

## Exotic states in the $S=1$ $N$ - $\pi$ - $K$ system and low-lying $1/2^+$ $S=-1$ resonances

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**Abstract.** In this manuscript we discuss about our study of the  $N\pi\bar{K}$  and the  $N\pi K$  systems made by solving the Faddeev equations with the two-body  $t$ -matrices obtained by solving the Bethe-Salpeter equations with the potentials obtained from chiral dynamics. In the strangeness = -1 case, we found that all the  $\Lambda$  and  $\Sigma$  resonances listed by the particle data group, with spin-parity  $1/2^+$ , in the 1550-1800 MeV region get generated due to the involved three-body dynamics. This motivated us to study the strangeness =1 three-body system, i.e.,  $N\pi K$ , where we did not find any evidence for the  $\Theta^+$  (1542) but found a broad bump around 1700 MeV which has a  $\kappa(800)N$  structure.

### 1 Introduction

According to the widely accepted theory of strong interactions, the baryons are made of three quarks and the mesons are made of a quark and anti-quark. However, there is nothing which prohibits the existence of hadrons with different structure. The advancement of excellent techniques and functioning of many new experimental facilities in the intermediate energy have lead to finding of many (excited) hadrons, which do not fit into the conventional hadron spectrum, based on the three-quark structure of the baryons and the quark-antiquark structure of the mesons. This has motivated several groups to scrutinize the unconventional configurations of hadrons, like, pentaquarks, tetra-quarks, (hadron) molecular states, glueballs, quark-gluon hybrids etc. Interestingly, several states fit into these unconventional schemes.

The importance of two-hadron dynamics, i.e., the meson-baryon or two meson interaction, in generation of resonant/bound states has already been proved, which has been useful in understanding the properties of many excited hadrons and hence the experimental data in the intermediate energy region. For example,

- A study of the  $\bar{K}N$ ,  $\pi\Sigma$ ,  $\pi\Lambda$ ,  $\eta\Lambda$ ,  $\eta\Sigma$  as coupled channels, in  $s$ -wave, lead to the dynamical generation of the  $\Lambda(1405)$  (see for example, [1,2]).
- The interaction of the  $\pi N$ ,  $\eta N$ ,  $K\Sigma$ ,  $K\Lambda$  in  $s$ -wave generates the  $N^*(1535)$  resonance [3].
- The  $\pi\Sigma(1385)$  and  $K\Sigma(1530)$  interaction gives rise to the  $\Lambda(1520)$  [4,5].
- In the meson sector  $f_0(980)$ ,  $a_0(980)$ ,  $\sigma(600)$ ,  $\kappa(800)$  have also been found to get dynamically generated in two meson interaction [6–8].

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It has been found that all these states contain an important meson-baryon and meson-meson component, respectively, in their wave functions, which play an important role in the description of their physical properties. Since the interaction in these two-body systems is attractive which gives rise to resonances or bound states, then the addition of one more hadron, either a meson or a baryon, could also lead to a new state in which the interaction of the three particles could be determinant in understanding the experimental data for such systems. There exist results which hint towards this possibility, e.g., in [9] it was found that the two meson cloud gave a sizeable contribution to the mass in the spectrum of the  $1/2^+$  baryon antidecuplet. An example of such states could be the  $N^*(1710)$ [10], which decays 40-90 % to  $\pi\pi N$  out of which 10-40% goes to the case where the two pions interact in the  $s$ -wave, in isospin 0. Other examples could be the  $\Lambda(1600)$  and  $\Sigma(1660)$ , which appear in reactions with three-body final state, i.e., in the experimental studies of the reactions  $K^- p \rightarrow \pi^0 \pi^0 \Lambda$  [11] and  $K^- p \rightarrow \pi^0 \pi^0 \Sigma$  [12]. Also new meson resonances like the  $X(2175)$  [13],  $Y(4260)$  [14],  $Y(4660)$  [15],  $X(1576)$ [16], etc. found by the BABAR, BES, CLEO, BELLE collaborations could also possess a three-body structure since they seem to couple strongly to three-mesons or to a system of a meson and a meson resonance, where the latter one in turn couples to two mesons or in other words it can be treated as a molecular state of two mesons.

