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**THE BI-OBJECTIVE POLLUTION-ROUTING PROBLEM
WITH MIXED LINEHAULS AND BACKHAULS**

**Master's dissertation submitted for the degree of Master of Industrial
Management and Engineering, supervised by the Professors Telmo Miguel Pires
Pinto and Carlos Alberto Henggeler de Carvalho Antunes, and presented to the
Department of Mechanical Engineering of the Faculty of Sciences and
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The Bi-objective Pollution-Routing Problem with Mixed Linehails and Backhails

Dissertation submitted for the degree of Master of Industrial Management and Engineering

O Problema de Optimização Bi-objectivo Poluição-Rotas com Entregas e Recolhas Indiferenciadas

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«fas est et ab hoste doceri»

P. Ovidii Nasonis, *Metamorphoseon*, IV, 428.

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Summary

The amount of greenhouse gases a vehicle emits when it travels is mainly a function of its load and speed, among several other factors, such as the driving behavior, road characteristics, etc. Reducing a vehicle's fuel consumption is an effective way of curbing pollutant emissions to the atmosphere. Several approaches have been proposed in the literature on how to ultimately achieve this goal.

The Pollution-Routing Problem is a variant of the Green Vehicle Routing Problems that aims at minimizing a cost objective function comprising the cost of fuel and emissions and labor costs. The problem consists of determining the optimal vehicle speed and load on each arc of a route, subject to vehicle capacity constraints, customers' time windows and service times.

In this dissertation, the Pollution-Routing Problem with Mixed Linehauls and Backhauls is addressed, considering a mixed pickup and delivery network. The single- and bi-objective versions of the problem were studied. In the single-objective approach, the economic and environmental impacts are monetized and the minimization of their total cost is sought. In the bi-objective approach, the minimization of the CO₂ emissions and the total driving time are considered. Efficient solutions were computed using the scalarizing ε -constraint technique for bi-objective optimization. The trade-offs between the two objectives are analyzed. The models were tested in benchmark instances adapted from the literature and the results obtained are discussed.

Keywords: Pollution-Routing Problem; Mixed Linehauls and Backhauls; Bi-objective Optimization; ε -constraint technique.

Resumo

A quantidade de gases com efeito de estufa que um veículo emite quando viaja é principalmente uma função da sua carga e velocidade, entre vários outros factores, como o comportamento de condução, as características da estrada, etc. A redução do consumo de combustível de um veículo é uma forma eficaz de reduzir as emissões de poluentes para a atmosfera. Várias abordagens têm sido propostas na literatura sobre como atingir esse objectivo.

O problema Poluição-Rotas é uma variante dos problemas de encaminhamento verde de veículos que visa minimizar uma função objectivo custo, compreendendo o custo com o combustível e emissões e o custo com a mão-de-obra. O problema consiste em determinar a velocidade e a carga óptimas em cada arco de uma rota, sujeito a restrições de capacidade dos veículos, janelas temporais e tempos de serviço nos clientes.

Nesta dissertação é abordado o problema Poluição-Rotas com entregas e recolhas indiferenciadas, considerando uma rede mista de entregas e recolhas. Foram estudadas as abordagens uni- e bi-objectivo do problema. Na abordagem uni-objectivo são monetizados os impactos económicos e ambientais e é procurada a minimização do seu custo total. Na abordagem bi-objectivo considera-se a minimização das emissões de CO₂ e do tempo total de condução. São calculadas soluções eficientes otimizando uma função escalar do tipo ε -*constraint* para o problema bi-objectivo. Os compromissos entre os dois objectivos são analisados. Os modelos foram testados em instâncias de referência adaptadas da literatura e os resultados obtidos são discutidos.

Palavras-chave: Problema Poluição-Rotas; Entregas e Recolhas Indiferenciadas; Optimização Bi-objectivo; Técnica ε -*constraint*.

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Chapter 1

Introduction

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1.1 Background

The rising awareness of environmental and social concerns due to climate change should be put at the center of today's economy. The advent of globalization ensued economic growth for many countries and paved the way for the development of global supply networks of products (whether goods or commodities), services, and information. Every day, these products traverse the land, water, and sky of the planet to fulfill the needs of billions of people all around the world. However, the extraction, production, storage, transportation, consumption, and dismantlement of these products has, on a par with the infrastructure required for their manufacture and distribution, taken both foreseen and unforeseen consequences on the environment.

One of the most significant human-induced impacts on the environment is climate change. The emission of greenhouse gases (GHG) to the atmosphere is of a particular concern since it constitutes the cause for global warming [1]. Despite a temporary break in the emissions in 2020, due to the COVID-19 pandemic-related measures, the atmospheric concentration of major GHG, such as carbon dioxide (CO₂), methane (CH₄), and nitrogen dioxide (NO₂), has been systematically rising over the past decades. For instance, CO₂'s average monthly concentration peaked at 419.13 parts per million in May 2021, a new historical maximum [2]. The average global temperature for 2020 was $1.2 \pm 0.1^\circ\text{C}$ above that of pre-industrial levels making it one of the three warmest years on record [3].

Aside the natural climatic oscillations, the scientists warn that even if all the anthropogenic emissions were to grind to a complete halt, the average global temperature would continue on a committed increase before stabilizing for many centuries, and eventually start decreasing, as it is now predominantly driven by decades-long CO₂ emissions [4]. Due to the long atmospheric lifetime of CO₂ [5], a changing carbon and water cycle, the oceans' thermal inertia [6], the temporary impacts of short-lived aerosols [7], and reactive GHG, global warming has now become an independent driver of climate change [4].

The anthropogenic CO₂ emissions, particularly those from the burning of fossil fuels, ought to receive a special attention. Although the *100 years Global Warming Potential*¹ of CO₂ is the least significant when compared to that of other major GHG (*e.g.*, CH₄ and NO₂ trap 21 and 310 times more the heat over time, respectively), it is by far the most emitted greenhouse gas. In 2019, a total of around 38 gigatons of CO₂ were emitted worldwide, amounting to around 64% of the global net anthropogenic GHG emissions [8]. The energy-related CO₂ emissions from coal, oil, and natural gas burning amounted to

¹The reference indicator for estimating a GHG's extra heat-trapping ability in the atmosphere over time in relation to CO₂.

around 91% (33.3 gigatons) of the global net CO₂ emissions in 2019 [9].

Especially over the past two decades, the scrutiny from various cohorts of society mounted on the governments and organizations and pressed them into reducing the negative environmental impacts of their economies and logistics operations. Numerous national and international policies, protocols, and agreements have since been put in place, from the establishment of the United Nations Framework Convention on Climate Change, in 1992, to the Kyoto Protocol, in 1997.

With the signing of the Paris Agreement, in 2015, the international community pledged to limit the increase in the global average temperatures to well below 2°C, in comparison to pre-industrial levels, and to pursue efforts to keep it under 1.5°C [10]. At the European level, in December 2019, the European Commission put forward the so-called «*European Green New Deal*» where it committed to becoming the first continent in the world to achieve climate neutrality by the year 2050. In June 2021, the goals inscribed in the deal were enshrined as a legally binding regulation under the European Climate Law.

On the one hand, the policy makers have been tightening the control on pollution as part of a strategic and competitive intent of transformation of the current economic paradigm into more modern, inclusive, and sustainable economies. The organizations, on the other hand, have been subject to both external and internal drivers that have influenced the adoption of *Green Logistics* (GL) and *Green Supply Chain Management* (GSCM) practices [11]. The adoption of these practices has been shown to lead to increased environmental and economic performance, which in turn positively impact the operational and organizational performance [12].

Transportation management, in particular, is a central activity in many companies' day-to-day operations. The companies engage in it to face their fixed and variable costs. Transportation alone has been shown to account for 33% to 66% of the total logistic costs of a company [13]. Managing transportation efficiently and economically also contributes to the development of larger economies of scale in production, stronger competition in the market, and helps to keep down the prices of goods [13].

Notwithstanding, in an increasingly globalized world, the consumers have come to expect a high availability of goods from all over the world and throughout the year. This not only presses the environment and the supply chains to deliver, but can only be achieved if transportation is carried out in a sustainable, reliable, fast, and cost-efficient manner. Transportation has been shown to represent up to 20% of the final cost of a good [14].

Undeniably, transportation shoulders a vital role in the economy and in the well-being and prosperity of billions of people in all the regions of the world. It grants citizens

mobility and ensures the flow of goods across national and international borders, thus contributing to a heightened quality of life. In the European Union (EU-27), the transport and storage sector accounts for more than 5% of the total employment and almost 5% of the gross domestic product [15]. In Portugal, the transport and storage sector accounts for around 20% of the country's total gross value added [16].

Transportation is, however, indissolubly linked to dire consequences to the environment, and consequently to human health. It not only contributes to the impairment of both the local and global air quality [17], the acidification of the soils and waters [18], the depletion of the ozone layer [11], but also generates noise, vibrations, accidents, congestion, is infrastructure-dependent (*e.g.*, roads, terminals, gas stations, among others) [19], and is a major contributor to GHG emissions. In the European Union (EU-27), transportation is responsible for around 30% of the Union's total GHG emissions [20]. In Portugal, a similar panorama is found with transportation being responsible for around 20% of the country's total GHG emissions [21].

Transportation is also an energy- and carbon-intensive sector. In fact, it shows the single highest energy and oil dependency of any other end-use sector, even greater than that of the industry's [19]. In 2018, transportation had a direct energy consumption of around 121 exajoules [22]. The transport vehicles, in particular, continue to be largely dependent on fossil fuels as their main energy requirement. In 2018, the transport vehicles' final energy consumption was responsible for around 25% of the direct CO₂ emissions from fossil fuel combustion [9].

Freight transportation, and especially the road and urban freight, ought to receive a special attention. Owing to the cross-border and commercial nature of freight, the policy makers' ambition towards decarbonizing this sector has been fairly overlooked when compared to passenger transport, even though freight accounts for around 44% of all the transport-related GHG emissions [19]. It is worrisome to note, as the global freight demand is projected to at least double by the year 2050, thrust, among other factors, by the growth of *e-commerce* [19].

The road and urban freight are held as decisive if transportation is to be decarbonized. The long- and short-haul, heavy- and light-duty trucks emit 65% of all the freight-related CO₂ emissions and are expected to remain the dominant mode of surface transport in the foreseeable future [19]. Despite the introduction of improvements in vehicle and fuel technologies (*e.g.*, increased engine efficiency, hybridization and electrification), no carbon-neutral solutions for long-haul and heavy-duty trucks are yet commercially available [19]. Owing to the expected growth of freight, such improvements are likely to be offset.

Over the recent years some companies began renewing their fleets of short-haul and

light-duty trucks with less pollutant electric vehicles. Although such vehicles are less pollutant from the point of consumption onwards (*i.e.*, no local emissions), some life-cycle assessments show that more pollution can actually be generated in their manufacture, in contrast to manufacturing vehicles operating on an internal combustion engine [23]. Other researchers argue that the electrification of the light-duty vehicles is not a one-off solution and that other pathways should be considered [24].

Despite the fleet renewals and electrification, the pace at which these are being carried out seems to be slow. The urban delivery trips still account for around 20% of all the freight-related CO₂ emissions, even though they only cover around 3% of the total freight activity, which really portrays its carbon intensity [19]. In 2020, an average of 150g of CO₂ per ton-kilometer were emitted in urban freight [19]. The road and urban freight combined are projected to represent around 72% of the total freight-related emissions by 2050 [19].

The urban deliveries take place in the cities and this is where the people are subject to the highest exposures of air pollution. Because the combustion that takes place inside the diesel and petrol engines is incomplete, not all the hydrogen and carbon present in these fossil fuels is converted into water vapor and CO₂, respectively. Consequently, carbon monoxide, nitrogen dioxides, non-methane volatile organic compounds, black carbon, lead, fine particulate matter (from tyre and brake ware), and ozone-depleting substances (from air conditioning units), among others, are emitted during transportation [4]. The exposure to both indoor and outdoor air pollution is the most concerning environmental risk factor to the global burden of disease and is responsible for around 7 million premature deaths annually [17].

It is against this progressively challenging background that Operations Research (OR) should heed the call to help to make the transport sector greener. OR has been described as the «*science of the better*» (the slogan of the INFORMS society) and has long held the tradition of improving the existing processes, chiefly through the reduction of their costs. Since its widespread implementation all across the world, OR has made a definitive contribution to the improvement of the efficiency in the organizations and the increase in the productivity of the economies [25]. The link of OR with GL in fostering a more rational and efficient use of the available resources, as well as in identifying trade-offs in multi-criteria decision analysis, has been addressed by the researchers [26]. At a more operational level, the link between the *Vehicle Routing Problem* and GL has been explored [27, 28].

Reducing economic and financial costs is undoubtedly important. They seem to remain the main drivers the organizations hold accountable when choosing between the imple-

mentation of different policies and strategies. The economic and financial costs, however, fall short of showing the whole picture. The organizations are also subject to regulatory and social pressures. These pressures directly influence the people, the structure, the environment, and the managerial decisions within the organizational context. In order to thrive in an increasingly resource-depleted world, the organizations will evermore need to reconcile their economic objectives with the environment and society. This «*triple-bottom-line*», as John Elkington called it, constitutes, after all, the pillars of sustainability.

1.2 Motivation and Objectives

The Vehicle Routing Problem plays a key role in transportation management, supply chain management and in the OR field in general. In its essence, the problem consists of routing a set of vehicles to serve a set of customers and calls for finding the optimal routes to do so. Since its introduction, numerous variants and analytical approaches have been proposed in the literature. This general problem has since been extended to account for a wide variety of operational and human constraints a real-world problem may arise. This helps to explain its success and relevance not only for the scientific community, but to the practitioners as well [39]. Notwithstanding, the generation of efficient route planning schemes without the use of proper analytical tools and computer-assisted solution methods is hard to achieve [29].

Especially over the last decade, the recognition of the negative externalities transportation has on the environment prompted many researchers to develop environmentally conscious models for vehicle routing. A new general class of problems known as *Green Vehicle Routing Problems* has since emerged. The application of these models has been shown to lead to significant reductions both in the total routing cost and in the amount of fuel/pollution the fleet consumes/generates while *en-service* [95, 109, 117].

In this dissertation, a variant of the Green Vehicle Routing Problems, known as the *Pollution-Routing Problem*, is addressed. This problem is a generalization of the *Vehicle Routing Problem with Time Windows* via the consideration of a more comprehensive cost objective function. It has spurred a considerable attention in the scientific community for two main reasons. On the one hand, it allows for the construction of more environmentally conscious routing schemes since the emissions of GHG, namely the $\text{CO}_{2(e)}$ emissions, are accounted for. However, it has been shown to be a challenging problem to solve to optimality even in the small- to the mid-sized instances [18].

The objective of this dissertation is to develop and test a bi-objective formulation for the *Pollution-Routing Problem with Mixed Linehauls and Backhauls* (PRPMB). The

proposed formulation is able to account for a heterogeneous fleet of vehicles and was solved using the scalarizing ε -constraint technique. In this problem, we wish to shed light on the existing trade-offs between the total driving time for the vehicles and their CO₂ emissions. As explained further, the two objectives are known to be conflicting due to the effect of the vehicle speed. The proposed problem then combines and subsumes the two well-known *Heterogeneous Vehicle Routing Problem*, where a heterogeneous fleet is accounted for, and the *Vehicle Routing Problem with Mixed Linehauls and Backhauls*, where a mixed pickup a delivery network is considered, into the context of the Pollution-Routing Problem.

To the best of our knowledge, the incorporation of mixed linehauls and backhauls in the Pollution-Routing Problem has heretofore not been addressed in the scientific literature. This dissertation fills this gap and provides an evaluation of the impacts of such a backhauling strategy. To this extent, the single-objective PRPMB has also been studied. In this problem, the minimization of a cost objective function comprising the cost of the fuel/emissions and the drivers' wages is considered. We set out to investigate and quantify how much the incorporation of mixed linehauls and backhauls can be beneficial or detrimental from a total cost perspective and emissions standpoint. Both models were tested in benchmark instances adapted from the literature and the results obtained are discussed.

1.3 Outline

This dissertation is divided into 6 chapters including this introductory one. At the beginning of each chapter, a brief introduction is made on the topics to be discussed. Similarly, at the end of each chapter a brief conclusion that sums up the findings is presented. A description of the following chapters and their contents is provided below.

In Chapter 2, the Pollution-Routing Problem is addressed. A general framing in the context of the Vehicle Routing Problems and Green Vehicle Routing Problems is provided first. Specifically, some important variants of these problems are introduced and discussed. A special emphasis is given to the Pollution-Routing Problem, which is completely defined and formulated. A comprehensive explanation of the function of fuel consumption and emissions, as well as of the whole model itself, is provided.

The Chapter 3 sets the state of the art on the Pollution-Routing Problem. The selected contributions are comprehensively reviewed and discussed. The approaches developed for this problem are emphasized.

In Chapter 4, the definition and formulation for the bi-objective Pollution-Routing Problem with Mixed Linehauls and Backhauls are presented. The mathematical formu-

lations on which our own is based are introduced and explained.

The Chapter 5 is dedicated to the computational results and their discussion. The results for the single-objective model are presented first. Subsequently, the results of the implementation of the scalarizing ε -constraint technique for the bi-objective model are presented.

Finally, in Chapter 6 some final conclusions are drawn. The main contributions and limitations of this dissertation are presented and possible future research directions are outlined.

Chapter 2

The Pollution-Routing Problem

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2.1 Introduction

In this chapter, the literature on the general classes of problems known as Vehicle Routing Problems (VRPs) and Green Vehicle Routing Problems (GVRPs) is reviewed and discussed. For the sake of brevity, an exhaustive coverage on all the variants is not offered herein. A selected few publications that represent much of the scientific research into the field of the VRPs and GVRPs are addressed. A comprehensive review is conducted on the Pollution-Routing Problem (section 2.4) as this is the variant upon which this dissertation is built. This problem is completely defined, reviewed, and discussed in depth. This chapter starts by addressing the VRPs.

