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THE MONEY MARKET DAILY SESSION: AN UHF-GARCH MODEL APPLIED TO THE PORTUGUESE CASE BEFORE AND AFTER THE INTRODUCTION OF THE MINIMUM RESERVE SYSTEM OF THE SINGLE MONETARY POLICY

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ABSTRACT:
The study of the determination of the overnight interest rate in the interbank market, and the behaviour of its volatility, gained new insights with contributions from the microstructure theory. The aim of this article is to study the effect of the trade intensity over the volatility of the overnight interest rate, using intra-daily data related to the Portuguese Money Market (MMI). The analysis is focused in two different periods of time, before and after the introduction of the minimum reserve rules of the Single Monetary Policy. We find that these rules have contributed to interest rate stability.

JEL CLASSIFICATION: E43, E58, G21.

KEYWORDS: Money market, Market Microstructure, Interest rate volatility, ACD models, UHF-GARCH models.

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INTRODUCTION

In modern economies, the money markets have a crucial role in monetary policy and in banks' reserve management. In these markets banks borrow and lend reserves, central bank money. These loans are short term loans, especially overnight loans, and contribute to a better allocation of central bank money. The shortest term interest rate in the economy is formed in this market and it influences the term structure of interest rates. Also, these markets have a role in the transmission of monetary policy. Therefore, it is of crucial importance to understand the functioning of the money markets, and how their interest rates emerge from the interaction of the market participants.

Over recent decades, several studies have been carried out that analyse the bank's demand for reserves in the interbank market, as well as the processes followed by the short-term interest rates, especially, the overnight interest rate. The works of Spindt and Hoffmeister (1988) and Hamilton (1996), which aim to study the evolution of the level and volatility of the overnight interest rate throughout the reserve maintenance period, have been developed further by numerous subsequent papers. The banks need to fulfil their reserve requirements, and this is a fundamental reason for the demand of Central Bank money, and a determinant factor in the formation of the overnight interest rate. Usually, these papers explain and test the formation of interest rates in the market according to the martingale hypothesis: that the reserves held by banks at different days of the maintenance period are perfect substitutes, doing the averaging provision for the reserve requirement, and the changes in the level of the overnight interest rate would be unpredictable. This hypothesis is compatible with heteroskedasticity and volatility clustering.

In recent years, the research focused on the interbank market gained new insights with contributions from the microstructure theory. That studies the process of price formation, taking in to account the rules and the characteristics of a specific market. The availability of high-frequency data on the money markets allowed the application of models of the microstructure theory and the study of the behaviour of the volume of activity, the interest rate and its volatility along the trading session. The empirical identification of interest rate behaviour and interest rate volatility behaviour is also important because it contributes to a better definition of strategies of intervention in these markets, both by banks and the central bank.

The main objectives of this article are the application of an Autoregressive Conditional Duration (ACD) model proposed by Engle and Russell (1997, 1998) and the application of an UHF-GARCH model proposed by Engle (2000) to the Portuguese Interbank Money Market (MMI). We analyse high-frequency data about MMI transactions and compare the trading session for two periods of time, before and after EMU. This comparison allows us to study the effect of the introduction of the Single Monetary Policy minimum reserve system on the durations of MMI loans and on the influence of the trade intensity over the volatility.

This article proceeds as follows. Section 1 provides a review of the literature. In Section 2 we describe the institutional features of the Portuguese domestic money market. Section 3 describes the data and Section 4 presents the empirical model specification. Section 5 presents the results of the estimation and finally the conclusion.

1. REVIEW OF THE LITERATURE

The study of the determination of the short-term interest rate, depending on the institutional features of the market and the behaviour of the market participants, is based on the seminal works of Ho and Saunders (1985) and Spindt and Hoffmeister (1988), which consider that the banks face a reserve management problem due to the uncertainty about the payments and receipts. Spindt and Hoffmeister (1988) assume that there is a “market maker”, always prepared to buy funds (at a bid rate), and to sell funds (at an asked rate) to other banks (price takers). The Central Bank does not participate directly in this trading market. However, it supplies or drains liquidity from the market through open market operations and determines the amount of pressure felt by the banks in their demand for reserves. The dealer wishes to maximize his expected trading profits, subject to his reserve requirement constraint, he determines his end-of-day reserve balance and chooses the asked and the bid rates that would maximize his profit. The reserves are calculated as an average, over a maintenance period of several days. So, the work-off rate increases as the settlement day approaches. The later any unexpected change in the funds of the bank occurs, the greater the funds’ demand/supply reactions to accommodate this change are. The daily funds rate variance is not constant throughout the reserves maintenance period; it increases towards the end of the maintenance period and is at its greatest value on the settlement day[1].

The martingale hypothesis along the maintenance period, proposed and tested by Hamilton (1996) received widespread attention in empirical literature. According to this hypothesis, the reserves held by banks at different days of the maintenance period are perfect substitutes, doing the averaging provision for the reserve requirement. Accepting this hypothesis, if $i_t > E_t(i_{t+1})$, where i_t is the overnight rate in day t and $E_t(i_{t+1})$ is the overnight interest rate expected in day t for the following day, the banks will lend in the market on day t, diminishing their excess reserves or augmenting the need to borrow later in the market. If all banks do this arbitrage, i_t decreases and i_{t+1} increases. The logical implication is that the interest rate changes are unpredictable, and that the interest rate should follow a martingale within the maintenance period[2]. This is compatible with the existence of a pattern in volatility. Hamilton (1996), Prati, Bartolini and Bertola (2003), Bartolini, Bertola and Prati (2002), Bartolini and Prati (2003)[3], Cyree and Winters (2001b) and Furfine (2000) tested this hypothesis in the Federal Funds market, Lee (2003) and Cyree, Griffiths and Winters (2003) studied the Eurodollar funds market and Quiróz and Mendizábal (2001), Gaspar, Quirós and Sicilia (2001) and Wurtz (2003) studied the European money market[4]. The main conclusion was that the specific regulation of each market influences the behaviour of overnight interest rate and overnight interest rate volatility along the maintenance period.

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[1] This model also predicts that the variance of the funds rate increases across the trading day. However, the available data does not permit a verification of this prediction.

[2] If t is the last day of the maintenance period, we can not follow this rationale. The reserve balances of day t and day t+1 are not substitutes. They can have only some degree of substitutability if the banks can carry over an amount for the next period.

[3] The main objective of this work was the comparison of the behaviour of money market interest rates in the USA and in Europe, due to the differences in operational framework of monetary policy of the two central banks; the Federal Reserve followed a *hands-on* approach while the ECB followed a *hands-off* approach.

[4] This hypothesis was tested using GARCH family models.

In recent years, the research on the formation of short-term interest rates focused on its behaviour along the day. This question was first posed by Spindt and Hoffmeister (1988)[5] whose work was inspired in Garman (1976), a pioneer work in market microstructure. The application of models developed by microstructure theory to the money market has some limitations: the organisation of money markets is not very sophisticated[6], the behaviour of market participants doesn't have the same motivations as the behaviour of agents in capital markets, and it depends on the framework of the monetary policy.

Empirical literature of the microstructure theory identifies patterns in the evolution of the market variables[7]: usually volume of activity, prices and volatility prices follow a U-shaped pattern along the trading session. Two different theories explain it: one based on the role of **private information** and the other based on **trading stoppages**[8]. The first approach, developed by Admati and Pfleiderer (1988) says that there are different types of traders in the market: those who have private information about futures assets returns, the informed traders, who trade to exploit their information, and the liquidity traders who trade for portfolio reasons. These are divided in two groups according to their capacity to choose the timing of their trades in discretionary and non-discretionary liquidity traders. The discretionary liquidity traders prefer to trade when their trade has little effect on prices, minimizing the expected cost of transactions, and the same applies to informed traders. Thus, there are incentives for trading to be concentrated. The clustering in activity by liquidity reasons induces informed-trading activity. Admati and Pfleiderer (1988) don't identify the moments in time when the clustering in activity tends to exist.