With this motivation we have studied three-hadron systems made of two mesons and a baryon and those made of three mesons. In this manuscript, however, we constraint the discussion to the strangeness -1 and +1 systems of the former kind. Before proceeding with the details of the formalism and calculations, we would like to call the attention of the readers to the fact that the status of the hyperon

resonances in the 1500-1800 MeV region looks quite poor [10]:

- In the isospin 1 case, the spin parity of the  $\Sigma(1560)$  is not known and it possesses two stars.
- For the  $\Sigma(1620)$ , the results of the partial wave analyses and the productions experiments is listed separately with the former one having a spin parity assignment  $1/2^-$  which for the latter one is not known.
- The case for the  $\Sigma(1670)$  is similar to the  $\Sigma(1620)$ .
- Then the  $\Sigma(1770)$  appears with a one star status.
- In the isospin 0, there are two resonances  $\Lambda(1600)$  and  $\Lambda(1810)$  and for both the cases there seems to be a possibility of being associated to more than one resonance.

We will show in the following sections that many of these hyperon resonances couple strongly to two-meson-one-baryon systems and that this finding helps in understanding their properties. We carry out this study by using the formalism explained briefly in the next section.

## 2 Formalism

Our formalism is based on the solution of the Faddeev equations

$$T = T^1 + T^2 + T^3, \quad (1)$$

where,

$$\begin{aligned} T^1 &= t^1 \delta^3(\mathbf{k}'_1 - \mathbf{k}_1) + g(T^2 + T^3) \\ T^2 &= t^1 \delta^3(\mathbf{k}'_2 - \mathbf{k}_2) + g(T^3 + T^1) \\ T^3 &= t^1 \delta^3(\mathbf{k}'_3 - \mathbf{k}_3) + g(T^1 + T^2). \end{aligned} \quad (2)$$

In Eq.(2),  $\mathbf{k}'_i(\mathbf{k}_i)$  is the final(initial) momentum of the  $i$ th particle and  $t^i$  is the  $t$ -matrix for the interaction of the  $jk$  pair where  $i \neq j \neq k$ . The  $T^i$  partitions represent the sum of all the three-body diagrams which have the particle  $i$  as a spectator in the end (see Fig.1 and Fig.2 for example).

These are not the only possible diagrams for three-body interactions, we could also have diagrams like the one shown in Fig.3b.

However, considering the fact that the chiral amplitudes can be separated into an on-shell part and an off-shell part, we find some cancellations between these diagrams. By saying the on-shell part we mean that, in the  $s$ -wave, this part of the two body amplitude is calculated as a function of the invariant mass of the interacting pair to obtain which we impose  $q^2 = m^2$ . When these particles are inside the loops in the Faddeev diagrams, where some or all the particles can be off shell, the full amplitude has an ‘‘off shell’’ part which goes as  $q^2 - m^2$  for mesons and  $q^0 - E(q)$  for baryons. This off shell part contains an inverse particle propagator and cancels one particle propagator in three-body equations rendering a Faddeev diagram with two two-body  $t$ -matrices, for example, the one shown in Fig.1a, into a three-body contact term shown in Fig.3a, which has the same topology as genuine three-body interactions that stem from the chiral Lagrangians (Fig.3b).

We have found that, in the  $s$ -wave and in the SU(3) limit, there is an exact cancellation between the contribution of the off shell part of the two-body amplitudes to the

three-body diagrams and the three-body force, of the kind shown in Fig.3b, generated by the same chiral Lagrangian. As a consequence, we need only the on shell two-body  $t$ -matrices and can ignore the three-body forces. This finding is novel for such studies and simplifies the work technically as has been discussed in detail in [17–21, 23].

Thus all the diagrams in Fig.1 can be written as  $t^i g^{ij} t^j$ , where the  $t$ -matrices are on-shell in nature and depend only on the Mandelstam variable  $\sqrt{s_{ij}}$  and the elements of the  $g^{ij}$  matrix are defined as

$$g^{ij}(\mathbf{k}'_i, \mathbf{k}_j) = N_l \frac{1}{\sqrt{s} - E_i(\mathbf{k}'_i) - E_l(\mathbf{k}'_i + \mathbf{k}_j) - E_j(\mathbf{k}_j)}, \quad (3)$$

where  $l \neq i, l \neq j, = 1, 2, 3$  and  $N_l = 1/E_l$  or  $M_l/E_l$  in case of a meson or a baryon propagator present in a diagram.

