Life-cycle asset allocation and the Peso Problem: Does ambiguity aversion matter?

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Abstract

The vast majority of previous studies on life-cycle consumption and asset allocation assumes that the equity premium is constant. In this paper, we evaluate the impact of rare disasters that shift the stock market to a low return state on investors' consumption and portfolio decisions. We assume that investors are averse to ambiguity relative to the current state of the economy and must incur a per period cost to participate in the stock market and solve their optimal consumption and asset allocation problem using dynamic programming. We aim to show that ambiguity aversion exerts a non-negligible effect on the investors' decisions, especially due to the possibility of sharp declines in stock prices. Our results show that most young investors choose not to invest in stocks because they have low accumulated wealth and the potential return from their stock market investments would not cover the participation costs. Furthermore, ambiguity averse investors hold considerably fewer stocks throughout their lifetime than ambiguity neutral ones. The fraction of wealth invested in stocks over the typical consumer's life is hump-shaped: it is low for a young individual, peaks at his early thirties, and then decreases until his retirement age. To the best of our knowledge, this is the first study that assesses the impact of negative stock price jumps on the optimal portfolio of an ambiguity averse investor.

JEL Classification: G11, D91, G01

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1. Introduction

Numerous financial researchers and practitioners addressed the issue of life-cycle allocation and portfolio selection. In a classical article, Samuelson (1969) shows that if returns are independent and identically distributed, preferences are homothetic and there is no labor income, then asset allocation should be constant over time. This problem becomes substantially more complex when the individual earns a salary, because the optimal asset allocation depends on the profile of labor income, as Bodie et al. (1992) show. According to these authors if labor income is riskless, then human capital is equivalent to an implicit investment in the riskless asset, and the worker should tilt his portfolio towards stocks. On the contrary, if labor income is strongly correlated with the risky asset return, then the worker should increase his holdings of the riskless asset. Viceira (2001) extends this model to an infinite horizon setting. He shows that if labor income is idiosyncratic, then workers should increase their holdings of stocks, especially if their expected retirement horizon is long. Cocco et al. (2005) develop a realistically calibrated life-cycle model of consumption and asset allocation with non-tradeable labor income. They find that labor income is almost uncorrelated with stock market returns, which implies that young workers, whose human capital is large and liquid wealth is low, should be fully invested in stocks. This prediction contrasts strongly with Ameriks and Zeldes (2000) who show that real-world young US workers hold almost no stocks. They also find a hump-shaped pattern in stock holdings over the life-cycle: the fraction of wealth invested in stocks is low for a young individual, it peaks for a middle-aged worker, and then decreases until retirement.

Several researchers provide explanations for the discrepancy between the investor behavior predicted in Cocco et al. (2005) and the observed investment pattern of US workers. Benzoni et al. (2007) argue, and provide some statistical evidence, that labor income and stock market returns are cointegrated, which implies that young workers are strongly exposed to stocks market shocks and choose to decrease their stock holdings. In a related article, Lynch and Tan (2011) argue that young investors should hold few stocks because labor income and stock markets are correlated at business cycle frequency. A different approach to explain this phenomenom was followed by Campanale (2011) and Peijnenburg (2018), who recognize that the true value of the equity premium is unknown and must be learned from observed returns. Assuming that the investor is averse to ambiguity about the equity premium and that this aversion may be modeled through a non-smooth utility function as in Gilboa and Schmeidler (1989), these authors can rationalize the moderate stock market investment reported in Ameriks and Zeldes (2000). Fagereng et al. (2017) propose an extension of the Cocco et al. (2005) model, which includes a per period participation cost and a small probability of a large stock market drop to replicate the investment pattern of Norwegian households.

Infrequent calamities, such as wars and the Great Depression, have wide and deep economic impacts, ranging from an increase in the unemployment rate and GDP contraction to a bust in asset prices. Usually, these periods are associated with stock market contractions (Barro, 2006, Berkman et al., 2011) driven by poor economic conditions. Thus, investors are legitimately concerned about the possibility a rare disaster occurs (Choi and Robertson, 2020) and should adapt their strategies accordingly. These phenomena have been successfully used to explain several asset-pricing puzzles, such as the equity premium and the risk-free rate ones (Barro, 2009, Liu et al., 2020, Tsai and Wachter, 2015, Wang and Mu, 2017). Alan (2012), using an investment framework featuring a non-ambiguity averse investor, shows that expectations of rare disasters can partially explain the investment pattern of US individuals who have less than a college degree.

Wars, natural catastrophes, and other disasters are sparsely distributed over time. Thus, they are inherently ambiguous, as investors find it difficult to assign probabilities to the occurrence of these events. Ellsberg (1961), in his seminal article, reports that individuals exhibit ambiguity aversion, as they prefer to incur risks they can quantify than risks whose probabilities of occurrence are unknown. The hypothesis people show aversion to unknown scenarios was confirmed in several studies using different experimental designs. In the last decades, the researchresearch focus shifted to the determinants of ambiguity aversion, such as personal traits (Chew et al., 2012) and gender. Regarding gender, there is no consense about its impact on ambiguity aversion: while many studies find that women are more ambiguity averse than men (Chew et al., 2012, Moore and Eckel, 2003, Powell and Ansic, 1997, Schubert et al. 1999), particularly in the gain dimension, other reach the opposite conclusion (Borghans et al., 2009, Friedl et al., 2017).

In this study, we offer an alternative explanation for the moderate stock market investment observed in the real world. Inspired by Barro (2006), who shows economies face rare disasters, such as wars and the Great Depression, that lead to severe stock market contractions, we modify the Cocco et al. (2005) model by assuming that the equity premium follows a hidden Markov chain. We assume investors do not know the true stock market state and are ambiguity averse. Unlike Campanale (2011) and Peijenburg (2018), we adopt a smooth recursive ambiguity aversion utility function as in Hayashi and Miao (2011) and Ju and Miao (2012). We think this utility function provides a more reasonable description of investor behavior than the Gilboa and Schmeidler (1989) one because it makes the investors' decisions depend on all the possible stock market returns and not only on the worst possible state. Assuming the investor faces a per-period participation cost, our model generates a reasonable hump-shaped life-cycle investment pattern that is compatible with the behavior of real-world households.

2. The Model

Our model is a modified version of Cocco et al. (2005) which incorporates both regime changes in the process driving stock market returns and ambiguity aversion. We also assume that the worker faces a per period cost to participate in the stock market.