Cyree and Winters (2001a) identify the banks as discretionary liquidity traders. They have the ability to time their loans in money market, along the maintenance period, because reserves are constituted according to an average provision; and along the trading day, because only the end-of-day balance matters for minimum reserves. Private information, if exists, is difficult to exploit because of the need to constitute minimum reserve balances dominates other strategies.

The approach based on **trading stoppages** says that the U-pattern identified in the variables is explained by the market stoppages, namely, the open and the close of the market[9]. This approach doesn't need to define types of traders or motivations to trade or trade organizations to explain the behaviour of the variables. It is the impossibility of trade that determines it.

Brock and Kleidon (1992) present a model of the determination of the bid-ask spread, where the increase in transactions, just after the open and before the close, is a consequence of the overnight close[10]. Just after the open, the traders react to the accumulated new information and recompose their portfolios. At the overnight close, the optimal portfolios are different from those during the preceding continuous trading, and the

[5] They predicted an increase of the volatility along the trading day, which empirical evidence was first found by Griffiths and Winters (1995).
[6] For instance, in the MMI the loans result of bilateral agreements.
[7] See, for instance, Wood, McInish and Ord (1985).
[8] Slezak (1994) and Hong and Wang (2000) conciliate the two approaches, based in private information and trading stoppages.
[9] See, for instance, the empirical work of Amihud and Mendelson (1991) for the Japanese Stock Market, Gerety and Mulherin (1994) for the NYSE(1994) or Huang, Liu and Fu (2000) for the Taiwan Stock Exchange.
[10] Note that, in some capital markets, there is a lunch break too.

activity is more intense. On these two occasions the demand[11] is more intense and less elastic. The model considers the existence of a market-maker, the dealer, which is able to respond to the orders to buy and to sell stocks. The functions of supply and demand faced by the market-maker are stochastic and expressed as Poisson processes: the supply is an increasing function of the bid price and the demand is a decreasing function of the asked price[12]. The dealer's objective is the maximization of expected profit, holding expected inventory constant, that is, keeping the offer and demand flows equal.

Brock and Kleidon (1992) suppose that the supply and demand functions, on different time intervals can be different, thus, leaving the possibility of these functions to increase at specific time intervals. If, at the opening of the session, there is a large volume of demand orders, the asset asked price increases. If, at the same time interval, there is a large volume of sell orders, the asset bid price decreases, implying a wider bid-ask spread. At this time interval, both the buy and sell side increase, and are less elastic. The increase in transactions and in the bid-ask spread also happens prior to the close[13]. The result is the U-shaped pattern identified in empirical literature, that is, the spread bid-ask is larger on the time interval which has the more intense demand[14].

Gerety and Mulherin (1992) link the increase in trading intensity with the need, by investors, to reallocate portfolios before the close, due to overnight volatility and the impossibility to trade during the night. At the open, it is the overnight information, accumulated during the night that leads to portfolio rebalancing. Gerety and Mulherin (1992) and Gerety and Mulherin (1994) present empirical evidence of the influence of trading stoppages on the behaviour of market variables.

The hypothesis (behaviour of agents, functions of demand and supply) of the Brock and Kleidon (1992) model are similar to the ones of Spindt and Hoffmeister (1988), which make the model adequate to applications on money markets. Also, Brock and Kleidon (1992) recognize the role of rules and market regulations (like the period of tax payments) that influence the market variables. In money markets the calculation of minimum reserves by end-of-day balances influences the increase in lending intensity at the end of the day[15].

Angelini (2000) states that the banks' intraday behaviour in the interbank market is explained as the outcome of their optimisation processes. Banks are risk-averse and are subject to two sources of uncertainty: their end-of-day position (payments are exogenous) and the interest rate that they pay/receive. Their objective function depends on two arguments[16]: the cost /gain of borrow/lend funds in the market and the cost of adjustment of its reserve position (due to an insufficient end-of-day balance). Their choice regarding the timing of operations in the market represents a trade-off between these two sources of uncertainty. The higher the uncertainty of the market rate relative to the variance of the

[11] On these moments, the demand and the supply can be larger, due the objective of portfolio composition.
[12] In Spindt and Hoffmeister (1988) the prices bid and asked correspond to bid and asked interest rates.
[13] In Spindt and Hoffmeister (1988), it is the pressure due to the increase in the rate of net borrowing of the market-maker (in order to hold minimum reserves) that leads to the increase in asked and bid interest rates that he sets. This tendency is observed throughout the day and leads to the increase in spread's volatility. Brock and Kleidon (1992) suppose that the dealer wants to maintain its inventory constant.
[14] The same arguments can be used to conclude about the consequences of a change in supply (with unchanged demand).
[15] The model of Brock and Kleidon (1992) can be applied to other kind of time intervals in which the trading intensity changes like, in the case of the money markets, the ending of reserve maintenance periods.
[16] The model is a reserve management model, usual used in the study of demand of reserves by banks.

end-of-day position, the sooner the banks prefer to operate in the market. Angelini (2000) studies the Italian money market (July 1993 and December 1996) and find that the trading volume and volatility present a U-shaped pattern along the day but not the level of overnight interest rate. On the latter days of the reserve period, the volatility of interest rate is substantially larger than during the rest of the period. In these days banks shift a significant share of transactions from the afternoon to the early morning. This evidence is consistent with the hypothesis of risk aversion.

Cyree and Winters (2001a) state that the approach based on trading stoppages is helpful to explain the behaviour of money market interest rates, due to the inexistence, in these markets of different traders with different motives to trade. They estimate the processes followed by the overnight interest rate in the Federal Funds Market (February 1984-January 1996)[17], and identify a U-shaped pattern in the overnight interest rate along the day. They conclude that the larger variations occur at the beginning and the end of the trading day. There is one exception: on the settlement day[18] the volatility rises throughout the trading day. Bartolini, Gudell, Hilton and Schwarz (2005) also note these patterns and role of the trading closures.

Hartmann, Manna and Manzanares (2001) analyse 5 months' of intraday data (November 1999-March 2000) recorded by 6 brokers from several European countries as well as on the Italian electronic market. Market activity (described by trading volume per hour) and overnight interest rate volatility show a U-shaped pattern. Finally, Sol Murta (2006) analyzed the Portuguese Interbank market (MMI) and did not identify the martingale hypothesis in 1998, due to the patterns related with moments of the trading session and monetary policy operations. In 2001, the martingale hypothesis could not be accepted; however the conclusion was less clear because the predictability in interest rate behaviour was almost zero[19]. The existence of lending and deposit standing facilities explained the absence of an interest rate pattern[20][21]. This was also why volatility decreased over the trading day. Only in the last few days of reserve maintenance periods, when there was pressure to hold minimum reserves, did the volatility increase during the afternoon (being above average throughout the day). In 1998 the difference between settlement day and the other days of maintenance period was less strong because the reserve period was smaller, banks faced more uncertainty about minimum reserves, and the pressure to hold reserves was more constant throughout the maintenance period.

