The effects of automation and lobbying in wage inequality: A Directed Technical Change model with routine and non-routine tasks*

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Abstract

We devise a Directed Technical Change (DTC) multisector Schumpeterian growth model in which both wage inequality and wage polarization are analysed. To that end, we introduced tasks in the model, some of which can be automated – replaced by robots or machines –, thus combining the DTC and task-based growth literature in an unified framework. This model produces positive relationships both (i) between the relative supply of high-skilled workers and the skill premium and (ii) between automation and wage polarization. Moreover, within the model, we analyse Lobbying as an activity that can affect the wage distribution and integrate it in the strategic interations between firms. We find that it can reduce the effects of automation on wage polarization, and through this channel possibly affecting the wage distribution without affecting the skill premium.

Keywords: Directed technical change; automation; routinization; lobbying power; economic growth; wage inequality; wage polarization.

JEL Classification: J31, P16, O30, O41.

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1 Introduction

Inequality has been rising in many advanced economies since the 1980s (Alvaredo *et al.* 2018). Initially, the predominant literature analyzed this phenomenon with a particular focus on the rise of the skill premium, the wage differential between high-skilled and low-skilled workers (*e.g.*, Akerman *et al.* 2015; McAdam and Willman 2018). This prompted the development of models that explain this wage differential as a result of directed technical change in favor of high-skilled workers in relation to low-skilled workers – the generalized Directed Technical Change (DTC) framework (*e.g.*, Acemoglu 1998; Acemoglu 2002; Bound and Johnson 1992; Katz and Murphy 1992; Juhn *et al.* 1993). In the canonical version of these models, low-skilled and high-skilled workers are complemented by specific types of technologies. An increase in the supply of a type of labor causes an expansion for the market size of the technologies it complements (market-size channel), and this creates additional incentives for R&D aimed at those technologies. Consequently, technological-knowledge changes toward those technologies,[1] which, in turn, increases the demand for the complementary type of labor. Thus the models were able to explain the rise of the skill premium as a consequence of the observed increase of the relative supply of high-skilled workers in the same period.

However, as more data became increasingly available a phenomenon known as wage polarization has been observed: a positive growth rate of wages of workers at the extremes of the skill distribution and negative for those at the middle (*e.g.*, Autor and Dorn 2013). This has contributed to a change in the paradigm in the literature, which began to be focused instead on how wage inequality can be affected by automation and the degree of the routinization of tasks (*e.g.*, Acemoglu and Autor 2011; Autor and Restrepo, 2018). This consists in the degree to which a task is codifiable into a set of specific and precise instructions and thus measures the extent to which it can be performed autonomously by robots or machines – automation. Several authors have found that middle-skilled workers are employed in routine tasks while low-skilled and high-skilled tasks are mostly employed in non-routine manual and abstract/cognitive tasks, respectively (*e.g.*, Acemoglu and Autor 2011; Autor and Dorn 2013). Both types of non-routine tasks are difficult to be reduced to a specific set of instructions, the former for requiring human

[1]The technological-knowledge change represents the overall process of invention / innovation as a result of R&D activity, and the technological-knowledge change can be biased/directed for some particular sector.

and physical elements (*e.g.*, service occupations) and the latter for requiring complex cognitive processes (*e.g.*, managers, technicians, etc.). In this context, one possible explanation for wage polarization is automation that, by leading to an increase of routine tasks performed by robots, decreases relative demand for middle-skilled workers (*e.g.*, Acemoglu and Autor 2011).

In spite of the increased importance of wage polarization and automation, the skill premium is a phenomenon that is still of interest to the literature (*e.g.*, Jaimovich *et al.* 2020). However, so far, no paper has attempted to propose a theoretical framework that permits the analysis of both phenomena simultaneously. Additionally, both literatures have so far not taken into consideration the importance attributed by the literature to lobbying in affecting the wage distribution, especially considering the dimension of this activity in developed economies with a high level of disparities, such as the United States.[2] For instance, Autor (2014) considers that, besides market conditions, increasing political capture of the policy-making process by elites may also affect inequality. Prettner and Rostam-Afschar (2020) argue that a more stringent tax policy that relocates resources from unproductive lobbying with the objective of rent-seeking to productive activity contributes to higher economic growth and lower inequality. Lesica (2018) shows that lobbying can influence policymakers to set the minimum wage according to the ideology of the former provided that labor demand is sufficiently elastic. Moreover, lobbying has been pointed out to negatively affect technology adoption (Comin and Hobjin 2009) and innovation (Akcigit et. al. 2018 and Bellettini et al. 2013). With firms allocating funds to lobbying, their relative profitability may be affected and, as a consequence, so, too, the R&D decisions that drive the technological-knowledge change and bias, which ultimately have an impact on wage differentials between workers.

In what concerns the most important characteristics of lobbyists, the literature suggests that most lobbying activities are conducted by large firms with innovation capabilities. For instance, Bellettini *et al.* (2013) argues that such firms resort to lobbying to obtain preferential treatments and create additional entry barriers for entrants. Figueiredo and Ritcher (2013) in a survey about the empirical regularities about lobbying state that "There is now overwhelming evidence to support a second general regularity in the data: corporations and trade associations comprise

[2] *"Lobbying organizations(...) spent $1.64 billion a figure which increased steadily until 2010."* (Washington Post, 21st April 2015).

the vast majority of the lobbying expenditures by interest groups." and also "large organized interest groups and groups that are supported by large corporations are more likely to lobby than are smaller groups and groups that are supported by smaller corporate interests.". Lux *et al.* (2011) considers that one of the predictors of firms engaging in Corporate Political Activities, which can be interpreted as lobbying activities in the context of this paper, is their size, with larger firms tending to engage more in these activities. This is also reinforced by Akcigit *et al.* (2018) who, in the context of an analysis of political connections and firm dynamics centered in Italy, found that it was a common practice for large market leaders to hire politicians.

Bearing all this in mind, we propose a new theoretical framework that addresses all the issues outlined above. In the new setup, aggregate output is produced both by a continuum of routine and non-routine tasks, with the latter being differentiated between tasks that require high-level abstract capacities (non-routine abstract/cognitive tasks) and others that require physical dexterity and proficiency in human interactions (non-routine manual tasks). Following the evidence outlined above, we consider that the first set is performed by middle-skilled workers and the latter by high-skilled and low-skilled workers, respectively, all of which are complemented by specific quality-adjusted machines. The latter, in turn, are made by individual firms, each allowed to conduct lobbying activities that end up benefiting firms in each sector that share common interests.

Our paper contributes to several literatures: (i) DTC literature (*e.g.*, Acemoglu 1998; Acemoglu 2002; Bound and Johnson 1992; Katz and Murphy 1992; Juhn *et al.* 1993; Sanchez-Carrera 2012, 2019) whose main goal was to explain the level and dynamics of the skill premium (measured by a ratio between earnings of high-skilled and low-skilled workers); (ii) robotization literature (*e.g.*, Acemoglu and Autor 2011; Autor and Restrepo 2018), which introduced task-based models with substitution between workers and 'robots' and wished to explain wage polarization; (iii) a lobbying literature (Afschar 2020; Akcigit *et al.* 2018; Heckelman and Wilson 2013; Acemoglu and Robinson 2008), in which different types of lobbying affects (un)employment, wage inequality, economic growth, and the overall dynamics of the economy and the society.

In relation to (i) and (ii), we innovate with the development of a framework that unifies both theoretical approaches and with the introduction of lobbying. In relation to (iii), we innovate by analyzing the specific impacts of lobbying on inequality, which are less frequently analyzed than effects on welfare, growth (*e.g.*, Akcigit *et al.* 2018; Heckelman and Wilson 2013), and economic institutions (Acemoglu and Robinson 2008) and by offering a completely different perspective on the impacts of lobbying on inequality in relation to the existing literature, which is focused on the economic effects of deviating resources into unproductive activities (*e.g.*, Prettner and Rostam-Afschar 2020; Akcigit *et al.* 2018).

Our findings are twofold. Firstly, we obtain a positive relationship the relative supply of high-skilled workers and the skill premium and a negative relationship between automation and relative wages of middle-skilled workers, which are in line with the DTC and robotization literature, respectively, and, therefore, demonstrates the validity of the unified theoretical framework that we propose. Secondly, we show that lobbying intensity can affect relative wages of middle-skilled workers.

The remainder of this paper is structured in the following way: section 2 outlines the model and the main static results each moment in time; section 3 describes the general equilibrium conditions of the model, with a particular focus on the Steady State; section 4 describes and justifies the calibration for the various parameters used to obtain quantitative results for steady variables and analysis the latter for various scenarios related to lobbying gains and automation; section 5 concludes.

2 Set up of the model

This Section describes the economic setup of the closed Economy in which infinitely-lived households inelastically supply labor, maximize the utility of consumption from the aggregate final good, and invest in a firm's equity. The inputs of the aggregate numeraire good, Y, are two final goods, Non-Routine (Y_A produced in the A-sector) and Routine (Y_B produced in the B-sector), which, in turn, require completion of a continuum of non-routine and routine tasks, respectively, in perfect competition.[3] Tasks in the non-routine sector include non-routine manual tasks, which require physical dexterity and situational adaptability, and also non-routine cognitive/abstract tasks, which require the dominion of high-level mental capacities. Therefore, both are performed by low-skilled and high-skilled labor, respectively, which are workers at

[3] At this point, we must clarify that by sector we do not mean sector of activity because both routine and non-routine tasks are prevalent across different parts of the economy.

the extremes of the skill distribution whose supply is represented by L_A^U and L_A^H. In turn, routine tasks in sector $s = B$ are performed by workers at the middle of the skill distribution, *i.e.*, medium-skilled labor type, whose supply is L_B^M. In both sectors, each task requires, in addition to the specific labor, a continuum of specific non-durable quality-adjusted machines that are complementary to the type of workers used in the corresponding production. Each machine/robotic sector, in turn, consists of a continuum of industries, $j \in (0, J_i]$, with $i = U, M, H$, and is characterized by monopolistic competition: the monopolist in industry j uses a design, sold by the R&D sector and protected by a patent, and numeraire to produce the highest quality level of machine j at a price that maximizes profits. In the R&D sector, each potential entrant devotes numeraire to invent successful vertical designs to be supplied a new monopolist machine firm/industry; *i.e.*, R&D allows increasing the quality of machines J_i and, thus, the technological knowledge. That is, some endogenous technological knowledge complements low-skilled labor, some complements medium-skilled labor, and some complements high-skilled labor.

2.1 Preferences

Infinitely-lived households obtain utility from the consumption, C, of the unique aggregate final good, whose price we normalize to 1, and collect income from investments in financial assets (equity) and from labor. They inelastically supply labor to the A-sector or to the B-sector according to whether they are low-skilled, middle-skilled or high-skilled, which results in an exogenous labor supply equal to L_A^U, L_B^M and L_A^H, respectively.[4] Preferences are identical across workers L_s^i. Thus, the Economy admits a representative household that maximizes preferences at time $t = 0$ given by $U_C = \int_0^\infty \left(\frac{C(t)^{1-\theta} - 1}{1-\theta} \right) e^{-\rho t} dt$, where $\rho > 0$ is the subjective discount rate, ensuring that U_C is bounded away from infinity if C were constant over time, and $\theta > 0$ is the inverse of the inter-temporal elasticity of substitution. This maximization is subject to the flow budget constraint $\dot{b}(t) = r(t) \cdot b(t) + \sum_{s=A,B} \sum_{i=U,M,H} w_i(t) \cdot L_s^i - C(t)$, where $b(t) =$

[4]In reality, workers have a wide-ranging of abilities. In this regard, we can interpret each of these quantities as the total sum of the supply of hours of workers with skills between certain thresholds. More specifically, considering that each individual has a different level of ability a which is exogenously distributed between 0 to 1, we can consider individuals as low-skilled if their ability a is between 0 and $\tilde{a}_1$, middle-skilled those whose ability lies between $\tilde{a}_1$ and $\tilde{a}_2$ and high-skilled those with a larger than $\tilde{a}_2$ and inferior to 1. In this regard, if we denote $z(a)$ as the number of hours exogenously supplied by individuals of ability level a, we have that the total number of hours supplied by workers with different skills are as follows: $L^U = \int_0^{\tilde{a}_1} z(a)da$, $L^M = \int_{\tilde{a}_1}^{\tilde{a}_2} z(a)da$, $L^H = \int_{\tilde{a}_2}^1 z(a)da$. Since low-skilled and high-skilled individuals are only employed in sector A while middle-skilled workers are employed in sector B, we have that $L_A^U = L^U$, $L_A^H = L^H$ and $L_B^M = L^M$.