2.1 Labor income

We consider an individual who starts working at age 20 and retirees at age 65. Following Carroll (1997) and Cocco et al. (2005) we assume that worker i's labor income at year t is subject to both temporary and permanent shocks:

$$log(Y_{i,t}) = f(t, Z_{i,t}) + v_{i,t} + \varepsilon_{i,t} \tag{1}$$

The deterministic component, $Z_{i,t}$, is a vector of individual characteristics, $\varepsilon_{i,t}$ is a zero mean normal shock with variance σ_{ε}^{2}, and $v_{i,t}$ is a persistent shock that follows a random walk:

$$v_{i,t} = v_{i,t-1} + u_{i,t} \tag{2}$$

where $u_{i,t}$ is uncorrelated with $\varepsilon_{i,t}$ and is distributed as $N(0, \sigma_u^2)$.

During retirement, the investor receives a deterministic income which is a constant fraction, $\emptyset_r$, of the labor income in the last year of his working life.

$$log(Y_{i,t}) = log(\emptyset_r) + f(t, Z_{i,65}) + v_{i,65} \tag{3}$$

2.2 Financial assets

We assume that the individual can invest in two assets: a riskless asset that offers a constant gross real return R_f, and a risky asset (stocks). The real return on stocks, R_t, depends on the state of the economy

$$R_{t+1} = R_f + \mu_{S_{t+1}} + \eta_{t+1} \tag{4}$$

where $\eta_{t+1} \sim N\left(0, \sigma^2_{S_{t+1}}\right)$ is an innovation to expected returns which is assumed to be uncorrelated with labor income shocks, and S_{t+1} represents the state of the economy in year t+1. S_t is modeled as a two-state Markov chain with transition probabilities

$$p_{11} = prob(S_{t+1} = 1 | S_t = 1) \tag{5}$$

$$p_{22} = prob(S_{t+1} = 2 | S_t = 2) \tag{6}$$

Investors do not observe the true state of the economy and must form an expectation about it based on past returns. Let $P^i_{1,t|t}$ and $P^i_{2,t|t}$ represent the prior probability, for investor i, that the economy is in state 1 and 2, respectively, given the parameter vector Θ, and the stock returns up to year t-1, Υ_{t-1} . After observing the stock return in year t, r_t, investors update their beliefs according to Bayes rule

$$P^i_{1,t|t} \equiv P(s_t = 1 | \Upsilon_t, \Theta) = \frac{P(s_t = 1 | \Upsilon_{t-1}, \Theta) \times Nd(r_t | s_t = 1, \Theta)}{Nd(r_t | s_t = 1, \Theta) + Nd(r_t | s_t = 2, \Theta)} \tag{7}$$

$$P^i_{2,t|t} \equiv P(s_t = 2 | \Upsilon_t, \Theta) = \frac{P(s_t = 2 | \Upsilon_{t-1}, \Theta) \times Nd(r_t | s_t = 2, \Theta)}{Nd(r_t | s_t = 1, \Theta) + Nd(r_t | s_t = 2, \Theta)} \tag{8}$$

where $Nd(r_t | s_t = 1, \Theta)$ and $Nd(r_t | s_t = 2, \Theta)$ represent the normal conditional densities of r_t given that the economy is in states 1 and 2, respectively. The perceived probabilities that the economy will be in the first and second states state in year t+1 are

$$P^i_{1,t+1|t} \equiv P(s_{t+1} = 1|\Upsilon_t, \Theta) = P_{1,t|t} \times p_{11} + P_{2,t|t} \times p_{21} \tag{9}$$

$$P^i_{2,t+1|t} \equiv P(s_{t+1} = 2|\Upsilon_t, \Theta) = P_{2,t|t} \times p_{22} + P_{1,t|t} \times p_{12} \tag{10}$$

where p_{21} represents the transition probability from state 2 to state 1 and p_{12} is the transition probability from the first to the second state.

2.3 Preferences

Investors are averse to ambiguity relative to the hidden state of the economy. We adopt the Miao and Hayashi (2011) and Ju and Miao (2012) recursive smooth ambiguity utility, which achieves separation between ambiguity aversion and risk aversion in an intertemporal setting. Let the hidden state s have a prior distribution p. Then, the consumer intertemporal utility over an adapted consumption plan, C=$(C_t)_{t\geq 0}$, is

$$J_t(C) = \left[C_t^{1-\rho} + \beta\left\{v^{-1}E_{p_t}v\; o\; u^{-1}E_{\pi_{s,t}}\left[u\big(J_{t+1}(C)\big)\right]\right\}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{11}$$

where

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \qquad \gamma > 0, \qquad \gamma \neq 1 \tag{12}$$

$$v(x) = \frac{x^{1-\eta}}{1-\eta} \qquad \eta > 0, \qquad \eta \neq 1 \tag{13}$$

, π_{s_t} is the probability distribution of the remaining parameters that characterize the economy, given that it is in state s_t, β is the discount factor, ρ is the reciprocal of the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, and η is the coefficient of relative ambiguity aversion. The investor exhibits ambiguity aversion if and only if η > γ.

This utility function subsumes, as particular cases, the Epstein and Zin (1989, 1991) recursive utility function, if η = γ, and the power utility function, if ρ = η = γ.

2.4 Optimization problem

We consider the consumption and asset allocation problem of an individual that starts working at age 20 and retirees at 65. We assume that he faces a cost if he wishes to invest in the stock market. This cost can include brokerage and commission fees, bid-ask spreads, and time spent gathering information about the stock market and filling tax forms.

The investor's next period wealth, $W_{i,t+1}$, before receiving period t+1 labor income is:

$$W_{i,t+1} = R^p_{i,t+1}\left(W_{i,t} + Y_{i,t} - C_{i,t}\right) - \mathbb{I}_{i,t+1} \times q \quad (14)$$

, where $\mathbb{I}_{i,t+1}$ is an indicator variable that equals one if the worker participates in the stock market in year t+1, q is the per year participation cost, and $R^p_{i,t+1}$ is the portfolio return given by:

$$R^p_{i,t+1} \equiv \alpha_{i,t} R_{i,t+1} + \left(1 - \alpha_{i,t}\right) R_F \quad (15)$$

We assume the investor can neither borrow against future labor income nor short-sell stocks. Thus, the fraction of wealth invested in the risky asset, $\alpha_{i,t}$, must be comprised between 0 and 1.