2.2 The Vehicle Routing Problem

The origins of the Vehicle Routing Problem (VRP) are inextricably linked to those of the *Traveling Salesman Problem* (TSP). As detailed further in the subsection 2.2.1, the VRP consists of a generalization of the TSP. Both are among the most well-known and extensively studied problems of combinatorial optimization in the OR field. The first mathematical formulation for the TSP is attributed to Karl Menger who described the problem as «*The Messenger Problem*» [30]. Throughout the years, a panoply of applications for the TSP has been found by the researchers: from Crystallography [31], to DNA sequencing [32], among many others. Laporte [33] reports other interesting applications for the TSP.

In its classical statement, the TSP consists of a salesman who wishes to visit several cities. Each city must be visited exactly once, the salesman must return to the initial city (the *depot*, as it is known in the VRP literature), and the total distance traveled should be minimal. If the problem is defined on a graph, the goal is to find the minimum distance circuit, or *Hamiltonian* circuit, that passes through all the vertices (cities) once and only once. Despite its seemingly simple statement, the TSP is a NP-Hard problem [33].

As the number of cities n increases, the number of feasible routes increases by $(n+1)!/2$ for the case where the distances between each pair of cities are symmetric. If 5 cities are considered, 12 feasible routes exist. However, if 50 cities are considered, around 10^{62} feasible routes exist. Enormous TSP instances have been solved to optimality, the largest of which in 2006 containing 85,900 cities [34]. Notable formulations for the TSP using *integer linear programming* (ILP) techniques include those of Dantzig *et al.* [35] and Miler *et al.* [36].

The VRP is also NP-Hard because it is an integration of two NP-Hard combinatorial optimization problems: the TSP and the *Bin-Packing Problem* (BPP). The BPP is also an

extensively studied problem in OR, with several applications: from the filling of containers or trucks [37], to computer memory allocation [38], among many others. As its name indicates, the BPP is a packing problem. In general terms, in the BPP one wishes to pack several different sized items into a finite number of bins, or containers, which have a maximum capacity. The goal is to find the optimal assortment of the items in the bins, so that the total number of bins used is minimal.

A well-known variant of the BPP is the *Knapsack Problem* where the number of bins is restricted to one and each item is characterized by a value and a weight. In this particular case, one wishes to maximize the value carried in the knapsack without exceeding a certain weight. Generalizations of the BPP can be considered if, for example, the items and the bins are characterized not by a one-dimensional property, such as a weight or a length, but by two-dimensional or three-dimensional properties, such as an area or a volume.

The VRP is at the intersection of the TSP and the BPP. If the vehicles and the customers are thought of as bins/salesmen and cities, respectively, a clearer definition arises as to what is the VRP. The VRP is a combinatorial optimization problem in which the best possible allocation of the items to the vehicles and subsequent scheduling of the vehicles to the customers is to be found, while satisfying all the problem's constraints, to pursue a given objective. Several different constraints and objectives can be considered and combined. The most common objective is the minimization of the total routing cost, and the most common constraint is the vehicle capacity constraint.

The VRP then consists of solving two sequential problems: a BPP and a TSP, in this order. The resulting problem is a challenging combinatorial optimization problem, one for which an optimal solution may not be found especially in large or complex instances. The state-of-the-art exact solution methodologies for the VRP can only consistently solve to optimality instances with up to 50 customers [39], although this depends on the overall complexity of the problem.

2.2.1 The First Publications

More than 60 years have elapsed since the first publication on the VRP. The first publication to emerge in the scientific literature is that of Dantzig and Ramser [40], in 1959.

In their seminal work, the authors described a real-world application of a generalized version of the TSP to the gasoline distribution context and named the problem as «*The Truck Dispatching Problem*» (TDP). The problem concerns the finding of the optimal routing scheme for a fleet of capacitated trucks that need to deliver gasoline from a bulk terminal to several service stations, returning back to the terminal. Each service station must be visited exactly once and has a known demand that must be fulfilled. However,

because the compound demand of all the service stations exceeds the capacity of any given vehicle, it is not possible to fulfill them all in a single visit. Thus, a set of routes is created. The goal is to find the set of minimum distance routes that guarantee the fulfillment of the stations' demands without exceeding the capacity of the vehicles. The authors proposed the first mathematical ILP formulation for the VRP. Noticeably, there are many similarities between the classical statements for the TSP and the VRP. In its simplest form, the VRP can be seen as a multiple TSP (m -TSP) with capacity constraints.

In 1964, another seminal work by Clarke and Wright [41] emerged in the scientific literature. In their work, a simple, yet effective, greedy heuristic procedure capable of finding near-optimal or even optimal solutions for the TDP of Dantzig and Ramser [40] was proposed.

The procedure starts by constructing back-and-forward routes starting at the depot and calculates the pairwise savings that are obtained if two customers are inserted into the same route, as opposed to visiting each one of them in separate routes. The savings obtained by this insertion procedure represent the difference in the total distance traveled when the two routing schemes are compared. The routes are subsequently created by greedily and iteratively choosing the pairs of customers with the highest savings and inserting them into a route, while respecting the capacity of the vehicles, until no further saving can be obtained. The procedure became known as *Clarke & Wright's Savings Algorithm* (C&W) and still is to this day a widely used heuristic for the VRP [42].

2.2.2 CVRP Definition and Formulation

In the previous subsection, the TDP of Dantzig and Ramser [40] was introduced and described. This problem is known today as the *Capacitated Vehicle Routing Problem* (CVRP). In this subsection, a classical CVRP is defined and formulated. The CVRP is the most studied variant of the VRP [39] and is introduced herein as an illustrative example.

The CVRP consists of routing a set of capacitated vehicles to serve a set of customers. The capacity of the vehicles is usually expressed in terms of the payload they can carry but can also be expressed in terms of the available area, volume, or maximum number of units, which are typically palletized items.

In the CVRP, a homogeneous fleet of vehicles is available at a single depot and each vehicle is operated at the same cost. Each customer has a non-negative demand for a single product. By convention, the depot has no demand and has all the units of that single product to satisfy the demands of all the customers. Each customer must be visited exactly once and its demand cannot be split. Each vehicle leaves from and must return to

the depot. Finally, each edge, or arc, is associated with a non-negative cost (or distance, or travel time), which represents the cost of traversing that edge. The goal is to find the set of minimum cost routes that guarantee the fulfillment of all the constraints.

According to this problem statement, the CVRP can be formulated either using a directed or an undirected graph. A two-index vehicle-flow formulation using a complete and directed graph is described next. The formulation uses two-index binary x_{ij} variables which measure if the arc (i, j) is traversed by a vehicle or not. The following formulation was introduced by Laporte [43].

Let then $G = (V, A)$ be a complete and directed graph. Let $V = \{0, \dots, n\}$ denote the set of nodes, with the node 0 as the depot. Let $N = \{1, \dots, n\}$ denote the set of customers. Let $A = \{(i, j) \in V \times V : i \neq j\}$ denote the set of arcs between each pair of nodes. Let c_{ij} denote the cost associated with traversing the arc going from the node i to the node j , $\forall (i, j) \in A$. It is assumed that $c_{ij} \leq c_{ik} + c_{kj}$, $\forall (i, j, k) \in V$, so that the triangle inequalities are satisfied. Let $q_i \geq 0$ denote the demand associated with each customer $i \in N$. Let $K = \{1, \dots, |K|\}$ denote the set of $|K|$ homogeneous vehicles available at the depot, each with a capacity $Q \geq \max \{q_i\}$, $\forall i \in N$.

Let $S \subseteq V$ denote an arbitrary subset of vertices. For directed graphs, let $\delta^- = \{(i, j) \in A : i \in S, j \notin S\}$ denote the set of in-arcs and $\delta^+ = \{(i, j) \in A : i \notin S, j \in S\}$ denote the set of out-arcs. Let $r(S)$ be the minimum number of vehicle routes needed to serve the subset S . Let x_{ij} denote a binary variable equal to 1 if the arc (i, j) is traversed by a vehicle and equal to 0 otherwise. The CVRP is formulated as an ILP program as follows:

$$\text{minimize } \sum_{(i,j) \in A} c_{ij} x_{ij} \quad (2.2.1)$$

subject to

$$\sum_{j \in \delta^+(0)} x_{0j} = |K| \quad (2.2.2)$$

$$\sum_{j \in \delta^+(i)} x_{ij} = 1 \quad \forall i \in N, \quad (2.2.3)$$

$$\sum_{i \in \delta^-(i)} x_{ij} = 1 \quad \forall j \in N, \quad (2.2.4)$$

$$\sum_{(i,j) \in \delta^+(S)} x_{ij} \geq r(S) \quad \forall S \subseteq N, S \neq \emptyset, \quad (2.2.5)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A. \quad (2.2.6)$$

The objective function (2.2.1) seeks the minimization of the total routing cost. The constraints (2.2.2) enforce the creation of exactly $|K|$ routes, so that the depot has $|K|$ successor vertices and is linked to $2|K|$ customers. In the CVRP, it is assumed that all the vehicles will be routed, thus exactly $|K|$ routes are created, one for each vehicle. If it is not required that all the vehicles be routed, the constraints (2.2.2) can be adapted to ≥ 1 .

The constraints (2.2.3)-(2.2.4) are known as *degree constraints* and ensure that, in a route, each vertex (customer) is linked to exactly two other vertices (customers), a so-called *successor* and a *predecessor*. The constraints (2.2.3)-(2.2.4) ensure that each customer is visited exactly once.

The constraints (2.2.5) serve simultaneously as vehicle capacity constraints and *sub-tour elimination constraints* (SEC). One of the challenges in most VRPs is to ensure that sub-tours are not created: those are routes $\langle v_i, v_j, \dots, v_i \rangle$ that do not start and end at a depot. The number $r(S)$ is computed by solving a BPP where each item (demand) has a weight q_i and each bin (vehicle) has a capacity Q .

For a given route, if $q(S) > Q$, the route is infeasible and $r(S)$ must be > 1 . This means that at least two routes, thus vehicles, are needed to serve the subset S and connect it with its complement $V \setminus S$. Since $\sum_{(i,j) \in \delta^+(S)} x_{ij} = 0$ is satisfied by any sub-tour over a non-empty subset $S \subseteq N$, due to $r(S) \geq 1$ this sub-tour is also eliminated. Finally, the constraints (2.2.6) are integrality constraints.

The constraints (2.2.5) may present a computational challenge as their number grows exponentially with the number of customers [39]. In order to reduce the cardinality of the constraint set to a polynomial number, a linear relaxation can be performed if an additional set of variables $f_i = (f_1, \dots, f_n)^\top$ is considered, which represent the accumulated demand up to a customer $i \in N$. The constraints (2.2.5) can then be replaced by the following:

$$f_i - f_j + Qx_{ij} \leq Q - q_j \quad \forall (i, j) \in A, \quad (2.2.7)$$

$$q_i \leq f_i \leq Q \quad \forall i \in N. \quad (2.2.8)$$

The constraints (2.2.7) serve as SEC constraints. If $x_{ij} = 1$, then $f_j \geq f_i + q_j > f_i$. Therefore, the presence of a sub-tour not containing the depot $\langle v_i, v_j, \dots, v_i \rangle$ leads to a contradiction whereby $f_i > f_j > f_i$. This sub-tour is eliminated.

The constraints (2.2.8) are vehicle capacity constraints and are used to bound the load carried in the vehicles as they travel along a route. In a route, the accumulated demand up to a customer $i \in N$ can neither exceed the capacity of the vehicle nor can it be less than the demand of that customer.

This formulation (2.2.7)-(2.2.8) is known as the *Miller-Tucker-Zemlin* (MTZ) formulation, introduced by Miler *et al.* [36] for the TSP. The MTZ formulation has the advantage of producing $\mathcal{O}(n^2)$ variables and constraints but the downside of producing weaker lower bounds comparatively to the formulation (2.2.1)-(2.2.6) [39].

2.2.3 Variants and Contributions

A very large number of contributions to the scientific body of literature on the VRP has been made since the problem was first introduced. The literature on the VRP is extensive and includes a multitude of different variants, extensions, and applications. Over the years, the researchers have sought to make their models ever more realistic through the incorporation of real-life complexities, objectives, and operational/human constraints. Ultimately, an increased proximity between the proposed models and the real-life contexts in which the VRPs arise is registered.

In fact, the fields of application for the VRP extend far beyond the context of the distribution of goods where these problems have traditionally been found. Such fields include, but are not limited to, healthcare [44], public transport [45], agriculture [46], and robotics [47]. Extensive bibliographies and surveys on VRPs can be found in [39] and [48]-[51]. Some VRP variants are introduced next.

2.2.3.1 Time Windows

The CVRP can be generalized to include specific time slots during which the customers must be served, as well as their corresponding service times. These time slots, or time windows, are usually expressed in terms of a time interval, composed of a lower and an upper bound, corresponding to the earliest and latest possible arrival times, respectively. The vehicles depart from the depot at the instant zero and all the customers must be served within their respective time window.

Additionally, the time windows can either be hard or soft. If hard time windows are considered, a vehicle that arrives early at a customer must wait until the service can start. It is often assumed that such a wait incurs no cost. Late arrivals, on the other hand, are not allowed. If the time windows are soft, both early and late arrivals are allowed but a penalty cost is incurred whenever they are violated.

This generalization of the CVRP is known as the *Vehicle Routing Problem with Time Windows* (VRPTW). Important contributions on the VRPTW include, among others, those in [52]-[56].

2.2.3.2 Heterogeneous Vehicles

In the CVRP, a homogeneous fleet is considered. In real-life settings, many transportation companies have a heterogeneous or mixed fleet at their disposal. This fleet can be comprised of vehicles from different car manufacturing companies, with different payload capacities, varying mileages and speeds, varying fixed and variable operating costs, among many other differences. The flexibility afforded by such a fleet may allow for a more cost-efficient routing scheme, in comparison to using a homogeneous fleet, since the type of vehicle deployed has an impact on the payload it can carry, the distance it can travel, the fuel consumption, and also the GHG emissions [57].

The vehicles and the customers may be partitioned into subsets and there may be further constraints on what subset of vehicles can serve a subset of customers. The corresponding problems are known as the *Heterogeneous Vehicle Routing Problem* (HVRP) and the *Fleet Size and Mix Vehicle Routing Problem* (FSMVRP).

In the HVRP, the fleet available at the depot is considered to be limited, while in the FSMVRP it is considered to be unlimited. In the FSMVRP, the number of vehicles to deploy is thus an inherent decision to be made so that the best fleet size and mix is found. In the FSMVRP, it is usually assumed that the fleet does not belong to the company in charge of the routing. Thus, a common objective is the minimization of the number of vehicles used, since the cost of renting and operating a certain vehicle type is usually accounted for. In the HVRP, it is usually assumed that the company owns the fleet and that the entire fleet will be routed. A review on heterogeneous routing can be found in [58].

2.2.3.3 Multiple Depots

In the CVRP, an assumption is made that the customers are served from a single depot. This may not be the case for the companies who have several depots and fleets available at such depots. Furthermore, in the CVRP it is also assumed that the vehicles depart from and arrive at the same depot.

First studied by Tillman [59], the problem known as the *Multiple Depot Vehicle Routing Problem* (MDVRP) allows for the possibility that the vehicles may start from and end at separate locations (depots) during their routes. Further constraints can be considered if the depots can only host a certain number or type of vehicles, and thus the assignment of the vehicles to the depots is needed, or if the depots can serve as replenishment stations for *en-route* deliveries. If the vehicles are not required to return to a depot after visiting the last customer in a route, the corresponding problem is known as an *Open Vehicle Routing Problem* (OVRP). Reviews on the MDVRP can be found in [60, 61].

Another challenge that many companies face is how to properly integrate the warehousing decisions with the routing/distribution decisions. It is commonly found that the logistical distribution networks are divided into multiple echelons, from the original manufacturer to the end-consumer (*e.g.*, *e-commerce*). In such networks, the intermediate depots, known as the *satellites*, can serve as warehousing stations, cross-docking stations or break-bulk stations. The corresponding problem is known as the *Multiple-Echelon Vehicle Routing Problem* (ME-VRP), with the most common variant being the *Two-Echelon Vehicle Routing Problem* (2E-VRP). A recent review on the 2E-VRP can be found in [62].

2.2.3.4 Pickup and Delivery

In the CVRP, a forward distribution system is considered where the items need to be delivered to the customers. In some contexts, the items also need to be collected from them. A fitting example emerges from the reverse logistics context where the empty loads must be collected after the consumption has taken place, in addition to the normal delivery schedules. Another example is that of the waste collection context where some damaged products, or products at the end of their life-cycle, must be collected either for maintenance/refurbishment or otherwise to recycle. These problems involving both distribution and collection are known as *Pickup and Delivery Problems* (PDPs).

Although the transportation of items (goods) is usually considered, the transportation of people is also an important field of investigation in the PDPs: from *Dial-a-Ride Problems* (DARP), to *School Bus Routing* (SBR), to *Car Pooling Problems* (CPP). The review herein is focused on the distribution of goods and merges the taxonomy for the PDPs as proposed by Parragh *et al.* [63, 64] with the one proposed by Toth and Vigo [39]. The Figure 2.2.1 illustrates the general taxonomy for these problems.

The Figure 2.2.1 shows that the general class of problems known as PDPs can be divided into three main categories: *many-to-many* (M-M) problems, *one-to-many-to-one* (1-M-1) problems, and *one-to-one* (1-1) problems. The M-M problems are addressed first.

In the M-M problems, any depot or customer may be the source or destination of any good. In these problems, the customers may request either a delivery or a pickup service from any of the other customers or from the depots. This implies that a customer is visited only once and that the goods can flow from a customer to a customer, from a customer to a depot, and vice-versa.

