

Physics of the fire piston and of the fog bottle

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Abstract

We analyze two irreversible adiabatic processes occurring in two thermodynamical devices: the fire piston and the “fog bottle”. In the first, an abrupt adiabatic compression of air inside a piston causes the burning of a small paper piece, since the final temperature reaches 700 K. In the second, an abrupt adiabatic expansion of a gas at high pressure leads to a temperature decrease with condensation of water vapour and fog formation. We show that, if the two processes were carried out reversibly, the final temperature would always be lower than in the irreversible processes.

Introduction

Count Rumford¹ was the first to recognize that mechanical methods could be equivalent to heat². We may confirm this with an easy experiment: a small aluminum bar frictioned with iron wool may heat up so that we get burned if we touch it with a bare hand.

The fire piston is a thermal device used in popular demonstrations³. The experiment done with it shows the conversion of mechanical energy into heat or, more precisely, internal energy, exemplifying the First Law of Thermodynamics⁴. The process is more controllable than that with the metallic bar.

On the other hand, producing fog in a bottle⁵ is another interesting experiment, where the effect is opposite. An abrupt expansion of air in a vessel leads to a temperature decrease, which causes condensation of the water vapor present in the air.

We describe here in detail these experiments, as done by us, and present the calculations associated with the two adiabatic irreversible processes. We show that if the processes were reversible the final temperatures in both cases would be lower than in the actual experiments.

¹ E. C. Watson, *Reproductions of prints, drawings and paintings of interest in the history of physics. 10. Gillray's caricature of Count Rumford*, Am. J. Phys. **8**, 120 (1940); Marcy S. Powell, *Count Rumford: soldier, statesman, scientist*, Am. J. Phys. **3**, 161 (1935)

² S. C. Brown, *The discovery of convection currents by Benjamin Thompson, Count of Rumford*, Am. J. Phys. **15**, 273 (1947).

³ A. Serrano, *Fire piston*, The Phys. Teacher **44**, 613 (2006)

⁴ M. E. Loverude, C H Kautz, P R L Heron, *Student understanding of the first law of thermodynamics: relating work to the adiabatic compression of an ideal gas*, Am. J. Phys. **70**, 137-148 (2002).

⁵ M. Talmage Graham, *Cloud formation*, The Phys. Teacher **42**, 301-304 (2004)

The fire piston

For the fire piston⁶, we used a transparent lucite tube, with height $h = 17.9$ cm and internal diameter $d = 1.0$ cm, tightly mounted vertically on a wood platform. A small piece of cotton or a very thin bit of paper was placed inside the tube which was closed by means of a steel piston. To better close the air inside the tube, an appropriate O-ring was mounted around the steel bar⁷. To originate a flame at the bottom of the cylinder, we simply pulled downwards the piston very quickly.

We placed the apparatus on a bathroom scale to estimate the force exerted on the piston. Using a small camera we made a movie which we viewed frame by frame. Figure 1 shows two pictures obtained in this way.

A thin and small piece of paper burns at $T_F = 451$ °F (using the famous novel *Fahrenheit 451* by the science-fiction writer Ray Bradbury⁸), equivalent to $T = 232.8$ °C ≈ 510 K.