The indirect utility function of a worker who chooses to hold stocks follows from equations (11) to (13) and the labor and asset markets dynamics:

$$V_{i,t}^{in}\left(X_{i,t}, Y_{i,t}, P_{1,t|t}^{i}\right) = \max_{C_{i,t}\geq 0, 0\leq\alpha_{i,t}\leq 1} \left[C_{it}^{1-\rho} + \delta \times p_{Surv_{t+1}} \left\{ p_{1,t+1|t}^{i} \left(E_{1,t}\left[V_{T+1}^{1-\gamma}\left(X_{i,t+1}, Y_{i,t+1}, P_{1,t+1|t+1}^{i}\right)\right]\right)^{\frac{1-\eta}{1-\gamma}} + \left(1 - p_{1,t+1|t}^{i}\right)\left(E_{2,t}\left[V_{T+1}^{1-\gamma}\left(X_{i,t+1}, Y_{i,t+1}, P_{1,t+1|t+1}^{i}\right)\right]\right)^{\frac{1-\eta}{1-\gamma}} \right\}^{\frac{1-\rho}{1-\eta}} \right]^{\frac{1}{1-\rho}} \tag{16}$$

, where $X_{it} = W_{i,t} + Y_{1,t}$ represents the *cash-on-hand* in period t, δ is the pure time discount factor, $p_{Surv_{t+1}}$ is the conditional probability that the worker is alive at year t+1, given that he was alive at year t, and $E_{1,t}$ and $E_{2,t}$ denote the expectation given that the economy is in states 1 and 2, respectively.

The indirect utility function for a worker who does not invest in the stock market is given by:

$$V_{i,t}^{out}\left(X_{i,t},Y_{i,t},P_{1,t|t}^{i}\right)=\max_{C_{i,t}\geq 0}\left[C_{it}^{1-\rho}+\delta\times p_{Surv_{t+1}}\left\{p_{1,t+1|t}^{i}\left(E_{1,t}\left[V_{T+1}^{1-\gamma}\left(X_{i,t+1},Y_{i,t+1},P_{1,t+1|t+1}^{i}\right)\right]\right)^{\frac{1-\eta}{1-\gamma}}+\left(1-p_{1,t+1|t}^{i}\right)\left(E_{2,t}\left[V_{T+1}^{1-\gamma}\left(X_{i,t+1},Y_{i,t+1},P_{1,t+1|t+1}^{i}\right)\right]\right)^{\frac{1-\eta}{1-\gamma}}\right\}^{\frac{1-\rho}{1-\eta}}\right]^{\frac{1}{1-\rho}} \tag{17}$$

Finally, the Bellman equation for the investor leads to the optimal participation decision:

$$V_{i,t}\left(X_{i,t},Y_{i,t},P_{1,t|t}^{i}\right)=max\left(V_{i,t}^{in}\left(X_{i,t},Y_{i,t},P_{1,t|t}^{i}\right),V_{i,t}^{out}\left(X_{i,t},Y_{i,t},P_{1,t|t}^{i}\right)\right) \tag{18}$$

2.5 Solution method

This problem does not have a closed-form solution. Thus, we must resort to numerical methods to derive the optimal policies.

First, following Cocco et al. (2005), we use the scaleability of the utility function which allows us to normalize v_{it} to one, to reduce the dimension of the state-space. Then, we discretize the state-space of the remaining state variables, cash-on-hand and the probability that the economy is in the first state, as in Tauchen and Hussey (1991), and we approximate the density functions of the shocks using Gaussian quadrature methods. The optimal policy in the last period is trivial- the worker consumes all his wealth. Using the last period value function, we compute the optimal policies in the previous year. To avoid numerical convergence problems, we discretize the decision variables space and use standard grid search to find the optimal policies. We use bi-cubic spline interpolation to evaluate the value function for values of the state variables that do not lie in the grid.

This procedure is repeated until the first period of the investor working life. Finally, we simulate an artificial panel of 50000 agents and save their optimal policies.

2.6 Parametrization

We choose the same labor income specification as Cocco et al. (2005). The deterministic component of labor income follows a third-order polynomial, and the variances of permanent and temporary shocks are 0.0106 and 0.0738, respectively. The conditional survival probabilities are obtained from the mortality tables of the National Center for Health Statistics. The remaining parameter values are presented in table 1.

We set the coefficient of relative ambiguity aversion, η, to 80. Chen et al. (2014) show that this value implies an ambiguity premium of 16.9% and 8.7% of the expected value of the bet, considering bets that represent 1% and 0.5% of the investor's wealth. Camerer (1999) reports that an ambiguity premium of 10% to 20% is reasonable in Ellsberg Paradox type experiments.

[Insert Table 1 around here]

Our choice of the stock return parameters is based on Barro (2006). This author reports 27 rare disasters, whose duration ranges from 2 to 9 years. The weighted average annual equity return is close to -15%. We set the average equity premium in the second state to -15%, and p_{22} to 0.7, which implies an expected duration of 3.33 years for the second regime. The transition probability from state 1 to state 2 equals 1.5%, and the expected return in the first state is set to a value that leads to an unconditional equity premium of 4%. The standard deviation of stock returns equals 16% in both states.

The per period stock market participation cost is set to 200 USD, which is compatible with the 50 to 350 USD implied participation cost estimated by Vissing-Jorgensen (2002), using Euler equation estimation methods.

The remaining parameters assume the same values as in Cocco et al. (2005).

3. Simulation results

In this section, we present the average consumption and portfolio choices across the 50000 simulated paths and compare them to the average policies in two alternative scenarios: no ambiguity aversion and no rare disaster.

Figure 1
Financial wealth threshold of stock market participation, with and without ambiguity aversion, for several ages
[Insert Figure 1 around here]

Figure 1 displays the financial wealth threshold of stock market participation, for both ambiguity averse and ambiguity neutral agents, at ages 25, 45, 65, 85, as a function of the perceived probability that the stock market will be in a low return state in the next year. No worker will ever invest in the stock market when the probability of a low return exceeds 25%, because the expected equity premium would be negative. At all ages, the financial wealth required to participate in the stock market is higher for an ambiguity averse investor than for an ambiguity neutral one. This pattern can be explained by the fact that an ambiguity averse investor makes decisions as if he had distorted believes, which attribute a higher weight to the low return state, relative to a Bayesian investor (see Chen et al. (2014)). The financial wealth threshold is U-shaped over the life-cycle, as in Fagereng et al. (2017). For a young worker, the wage is low and consumption tracks labor income closely. Thus, he is only willing to risk his savings in the stock market if his financial wealth buffer is sufficiently high. The threshold is also higher for an old agent relative to a middle-aged worker, because the expected lifetime of the former may be insufficient to recover from a stock market bust.

