.w.} = 3.79 \times 10^2 \text{ Pa}. \quad (4)$$

[As pressure unit Carnot used the meter of water (m.w.): 1 atm = 10.4 m.w.] Thus, from Eq. (2), Carnot calculated the useful work done in cycle 1234:

$$W_N^{(1)} = 3.72 \times 10^{-4} \text{ m}^3 \text{ m.w.} = 3.61 \text{ J}. \quad (5)$$

To evaluate the absorbed heat, Carnot assumed that the heat capacity of air was 0.267 times the heat capacity of an equal mass of water. Taking $c_{\text{air}} = 0.267 \text{ kcal kg}^{-1} \text{ }^\circ\text{C}^{-1}$, his estimate for the heat provided by the hot body was

$$Q_A^{(1)} = m c_{\text{air}} \Delta T = 0.267 \text{ kcal} = 1116.1 \text{ J}. \quad (6)$$

The work (5) was correctly determined but not the heat (6). In Appendix B we compute the work and the heat absorbed in the cycle and compare them with Carnot's results.

From (5) and (6), Carnot obtained the efficiency of the cycle:

$$\eta^{(1)} = \frac{W_N^{(1)}}{Q_A^{(1)}} = 3.25 \times 10^{-3}. \quad (7)$$

(Although efficiencies are in fact dimensionless, those reported by Carnot were expressed in $\text{m}^3 \text{ m.w. kcal}^{-1}$. Because $1 \text{ m}^3 \text{ m.w.} = 9800 \text{ J}$ and $1 \text{ kcal} = 4180 \text{ J}$, Carnot's efficiencies should be multiplied by 2.34 to become pure numbers.)

B. Cycle with water between 99 and 100 °C

The second cycle used 1 kg of water. The reservoir temperatures were $t_A = 100^\circ\text{C}$ and $t_B = 99^\circ\text{C}$. During the cycle, water undergoes liquid-vapor phase transitions. In order to evaluate the useful work, Carnot had to calculate both the volume increment (from liquid to steam) and the pressure increment (from steam pressure at 99 °C to steam pressure at 100 °C).

For steam, he assumed the ideal gas equation (see Appendix A 2). The specific volume of steam at 100 °C and atmospheric pressure is $v_V(100^\circ\text{C}) = 170 \text{ m}^3 \text{ kg}^{-1}$. Taking the specific volume of liquid water $v_L(99^\circ\text{C}) = 1 \times 10^{-3} \text{ m}^3 \text{ kg}^{-1}$, he estimated the volume increment in the cycle to be $\Delta V \approx V_V(100^\circ\text{C}) = 170 \text{ m}^3$. To obtain the pressure increment, Carnot used liquid-vapor equilibrium data from Biot¹⁸ (see Appendix A 2). He fitted the temperature dependence of the steam volume, v_V , by the function [see Eq. (A7)]

$$v_V = 10^{a(t)}, \quad a(t) = \frac{2268 - 19.64t}{1000 + 3.30t}, \quad (8)$$

where v_V is in liters per gram and t in degrees Celsius. To calculate the steam pressure at 99 °C, Carnot first inserted this temperature in Eq. (8) and then introduced the resulting specific volume, $v_V(99^\circ\text{C}) = 1.754 \text{ l g}^{-1} = 1.754 \text{ m}^3 \text{ kg}^{-1}$, together with the temperature, in the equation of state [see Eq. (A6)],

$$P_V(t) = \frac{3.52}{v_V} (267 + t), \quad (9)$$

obtaining $P_V(99^\circ\text{C}) \approx 734 \text{ mm Hg}$. Because, on the other hand, $P_V(100^\circ\text{C}) = 760 \text{ mm Hg}$, he got the pressure increase: $\Delta P = 26 \text{ mm Hg} = 3.465 \times 10^3 \text{ Pa}$.

In Table II (Appendix A 2) we summarize, in SI units, the data used by Carnot, not only for the present cycle but also for other ones.

The net work produced in the cycle by 1 kg of liquid water at 99 °C, which transforms into steam at 100 °C, was $W_N^{(2)} = 1.70 \times 0.36 = 5.890 \times 10^3 \text{ J}$.