$\sum_{s=A,B} \sum_{i=U,M,H} b_s^i(t)$ denotes household's real financial assets/wealth holdings (composed of equity of machine producers, considering the profits seized by the top-quality producers), $r(t)$ is the real interest rate, and $w_i(t)$ is the wage for labor type $i = \{U, M, H\}$, with $s = A$ if $i \in \{U, H\}$ and $s = B$ if $i = M$. The initial level of wealth $b(0)$ is given and the non-Ponzi games condition $\lim_{t\to\infty} e^{-\int_0^t r(s)ds} b(t) \geq 0$ is imposed. The representative household chooses the path of aggregate consumption $[C(t)]_{t\geq 0}$ to maximize the discounted lifetime utility, resulting in the following optimal consumption path Euler equation,

$$\frac{\dot{C}(t)}{C(t)} = g = \frac{1}{\theta} \cdot [r(t) - \rho]\,. \tag{1}$$

Moreover, the transversality condition is also standard: $\underset{t\to\infty}{lim}\, e^{-\rho t} \cdot C(t)^{-\theta} \cdot b(t) = 0$.

2.2 Technology, output and prices

Aggregate economy. In the Economy, aggregate output Y is produced with a CES aggregate production function of A and B competitively produced final goods:

$$Y(t) \quad = \left[\sum_{s=A,B} \chi_s \cdot Y_s(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad , \quad \varepsilon \in (0, +\infty)\,, \tag{2}$$

where: Y_A and Y_B are the total outputs of the A- and the B-sectors, respectively; χ_A and χ_B, with $\sum_{s=A,B} \chi_s = 1$, are the distribution parameters, measuring the relative importance of the sectors; $\varepsilon \geq 0$ is the elasticity of substitution between the outputs produced by the two sectors, wherein $\varepsilon > 1$ ($\varepsilon < 1$) means that they are gross substitutes (complements) in the production of Y. The assumption of competitive final-good firms implies the following maximization problem:[5] $\max_{Y_s} \Pi_Y = P_Y \cdot Y - \sum_{s=A,B} P_s \cdot Y_s$, where P_s is the price of outputs from sector s. If we normalize the price of output to 1, from the first-order conditions emerge the inverse demand for Y_s, $s = \{A, B\}$:[6]

$$P_s = \chi_s \left(\frac{Y}{Y_s}\right)^{\frac{1}{\varepsilon}} \Leftrightarrow Y_s = \left(\frac{P_s}{\chi_s}\right)^{-\varepsilon} Y. \tag{3}$$

[5]We must clarify that all the maximization problems of profits we present in this section are maximization of profits over the entire lifetime of the producers because agents are forward-looking. However, since these problems are static by nature their solution is the same as the maximization of the corresponding profits at each point in time. Since this is more simple, we opt for presenting the latter instead of the former.

[6]We suppress the time argument t and will do so throughout as long as this causes no confusion.

Thus, we obtain the following expression for relative demand for output from the A-sector:

$$\frac{Y_A}{Y_B} = \left(\frac{\chi_A}{\chi_B}\right)^{\epsilon} \left(\frac{P_A}{P_B}\right)^{-\epsilon}, \tag{4}$$

which depends positively on the relative share in production and negatively on the relative price of output from this sector. Higher values for the elasticity of substitution ϵ imply that a higher relative price of output of sector A produces a higher decrease in relative output. Replacing (3) in (2) we have that:

$$\sum_{s=A,B} \chi_s^{\varepsilon} \cdot P_s^{1-\varepsilon} = 1. \tag{5}$$

From (3) we $P_s \cdot Y_s = \chi_s \cdot Y_s^{\frac{\epsilon-1}{\epsilon}} \cdot Y^{\frac{1}{\varepsilon}}$ which, summing across sectors, results in $Y = P_A \cdot Y_A + P_B \cdot Y_B$.

Sectors of the economy. The output Y_s of each sector $s = \{A, B\}$ is produced in perfect competition by the following production function with constant returns to scale $Y_s = \exp\left(\int_0^1 \ln Y_{v_s} dv_s\right)$, *i.e.*, Y_s is a continuum of the output produced by tasks Y_{v_s} indexed respectively, by $v_A \in [0,1]$ and $v_B \in [0,1]$. The producer of Y_s maximizes profits given by $\Pi_s = P_s \cdot Y_s - \int_0^1 P_{v_s} \cdot Y_{v_s} dv_s$, subject to the restriction imposed by the functional form of the production function of Y_s. Assuming perfect competition, the maximization problem results in the following first-order conditions: $\frac{\partial \Pi_s}{\partial Y_{v_s}} = 0 \Rightarrow Y_{v_s} = \frac{P_s \cdot Y_s}{P_{v_s}}$. Therefore, from here $P_{v_s} \cdot Y_{v_s} = P_s \cdot Y_s$ is a constant, which replaced in the profits function and in the production function results, respectively, in $\Pi_s = P_s \cdot Y_s - \int_0^1 P_s \cdot Y_s dv_s = 0$ and also in

$$Y_s = \exp\left(\int_0^1 \ln \frac{P_s \cdot Y_s}{P_{v_s}} dv_s\right) \Leftrightarrow P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right). \tag{6}$$

Tasks in each sector. In turn, the producer of tasks in the non-routine A-sector must choose to produce them either with low-skilled ($i = U$) or high-skilled ($i = H$) workers, which implies choosing between the following Cobb-Douglas production functions:

$$Y^U_{v_A}(t) = \left[\int_0^{J_U} \left(q_A^{k(j,t)} \cdot x^U_{v_A}(k,j,t)\right)^{1-\alpha_A} dj\right] \left[\psi_U(v_A) \cdot l^U_A \cdot L^U_{v_A}\right]^{\alpha_A}, \tag{7}$$

$$Y^H_{v_A}(t) = \left[\int_0^{J_H} \left(q_A^{k(j,t)} \cdot x^H_{v_A}(k,j,t)\right)^{1-\alpha_A} dj\right] \left[\psi_H(v_A) \cdot l^H_A \cdot L^H_{v_A}\right]^{\alpha_A}. \tag{8}$$

Each uses two factors: labor of type H or U (the second term on the right-hand side) and machines (the first term on the right-hand side) with a share in the income of α_A and $1-\alpha_A$, respectively. Each machine $j \in [0, J_i]$ used is quality-adjusted: the constant quality upgrade is $q_A > 1$ and is constant, k is the top-quality rung at t and $x^H_{v_A}(k,j,t)$ and $x^U_{v_A}(k,j,t)$ represent the units of machines demanded for task v_A if it is produced using high-skilled ($i = H$) and low-skilled ($i = U$) workers, respectively. Similarly, the labor term includes the quantities employed in the production of each task according to the type used, $L^H_{v_A}$ or $L^L_{v_A}$, and two types of corrective factors accounting for productivity differentials; *i.e.*,

- The first type is l^i_A, a specific term that reflects the absolute efficiency of labor of type i in producing each task.
- The second type, following the point of view proposed by, *e.g.*, Acemoglu and Autor (2011), is a measure of the relative productivity advantage of either type of labor, captured by the terms $\psi_U(v_A)$ and $\psi_H(v_A)$, which we use to connect the relative advantages of workers with and without qualifications in performing each of these tasks. To do so, we assume that tasks are characterized by two attributes that vary in different ways along the indexes: (i) the degree to which they require problem-solving, analytical thinking, creativity, and high-level mental capacities, which increases as the index becomes closer to 1; (ii) the degree to which they require physical dexterity, situational adaptability, and language proficiency, which increases as the index becomes closer to 0. Following the literature, (*e.g.*, Acemoglu and Autor 2011; Acemoglu and Rastrepo 2018) we consider that tasks with a high level of attribute (i) and a low level of attribute (ii), *i.e.,* non-routine cognitive/abstract tasks, requires high levels of education while tasks with the opposite profile, *i.e.,* non-routine manual tasks, (ii) do not. We reflect this by considering that $\psi_U(v_A) = 1 - v_A$ and

$\psi_H(v_A) = v_A$ which implies that high-skilled workers are relatively more productive in tasks indexed by larger v_A, and vice-versa.

In the routine sector $s = B$, each task is produced by middle-skilled workers and by a continuum of robots indexed by $j \in [0, J_M]$ which are also quality-adjusted: $q_B > 1$ is the constant quality upgrade, k is the top-quality rung at t and $x^M_{v_B}(k, j, t)$ represents the units of machines demanded for the task. The output of v_B, Y_{v_B}, at time t is,

$$Y^M_{v_B}(t) = \left[\int_0^{J_M} \left(q_B^{k(j,t)} \; x^M_{v_B}(k, j, t)\right)^{1-\alpha_B} dj\right] \cdot \left[\psi_M(v_B) \cdot l^M_B \cdot L^M_{v_B}\right]^{\alpha_B}, \tag{9}$$

where $\psi_H(v_B) = 1$ reflects the assumption that middle-skilled workers are equally good at different types of routine tasks. In similarity to sector α_A the income share of middle-skilled workers. We assume that $\alpha_B \leq \alpha_A$ to reflect the fact the role of labor is less important than machines relative to routine tasks in relation to non-routine tasks.

In both sectors, the maximization problem of the producer of a task v_s can be described as follows:

$$\max_{x^i_{v_s}(k,j,t), L^i_{v_s}} \Pi^i_{v_s}(t) = P^i_{v_s}(t) \cdot Y^i_{v_s}(t) - \int_0^{J_i} p^i_s(k, j, t) \cdot x^i_{v_s}(k, j, t) \cdot dj - w^i_s(t) \cdot L^i_{v_s}, \; i = \{U, M, H\}, \tag{10}$$

with $s = A$ for $i \in \{U, H\}$ and $s = B$ for $i = M$ and where $P^i_{v_s}(t)$ is the price of task v_s produced by i-labor type in the s-sector at time t, $p^i_s(k, j, t)$ denotes the price paid for the machine j with quality k at time t by a producer of a task v_s that in the s-sector uses i-labor type, and $w^i_s(t)$ is the price of each unit of i-labor type in the s-sector at time t, which are taken as given by the perfectly competitive producers of the tasks. The first-order conditions with respect to machines allow us to obtain the following:

$$x^i_{v_s}(k, j, t) \;\; = \;\; \left[\frac{P^i_{v_s}(t) \cdot (1 - \alpha_s)}{p^i_s(k, j, t)}\right]^{\frac{1}{\alpha}} \cdot q_s^{k(j,t)\frac{1-\alpha_s}{\alpha_s}} \cdot \psi_i(v_s) \cdot l^i_s \cdot L^i_{v_s}. \tag{11}$$

Replacing (11) in the corresponding production functions (7), (8), or (9), we have that:

$$Y_{v_s}^i(t) \quad = \quad \left[\frac{P_{v_s}^i(t)\cdot(1-\alpha_s)}{p_s^i(k,j,t)}\right]^{\frac{1-\alpha}{\alpha}} \cdot Q_s^i(t)\cdot\psi_i(v_s)\cdot l_s^i\cdot L_{v_s}^i, \tag{12}$$

where $Q_s^i \equiv \int_0^{J_i} q_s^{k(j,t)\frac{1-\alpha_s}{\alpha_s}} dj$ is a measure of the quality level of machines used in sector s to be endogenously determined in Section 3, thereby originating the dynamic effects of the model as will be shown further ahead.