Figure 2
Stock market participation
[Insert Figure 2 around here]
Figure 3
Portfolio share invested in stocks
[Insert Figure 2 around here]

Figures 2 and 3 show the participation rates and the proportions of financial wealth invested in stocks over the life-cycle. The participation rate is low for a young worker in all the scenarios because his accumulated financial wealth is also low. Thus, the expected return from holding stocks is insufficient to pay the participation cost. As the agent ages, he accumulates wealth and the participation rates increases to the 60%-70% level during his working life. During the retirement years, the participation rate decreases due to the combination of a decrease in wealth and in the expected remaining lifetime. The benchmark scenario generates lower participation rates over the life-cycle than both the no ambiguity aversion case (1% to 13% lower) and the constant risk premium case (2% to 14% lower), which is consistent with the financial wealth threshold for stock market participation depicted in figure 1. The fraction of wealth invested in stocks presents a hump-shaped pattern: it is low for a very young worker, peaks around 60% in the benchmark case for a worker in his early thirties, and the decreses throughout the remainder of his lifetime. It is also noticeable that ambiguity aversion causes a substantially higher reduction in stock allocation relative to the constant risk premium scenario (3% to 15%) than the mere possibility of rare disasters (less than 4%).

Wealth evolution over the life-cycle (figure 4) displays a pattern similar to the one reported in Cocco et al. (2005). Wealth accumulates slowly during the early years of the agent working life because consumption follows labor income closely, then he starts accumulating wealth faster, as his labor income increases, and, finally, in the retirement years wealth declines as he draws wealth to pay for his consumption. The agent accumulates less wealth in the benchmark scenario relative to the constant equity premium case because the allocation to stocks is lower in the former case, which generates a lower return on wealth.

Figure 4
Financial wealth
[Insert Figure 4 around here]

3.1 Sensitivity analysis

In this subsection, we evaluate the sensitivity of our results to several parameters changes, namely, risk aversion, ambiguity aversion, participation cost, the elasticity of intertemporal substitution, risk premium, the standard deviation of stock returns and regime duration.

Figures 5 and 6 show the participation rates and the fraction of wealth invested in stocks for different risk aversion levels. A decrease in risk aversion causes an increase in stock market participation of young workers but generates a higher participation rate for middle-aged and old workers. This apparent contradiction is explained by the fact that low risk-aversion investors accumulate wealth more rapidly, because they invest a larger fraction of their wealth in the stock market. This pattern reverses again for very old individuals because, as they draw down their wealth rapidly risk aversion becomes the determinant factor for stock market participation. The fraction of wealth invested in stocks is monotonously decreasing in risk aversion, especially for old agents.

Figure 5
Stock market participation for different risk aversion levels
[Insert Figure 5 around here]

Figure 6
Portfolio share invested in stocks for different risk aversion levels
[Insert Figure 6 around here]

The effect of a decrease in ambiguity aversion in the participation rates (figure 7) is similar to the effect of a reduction in risk aversion: it increases participation rates for very young and old individuals and increases it for middle-aged workers. A lower ambiguity aversion leads to an increase in stock investment (figure 8) that is more uniformly distributed over the life-cycle than the one that results from a decrease in risk aversion.

The participation cost is a fundamental variable driving portfolio choice, particularly for young investors. Figures 9 and 10 display the portfolio choices for different participation costs. A decrease (an increase) in the participation cost causes a sizeable increase (reduction) in stock market participation, particularly for young investors. Then, its impact becomes less relevant for middle-aged and old workers because their higher wealth reduces the relative weight of participation costs on the expected portfolio return. Finally, it increases again for retired investors as they gradually decrease their wealth. As expected, a higher participation cost decreases stock allocation, especially for young investors.

Figure 9
Stock market participation with several participation costs
[Insert Figure 9 around here]

Figure 10
Portfolio share invested in stocks with several participation costs
[Insert Figure 10 around here]

Table 2 shows the participation rate and the fraction of wealth invested in stocks at several ages, in the benchmark case and is four alternative scenarios. A higher elasticity of intertemporal substitution (lower ρ) leads a young worker to decrease his consumption. Thus, he accumulates wealth more rapidly, and both his participation rate and stock investment increase. This pattern is reverted as he ages: an old individual with a higher elasticity of intertemporal substitution invest substantially less in stocks than in the benchmark case. The third column displays the optimal portfolio decision of an investor who expects the equity premium to be lower. Even though a 3% equity premium may seem to low compared to the historical performance of the US equity markets, it should be noted that individuals base their investment decisions on the expected equity premium which, according to several authors such as Claus and Thomas (2001) and Arnott and Bernsetein (2002) is quite lower than 4%. A 1% decrease in the expected equity premium leads to a considerable reduction in both participation and allocation to stocks for young investors, but its impact becomes more moderate as he ages. An increase in the standard deviation of stock returns to 20% generates similar changes, both in direction and magnitude. The final column of table 2 shows the optimal investor decisions when the expected regime durations are longer, and the transition probability to the low return state is lower. For these transition probabilities the participation rates and the proportion of financial wealth invested in stocks are slightly higher than in the benchmark case. That is, agents are more sensitive to the decrease in the probability of a low return state than to the increase in its duration.

[Insert Table 2 around here]

4. Concluding remarks

In this paper, we developed a realistically calibrated life-cycle consumption and asset allocation model that features both the possibility of rare disasters and participation costs. We showed that an ambiguity averse worker invests a moderate fraction of his wealth in stocks throughout his lifetime. Participation costs are crucial to restrain stock market investments of young workers, whose wealth is low, and ambiguity aversion causes a sensible reduction in stock investment relative to the framework in which the agent is ambiguity neutral.

Several empirical studies reported that many individuals tend to exhibit aversion to ambiguity in a wide variety of situations. Thus, it would certainly be interesting to evaluate the impact of recognizing the ambiguity of other model parameters, besides the equity premium, on investors' optimal choices.