Note that, nowadays, short-term interest rate stability is a subject of crucial importance: central banks, and ECB in particular, use the minimum reserve system to create a stable demand for reserves in order to control the short-term interest rate. Bindseil (2000) discuss the contribution of the characteristics of the minimum reserve system to the objective of smoothing interest rates. The main concern about short-term interest rate volatility is its

[17] The authors estimate the asymmetric GARCH models proposed by Glosten, Jagannathan and Runkle (1993).
[18] The settlement day is the last day of the reserves maintenance period.
[19] This conclusion is similar, in a intra-daily level, to the conclusion of Quiróz and Mendizábal (2001) that reject the martingale hypothesis in Germany, before January 1999, but do not reject it in the EONIA, after EMU.
[20] To grant access to the lending facility, the counterparties need to possess eligible assets. Therefore, this facility is not a perfect substitute for borrowing from the MMI, where the loans are unsecured. However, until 2004, Portuguese banks had "Títulos de Depósito do Banco de Portugal" that were securities first issued by the Banco de Portugal in 1994 at a time when the reserve ratio changed from 17% to 2% in order to soak up the banks' excess liquidity. These securities were immobilised, so they were perfect for using as eligible assets.

[21] Quirós and Mendizábal (2001) have already presented the existence of standing facilities to justify the non-rejection of the martingale hypothesis in the European money market.

transmission to other interest rates of longer maturity and the potential real effects in investment decisions[22].

In conclusion, we can say that the patterns identified in overnight volatility are determined by the behaviour of the trading institutions and the institutional characteristics of the market. The understanding of these features is essential to the study of interbank money markets.

2. THE PORTUGUESE INTERBANK MONEY MARKET

The Portuguese Money Market, known as the Mercado Monetário Interbancário- MMI is the domestic interbank market. It was created in the seventies, when the banking system was extremely closed and tightly controlled by the Central Bank- the *Banco de Portugal*. Since its creation, the MMI has continuously adapted to a more competitive environment based on the free interaction of supply and demand[23].

Nowadays, on the MMI, and in accordance with Instruction nº 51/98 from the *Banco de Portugal*, only the institutions subject to the minimum reserve requirements of European System of Central Banks (ESCB), and others whose activity or volume of operations justify the participation in the market, can trade. Here, the banks borrow and lend Central Bank money. These loans are unsecured[24] and their terms can run for up to a year. However, the vast majority of these loans are very short term, and often only run overnight. The loans can also be forward loans with one or two days to run before the delivery date. The conditions set for the loans, their amount, the interest rates and the terms are freely agreed between the lending bank and the borrowing bank. The MMI is an over-the-counter (OTC) market.

The trading is executed through SITEME[25], an electronic infrastructure created for the processing of interbank transfers and payments between the Central Bank and other banks. When two banks agree on a loan, this operation is communicated to the *Banco de Portugal*, which transfers the funds between the accounts of the two banks. On the repayment day, the Central Bank reverses the transfer of funds along with the interest accrued.

The institutional features of the reserve requirements faced by Portuguese banks changed with the coming of the EMU. Although, the reserve ratio remained constant (2%), the reserve base maintained its main characteristics and the required reserves were still specified in terms of an average level over a maintenance period; two important distinctions were introduced by the minimum reserve system of single monetary policy[26]: one, the reserve period of 7-9 days length changed to one month[27] and two, the system evolved from being semi-lagged, that is, having the calculation period partly overlapping the maintenance period, to becoming lagged, meaning that the former precedes the latter. That is, until 1998, each calendar month had four periods of calculation and four periods of maintenance. The calculation periods were days 1-8, 9-15, 16-22 and 23-last day of month,

[22] See Lee (2002) and Bindseil (2000).
[23] The creation and evolution of MMI are described in Sol (1996).
[24] In the MMI banks can also make secured loans. However, the majority of the loans are unsecured.
[25] The SITEME features are described in *Banco de Portugal* (2000).
[26] The detailed description of Single Monetary Policy instruments can be found in European Central Bank (2005).
[27] After 9 March 2004 the reserve maintenance period has a length of, approximately, one month.

and the reserves were calculated by applying the reserve ratio to average end-of day sheet balances. The maintenance periods began 3 days after the calculation period, that is, they were, respectively, 4-11, 12-18, 19-25 and 26-3 of following month.

Nowadays, it is the balance sheet data referring to the end of the calendar month that are used to determine the reserve base. After 9 March 2004 the reserve maintenance period starts and ends according to a calendar defined by the ECB. [28]

Also, in the 1998 period, the trading session was open from 8am until 3pm (it closed before the end of the business day). Since 2001, it is open from 7am until 5pm, and hence, the banks have more time to trade, including, after the open hours of business.

In a nutshell, the features of the Portuguese reserve system before EMU were much less flexible for the management of reserves by banks: the maintenance period was shorter, in its first part (3 days) the banks were uncertain about the minimum balance level to hold, and in each day they had less time to trade.

3. DATA

This paper analyses the MMI and the way trading takes place throughout the day. Our data is intraday data collected by the Markets and Reserves Management Department of the *Banco de Portugal*. The data covers two periods of time: the period from July until 3 November 1998[29] and the period from May to October 2001. This data is not public and it was kindly supplied by the *Banco de Portugal*. It allows the comparison between two periods of time, where the rules of the minimum reserve system that influence the behaviour of banks are different: the last months before the introduction of the 3rd stage of Monetary Union and a semester in which banks were already perfectly adapted to the rules of the minimum reserve system of the single monetary policy[30]. For each of the loans in the MMI, we have the following information: the day and time (in hours, minutes and seconds) of the operation, the size of the loan, the interest rate and the term of the loan. This database includes 5230 overnight transactions in July-November 1998 and 4542 overnight transactions between 2 May and 31 October 2001.

For the period of 1998, our data spans 16 calculation and reserve maintenance periods, and 3 days (1,2 and 3 July) corresponding to the end of a maintenance period that is not complete. In 2001, our data period spans seven reserve maintenance periods, completely covering five of them, and corresponding approximately to a total of six periods. When the settlement day occurs at weekends, the last day on which banks can operate in the money market is the preceding Friday, which is then regarded as the settlement day.

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[28] In the period that we studied it always started on the 24rd of the month.

[29] Note that a transition period, between 4 November 1998 and 31 December 1998, was established to allow the Portuguese banks to adapt to the new rules of Single Monetary Policy.

[30] After that, in March 2004, some changes were introduced in the procedures of the ECB. However, they didn't modify the main characteristics of the reserve system. For instance, the days in which the maintenance periods begin and end were modified, but not the length of the period.

4. EMPIRICAL MODEL SPECIFICATION

4.1. THE ACD MODEL

The availability of intraday databases has allowed to new models which attempt to describe the characteristics of the trade and price processes. One of the features of these databases is that it keeps record of all transactions registered in the market, usually not regularly time-spaced. In order to study the price or the volatility process a transformation of data is necessary: to divide the data in regular time intervals and retain information by interval (usually, the information related with the last transaction). However, the clock time is informative: the absence of trading during a long period of time may be caused by the lack of news; a period of intensive trades can be related to the reaction of traders to the arrival of new information[31]

The need to consider the time elapsed between transactions was taken in account by Engle and Russell (1997, 1998) who proposed the Autoregressive Conditional Duration (ACD) model, which models the time gaps (trade durations[32]) between irregularly spaced events. The duration is treated like a random variable and follows an autoregressive process. To each moment where a trade happens a vector of marks like the price, the *bid-ask spread*, the volume, etc is associated.