In this way the diagrams in Fig.2, for example, the first one can be written as

$$\begin{aligned} &t^1(\sqrt{s_{23}}) \left( \int \frac{d\mathbf{q}}{(2\pi)^3} g^{12}(\mathbf{k}'_1, \mathbf{q}) t^2(\sqrt{s_{31}}(q)) \right. \\ &\left. g^{21}(\mathbf{q}, \mathbf{k}_1) \right) \times t^1(\sqrt{s_{23}}). \end{aligned}$$

However, introducing an identity (underlined in the next equation) in the above equation we can re-write it as

$$\begin{aligned} &t^1(\sqrt{s_{23}}) \left( \int \frac{d\mathbf{q}}{(2\pi)^3} g^{12}(\mathbf{k}'_1, \mathbf{q}) t^2(\sqrt{s_{31}}(q)) \right. \\ &\left. g^{21}(\mathbf{q}, \mathbf{k}_1) \times \underline{[g^{21}(\mathbf{k}'_2, \mathbf{k}_1)]^{-1} \times [t^2(\sqrt{s_{31}})]^{-1}} \right) \\ &\underline{t^1(\sqrt{s_{31}}) g^{21}(\mathbf{k}'_2, \mathbf{k}_1) t^1(\sqrt{s_{23}})}. \end{aligned} \quad (4)$$

We call the loop dependent term in the bracket as the  $G^{121}$  function which makes the Eq.(4) equal to

$$t^1 G^{121} t^2 g^{21} t^1, \quad (5)$$

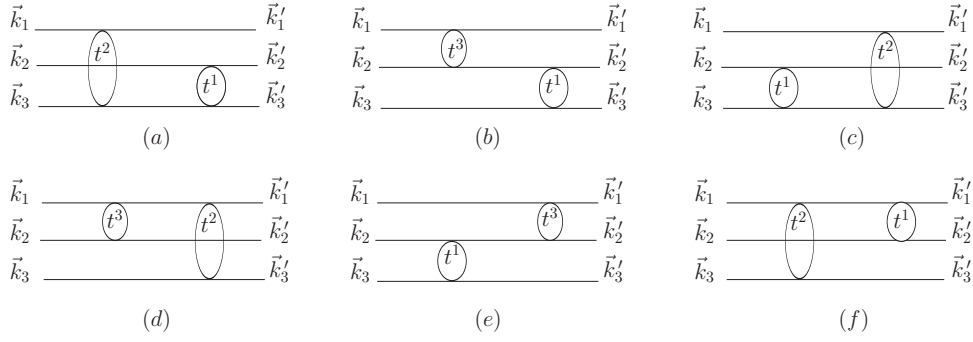
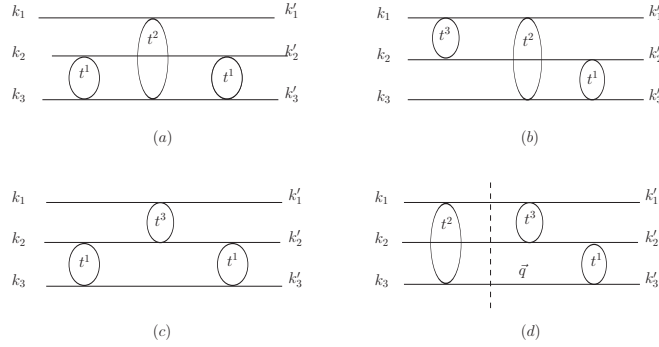
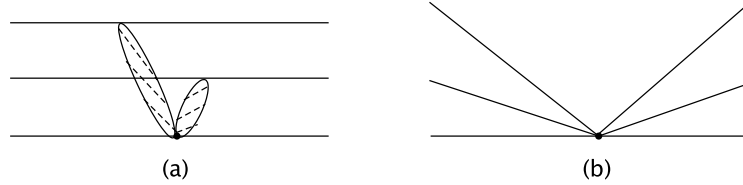
where all the terms except  $G^{121}$  depend on the on-shell variables. Proceeding in this way, we obtain the following equations