The first-order conditions with respect to labor units allow us to obtain the following:

$$w_s^i(t) \quad = \quad \frac{\alpha_s\cdot P_{v_s}^i(t)\cdot Y_{v_s}^i(t)}{L_{v_s}^i} = \left[P_{v_s}^i(t)\right]^{\frac{1}{\alpha_s}}\cdot\left[\frac{1-\alpha_s}{p_s^i(k,j,t)}\right]^{\frac{1-\alpha_s}{\alpha_s}}\cdot Q_s^i(t)\cdot\psi_i(v_s)\cdot l_s^i, \tag{13}$$

Relative prices and threshold task in sector A. We assume that workers of the same type receive the same wage. In the case of sector A,[7] in order to ensure this, we define the constants $P_A^U(t) = P_{v_A}^U(t)\cdot(1-v_A)^{\alpha_A}$ and $P_A^H(t) = P_{v_A}^H(t)\cdot(v_A)^{\alpha_A}$ which imply the following relative price of tasks:

$$\frac{P_{v_A}^U}{P_{v_A}^H} = \left(\frac{v_A}{1-v_A}\right)^{\alpha}\frac{P_A^U}{P_A^H}, \tag{14}$$

which depends positively on an advantage of producing task v_A using high-skilled labor that is specific to the index of the task, represented by by $\frac{v_i(t)}{1-v_i(t)}$, and relative advantage of using this type of workers that is common to all tasks, regardless of their index, which is represented by $\frac{P_A^U}{P_A^H}$ since an increase causes the relative prices of all tasks using low-skilled labor to be higher.[8]

As shown in appendix A.1, we can prove the existence of a threshold task $\overline{v}_A$ defined by (15) such that for $v_A < \overline{v}_A$, $\frac{P_{v_A}^U}{P_{v_A}^H} < 1$, which implies that it is more advantageous to produce such tasks using low-skilled labor, with the converse holding for $v_A > \overline{v}_A$:

[7] In the case of sector B, as we show in appendix A.2, this step is not required as $P_{v_B}^B$ is constant with respect to v_B and, therefore, wages of middle-skilled workers, w_M, are also constant with respect to v_B.

[8] For sake of clarity, henceforth we refer to the first as a specific relative advantage and the second as a relative advantage.

$$\overline{v}_A = \left[1 + \left(\frac{Q_A^H \cdot l_A^H \cdot L_A^H}{Q_A^U \cdot l_A^U \cdot L_A^U}\right)^{\frac{1}{2}}\right]^{-1}. \tag{15}$$

For a given $\frac{Q_A^H}{Q_A^U}$, bearing in mind the labor levels, which are supplied inelastically, and the net absolute productivity advantage of labor L_A^H over labor L_A^U, $\frac{l_A^H}{l_A^U}$, results a $\overline{v}_A$, and, thus, a given number of tasks produced by each type of labor in the A-sector which reflects the "comparative advantage" of high-skilled over low-skilled workers. In particular, a decrease in $\overline{v}_A$ means a larger space for production with high-skilled workers, and therefore, any changes that contributes to this, such as an increase of $\frac{l_A^H}{l_A^U}$, increases the high-skilled workers "comparative advantage".

2.3 Machines/Robots: technology, output and prices

In the machines sector, the production of the top quality k of each machine j requires a start-up cost of R&D to reach the new design, which can only be recovered if profits at each date are positive for a certain time in the future. This is assured by a system of Intellectual Property Rights that protects the leader firm's monopoly, while at the same time, disseminating, almost without costs, acquired technological knowledge to other firms. Hence, each firm that holds the patent for the top quality k of j at t supplies all respective tasks, v_s, in the s-sector.

Assuming that each unit of machine $j \in [0, J_i]$ requires one unit of final output Y, since its price is normalized to 1, the producer of j gets profits $\left[p_s^i(k,j,t) - 1\right] \cdot x_s^i(k,j,t))$, where $x_s^i(k,j,t) = \left(\frac{P_{v_s}^i(1-\alpha_s)}{p_s^i(k,j,t)}\right)^{\frac{1}{\alpha_s}} q_s^{k(j,t)\frac{1-\alpha_s}{\alpha_s}} \cdot \psi_i(v_s) \cdot l_s^i \cdot L_s^i$ is the demand for machine j from all the producers of tasks v_s that use i-type of labor in the s-sector that is complementary with this type of machine, which is defined, for example in the case $i = U$, as $x_A^U(k,j,t) = \int_0^{\overline{v}_A} x_{v_A}^U(k,j,t)dv_A$. From the maximization problem results the following mark-up price and, consequently, profits:

$$p_s^i(k,j,t) = p_s = q_s = \frac{1}{1-\alpha_s}, \tag{16}$$

which is equal to the quality jump q_s of the corresponding sector by assuming a binding limit pricing strategy.[9] In turn, this implies the following profits:

$$\pi_s^i(k,j,t) \equiv \left(\frac{\alpha_s}{1-\alpha_s}\right) \cdot x_s^i(k,j,t) = (q_s - 1) \cdot q_s^{-\frac{1}{\alpha_s}} \cdot (1-\alpha_s)^{\frac{1}{\alpha_s}} \cdot q_s^{k(j,t)\frac{1-\alpha_s}{\alpha_s}} \cdot \left(P_s^i\right)^{\frac{1}{\alpha_s}} \cdot l_s^i \cdot L_s^i. \quad (17)$$

2.4 Lobbying Setup

We envision lobbying as an activity whereby individual firms[10] conduct efforts to influence entities, whose activity allows them to establish barriers to the correct allocation of resources or, alternatively, to prevent the establishment of barriers (*e.g.*, Grossmann and Steger 2008; Mathur *et al.* 2013; Cothren and Radhakrishnan 2017; Bellettini et. al., 2013; Figueiredo and Ritcher, 2013). These firms, in turn, belong to the machine sector because there is substantial evidence in the literature that lobbying activities are conducted by firms that can only exist in this sector, which are large firms with innovation capabilities and market power that conduct such efforts to further increase it (*e.g.*, Belletini *et al.* 2013; Figueiredo and Ritcher 2013; Lux *et al.* 2011).[11]

We assume that each firm that produces machines from each sector s, spends a fraction z_s of their profits in order to obtain gains in the same period up to a maximum of $\gamma \cdot \pi_s^i(k,j,t)$.[12] Therefore, the parameter γ determines the maximum possible gains from lobbying. Moreover, in line with the existing literature (*e.g.*, Lux *et al.* 2011, and references therein), we also consider

[9] We now explain this assumption in more detail. In this setup, we assume that only the top quality rung of each machine is used in the production. If we generalize and consider that the producer of task v_s actually uses several quality levels of machine j, *i.e.*, $\tilde{x}_{v_s}^i(j,k,t) = \sum_0^{k(j,t)} q_s^{k(j,t)} x_{v_s}^i(j,k,t)$, then the price that each producer can apply is bound by the inequality $p(j,k,t) \leq \frac{p(j,k-1,t)}{q_s}$. The intuition behind this result is that since a machine of quality $k-1$ corresponds to $\frac{1}{q_s}$ units of a machine with quality k, its price to the consumer can be, at most, $\frac{1}{q_s}$ of the price of the machine with superior quality. Therefore, assuming that k is the highest quality rung of the machine j, the producer of a machine with quality rung k can adopt a limit pricing strategy and drive other firms producing lower quality machines off the market. Bearing in mind that the marginal cost of all firms is 1, this can be accomplished by setting the price to $q_s - \epsilon$, where ϵ is an infinitesimal, because this would imply that none of the inferior qualities would be able to survive since $p_s^i(k-1,j,t) \leq \frac{q_s - \epsilon}{q_s} \leq 1$, which is smaller then marginal costs, and thus implies negative profits. Since the monopoly optimal price is $p_s^i(k,j,t) = \frac{1}{1-\alpha_s}$, assuming that the limit pricing strategy is binding implies that $p_s^i(k,j,t) = q_s = \frac{1}{1-\alpha_s}$ – for additional details see Barro and Sala-i-Martin (2004).

[10] We must clarify that the fact that lobbying is conducted by representative firms is not meant to signify that lobbying is conducted at the industry level because in this setting, as we explained before, the meaning of sector we adopt is not sector of activity or industry since both routine and non-routine tasks are prevalent in different occupations in all parts of the economy. We merely consider a representative firm to avoid unnecessary complications to the analysis. It is important to note that according to Figueiredo and Ritcher (2013), there is overwhelming evidence to support that corporations and trade associations comprise the vast majority of the lobbying expenditures by interest groups.

[11] This is possible due the monopoly each one has on the production of machines, whereas in the remaining sectors this is not possible due to the existence of perfect competition.

[12] As shown further ahead, this implies that the maximization problem of the monopolists in relation to the lobbying effort is a static problem, which simplifies the analysis.

that the effectiveness of the lobbying efforts depends on the sum of individual efforts made by firms with similar interests in relation to total efforts made by all firms. In the context of this model, we consider similar firms those that produce machines complementary to tasks with the same degree of routine intensity, *i.e.*, those that belong to the same sector (A or B). This implies the assumption that firms engaged in the production of machines to be used in routine tasks (sector A) have common interests that are distinct from the common interests of those that produce machines used in non-routine tasks (sector B).[13] Therefore, the increased gains from lobbying depend on the relative share of lobbying expenditures of companies in each sector, $\mathcal{S}_s(z_A, z_B)$, which implies that the gains from lobbying in each period are given by $\gamma \cdot \mathcal{S}_s(z_A, z_B) \cdot \pi_s^i(k, j, t)$.[14]

Bearing this in mind the general expression for profits of machine firms in both sectors is the following:

$$\tilde{\pi}_s^i(k, j, t) = \pi_s^i(k, j, t) + \underbrace{\gamma \cdot \mathcal{S}_s(z_A, z_B) \cdot \pi_s^i(k, j, t)}_{\text{Lobbying gains}} - \underbrace{z_s \cdot \pi_s^i(k, j, t)}_{\text{Lobbying costs}}, \tag{18}$$

where

$$\mathcal{S}_A(z_A, z_B, \mathcal{R}_A) = \frac{z_A \cdot \left(\int_0^{J_L} \pi_A^U(k,j,t)dj + \int_0^{J_H} \pi_A^H(k,j,t)dj\right)}{z_A \cdot \left(\int_0^{J_L} \pi_A^U(k,j,t)dj + \int_0^{J_H} \pi_A^H(k,j,t)dj\right) + z_B \cdot \int_0^{J_M} \pi_B^M(k,j,t)dj} = \frac{\mathcal{R}_A \cdot z_A}{\mathcal{R}_A \cdot z_A + z_B}, \tag{19}$$

$$\mathcal{S}_B(z_A, z_B, \mathcal{R}_A) = \frac{z_B \cdot \int_0^{J_M} \pi_B^M(k,j,t)dj}{z_A \cdot \left(\int_0^{J_L} \pi_A^U(k,j,t)dj + \int_0^{J_H} \pi_A^H(k,j,t)dj\right) + z_B \cdot \int_0^{J_M} \pi_B^M(k,j,t)dj} = \frac{z_B}{\mathcal{R}_A \cdot z_A + z_B}, \tag{20}$$

with $\mathcal{R}_A = \frac{\int_0^{J_L} \pi_A^U(k,j,t)dj + \int_0^{J_H} \pi_A^H(k,j,t)dj}{\int_0^{J_M} \pi_B^M(k,j,t)dj}$ representing the relative profitability of firms in sector A. The first-order conditions of the maximization of profits with respect to lobbying results in the following lobbying best response functions for each firm:

[13] This assumption is a simplification of the reality, which is far more complex since the similarity of firms is related to other dimensions, namely the sector of activity/industry to which each firm belongs. Nonetheless, we make it because it allows to make the model tractable and is also plausible considering that there is evidence of a connection between these two dimensions, sector of activity and share of routine/non-routine tasks. For instance, there is evidence in the literature that manufacturing firms conduct lobbying activities (*e.g.*, Fuller-Love and Thomas, 2004) and are more intensive in routine intensive tasks than the services sector (Marcolin *et al.* 2019; Keister and Lewandowski 2017).