Considering the stochastic process composed by a sequence of points in time, the arrival times, which are represented by

$\{\tau_1, \tau_2, ..., \tau_n ...\}$ where $\tau_1 < \tau_2 < ... < \tau_n < ...$

Associated with it is a counting function represented by $N(\tau)$ which is the number of events which have already occurred by the time τ. Thus, $N(\tau)$ is a non-decreasing function of time.

Let x_t be the duration between two consecutive events τ_{t-1} and τ_t, $x_t = \tau_t - \tau_{t-1}$. The ACD model considers that the t^{th} conditional expected duration ψ_t, is a function of past durations:

$$\psi_t = E(x_t \mid I_{t-1}) \tag{1}$$

where I_{t-1} is the information available at time $t-1$; we assume that x_t / ψ_t is independent and identically distributed in t. According Engle and Russell, the ACD model can be specified by:

$$x_t = \psi_t \varepsilon_t \tag{2}$$

[31] The importance of time in this context has been noted by Easley and O'Hara (1992).
[32] This model can be applied to other type of durations like price durations or volume durations. See, for instance, Bauwens and Giot (2001).

with $\varepsilon_t \approx i.i.d.$, $E(\varepsilon_t) = 1$ and $Var(\varepsilon_t) = \sigma^2$, allowing that $E(x_t \mid I_{t-1}) = \psi_t$. Therefore, the ACD model considers that the error is multiplicative and that past information influences duration by the average conditional duration. The complete specification of the model depends on the dynamics of conditional duration and the error distribution.

Engle and Russell (1997, 1998) suppose that the conditional expected duration follows a linear autoregressive model:

$$\psi_t = \omega + \sum_{j=1}^{m} \alpha_j x_{t-j} + \sum_{j=1}^{q} \beta_j \psi_{t-j} \tag{3}$$

named ACD(m,q) where m and q are the orders of the autoregressive process, $\alpha_j \geq 0$, $\beta_j \geq 0$, $\omega > 0$ and $\sum_{j=1}^{m} \alpha_j + \sum_{j=1}^{q} \beta_j < 1$. The two first constraints ensure that the conditional duration is positive; the last one guarantees the existence of the unconditional mean of the duration. Engle and Russell (1998) assume that the error follows a Weibull distribution with parameters (κ, γ) [33].

The ACD models are similar to GARCH models and describe the clustering in the durations: long (short) durations are followed by long (short) durations. Other variables with economic relevance can be included. Dufour and Engle (2000), Manganelli (2005) and Spierdijk (2004) have applied this and other ACD models[34] using NYSE data.

4.2. MODELING SIMULTANEOUSLY THE DURATION AND THE VOLATILITY

The analysis of the time elapsed between transactions is useful to the study of other market variables, specially the prices, and to test market microstructure theories[35]. Engle (2000) analyses simultaneously the passing of time, the returns and the volatility. First, he modeled the process of the arriving of trades independently, and then he modeled, conditionally to this process, the important variables. Let y_t be a vector of variables, marks, of the stochastic process of trade, then:

$$(x_t, y_t) \mid I_{t-1} \sim f(x_t, y_t \mid \tilde{x}_{t-1}, \tilde{y}_{t-1}; \theta) \tag{4}$$

where $\tilde{x}_{t-1}$ and $\tilde{y}_{t-1}$ represent the past value of the variables and θ represents the parameters of the model.

[33] Usually this distribution adjusts well to the data; however other distributions can be used, like Gamma or Burr distributions. See, for instance, Bauwens and Giot (2001).

[34] Bauwens, Giot, Grammig and Veredas (2004) and Kamionka (2000) survey the extensions of the ACD model.

[35] Kamionka (2000) presents a survey of the simultaneous modelisation of durations and trades features.

Engle (2000) shows that the joint density can be written as the product of the marginal density of the duration times the conditional density of the marks given the duration, all conditioned upon the past transactions:

$$f(x_t, y_t \mid \tilde{x}_{t-1}, \tilde{y}_{t-1}; \theta) = g(x_t \mid \tilde{x}_{t-1}, \tilde{y}_{t-1}; \theta_1) q(y_t \mid x_t, \tilde{x}_{t-1}, \tilde{y}_{t-1}; \theta_2) \tag{5}$$

In this work, Engle combines an ACD model with a modified GARCH model and calls it Ultra-High-Frequency Data GARCH, UHF-GARCH. Its estimation is done according two stages: in the first one, an ACD model is applied to the time-of-day adjusted durations, assuming that the error follows a specific distribution with positive support; in the second stage a GARCH model, augmented with the duration, is estimated. Therefore, the passing of time is accounted in the characterization of the price volatility and the volatility becomes non-linearly dependent of transactions.

Engle suggests GARCH (1,1) models where the mean equation is an ARMA(1,1) and the dependent variable is the return series *r* divided by the series of the square root of the durations $r_t / \sqrt{x_t}$, that is, the series of the return per unit of time, per second[36]. One of the specifications of the variance by Engle is:

$$\sigma_t^2 = a_0 + a_1 e_{t-1}^2 + a_2 \sigma_{t-1}^2 + a_3 x_t^{-1} + a_4 \frac{x_t}{\psi_t} + a_5 \xi_{t-1} + a_6 \psi_t^{-1} \tag{6}$$

In this equation the influence of the durations over the volatility depends on three elements: the reciprocal of the duration (x_t^{-1})[37], the surprises in the duration (x_t / ψ_t) and the reciprocal of the expected duration (ψ_t^{-1}), that is, the expected intensity of trade. The long run volatility (ξ_t) is also considered and it is computed by exponentially smoothing r^2 / x using the parameter 0,005, as:

$$\xi_t = 0{,}005 \left(r_{t-1}^2 / x_{t-1} \right) + 0{,}995 \xi_{t-1}$$ [38]

[36] Note that, if $h_t = V(r_t \mid x_t)$ is the conditional variance (per transaction), and $\sigma_t^2 = h_t / x_t$ is the conditional variance per transaction and per unit of time, then $\sigma_t^2 = V\left(\frac{r_t}{\sqrt{x_t}} \mid x_t \right)$.

[37] A positive estimated coefficient means that long durations (slow trading) will be related with low volatility.

[38] Grammig and Wellner (2002) also use this long run volatility specification.

The estimation of equation (6) requires the previous estimation of the coefficients of the ACD model. The introduction of exogenous variables, with economic relevance over the volatility, is possible.

Engle (2000) applies the UHF-GARCH model to IBM stocks transactions (NYSE data) and concludes that long durations (low trading intensity) are associated with low volatilities. Also, surprises in the durations (non expected low trading intensity) have a negative effect over the volatility and, with similar economic meaning, also find evidence that lower expected duration is associated with higher volatility.

Concluding, Engle's UHF-GARCH model shows the effect of the trade process over the volatility but it doesn't study a possible effect of the volatility over the trading intensity. The interdependence between these two variables is analyzed by Ghysels and Jasiak (1998) and Grammig and Wellner (2002) and in the survey of Kamionka (2000). The simultaneous modelization of the duration and other characteristics of the transactions, like the prices, the returns or the volume, is developed by Dufour and Engle (2000), Spierdijk (2004) and Manganelli (2005) and the main conclusion is that periods of time with high trading intensity are associated with higher price changes, especially in the more frequently traded stocks.

5. ESTIMATION AND RESULTS

5.1. The characteristics of MMI's durations

Loans of reserves in the MMI are not regularly spaced. Thus, we computed, in seconds, for the two semesters considered, the two series of the durations $x_t = \tau_t - \tau_{t-1}$ where τ_t is the moment in time when the t^{th} loan is communicated to the *Banco de Portugal*. As in similar studies, we have not considered the time elapsed between the last transaction of the day and the first one of the following day. Therefore, the first duration of each day is the time elapsed between the first and the second loan of the market session.