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Table 1

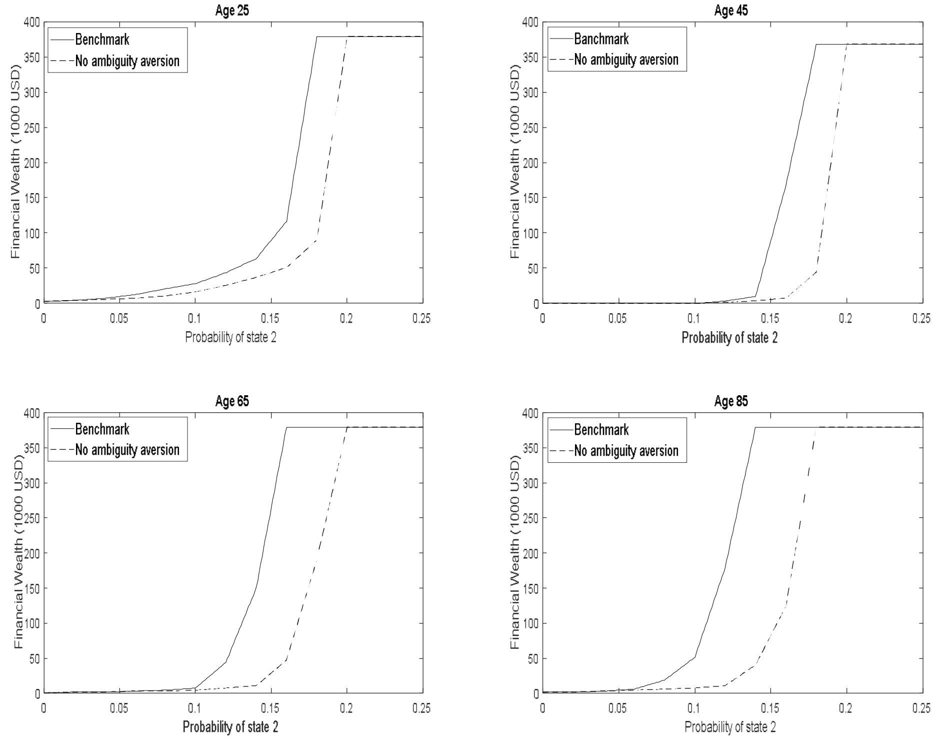
Benchmark parameter values

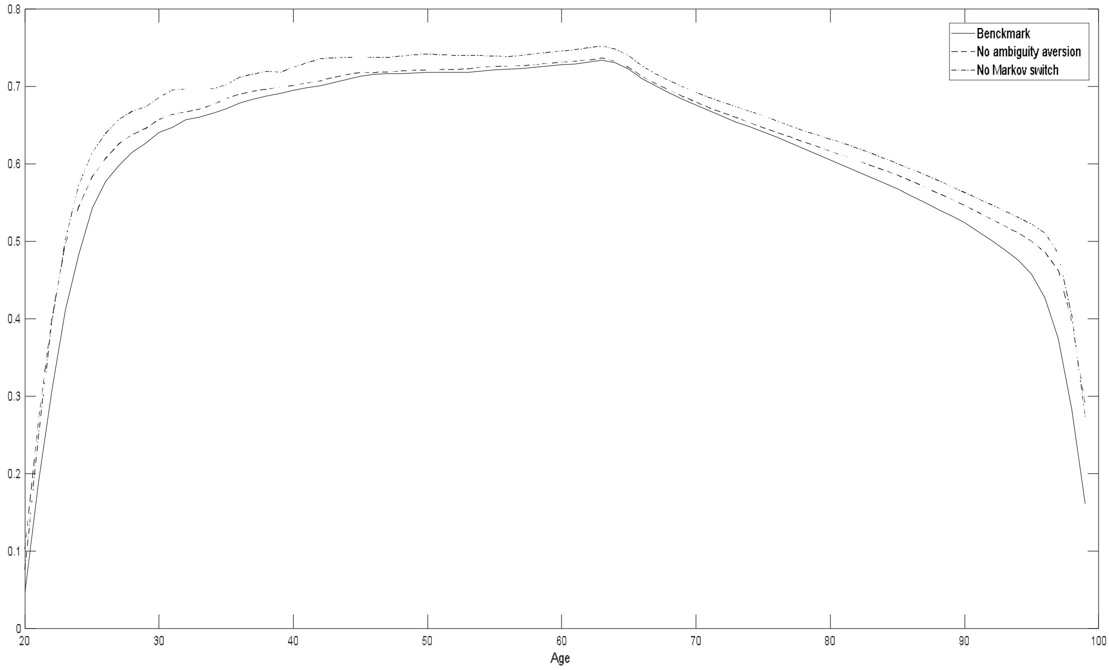
Parameter	Value
Retirement age	65
Replacement ratio ($\emptyset_r$)	0.68212
Discount factor (δ)	0.96
Risk Aversion (γ)	10
Inverse of EIS (ρ)	10
Ambiguity aversion (η)	80
Riskfree rate (R_F-1)	0.02
Equity premium in the first state (μ_1)	0.049496
Equity premium in the second state (μ_2)	-.15
Transition probability state 1→ state 1 (p_{11})	0.985
Transition probability state 2→ state 2 (p_{22})	0.7
St. deviation stock returns in first state (σ_1)	0.16
St. deviation stock returns in second state (σ_2)	0.16
Participation cost (q)	200

Table 2

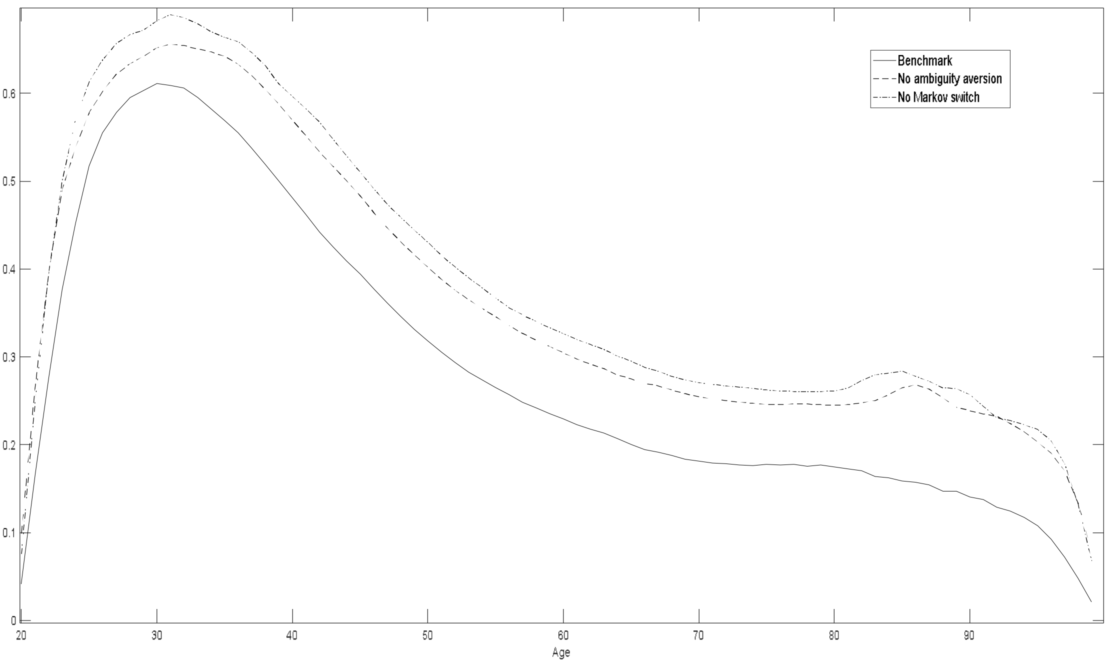
Participation rates (left number on each cell) and portfolio share invested in stocks (right number on each cell), in percentage points, for several alternative scenarios: Inverse of the elasticity of intertemporal substitution equal to five ($\rho = 5$); 3% equity premium (E.P. = 3%); stock return standard deviation equal to 20% (S.D. = 20%); probabilities of remaining in states 1 and 2 equal to 99% and 80%, respectively ($p_{11} = 0.99$, $p_{22} = 0.8$).