Toth and Vigo [39] report that the single-commodity case is the most studied variant of the M-M problems, which is known as the *One-Commodity M-M Pickup and Delivery Vehicle Routing Problem* (1-PDVRP). If more than one commodity is considered, the

corresponding problem is known as the *N-Commodity M-M Pickup and Delivery Vehicle Routing Problem*.

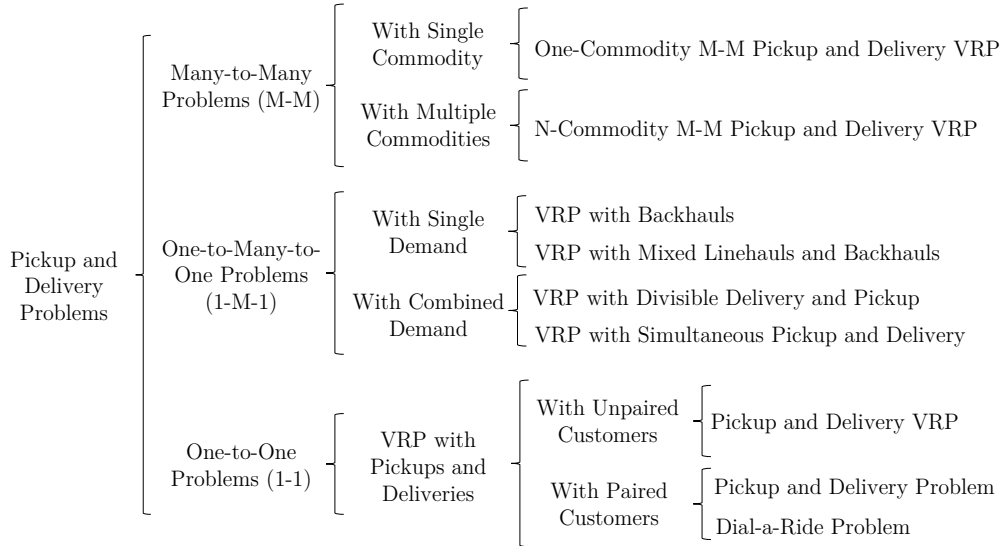


Figure 2.2.1: General taxonomy for the PDPs (based on [39, 63, 64]).

In 1-M-1 problems, all the deliveries are supplied by the depot and all the pickups must return to the depot. Thus, only the depot serves as the source and destination of all the delivered and picked goods. In other words, the demand of a customer cannot be fulfilled by another customer. The customers may request a delivery or a pickup service, or both. The customers who request a delivery service are designated as *linehaul* customers and those who request a pickup service are designated as *backhaul* customers. The customers who request both services may be referred to as *mixed* ones.

In the case where the customers have a single demand for a service, they are partitioned into two disjoint subsets. Put simply, a given customer can only be a linehaul customer or a backhaul customer, but not the two at the same time. Thus, each customer will only be visited once, and only one type of service (a delivery or a pickup) is performed at each one. The corresponding problems are known as the *Vehicle Routing Problem with Backhauls* (VRPB) and the *Vehicle Routing Problem with Mixed Linehauls and Backhauls* (VRPMB).

In the VRPB, a precedence constraint is considered whereby in any given route all the linehaul customers are visited before any of the backhaul customers. Such a constraint is considered in order to avoid loading or cargo-rearrangement problems at the delivery locations [65]. In the VRPMB, the precedence constraint does not apply, hence any customer route sequence is allowed.

In the VRPB, an additional routing constraint is considered whereby routes with backhaul customers only are not permitted. This constraint is used to reflect the importance of the linehaul customers over that of the backhaul customers [65]. Conversely, in the VRPMB, routes with backhaul customers only may exist. Finally, in both problems, in any given route, the total deliveries and pickups, considered separately, must not exceed the capacity of the vehicle.

The main goal in the VRPB/VRPMB is the minimization of the total routing cost: that is the cost of servicing both the linehaul and the backhaul customers. Notwithstanding, making the most use of the capacity of the vehicles and avoiding empty return trips are also two important metrics, which can contribute to increased economic and environmental benefits when a backhauling strategy is considered [66]. Recent reviews on the VRPB/VRPMB include those in [65, 67].

If the customers have a combined demand for both a delivery and a pickup service, the partitioning into disjoint subsets does not apply. Thus, each customer can be a linehaul customer and a backhaul customer at the same time. In such a case, one of two distinct situations can occur: either the delivery and pickup services can be performed in separate visits or they are performed in the same visit and at the same time. The corresponding problems are known as the *Vehicle Routing Problem with Divisible Delivery and Pickup* (VRPDDP) and *Vehicle Routing Problem with Simultaneous Pickup and Delivery* (VRPSPD).

In the VRPDDP, the customers are visited twice, once for each type of service. In VRPSPD, the customers are visited only once and both services are performed in the same visit and at the same time. The literature on the VRPDDP is rather scarce but an important contribution can be found in [68]. A recent review on the VRPSPD can be found in [69].

The final subclass of the PDPs is the 1-1 problems. These problems can be understood as point-to-point transportation schemes, in that the goods are transported from a single source (a pickup vertex) to a single destination (a delivery vertex). The 1-1 problems are also commonly designated as *Vehicle Routing Problems with Pickups and Deliveries* (VRPPD).

In these problems, one of two distinct situations can occur: either the picked goods from a specific backhaul customer are to be delivered to a specific linehaul customer, or the picked goods from any backhaul customer can be delivered to any linehaul customer. In the former, the customers are said to be paired, whereas in the latter the customers are said to be unpaired. If the customers are unpaired, the corresponding problem is the *Pickup and Delivery Vehicle Routing Problem* (PDPVRP). If the customers are paired,

the corresponding problems are the *Pickup and Delivery Problem* (PDP), if one refers to the transportation of goods, and the *Dial-a-Ride Problem* (DARP), if one refers to the transportation of people.

2.2.3.5 Time Dependency

In the CVRP, the cost (or the travel time) between two nodes is assumed to be time-invariant and constant. However, the real-life driving conditions usually present congestion during certain periods of the day, namely in urban areas. Thus, the cost/travel time between two nodes is usually not a linear function of the distance traveled because the speeds are not constant. Therefore, in congested settings, the cost/travel time is dependent both on the distance traveled and on the period of the day.

Naturally, the congestion due to the rush hours is the most predictable of the possible traffic-related complications that may arise. Other complications include adverse weather conditions, accidents, poor road quality, seasonality, among other non-predictable and random events [70].

The problem known as the *Time-Dependent Vehicle Routing Problem* (TDVRP) seeks to model this dependency in the cost/travel times by dividing the day into periods. The cost/travel time is then modeled as a step function of time, with different costs/travel times associated to different periods of the day.

In the TDVRP, the objective is usually the minimization of the total driving time, subject to time windows and service times. The departure time from a given a node is held as a decision variable, since it may be advantageous, from a cost perspective, to wait until the congestion period dissipates before departing. A review on the TDVRPs can be found in [71].

2.2.3.6 Stochastic Information

In the CVRP, it is assumed that all the relevant data for problem is readily available and known *a priori* at the planning stage. Specifically, all the necessary parameters for the model are assumed to be deterministic.

In practice, most of the information required for route planning has some level of uncertainty associated to it, albeit such uncertainty may be captured in ways other than stochastically (*e.g.*, fuzziness). This uncertainty may originate from numerous sources, such as the customers' demands, their number, or travel times, to name a few.

Such an uncertain environment may generate poor-quality solutions, or even infeasible ones, if a deterministic model is used. For example, a customer may request the delivery of a higher amount than was previously expected and its corresponding route may turn

out to be capacity-infeasible due to such increase. Therefore, robust models which can take uncertainty into account are needed.

The problems known as *Stochastic Vehicle Routing Problems* (SVRPs) integrate this type of uncertainty by either considering a recourse function or probabilistic constraints.

When probabilistic constraints are considered, a model is said to be *chance constrained* if one or more constraints are satisfied in spite of a maximum pre-specified probability α of failing to be met. In this case, the problem is solved with the assurance that the potential failures are below α (*i.e.*, the constraints must be met with probability $\geq 1 - \alpha$), and the cost of the failures is typically overlooked.

The decision-maker can also specify a given recourse policy. This policy models the failure costs, which are taken into account in the overall objective function, minimizing the total expected cost, comprising the routing cost and the failure costs. Since in this case failures are allowed, the solution must be repaired. For example, the decision-maker may specify that the vehicles return to the depot to re-load if their capacity is exceeded. Recent reviews on the SVRPs can be found in [72, 73].

2.2.3.7 Dynamic VRP

In the CVRP, the route planning is done at the planning stage and in a deterministic environment. However, in many real-life settings some of the information may be unknown, uncertain, or otherwise become available as the time progresses. This implies that this new information must be relayed to the drivers who have been dispatched and the (re-)optimization process must be conducted in real time.

The problem known as the *Dynamic Vehicle Routing Problem* (DVRP) copes with such environments where the decisions must be made on an ongoing basis. An example is that of the emergency response context where a vehicle may need to be detoured to respond to a critical situation. Another example is that of some courier services where new customers may be added to or suppressed from a route along the day.

The dynamics present in these situations usually imply that not all the customers will be served, hence metrics such as the service level, throughput or maximum revenue may be utilized to evaluate the overall performance of the system. Reviews on the DVRP can be found in [74, 75].

2.2.3.8 Other Variants and Considerations

In the previous subsections, some VRP variants have been briefly reviewed and discussed. Although representative of much of the typical constraints considered in the literature, it should be stressed that these are just some of the many VRP variants. The VRP family is

rich and varied, and general taxonomies for the classification of these problems have been suggested [76]. These mainly classify the problems according to their scenario-, problem- and information-specific characteristics.

For instance, in the CVRP a graph comprising nodes and arcs is considered. The service is performed at the nodes and the arcs represent the links between the nodes. However, there may be a request for the service to be performed along an arc where several customers may be located. An example of such a service takes place in cities where the street cleaning is carried out along a street rather than at a discretized location.

Such problems are known as *Arc Routing Problems* (ARPs). If a mixture of services at the nodes and along the arcs is considered, the corresponding problem is known as a *General Routing Problem* (GRP). Still, should a service have some periodicity associated to it, one refers to the problem as a *Periodic Vehicle Routing Problem* (PVRP).

Another assumption in the CVRP considers the vehicles to have a single compartment where the goods can be transported side-by-side. In some contexts, however, such a consideration is not permitted. For example, in the pharmaceutical industry some chemicals cannot be transported in the same compartment due to the danger of chemical reactions. Another example sprouts from the food industry where some foods must be transported in separate compartments due to public health regulations to prevent cross-contamination.

When more than one compartment must be considered, the corresponding problem is known as the *Multiple Compartment Vehicle Routing Problem* (MCVRP). Additional constraints can be considered if, for example, certain goods can only be transported in certain compartments, or otherwise if certain compartments can only be filled up to a certain capacity to prevent overflowing issues.

In the CVRP, a simplification is made when the loading of the vehicles is overlooked. The vehicles and the items are characterized by a one-dimensional property (usually a payload and a weight, respectively), rather than by two- or three-dimensional properties. In practice, the items inside the vehicles must be arranged before departing.

In fact, it may happen that when the vehicles are loaded to near capacity, a feasible vehicle capacity with an infeasible layout for the items may be obtained [77]. Also, the vehicles are usually loaded/unloaded from the rear side. Thus, when visiting a customer, the items must usually be unloaded in a last-in-first-out (LIFO) basis. A proper load configuration inside the vehicles is thus important to avoid handling issues and wasting time with cargo rearrangements mid-tour.

Several other constraints can be accounted for, such as the load stability and fragility. With respect to the CVRP, the corresponding problems are known as the *Capacitated Vehicle Routing Problem with Two-Loading Constraints* (2L-CVRP) and the *Capacitated*

Vehicle Routing Problem with Three-Loading Constraints (3L-CVRP).

Another possible generalization of the CVRP is to consider that the customers have a known rate of consumption of a good rather than a one-time demand for that good. The customers are supplied and have an initial stock, as well as a storage capacity. The problems known as *Inventory-Routing Problems (IRPs)* seek to periodically replace the customers' inventory, making sure that shortages do not happen.

Finally, it is worth mentioning that, in recent years, the subclass of problems known as *Rich Vehicle Routing Problems (RVRPs)* has been receiving increased attention. There is a trend in the scientific community to apply the VRP to real-life case studies by combining several constraints from classical VRP variants in the same optimization model. The goal is to develop very realistic models that can account for the numerous specificities a real-world problem may arise, albeit the debate is still ongoing as to what exactly defines a VRP as «*rich*» [78].

2.3 Green Vehicle Routing Problems

Especially since 2006, green freight transportation has enjoyed a growing preponderance in the OR field and has captured the attention of many researchers. The emergence and dissemination of GL and GSCM practices reflect the need to address the challenges the companies face when trying to balance the economic costs and the negative environmental externalities of their operations. Transportation lies at the forefront of the logistic distribution networks, is a significant cost, and a major contributor to such externalities.

As an emergent branch of research, the general class of problems designated as *Green Vehicle Routing Problems (GVRPs)* pertains to the inclusion of different cost metrics, namely environmental but also social ones, alongside with the more traditional economic metrics found across most of the VRP literature. The GVRPs thus reflect the increasing sensitivity towards the environmental concerns and the consideration of wider objectives and operational/human constraints in the context of vehicle routing [79]. Ultimately, the construction of models for route planning which generate more sustainable routing schemes is intended.

In the GVRPs, the main goal is to harmonize the different metrics so that the negative externalities associated to transportation are minimized. Most of the research on the GVRPs has been devoted to topics such as energy/fuel consumption [80], GHG emissions [81], electric vehicles [82], reverse logistics [83], and waste collection [84]. The latter two topics have a strong connotation with the PDPs, which have been previously reviewed. The review herein is thus focused on the first three topics. Comprehensive surveys on the

GVRPs can be found in [85, 86].

2.3.1 The Green-Vehicle Routing Problem

The research on the *Green-Vehicle Routing Problem* (G-VRP) focuses on the optimization of the vehicles' energy consumption during transportation. The consumption of fossil fuels is still the main source of today's freight road vehicles energy requirements and its minimization can help to curb the GHG emissions to the atmosphere, alongside with the creation of more operationally efficient routing schemes for the companies.

Another problem that arises is the necessity of refueling (or recharging, in the case of the electric vehicles) to overcome the fuel tank (or the battery) capacity constraint. Either fossil fuel-powered vehicles or alternative-fuel powered vehicles can be considered. The problem very much suits the companies that serve a set of geographically dispersed customers and need to plan ahead for the refueling, for there may exist a limited refueling infrastructure.

Erdoğan and Miller-Hooks [87] introduced the G-VRP allowing for the possibility that the vehicles may stop to refuel/recharge along their routes. In each route, the vehicles may stop at one or more alternative fuel stations (AFSs), so as to extend their driving range. The depot can also serve as an AFS. A homogeneous fleet of vehicles, whose fuel consumption rate is assumed to be constant and known *a priori*, was considered. The travel speeds were assumed to be constant over an arc, no limit was imposed on the number of refueling stops a vehicle could make and the vehicles were refueled to full capacity whenever they stopped at an AFS. The service times at the AFSs and at the customers were also considered.

The model sought to find the set of minimum distance routes, subject to time windows and fuel tank capacity constraints. A *mixed-integer linear programming* (MILP) formulation for the problem, which was solved through CPLEX, was proposed. Two heuristics were also proposed for solving the larger instances: one was based on a modified C&W heuristic and the other on the *Density-Based Spatial Clustering of Applications with Noise* (DBSCAN) algorithm. To improve a feasible solution found by the former heuristics, an improvement heuristic was run.

The results indicated that the total distance traveled decreased as the number of AFSs increased, particularly when the AFSs were located near the customers. When this happened, the model could more efficiently choose when and where the vehicles stopped to refuel. The results also showed a superior performance of the heuristics comparatively to the exact approach, especially on the larger instances.

Since the introduction of the G-VRP, numerous other approaches have been developed for the problem. One example is the work by Koç and Karaoglan [88] who proposed a matheuristic approach. The matheuristic combines a *Branch-and-Cut* (B&C) algorithm and a *Simulated Annealing* (SA) metaheuristic. In each iteration, SA finds a feasible solution which is passed onto the B&C. The B&C then tries to improve the solution. In this case, SA serves as an outer-level procedure defining both the acceptance criteria for a solution and the stopping criteria for the overall procedure.

The authors noted that SA was in fact able to improve the performance of the B&C. Improved solutions with tighter lower bounds than those of Erdoğan and Miller-Hooks [87] were reported within a reasonable computation time. It was also concluded that the G-VRP was easier to solve when the number of AFSs increased.

Recently, Yu *et al.* [89] proposed the first improved *Branch-and-Price* (B&P) algorithm for solving the G-VRP with a heterogeneous fleet and time windows. The authors explicitly considered the CO₂ emissions as a function of the fuel consumption rate. To speed up the finding of a solution for the pricing sub-problem, a *Multi-Vehicle Approximate Dynamic Programming* (MVADP) algorithm was used. This method was found to be highly efficient, reducing the total required computation time. Several other publications on the G-VRP can be found, for example, in [90]-[92].

2.3.2 The Energy-Minimizing Vehicle Routing Problem

The research on the *Energy-Minimizing Vehicle Routing Problem* (EMVRP) focuses on the minimization of the vehicles' energy/fuel consumption through the optimization of the load carried. Although a vehicle's fuel consumption depends on numerous factors other than the load, which are detailed further in the section 2.4 and in the subsection 2.4.2, the load is known to significantly impact it. A heavier vehicle, or a higher loaded vehicle, will consume more fuel than a lighter or a less loaded one.

Through the optimization of the load carried, it is expected that savings can be obtained in the energy/fuel consumption in comparison to the more traditional distance- or time-minimizing objectives.