Table 1 shows the characteristics of the two series of durations, usually presented in the literature[39]. The MMI is more liquid in 1998 than in 2001: in 4 months of 1998 the number of loans is bigger than in 6 months of the year 2001. The intensity of transactions is larger in 1998, as we can see through the values of the average, median, mode and maximum of the duration. The minimum value of the duration is zero, in both cases, that is, in the two periods of time there are simultaneous loans in the market. However, in 2001 this situation occured only once, whereas in 1998, it occured 4 times.

[39] See, among others, Bauwens and Giot (2001) or Giot (2001).

TABLE 1. TRADE DURATIONS IN THE MMI

	1998 1 July-3 November	2001 May- October
Number of durations	5141	2352
Durations (in seconds):		
maximum	4889	12110
minimum	0	0
average	356	1387
median	196	752
mode	24	77
Standard-deviation	454	1727
Skewness	2,933	2,301
Kurtosis	13,432	6,520
Jarque-Bera test	46017,0	5241,9
Dispersion index	1,275	1,245
Q(10)	57,054	49,608
	(0,000)	(0,000)

Source: *Banco de Portugal.*
Values in parentheses indicate the significance level.

The dispersion index (equal to the ratio standard deviation to mean) is similar for both periods of time and indicates overdispersion (>1). As usual in the series of trade durations, the Ljung-Box statistic Q(10) shows a high value (with p-value equal to 0,000) indicating a strong autocorrelation in the durations. The autocorrelation function (ACF) and the partial autocorrelation function (PACF) of durations are presented in table 2 and show the usual features of the trade durations series.

TABLE 2. AUTOCORRELATION FUNCTION (ACF) AND PARTIAL AUTOCORRELATION FUNCTION (PACF) OF DURATIONS IN THE MMI

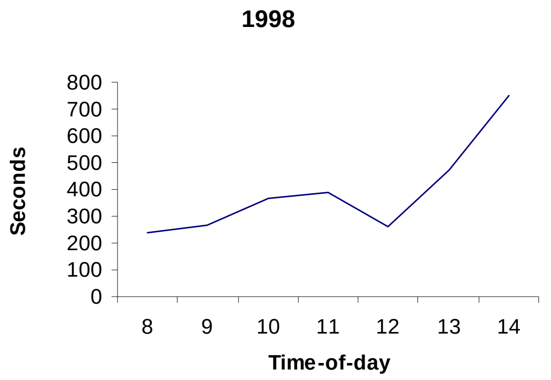
	1998 1 July-3 November		2001 May- October	
	ACF	**PACF**	**ACF**	**PACF**
1	0.0721890	0.0721890	0.0207843	0.0207843
2	0.0424153	0.0373989	0.0178223	0.0173978
3	0.0367578	0.0313174	0.0196505	0.0189389
4	0.0334279	0.0273693	-0.0154912	-0.0166010
5	0.0008161	-0.0059727	-0.0236322	-0.0236867
10	-0.0152342	-0.0110793	-0.0522510	-0.0483290
15	-0.0201785	-0.0112163	0.0361596	0.0171177
20	-0.0151427	-0.0130099	0.0543404	0.0448782
25	-0.0041191	-0.0089281	0.0220293	0.0470166

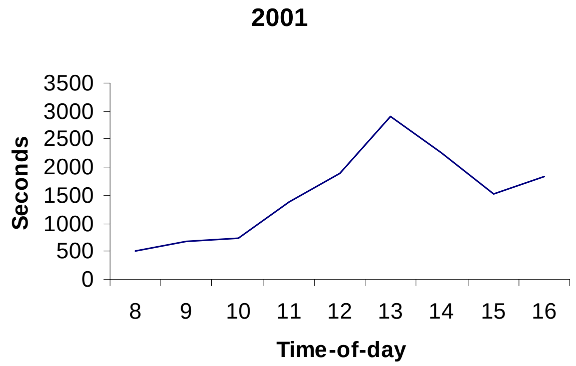
Source: *Banco de Portugal.*

It often happens that the trade durations show strong intra-day seasonality. The figure 1 presents the intra-day behaviour of the MMI's durations and shows a difference between its behaviour in 1998 and in 2001. In 1998, after the open of the market, the duration presents its minimum value, and then it increases smoothly. In the middle of the market session, between 12 a.m. and 1 p.m. it falls, and then it increases strongly until the end of the session. In 2001, the open also shows the smallest duration, but then it increases, reaching the maximum between 1 p.m. and 2 p.m. and then it decreases. These differences are compatible with the rules of the market and the institutional environment of each of the two periods analyzed.

In 1998 the maintenance of required reserves was much more difficult due to a shorter reserve maintenance period, the uncertainty about the minimum reserves to hold and a shorter market session. Therefore, in 1998, we don't find the peak in durations that is usual in several markets at lunch time an that we find in 2001 in the MMI. In 1998, at lunch time, the duration is small, due to the need, by banks, to intervene in the market and to not postpone the loans until the close. In 2001, the banks have more flexible conditions for the maintenance of minimum reserves: the maintenance period is longer and the market session is also longer, ending after the business close. Thus, the banks can postpone, to the last hours of the trading session, their intervention in the market; the trading intensity is stronger between 3 p.m. and 4 p.m.

FIGURE 1. AVERAGE TRADE DURATION ALONG THE MMI SESSION





Source : Banco de Portugal.

As usual in the literature, we computed the time-of-day adjusted durations[40][41] x_t^a as :

[40] See, among others, Engle and Russell (1998) and Bauwens and Giot (2001).

[41] Before this transformation, we added 1 second to all the durations, therefore eliminating the zeros of the series and maintaining its dynamic structure.

$$x_t^a = x_t \big/ \phi(\tau_t) \qquad (7)$$

where x_t is the original duration. In this equation $\phi(\tau_t)$ represents the time-of-day effect at time τ_t and it is defined as the expected duration conditioned on time-of-day $\phi(\tau_t) = E(x_t \mid \tau_t)$. The procedure usually adopted in the literature is the following: the daily session is divided in several parts (half-hour or one hour intervals), then linear splines[42] or cubic splines[43] are used to compute "smooth" durations. This method computes regressions to 1st order polynomials[44] in the case of the linear splines or 3rd order, in the case of the cubic splines. The original series is then divided by the estimated series and an adjusted time-of-day series, with mean near 1, is computed.

However, the MMI presents a small number of transactions (compared with the trades in the equity markets), especially in 2001. The existence of one hour intervals (and even two hours) without any loan is frequent. In the table 1, we see that the larger duration in the MMI, in 2001, is about 3 hours and 36 minutes. As a consequence, to compute regressions on the MMI durations is technical possible, but it means that the regular intervals must be very large and a problem of lost of information would emerge. Therefore, the MMI's trade durations were adjusted trough a simple method: we calculated the average of the duration over each hour of the market session, taking in account all the observations of the semester; this average was used as the expected duration conditioned on time-of-day. Then, the original series was divided by the series of conditioned expected durations and we obtained the series of adjusted durations. This procedure has been used by Taylor and Xu (1997); Areal and Taylor (2002) compute the time-of-day effects of the volatility of the futures contracts using alternative methods and conclude that the calculation of the simple multiplier produces estimations similar to the ones obtained trough more sophisticated methods[45].