Scenario → Age↓	Benchmark	$\rho = 5$	E.P. = 3%	S.D. = 20%	$p_{11} = 0.99$ $p_{22} = 0.8$
25	54.9/52.1	58.4/57.4	36.9/32.2	41.8/32.2	56.6/54.4
35	67.1/57	65.9/56.6	62.4/50.4	65.4/50.9	67.8/59.1
45	71.3/39.3	68.5/39.7	68/34.6	69.8/33.8	70.8/40.8
55	72.3/26.6	69.5/27.9	70/23.6	71/22.6	72.1/27.9
65	72/19.6	68.5/20.7	69.5/16.8	70.2/15.78	71.6/20.8
75	64.2/17.7	59.2/19.5	61/15.9	61.7/14.1	63.8/18.5
85	56.7/16.1	50.2/15.6	53/12.8	54.4/11.7	56.7/17.1
95	46.1/11	37.8/10.3	38.1/6.6	40.7/6.9	46.9/12.7

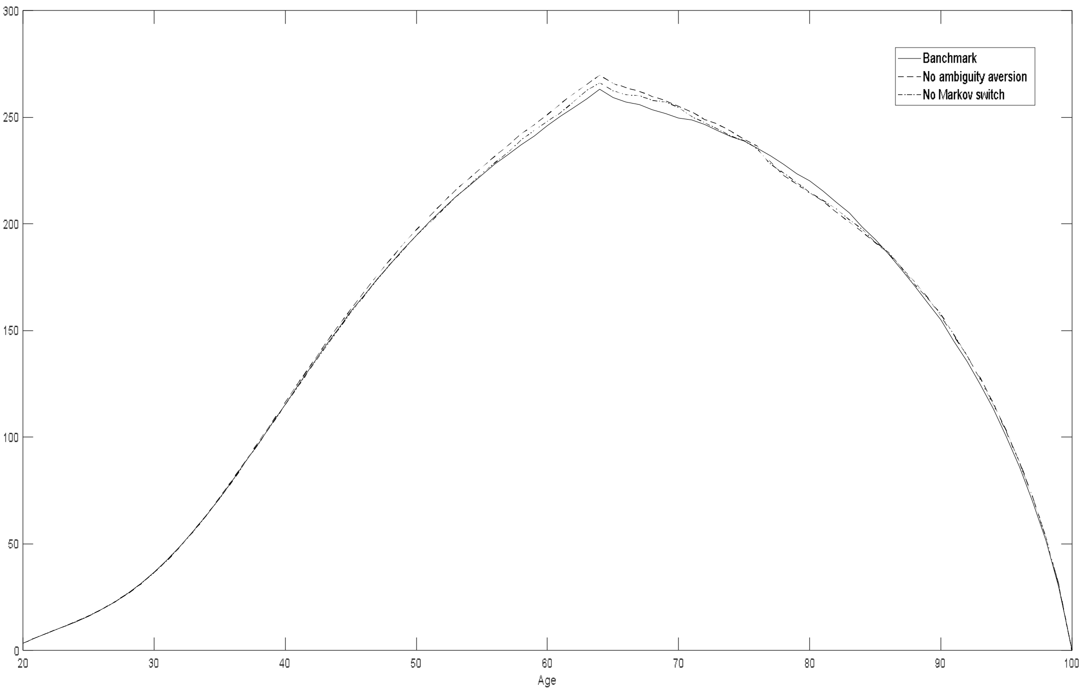

Age 25
Age 45
Age 65
Age 85
Benchmark
No ambiguity aversion
Banchmark
Financial Wealth (1000 USD)
Probability of state 2
0
0.05
0.1
0.15
0.2
0.25
0
50
100
150
200
250
300
350
400

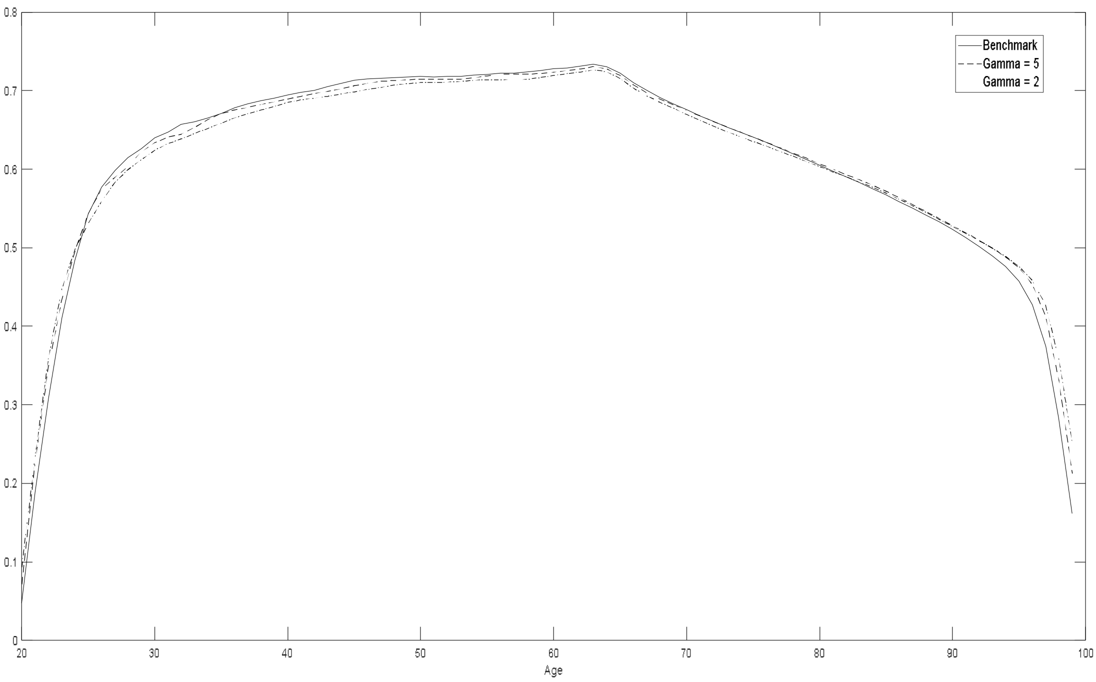


Benckmark
No ambiguity aversion
No Markov switch
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90
100
Age