$$\begin{aligned} T_R^{12} &= t^1 g^{12} t^2 + t^1 \left[ G^{121} T_R^{21} + G^{123} T_R^{23} \right] \\ T_R^{13} &= t^1 g^{13} t^3 + t^1 \left[ G^{131} T_R^{31} + G^{132} T_R^{32} \right] \\ T_R^{21} &= t^2 g^{21} t^1 + t^2 \left[ G^{212} T_R^{12} + G^{213} T_R^{13} \right] \\ T_R^{23} &= t^2 g^{23} t^3 + t^2 \left[ G^{231} T_R^{31} + G^{232} T_R^{32} \right] \\ T_R^{31} &= t^3 g^{31} t^1 + t^3 \left[ G^{312} T_R^{12} + G^{313} T_R^{13} \right] \\ T_R^{32} &= t^3 g^{32} t^2 + t^3 \left[ G^{321} T_R^{21} + G^{323} T_R^{23} \right], \end{aligned} \quad (6)$$

which are summed to get the full three-body  $t$ -matrix

$$T_R = T_R^{12} + T_R^{13} + T_R^{21} + T_R^{23} + T_R^{31} + T_R^{32}. \quad (7)$$

The  $T_R^{ij}$  equations can be interpreted as the sum of all the diagrams of different possible interactions between the


**Fig. 1.** Three-body diagrams with involving two t-matrices.

**Fig. 2.** Different possible diagrams for three-body interaction with two t-matrices.

**Fig. 3.** More diagrams for three-body interaction.

three particles where the last term is written in terms of  $t^i$  and  $t^j$ .

This formulation, which has been shown to be reasonably close to the exact calculation in [19], simplifies the numerical solution of the Eqs.(6).

We shall now discuss the results of the study of the  $N\pi\bar{K}$  and the  $N\pi K$  systems made with the formalism explained in this section.

### 3 Results(I): S=-1 three-body system

To study the  $N\pi\bar{K}$  system we take the following coupled channels into account:  $\pi^0 K^- p$ ,  $\pi^0 \bar{K}^0 n$ ,  $\pi^0 \pi^0 \Sigma^0$ ,  $\pi^0 \pi^+ \Sigma^-$ ,  $\pi^0 \pi^- \Sigma^+$ ,  $\pi^0 \pi^0 \Lambda$ ,  $\pi^0 \eta \Sigma^0$ ,  $\pi^0 \eta \Lambda$ ,  $\pi^0 K^+ \Xi^-$ ,  $\pi^0 K^0 \Xi^0$ ,  $\pi^+ K^- n$ ,  $\pi^+ \pi^0 \Sigma^-$ ,  $\pi^+ \pi^- \Sigma^0$ ,  $\pi^+ \pi^- \Lambda$ ,  $\pi^+ \eta \Sigma^-$ ,  $\pi^+ K^0 \Xi^-$ ,  $\pi^- \bar{K}^0 p$ ,  $\pi^- \pi^0 \Sigma^+$ ,  $\pi^- \pi^+ \Sigma^0$ ,  $\pi^- \pi^+ \Lambda$ ,  $\pi^- \eta \Sigma^+$ ,  $\pi^- K^+ \Xi^0$ . We label the particles as 1,2 and 3 in the order in which they are written. This channels have been chosen in such a way that:

- The sub-system of the two pseudoscalars dynamically generates  $f_0(980)$ ,  $a_0(980)$ ,  $\sigma(600)$ ,  $\kappa(800)$ .

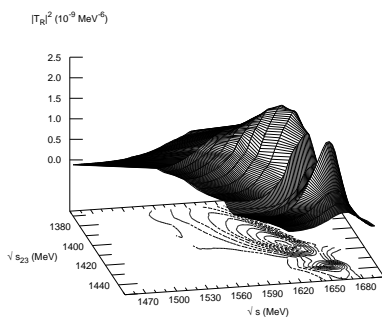
- The  $N^*(1535)$  get generated in the system of particles 1 and 2.
- And the interaction of the particles 1 and 3 gives rise to the  $\Lambda(1405)$ .

We solve the Eqs.(6) for these systems as a function of the total energy of the three-body system ( $\sqrt{s}$ ) and the invariant mass of the particles 2 and 3 ( $\sqrt{s_{23}}$ ). All the interactions in this study are considered in the s-wave and hence, the Eqs.(6) are projected in the s-wave. Finally, in order to identify the isospin of the resonances found in our work, we project the  $T_R$ -matrix (Eq.(7)) on to an isospin base defined in terms of the total isospin of the three-body system and that of a sub-system. We label such a base as  $|I, I_{ij}\rangle$ .