The EMVRP was first proposed by Kara *et al.* [93]. The cost objective function was load-based. It accounted for the load carried in the vehicles, as well as the distance traveled, rather than just the distance. The objective was to minimize this weighted load function: the amount of load carried multiplied by the distance it is carried.

This approach, however, simplified the energy a vehicle consumed as the work it produced when traveling a certain distance. Although work is a form of energy, such an approximation inaccurately reflected the actual energy consumed as it failed to account

for several vehicle and road specific characteristics, among other factors, which also significantly impact the energy/fuel consumption.

The results indicated that if the total energy was minimized, the distance traveled increased. Conversely, if the total distance traveled was minimized, the total energy consumed increased. Such a trade-off occurred because the average load carried was higher when the distance traveled was minimized: it could happen that more customers were inserted into the same route, thus aggravating the energy/fuel consumption.

Recently, Ghannadpour [94] presented a multi-objective formulation for the EMVRP and proposed an evolutionary algorithm to solve it. Time windows and the customers' satisfaction were considered. The vehicles' energy/fuel consumption was also considered to be equivalent to the work done, although the formulation was able to account for the roads' friction coefficients and slopes.

The customers were divided into very important, important, casual, and unimportant. Different time windows for each customer type were considered. It is argued that classical time windows disregard the customers' preference to be served on a specific time as they assume a uniform 100% satisfaction pattern throughout the considered interval. The time windows were modeled as fuzzy, with stated preferences, and with tighter bounds the more important a customer was deemed. The customers were served according to their importance.

Three objectives were considered: the minimization of the total energy/fuel consumption, the minimization of the number of vehicles used, and the maximization of the customers' satisfaction, which was gauged by time window compliance. The model was solved with the proposed evolutionary algorithm and its performance was compared against a *Non-Dominated Sorting Genetic Algorithm-II* (NSGA-II) and CPLEX.

The evolutionary algorithm outperformed the NSGA-II, although some instances were found to lead to longer waiting times at the customers, which can translate into costs in some transportation systems. The results indicated that a total fuel consumption-minimizing objective yielded an average of 5.6% savings in the total fuel consumption comparatively to using a distance-minimizing objective. However, the total distance traveled increased by 10.6% in this setting. Several other publications on the EMVRP can be found in [95]-[97].

2.3.3 The Electric Vehicle Routing Problem

Electric vehicles (EVs) use stored battery energy for their locomotion. Regarded as the cleanest vehicles for achieving zero GHG emissions during their use, the problem of routing EVs has been receiving considerable attention by the researchers, becoming known as the

Electric Vehicle Routing Problem (EVRP). The EVRP suits, for example, companies that have the ability to deploy the EVs in urban areas where the delivery of less-than-truckload quantities may be required.

Companies, as well as public entities all over the world, have been renewing and/or converting their fleets from the traditional internal combustion engine (ICE) to battery-powered engines, which use renewable and thus more sustainable energy. However, the EVs present some additional routing challenges than their ICE counterparts: from the vehicles' driving range, to the duration (and price) of the batteries, the duration of the charging process, or even the availability of charging stations.

Gonçalves *et al.* [98] presented a case study for the optimization of a portuguese battery supplier distribution network. Both ICE vehicles and EVs were considered. A VRPPD was studied under four scenarios: (i) the company's current policy, which used ICE vehicles only; (ii) the optimization of the company's current policy; (iii) a mixed fleet policy; (iv) a policy with EVs only. MILP models were proposed for all the scenarios. The driving range and the recharging time for the EVs were considered. However, the trips to the charging stations were not accounted for. It was assumed that the EVs could recharge at the customers' location. A p -median approach was followed in order to divide the customers into clusters, so as to reduce the computational burden.

Optimizing the company's current policy was found to be the most cost-effective one. This stemmed from the fact that the implied cost for leasing the EVs was considered to be significantly higher than the daily operating cost for the ICE vehicles, which the company already owned. Under the mixed policy, and despite a slight increase in the total cost with regard to the company's current policy, both the total driving time, the total distance traveled and the number of vehicles used was lower. The policy with EVs only was the most costly one, which is explained by the implied cost of leasing these vehicles. Under this policy, the total driving time was also extended as the EVs would take time to recharge.

Hiermann *et al.* [99] introduced the *Fleet Size and Mix Electric Vehicle Routing Problem with Time Windows and Recharging Stations*. This work allowed for the possibility that the vehicles could stop at several charging stations (RSs). It was assumed that the vehicles were recharged to full capacity whenever they stopped at a RS. The charging time was accounted for. Each vehicle type was characterized by its respective payload, maximum energy capacity, rate of energy consumption per unit distance traveled, and charging time per energy unit. The rate of energy consumption and the charging time were assumed to be constants and known *a priori* for each vehicle type. Thus, no function of energy consumption/charging was incorporated in the model.

The minimization of a cost objective function comprising the total acquisition cost of the EVs and cost with the total distance traveled was considered. A B&P algorithm to solve the smaller instances and an *Adaptive Large Neighborhood Search* (ALNS) meta-heuristic to tackle the larger ones were proposed. The results indicated that both the B&P and the ALNS were able to find the optimal solution within a short computational time for the instances containing 15 customers. As for the larger instances, the ALNS derived solutions with a maximum average difference of 1.9% from the optimal one.

Recently, Zhen *et al.* [100] explored the mode selection (electric or petrol) in hybrid electric vehicles (HEVs). The mode selection was formulated as a MILP model to minimize the energy consumption cost. The mode selection is important when considering HEVs since these vehicles can operate either on the ICE or the electric engine and decisions have to be made on what kind of stop is required (*i.e.*, for petrol refueling or electric recharging).

A MILP formulation for the problem was proposed and small- to mid-sized instances were solved. An *Improved Particle Swarm Optimization* (IPSO) procedure was proposed to tackle the larger instances. This hybrid procedure combines PSO with *Local/Variable Neighborhood Search* (L/VNS) and a labelling procedure.

The minimization of the total energy consumption cost (fuel and electricity combined) was considered. The fuel and electricity prices were considered to be known *a priori*. The energy consumption depended on the traveling speed of the vehicles and two consumption functions (for fuel and electricity) per unit distance were developed, which in turn lead to the four possible driving modes. The results indicated that IPSO was capable of obtaining the optimal solution even in some large-sized instances within 1 hour of computation. Other publications on the EVRP can be found, for example, in [82, 101]. Recent reviews on the EVRP can be found in [102, 103].

2.4 The Pollution-Routing Problem

The pressing necessity to curb the transport-related CO₂ emissions through the reduction of the vehicles' fuel consumption, either by optimizing the total distance traveled, the speeds, or the load carried, incentivized many researchers to develop optimization models that could contribute to more sustainable routing decisions.

Prior to the formal introduction of the Pollution-Routing Problem by Bektaş and Laporte [18], in 2011, other researchers had been developing optimization models which considered different constraints and objective functions to minimize the emissions.

Palmer [104] first integrated transportation planning with environmental modeling. The impact of the speed on the CO₂ emissions under several scenarios considering congestion and time windows was studied. The speed was held as a decision variable. The minimization of the total amount of fuel consumed, which was used as a proxy for the emissions, was considered. Although the effect of the load was not accounted for, the results indicated that up to 5% savings in the emissions could be obtained with a fuel consumption-minimizing objective with respect to the more traditional time- or distance-minimizing objectives.

From a similar perspective, Jabali *et al.* [105], Figliozzi [106], and Kuo [107] studied a TDVRP to minimize the total fuel consumption holding the speed as a decision variable. A similar approach is found in Fagerholt *et al.* [108] where the speed was considered as a decision variable and the minimization of the total fuel consumption was sought, in the context of maritime shipping.

As discussed in the subsection 2.3.2, Kara *et al.* [93] introduced the EMVRP. In the EMVRP, the minimization of the total energy/fuel consumed, which translates into emissions, is sought through the optimization of the load carried in the vehicles. Other studies pursued a similar approach, such as the ones of Peng and Wang [109] and Xiao *et al.* [80].

For instance, Xiao *et al.* [80] introduced a so-called *Fuel Consumption Vehicle Routing Problem* (FCVRP), an extension of the classical CVRP in which a fuel consumption estimation model was incorporated. The minimization of the total fuel consumption, which depended on the load carried in the vehicles, was sought. Although the authors did not consider the effect of the speed, the results indicated that around 5% savings in the fuel consumption could be obtained in the FCVRP, comparatively to using a distance-minimizing objective.

The previously cited publications show a trend in the scientific community. A need for more comprehensive optimization models that could simultaneously account for the numerous factors that influence a vehicle's fuel consumption and emissions was at hand. The aforementioned works considered the optimization of the vehicles' load or speed separately, not concurrently, while other factors were largely overlooked.

Although the load and speed are known to significantly impact a vehicle's fuel consumption, a recent report by Zacharof *et al.* [110] for the European Commission shows that, apart from these two factors, the auxiliary systems (*e.g.*, air conditioning (A/C) units and steering assist systems), aerodynamics (*i.e.*, the shape of the vehicle and addons), the weather conditions (*e.g.*, temperature, wind, rain, snow), the driving behavior, the vehicle's condition (*e.g.*, the engine friction factor, the rolling resistance of the tires),

the road's condition and morphology (*e.g.*, altitude, road grade, roughness), the traffic condition, the fuel's characteristics, and the route distance and duration are all important factors which influence a passenger vehicle's fuel consumption. For instance, the use of the A/C unit is estimated to increase the fuel consumption up to 9% [110]. An aggressive driving behavior may increase the fuel consumption as much as 24% [110].

Naturally, some of this information is dynamic in nature and may not be readily, or even easily, incorporated in a mathematical programming model. For example, the road grade can be seen as relatively constant for roads of the same type (*e.g.*, asphalt) and some of the available fuel consumption/emissions estimation models can account for such information. However, modeling the expected weather conditions for the day and the driving behavior (which may be conditioned by the former), for example, may not be so straightforward, and the available fuel/emissions estimation models may not be able to account for such information.

In light of this reality, and following the previous works, Bektaş and Laporte [18] proposed a new GVRP variant, which was designated as the *Pollution-Routing Problem*. The authors sought to incorporate some of the aforementioned factors concurrently in a more comprehensive optimization model for route planning. Their work is introduced and reviewed in the next subsections.

2.4.1 Statement and Main Findings

The *Pollution-Routing Problem* (PRP) was introduced by Bektaş and Laporte [18] as an extension of the classical VRPTW including a more comprehensive cost objective function. The objective function comprises the costs of the fuel/emissions and the drivers' wages. In the PRP, the vehicles' fuel consumption, which further translates into CO_{2(e)} emissions, is explicitly considered.

The PRP then consists of routing a set of capacitated vehicles to serve a set of customers within their respective time windows. The problem calls for the determination of the optimal speed and load on each arc of a route so that the overall cost objective function is minimized. The load and speed are decision variables in order to find their optimal values for each arc.

As far as the emissions are concerned, the amount of GHG a vehicle emits when it travels over an arc (i, j) was considered as a function of its load and speed, among other parameters. The emissions are linearly dependent on the vehicle's fuel consumption rate, which in turn is non-linearly dependent on several of the vehicle's characteristics.

The function of emissions per unit distance traveled for a light-weight vehicle was constructed in accordance with a simplified version of the *Comprehensive Modal Emissions*

Model (CMEM) of Barth *et al.* [111] and Barth and Boriboonsomsin [112, 113]. Bektaş and Laporte [18] assumed that all the parameters remain constant on a given arc of a route, although the load and speed may vary from arc to arc. This simplified version of the CMEM is introduced and explained in the next subsection.

In the main formulation for the PRP, the authors developed a non-linear MIP model. The model can, however, be linearized, as explained further in the subsection 2.4.3. A single depot, a capacitated and homogeneous fleet of vehicles, time windows and service times at the customers were considered. In this formulation, the objective is the minimization of the total routing cost, comprised of the formerly mentioned costs.

In addition to this formulation, three variants of the PRP were tested, regarding the inclusion of different objectives: the total distance, the total weighted load, and the total energy were, respectively, minimized. Notwithstanding, all the four formulations were tested with and without time windows, using realistic instances with 10, 15 and 20 cities of the United Kingdom.

The results showed that the PRP was significantly hard to solve even in the small-sized instances on which it was tested. Although the focus of the work was not the algorithmic performance, some processing times were provided to illustrate the behavior of the main formulation for the PRP.

For example, on the 10-node instances, CPLEX took an average of 3165.85 CPU seconds to find the optimal solution, with all the instances solved to optimality. However, none of the 15- and 20-node instances were solved to optimality within 3 hours of computational time limit. On the 15- and 20-node instances, the optimal solution was actually found early in the optimization. CPLEX spent the rest of the time trying to improve the lower bound so that the solutions' optimality was proven.

Several important insights were derived from this study. On the one hand, the drivers' wages dominated the total cost figure, not the CO₂ emissions. The reason for this was that the cost considered for the CO₂ was significantly lower than that of the drivers' wages. Furthermore, the estimates on the actual cost of the CO₂ may vary considerably (*e.g.* 10\$-20\$/ton [136]; 93\$/ton [137]). No consensus exists on a single price to put on it.

Unsurprisingly, the results showed that an energy-minimizing solution will only be of interest should the cost of CO₂ be of concern. The energy requirements from fuel, and its respective cost, are more prevalent in the total cost figure. Hence, a total cost-minimizing solution can yield a worse energy consumption value. In other words, the model seeks to minimize the total driving time, and thus the cost of drivers' wages, by increasing the speeds. However, increasing the speeds increases the fuel consumption/emissions.

Another important insight showed that minimizing the total distance traveled did not guarantee a solution where either the energy or the cost of drivers' wages were minimized. In fact, it could happen that the vehicles ended up visiting more customers in a route. When this happened, the vehicles carried a higher average load, thus consuming more fuel, and the total route duration could be extended, in turn increasing the cost of the drivers' wages.

Finally, as far as load-minimization is concerned, minimizing the total cumulative vehicle load did not necessarily imply energy minimization, particularly when the time windows were in place. Although the vehicles generally traveled with lower average loads to minimize the fuel consumption, it could happen that the total distance traveled was higher. Especially when the time windows were narrow, it could happen that the vehicles would need to travel at higher speeds to meet their customers' schedules, thus increasing the fuel consumption/emissions.

2.4.2 Fuel Consumption and Emissions

In this subsection, the simplified version of the function of emissions and fuel consumption based on the CMEM and used by Bektaş and Laporte [18] is explained. The instantaneous engine-out emission rate E , in g/s , for a given GHG is defined as:

$$E = \delta_1 F + \delta_2, \quad (2.4.1)$$

where δ_1 and δ_2 are GHG-specific emission indexes and F is the vehicle's fuel use rate (g/s). From (2.4.1), it is seen that the emissions are linearly dependent on the fuel use rate. A higher fuel consumption will directly translate into higher GHG emissions. The parameter F is calculated as follows:

$$F \approx \xi \left(kNV + \frac{P_t/\varepsilon + P_a}{\eta} \right) U, \quad (2.4.2)$$

where ξ is the fuel-to-air mass ratio, k is the engine friction factor ($kJ/rev/l$), N is the engine speed (rev/s), V is the engine displacement (l), P_t is the total tractive power demand requirement (kW) placed on the vehicle's wheels, ε is the vehicle drive train efficiency, P_a is the engine power demand (kW) from engine running losses and vehicle accessories (*e.g.*, A/C unit), η is an efficiency parameter for diesel engines, and U is the heating value (kJ/g) of a typical diesel fuel. Additionally, it can be referred that $P = P_t/\varepsilon + P_a$ is the engine power output ($kW/s = kJ$). The parameter P_a is assumed to

be equal to zero. The parameter P_t is calculated as follows:

$$P_t = \frac{(Mav + Mgv\sin(\theta) + 0.5C_dA\rho v^3 + MgvC_r\cos(\theta))}{1000}, \quad (2.4.3)$$

where M is the total mass (kg) of the vehicle (*i.e.*, curb weight plus carried load), a is the acceleration (m/s^2), v is the speed (m/s), g is the gravitational constant (m/s^2), θ is the road grade angle ($^\circ$), C_d is the coefficient of aerodynamic drag, A is the frontal surface area (m^2) of the vehicle, ρ is the air density (kg/m^3), and C_r is the coefficient of rolling resistance. The total amount of energy (kJ) consumed on a given arc (i, j) can be approximated as:

$$P_{ij} \approx P_t \frac{d_{ij}}{v_{ij}} \quad (2.4.4)$$

$$\approx \alpha_{ij}(w + f_{ij})d_{ij} \quad (2.4.5)$$

$$+ \beta v_{ij}^2 d_{ij}, \quad (2.4.6)$$

where $\alpha_{ij} = a + g\sin(\theta_{ij}) + gC_r\cos(\theta_{ij})$ is an arc specific constant and $\beta = 0.5C_dA\rho$ is a vehicle specific constant. The parameters w and f_{ij} are, respectively, the curb weight (kg) of the vehicle and the load (kg) carried for the arc (i, j). The term (2.4.5) is the load-induced energy requirement and the term (2.4.6) is the speed-induced energy requirement.

As stated earlier, it is assumed that all the parameters will remain constant on a given arc, although the load and speed may vary from arc to arc. Thus, for the arc (i, j), with length d_{ij} and road angle θ_{ij} , the vehicles will travel at an average speed v_{ij} , with a total mass of $M = w + f_{ij}$. In order to visualize the impact that the speed has on the fuel consumption, (2.4.2) can be rewritten as:

$$F(v) = \frac{kNV\lambda d}{v} \quad (2.4.7)$$

$$+ \frac{P_t\gamma\lambda d}{v}, \quad (2.4.8)$$

where $\lambda = \xi/v\psi$ and $\gamma = 1/1000\varepsilon\eta$ are constants. The parameter ψ is the conversion factor from g/s to l/s , so that (2.4.7)-(2.4.8) yields the fuel consumption in liters per second. The former expression can be further expanded as:

$$F(v) = \frac{\lambda d(kNV + w\gamma\alpha v + f\gamma\alpha v + \beta\gamma v^3)}{v}. \quad (2.4.9)$$

The indices (i, j) in (2.4.7)-(2.4.9) have been omitted from the variables v, d, f and α for the sake of simplicity in the notation.