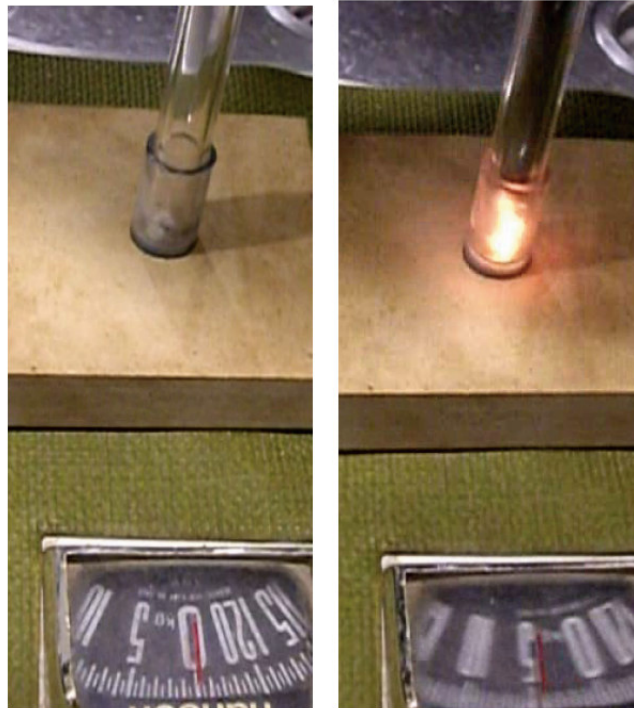


Figure 1: Fire piston. The air inside the cylinder is compressed in adiabatic conditions. On the right, the cotton placed in the interior burns due to the high temperature. The scale measures the force.

⁶ H. Hayn, S. C. Baird, *Adiabatic compression in a fire syringe piston*, The Physics Teacher **23**, 101-102 (1985).

⁷ Fire Syringe Ref. FIR-150 in Educational Innovations Inc., see www.teachersource.com.

⁸ This novel describes a society in which books are forbidden, being burned by the police. The temperature at which the paper is burned is 451 °F

The formation of the flame is explained using the First Law. We first show that the air compressed inside the cylinder reaches the ignition temperature of the paper.

The variation of the internal energy is given by

$$\Delta U = n c_v (T_f - T_i),$$

where $T_i \approx 300$ K is the initial temperature of the air, T_f its final temperature, n is the number of moles and $c_v \approx \frac{5R}{2} \approx 20 \text{ J mol}^{-1} \text{ K}^{-1}$ is the heat capacity of the air at constant volume⁹ ($R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ is the ideal gas constant). The volume occupied by the air is $V = V_i = \pi(d/2)^2 h$, so that the number of moles is given by

$$n = \frac{P_i V_i}{RT_i} = 0.57 \times 10^{-3} \text{ mol.}$$

The First Law states

$$\Delta U = Q + W,$$

where Q stands for heat and W for work. The system is contained in plastic wall and the process is so fast that there is no heat exchange, *i.e.* it is adiabatic: $Q = 0$. The configuration work can be expressed as a function of the pressure and the volume. Assuming that the work is done at constant external pressure P_E ¹⁰ we have

$$W = F_E \Delta L = -P_E \Delta V,$$

where F_E it is the external force, ΔL is the piston displacement and $\Delta V = A \times \Delta L$ is the (negative) variation of the volume of the gas ($A = \pi(d/2)^2$ is the area of the internal section of the tube). According to our measurement, we take $F_E = 50$ N as if a body of 5 kg were placed on the system. The corresponding pressure is

$$P_E = \frac{F_E}{A} = \frac{50}{\pi(10^{-2}/2)^2} = 6.37 \times 10^5 \text{ Pa.}$$

This is also the final pressure of the gas.

The work done on the gas is

$$W = -P_E (V_f - V_i).$$

⁹ According to K. Raznjevic, *Tables et Diagrammes Thermodynamiques*, Editions Eyrolles, Paris (1970), at 0 °C one has for the air $c_p = 1.227 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and $\gamma = 1.40$. Assuming the molecular weight 28.96 g mol^{-1} , we have $c_v = c_p / \gamma = 25.38 \text{ J mol}^{-1} \text{ K}^{-1}$.

¹⁰ C E Mungan, *Irreversible adiabatic compression of an ideal gas*, The Phys. Teacher 41, 450-451 (2003). It is the external pressure that matters. The process is non-static and irreversible so that the pressure of the system is not even defined.