[14] This simple setup can be interpreted as various types of lobbying. For instance, if we consider legislative lobbying, whereby firm from different sectors compete for passing opposite pieces of legislation, we can interpret $\gamma \cdot \pi_s^i(k, j, t)$ as the monetary gains of such legislation being approved, with $\mathcal{S}_s(z_A, z_B)$ as the probability of this occurring, which varies between 0 and 1.

$$z_A(z_B, \mathcal{R}_A, \gamma) = \left(\frac{z_B}{\mathcal{R}_A}\right)^{0.5} \cdot \left[\gamma^{0.5} - \left(\frac{z_B}{\mathcal{R}_A}\right)^{0.5}\right], \tag{21}$$

$$z_B(z_A, \mathcal{R}_A, \gamma) = (\mathcal{R}_A z_A)^{0.5} \cdot \left[\gamma^{0.5} - (\mathcal{R}_A z_A)^{0.5}\right]. \tag{22}$$

We summarize the sign of the derivatives of these functions in table 1. Its analysis shows interesting dynamics between firms in both sectors. An increase in maximum lobbying gains always increases the lobbying effort from firms in both sectors. Both also depend on the lobbying effort from the other firms and negatively on their own relative profitability, which, in the case of firms of sector B, is $1/\mathcal{R}_A$, which is positive or negative under certain conditions, as described in Table 1.

We now provide some intuition for these results. The first result is straightforward because an increase in maximum gains creates additional incentives to increase lobbying intensity. Regarding the second, if a firm is confronted with an increase of lobbying efforts from firms in the other sector or with a decreased relative profitability, it will see a decrease in the share of gains as can be easily seen in equations (19) and (20). In both sectors, the share of gains that accrues to each firm has decreasing marginal returns with respect to the corresponding lobbying efforts. Bearing this in mind, if lobbying efforts from firms in the other sector are relatively small, then, the marginal benefits from increasing lobbying efforts will exceed its marginal costs, driving the original firms to increase lobbying efforts. Otherwise, they will decrease them.

	x				Conditions
	γ	z_A	z_B	$\mathcal{R}_A$	
$\frac{\partial z_A(\cdot)}{\partial x}$	$+$	NA	$+$	$-$	$z_B < \frac{\gamma \mathcal{R}_A}{4}$
			$-$	$+$	$z_B > \frac{\gamma \mathcal{R}_A}{4}$
$\frac{\partial z_B(\cdot)}{\partial x}$	$+$	$+$	NA	$+$	$z_A < \frac{\gamma}{4\mathcal{R}_A}$
		$-$		$-$	$z_A > \frac{\gamma}{4\mathcal{R}_A}$

Table 1: Sign of the derivatives of the best response functions of the lobbying efforts of firms in each sector.

In equilibrium we obtain the following lobbying effort from both firms:

$$z_A^* = z_B^* = z^* = \max\left\{0, \frac{\gamma \mathcal{R}_A}{(1+\mathcal{R}_A)^2}\right\}, \tag{23}$$

which results from imposing that lobbying efforts are non-negative. From here, we derive the following proposition:

Proposition 1. *For $\gamma > 0$, the optimal lobbying effort is symmetric and depends (i) positively on γ, (ii) positively on $\mathcal{R}_A$ if $\mathcal{R}_A > 1$, and negative otherwise. For $\gamma \leq 0$, no firm conducts a lobbying effort.*

The proposition results from the strategic interactions between firms described before and from lobbying efforts being necessarily by definition non-negative. Substituting in profits, we obtain the following:

$$\tilde{\pi}_s^i(k,j,t) = (1 + \mathcal{L}_s(\mathcal{R}_A, \gamma)) \cdot \pi_s^i(k,j,t), \tag{24}$$

where

$$\mathcal{L}_A(\mathcal{R}_A, \gamma) = \max\left\{0, \gamma\left(\frac{\mathcal{R}_A}{1+\mathcal{R}_A}\right)^2\right\}, \tag{25}$$

$$\mathcal{L}_B(\mathcal{R}_A, \gamma) = \max\left\{0, \frac{\gamma}{(1+\mathcal{R}_A)^2}\right\}, \tag{26}$$

represent the net benefits for the firm in sector s to conduct lobbying efforts. These expressions result from considering that since lobbying efforts imply a cost, firms only have incentives to engage in lobbying activities iff $\gamma > 0$. Otherwise, lobbying effort is zero for all firms in all sectors. From here we derive the following proposition:

Proposition 2. *For $\gamma > 0$, the optimal level of profits of machine firms in both sectors depends (i) positively on the parameter that determines maximum gains from lobbying, γ, (ii) positively on the relative profitability of firms of their own sector, which is $\mathcal{R}_A$ in the case of firms of sector A and $1/\mathcal{R}_A$ in the case of firms of sector B. For $\gamma \leq 0$, there is no lobbying effort and hence profits remain equal to the baseline scenario without lobbying.*

Proof. Directly from (25) and (26).

□

We now explain the intuition of Proposition 2. An increase in the parameter γ increases lobbying gains and therefore increases net profits for lobbying for every level of effort, as explained before. On the other hand, an increase in $\mathcal{R}_A$, which is an endogenous variable of the model, results in an increase of the equilibrium share of gains that accrue to firms in sector A and a decrease for firms in sector B, which results in a corresponding increase and decrease of profits of firms in each of these sectors.

2.5 Resource allocation and some static results

Once the threshold task for the A-sector has been determined as well as the price of machines, we can now move on to determine, for a given factor/input levels: (i) the price indexes of each sector, P_A and P_B; (ii) the price and output of each sector both in levels and relative terms (iii) the wage differences between types of labor in the various contexts. This analysis is conducted without the adjustment of the technological-knowledge progress of the sectors and, as a result, allows for some static results.

Prices and outputs of each sector. In order to obtain an expression for the prices and outputs of each sector, we need first to derive the expressions that determine the absolute values of price indexes in sector A. To do so, we consider the relation between implied by (6) between P_s and the prices of tasks P_{v_s}, which is $P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right)$, the result that the value of each task, $P_{v_s} \cdot Y_{v_s}$, is a constant for all v_s, and, also, make use of (14) applied to the threshold task to obtain $P_A^H = \left(\frac{\overline{v}_A}{1-\overline{v}_A}\right)^{\alpha} \cdot P_A^U$. From this analysis, we obtain the following expressions – see Appendix A.2:

$$P_A^U = P_A \cdot \exp\left(-\alpha_A\right) \cdot \overline{v}_A^{-\alpha_A} \; and \; P_A^H = P_A \cdot \exp\left(-\alpha_A\right) \cdot \left(1 - \overline{v}_A\right)^{-\alpha_A}, \tag{27}$$

from where we derive the following proposition, which is consistent with the interpretation of $\frac{P_A^U}{P_A^H}$ described in the previous section:

Proposition 3. *In the A-sector, the price index of the tasks produced by a certain type of labor, P_A^U or P_A^H, depends positively on the price of the output of sector A, P_A, and depends negatively on the relative number of tasks produced by the corresponding type of labor which is $\overline{v}_A$ in the case of low-skilled labor and $1-\overline{v}_A$ in the case of high-skilled labor.*

Still in sector A, from the profit maximization problem of the producer of Y and since in each sector part of the tasks is done by labor L_A^H and other part is performed by labor L_A^U, the aggregate output is the following: $P_A Y_A = \int_0^1 P_{v_A} Y_{v_A} dv_A = \int_0^{\overline{v}_A} P_{v_A}^U Y_{v_A}^U dv_A + \int_{\overline{v}_A}^1 P_{v_A}^H Y_{v_A}^H dv_A$. On the basis of these definitions and taking into account (12), (14), and (27), we have:

$$P_A Y_A = (P_A)^{\frac{1}{\alpha_A}} \cdot \exp(-1) \cdot \left(\frac{1-\alpha_A}{p_A}\right)^{\frac{1-\alpha_A}{\alpha_A}} \cdot M_A, \tag{28}$$

where $M_A \equiv \frac{Q_A^U \cdot l_A^U \cdot L_A^U}{\overline{v}_A} + \frac{Q_A^H \cdot l_A^H \cdot L_A^H}{1-\overline{v}_A} = \left[\left(Q_A^U \cdot l_A^U \cdot L_A^U\right)^{\frac{1}{2}} + \left(Q_A^H \cdot l_A^H \cdot L_A^H\right)^{\frac{1}{2}}\right]^2$ and, therefore, the expression for output in A is:

$$Y_A = \exp(-1) \cdot \left[\frac{P_A \cdot (1-\alpha_A)}{p_A}\right]^{\frac{1-\alpha_A}{\alpha_A}} \cdot M_A. \tag{29}$$

In the case of sector B, as we show in appendix A.2, we have that $P_B Y_B = \int_0^1 P_{v_B} Y_{v_B} dv_B = P_{v_B} Y_{v_B}$ and also that $P_B = P_{v_B}^M$ and $L_B^M = L_{v_B}^M$. Considering this, we have

$$P_B Y_B = (P_B)^{\frac{1}{\alpha_B}} \cdot \left(\frac{1-\alpha_B}{p_B}\right)^{\frac{1-\alpha_B}{\alpha_B}} \cdot M_B, \tag{30}$$

where $M_B \equiv Q_B^M \cdot l_B^M \cdot L_B^M$ and, thus, the expression for output in B is:

$$Y_B = \left[\frac{P_B \cdot (1-\alpha_B)}{p_B}\right]^{\frac{1-\alpha_B}{\alpha_B}} \cdot M_B. \tag{31}$$

Now, dividing the output between sectors; *i.e.*, bearing in mind (29) and (31), yields:

$$\frac{Y_A}{Y_B} = \mathcal{C} \cdot \mathcal{M} \cdot \frac{P_A^{\frac{1-\alpha_A}{\alpha_A}}}{P_B^{\frac{1-\alpha_B}{\alpha_B}}} \tag{32}$$

where $\mathcal{C} \equiv \exp(-1) \cdot \frac{\left(\frac{1-\alpha_A}{p_A}\right)^{\frac{1-\alpha_A}{\alpha_A}}}{\left(\frac{1-\alpha_B}{p_B}\right)^{\frac{1-\alpha_B}{\alpha_B}}}$ and $\mathcal{M} \equiv \frac{M_A}{M_B} = \left[\left(\frac{Q_A^U}{Q_B^M} \cdot \frac{l_A^U}{l_B^M} \cdot \frac{L_A^U}{L_B^M}\right)^{\frac{1}{2}} + \left(\frac{Q_A^H}{Q_B^M} \cdot \frac{l_A^H}{l_B^M} \cdot \frac{L_A^H}{L_B^M}\right)^{\frac{1}{2}}\right]^2$.