From (2.4.7)-(2.4.8) it is seen that the fuel consumption as a function of speed has two terms. The first term measures the effect of kNV (which is designated as the engine module) on the fuel consumption and is mainly dependent on the characteristics of the vehicle's engine. The second term measures the speed and load-induced effects (*i.e.*, the effect of P_t) on the fuel consumption and its relative contribution is mainly dictated by how fast a vehicle travels, but also by its total mass.

The behavior of the fuel consumption as a function of speed (2.4.7)-(2.4.8) for two different light-weight diesel-powered vehicles is shown in the Figure 2.4.1.

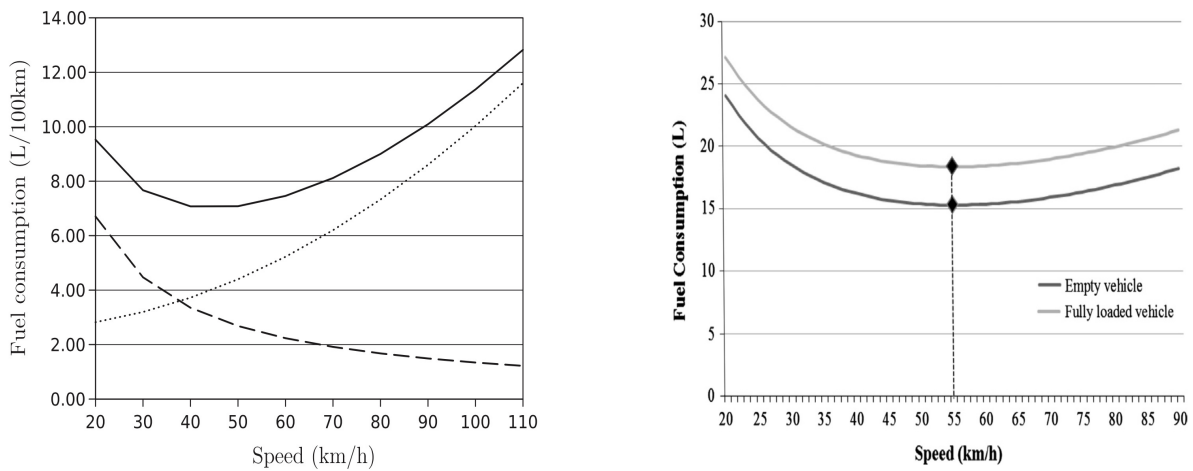


Figure 2.4.1: Fuel consumption as a function of speed ([18, 139]).

The Figure 2.4.1 shows the non-linear behavior of the fuel consumption as a function of speed. On the left-hand side figure, the dashed line shows the contribution of kNV and the dotted line the contribution of P_t . It is seen that the contribution of kNV is mainly significant for speeds lower than $40\text{km}/h$, whereas the contribution of P_t is more significant for higher speeds. When added, these two terms result in the continuous line which shows the total fuel consumption. It is noted that, for the considered light-weight vehicle, the optimal speed at which the fuel consumption (and consequently the emissions) is minimal is $40\text{km}/h$. Bektaş and Laporte [18] assumed speeds of at least $40\text{km}/h$.

Naturally, the speed at which the fuel consumption is minimal depends on the type of vehicle considered and its characteristics. On the right-hand side figure, this corresponds to $55\text{km}/h$. This figure also shows that the fuel consumption is linearly dependent on the load. Hence, a higher loaded vehicle consumes more fuel than a lighter loaded one.

In the Table A.1, provided in the Appendix A, the parameters and their typical values used in the PRP model of Bektaş and Laporte [18] are summarized. The vehicle and engine parameters are reported for a light-weight diesel-powered vehicle.

As stated at the beginning of this subsection, this fuel consumption model is a simplified version of the CMEM. The CMEM is an instantaneous model: it permits the calculation of the second-by-second fuel consumption rate if the second-by-second data is readily available at the planning stage.

Bektaş and Laporte [18] stress the two reasons for their choice of this fuel consumption/emissions model. The CMEM is able to «reflect the change in the vehicle load as it travels», whereas other models «assume a static load profile». It is «also applicable to heavy-duty vehicle emission estimations which apply to freight transportation». Notwithstanding, the modal emissions models are more accurate in the estimation of the fuel consumption/emissions than their counterparts (*i.e.*, emissions factor models and average speed models), since they can take microscopic (second-by-second) data as input [39].

Two simplifications are made when using this model. The parameter P_a is assumed to be equal to zero. It is assumed that no engine running losses exist. Still, from (2.4.3) it is seen that the instantaneous acceleration a of the vehicle is a parameter of the model. Bektaş and Laporte [18], and several studies that ensued, assume the acceleration and deceleration (A/D) rates to be equal to zero since average speeds for each arc (i, j) are considered.

As stated in Raeesi and Zografos [116], most studies ignore the instantaneous A/D rates because this microscopic data is «usually unavailable at the planning stage». However, as stressed by these authors, ignoring the A/D rates can result in misleading routing decisions because the error in the estimation of the fuel consumption can be significant. The results indicate that the error in the estimation can be of around 45% to as high as 80%. A strategy to cope with the lack of this information is further discussed in Chapter 3 when the authors' work is presented.

2.4.3 Formulation

According Bektaş and Laporte [18], the PRP is defined on a complete and directed graph $G = (N, A)$. A single depot, a homogeneous fleet, and hard time windows were considered. The indices, parameters (see also the Appendix A), sets, and decision variables are described and summarized below.

Indexation:

$i, j \rightarrow$ Indices for the customers/arcs;

$r \rightarrow$ Index for the speed levels.

Parameters:

$n \rightarrow$ number customers;

$a_i \rightarrow$ time window lower bound of the customer i ;

$b_i \rightarrow$ time window upper bound of the customer i ;

$t_i \rightarrow$ service time of the customer i ;

$q_i \rightarrow$ demand of the customer i ;

$d_{ij} \rightarrow$ distance between the node i and the node j ;

$m \rightarrow$ number of vehicles available;

$Q \rightarrow$ maximum payload of a vehicle.

Sets:

$N \rightarrow$ Set of nodes: $N = \{0, 1, \dots, n\}$. The node 0 is the depot;

$N_0 \rightarrow$ Set of customers: $N_0 = N \setminus \{0\}$;

$A \rightarrow$ Set of arcs: $A = \{(i, j) : i, j \in N, i \neq j\}$;

$K \rightarrow$ Set of homogeneous vehicles: $K = \{1, \dots, |K|\}$;

$R \rightarrow$ Set of speed levels: $R = \{1, \dots, |R|\}$.

Decision Variables:

$x_{ij} \rightarrow$ Binary variable equal to 1 if a vehicle traverses the arc (i, j) . Equal to zero otherwise;

$z_{ij}^r \rightarrow$ Binary variable equal to 1 if a vehicle traverses the arc (i, j) at the speed level r . Equal to zero otherwise;

$f_{ij} \rightarrow$ Continuous variable representing the load carried by a vehicle for the arc (i, j) ;

$\bar{v}^r \rightarrow$ Continuous variable representing the *average speed* of the speed level r ;

$y_i \rightarrow$ Continuous variable representing the service time start at the customer i ;

$s_j \rightarrow$ Continuous variable representing the total driving time for a route that has the customer j as the last customer.

The non-linearity of the program is due to the expression of the fuel consumption and is handled by a discretization of the speeds. Firstly, it is assumed that the speed limitations are the same for every arc. Thus, $l_{ij} = l = v_{min}$ and $u_{ij} = u = v_{max}$, $\forall (i, j) \in A$. Let then $R = \{1, \dots, |R|\}$ denote a set of non-decreasing speed levels. For a given arc (i, j) , each $r \in R$ corresponds to a speed interval $[l^r, u^r]$ where $l^1 = l$ and $u^{|R|} = u$. The *average speed* for each $r \in R$ is then calculated as $\bar{v} = (l^r + u^r)/2$. Let then z_{ij}^r denote a binary variable equal to 1 if a vehicle traverses the arc (i, j) at the speed level r and equal to 0 otherwise. The PRP is formulated as a MILP model as follows:

$$\text{minimize } \sum_{(i,j) \in A} (c_f + e)w\alpha_{ij}d_{ij}x_{ij} \quad (2.4.10)$$

$$+ \sum_{(i,j) \in A} (c_f + e)f_{ij}\alpha_{ij}d_{ij} \quad (2.4.11)$$

$$+ \sum_{(i,j) \in A} (c_f + e)\beta d_{ij} \left(\sum_{r \in R} \bar{v}^2 z_{ij}^r \right) \quad (2.4.12)$$

$$+ \sum_{j \in N_0} ps_j \quad (2.4.13)$$

subject to

$$\sum_{j \in N_0} x_{0j} = m \quad (2.4.14)$$

$$\sum_{j \in N} x_{ij} = 1 \quad \forall i \in N_0, \quad (2.4.15)$$

$$\sum_{i \in N} x_{ij} = 1 \quad \forall j \in N_0, \quad (2.4.16)$$

$$\sum_{j \in N} f_{ji} - \sum_{j \in N} f_{ij} = q_i \quad \forall i \in N_0, \quad (2.4.17)$$

$$q_j x_{ij} \leq f_{ij} \leq (Q - q_i) x_{ij} \quad \forall (i, j) \in A, \quad (2.4.18)$$

$$y_i - y_j + t_i + \sum_{r \in R} \frac{d_{ij}}{\bar{v}^r} z_{ij}^r \leq M_{ij}(1 - x_{ij}) \quad \forall i \in N, \forall j \in N_0, i \neq j, \quad (2.4.19)$$

$$a_i \leq y_i \leq b_i \quad \forall i \in N_0, \quad (2.4.20)$$

$$y_j + t_j - s_j + \sum_{r \in R} \frac{d_{j0}}{\bar{v}^r} z_{j0}^r \leq L(1 - x_{j0}) \quad \forall j \in N_0, \quad (2.4.21)$$

$$\sum_{r \in R} z_{ij}^r = x_{ij} \quad \forall (i, j) \in A, \quad (2.4.22)$$

$$f_{ij} \geq 0 \quad \forall (i, j) \in A, \quad (2.4.23)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A, \quad (2.4.24)$$

$$z_{ij}^r \in \{0, 1\} \quad \forall (i, j) \in A, r \in R. \quad (2.4.25)$$

The objective function (2.4.10)-(2.4.13) comprises four terms and seeks the minimization of the total routing cost. The term (2.4.10) measures the total cost incurred due to the vehicles' curb weight. The term (2.4.11) measures the total cost incurred due to the load carried in the vehicles. The term (2.4.12) measures the total cost incurred due to speed variations. The term (2.4.13) measures the total cost incurred due to the drivers' wages. The terms (2.4.10)-(2.4.12) directly measure the total cost of the fuel consumption and emissions as they are multiplied by their respective unit costs $(c_f + e)$.

The constraints (2.4.14) ensure that exactly m routes are created, one for each vehicle. The constraints (2.4.15)-(2.4.16) are degree constraints and ensure that each customer is visited exactly once. The constraints (2.4.17) ensure the balance of flow. The constraints (2.4.18) are capacity constraints and serve also as SEC. The constraints (2.4.19)-(2.4.20) enforce the time windows and M_{ij} is a sufficiently large number computed as $M_{ij} = \max\{0, b_i + s_i + d_{ij}/l_{ij} - a_j\}$. The constraints (2.4.21) are used to calculate the total driving time s_j for each route (thus for each vehicle) and L is a sufficiently large and positive constant. The constraints (2.4.22) ensure that only one speed level is chosen per arc. The constraints (2.4.23) ensure that the load carried for an arc cannot be negative. Finally, the constraints (2.4.24)-(2.4.25) are integrality constraints.

2.5 Conclusion

In this chapter, the Pollution-Routing Problem was introduced and described in detail. This problem was framed in the general context of the VRPs and GVRPs. Several different and important variants have been introduced and discussed. The specific constraints of each one and the typical objectives considered have been explained. Some solution methodologies used to solve these problems have been briefly introduced. The importance of the GVRPs in allowing for the reduction of the negative environmental externalities of transportation was also underlined.

The PRP, as an extension of the VRPTW, is a NP-Hard problem. To this extent, several different approaches have been developed. In the next chapter, the state of the art on the PRP is presented and a selection of publications is reviewed. A special emphasis is given to the solution methodologies employed to solve this problem.

Chapter 3

State of the Art

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3.1 Introduction

In the previous chapter, the PRP was formally introduced. In this chapter, the selected contributions on this problem are reviewed and discussed.

The review herein divides the contributions on this problem either according to the solution methodology employed or the formulation provided. This chapter is organized as follows: (i) the sections 3.2 and 3.3 address the exact and heuristic/metaheuristic approaches, respectively. In these sections, only single-objective approaches are referred to; (ii) the section 3.4 addresses the bi- and multi-objective approaches; (iii) the section 3.5 addresses the matheuristic approaches; (iv) the section 3.6 addresses bi-level optimization.

As an extension of the VRPTW, the PRP is a NP-Hard combinatorial optimization problem. This renders the use of exact methods unsuitable for dealing even with some small- to mid-sized instances. Some exact approaches are found in the literature, although the metaheuristic approaches are particularly common.

3.2 Exact Approaches

The first category of approaches for the PRP consists of exact methods. These methods guarantee that the optimal solution for the problem is found. However, they may become unsuitable for tackling even some small- to mid-sized instances, depending on the overall complexity of the problem. They may, otherwise, take a prohibitive long time to run. Most commonly, the researchers have resorted to general exact solvers of which IBM's® ILOG® CPLEX® is the most widely used. CPLEX is a state-of-the-art solver and offers several algorithms designed to tackle a wide range of ILP, MILP, and quadratic problems.

3.2.1 General Solver Approaches

3.2.1.1 A Time-Dependent PRP

The first exact approach for the PRP is due to Franceschetti *et al.* [117] who introduced the time-dependent version of the problem. The authors proposed a MILP formulation which was solved using CPLEX. Similarly to the TDVRP, in the *Time-Dependent Pollution-Routing Problem* (TDPRP) the traffic congestion is explicitly considered and the departure time from a given node is a decision variable.

It was assumed there existed an initial period of congestion. This period lasted for a units of time and represented the morning rush hour. A period of free-flow ensued. No congestion period for the afternoon rush hour was considered, although such a consideration was offered as a possible future research direction.

The vehicles could travel at two discretized speed levels: a so-called *congestion speed* and a *free-flow speed*, depending on the period of the day. The congestion speed v_c was assumed to be a constant. Naturally, such a consideration constrains the vehicles to only one possible speed during the congestion period. When congestion is being accounted for, it can be argued that it is important to model the A/D rates. The drivers are forced to constantly accelerate and decelerate and change gears often. As noted earlier, the A/D rates may have a significant impact on the fuel consumption. However, such an approach was not followed. The free-flow speed v_f varied within discrete values between a minimum and a maximum speed, similarly to the speed levels considered in Bektaş and Laporte [18]. A transient region existed between the congestion period and the free-flow period. In the transient region, the vehicles traveled at v_c until the congestion period dissipated. Afterwards, the vehicles traveled the remaining distance at a chosen v_f . The speeds were modeled as a step-function of time and the travel time varied linearly in the transient region.

Two remuneration policies for the drivers' wages were considered: one in which the drivers were paid from the beginning of the time horizon and until they returned to the depot, and another where the drivers were paid only for the time they spent *en-route* and serving the customers. The difference between the two is that, in the latter policy, the drivers were not paid for the time they had to wait at the depot before departing.

The MILP formulation for the TDPRP was tested on the same instances of Bektaş and Laporte [18]. The results indicated that the TDPRP outperformed it, both in computational time and in finding solutions for some 15- and 20-node instances.

When traffic congestion must be considered, using a time-dependent formulation could yield significant savings in the total cost. An average of 5% to 15% over a formulation that does not incorporate this dependency could be obtained. Also, the potential for cost reduction increased proportionally to the length of the congestion period. The longer the congestion period, the more advantageous it was, from a cost perspective, for the drivers to wait that period at the depot, or at a customer, before departing. This was especially true if the drivers were paid from their departure time. If this policy was considered, the savings could more than double.

3.2.1.2 A Pickup and Delivery PRP

Tajik *et al.* [126] introduced a *Pickup and Delivery* PRP and proposed a robust MILP model to deal with the uncertain input data. The model was solved using GAMS.

The customers were partitioned into two disjoint subsets: those whose loads needed to be picked up, and those to whom such pickups needed to be delivered. In each route,

the pickup customers were visited first. Subsequently, the vehicles distributed the loads to the delivery customers. It was assumed that extra loads could return to the depot at the end of each route. Thus, the compound demand of all the delivery customers did not have to be equal to the total load picked up from the pickup customers. Additionally, the vehicles could also load up at the depot.

The fuel cost, the emissions cost, the service time at each customer, and the speeds (thus the traveling times) were considered to be uncertain parameters. Their values depended on a normalized value and the level of uncertainty associated to each parameter. A uniform random distribution was used to model this uncertainty. By using such a distribution, it was assumed that all the possible outcomes were equally likely.

The authors tested their MILP model considering both a scenario where there was no uncertainty (a deterministic model) and one where there was (the robust model). As expected, the robust model derived solutions with higher total costs than its deterministic counterpart.

As far as network balance is concerned, on an instance with 9 nodes comprising 8 pickup nodes and 1 delivery node, the total cost was at its highest. This happened because the average load carried and the distance it was carried were high, thus impacting on the fuel/emissions costs.