The initial volume of the air is

$$V_i = h\pi\left(\frac{d}{2}\right)^2 = 14.1 \times 10^{-5} \text{ m}^3.$$

Using $\Delta U = W = nc_V(T_f - T_i)$ and since (we accept that air is an ideal gas)

$$T_i = \frac{P_i V_i}{nR} \quad \text{and} \quad T_f = \frac{P_f V_f}{nR}$$

we obtain

$$T_f = T_i \frac{c_V + R(P_E / P_i)}{c_V + R}.$$

The final volume is given by

$$V_f = V_i \frac{P_i T_f}{P_E T_i} = V_i \frac{c_V(P_i / P_E) + R}{c_V + R}.$$

As $P_E \rightarrow \infty$, the volume does not vanish but approaches its minimum value, given by (we use the relation $c_p = c_V + R$)

$$V_{\min} = \frac{V_i}{c_p/R} = \frac{2}{7} V_i.$$

For air initially at 300 K, and taking $P_E/P_i = 6.3$, one obtains $T_f = 2.51 T_i = 754.3$ K, corresponding to the temperature variation $\Delta T = 454$ K. This value explains the paper ignition¹¹. The final volume is then

$$V_f = 14.1 \times 10^{-6} \times \frac{2.51}{6.3} = 5.61 \times 10^{-6} \text{ m}^3.$$

We may calculate the temperature variations if an adiabatic reversible processes would bring the gas from the same initial state to a final state in which one of the state variables is the same as in the final state of the irreversible adiabatic process. The temperature corresponding to the final state ($P_f = 6.3 \times 10^5$ Pa, V_f' , T_f') after the adiabatic process is

$$T_f' = T_i \left(\frac{P_i}{P_f} \right)^{(1-\gamma)/\gamma} = 300 \times 6.3^{0.285} = 507.3 \text{ K}.$$

¹¹ H. Hayn, S. C. Baird, *Adiabatic compression in a fire syringe piston*, The Physics Teacher **23**, 101-102 (1985).

This temperature is lower than the final temperature of the irreversible process. The corresponding final volume, which can be obtained from the equation of state or from the equation of the adiabatic process, is $V_f' = 3.79 \times 10^{-6} \text{ m}^3$.

If, instead, we consider the final state ($P_f'', V_f = 5.61 \times 10^{-6} \text{ m}^3, T_f''$) reached from the initial state through another adiabatic reversible process, the final temperature is

$$T_f'' = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = 300 \times \left(\frac{14.1}{5.6} \right)^{0.4} = 434 \text{ K},$$

also lower than the temperature reached in the irreversible process. The final pressure is now $P_f'' = 3.63 \times 10^5 \text{ Pa}$. Let us consider the work in an adiabatic process:

$$W = - \int_{V_i}^{V_f} P_{\text{ad}} dV = \frac{P_f V_f - P_i V_i}{\gamma - 1}.$$

For the two reversible adiabatic processes, one obtains $W' = 2.43 \text{ J}$ and $W'' = 1.57 \text{ J}$. Both are smaller than the work done for the irreversible process,

$$W_1 = -6.3 \times 10^5 (5.61 - 14.1) \times 10^5 = 5.35 \text{ J},$$

explaining the smaller temperature increase in reversible processes.

The process described here is the basis of the Diesel engine, in which the mixture of Diesel oil and air become inflamed spontaneously when putting it under a high pressure, without a spark plug.

Fog bottle

An experiment closely related to the fire piston¹² is the production of fog in a bottle¹³. We are still in the presence of an adiabatic irreversible process, but now we have an expansion¹⁴. Using an air pump the pressure is increased inside a bottle (the fog bottle) with a lateral entrance (the walls of the bottle should be resistant enough to avoid blowing up of the glass with the pressure increase) and with a plastic cork firmly attached to its mouth. A T-junction allows the connection of a manometer.

In our experiment, when the pressure was around 4 bar, the cork of bottle suddenly flied (the bottle was firmly mounted in a rigid structure to prevent its backwards motion). We observed a cloud inside the bottle, which lasted for a few seconds (see Figure 2).

¹² R. D. Russell, *Demonstrating adiabatic temperature changes*, The Phys. Teacher **25**, 450-451 (1987).

¹³ J. J. Coop, *A demonstration of fog production*, Am. J. Phys. **9**, 242 (1941).