Moreover, Carnot knew that the heat required to transform into steam 1 g of liquid water at 100 °C was 550 kcal kg⁻¹ (specific enthalpy of vaporization in Table II). Thus, $Q_A^{(2)} = 2299.0 \times 10^3$ J. The cycle efficiency was

$$\eta^{(2)} = \frac{W_N^{(2)}}{Q_A^{(2)}} = 2.57 \times 10^{-3}. \quad (10)$$

C. Cycle with water between 0 and 1 °C

Carnot next considered a cycle with liquid water in equilibrium with its steam, between 0 and 1 °C. He compared this efficiency with that of the first cycle with air between the same temperatures.

Taking $t=1$ °C in Eq. (8), one obtains the steam specific volume $v_V(1\text{ °C})=174.16\text{ l g}^{-1}=174.16\text{ m}^3\text{ kg}^{-1}$. Inserting this value and $t=1$ °C in Eq. (9), one has $P_V(1\text{ °C})=5.418\text{ mm Hg}$. On the other hand, because from Biot's data $P_V(0\text{ °C})=5.06\text{ mm Hg}$, one finds $\Delta P=0.358\text{ mm Hg}=47.71\text{ Pa}$.

Given $v_L(0\text{ °C})\approx 1 \times 10^{-3}\text{ m}^3\text{ kg}^{-1}$, one has $\Delta V \approx V_V(1\text{ °C})=174.16\text{ m}^3$. Introducing these pressure and volume increments in Eq. (2), the net work is $W_N^{(3)}=8.310 \times 10^3$ J. Carnot did not explicitly present these calculations but obtained a value that is similar after convenient translation.

He assumed that the heat absorbed in the cycle was the sum of two contributions: One part would increase the water temperature from 0 to 100 °C at normal pressure, that is, 100 kcal kg⁻¹; and the other would evaporate water at 100 °C, that is, 550 kcal kg⁻¹.¹⁹ At his time, it was believed that the heat required to vaporize water was independent of the pressure: to vaporize water at 1 °C, one would need the same heat that would be required first to raise the water temperature, up to the normal boiling point, and then to vaporize it completely, always at normal pressure. The total heat absorbed in the cycle should be 650 kcal kg⁻¹=2717 kJ kg⁻¹.

Finally, for this water cycle, the efficiency [Eq. (1)] was (Carnot arrived at a slightly different value)

$$\eta^{(3)} = \frac{W_N^{(3)}}{Q_A^{(3)}} = 3.06 \times 10^{-3}. \quad (11)$$

D. Cycle with alcohol between 77.7 and 78.7 °C

The next system studied by Carnot was 1 kg of ethylic alcohol. For alcohol, whose normal boiling point is 78.7 °C, he assumed that the vapor pressure at 77.7 °C was reduced by 760/25 mm Hg (see Appendix A 3): $\Delta P=30.4\text{ mm Hg}=4.05 \times 10^3$ Pa.

To obtain the volume variation when alcohol goes from liquid to vapor, one may use Eq. (9) adapted for alcohol. The overall numerical factor of that equation should be multiplied by the ratio of water and alcohol molecular weights:

$$v(P,t) = 1.37 \frac{267+t}{P}. \quad (12)$$

Actually, this equation was not used by Carnot. According to Eq. (12), the volume occupied by alcohol vapor near its boiling point, $t=78.7$ °C, is $v_V(78.7\text{ °C})=0.626\text{ m}^3\text{ kg}^{-1}$. Thus, $\Delta V \approx V_V(78.7\text{ °C})=0.626\text{ m}^3$, because $v_L(77.7\text{ °C})\approx 0.74 \times 10^{-3}\text{ m}^3\text{ kg}^{-1}$.²⁰ The net work in this cycle was $W_N^{(4)}$

$=2.54 \times 10^3$ J. Again, this work was not explicitly given by Carnot.

On the other hand, Carnot knew that the heat required to evaporate 1 kg of alcohol at its boiling point was $Q_A^{(4)} = 207\text{ kcal}=865.3 \times 10^3$ J. The efficiency of the cycle was then

$$\eta^{(4)} = \frac{W_N^{(4)}}{Q_A^{(4)}} = 2.94 \times 10^{-3}. \quad (13)$$

Carnot again presented a slightly different value.