As the number of delivery nodes increased, the total cost decreased, reaching its minimum when there were 5 pickup nodes and 4 delivery nodes. This granted a better network balance and the vehicles could more efficiently pick up and deliver during the routes, instead of loading up at the depot. If the number of delivery nodes continued to increase, the total cost also increased, as the vehicles would load up more at the depot and carry a higher average load during the routes.

3.2.2 Branch-and-Price

Dabia *et al.* [127] proposed a different PRP variant and developed a B&P algorithm. The speed was not held as a decision variable. Instead, an average speed was fixed for each arc of a route. It was argued that defining an average speed is more practical from the drivers' point of view, but also because it makes the pricing problem easier to solve. Additionally, the departure time from the depot was considered as a decision variable. It is argued that this decision is more practical if one wishes to implement a remuneration policy whereby the drivers are paid only for the actual time spend *en-route* and serving the customers.

The proposed B&P algorithm comprised a master problem, which was a *Set-Partitioning* problem (SP), and a pricing sub-problem, which was a *Speed and Start-time Elementary*

Shortest Path Problem with Resource Constraints. The SP found the minimum cost routes while guaranteeing that each customer was visited by only one path. The pricing sub-problem was solved using a tailored labelling algorithm and found the departure time from the depot, as well as the speed for each individual route.

The pricing sub-problem searched for the variables (paths) with a negative reduced cost, whose column was then added to the master problem (SP). A LP-relaxation of the master problem was solved using column generation. Several branching rules were used to enumerate the candidate solutions. Prior to making a branching decision, each branch candidate was evaluated by solving the corresponding LP-relaxation for the two child nodes. The branch which maximized the lower bound was selected.

The algorithm was tested on instances containing 10, 15 and 20 nodes. Two different problems were compared. The first problem (P2) considered fixed speeds for all the arcs of a route. The second problem (P3) considered the same assumption but also that the departure time from the depot could be postponed. The results indicated that P2 allowed for a marginal decrease (0.2%, on average) in the total cost comparatively to the values reported by Demir *et al.* [129]. P3 allowed for a 1.6% decrease, on average.

3.2.3 An ε -accurate approach

Xiao *et al.* [128] introduced the *Continuous* PRP (CPRP) and proposed an ε -accurate MILP model which optimized the decision variables in their piecewise linear domain. The minimization of the total routing cost, comprising the fuel cost and the cost of the drivers' wages, was considered. The cost of the drivers' wages was studied under the two policies previously discussed in Franceschetti *et al.* [117].

Unlike the previous studies, the authors did not consider discretized speeds. Instead, the non-linear relations between the travel time and speed ($t = d/v$), and the fuel consumption as a function of speed were linearized using secant lines. These relations were transformed into piecewise linear curves instead.

Naturally, when the objective function is minimized (or, respectively, maximized), such an approximation is only valid if the relations are concave (or convex), which is the case. The minimum number of secant lines to use depends on the desired maximum deviation ($\varepsilon\%$) from the actual non-linear functions.

This linearization method was proposed so that the speeds in the solutions were more closely optimized. That is, a model using the speed as a continuous variable likely achieves better solutions than one using discretized speeds. Also, there is no need to use binary decision variables to model the selection of a speed level if the speed is held as a continuous decision variable. This can help to lighten the computational burden.

Since the proposed linearization method derived solutions which were only approximations, *i.e.*, an optimal solution for the ε -CPRP (linearized) is only a feasible solution for the CPRP (with the non-linear relations), a post-optimization procedure was conducted on a feasible solution to eliminate the approximation errors.

Extensive proof was provided that there always existed a feasible solution for the non-linearized model which was at least as good as (or better) than the one found for the linearized model, the same applying to the post-optimization procedure.

The proposed method was computationally efficient on instances with up to 25 customers. Most instances were solved to optimality in just a few seconds. New optimal values were obtained on some instances. The authors concluded that their method could yield better solutions than the discretized PRP models.

3.3 Heuristic and Metaheuristic Approaches

Heuristics and metaheuristics are approximate solution methods which may, or not, generate feasible solutions for a problem. Nevertheless, these methods can derive good-quality sub-optimal solutions within a more reasonable computation time than that of the exact resolution.

Decisions must often be made on-the-spot and a model that takes a prohibitive long time to run may lose its practical relevance. Obtaining sub-optimal solutions can be of interest, provided that some assurance can be given about their quality. These methods do not guarantee the quality of a solution, but usually derive near-optimal ones [25]. The quality of a solution is commonly assessed in benchmark instances where the optimal solution is known.

Heuristics are iterative algorithms that are likely to discover good-quality feasible solutions. In each iteration, the solution space is searched for a new solution that is better than the previous one. These algorithms use a set of user-specified rules on how to search for these solutions, often based on common-sense or empiric ideas. They start from an initial solution and search its neighborhood. Ultimately, the process converges to a *best-known* solution, which is the best the algorithm was able to find.

In broad terms, the heuristics are divided into two main categories: constructive heuristics and improvement heuristics. The constructive heuristics start from an empty solution and iteratively construct a feasible solution. The improvement heuristics start from a feasible solution and iteratively try to improve it. This is usually done by performing intra- and/or inter-route exchanges. Hybridizations of these two categories are possible. An algorithm may comprise a constructive and an improvement phase.

Among others, two examples of constructive heuristics include the C&W heuristic and petal algorithms [118]. Among others, two examples of improvement heuristics include the λ -opt exchange, where λ nodes are removed from a route and replaced by other λ nodes, and the swap move, where consecutive customers are swapped between different routes.

Metaheuristics are high-level procedures which control lower-level heuristics. These procedures provide the general framework and guidelines on how to search the solution space. The metaheuristics are divided into two main categories: local search methods and population-based methods. The local search methods explore the solution space one solution per iteration and focus on improving a single candidate solution. The population-based methods explore it by iteratively improving several candidate solutions which are part of a population. They often use diversification strategies to modify the solutions in the population to guide the search.

Examples of local search methods include algorithms such as SA [119], *Tabu Search* (TS) [121], ALNS [131] and *Iterated Local Search* (ILS) [151], among several others. Examples of population-based methods include *Genetic Algorithms* (GA) [123], *Particle Swarm Optimization* (PSO) [120], *Ant Colony Optimization* (ACO) [124], *Differential Evolution* (DE) [122], among several other bio-inspired algorithms [125].

3.3.1 Adaptive Large Neighborhood Search

The ALNS metaheuristic was first introduced by Ropke and Pisinger [131] as an extended version of the *Large Neighborhood Search* (LNS) heuristic proposed by Shaw [132]. Unlike the LNS, which uses just one heuristic operator to search the (large) neighborhood structure of a solution, the ALNS combines several destroy and repair operators.

In each iteration, one destroy and one repair operator are dynamically and probabilistically selected according to their past performance using a roulette wheel mechanism. Each operator is assigned a score. If it improves the current best-known solution, it is rewarded, and will more likely be selected in the next iteration. In other words, these sub-heuristics compete with each other in order to improve the solution. This iterative procedure stops when the stopping criteria is met.

Ropke and Pisinger [131] demonstrated the efficacy of the proposed method when applied to derive solutions for a PDP. The results showed that the ALNS improved the best-known solutions in more than 50% of the instances on which it was tested.

3.3.1.1 ALNS applied to the PRP

The first metaheuristic approach for the PRP is due to Demir *et al.* [129]. An extended version of the ALNS was proposed. This extended version considered 3 new destroy operators and 2 new repair operators. The remaining operators were taken and adapted from Shaw [132] and Ropke and Pisinger [131]. In total, 12 destroy operators and 5 repair operators were considered.

The destroy operators consist of removing one to several nodes, or even an entire route from the solution, based on a set of predefined rules. For example, a node may be removed from a route if a high cost is incurred in serving it in that route. The repair operators consist of adding one to several nodes to a route, also based on a set of predefined rules. For example, a node may be greedily inserted in the position that yields the least cost increase. In short, a destroy operator partially deconstructs the solution and a repair operator rebuilds it differently.

The extended ALNS was embedded in a SA framework. SA served as an outer-level procedure and defined the criteria via which a solution found by the ALNS was accepted or rejected, as well as the stopping criteria for the overall procedure. SA is a greedy local search procedure which always accepts a current solution if it has a better objective value than the incumbent one. To try to escape local optima, a current solution which is worse than the incumbent one may be accepted with a given probability. This probability is modeled using a *temperature* variable, which decreases in each iteration, so that the probability of accepting a worse solution than the incumbent one decreases over time. When the temperature reaches its minimum and pre-specified value, the procedure stops. When the overall procedure (SA+ALNS) stopped after having found a feasible solution, a *Speed Optimization Algorithm* (SOA) was run to possibly improve it.

The method then operated in two distinct stages. At the first stage, an initial feasible solution was computed using a modified C&W heuristic. This solution was generated while ensuring the feasibility for the vehicles' capacity and the time windows. This solution was then fed to the ALNS. The VRPTW part of the PRP was solved using the ALNS. At this stage, fixed speeds were used. This was an iterative process whereby fixed speeds were fed to the ALNS, which used them as input for solving the VRPTW. At the second stage, when the SA+ALNS procedure stopped after having found a feasible solution, the SOA was run as a post-optimization procedure. The SOA then sought to optimize the speeds for each arc of a route in order to improve the solution.

This metaheuristic was proposed so that good-quality solutions were obtained within a comparatively short computational time to that of the exact resolution. The authors also proposed the first sets of PRP benchmark instances, which range from 10 nodes to

200 nodes. Some of these sets are used in our computational experiments, in Chapter 5.

The results showed that the extended ALNS was highly effective and able to find good-quality solutions. For example, on the 10-node instances, CPLEX took an average time of around 509 seconds to find the optimal solutions. The extended ALNS found them on an average of just 2.3 seconds. On some 200-node instances, the results were even more significant. In just over 10 minutes of computation, the ALNS metaheuristic found solutions that were up to 30% better than those CPLEX could find in 3 hours.

3.3.1.2 A Fleet Size and Mix PRP

Koç *et al.* [134] introduced the *Fleet Size and Mix* PRP. Three vehicle types were considered: light-duty, medium-duty, and heavy-duty vehicles. Each vehicle type had its own specific characteristics (*e.g.*, payload, curb weight, among others). The cost of using a certain vehicle type was accounted for in the objective function. A hybrid algorithm was used to solve the model. This hybridization combined a GA with the ALNS and a SOA.

The method started by constructing an initial population using a modified C&W heuristic and the ALNS. Two parents were selected from the initial population using a binary tournament. Subsequently, the parents underwent a crossover operator. An offspring solution was generated from the crossover operator. The offspring solution was fed in the form of a giant tour to a SPLIT algorithm with the SOA. The SPLIT algorithm split the offspring solution into several vehicles routes. These routes were subsequently fed to the ALNS+SOA. If the resulting solution was feasible, it was stored in the population. Otherwise, the ALNS+SOA was run until feasibility was ensured.

An intensification procedure ensued. This intensification procedure applied the overall ALNS+SOA to the elite solutions in the population to improve them. Whenever a pre-specified population size limit was reached, a survivor selection mechanism was applied. The mechanism selected the best solutions in the population and deleted the remaining ones. The surviving solutions were then mutated according to a pre-specified probability. The mutated solutions then underwent a regeneration procedure if no improvement was registered in the best-known solution of the population after a certain number of consecutive iterations. The overall procedure stopped when the maximum and pre-specified number of iterations was reached.

The model was tested on large instances containing 75, 100, 150, and 200 nodes. The results indicated that the hybrid algorithm outperformed, in terms of solution quality and on all the considered instances, the ALNS proposed by Demir *et al.* [129], if a homogeneous fleet was considered. Still, if a homogeneous fleet was considered, medium-duty vehicles

were preferable in terms of the total cost impact. If a heterogeneous fleet was considered, optimizing its speed was preferable to using a homogeneous fleet with speed optimization.

Finally, an interesting insight was derived from this study. On some instances, using a heterogeneous fleet without speed optimization allowed the total cost to be reduced comparatively to using a homogeneous fleet with speed optimization. Setting an adequate speed for all the arcs could yield solutions with a marginally higher total cost (as low as 0.07% and as high as 4.5%, with an average of 1.6%) than optimizing the speed for each arc.

3.3.1.3 A PRP with Simultaneous Pickup and Delivery

Majidi *et al.* [130] studied a PRP *with Simultaneous Pickup and Delivery* and proposed an ALNS metaheuristic to solve the problem. The ALNS was also embedded in a SA framework similarly to Demir *et al.* [129].

Unlike Demir *et al.* [129] who used a modified C&W heuristic to find an initial feasible solution, Majidi *et al.* [130] proposed a parallel insertion-based constructive heuristic. This heuristic considered the cost of inserting a node v_k between the nodes (v_i, v_j) in a route $\langle v_1, v_2, \dots, v_n \rangle$. The cost of inserting the node v_k was a weighted function of the load carried and the distance. This cost was calculated for all the available positions while ensuring the feasibility of the vehicles' capacity and the time windows. The node v_k was then inserted in the position that yielded the least cost increase. This heuristic was repeated until all the nodes were inserted into routes.

Unlike the C&W heuristic, which considers only the distances between the nodes, the constructive heuristic considers a weighted function of the load and the distance. In the PRP, minimizing the load carried can contribute to reduce the overall fuel consumption. This heuristic was proposed to try to obtain a better initial feasible solution. It should be stressed that a good-quality initial feasible solution can reduce the computational burden.

The performance of the ALNS with the constructive heuristic was tested against that of the ALNS with the C&W heuristic of Demir *et al.* [129]. On the 10-node instances, the results showed that no difference was registered in terms of the solution found. All the instances were solved to optimality. As for the 100-node instances, improvements were obtained both in the total costs and in the total distances for most instances. The computation times were, however, not presented. It is not possible to assess the relative fastness of the proposed heuristic.

3.3.2 Scatter Search

Pradenas *et al.* [133] studied a PRP *with Backhauls* and proposed a *Scatter Search* (SS) metaheuristic to solve the problem. The proposed method is a population-based metaheuristic which aims at generating good-quality solutions from an initial reference set.

The SS comprises five distinct stages: diversification, improvement, reference set update, subset generation, and solution combination. At the diversification stage, an initial population, known as the reference set, is generated using a constructive heuristic. This constructive heuristic inserts the nodes one-by-one into routes based on three different criteria: randomness, closeness, and multiple criteria. If the randomness criterion is selected, a node v_j is randomly selected and inserted after the node v_i . If the closeness criterion is selected, should the node v_j be the closest one to the node v_i , it is inserted after v_i . If the multiple criteria is selected, a node's proximity and urgency (time window) are assessed. The node with the highest priority is inserted.

At the improvement stage, several local search methods, which perform intra- or inter-route exchanges, may be applied to the feasible solutions obtained at the diversification stage. At reference set update stage, the best solutions in the population are selected to update the initial reference set. Once it is updated, the solutions in the reference set are combined into subsets containing 2- to $(n + 1)$ -elements to exhaustively evaluate all the possible combinations. Finally, at the solution combination stage, the subsets are combined and several local search methods may be applied to improve them. The overall procedure stops when no new solutions different from the ones in the reference set are obtained.

Pradenas *et al.* [133] followed an approach of cooperative gaming analysis. Three scenarios were studied: (i) total cooperation; (ii) total competition; (iii) a mixed system. In the total cooperation scenario, an unrestricted flow of information exists. The customers' location, demands, and time windows are shared between the companies. They may also share transport logistics (*i.e.*, the customers are not allocated to a particular company). Conversely, no such cooperation exists in the total competition scenario. The mixed system allows for the exchange of logistics but not of transportation.

The premise behind this approach is that it could be advantageous for a company to participate in a so-called coalition in order to minimize the total compound energy/fuel consumption of the coalition as a whole. There is an implied cost for the company who opts in. However, potential savings can be obtained in the overall coalition cost or even at the individual company level.

In total, a network comprising 100 nodes and 4 companies, with 25 nodes assigned to each one, was considered. For each company, the backhaul customers were present in a 30% proportion in relation to the total number of customers assigned to each one.

The results indicated that the cooperation between the companies was mostly beneficial from a total cost perspective. In some cases, as much as 30% in the total cost could be reduced. However, the total cost figures for the various coalitions were not decomposed in their respective terms (operating costs and environmental costs). It is not possible to accurately evaluate and contrast the actual reductions or increases in these costs.

3.3.3 Local Search with Cost Estimates

Saka *et al.* [135] developed a local search heuristic for a *Heterogeneous PRP with Time Windows*. The method used three types of moves to explore the solution space: the inter-relocate, the intra-relocate, and exchange moves. Each move was associated with a cost estimate. The moves that yielded the least cost increases were chosen first. This enabled the local search to be hastened and the solution space searched more efficiently. The optimality properties of the speed optimization problem were used to build such cost estimates.

Three versions of the local search heuristic were tested: the first used cost estimates (H1), the second solved the speed optimization problem for all possible moves (H2), and the third was a hybrid approach (H3). The results indicated that H1 saved a significant amount of CPU time in comparison to H2, while all the versions provided very close total cost figures.

3.4 Bi- and Multi-Objective Approaches

In single-objective programming problems, the optimization process leads to either a single optimal solution or alternative optimal solutions. No decision is at stake since there is a single measure of what is best. The model is solved with the appropriate linear, mixed-integer, or non-linear solver.

However, in multi-objective optimization (MOO) problems, the objective functions are generally conflicting, incommensurate (*i.e.*, measured in different units), and no single solution simultaneously optimizes all of them. The concept of optimality is replaced with the concept of efficiency (non-dominance, non-inferiority, or Pareto-optimality).

A solution is said to be (strictly) efficient (non-dominated, non-inferior, or Pareto-optimal), if and only if no other feasible solution exists which simultaneously yields a better value for all the objective functions. A solution is said to be weakly efficient

(weakly non-dominated, weakly non-inferior, or weakly Pareto-optimal), if there exists another feasible solution which has a better value for at least one of the objective functions without degrading the values for the remaining.