¹⁴ Z. Błaszczak, P. Gauden, *Application of a laser beam in demonstration of adiabatic gas decompression*, Am. J. Phys. **58**, 1112-1113 (1990).



Figure 2: Fog in a bottle. The pressure is increased inside the bottle. When the cork flies apart from the bottle, the abrupt decrease of the pressure originates a cloud. The experiment works better if few drops of water are placed inside the flask (as suggested by Coop, see footnote 13).

The volume inside the bottle is $V_i = 250 \text{ cm}^3$, and the gas (supposed to be ideal) is initially at pressure $P_i = 4 \times 10^5 \text{ Pa}$ and temperature $T_i = 300 \text{ K}$. The abrupt expansion against the atmosphere lowers the pressure to $P_0 = 1,013 \times 10^5 \text{ Pa}$, and the work made by the gas is $W = -P_0 \Delta V$, where $\Delta V = V_f - V_i$ is the increase of the gas volume. The expansion is so quick so that it may be considered adiabatic. The internal energy variation is

$$\Delta U = W = n c_v (T_f - T_i),$$

with $T_f = P_0 V_f / nR$ and $T_i = P_i V_i / nR$. The final temperature is

$$T_f = T_i \frac{c_v + R(P_0 / P_i)}{c_v + R} \approx \frac{T_i}{\gamma},$$

while the final volume is

$$V_f = V_i \frac{c_v (P_i / P_0) + R}{c_v + R}.$$

We have $T_f \rightarrow T_i / \gamma$ as $P_0 \rightarrow 0$ ¹⁵. And the volume tends to infinity as $P_0 / P_i \rightarrow 0$ ¹⁶.

¹⁵ The external pressure can be low but never zero. For $P_0 = 0$, the work would be zero, the expansion would be free (Brad Baker, *An easy to perform but often counterintuitive demonstration of gas expansion*,

For air at 300 K, and taking $P_0/P_f = 1/4$, one obtains the final temperature $T_f = 0.785 T_i = 235.7$ K, so that the temperature decrease is $\Delta T = -64.2$ K.

This temperature reduction explains the formation of fog inside the bottle. For an initial volume 250 cm^3 the number of moles is

$$n = \frac{4.0 \times 10^5 \times 250 \times 10^{-6}}{8.314 \times 293} = 4.08 \times 10^{-2} \text{ mol.}$$

The vapour pressure at the final temperature may be calculated from the Clausius-Clapeyron equation¹⁷. The vapour pressure of water at temperature T is given by

$$P_v(T) = P_0 \exp\left[-\frac{\Delta h_v}{R} \left(\frac{T_0 - T}{T_0 T}\right)\right],$$

where $\Delta h_v = 40\,500 \text{ J mol}^{-1}$ is the molar enthalpy variation for the vaporization of water (assumed to be constant), and $T_0 = 373.15$ is the boiling point of water at normal pressure, $P_0 = 1.013 \times 10^5 \text{ Pa}$. Thus, at 20°C , $P_v(293) = 0.032 \text{ bar}$, $P_v(T) = 0.023 \text{ bar}$ for a relative humidity of 70%, and the number of moles of water molecules is

$$n = \frac{4.0 \times 0.02339 \times 10^5 \times 250 \times 10^{-6}}{8.314 \times 293} = 9.16 \times 10^{-4} \text{ mol.}$$

Since at such low temperatures the vapour pressure is basically zero, in agreement with the Clausius-Clapeyron equation, all water vapour condensates, giving rise to fog. This phenomenon is better observed with the bottle filled with smoking particles and a few drops of water.

The final air volume is given by

$$V_f = V_i \frac{P_i}{P_f} \frac{T_f}{T_i} \approx V_i \frac{P_i}{\gamma P_f}.$$

In our experiment, $V_f \approx 3V_i$.

As before, let us now evaluate the temperature variations if the gas were submitted to a reversible adiabatic expansion between the same initial state ($P_i = 4 \times 10^5$

Am. J. Phys. **67**, 712 (1999)) and there would be no temperature variation for an ideal gas (J.-O. Goussard and B. Roulet, *Free expansion for real gases*, Am. J. Phys. **61**, 845-848 (1993)).