E. Cycle with water between 77 and 78 °C

To compare his results for alcohol and water, Carnot studied a fifth cycle with liquid water at 77 °C, which evaporates at 78 °C. First, he sought the specific volumes occupied by steam at those temperatures. Using Eq. (8), he found the specific volumes $v_V(77\text{ °C})=4.005\text{ l g}^{-1}$ and $v_V(78\text{ °C})=3.849\text{ l g}^{-1}$, respectively. Using Eq. (9) he could obtain the corresponding vapor pressures for water steam at those temperatures: $P_V(77\text{ °C})=302.3\text{ mm Hg}$ and $P_V(78\text{ °C})=315.5\text{ mm Hg}$, respectively. Thus $\Delta V \approx V_V(78\text{ °C})=3.849\text{ m}^3$, and the pressure difference for water steam between 77 and 78 °C is $\Delta P=13.2\text{ mm Hg}=1.76 \times 10^3$ Pa. So the useful work performed in this cycle was $W_N^{(5)}=6.77 \times 10^3$ J.

Following the method described in Sec. III C to calculate the heat absorbed by water at 77 °C that evaporates at 78 °C, Carnot obtained $Q_A^{(5)}=572\text{ kcal}=2390 \times 10^3$ J. Thus, the efficiency of this cycle was

$$\eta^{(5)} = \frac{W_N^{(5)}}{Q_A^{(5)}} = 2.83 \times 10^{-3}. \quad (14)$$

These calculations were not presented by Carnot. He just gave the cycle efficiency.

IV. DISCUSSION AND CONCLUSIONS

In Table I we list the efficiencies η obtained by Carnot for his modified cycles (Sec. III), the efficiencies η' obtained using Carnot's reasoning but contemporary data, and the efficiencies η_{CT} of the Carnot cycles calculated from Carnot's theorem: $\eta_{CT}=1-T_A/T_B$. (The data for water and alcohol are also summarized in Table II.) For air, η' is obtained in Appendix B.

For cycles with liquid–vapor phase transitions, Carnot obtained reasonable or even accurate values of the work, heat, and efficiencies. For the cycle with air, Carnot's result deviates from the modern one (in this case, the Carnot cycle cannot be simply replaced by the modified cycle). In Appen-

Table I. Comparison between Carnot's original efficiencies, η , efficiencies obtained from Eq. (1) using modern data for the modified Carnot cycles, η' , and efficiencies obtained from Carnot's theorem for the Carnot cycles, η_{CT} .

Cycle	$\eta \times 10^3$	$\eta' \times 10^3$	$\eta_{CT} \times 10^3$
(1) Air, 0–1 °C	3.25	0.89	3.65
(2) Water, 99–100 °C	2.57	2.65	2.68
(3) Water, 0–1 °C	3.06	3.56	3.65
(4) Alcohol, 77.7–78.7 °C	2.94	2.83	2.84
(5) Water, 77–78 °C	2.83	2.81	2.85

Table II. Data in SI units used by Carnot for water and alcohol. Listed are vaporization pressures, vapor specific volumes, and specific enthalpies of vaporization at various temperatures. Modern thermodynamical data are given in parentheses. Water pressures were obtained from Eq. (A8). The steam specific volume and specific enthalpy variations of water were obtained from Ref. 21. Alcohol data are from Ref. 20.

t (°C)	P_V (kPa)	v_V (m ³ kg ⁻¹)	Δh_V (kJ kg ⁻¹)
Water			
0	0.6744 (0.608 54)		
1	0.7224 (0.654 46)	174 (193.83)	2717 (2498.2)
77	40.29 (41.843)		
78	42.05 (43.603)	3.85 (3.696)	2390 (2313.2)
99	97.83 (97.805)		
100	101.3 (101.38)	1.70 (1.6736)	2299 (2256.6)
Alcohol			
77.7	97.25 (97.645)		
78.7	101.3 (101.38)	0.626 (0.627)	865 (841.55)

dix B we compare results for the modified Carnot cycle with modern data and for the Carnot cycle with the same initial state and the same volume increment.

By comparing the efficiencies of cycles 1 and 3, Carnot came to the conclusion that the efficiencies did not depend on the substance, but only on the two reservoir temperatures.²² A similar reasoning applied to cycles 4 and 5. He was so convinced of his “fundamental law” that he overlooked that this calculations could only support his guess with some difficulties.

Carnot was a genius of thermodynamics. It took some time until his *Reflections* were understood and the quality of his work was fully recognized. Because we are not aware of work analyzing the original calculations that led him to formulate the important Carnot’s principle—the efficiency of a Carnot cycle just depends on the two reservoir temperatures and not on the operating substances—we have presented them with a pedagogical perspective. We should note the ability of Carnot to use the scarce and scattered data that were available. In particular, the quality of his results for water cycles was due to this ability to fit steam pressure data (he did not know about the Clausius–Clapeyron equation).