Depending on the purpose of its application, in MOO one can either be interested in: (i) selecting one efficient solution; (ii) characterizing totally, or at least potentially, the set of efficient solutions. Since no solution optimizes all the objective functions simultaneously, it is important the the decision-maker (DM) be involved in the process by taking into account his/her preferences.

The DM may play a more active or passive role in the process. The DM may express a series of quantitative or qualitative preferences which can help to narrow the scope of the search. The process ultimately converges to a region of the solution space where his/her most preferred solutions are located. Conversely, the DM may not state preferences and wish to be provided with a representative (full, if possible) set of efficient solutions he/she can choose from.

Several methods are available to generate efficient solutions. One possible classification considers whether many solutions are to be generated or if a single solution is to be obtained [139]. Another possible classification takes into account the stage of the decision-making process at which the DM expresses his/her preferences [140].

The methods for MOO can be divided into two main groups: (i) the *a posteriori* generating methods; (ii) the *a priori* preference-based methods. In the former, many efficient solutions are generated without any prior input from the DM. In the latter, the DM expresses his/her preferences to narrow the scope of the search for efficient solutions.

The *a posteriori* generating methods use a scalarization approach to generate many solutions by transforming the multi-objective problem into a single-, or a series of single-objective ones. These single-objective problems are solved to find the efficient solutions. Two well-know *a posteriori* methods which use a scalarization approach include the weighting method [139] and the ε -constraint technique [140].

The *a priori* preference-based methods rely on extra information from the DM as an input for the initial formulation of a single-objective problem. The DM can specify, for example, the weights for the objective functions or set minimum/maximum goals to be obtained in each one. There are usually dialogue phases in-between computation phases. The process tends to converge to a *most-preferred* solution. These methods typically generate fewer solutions and may not provide such a rich representation of the efficient set. Two examples of *a priori methods* include lexicographic ordering [136] and goal programming [137].

3.4.1 Adaptive Large Neighborhood Search

The first bi-objective approach for the PRP was proposed by Demir *et al.* [139]. The minimization of both the total fuel consumption and the total driving time were considered. The objectives are partially conflicting. The fuel consumption depends, among other factors, on the vehicle speed. The driving time is minimized if the speed is increased. The conflict is apparent: either the vehicles are driven at higher speeds to minimize the total driving time, albeit at the expense of a higher fuel consumption, or the vehicles are driven at lower speeds to minimize the fuel consumption, albeit at the expense of increased route durations and, consequently, the drivers' wages. Hence, a bi-objective formulation was proposed to shed light on the existing trade-offs between the total driving time and the total fuel consumption.

A similar approach to the one previously described in Demir *et al.* [129] was followed. In this case, the extended ALNS served as an outer-level search engine to find and store efficient solutions to which the SOA was then applied. Instead of using heuristic operators to find the solutions, the authors embedded in the ALNS four *a posteriori* multi-objective mathematical programming methods which used a scalarization of the two objective functions: the weighting method (WM), the weighting method with normalization (WMN), the ε -constraint technique (EC), and a hybrid method (HM), combining the WMN and ε -constraint technique, were used. Note that each method was used separately. Only one method was embedded at a time inside the ALNS. Two indicators were used to compare the performance of the four methods: the hyper-volume indicator and the ε -indicator. The performance of the four methods was tested on realistic instances with 100 nodes.

The results indicated that the HM outperformed the remaining methods, despite a good and fast performance throughout all of them. According to the considered indicators, the HM was able to find more solutions and these solutions showed overall better values for the objectives than the remaining methods. As expected, the WM/WMN showed the worst performance. This owes to the fact that the WM/WMN are known for their pitfalls when applied to MOILP and MOMILP problems. When comparing them to the ε -constraint technique, Mavrotas [140] stresses four reasons for this: (i) the WM cannot generate non-extreme efficient solutions; (ii) the WM cannot generate unsupported (interior to the convex hull) efficient solutions; (iii) the scaling of the objective functions influences the results obtained; (iv) due to (i), the number of solutions generated is not easily controllable.

On a test instance with 30 nodes, the results showed the anticipated trade-off between the fuel consumption and the total driving time. In one solution, a 9.7% increase in the total driving time led to a 27% saving in the fuel consumption. In another solution, a

reduction of 8.8% in the total driving time led to a 37.7% increase in the fuel consumption. In light of the results obtained, it was argued that one objective did not have to be greatly compromised to improve the other. Considerable reductions could be obtained in the total driving time if the emissions were slightly increased. The inverse also holds, whereby considerable reductions in the emissions could be achieved if the total driving time was slightly increased.

3.4.2 Simulated Annealing with Tabu Search

Suzuki [141] studied a PRP from the user's perspective and proposed a dual-objective metaheuristic to solve the problem. No time windows were considered. It is argued that the PRP model uses a high number of parameters and that this makes it too inconvenient for the practitioners to use in real-life decision-making settings.

On-site interviews with three trucking companies showed that the fleet size and the drivers' operating habits were regarded as trivial. The fleet size was held as a strategic decision and could be irrelevant to the day-to-day operations. The drivers' (bad) operating habits were believed to be eliminated, or minimized, through driver education and training. The speed, road gradient, and congestion were considered important and included in the model, although not as decision variables. It is argued that effects of these aspects can be captured using arc-specific constants. For example, the roads' gradients were modeled by adjusting the arcs' lengths. If a road had a positive gradient and the fuel consumption was increased in, for example, 10% because of it, this would correspond to increasing the arc's length 1.1 times. This approach, however, implies that the traffic congestion (and hence the speeds) are modeled as time-invariant, which is hardly the case in a congested network.

The objective function was divided into two terms: one measured the fuel consumption due to the distance traveled (*i.e.*, with zero payload) and the other measured the extra load-induced fuel consumption. The two objectives are partially conflicting. Minimizing the total distance traveled does not imply fuel consumption minimization due to the effect of the load.

The proposed metaheuristic combined SA and TS and operated in two stages. At the first stage, SA was used to approximate the set of efficient solutions. TS was used afterwards to improve each solution in the set. In SA, two local search methods were embedded. One method incorporated three exchange rules, which were randomly selected in each iteration. A customer insertion rule, a two-exchange rule, and a *2-opt* rule were used. The other method forbade the insertion of costly customers when a neighbor solution was constructed. The results showed a good efficiency of SA+TS on large instances.

3.4.3 Non-Dominated Sorting Genetic Algorithm-II with Multi-Factorial Optimization

Rauniyar *et al.* [144] introduced a novel solution approach for a multi-objective PRP based on a *Multi-Factorial Evolutionary Algorithm* (MFEA). No time windows were considered. MFEA integrated Multi-Factorial Optimization and the NSGA-II.

The premise behind the approach was to run parallel optimization of distinct solution spaces instead of a sequential and unified search of one solution space. It is argued that this parallel search of distinct solution spaces allows for a faster convergence and better solutions to be obtained in comparison to a simple stand-alone NSGA-II or *Strength Pareto Evolutionary Algorithm*2 (SPEA2) methods commonly found in MOO.

The NSGA-II was run to divide the nodes into different routes. These routes were then passed to the MFEA. In the MFEA, each route was a separate task to be optimized and the minimum spanning path for each one was to be found. The distance-minimized routes found by the MFEA were then relayed to the NSGA-II for further optimization taking into account the minimization of the fuel consumption and the minimization of the distance traveled.

The performance of the MFEA was tested against a simple NSGA-II and SPEA2 and on instances containing 10, 50, 100, and 200 customers. The results indicated that the MFEA outperformed both the NSGA-II and SPEA2 in terms of solution quality, assessed by the hyper-volume indicator, and fastness of convergence.

On a related study, Rauniyar *et al.* [145] proposed another solution approach for the (single-objective) PRP, based on a *Modified Brainstorm Optimization Algorithm in Objective Space* (BSO-OS), a novel population-based swarm intelligence metaheuristic. The performance of the method was tested against a simple GA. The results showed that BSO-OS was considerably faster than the GA but derived solutions with marginally higher fuel consumption.

3.4.4 Path Elimination Procedure

Raeesi and Zografos [116] introduced the *Steiner PRP* (SPRP). Three objectives were considered: the minimization of the vehicles' hiring cost, the total fuel consumption, and the total duration of the routes. The SPRP takes into account time- and load-dependency, the fleet size and mix, time windows, flexible departure times, and multi-trips on congested urban roads.

The instantaneous, time-dependent, second-by-second speed and A/D rates were incorporated into the travel time and fuel consumption estimation models. The previous

works had considered the A/D rates to be equal to zero over an arc and the speed had been considered as a step function of time [117]. Raeesi and Zografos [116] considered the speed to be a piecewise linear function of time. It is argued that ignoring the A/D rates can lead to misleading routing decisions because the error in the estimation of the fuel consumption can be significant. The results indicate that ignoring such a consideration can lead to an estimation error of around 45% up to 80%. However, as pointed by the authors, this microscopic data is many times unavailable at the planning stage.

To face the issue of this unavailability, a model that constructed synthetic worst-case driving cycles for a given arc was proposed. If the time-dependent travel time between two nodes is known from the macroscopic data, if the maximum and minimum possible A/D rates for a vehicle are also known, and given the maximum possible speed for that arc, then worst-case second-by-second A/D rates can be constructed by determining speed levels for every second during a given time period.

A *Path Elimination Procedure* (PEP) to cope with the difficulty of the multi-objective time-, load-, and vehicle-dependent problem was proposed. The premise behind the procedure is that redundant paths should be eliminated from the network and eligible paths should be retained. An eligible (or feasible) path is defined as one that may be traversed by at least one vehicle, for at least one time instant, carrying some level of load. This pre-computation of eligible paths sought to reduce the network's size, and expectedly the computational burden, while making sure that no efficient solutions were eliminated *a priori*.

The results showed that when the total driving time was minimized, less sacrifice had to be made to the other two objectives. When the fuel consumption was minimized, a 281% increase was registered in the total driving time because the vehicles traveled slower to consume less fuel. The vehicle costs also rose significantly (70%) in a fuel consumption-minimizing setting. When the cost of the vehicles was minimized, the fuel consumption increased 33% and the travel time increased 45%. The main limitation of the PEP was its inability to cope even with small-sized instances. Only 5 nodes were considered during the tests.

3.4.5 Simulated Annealing and Non-Dominated Sorting Genetic Algorithm-II

Tirkolaee *et al.* [142] proposed a bi-objective formulation for a multi-period PRP with cross-docking stations. The network comprised several suppliers, cross-docks, and customers. The minimization of the total cost and the maximization of the customers' satisfaction were considered. The total cost comprised the fuel cost and the drivers'

wages for both areas of the service (*i.e.*, the pickup and delivery areas).

It was assumed that the suppliers had a limited capacity and might be subject to potential failures. A supplier's reliability was assessed by means of an exponential distribution. Since the suppliers could be unable to deliver, the customers' satisfaction was gauged by the average amount of goods transported to the cross-docks. It was further assumed that the vehicles were loaded to full capacity before departing to serve the customers.

The planning horizon comprised several time periods (days) during which the customers must be served. A homogeneous fleet with a high capacity performed the pickup activities at the suppliers. The goods were then transported to the cross-docks where break-bulking was performed. The smaller quantities were then delivered to the customers using a homogeneous fleet with a smaller capacity.

Three types of traffic condition were considered: heavy, light, and free-speed. The speeds in each traffic condition varied within discrete values. In the heavy and free-speed conditions, a speed was selected and the vehicles always traveled at such a speed until the end of that time period. In light traffic, the vehicles could adjust their speed according to a speed increase gradient. This gradient modeled a transient region between the heavy and free-speed traffic conditions.

The ε -constraint technique was used to solve the small-sized instances. The larger instances were tackled with a multi-objective SA (MOSA) and a NSGA-II. The NSGA-II showed a better overall performance than MOSA. In the small-sized instances, both MOSA and NSGA-II performed similarly. The efficient sets generated were approximately equal to those generated by the ε -constraint technique. However, as the dimension of the problems increased, the ε -constraint technique became unsuitable and only the metaheuristics could derive solutions for the problem.

3.4.6 Variable Neighborhood Search

Paul *et al.* [146] proposed a bi-objective *Two-Echelon PRP with Simultaneous Pickup and Delivery* under multiple time windows. Two objectives were considered: the minimization of the fuel consumption/emissions and the total driving time.

The first echelon comprised the depots and intermediate depots. The second echelon comprised the intermediate depots and the customers. Two heterogeneous fleets, one for each echelon, were considered. In both echelons, the vehicles performed pickup and delivery activities. All the picked goods had to be transported back to the first echelon depots. Each customer could be visited more than once, hence multiple time windows were considered.

A modified version of the *Variable Neighborhood Search* (VNS) metaheuristic for the multi-objective case (MOVNS) was proposed. The method considered an initial population and its neighborhood was constructed. The neighboring solutions were iteratively destroyed and repaired through a series of six randomly chosen operators.

Computational experiments were performed on instances involving up to 100 customers and 5 intermediate depots. The graphical representation of the solutions showed the anticipated trade-off between the fuel consumption and the total driving time. As the number of satellites increased, substantial savings could be obtained both in the total fuel consumption and in the total driving time. For example, on an instance with 100 customers and 2 satellites, the extreme solutions yielded a value of around 44 hours and 375 liters, when the total driving time was at its maximum, and of around 34 hours and 395 liters, when the fuel consumption was at its maximum. If one satellite was added, these solutions yielded the values of around 38.5 hours and 334 liters, and 32 hours and 348 liters, respectively.

3.5 Matheuristic Approaches

Matheuristics are hybrid procedures that combine mathematical programming (MP) techniques with heuristic/metaheuristic methods [147]. The fundamental premise behind these hybrid procedures is to improve the performance of the MIP solver, which is generally used to solve the MP model, by designing a heuristic which serves as outer-level procedure to guide the search. As stressed by Boschetti *et al.* [148], this approach is the most studied one, although the reverse can also happen. Two distinct approaches thus exist: either the heuristic is used to improve the performance of the MIP solver, or the MIP solver is used to improve the performance of the heuristic. Matheuristics are also usually referred to as *model-based heuristics*, in that the MP model plays a central role in the optimization process.

3.5.1 Iterated Local Search with a Set Partitioning Formulation

The only publication found resorting to a matheuristic approach to solve the PRP is due to Kramer *et al.* [149]. A hybrid *Iterated Local Search* (ILS) algorithm was proposed. This algorithm combined the ILS metaheuristic, in which an ILP-based SP formulation was embedded, and a SOA. The ILS metaheuristic was based on the *Randomized Variable Neighborhood Descent* heuristic of Subramanian *et al.* [150]. It used five local search heuristics which performed intra- and inter-route exchanges to create the neighborhood structures of a solution. Additionally, the authors proposed new PRP instance sets with

narrower time windows. Some of these sets are used in our computational tests, in Chapter 5.

The ILS metaheuristic began by finding an initial feasible solution. This solution was computed using the modified *Cheapest Insertion* (CI) heuristic of Penna *et al.* [152]. The CI heuristic started with a two-node loop $\langle v_i, v_j \rangle$ and $\langle v_j, v_i \rangle$ and iteratively calculated for all the arcs the cost of inserting a node v_k . The node v_k was inserted in the position that yielded the minimum cost increase. The process was repeated until all the nodes were inserted into routes. This initial feasible solution was computed using the maximum allowable speed (to ensure feasibility), relaxed time windows, and while also taking into account the PRP objective (*i.e.*, the fuel consumption and the drivers' wages).

The matheuristic operated in two stages. At the first stage, after an initial feasible solution had been found, a speed matrix was initialized with the speeds for each arc set to their maximum value. The value of the cost objective function for this initial solution will be high as maximum speeds are being considered. To improve the initial solution, the ILS and the SOA were applied. A current solution was derived and the speed matrix was updated with the newly optimized speeds.

At the second stage, an incumbent solution was derived from the current one by iteratively applying the ILS and the SOA. In each iteration, one of three randomly selected perturbation mechanisms was applied. If no improvement is registered after a certain number of consecutive iterations, it is likely that a local optimum has been reached. The SP formulation, which was solved using a MIP solver (CPLEX), was called whenever a local optimum was found in order to possibly generate better solutions. In other words, the SP formulation tried to create better solutions from a temporary pool, comprised of the local optima from the local search, and from a permanent pool, comprised of the best-known solutions found at each restart phase of the algorithm.

The results showed that the hybrid ILS outperformed the ALNS of Demir *et al.* [129] both in terms of solution quality and fastness. The convergence time of the hybrid ILS was also reduced in comparison to the ALNS of Demir *et al.* [129]. This owed to the fact that a longer sequence of high-quality solutions was found due to the interplay between the local search and the speed optimization procedures. Demir *et al.* [129] considered the SOA as a post-optimization routine, whereas Kramer *et al.* [149] considered it within the search. The results indicated that using the SOA within the search was beneficial to obtain high-quality solutions, especially on the larger instances.

On a related study, Kramer *et al.* [153] proposed a new speed and departure algorithm for the PRP. The departure time from the depot was held a decision variable for the individual routes. The algorithm was embedded in the previously described matheuristic.

The results showed that when the departure time from the depot could be postponed, up to 8.36% savings could be obtained in the total cost.

3.6 Bi-level Approaches

Bi-level optimization (BLO) models are an adequate way of modeling partially conflicting objectives between two non-cooperative and hierarchical decision-makers [154]. These models are also referred to as *nested*, in that one optimization model is nested within the other. Two levels then exist: an upper-level problem (UL), also known as the *leader*, and a lower-level problem (LL), known as the *follower*. The LL problem is part of the constraint set of the UL problem. The decision process is hierarchical and both parties control different sets of variables. Some variables and constraints are common to both levels making the problems interdependent. The leader instantiates the values for his/her decision variables and the follower responds by optimizing his/her objective function. Another relevant aspect of BLO models is that the visibility is usually not symmetric for the two levels. Usually, the UL has full knowledge of the LL problem, while the LL's attitude is mainly a responsive one, reacting to the decisions made at the UL [155].