Replacing this in the expression for relative demand of output in each sector (4), we can determine the relationship between the prices of each sector:

$$P_A = (\mathcal{C} \cdot \mathcal{M})^{-\frac{\alpha_A}{\sigma_A}} \cdot \left(\frac{\chi_A}{\chi_B}\right)^{\frac{\epsilon\alpha_A}{\sigma_A}} \cdot P_B^{\frac{\sigma_B}{\alpha_B}\frac{\alpha_A}{\sigma_A}}, \tag{33}$$

where: $\sigma_A = 1 - \alpha_A + \epsilon \cdot \alpha_A$ e $\sigma_B = 1 - \alpha_B + \epsilon \cdot \alpha_B$. Considering this and the relation between prices implied by (5) from the maximization problem of the producer of Y, we can determine the absolute values of P_A and P_B by solving numerically the following equations for a given value of the technological-knowledge bias:

$$\chi_A^\epsilon \cdot (\mathcal{C} \cdot \mathcal{M})^{-\frac{\alpha_A}{\sigma_A}(1-\epsilon)} \cdot \left(\frac{\chi_A}{\chi_B}\right)^{\frac{\alpha_A}{\sigma_A}\epsilon(1-\epsilon)} \cdot P_B^{\frac{\sigma_B}{\alpha_B}\frac{\alpha_A}{\sigma_A}(1-\epsilon)} + \chi_B^\epsilon \cdot P_B^{1-\epsilon} = 1, \tag{34}$$

$$\chi_B^\epsilon \cdot P_A^{(1-\epsilon)\frac{\alpha_B}{\sigma_B}\frac{\sigma_A}{\alpha_A}} \cdot (\mathcal{C} \cdot \mathcal{M})^{\frac{\alpha_B(1-\epsilon)}{\sigma_B}} \cdot \left(\frac{\chi_A}{\chi_B}\right)^{-\frac{\epsilon\alpha_B(1-\epsilon)}{\sigma_B}} + \chi_A^\epsilon \cdot P_A^{1-\epsilon} = 1. \tag{35}$$

Having determined the prices and output of each sector, we can now determine aggregate output, $Y = P_A \cdot Y_A + P_B \cdot Y_B$, which is given by:

$$Y = (P_A)^{\frac{1}{\alpha_A}} \exp(-1) \left(\frac{1-\alpha_A}{p_A}\right)^{\frac{1-\alpha_A}{\alpha_A}} M_A + \frac{(P_B)^{\frac{1}{\alpha_B}}}{P} \left(\frac{1-\alpha_B}{p_B}\right)^{\frac{1-\alpha_B}{\alpha_B}} M_B. \tag{36}$$

Wage differentials. Finally, in this Subsection, the differences in wages still need to be addressed. Bearing in mind the wages in (13), by considering (27) and (15) we can obtain the following expression for the skill premium or intra-sector A wage inequality, which is

$$\frac{w_A^H}{w_A^U} = \left(\frac{Q_A^H}{Q_A^U} \cdot \frac{l_A^H}{l_A^U} \cdot \frac{L_A^U}{L_A^H}\right)^{\frac{1}{2}}. \tag{37}$$

Proposition 4. *The wage differential of high-skilled to low-skilled workers – skill premium – depends (i) positively on an increase of the technological-knowledge bias favorable to low-skilled* *workers relative to high-skilled workers in sector A,* $\frac{Q_A^H}{Q_B^M}$*. For a given intra-sector technological-knowledge bias, it depends (ii) positively on an increase of the absolute advantage of high-skilled over low-skilled workers* $\frac{l_A^H}{l_A^U}$*; (iii) positively on the increase of the relative supply of low-skilled to high-skilled workers.*

Proof. Directly from (37).

□

From the same equations we can also obtain an expression of the inter-sector wage inequality, which is the following:

$$\frac{w_A^i}{w_B^M} = \frac{\alpha_A \left(1-\alpha_A\right)^{\frac{2-2\alpha_A}{\alpha_A}}}{\alpha_B \left(1-\alpha_B\right)^{\frac{2-2\alpha_B}{\alpha_B}}} \cdot \frac{l_A^i}{l_B^M} \cdot \frac{\left(P_A^i\right)^{\frac{1}{\alpha_A}}}{\left(P_B\right)^{\frac{1}{\alpha_B}}} \cdot \frac{Q_A^i}{Q_B^M}, \tag{38}$$

Proposition 5. *The wage differential of worker type i in sector A, high-skilled and low-skilled, to middle-skilled workers is affected positively by (i) an increase of the technological-knowledge bias favorable to workers in sector A relative to middle-skilled workers in sector B,* $\frac{Q_A^i}{Q_B^M}$ *– the technological-knowledge bias channel. Moreover, for a given inter-sector technological-knowledge bias, it is affected (ii) positively by an increase of the labor share of workers in sector A relative to sector B, (iii) an increase of the relative price of output produced in sector A,* $\frac{\left(P_A^i\right)^{\frac{1}{\alpha_A}}}{\left(P_B\right)^{\frac{1}{\alpha_B}}}$ *– the price channel, (iv) an increase of the relative efficiency of labor units* $\frac{l_A^i}{l_B^M}$*.*

Proof. Directly from (38)

□

2.6 R&D sector: technology, patent value, and expenditures

The probability of successful innovation is, thus, at the heart of the R&D activity. Let $\mathcal{I}_s^i(k,j,t)$ denote the instantaneous probability at time t – a Poisson arrival rate – of innovation in the quality rung a machine j complementary with labor type i, with $s = A$ for $i \in \{U, H\}$

and $s = B$ for $i = M$, that results in a jump from quality rung $k(j,t)$ to a higher quality level $[k(j,t)+1]$. We define it as follows:

$$\mathcal{I}_s^i(k,j,t) = e_s^i(k,j,t) \cdot \beta q^{k(j,t)} \cdot \zeta^{-1} q^{-\alpha_s^{-1} k(j,t)} \cdot \left(L_s^i\right)^{-\xi} \cdot f_s(i) \tag{39}$$

where: (i) $e_s^i(k,j,t)$ is the flow of domestic final-good resources devoted to R&D by firms producing machine j complementary to labor type i, with $s = A$ for $i \in \{U,H\}$ and $s = B$ for $i = M$, which defines our framework as a lab-equipment model (Afonso 2012); (ii) $\beta q^{k(j,t)}$, $\beta > 0$, is the learning-by-past domestic R&D, as a positive learning effect of public knowledge accumulated from past successful R&D (Grossman and Helpman 1991, ch. 12; Afonso 2012); (iii) $\zeta^{-1} q^{-\alpha_s^{-1} k(j,t)}$, $\zeta > 0$, is the adverse effect – cost of complexity – caused by the increasing complexity of quality improvements (Afonso 2012);[15] (iv) $\left(L_s^i\right)^{-\xi}$ is the adverse effect of market size. The larger the market size, the larger the profits obtained by incumbents as can be seen in (17), which increase the incentives of incumbents to adopt strategies meant to create technical and other types of barriers to entrants in order to protect their economic rents;[16] (v) the term $f_s(i)$ reflects an absolute advantage of workers of type i in sector s to learn, implement and improve existing technological knowledge. In this term, we reflect the widely accepted complementary in the literature between R&D and human capital accumulation (*e.g.*, Lucas 1988; Sanchez-Carrera 2012, 2019). Since workers at the top of the skill distribution are those that have higher human capital, following the literature (*e.g.,* Afonso 2006) we consider a specification which benefits innovations directed at machines complementary to high-skilled workers, which depends positively

[15]The complexity cost is modeled in such a way that, together with the positive learning effect (ii), it exactly offsets the positive effect of the quality rung on profits of each leader machine firm; this is the reason for the presence of the production function parameter α in (39) – *e.g.*, Barro and Sala-i-Martin (2004, ch. 7).

[16]According to empirical evidence obtained by several authors in the literature over the years (*e.g.*, Eisenhardt and Brown 1998, Cohen *et al.* 2003) these activities include, but are not limited to, trade secrecy (*e.g.*, the secrecy of the formula of Coca-Cola), increasing the degree of complexity of products to camouflage researches processes (*e.g.,* the increased features added to the Microsoft operating system over the years, the increasingly smaller and more sophisticated produced by Intel), patent blocking, *i.e.*, patenting similar inventions to the original one without introducing them in the market, among others.

on the ratio of high-skilled to low-skilled workers,[17] resulting in the following specification:

$$f_s(i) = \begin{cases} 1 & \text{if} \quad i \in \{U, M\} \\ \left(1 + \frac{L_A^H}{L_A^U + L_A^H}\right)^{1+\frac{L_A^H}{L_A^U}} & \text{if} \quad i = H \end{cases}, \tag{40}$$

An innovator that produces a a machine j with quality rung k will receive monopoly profits during the time in which the patent is valid from time t to τ, when a (new) machine firm introduces the new quality of j, $q_s^{k(j)+1}$. Bearing this in mind, the present value of the profits of the producer of machine j during the time in which the patent is valid is given by the following expression:

$$V_s^i\left(k, j, t, \tau\right) = \int_t^\tau \tilde{\pi}_s^i(k, j, v) \cdot \exp\left[-\int_t^\omega r\left(\omega\right) d\omega\right] dv. \tag{41}$$

However, the duration of the patent, $\tau - t$, is stochastic and depends on the probability of a superior quality rung being achieved by an entrant at any time t, which is determined by $\mathcal{I}_s^i\left(k, j, t\right)$ and depends on the present quality rung k. Bearing this in mind and considering that, in equilibrium, the interest rate between t and τ is constant, the expected value of $V_s^i\left(k+1, j, t, \tau\right)$ is:

$$V_s^i\left(k+1, j, t\right) \equiv E_\tau\left[V_s^i\left(k+1, j, t, \tau\right)\right] = \frac{\tilde{\pi}_s^i(k+1, j, t)}{r\left(t\right) + \mathcal{I}_s^i\left(k, j, t\right)}. \tag{42}$$

Equation (42) can be seen as the no-arbitrage condition, where $V_s^i\left(k+1, j, t\right) \cdot r\left(t\right)$, the expected income generated by a successful improvement of the quality rung of machine j to $k+1$, equals the profit flow, $\tilde{\pi}_s^i(k+1, j, t)$, minus the expected capital loss that results from a displacement of the incumbent by an entrant firm, $V_s^i\left(k+1, j, t\right) \cdot \mathcal{I}_s^i\left(k, j, \tau\right)$. We can make this interpretation because innovation is always achieved by entrants rather than incumbents. This is the Arrow effect (*e.g.,* Aghion *et al.* 1998, ch. 2), and results from the fact that the variation in profits of introducing a higher quality rung for a machine j is always lower for an incumbent than for an entrant.

Finally, from the definition of the probability of achieving higher quality rungs (39), we have

[17] We do not consider the ratio of high-skilled workers to other workers in the distribution because we consider that middle-skilled workers have a neutral role with respect to innovation.

that $e_s^i(k,j,t) = \mathcal{I}_s^i(k,j,t) \cdot \frac{\zeta}{\beta} \cdot q^{(1-\alpha_s)\cdot\alpha_s^{-1}k(j,t)} \cdot \left(L_s^i\right)^{\xi}$. Since, by definition, $\mathcal{I}_s^i(k,j,t)$ does not differentiate between different machines belonging to the same sector, we have that:

$$E_s(t) = \sum_i \int_0^{J_i} e_s^i(k,j,t) \cdot dj = \sum_i \mathcal{I}_s^i(k,j,t) \cdot \frac{\zeta}{\beta} \cdot Q_s^i \cdot \left(L_s^i\right)^{\xi}, \tag{43}$$

with $i \in \{U, H\}$ for $s = A$ and $i = M$ for $s = B$. Thus, more resources devoted to R&D are needed as Q_s^i rises to offset the greater difficulty of R&D when Q_s^i increases.

3 General equilibrium

As the economic structure has been characterized for given states of technological knowledge represented by indexes Q_A^L, Q_A^H and Q_B^M in the two existent sectors of activity, we now proceed to characterize the general equilibrium. For this purpose we first derive the equilibrium of achieving higher quality rungs, where it is embodied that households and firms are rational and solve their problems, free-entry R&D conditions are met, and markets clear. Then we derive the aggregate resource constraint of the economy and show that all variables including consumption depend on the dynamics of technological-knowledge indexes. Finally, we proceed to characterize the transitional dynamics, where we start by deriving the law of motion of each index, and the steady-state growth of the model.