We believe that revisiting original calculations like this in courses of thermodynamics illustrates the power of the thermodynamical formalism, while clarifying the role of the distinguished individuals who helped to found thermal science.

APPENDIX A: DATA USED BY CARNOT

1. Air

For temperature increases of 1 °C, we quote Carnot:²³

By M. Gay-Lussac’s rule, direct heating at constant pressure must increase the volume of the air by 1/267 of its value at 0° centigrade.

This increment of volume at constant pressure is given by $\Delta V = (\partial V/\partial T)_P \Delta T$. For an ideal gas, $\Delta v = (v/T)\Delta T$. Thus, for $\Delta T = 1$ K and $T = 273.15$ K,

$$\frac{\Delta v}{v} = \frac{1}{273.15}, \quad (\text{A1})$$

close to the experimental value used by Carnot.

In regard to the adiabatic compression of air, Carnot wrote:²³

The temperature of air will always rise by 1° centigrade when its volume is suddenly reduced by 1/116 of its original value.

In modern language, assuming that the process is isentropic,

$$\Delta T = \left(\frac{\partial T}{\partial v} \right)_s \Delta v \quad (\text{A2})$$

($\Delta v < 0$). If we use the reciprocity theorem and assume air to be an ideal gas, we have

$$\left(\frac{\partial T}{\partial v} \right)_s = - \left(\frac{\partial P}{\partial T} \right)_v \frac{T}{c_V} = - \frac{R}{v} \frac{T}{c_V}. \quad (\text{A3})$$

From $\Delta T = 1$ K, $c_V = 5R/2$, and $T = 273.15$ K, one finds

$$\frac{\Delta v}{v} = - \frac{c_V}{RT} \Delta T \approx - \frac{1}{109}, \quad (\text{A4})$$

close to Carnot’s value.

Again supported by a result by Gay-Lussac, Carnot wrote about the isochoric heating of a gas:²⁴

By M. Gay-Lussac’s rule, this difference [between the air pressure at 1 °C and 0 °C] is one-thousandth of 1/267 of the elastic force of the gas....

For an isochoric transformation of an ideal gas with $\Delta T = 1$ K and $T = 273.15$ K, we have

$$\frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{1}{273.15}, \quad (\text{A5})$$

very close to the experimental value used by Carnot.

2. Water

Carnot used the pressure data for equilibrium liquid–vapor water at various temperatures given in the *Traité de Physique* by Biot (1816).¹⁸ He assumed that steam obeys the ideal gas equation

$$P = c \frac{267 + t}{v} \quad (\text{A6})$$

with $c = 3.52$. This value of c follows from the observation that vapor at atmospheric pressure occupies a volume 1700 times bigger than the volume of the corresponding liquid. The data for the steam specific volume and temperature were then fitted by Carnot using

$$t = \frac{2268 - 1000 \log_{10} v_V}{19.64 + 3.30 \log_{10} v_V}, \quad (\text{A7})$$

which is Eq. (8). From here, Carnot was able to obtain steam volumes and pressures at different temperatures. In Table II we list Carnot’s data for water and also for alcohol.

Antoine’s equation,²⁵

$$\ln P_V = 16.573 - \frac{3988.842}{(T - 39.47)}, \quad (\text{A8})$$

with P_V in kPa and T in K, fits modern data. In Fig. 2 we compare vapor pressures fitted by Carnot and vapor pressures obtained from Eq. (A8). Modern data used to compute the efficiencies η' in Table I are displayed in parentheses in Table II.