Table 3 reports the statistics of the series of adjusted durations. Taking into account the intraday seasonality leads to a drop in the autocorrelation exhibited by the durations, but does not eliminate it completely. In 2001, the Ljung-Box statistic Q(10) is not significant but the same statistic presents significant values to low number of lags. Just like the original series, this one also presents overdispersion.

[42] Dufour and Engle (2000), Manganelli (2002) and Engle (2000) use linear splines.
[43] Engle and Russell (1998) and Giot (2001) use cubic splines.
[44] The linear regression allows the estimation of the mean and the tendency over each one of the hours of the daily session.
[45] Areal and Taylor (2002) compute the effects time-of-the-day using Fourier flexible functions, according with Andersen and Bolllerslev (1997).

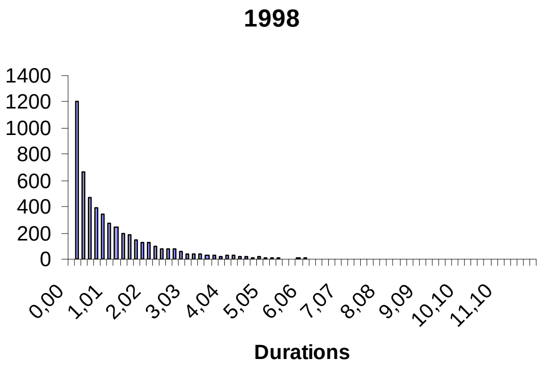
TABLE 3. ADJUSTED TRADE DURATIONS IN THE MMI

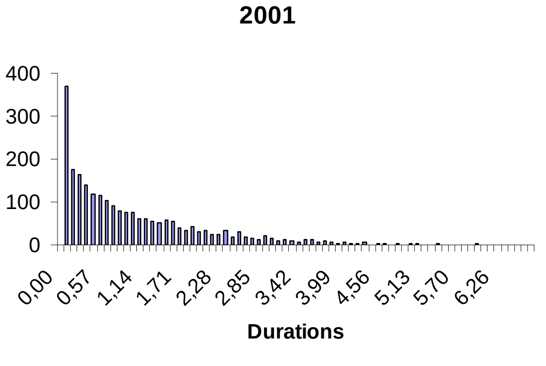
	1998 **1 July-3 November**	**2001** **May- October**
Number of durations	5141	2352
Average	1,000	1,000
Standard-deviation	1,147	1,039
Skewness	2,284	1,664
Kurtosis	8,124	3,419
Jarque-Bera test	18607,9	2231,1
Dispersion index	1,147	1,039
Q(10)	22,973	14,243
	(0,011)	(0,162)

Source: *Banco de Portugal*.
Values in parentheses indicate the significance level.

Figure 2 shows the histograms of the adjusted durations confirming the positive asymmetry of the series. These characteristics are usual in the series of trade adjusted durations.

FIGURE 2. HISTOGRAM OF THE MMI ADJUSTED DURATIONS





Source: *Banco de Portugal*.

After accounting for the deterministic intraday seasonality, it still exist a dynamic structure of trade durations in the MMI. In the next sections we will take as input the series of adjusted durations[46].

5.2. Estimation

It is not frequent to apply ACD models to the money markets; the exception is Fischer (2000) who modeled the durations in the Federal Funds Market, aiming to study the duration-intervention of the target funds rate, that is, the probability of a change in this rate[47].

In this work, we apply an ACD(1,1) model, like the one proposed by Engle e Russell (1998), on the MMI data, and analyze the overnight interest rate dynamics, taking in account the trading intensity. In order to estimate the model we need to assume a distribution of the error term and, after its estimation, we need to test it. To evaluate the model we tested the absence of autocorrelation of the residuals and of squared residuals with Ljung-Box Q-statistics. We also checked the distributional assumption, using the procedure proposed by Diebold, Gunther and Tay (1998) and used by Bauwens, Giot, Grammig and Veredas (2004) and Bauwens and Giot (2001). This method consists in the analysis of the density of the forecasts, that is, taking in account the historical durations, to analyse the distribution of future durations, with the aim to minimize the loss function. The appendix 1 presents this method with more detail.

First, we assumed a Weibull distribution for the error, which fits a positive series. The results obtained with WACD were similar to the ones usually obtained to the duration-trade series and its application eliminated the autocorrelation of the residuals. However, the result on the chi-square goodness-of-fit statistics led to the rejection of the uniform distribution of z. Thus, we assumed others distributions: the exponential distribution, the Burr distribution[48] and the Pareto distribution[49]. The test always rejected the null hypothesis of the uniform distribution of z. Therefore, we used the Weibull distribution and we present its results because the WACD model is parsimonious, it is widely used in the literature and its implementation is easy. Its results are similar to the ones obtained with different distributional assumptions. The estimated parameters are stable, there is no autocorrelation of the residuals, and it characterizes very well the clustering feature of the durations. The main objective of this work is to analyse the behaviour of the volatility and the effect of the passing of the time over the volatility, and not to obtain a perfect adjustment of the duration series. We present the results in the next section. Its equations, the characteristics of the Weibull function and the derivation of the log-likelihood function are described in Appendix 2.

[46] For writing and reading simplicity, we will drop the coefficient *a* that signals the adjusted duration.
[47] In the USA, the Federal Reserve intervenes daily in the market, the target funds rate is changed frequently.
[48] The Burr distribution, unlike Weibull distribution, allows a non monotonic hazard function, that is, it allows that the probability of a trade in the next second does not behave monotonically. This was the reason that motivated its use by Grammig and Maurer (2000).
[49] This distribution is suitable in the modelation of extreme values, so it can be applied to duration series with fat tails.

5.3. Results of the application of the ACD model to the MMI's durations

The results obtained in the estimation of the WACD model, applied to the MMI's durations, can be observed in the table 4[50]. These results are similar to the ones usually obtained on the stocks' trade durations. All the parameters estimated are significant, confirming the clustering in the durations of the MMI, that is, long (short) durations are followed by long (short) durations. In both cases we verify that $\alpha + \beta < 1$, and its sum is approximately equal to 0,80 in 1998 and approximately equal to 0,98 in 2001, so the persistence is higher in 2001. The estimated parameter γ of the Weibull distribution is inferior to 1 indicating overdispersion of the series: a long duration means that is less probable that a trade takes place in the next moment of time[51]. The Ljung-Box statistics (1 and 10 lags) allow the conclusion that the application of the model eliminates the autocorrelation in the durations. Similar estimations, including a dummy variable related to the last day(s) of the reserves maintenance period[52], were carried out. The results, not presented here, were similar to these ones, allowing the observance of the stability of the coefficients estimated. The coefficients estimated for the dummy variable were in both cases, and as expected, negative.

TABLE 4. RESULTS OF THE WACD MODEL IN THE MMI

	1998 1 July-3 November			**2001 May- October**		
	Estimated coefficient	Standard-deviation		Estimated coefficient	Standard-deviation	
ω	0.202590	0.001320	**	0.018173	0.000356	**
α	0.043982	0.010713	**	0.004340	0.000198	**
β	0.752918	0.009734	**	0.977565	0.000731	**
γ	0.892933	0.009627	**	0.887597	0.014650	**
Value of log-likelihood function	-5068.248			-2324.731		
Autocorrelation of residuals	Q(1)=0.0578 (0.809985) Q(10)=7.2358 (0.703011)			Q(1)=2.6766 (0.101835) Q(10)=15.0145 (0.131532)		
Autocorrelation of $(\text{residuals})^2$	Q(1)=0.7739 (0.379007) Q(10)=8.5840 (0.571981)			Q(1)=2.0789 (0.149353) Q(10)=10.9657 (0.360196)		
Test of the uniform distribution	$Q^2(9)$=351.859366 (0.000000)			$Q^2(9)$=169.8793 (0.000000)		

[50] We also estimated WACD(2,2) with results similar to the WACD(1,1).