In case of the total isospin 1, we find four resonances at:

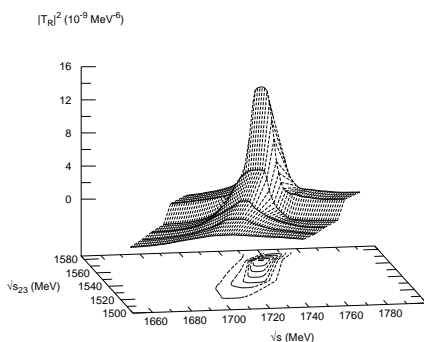
1.  $\sqrt{s} = 1590$  MeV with with the full width at half maximum  $\sim 70$  MeV,
2.  $\sqrt{s} = 1630$  MeV with  $\Gamma = 39$  MeV,
3.  $\sqrt{s} = 1656$  MeV with  $\Gamma \sim 30$  MeV,
4. and  $\sqrt{s} = 1790$  MeV with  $\Gamma = 24$  MeV.

We associate the first one with the  $\Sigma(1560)$ , for which the spin-parity is not known [10]. Our results would associate a  $1/2^+$  to the spin-parity of this resonance. We relate the next one with the  $\Sigma(1660)$ , for which an indication has also been found in the data on the  $K^- p \rightarrow \pi^0 \pi^0 \Sigma$  reaction [12]. We associate the third one with the  $\Sigma(1620)$ , which as mentioned in the introduction is controversial. The partial wave analyses groups assign a spin parity  $1/2^-$  to this resonance but from the production experiments, it seems that its not clear if there is one or more resonance present in this region [10]. We relate the fourth one with the  $\Sigma(1770)$ . We show the squared three-body amplitude for the first two resonances as a function of the two variables of the formalism in Fig.4.



**Fig. 4.** The squared amplitude for the  $\pi\pi\Sigma$  channel projected in the isospin 1. These resonances show that the  $\Sigma(1660)$  and  $\Sigma(1620)$  couple strongly to three-hadron channels.

In the case of the total isospin = zero, we find three resonances: one at the  $\sqrt{s} = 1568$  with a width of 60 MeV, another one at 1700 MeV with  $\Gamma=136$  MeV and one more at 1740 MeV with the full width at half maximum being 20 MeV (which is shown in Fig.5). We found the first two peaks in the  $\pi\bar{K}N$  amplitude with  $I_{\pi\bar{K}} = 1/2$  and the second one in the the  $\pi\pi\Lambda$  amplitude with  $I_{\pi} = 0$ . We relate the first two peaks with the  $\Lambda(1600)$  and the second one with the  $\Lambda(1810)$ .



**Fig. 5.** The squared amplitude for the  $\pi\pi\Lambda$  channel projected in the isospin 0. The resonance shown here corresponds to the  $\Lambda(1810)$ .

Thus we find dynamical generation of four resonances in the isospin 1 case and three in the isospin zero, which can all be related to known hyperons[10]. Our findings imply that these resonances couple strongly to the two-meson–one-baryon channels and that their wave functions thus have large component of such channels.

It were these findings which motivated us to study the corresponding strangeness =1 system and look for the exotic resonances.

## 4 Results(II): S=1 three-body system

The observation of a peak in the  $K^+n$  invariant mass in the  $\gamma n \rightarrow K^+ K^- n$  reaction on a  $^{12}\text{C}$  target at Spring8/Osaka [24] raised great hopes that for the first time a strangeness S=1 narrow exotic baryon could be found. The peak was thus associated to a pentaquark, since the standard  $3q$  states cannot produce S=1. Subsequently, many experiments were done, some which reproduced this peak and others which did not. A new experimental analysis has been done at LEPS confirming the original peak, now on a deuteron target and with more statistics [25]. Although one cannot rule out an interpretation of the peak as a consequence of the particular set up of LEPS, no alternative conventional explanation for this peak has been provided. Hence, one can say that the status of existence of this pentaquark state is still unclear.

In any case, the  $KN$  interaction obtained from chiral Lagrangians is basically repulsive in nature [26], and hence it is not appealing to look for a narrow (long lived) resonance, as the one claimed in [24], in this system. This is why very early there were suggestions that if the peak represented a new state, it could be a bound state of three hadrons,  $K\pi N$ , with the pion acting as a glue between the nucleon and the K. However, investigations along this line, weakly concluded the difficulty to have this system as a bound state [27,28].