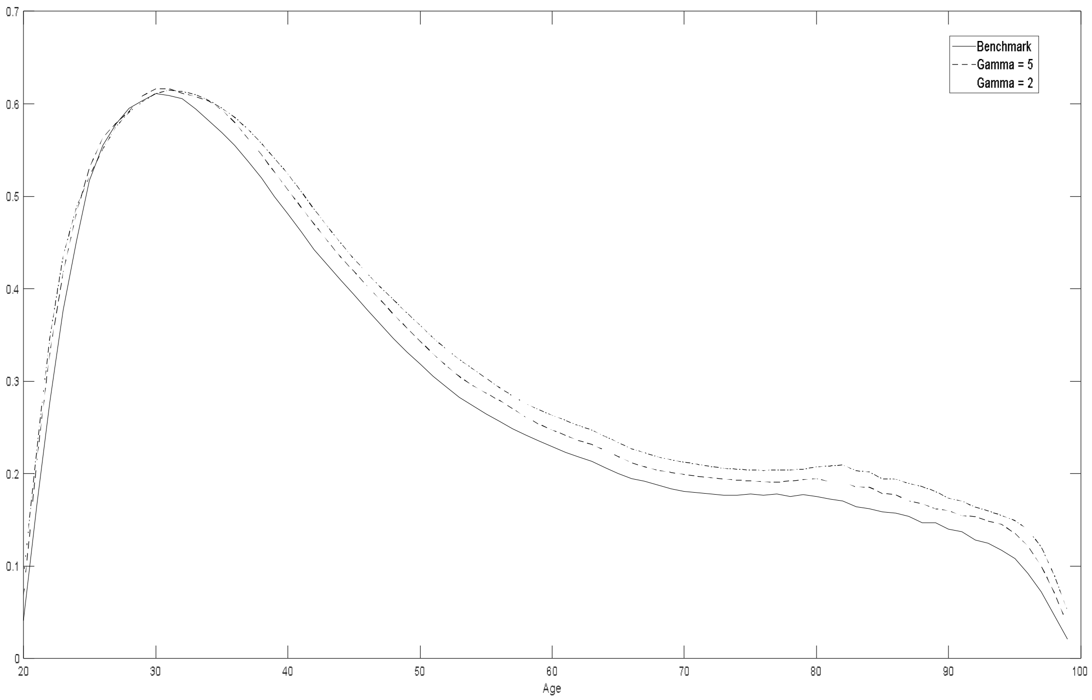


Benchmark
No ambiguity aversion
No Markov switch
0
0.1
0.2
0.3
0.4
0.5
0.6
20
30
40
50
60
70
80
90
Age

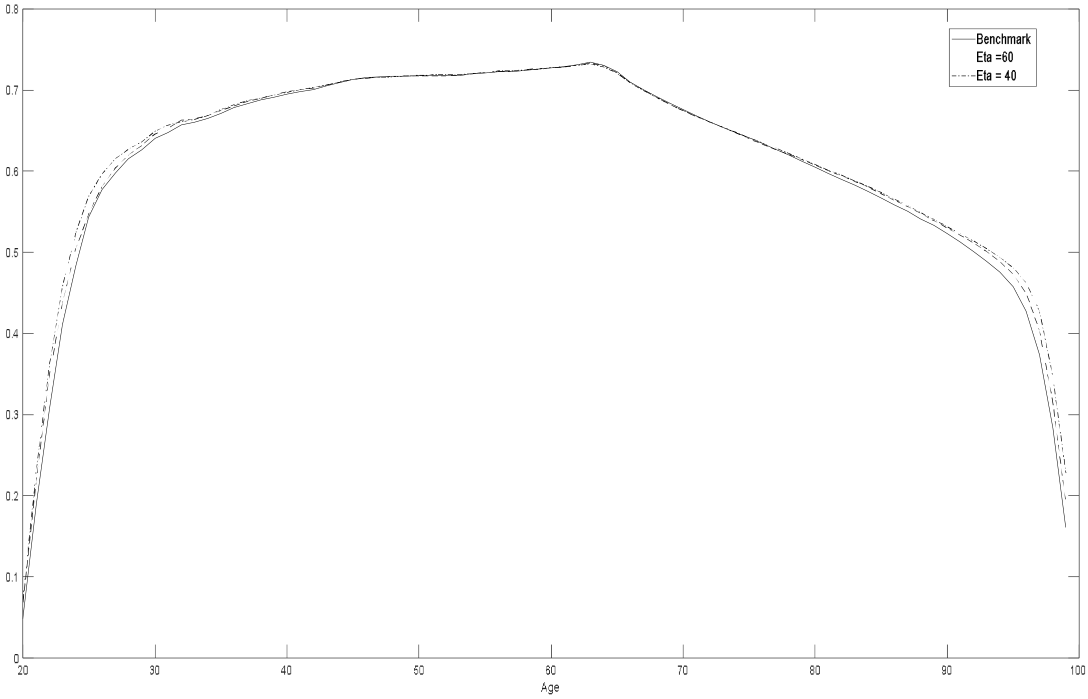

Banchmark
No ambiguity aversion
No Markov switch
Age
300
250
200
150
100
50
0
20
30
40
50
60
70
80
90
100