3.1 R&D equilibrium and law of motion of technological knowledge

From the free-entry condition we have that the expected payoff generated by the innovation in j should be equal to the R&D spending to improve j; *i.e.*, $\mathcal{I}_s^i(k,j,t) \cdot V_s^i\,(k+1,j,t) = e_s^i(k,j,t)$,[18] where $\mathcal{I}_s^i(k,j,t)$ and $V_s^i\,(k+1,j,t)$ are given by (39) and (42), respectively, and we can determine – by using also (16), (17) and (18) –, the equilibrium probability:

$$\mathcal{I}_s^i(k+1,j,t) = \frac{\beta}{\zeta} \cdot (1 + \mathcal{L}_s(\mathcal{R}_A, \gamma)) \cdot \alpha_s \cdot (1-\alpha_s)^{\frac{1}{\alpha_s}} \cdot \left(P_s^i\right)^{\frac{1}{\alpha_s}} \cdot l_s^i \cdot \left(L_s^i\right)^{1-\xi} \cdot f_s(i) - r(t). \tag{44}$$

Hence, $\mathcal{I}_s^i(k+1,j,t)$ is independent of the quality level k and j, which implies that $\mathcal{I}_s^i(k+1,j,t) =$

[18] At this point we must highlight the fact that the free entry condition only implies that entrants can freely devote resources to R&D in order to develop higher quality rungs for existing machines. However, they face technical and other kind of barriers from incumbent which, as we have explained above, are embodied in the probability of achieving higher quality rungs.

$\mathcal{I}_s^i(t)$. Moreover, if a new quality of machine j is introduced the rate of change in the quality index of sector s will be the following: $\Delta Q_s^i = Q_s^i(k+1,t) - Q_s^i(k,t) = \int_0^{J_i} q_s^{[k(j,t)+1]\left(\frac{1-\alpha_s}{\alpha_s}\right)} - \int_0^{J_i} q_s^{k(j,t)\left(\frac{1-\alpha_s}{\alpha_s}\right)}$ and thus $\frac{\Delta Q_s^i}{Q_s^i} = \left[q_s^{\left(\frac{1-\alpha_s}{\alpha_s}\right)} - 1\right]$. Since the probability of this occurring per unit of time if given by $\mathcal{I}_s^i(t)$, we have that $\frac{\dot{Q}_s^i(t)}{Q_s^i(t)} = \mathcal{I}_s^i(t) \cdot \left[q_s^{\left(\frac{1-\alpha_s}{\alpha_s}\right)} - 1\right]$, which results in the following growth path of each technological-knowledge index:

$$\frac{\dot{Q}_s^i(t)}{Q_s^i(t)} = \left[q_s^{\left(\frac{1-\alpha_s}{\alpha_s}\right)} - 1\right] \cdot \left\{ \begin{array}{c} \frac{\beta}{\zeta} \cdot (1 + L_s(\mathcal{R}_A, \gamma)) \cdot \alpha_s \cdot (1-\alpha_s)^{\frac{1}{\alpha_s}} \times \\ \left(P_s^i\right)^{\frac{1}{\alpha_s}} \cdot l_s^i \cdot \left(L_s^i\right)^{1-\xi} \cdot f_s(i) - r(t) \end{array} \right\}. \tag{45}$$

3.2 Transitional dynamics and steady-state results

In this section we characterize the transitional dynamics of the economy and the steady state, where the growth rate of all variables and the real interest rate are constants.

Taking into account the aggregate expenditures in the final good are given by $Y = P_A Y_A + P_B Y_B$ from the profit maximization problem of the producer of aggregate output, considering that aggregate expenditures in machines and R&D activities are the sum of aggregates in both sectors already derived in equilibrium in the previous sections, $X \equiv X_A + X_B$ and $E \equiv E_A + E_B$ and that assets in the economy are the present value of the patent of all producers of machines, we can prove that in equilibrium the aggregate flow constraint of households can be expressed as $Y = C + X + E$. Therefore, since Y, X and E are all multiples of the quality indexes Q_A^U, Q_A^H and Q_B^M, the aggregate flow constraint implies that consumption C is also a constant multiple of these variables. Therefore in the steady-state we have the following:

$$g^* \equiv \left(\frac{\dot{Q}_s^i}{Q_s^i}\right)^* = \left(\frac{\dot{Y}}{Y}\right)^* = \left(\frac{\dot{X}}{X}\right)^* = \left(\frac{\dot{E}}{E}\right)^* = \left(\frac{\dot{C}}{C}\right)^* = \frac{r^* - \rho}{\theta}. \tag{46}$$

From this expression we can determine an expression for the steady-state economic growth rate by considering that $r^* = \theta g^* - \rho$ and $g^* \equiv \left(\frac{\dot{Q}_s^i}{Q_s^i}\right)^*$ in (45) and solve for g^*, resulting in the following expressions for the growth rate:

$$g^* \equiv \left(\frac{\dot{Q}_s^i}{Q_s^i}\right)^* = \frac{\left(q_s^{\frac{1-\alpha_s}{\alpha_s}} - 1\right) \left\{ \begin{array}{c} \frac{\beta}{\zeta} \cdot (1 + \mathcal{L}_s(\mathcal{R}_A, \gamma)) \cdot \alpha_s \cdot (1-\alpha_s)^{\frac{1}{\alpha_s}} \times \\ \times \left(P_s^i\right)^{\frac{1}{\alpha_s}} \cdot l_s^i \cdot \left(L_s^i\right)^{1-\xi} \cdot f_s(i) - \rho \end{array} \right\}}{\theta q_s^{\frac{1-\alpha_s}{\alpha_s}} + 1 - \theta}, \tag{47}$$

for $i \in \{U, M, H\}$, with $s = A$ if $i \in \{U, H\}$ and $s = B$ if $i = M$. The growth rate of the technological-knowledge indexes depends on P_s^i, which in turn depends on the low to medium and high to medium technological-knowledge bias, $Q^{U/M}$ and $Q^{H/M}$, respectively. Therefore by equalizing these expressions for the growth rate we can determine the steady-state values of $Q^{U/M}$ and $Q^{H/M}$ that ensure that the growth rate is unique and constant. For this purpose, we can determine the relation between $Q^{U/M}$ and $Q^{H/M}$ by equalizing the expression that results from considering $\left(\frac{\dot{Q}_A^U}{Q_A^U}\right)^* = \left(\frac{\dot{Q}_A^H}{Q_A^H}\right)^*$, resulting in the following:

$$Q_A^{H/M} = f_A^2(H) \cdot \frac{l_A^H}{l_A^U} \cdot \left(\frac{L_A^H}{L_A^U}\right)^{1-2\xi} \cdot Q_A^{U/M}. \tag{48}$$

Hence, we need to determine the steady-state value of $Q_A^{U/M}$, $Q_A^{H/M}$, P_B, and P_A by numerically solving the following equations (34), (35), (47), and (48), which can then be used to determine the growth rate of output and wage differentials between sectors. From (48), we can determine the following expression for the Skill Premium in the steady-state:

$$\frac{w_A^H}{w_A^U} = f_A(H) \cdot \left(\frac{l_A^H}{l_A^U}\right) \cdot \left(\frac{L_A^H}{L_A^U}\right)^{-\xi}. \tag{49}$$

From here we derive the following proposition:

Proposition 6. *In the steady-state equilibrium, the premium between high-skilled and low-skilled workers depends (i) positively on the differences in absolute advantages of high-skilled over low-skilled workers; (ii) positively on the relative supply of high-skilled workers.*

Proof. Directly from equation (49) and considering (40). □

This important proposition shows that the model can produce the same positive relationship between labor supply and skill premium obtained by papers that adopt the DTC approach (*e.g.*, Acemoglu 1998; Acemoglu 2002). This results from the canonical models that follow this approach being, in the context of our approach, a particular case of an economy where aggregate output is only produced by non-routine tasks. Moreover, we can also conclude that the steady-state value of the Skill Premium is affected neither by γ nor by α_B and, therefore, are affected neither by lobbying nor automation, respectively.

4 Results

In this section, we analyze the quantitative implications of automation and lobbying on wage differentials, technological-knowledge bias, and other variables in the steady-state in subsection 4.2 after presenting and justifying the corresponding calibration of the model in section 4.1. As we have already concluded from Proposition 6, the steady-state value of the Skill Premium is affected neither by γ nor by α_B and, therefore, we do not include this variable in this analysis.

4.1 Calibration

We set $\rho = 0.0101$ and $\theta = 60$ based on Chen *et al.* (2013) and $\beta = 1.4$, $\zeta = 2$ in accordance with the usual practice in the literature (*e.g.*, Jones and Williams 2000; Barro and Sala-i-Martin 2004; Afonso 2006).

In the baseline scenario, we set the relative importance of the non-routine and routine tasks in the economy, measured by χ_A and χ_B, respectively, as equal to 0.4 and 0.6. We consider that these values are consistent with the intuitive notion that most of the tasks required to produce output in the economy are routine-based. Moreover, we set $\epsilon = 0.5$ because we consider that routine and non-routine tasks are part of every sector that complement each other in final outputs. The parameter ξ governs the strength of scale effects in the economy. As we are considering the calibration of a developed economy, we consider that they should be very small or zero – as suggested by Jones (1995a, b) and Peretto (1998). Therefore we set $\xi = 1$.

In what concerns labor parameters, we set $L_A^U = 24521$, $L_B^M = 143405$, $L_A^H = 74213$ based on time averages of working hours of the period 1995-2009 for the United States obtained from the Social Economic Account Database (Timmer *et al.* 2015), which considers 35 sectors of activity and, within each one, three labor types based in the International Standard Classification of Education (ISCED): low-skilled (ISCED categories 1 and 2), medium-skilled (ISCED 3 and 4), and high-skilled (ISCED 5 and 6). We consider the calibration $l_A^U = 10.4549$, $l_B^M = 82.3852$ and $l_A^H = 9.0170$ in order to replicate the steady-state values of the wage differentials and growth rate obtained from the same database, which were $\left(\frac{w_A^U}{w_B^M}\right)^* = 0.6649$, $\left(\frac{w_A^H}{w_B^M}\right)^* = 1.8104$, $g^* = 0.0261$. Finally, in the baseline scenario, we consider that $\alpha_A = \alpha_B = \alpha = 0.6$, which is the labor share of income obtained from the same source.