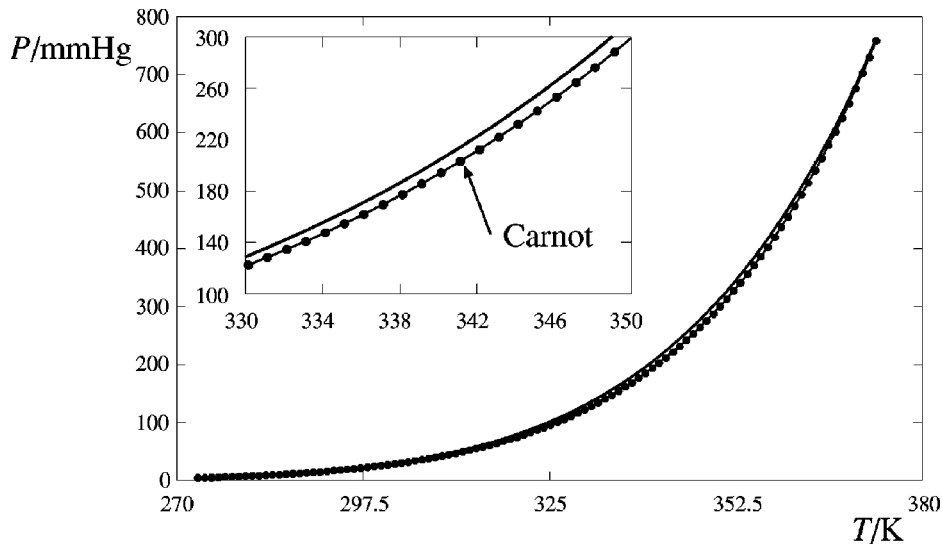


Fig. 2. Comparison between steam equilibria pressures fitted by Carnot (dots), using Eqs. (8) and (9) and modern steam pressures using Eq. (A8) (solid line).

3. Alcohol

In Carnot's words:²⁶

At normal pressure pure alcohol boils at 78.7 centigrade. At this temperature, according to MM. Delaroche and Brard, 207 units of heat [1 unit of heat=1 cal] are required in order to vaporize one kilogram of alcohol. The vapor pressure of alcohol at a temperature 1 °C below the boiling point is reduced by 1/25: it is 1/25 less than atmospheric pressure.

For alcohol, Carnot assumed that the vapor pressure increment between 77.7 and 78.7 °C was $\Delta P = 760/25 = 30.4$ mm Hg.

The boiling point of ethylic alcohol is 78.3 °C = 351.45 K. The specific enthalpy variation in vaporization²⁰ is $\Delta h_V = 841.55$ kJ kg⁻¹. Because the alcohol molecular weight is 46.07 g mol⁻¹ and it is assumed to obey the Clausius–Clapeyron's equation near the boiling point, one has

$$P(77.7\text{ °C}) \approx 760 \exp\left[-\frac{841.55 \times 46.07}{8.314 \times (273.15 + 78)^2}\right], \quad (\text{A9})$$

where $P(78.7\text{ °C}) = 760$ mm Hg, so that $P(77.7\text{ °C}) \approx 732$ mm Hg. Thus $\Delta P = P(78.7\text{ °C}) - P(77.7\text{ °C}) = 28$ mm Hg, very close to Carnot's value.

APPENDIX B: CYCLE WITH AIR

Taking 1 kg of air (molecular weight 29 g mol⁻¹), assumed to be an ideal gas, one has for states 1, 2, 3, 4 in Fig. 1(c) (volumes in m³, pressures in 10⁵ Pa, and temperatures in K):

$$\begin{aligned} V_1 = 0.7759, \quad V_2 = 0.7859, \quad V_3 = 0.7859, \quad V_4 = 0.7759, \\ P_1 = 1.0130, \quad P_2 = 1.0130, \quad P_3 = 1.0092, \quad P_4 = 1.0092, \quad (\text{B1}) \\ T_1 = 274.15, \quad T_2 = 277.69, \quad T_3 = 276.66, \quad T_4 = 273.15, \end{aligned}$$

and the net work

$$W_N^{(1)} = 3.8 \text{ J}. \quad (\text{B2})$$

Assuming air to be a diatomic ideal gas, the absorbed heats in the process at constant volume and at constant pressure are (c_V and c_P are molar specific heats)

$$Q_V^{(1)} = n_{\text{air}} c_V \Delta T = 716.7 \text{ J}, \quad Q_P^{(1)} = n_{\text{air}} c_P \Delta T = 3552.1 \text{ J}. \quad (\text{B3})$$

We note, on the one hand, that the first heat cannot be neglected in comparison with the second, and on the other hand, that the second heat was not correctly estimated by Carnot (in process 1–2 the temperature increase is $\Delta T \sim 3.5$ K and not $\Delta T = 1$ K as assumed by him). The efficiency of this cycle is

$$\eta'^{(1)} = \frac{W_N^{(1)}}{Q_V^{(1)} + Q_P^{(1)}} = 0.89 \times 10^{-3}, \quad (\text{B4})$$

quite different from Carnot's value [Eq. (7)].