¹⁶ For an irreversible adiabatic compression, the temperature goes to infinity as the pressure increases, but the volume tends to a minimum, whereas in an irreversible adiabatic expansion, the volume goes to infinity as the pressure decreases and the temperature reaches a minimum. For reversible adiabatic processes,

$$T_f = T_i \left(\frac{P_f}{P_i}\right)^{\frac{\gamma-1}{\gamma}}; \quad V_f = V_i \left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}}$$

leading to $\lim_{P_i \rightarrow \infty} T_f = \infty$; $\lim_{P_i \rightarrow \infty} V_f = 0$; and $\lim_{P_i \rightarrow 0} T_f = 0$; $\lim_{P_i \rightarrow 0} V_f = \infty$, in contrast with the irreversible processes.

¹⁷ S. Velasco, J. Faro, and F. L. Román, *An experiment for measuring the low temperature vapor line of water*, Am. J. Phys. **68**, 1154-1157 (2000).

Pa, $V_i = 0.25 \times 10^{-3} \text{ m}^3$, $T_i = 300 \text{ K}$) and a final state for which one of the state variables is the same of the irreversible process. The final temperature T_f' of the state ($P_f = 1.013 \times 10^5 \text{ Pa}$, V_f' , T_f') reached by an adiabatic reversible process is given by

$$T_f' = T_i \left(\frac{P_f}{P_i} \right)^{\frac{1-\gamma}{\gamma}} = 300 \times 4^{-0.285} = 200 \text{ K}.$$

This temperature is lower than that obtained after the irreversible process. For the volume one finds

$$V_f' = V_i \times 4^{0.71} = 0.672 \times 10^{-3} \text{ m}^3.$$

On the other hand, the final temperature T_f'' of the state (P_f'' , $V_f = 0.75 \times 10^{-3} \text{ m}^3$, T_f'') reached by an adiabatic reversible process is given by

$$T_f'' = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1} = 300 \times 3.0^{-0.4} = 190.8 \text{ K}.$$

This temperature is also lower than that obtained at the end of irreversible process. For the pressure one finds

$$P_f'' = P_i \left(\frac{1}{3} \right)^{1.40} = 0.85 \times 10^5 \text{ Pa}.$$

For the reversible process between the initial state ($P_i = 4.0 \times 10^5 \text{ Pa}$, $V_i = 0.25 \times 10^{-3} \text{ m}^3$, $T_i = 300 \text{ K}$) and the first final state the work is $W' = -82 \text{ J}$, which is higher, in absolute value, than the work in the irreversible process:

$$W_1 = -1.01 \times 10^5 (0.75 - 0.25) \times 10^{-3} = -50 \text{ J}.$$

This explains the larger temperature decrease. For the reversible process between the initial state and the second final state, the work is $W'' = -90.6 \text{ J}$, again higher in absolute value than in the irreversible case.

In conclusion, in spite of the particular character of the described experiments – both the fire piston and the fog in a bottle – they are examples of a general, probably counter-intuitive conclusion: the final temperature in any adiabatic reversible process is always lower than in the irreversible one with equal initial and final displacement coordinates (pressure or volume). This result is quite general since the Second Law for a *PVT* system imposes:

(i) in any irreversible process the system only can reach states to the right of the line of the adiabatic reversible process, in a $P - V$ diagram. This is the Carathéodory Principle¹⁸

(ii) $c_P > c_V$ and $\gamma > 1$, implying that the slope of the adiabatic line in a $P - V$ diagram is always smaller (larger in absolute value, but negative) than the slope of the isothermal at the same point. This is the Le Châtelier Principle¹⁹.

¹⁸ Mark W. Zemansky, *Kelvin and Caratheodory—A Reconciliation*, Am. J. Phys. **34**, 914-920 (1966).

¹⁹ Robert Gilmore, *Le Châtelier reciprocal relations and the mechanical analog*, Am. J. Phys. **51**, 733-744 (1983).