[51] The hazard function represents the instantaneous rate of occurrence of a trade per second. In the Weibull, with $\gamma < 1$, this function is decreasing. See, for instance, Bauwens and Giot (2001).

[52] The dummy variable is 1 in the last day of the maintenance period in 1998 and it is 1 in the last two days of the maintenance period in 2001.

According with the WACD representation, Appendix 1, ω is the *constant*, α is the estimated coefficient of the variable *past duration*, β is the estimated coefficient of the variable *past expected duration* and γ is the estimated parameter of the *error term* distribution.
Values in parentheses indicate the significance level
* Significant at 5% level **Significant at 1% level

5.4. Results of the application of the UHF-GARCH model to the MMI

The aim of this work is to study the effect of the trading intensity in the MMI over the overnight interest rate volatility: we apply the approach proposed by Engle (2000). This analysis considers the effect of the trade process over the volatility, but doesn't study a possible effect of the volatility over the trading intensity. We choose this simple approach by two reasons: first, the aim of this work is the study of the volatility of the overnight interest rate and, second, the application of market microstructure theories (which these models confirm or don't confirm) in the money market must be done carefully. Many of their essential hypotheses don't apply to this market as, for instance, the realization of trades by speculative reasons.

Following Engle (2000), we constructed the series of the changes in the overnight interest rate[53], and we divided it by the series of the square root of the durations, obtaining the series of the overnight interest rate change per unit of time, per second. Then, these series were adjusted according to the same method applied to the durations.

Finally, we estimated the UHF-GARCH model proposed by Engle. The mean equation is specified as an ARMA process and the variance equation is specified as equation 6 augmented with V_j, a vector of dummies variables: a dummy variable that takes value 1 in the last day(s) of reserve maintenance periods and a dummy for the days that followed the terrorist attacks of September 11th in the USA (due to the economic instability that followed it). The series of the expected duration ψ_t, in each of the two periods considered, is obtained using the parameters of the estimated WACD model. The results obtained in the estimation of the UHF-GARCH model, applied to the MMI are presented in the table 5. Some coefficients have a high value, due to the transformations in the durations and in the overnight changes.

[53] We eliminated the first change of each day, that is, we do not considered the night change in the interest rate. The same procedure was adopted in the case of the durations. Thus, the series of durations and changes in the interest rate can be used simultaneously in the estimations.

TABLE 5. RESULTS OF THE UHF-GARCH MODEL IN THE MMI

	1998 1 July-3 November			**2001 May- October**		
	Estimated coefficient	Standard-deviation		Estimated coefficient	Standard-deviation	
Mean						
Constant	1.99900	0.21053	**	0.12422	0.01581	**
AR(1)	-0.18717	0.01152	**	0.41987	0.01368	**
AR(2)	-	-		0.04744	0.06438	
AR(3)	-	-		0.52491	0.06545	**
MA(1)	-0.20280	0.01112	**	-0.67346	0.02903	**
MA(2)	-	-		0.06473	0.05650	
MA(3)	-	-		-0.38114	0.06395	**
Variance						
constant	9929.89703	78.53375	**	11077.35124	587.50430	**
ARCH(1)	0.98050	0.00449	**	0.46746	0.01553	**
GARCH(1)	0.01950			0.12752	0.02589	**
1/duration	75.05274	1.17322	**	-1.16413	0.37271	**
Duration/expected duration	-133.21266	1.65308	**	65.02765	8.90805	**
Long run volatility (t-1)	-0.00025	0.00002	**	0.06094	0.00186	**
1/ expected duration	-7996.33305	69.43583	**	-10932.2926	580.48875	**
last day(s) of m.p.	2357.13033	26.37833	**	1470.41138	368.65650	**
September	-	-		810.79610	80.58688	**
Value of log-likelihood function	-28640.612			-12442.071		

The series of the overnight interest rate change per second is modeled using an ARMA(3,3) for the mean equation. The estimated parameters are the constant, the AR(1), AR(2), AR(3), MA(1), MA(2) and MA(3). The variance equation is the equation 6 augmented with two dummies variables: the variable *last day(s) of m.p.* and the variable *September*.
Values in parentheses indicate the significance level
* Significant at 5% level
**Significant at 1% level
With these estimations, the test results fail to reject the hypothesis of non autocorrelation of residuals, of the inexistence of additional ARCH effects and of the inexistence of signal effects.

The estimations performed to the period of 1998 showed that an IGARCH equation would be indicated, because the sum of ARCH e GARCH coefficients presented values close to 1. Therefore, we concluded that the persistence of the interest rate changes is strong; the shocks produce long lasting effects. Note that, after removing the intra-day seasonality, we can expect that the effect of the shocks would be high and persistent, since the rules of the maintenance of minimum reserves, that influence the behaviour of the banks, are very strict. In 2001, the conditional variance was estimated as a GARCH(1,1) because the persistence of the shocks was smaller: its effect over the bank's reserve position has a long period to fade away, it does not force to an immediate reaction in the demand/supply of reserves with effects over the interest rate. Nonetheless, these results are still higher than the results of Engle (2000) and Grammig and Wellner (2002), who found evidence that the persistence in the GARCH models estimated was small. However, the coefficients estimated for the variable *long run volatility* (ξ_t) are small, especially in 1998, and negative, as in Grammig and Wellner (2002).

The reaction of the overnight rate volatility to the trade intensity can be expressed through several factors: the first one is the reciprocal of the duration (x_t^{-1}) or, the current trade intensity. Engle (2000) and Grammig e Wellner (2002) obtain positive coefficients in this variable and interpret them according to the Easley and O'Hara (1992) model, in which there is asymmetrical information. In that, trade intensity is related with new information: the agents who possess private information trade in the market, selling (when the news are bad) or buying (when the news are good). Therefore, long durations are associated with the lack of new information, that is "no trade means no news". We classified banks as *liquidity traders*, because they trade in the money market in order to hold minimum reserves and working balances. Therefore, the issue of private information is not relevant in this market; the portfolio needs of the banks determine their loans. As for the time variable being relevant, Dufour and Engle (2000) say that is essentially an empirical question. In the MMI's case, we can say that, in 1998, more intense trades have the effect of raising the overnight rate volatility; in 2001 they have the opposite effect. Taking in account the reserve maintenance rules, in each of these two periods of time, how can we explain this evidence? In both periods, banks make loans due to their reserve needs, but in 1998 banks had less discretionarity, they had less time to balance their average reserve balances. Thus, more intense trades mean there were many banks with urgent needs, willing to pay/receive the price asked/bid in order to balance their portfolio, leading to higher interest rate volatility. In 2001, the banks have higher discretion, they can satisfy their needs when it is most convenient: banks who need reserves, want to borrow cheap, and those with excess liquidity want it the other way round. Thus, discretion contributes to a better result of the reserve management of a bank, to a higher difference between the gains from the loans made and the costs of borrowing reserves in the market[54].

These canceling desires of different banks, along with less urgency, contribute to make concentration periods less acute when they do occur; which means they have lower volatility. Note that, according with Admati and Pfleiderer (1988), the discretionary liquidity traders concentrate their trades in the moments of time when their costs are lower, meaning no big gains or losses; the concentration also means a low proportion of informed traders, that is, it is not associated with new information, so, both factors push the volatility down.