We did the calculation with the same formalism which, as discussed in the previous section, lead to dynamical generation of several low-lying  $1/2^+$  resonances in the corresponding three-body  $S = -1$  channels. We did not find any structure in the energy region close to 1542 MeV, therefore, the interpretation of the  $\Theta^+$  as a possible  $N\pi K$  bound state is ruled out. We did not find any resonance in the isospin 1 and 2 configurations also. However, we did obtain a peak with a broad structure in the isospin zero amplitude (i.e., when the  $\pi K$  subsystem is in isospin  $1/2$ ) around 1720 MeV. The full width at half maximum of the peak is of the order of 200 MeV. These features are far from the resonance claimed in [24]. The value of  $\sqrt{s_{23}}$ , for which this bump is found, is around the mass of the  $\kappa(800)$  resonance. Thus it can be interpreted as a  $\kappa(800)N$  resonance, which reveals the underlying chiral dynamics of the three-body system, and that we hope can be seen in  $K^+N$  scattering, but much better in reactions producing  $\pi K N$  in  $I = 0$  in the final state.

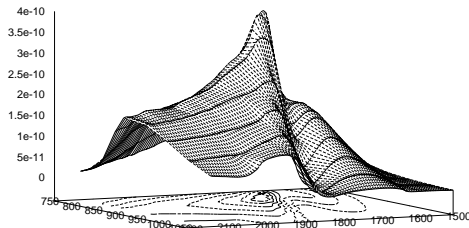


Fig. 6. A broad resonance found in the  $N\pi K$  amplitude.

## 5 Summary

We have studied the three-body systems made of two-mesons and one baryon. In this work, we have made a review of our study of the  $\pi\bar{K}N$  and  $\pi KN$  systems and the corresponding coupled channels. We find that the three-body dynamics generates several hyperon resonances whose status is poor or controversial. The reason for such a status becomes clear by considering our finding that these resonances couple strongly to three-body decay channels. Indeed, more experimental information on such channels should help in better understanding of these resonances. In the positive strangeness case, the calculations with our formalism did not result in any structure in the energy region close to 1542 MeV. However, we find a peak with a broad structure in the isospin zero amplitude around 1720 MeV with a  $\kappa(800)N$  structure.

We have also studied the  $S = 0$  meson-meson-baryon system with our formalism, where the  $N^*(1710)$  appears neatly as resonances of the  $\pi\pi N$  [19]. We also found that it actually has a  $\sigma - N$  structure and that no other coupled channel played any role in generation of this resonance. But there are other  $J^P = 1/2^+$  states, like the  $N^*(2100)$  and the  $\Delta(1910)$  which did not appear with the use of the amplitudes obtained with the lowest order chiral Lagrangians. From the work of [3] we know that the chiral unitary approach using the lowest order chiral Lagrangian provides a fair amplitude up to  $\sqrt{s} = 1600$  MeV but fails beyond this energy. For instance, the  $N^*(1650)$  does not appear in the approach. As a consequence, any three body states which would choose to cluster a  $\pi N$  subsystem into this resonance would not be obtained in the approach of [19].

We then made a new study of the  $S = 0$  systems, by using the experimental  $\pi N$  amplitudes as the input. In this work we found three more  $1/2^+$  baryon resonances with  $S = 0$ ; one corresponding to the  $\Delta(1910)$  with a  $\pi K\Lambda$  structure and another to the  $N^*(2100)$  and yet another around 1920 MeV with isospin 1/2. There is no known  $N^*$  resonance around 1920 MeV but there are many speculations of existence of one as we discuss in [8].

Other kind of systems which we have also investigated are those made of three mesons. To be explicit we have studied two systems of vector-pseudoscalar-pseudoscalar meson system;  $\phi K\bar{K}$  with  $\phi\pi\pi$  as coupled channels and

$J/\psi K\bar{K}$  with  $J\psi\pi\pi$  as coupled channels. This lead to finding of dynamical generation of the newly discovered  $X(2175)$  [20] and  $Y(4260)$  [29] in the respective cases.

There are many more resonances which couple to three-hadron systems and we are studying some of them and shall take up more in future.

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