Table 2: Calibration used in the model

Parameter	Value	Description
ρ	0.0101	Subjective discount rate
θ	60	Inter-temporal elasticity of substitution
ϵ	0.5	Elasticity of substitution between routine and non-routine tasks
β	1.4	Learning-by-past parameter
ζ	2	Fixed cost of innovation
ξ	1	Parameter that determines the presence of scale effects.
χ_A	0.4	Distribution parameter measuring the relative importance of the A-sector
χ_B	0.6	Distribution parameter measuring the relative importance of the B-sector
L_A^U	24521	low-skilled labor supply (in hours)
L_B^M	143405	Medium-skilled labor supply (in hours)
L_A^H	74213	high-skilled labor supply (in hours)
α_s	0.60	Exogenous labor share in sector s
l_A^U	10.4549	Exogenous efficiency parameter of low-skilled labor
l_B^M	82.3852	Exogenous efficiency parameter of Middle-skilled labor
l_A^H	9.0170	Exogenous efficiency parameter of high-skilled labor

4.2 Quantitative Results

We depict the quantitative results of the model in figure 1, where we plot the steady-state value of the variables of interest of the model for different values of α_B. We do so in order to analyze the impacts of automation which we interpret as a decrease of α_B since this parameter, by definition, measures the relative importance of labor in the routine sector. In order to analyze the impacts of lobbying we distinguish between the baseline scenario, where $\gamma = 0$, and an additional scenario where we set $\gamma = 1$ because, by proposition 1, the latter implies the existence of lobbying effort by firms in both sectors.[19]

In the baseline scenario, we can observe that automation in the form of a decrease in α_B contributes to increased wage differentials of low-skilled and high-skilled workers in relation to

[19]In the Appendix A3, for completeness, we present an alternative quantitative exercise in which we compare $\gamma = 0$, and an additional scenario where we set $\gamma = 0.5$.

middle-skilled workers, *i.e.*, wage polarization.[20] This in turn has different impacts on inequality depending on α_B, in the following manner:

1. For a range of values of α_B close to the calibrated value of 0.6 there are opposing effects on inequality. In this scenario, the steady state values for these ratios are, respectively, less and greater than 1, i.e., $\left(w_A^U/w_B^M\right)^* < 1 \wedge \left(w_A^H/w_B^M\right)^* > 1$, Therefore, the relative decrease of wages of middle-skilled workers leads to an increase in w_A^U/w_B^M, which contributes negatively to inequality approximating w_A^U/w_B^M to 1, while an increase in w_A^H/w_B^M has the opposite effect by leading w_A^H/w_B^M further from 1. In this case the ultimate impact on inequality is ambiguous since it will depend on the magnitude of the increase in these ratios and the proportion of low-skilled workers in relation to high-skilled. For example, if the latter is sufficiently high, and the increase in both ratios is not substantially different, the first effect can surpass the second, leading to a decrease in inequality.

2. For sufficiently low values of α_B both wage ratios are superior to 1, i.e., $\left(w_A^U/w_B^M\right)^* > 1 \wedge \left(w_A^H/w_B^M\right)^* > 1$. This implies that an increase of the relative wage of middle-skilled workers contributes positively to inequality.

We now explain the mechanisms driving this effect on the relative wage of middle-skilled workers. A decrease in the labor share of the middle-skilled workers contributes to increasing the jumps in the quality index of machines that complement this type of workers, increasing incentives to innovate in this sector. This leads to a decrease of the technological advantage of sector A over sector B, as is made evident by the continuous decrease of both $\frac{Q_A^U}{Q_B^M}$ and $\frac{Q_A^H}{Q_B^M}$ as α_B decreases, and also of the relative price of output produced in this sector, $\frac{P_A}{P_B}$. Both these outcomes have a negative impact on the relative wage of workers of sector A relative to those in sector B, but, as we explained in proposition 5, relative wages of workers in sector A are also affected directly by a decrease in α_B. This positive effect supplants the former, thus resulting in an increase in the relative wage of workers in sector A. For very low values of α_B, a decrease of this parameter has a negative effect through the technological-knowledge bias as well since, in this, case, despite leading to a lower drop in relative prices, it contributes to an increase in the

[20] This definition of wage polarization is consistent with the definition adopted in the literature (*e.g.*, Autor and Dorn 2013).

technological-knowledge bias that is relatively favorable to sector A.

In the second scenario, where we consider $\gamma = 1$, we can observe that now a decrease of α_B increases the technological advantage of sector A over sector B and leads to a higher drop in relative prices in relation to the baseline model. This can be explained by the fact that a decrease in α_B leads to an increase of the relative profitability of firms in sector A, $\mathcal{R}_A$, which leads to higher relative net benefits from lobbying that cause an increase of the relative profitability of producers of machines in sector A. This, in turn, leads to an increase in the probability of achieving higher qualities, which results in a higher relative technological-knowledge index in sector A relative to sector B. Thus the presence of lobbying within the model is crucial for the oscillatory behavior of the technological-knowledge bias attaining a local maximum for intermediate level of automation. The combined effects on the technological-knowledge bias and relative prices cause relative wages in sector A to increase much slower as α_B declines than in the baseline model. Therefore, the existence of lobbying activities dampens the effects of wage polarization caused by a decrease in the labor share in the routine sector. The alternative exercise for $\gamma = 0.5$, that we present in Appendix A.3, demonstrates that the same qualitative effects we have described concerning the effect of lobbying in the relationship between automation and the relevant variables of the model are still present even when there are lower gains from lobbying and, hence, firms conduct less intensive lobbying efforts.

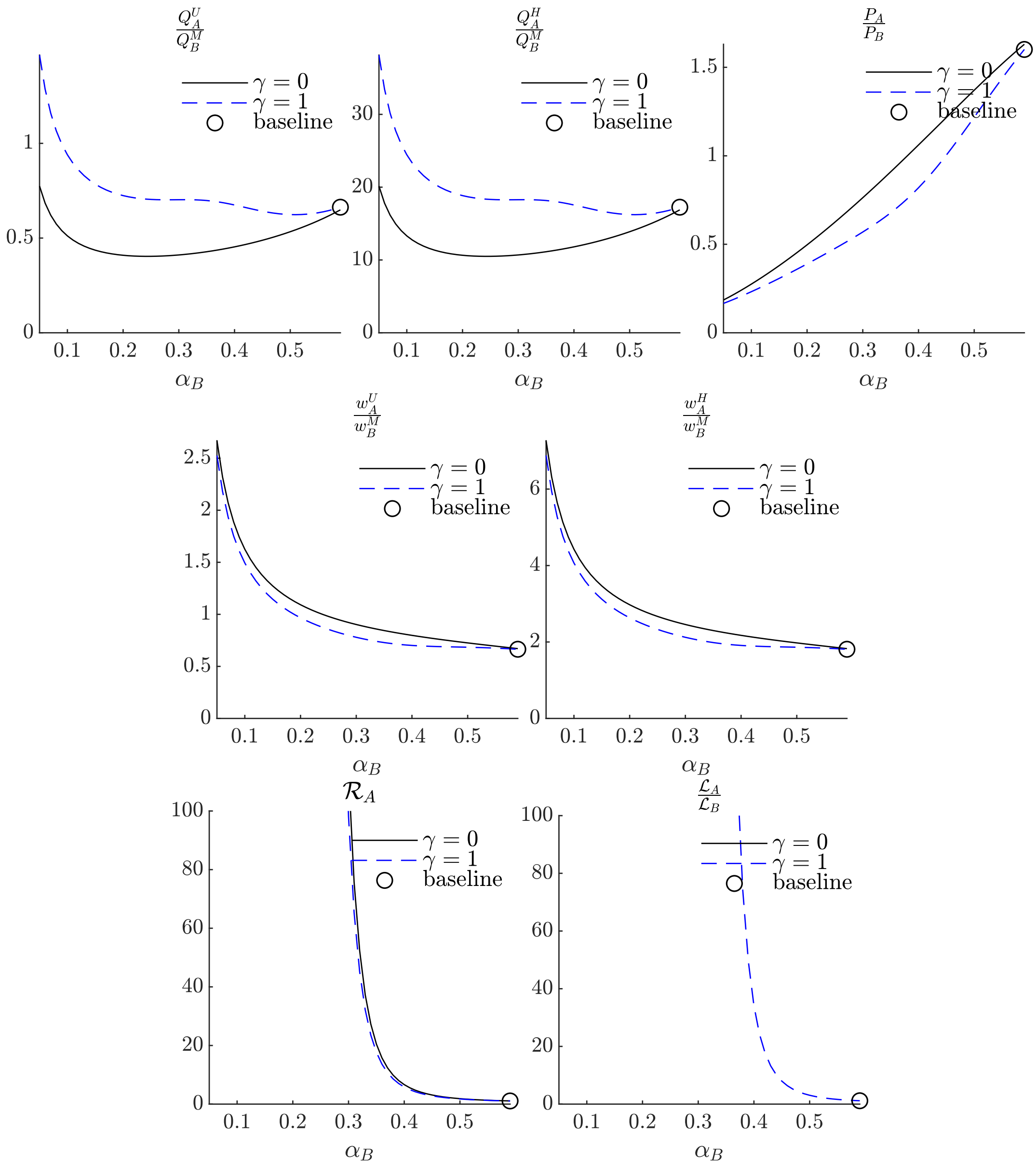

Figure 1: Steady state values of wage differentials and other variables of interest of the model for various values of α_B, considering a scenario without and with lobbying represented, respectively by $\gamma = 0$ and $\gamma = 1$.

5 Concluding remarks

In this paper, we have developed a model that unifies three strands of the literature related to the explanation of wage (or income) inequality. The first is the Directed Technical Change (DTC) literature, which is mainly focused on explaining the skill premium as a result of an increase of the relative supply of high-skilled workers that complements technology that increases their productivity (*e.g.*, Acemoglu 2002). The second consists in task-based modeling approaches that are more concentrated in wage polarization and explain it as a result of, for instance, automation decreasing the relative demand for middle-skilled workers, the ones typically executing tasks that are more susceptible to being automated (*e.g.*, Acemoglu and Autor 2011; Autor and Restrepo 2018). The third is related to the theoretical connection between lobbying and wage inequality found in the literature (*e.g.,* Lesica 2018; Prettner and Rostam-Afschar 2020). With this, we also recognize the importance of non-market factors such as lobbying in shaping inequality and do not neglect the overwhelming evidence of lobbying activities in countries with high wage inequality (*e.g.*, the USA) and the strategic behavior of firms when taking decisions concerning productive and non-productive activities. Moreover, in doing so, we innovate in relation to the extant literature with a perspective that differs completely from the standard approach in this literature, which is usually focused on the impacts on lobbying on inequality through rent-seeking and resulting deviation of resources from productive applications.

Our contributions are twofold. Firstly, we find a positive relationship between the skill premium and the relative supply of high-skilled workers and that an increase in automation, measured by a decrease of the importance of labor in the execution of routine tasks, contributes to an increase of the wage differential of low-skilled and high-skilled workers relative to middle-skilled workers. These findings are in line, respectively, with the DTC and task-based literatures, and demonstrate the validity of the unifying approach we develop. Secondly, we find that an increase in lobbying intensity can actually decrease the effects of the polarization of wages and has no effect on the skill premium.

We acknowledge that in the lobbying setup we make simplifying assumptions of reality, namely that similar firms are those that produce machines to be used by tasks with the same degree of routine intensity. Even so, we consider that this does not jeopardize the validity of our results considering the evidence of a connection between sector of activity, which determines more the similarity of firms, and share of routine/non-routine tasks (Marcolin *et al.* 2019; Keister and Lewandowski 2017). Moreover, by bearing this in mind, we provide a new important perspective on the impacts of lobbying in inequality, which so far, to the best of our knowledge, has not been explored in the literature and contributes to the evolutionary nature of this paper: If opposing lobbies are constituted by firms (*e.g.*, industries) that have different shares of routine-intensive tasks, their lobbying activities can have impacts on relative wages of middle-skilled workers and hence wage polarization. In turn, depending on the relative wages and share of low-skilled workers engaged in non-routine manual tasks, this can either increase or reduce inequality. As an example, lobbying activities conducted by manufacturing firms can result in a decrease of wage polarization due to the high share of routine-intensive tasks that characterizes this sector.

We consider that our findings have some important implications in terms of the policy. Firstly, it reinforces the degree to which automation can increase wage polarization, which highlights the necessity for governments to pay special attention to middle-skilled workers affected by this phenomenon. Secondly, the fact that lobbying attenuates the reduction of the wages of middle-skilled workers caused by automation suggests that governments, when addressing the need to protect real wages of middle-skilled workers, should prioritize sectors with less lobbying since workers in this sector will be more exposed to the negative effects of automation. Moreover, these policies aimed at reducing wage polarization, in turn, should not result in the reduction of growth rates of wages of low-skilled and high-skilled workers but rather on keeping or increasing the growth rates of wages of middle-skilled workers to keep them from falling even further in relation to the earnings of the aforementioned groups.