In 2001, the effect of the expected trade intensity reinforces the one of the trade intensity, in 1998 the effect is inverse of the one of the trade intensity (in 1998, strong expected intensity of trades as a negative effect over the volatility).

The surprise in the durations also has opposite effects; in 1998 the estimated coefficient of this variable is negative, like in Engle (2000) and Grammig and Wellner (2002): the higher the duration over its expected value (the lower the trade intensity over its expected value), the lower the volatility. The opposite happens in 2001 where the estimated coefficient of the same variable is positive: the higher the duration over its expected value, the higher the volatility. Therefore, in both periods, the surprise in the trade intensity reinforces the effect of the trade intensity.

The dummies variables have, as we expected, positive estimated coefficients. In the last days of maintenance periods, the overnight interest rate volatility is higher, due to the

[54] According to Angelini (2000) the cost/profit of borrow/lend in market enters in the objective function of a bank.

pressure to hold minimum reserves which is stronger in these last days. The volatility is also higher in the days that followed the terrorist's attacks of September 11th, due to the instability verified in financial markets.

As for the mean, in 2001 we needed to estimate an ARMA(3,3) for the mean equation. For the period of 1998, we estimated an ARMA(1,1). The estimated coefficients to the factors AR and MA are significant (with two exceptions) thus, we can not accept the martingale hypothesis to the overnight interest rate changes per second.

The main conclusion of this work is that the minimum reserve system introduced by the Single Monetary Policy and the enlargement of the trading session mean less uncertainty to Portuguese banks, and contributed to their higher discretion in the choice of the time to trade. The policy implications of this evidence for monetary authorities and for the design of monetary policy are clear: if central banks have a short-term interest rate objective, characteristics of the reserve system like these – knowledge about the amount of reserves to hold, a long period of time to hold them, a long daily session to intervene - are a good contribution to obtain interest rate stability. They ease the management of reserves by banks and bring a higher discretion regarding the time to trade in the market, leaving the role of reserves in monetary policy unchanged.

CONCLUSION

The aim of this work is the application of ACD models to the durations of the Portuguese money market (MMI) and the study of the influence of these durations over the volatility of the overnight interest rate. The ACD models, originally proposed by Engle and Russell (1997, 1998), study the processes that regulate the arrival of new trades and the time elapsed between them.

Our data is intraday data related to all operations of the MMI, irregularly spaced, and covers two periods of time where the rules of maintenance reserves were not the same. We constructed the two series of durations, that exhibit strong intra-day seasonality and we computed the time-of-day adjusted durations. Then we applied an WACD(1,1) model to these data and concluded that the parameters estimated are significant, confirming the clustering in the durations: long (short) durations are followed by long (short) durations. This means the MMI's trade session is characterized by some periods with high trade intensity and periods with low trade intensity.

Finally, we estimated an UHF-GARCH model and we concluded that the trade intensity exerts influence over the overnight interest rate volatility. The influence over the volatility is different between the two periods of time and it is compatible with the rules of the required reserves systems. The introduction of the Single Monetary Policy features and the enlargement of the MMI trading session contributed to bank's higher discretion, and the possibility of choosing the moment to trade means higher interest rate stability and lower volatility.

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APPENDIX 1. TESTING THE DISTRIBUTIONAL ASSUMPTION OF THE ACD MODEL

In order to evaluate the estimated ACD model it is useful to check the distributional assumption. The procedure of evaluation was proposed by Diebold, Gunther and Tay (1998) and used by Bauwens, Giot, Grammig and Veredas (2000) and Bauwens and Giot (2001). This method consists in the analysis of the density of the forecasts, that is, taking in account the historical durations, to analyse the distribution of future durations, with the aim to minimize the loss function.

Supposing that :

$$\{f_t(x_t \mid I_t)\}_{t=1}^{m}$$

is the sequence of densities that define the duration series, and that

$$\{p_t(x_t \mid I_t)\}_{t=1}^{m}$$

is the sequence of one-step-ahead density forecasts, it must be determined if:

$$\{p_t(x_t \mid I_t)\}_{t=1}^{m} = \{f_t(x_t \mid I_t)\}_{t=1}^{m} \qquad \text{(A.1.1)}$$

The evaluation of the relationship between the true density and the forecasts' density is done using the *probability integral transform* z_t that is the cumulative density function of

$p_t(x_t)$:

$$z_t = \int_{-\infty}^{x_t} p_t(u)du = P_t(x_t) \qquad \text{(A.1.2)}$$

Under the null hypothesis $p_t = f_t$, thus, z_t is uniformly distributed (*U(0,1)*).

To verify the null hypothesis, the authors propose graphical analysis (the histogram of the empirical z-sequence complemented with confidence intervals, in order to compare the estimated density with *U(0,1)* and to compute chi-square goodness-of-fit statistics for the uniform distribution of *z*, in order to test dependence patterns in *z*.

APPENDIX 2. THE WACD MODEL

The WACD(1,1) model is defined by the equations:

$$x_t = \psi_t \varepsilon_t \qquad \text{(A.2.1)}$$

$$\psi_t = \omega + \alpha x_{t-1} + \beta \psi_{t-1} \qquad \text{(A.2.2)}$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta < 1$.

A variable *z* follows a Weibull distribution, with parameters (γ, c) when:

$$f(z \mid \gamma, c) = \frac{\gamma}{c^{\gamma}} z^{\gamma-1} \exp\left(-(z/c)^{\gamma}\right) \qquad \text{with } z > 0 \tag{A.2.3}$$

The uncentered moments of order *p* are represented by:

$$\mu_p = c^p \Gamma(1 + p/\gamma) \tag{A.2.4}$$

where Γ is the gamma function.

The γ parameter is related with the degree of dispersion of the variable. If $\gamma < 1$ the variable is overdispersed and if $\gamma > 1$ it indicates underdispersion. If $\gamma = 1$ the distribution is the exponential where $f(z) = \frac{1}{c}\exp\left(-(z/c)\right)$.

Normalizing $\varepsilon = z \big/ (c\Gamma(1+1/\gamma))$ and taking in account that the expected value of the error term is 1, we can write:

$$f(\varepsilon \mid \gamma) = \gamma \left[\Gamma(1+\frac{1}{\gamma})\right]^{\gamma} \varepsilon^{\gamma-1} \exp\left[\left(-\Gamma(1+\frac{1}{\gamma})\varepsilon\right)^{\gamma}\right] \qquad \text{with } \varepsilon > 0 \text{ [55]} \tag{A.2.5}$$

Assuming these hypotheses for the distribution of the error term, we can describe the log-likelihood function[56]. The density function of x_t is:

$$f_x(x_t \mid I_{t-1}, \theta) = f_{\varepsilon}(\frac{x_t}{\psi_t}) \left|\frac{d\varepsilon_t}{dx_t}\right| = f_{\varepsilon}(\frac{x_t}{\psi_t})(\psi_t)^{-1} \tag{A.2.6}$$

In logarithmic form the log-likelihood function is:

[55] For sake of simplicity, we drop *t* in the equations.
[56] About the Weibull and the log-likelihood function see Bauwens and Giot (2001) or Kamionka (2000).

$$L = \sum \left[\ln f_{\varepsilon}(\frac{x_t}{\psi_t}) - \ln \psi_t \right] = \sum \left[\gamma \ln\left(\Gamma(1+\frac{1}{\gamma}) \right) + \ln(\frac{\gamma}{x_t}) + \lambda \ln(\frac{x_t}{\psi_t}) - \left(\frac{\Gamma(1+\frac{1}{\gamma}) x_t}{\psi_t} \right)^{\gamma} \right]$$

(A.2.7)