The efficiency of the standard Carnot cycle with two isothermics at $t_B = 0$ °C and $t_A = 1$ °C can be obtained directly from Carnot's theorem:

$$\eta_{\text{CT}}^{(1)} = 1 - \frac{273.15}{274.15} = 3.65 \times 10^{-3}. \quad (\text{B5})$$

Curiously, this value is closer to that found by Carnot. Analyzing the modified cycle, he was fortunate enough to obtain an efficiency that deviates only $\approx 10\%$ from η_{CT} .

^{a)}Electronic mail: guemezj@unican.es

^{b)}Electronic mail: tcarlos@teor.fis.uc.pt

^{c)}Electronic mail: tmanuel@teor.fis.uc.pt

¹S. Carnot, *Reflections on the Motive Power of Fire. A Critical Edition with the Surviving Manuscripts*, edited by Robert Fox (Manchester U.P. and Lilian Barber Press, New York, 1986).

²See Ref. 1, p. 63.

³See Ref. 1, p. 67.

⁴H. Erlichson, "Sadi Carnot, 'Founder of the Second Law of Thermodynamics,'" *Eur. J. Phys.* **20**, 183–192 (1999); Ref. 1, p. 70.

⁵See Ref. 1, pp. 76–77.

⁶Although Carnot refers to the *caloric theory of heat*, his conclusions on heat engines do not depend on any caloric or mechanical theory of heat. For this discussion, see, for example, Refs. 7 and 14, and V. K. La Mer, "Some current misinterpretations of N. L. Sadi Carnot's memoir and cycle. II," *Am. J. Phys.* **23**, 95–102 (1955); M. A. Hirshfeld, "Some current misinterpretations of Carnot's memoir," **23**, 103–105 (1955).

- ⁷V. K. La Mer, "Some current misinterpretations of N. L. Sadi Carnot's memoir and cycle," *Am. J. Phys.* **22**, 20–27 (1954).
- ⁸M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics* (McGraw–Hill, Singapore, 1997), 7th ed., Chap. 7.
- ⁹C. Treusdell, *The Tragicomical History of Thermodynamics 1822–1854* (Springer-Verlag, New York, 1980).
- ¹⁰See Ref. 1, p. 14.
- ¹¹See Ref. 1, pp. 74–75.
- ¹²L. Tansjö, "Comments on the closure of the Carnot Cycle," *J. Chem. Educ.* **62**, 585–591 (1985).
- ¹³See Ref. 1, p. 77.
- ¹⁴T. S. Kuhn, "Carnot's version of "Carnot's cycle," *Am. J. Phys.* **23**, 91–95 (1955).
- ¹⁵See Ref. 1, pp. 96–97.
- ¹⁶See Ref. 1, p. 22.
- ¹⁷See Ref. 1, pp. 94–95.
- ¹⁸See Ref. 1, pp. 90–91.
- ¹⁹See Ref. 1, p. 9.
- ²⁰K. Raznjevic, *Tables et Diagrammes Thermodynamiques* (Editions Eyrolles, Paris, 1970), p. 71.
- ²¹J. B. Jones and R. E. Dugan, *Engineering Thermodynamics* (Prentice–Hall, New York, 1996).
- ²²Carnot wrote that "Our fundamental law is therefore confirmed for one particular case," Ref. 1, p. 99.
- ²³See Ref. 1, p. 79.
- ²⁴See Ref. 1, p. 95.
- ²⁵S. Gesari, B. Irigoyen, and A. Juan, "An experiment on the liquid-vapor equilibrium for water," *Am. J. Phys.* **64**, 1165–1168 (1996).
- ²⁶See Ref. 1, pp. 99–100.

EINSTEIN'S DOUBTS

The finality of death took five years. Einstein's aneurysm had been diagnosed as early as 1950, and when doctors informed him that an operation might save his life, he said, "Let it burst." Like Newton before his own death, Einstein was less worried about his physical demise than the constant nagging doubts about his theories. "There is not a single concept of which I am convinced that it will stand firm," he wrote to an old friend, Maurice Solovine, in 1949, "and I feel uncertain whether I am in general on the right track." And while his brain kept whirring with the problems of a unified theory, his body became frail and thin, wrinkles fell over him like elephant skin, and he began to slouch, as if with each new day he was carrying a slightly heavier pack on his back. "The strange thing about growing old is that the intimate identification with the here and now is slowly lost," he wrote to the Queen Mother of Belgium in 1953, "one feels transposed into infinity, more or less alone, no longer in hope or fear, only observing."

Michael Paterniti, *Driving Mr. Albert: A Trip Across America with Einstein's Brain* (Dell Publishing, New York, NY, 2000), p. 185.