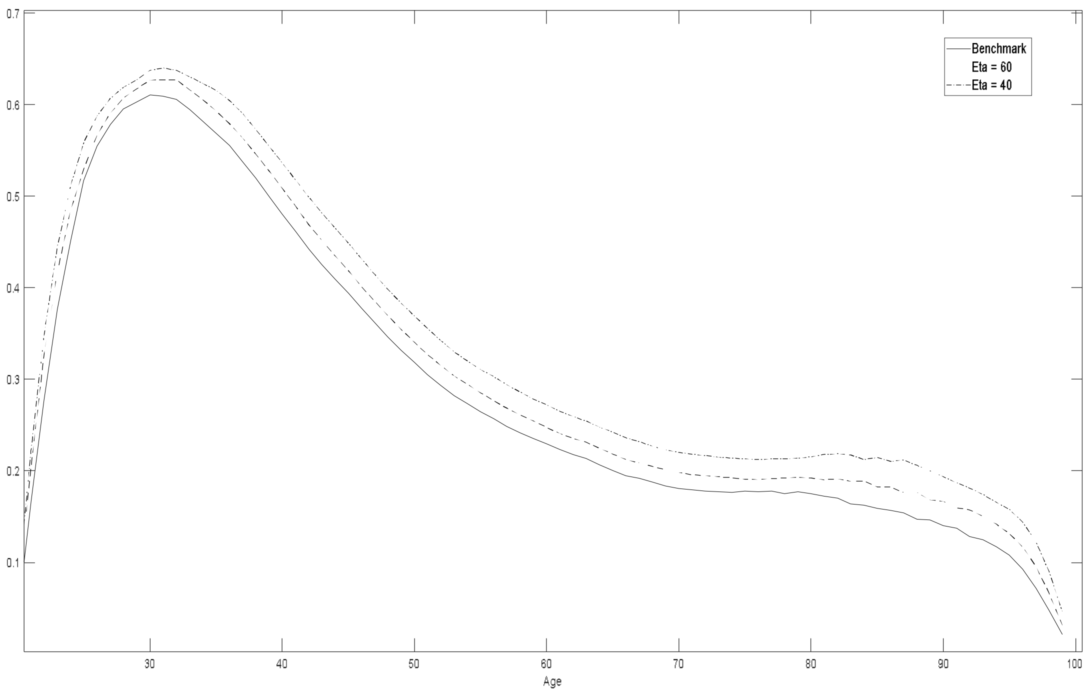


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90
100
Age
Benchmark
Gamma = 5
Gamma = 2

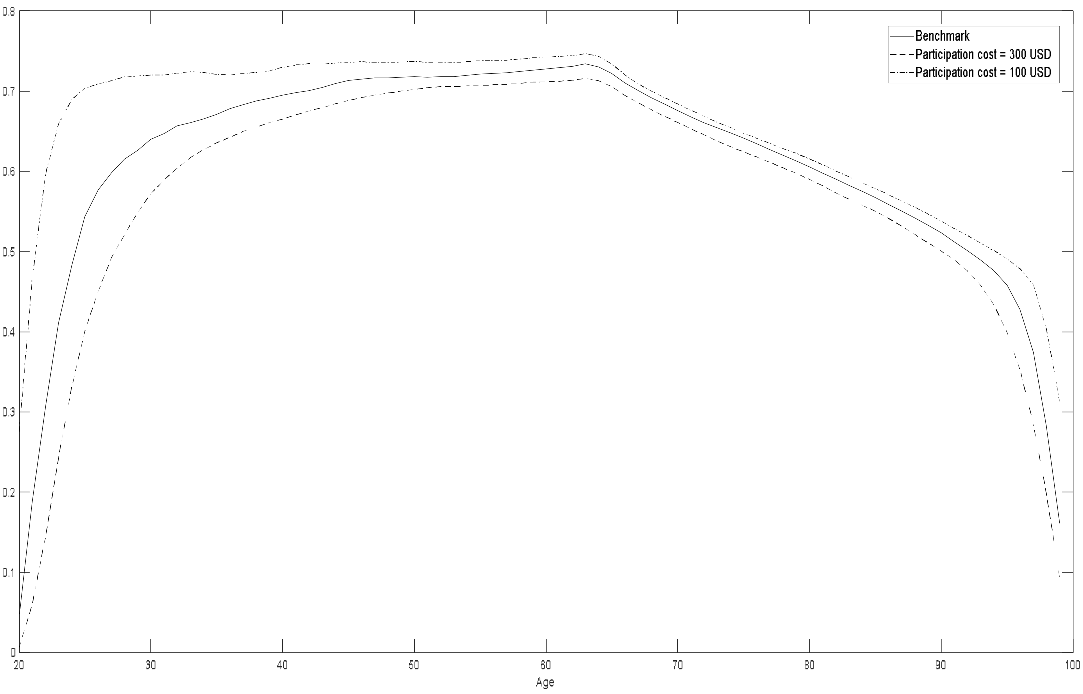


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90
100
Age
Benchmark
Gamma = 5
Gamma = 2

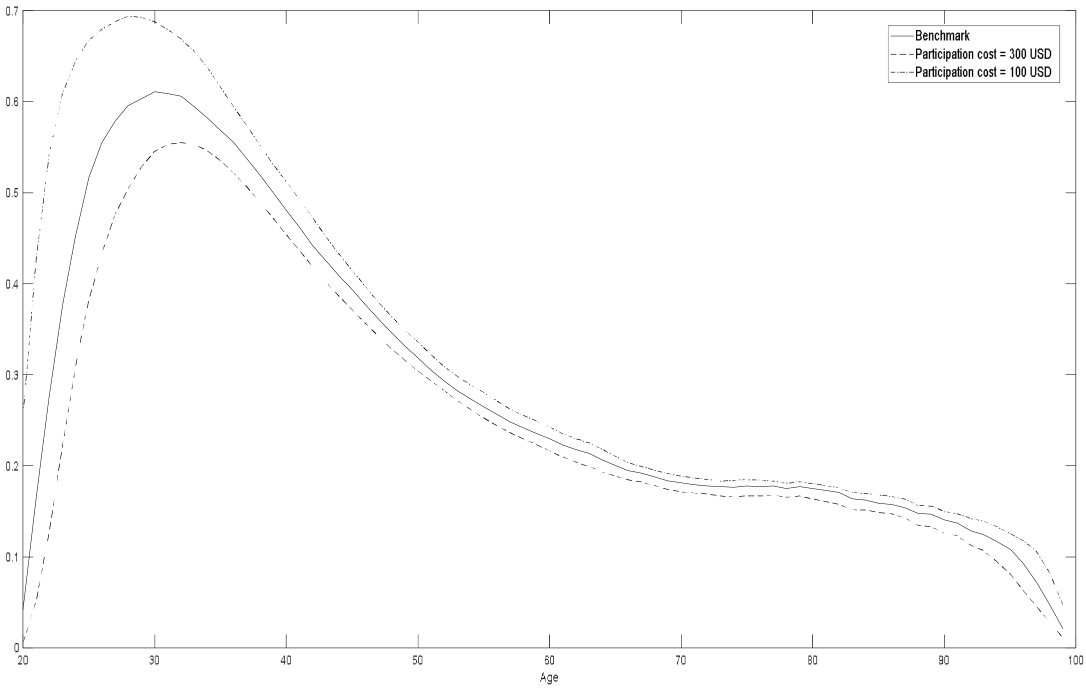

Benchmark
Eta =60
Eta = 40
Age
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90
100



Benchmark
Eta = 60
Eta = 40
0.7
0.6
0.5
0.4
0.3
0.2
0.1
30
40
50
60
70
80
90
100
Age



Benchmark
Participation cost = 300 USD
Participation cost = 100 USD
Age
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
20
30
40
50
60
70
80
90
100



Benchmark
Participation cost = 300 USD
Participation cost = 100 USD
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
20
30
40
50
60
70
80
90
100
Age