Finally, we consider that our paper opens interesting avenues for future research to explore. Firstly, it would be interesting to explore the degree to which opposing lobbies differ with respect to the degree to which their activities can be automated. Secondly, more advanced lobbying setups can be made where more complex coalitions are formed, which would be relevant to analyze the extension to which characteristics of lobbies can affect wage polarization. Thirdly, our paper only demonstrated the existence of a qualitative effect of lobbying characterized by a positive impact on relative wages of middle-skilled workers and, therefore, future research could focus on quantifying the magnitude of these effects, by developing more advanced extensions of our model as suggested in the previous point. Fourthly, we do not divide the economy into sectors of activity/industries as non-routine and routine tasks are commonplace through many activities. Nonetheless, we consider that it would be relevant to consider this possibility to analyze the extent to which automation and lobbying can affect inter-sector wage differentials and other variables of interest.

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A Appendix

A.1 Threshold task and labor units in sector A

From the definition of price indexes in (14) we have that (i) $\frac{P^H_{v_A}}{P^U_{v_A}}$ is a continuous function of v_A; (ii) Since $\frac{P^H_A}{P^U_A}$ is assumed to be a positive constant, $\frac{P^H_{v_A}}{P^U_{v_A}}$ varies negatively with v_A, *ceteris paribus*; (iii) $\lim_{v_A\to 1}\frac{P^H_{v_A}}{P^U_{v_A}}=0$; and (iv) $\lim_{v_A\to 0}\frac{P^H_{v_A}}{P^U_{v_A}}=\infty$. Using (i)-(iv) by the Intermediate Value Theorem there is a $\overline{v}_A\in[0,1]$ such that $\frac{P^H_{v_A}}{P^U_{v_A}}=1\Leftrightarrow P^H_{v_A}=P^U_{v_A}$. Moreover by (i) for $v_A>\overline{v}_A$, $P^H_{v_A}<P^U_{v_A}$ and for $v_A<\overline{v}_A$ we have that $P^H_{v_A}>P^U_{v_A}$. Since the output of each variety v_A is produced in perfect competition, firms opt for producing task v_A with the lowest price. Therefore, for $v_A=\overline{v}_A$ they are indifferent between labor types, but for $v_A<\overline{v}_A$ $(v_A>\overline{v}_A)$ they choose L^U_A (L^H_A). Considering (14) and the condition that characterizes the threshold task as $\frac{P^H_{v_A}}{P^U_{v_A}}=1$, we can determine the following expression for the threshold task $\overline{v}_A=\left[1+\left(\frac{P^U_A}{P^H_A}\right)^{\frac{1}{\alpha_A}}\right]^{-1}$.

We now only need to determine the expression for $\frac{P^U_A}{P^H_A}$. Considering that the value of each task produced in sector s is constant for all v_A, we have that (i) $P^i_{v_A}Y^i_{v_A}(t)=\left(P^i_A\right)^{\frac{1}{\alpha_A}}\left(\frac{1-\alpha_A}{p_A}\right)^{\frac{1-\alpha_A}{\alpha_A}}Q^i_A\cdot l^i_A\cdot L^i_{v_A}$ for $i\in\{U,H\}$ are constants which, since the terms $\left(P^i_A\right)^{\frac{1}{\alpha_A}}\left(\frac{1-\alpha_A}{p_A}\right)^{\frac{1-\alpha_A}{\alpha_A}}Q^i_A\cdot l^i_A$ are constants with respect to v_A implies necessarily that $L^i_{v_A}$ are also constants that are defined as $L^U_{v_A}=\frac{L^U_A}{\overline{v}_A}$ and $L^H_{v_A}=\frac{L^H_A}{1-\overline{v}_A}$ as a result of considering that $L^U_A=\int_0^{\overline{v}_A}L^U_{v_A}dv_A$ and $L^H_A=\int_{\overline{v}_A}^1 L^H_{v_A}dv_A$; (ii) $P^U_{v_A}Y^U_{v_A}=P^H_{v_A}Y^H_{v_A}$ from where we derive that $\frac{P^U_A}{P^H_A}=\left(\frac{Q^H_A}{Q^U_A}\frac{l^H_A\cdot L^H_A}{l^U_A\cdot L^U_A}\frac{\overline{v}_A}{1-\overline{v}_A}\right)^{\alpha_A}$. Replacing this expression in the previous we obtain $\overline{v}_A=\left[1+\left(\frac{Q^H_A\cdot l^H\cdot L^H_A}{Q^U_A\cdot l^U\cdot L^U_A}\right)^{\frac{1}{2}}\right]^{-1}$.

A.2 Price indexes of tasks and price of the output in each sector

In this appendix we determine the values for price indexes of tasks produced with each type of labor. We start from $P_s = \exp\left(\int_0^1 \ln P_{v_s} dv_s\right)$, to write, for sector A, $\ln P_A = \int_0^{\overline{v}_A} \ln P_{v_A}^U dv_A + \int_{\overline{v}_A}^1 \ln P_{v_A}^H dv_A$, which by considering $P_A^U = P_{v_A}^U \cdot (1 - v_A)^{\alpha_A}$ and $P_A^H = P_{v_A}^H \cdot (v_A)^{\alpha_A}$ results that $\ln P_A = \int_0^{\overline{v}_A} \ln\left[P_A^U (1 - v_A)^{-\alpha_A}\right] dv_A + \int_{\overline{v}_A}^1 \ln\left[P_A^H v_A^{-\alpha_A}\right] dv_A$. Further developing this expression we obtain $\ln P_A = \overline{v}_A \ln P_A^U + (1 - \overline{v}_A) \ln P_A^H - \alpha\left[\int_0^{\overline{v}_A} \ln(1 - v_A) \cdot dv_A + \int_{\overline{v}_A}^1 \ln v_A \cdot dv_A\right]$. Now, considering that $\int_0^{\overline{v}_A} \ln(1 - v_A) \cdot dv_A = (\overline{v}_A - 1)\ln(1 - \overline{v}_A) - \overline{v}_A$, $\int_{\overline{v}_A}^1 \ln v_A dv_A = -1 - \overline{v}_A \ln \overline{v}_A + \overline{v}_A$, and the relation between price indexes implied by (14) when for the threshold task, *i.e.*, $P_A^H = \left(\frac{\overline{v}_A}{1-\overline{v}_A}\right)^{\alpha_A} P_A^U$, we have that $P_A^U = P_A \cdot \exp(-\alpha_A) \cdot \overline{v}_A^{-\alpha_A}$, and, replacing in the relation between price indexes, we obtain $P_A^H = P_A \cdot \exp(-\alpha_A) \cdot (1 - \overline{v}_A)^{-\alpha_A}$.

In the case of sector B, from the profit maximization problem of the producer of Y_B we have that $P_B Y_B = P_{v_B} Y_{v_B}$, which implies that $L_B^M = \int_0^1 L_{v_B}^M dv_B = L_{v_B}^M$. In turn, $P_{v_B} Y_{v_B} \equiv \left(P_{v_B}^M\right)^{\frac{1}{\alpha_B}} \cdot \left(\frac{1-\alpha_B}{p_B}\right)^{\frac{1-\alpha_B}{\alpha_B}} \cdot Q_B^M \cdot l_B^M \cdot L_B^M$ is a constant with respect to v_B and since $\left(\frac{1-\alpha_B}{p_B}\right)^{\frac{1-\alpha_B}{\alpha_B}} \cdot Q_B^M \cdot l_B^M \cdot L_B^M$ are also constants with respect to this variable, it follows that P_{v_B} does not vary with v_B, from which it results that $P_B = P_{v_B}$. Finally, we have $P_B = \int_0^1 P_{v_B} dv_B$ which, bearing in mind the previous result, implies that that $P_B = P_{v_B}$.

A.3 Alternative Calibration Exercise

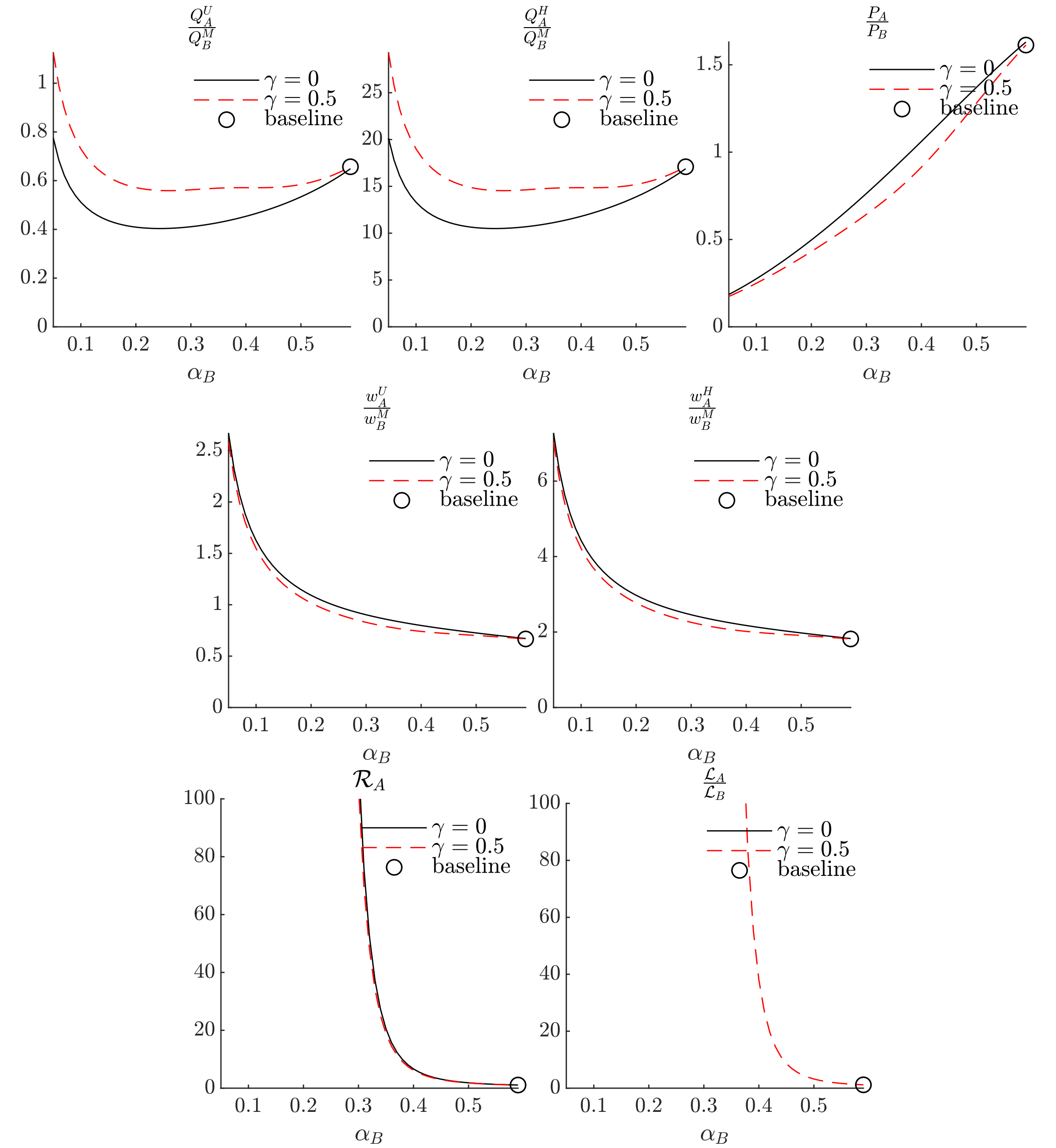

Figure 2: Steady state values of wage differentials and other variables of interest of the model for various values of α_B, considering a scenario without and with lobbying represented, respectively by $\gamma = 0$ and $\gamma = 0.5$.