3.6.1 Non-Dominated Sorting Genetic Algorithm-II and Genetic Algorithm

Nath *et al.* [156] proposed a bi-level formulation for a multi-objective PRP and solved the problem with a hybrid evolutionary algorithm. Both levels were formulated as MILP models. A NSGA-II was used to solve the UL, whereas the LL was solved with a GA. The time windows and the drivers' wages were not considered.

It is argued that the formulation for the PRP is bi-level in nature, in that the relationship leader-follower is present. The depot acted as the leader, assigning the vehicles to the customers. The UL determined which vehicle served which customers and how many vehicles would be necessary to serve all the customers. The LL decided the order in which the customers were visited so that the total distance traveled was minimized. Once the establishment of these routes was obtained, the speeds, and thus the fuel consumption, were computed and optimized at the UL, while also taking into account the minimization of the distance.

The approach, however, overlooked the fact that the decision-making process was, in reality, in the hands of only one party (the freight company). By formulating the problem as bi-level, the cooperation between the freight company and the freight company itself was assumed to be antagonistic, a rather dichotomous view.

Notwithstanding, the use of a metaheuristic at the LL does not guarantee the optimality of the solution found for this level. A solution is only guaranteed to be feasible to the BLO (the UL problem) provided that it is optimal for the LL [154]. The approach may, therefore, raise concern regarding the feasibility of the solutions obtained, namely in the large instances.

It can be argued that a bi-objective formulation, where cooperation exists, is better suited for the problem at hand. The conflicting objectives can be modeled and exact methods applied, albeit up to a certain instance size and complexity. The DM can be provided with a (full) efficient set and explore the trade-offs between the solutions. It should be stressed that even if the BLO model were to converge to single optimal solution, this is generally not an efficient solution to the corresponding bi-objective model [154]. In practice, one can be denying the company better solutions.

Computational experiments were performed against a single-level version of the model using the NSGA-II. The results suggested that the bi-level formulation derived better solutions than its single-level counterpart, especially on the large instances. In particular, the improvement in the fuel consumption could be as low as 6.9%, on the 10-node instances, and as high as 22.7%, on the 150-node instances.

3.6.2 Particle Swarm Optimization and Adaptive Large Neighborhood Search

Qiu *et al.* [157] presented a carbon pricing initiatives-based bi-level PRP. Unlike the previous study, a freight company and an authority were considered. The latter sets a carbon tax price for the actual emissions and a subsidy price to minimize them. From the authority's point of view, it is desirable to minimize the emissions, whereas the freight company wishes to minimize its total cost. These objectives are partially conflicting: a total cost-minimizing solution for the freight company may yield a higher fuel consumption/emissions value.

The authors developed a MIP model in which the UL (the authority's decision on the carbon price) was non-linear with continuous variables. The LL (the company's routing decision) was the traditional PRP. These decisions were integrated with a feedback loop mechanism, whereby the carbon tax price was dynamically adjusted with the information from the company's routing decisions. The routing decisions were subsequently influenced by the carbon tax price set. Fuzzy logic controlled PSO was used to solve the UL, whereas the LL was solved using ALNS.

The model was tested on instances with 10, 50, and 100 cities. It was concluded that the carbon pricing initiatives could help to produce lower emissions (as much as 6%, on

average) with relative few cost increases for the company (as much as 4%, on average). Although the combined average figure for the carbon tax and carbon subsidy prices more than doubled that of fuel (2.44 £/l, while the fuel was set to 1.2 £/l), the average carbon-related cost in the total cost figure did not exceed 1%. Ultimately, this meant that by dynamically and mutually influencing each other's decisions, a convergence to a solution which was at least satisfactory for both parties could be obtained.

3.7 Conclusion

In this chapter, the selected contributions on the Pollution-Routing Problem were reviewed and discussed. Several different approaches and solution methodologies were introduced and explained. The metaheuristic approaches and the bi- or multi-objective approaches are particularly common. The researchers resort to these approaches because the PRP is NP-Hard and complex, also comprising several conflicting objectives.

Some contributions considered a pickup and delivery network where linehaul and backhaul customers were present, either employing a homogeneous fleet or a heterogeneous one. All these contributions either assumed that the linehaul or backhaul customers were served first, or otherwise a simultaneous pickup and delivery policy was considered. The incorporation of mixed linehails and backhails is a gap in the literature.

In the next chapter, the bi-objective Pollution-Routing Problem with Mixed Linehails and Backhails is addressed.

Chapter 4

The Pollution-Routing Problem with Mixed Linehails and Backhails

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4.1 Introduction

In this chapter, the bi-objective Pollution-Routing Problem with Mixed Linehauls and Backhauls (PRPMB) is defined and formulated. A MILP model, which is able to account for a heterogeneous fleet, is proposed. The problem statement is defined and the model is explained. This chapter starts by introducing the works the formulation for the bi-objective PRPMB is based on.

4.2 The Vehicle Routing Problem with Mixed Linehauls and Backhauls

An example of a VRP with Mixed Linehauls and Backhauls is the work by Belmecheri *et al.* [158]. The problem is defined on a complete and directed graph $G = (V, A)$ and uses a copy of the depot. A single depot, a heterogeneous fleet, and time windows were considered. The indices, parameters, sets, and decision variables are described and summarized below.

Indexation:

$i, j \rightarrow$ Indices for the customers/arcs;
 $k \rightarrow$ Index for the vehicles.

Parameters:

$n \rightarrow$ number of customers;
 $n_d \rightarrow$ number of linehaul customers;
 $a_i \rightarrow$ time window lower bound of the customer i ;
 $b_i \rightarrow$ time window upper bound of the customer i ;
 $s_i \rightarrow$ service time of the customer i ;
 $d_i \rightarrow$ demand of the linehaul customer i : $d_i = 0, \forall i \in \{0\} \cup P_c \cup \{n + 1\}$;
 $p_i \rightarrow$ supply of the backhaul customer i : $p_i = 0, \forall i \in \{0\} \cup D_c \cup \{n + 1\}$;
 $c_{ij} \rightarrow$ cost of traversing the arc (i, j) ;
 $t_{ij} \rightarrow$ travel time of the arc (i, j) ;
 $Q^k \rightarrow$ maximum payload of the vehicle k ;
 $e^k \rightarrow$ variable cost per unit distance traveled of the vehicle k .

Sets:

$D_c \rightarrow$ Set of linehaul customers: $D_c = \{1, \dots, n_d\}$;

$P_c \rightarrow$ Set of backhaul customers: $P_c = \{n_d + 1, \dots, n\}$;

$V \rightarrow$ Set of nodes: $V = \{0\} \cup D_c \cup P_c \cup \{n + 1\}$. The nodes 0 and $n + 1$ are copies of the depot;

$A \rightarrow$ Set of arcs: $A = \{(i, j) : i, j \in V, i \neq n + 1, j \neq 0, i \neq j\}$;

$K \rightarrow$ Set of heterogeneous vehicles: $K = \{1, \dots, |K|\}$.

Decision Variables:

$x_{ij}^k \rightarrow$ Binary variable equal to 1 if the vehicle k traverses the arc (i, j) . Equal to zero otherwise;

$L_i^k \rightarrow$ Continuous variable representing the load carried in the vehicle k after the customer i is visited;

$T_i^k \rightarrow$ Continuous variable representing the service time start of the vehicle k at the customer i .

The problem is formulated as a MILP model as follows:

$$\text{minimize } \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ij}^k e^k \quad (4.2.1)$$

subject to

$$\sum_{k \in K} \sum_{j: (i,j) \in A} x_{ij}^k = 1 \quad \forall i \in D_c \cup P_c, \quad (4.2.2)$$

$$\sum_{i: (0,i) \in A} x_{0i}^k = 1 \quad \forall k \in K, \quad (4.2.3)$$

$$\sum_{i: (i,n+1) \in A} x_{in+1}^k = 1 \quad \forall k \in K, \quad (4.2.4)$$

$$\sum_{j: (j,i) \in A} x_{ji}^k = \sum_{j: (i,j) \in A} x_{ij}^k \quad \forall i \in D_c \cup P_c, \forall k \in K, \quad (4.2.5)$$

$$a_i \leq T_i^k \leq b_i \quad \forall i \in V, \forall k \in K, \quad (4.2.6)$$

$$T_i^k + s_i + t_{ij} \leq T_j^k + M(1 - x_{ij}^k) \quad \forall (i, j) \in A, \forall k \in K, \quad (4.2.7)$$

$$p_i \leq L_i^k \leq Q^k \quad \forall i \in P_c, \forall k \in K, \quad (4.2.8)$$

$$0 \leq L_i^k \leq Q^k - d_i \quad \forall i \in D_c, \forall k \in K, \quad (4.2.9)$$

$$L_i^k + p_j - d_j \leq L_j^k + M(1 - x_{ij}^k) \quad \forall (i, j) \in A, \forall k \in K, \quad (4.2.10)$$

$$L_0^k = \sum_{i \in D_c} d_i \sum_{j: (i,j) \in A} x_{ij}^k \quad \forall k \in K, \quad (4.2.11)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in A, \forall k \in K. \quad (4.2.12)$$

The objective function (4.2.1) seeks the minimization of the total routing cost, which is weighted by the costs of the arcs traversed and the variable costs of the vehicles used. The constraints (4.2.2) are degree constraints and ensure that each customer is visited exactly once and by one vehicle only. The constraints (4.2.3)-(4.2.4) ensure that each vehicle departs from and arrives at the depot. The constraints (4.2.5) are used to model the connectivity of the routes and state that if a vehicle arrives at a node, it must also depart from it. The constraints (4.2.6)-(4.2.7) enforce the time windows and M is a sufficiently large and positive constant.

The constraints (4.2.8)-(4.2.9) are capacity constraints and are used to limit the weight of the load carried in the vehicles. The constraints (4.2.8) ensure that the weight carried in a vehicle after a backhaul customer is visited is at least the weight of the load picked up at that customer and cannot exceed the capacity of that vehicle. The constraints (4.2.9) ensure that the weight carried in a vehicle after a linehaul customer is visited is at most the amount of capacity of the vehicle minus the weight of the demand dropped off at that customer. Note that the load carried after a linehaul customer is visited can be equal to zero but such a situation never occurs after visiting a backhaul customer.

The constraints (4.2.10) model the increase, or respectively the decrease, of the load carried after a customer is visited. The constraints (4.2.11) ensure that, in a route, a vehicle departs from the depot carrying the total demand of its respective linehaul customers for that route. Finally, the constraints (4.2.12) are integrality constraints.

4.3 The Fleet Size and Mix Pollution-Routing Problem

Koç *et al.* [134] addressed the Fleet Size and Mix Pollution-Routing Problem. The problem is defined on a complete and directed graph $G = (N, A)$. A single depot, a heterogeneous fleet, and time windows were considered. The indices, parameters, sets, and decision variables are described and summarized below.

Indexation:

$i, j \rightarrow$ Indices for the customers/arcs;

$h \rightarrow$ Index for the vehicle types;

$r \rightarrow$ Index for the speed levels.

Parameters:

$n \rightarrow$ number customers;

$a_i \rightarrow$ time window lower bound of the customer i ;

$b_i \rightarrow$ time window upper bound of the customer i ;

$t_i \rightarrow$ service time of the customer i ;

$q_i \rightarrow$ demand of the customer i ;

$d_{ij} \rightarrow$ distance between the node i and the node j ;

$m^h \rightarrow$ number of vehicles of each type h : $m_h = |N_0|$;

$Q^h \rightarrow$ maximum payload of the vehicle of type h ;

$f^h \rightarrow$ daily fixed operating cost of the vehicle of type h .

Sets:

$N \rightarrow$ Set of nodes: $N = \{0, 1, \dots, n\}$. The node 0 is the depot;

$N_0 \rightarrow$ Set of customers: $N_0 = N \setminus \{0\}$;

$A \rightarrow$ Set of arcs: $A = \{(i, j) : i, j \in N, i \neq j\}$;

$H \rightarrow$ Set of vehicle types: $H = \{1, \dots, |H|\}$;

$R \rightarrow$ Set of speed levels: $R = \{1, \dots, |R|\}$.

Decision Variables:

$x_{ij}^h \rightarrow$ Binary variable equal to 1 if a vehicle of type h traverses the arc (i, j) . Equal to zero otherwise;

$z_{ij}^{rh} \rightarrow$ Binary variable equal to 1 if a vehicle of type h traverses the arc (i, j) at the speed level r . Equal to zero otherwise;

$f_{ij}^h \rightarrow$ Continuous variable representing the load carried in a vehicle of type h for the arc (i, j) ;

$\bar{v}^r \rightarrow$ Continuous variable representing the *average speed* of the speed level r ;

$y_i \rightarrow$ Continuous variable representing the service time start at the customer i ;

$s_j \rightarrow$ Continuous variable representing the total driving time for a route that has the customer j as the last customer.

Similarly to Bektaş and Laporte [18], it is assumed that the speed limitations are the same for every arc. The calculation of the *average speed* for each speed level $r \in R$ has been previously explained (see subsection 2.4.3).

Unlike Bektaş and Laporte [18], Koç *et al.* [134] allow for the vehicles to travel at speeds lower than 40km/h . An additional term in the objective function, which measures the contribution of $k^h N^h V^h$ (the vehicles' engine module), is added. Recall that this contribution is mainly significant for lower speeds (see subsection 2.4.2).

The problem is formulated as MILP model as follows:

$$\text{minimize } \sum_{h \in H} \sum_{(i,j) \in A} f_c \lambda k^h N^h V^h d_{ij} \sum_{r \in R} \frac{z_{ij}^{rh}}{\bar{v}^r} \quad (4.3.1)$$

$$+ \sum_{h \in H} \sum_{(i,j) \in A} f_c \lambda \gamma^h \alpha_{ij} d_{ij} (w^h x_{ij}^h + f_{ij}^h) \quad (4.3.2)$$

$$+ \sum_{h \in H} \sum_{(i,j) \in A} f_c \lambda \gamma^h \beta^h d_{ij} \sum_{r \in R} (\bar{v}^r)^2 z_{ij}^{rh} \quad (4.3.3)$$

$$+ \sum_{j \in N_0} f_d s_j + \sum_{h \in H} \sum_{j \in N_0} f_h x_{0j}^h \quad (4.3.4)$$

subject to

$$\sum_{j \in N_0} x_{0j}^h \leq m^h \quad \forall h \in H, \quad (4.3.5)$$

$$\sum_{h \in H} \sum_{j \in N} x_{ij}^h = 1 \quad \forall i \in N_0, \quad (4.3.6)$$

$$\sum_{h \in H} \sum_{i \in N} x_{ij}^h = 1 \quad \forall j \in N_0, \quad (4.3.7)$$

$$\sum_{h \in H} \sum_{j \in N} f_{ji}^h - \sum_{h \in H} \sum_{j \in N} f_{ij}^h = q_i \quad \forall i \in N_0, \quad (4.3.8)$$

$$q_j x_{ij}^h \leq f_{ij}^h \leq (Q^h - q_i) x_{ij}^h \quad \forall (i,j) \in A, \forall h \in H, \quad (4.3.9)$$

$$y_i - y_j + t_i + \sum_{r \in R} \frac{d_{ij}}{\bar{v}^r} z_{ij}^{rh} \leq M_{ij} (1 - x_{ij}^h) \quad \forall i \in N, \forall j \in N_0, i \neq j, \forall h \in H, \quad (4.3.10)$$

$$a_i \leq y_i \leq b_i \quad \forall i \in N_0, \quad (4.3.11)$$

$$y_j + t_j - s_j + \sum_{r \in R} \frac{d_{j0}}{\bar{v}^r} z_{j0}^{rh} \leq L_{ij} (1 - x_{j0}^h) \quad \forall j \in N_0, \forall h \in H, \quad (4.3.12)$$

$$\sum_{r \in R} z_{ij}^{rh} = x_{ij}^h \quad \forall (i,j) \in A, \forall h \in H, \quad (4.3.13)$$

$$f_{ij}^h \geq 0 \quad \forall (i,j) \in A, \forall h \in H, \quad (4.3.14)$$

$$y_i \geq 0 \quad \forall i \in N_0, \quad (4.3.15)$$

$$x_{ij}^h \in \{0, 1\} \quad \forall (i,j) \in A, \forall h \in H, \quad (4.3.16)$$

$$z_{ij}^{rh} \in \{0, 1\} \quad \forall (i,j) \in A, r \in R, \forall h \in H. \quad (4.3.17)$$

The formulation is similar to that of the PRP, adapted to handle a heterogeneous fleet. The objective function (4.3.1)-(4.3.4) comprises four terms and seeks the minimization of the total routing cost. The term (4.3.1) measures the total cost incurred due to the vehicles' engine module. The term (4.3.2) measures the total cost incurred due to the

vehicles' curb weight and load carried. The term (4.3.3) measures the total cost incurred due to the speed variations. The term (4.3.4) is comprised of two parts. The first part measures the total cost incurred due the drivers' wages. The second part measures the total cost incurred due to the vehicles' daily operating costs. The terms (4.3.1)-(4.3.3) directly measure the total cost of the fuel consumption and emissions as they are multiplied by its respective unit cost f_c . Both the cost of fuel and the cost of the CO₂ emissions are included in f_c .

The constraints (4.3.5) ensure that at most m^h vehicles of each type depart from the depot. The constraints (4.3.6)-(4.3.7) are degree constraints and ensure that each customer is visited exactly once and by one vehicle type only. The constraints (4.3.8) ensure the balance of flow. The constraints (4.3.9) are capacity constraints and serve also as SEC. The constraints (4.3.10)-(4.3.11) enforce the time windows and M_{ij} is a sufficiently large number computed as $M_{ij} = \max\{0, b_i + s_i + d_{ij}/l_{ij} - a_j\}$.

The constraints (4.3.12) are used to calculate the total driving time s_j for each route (thus for each vehicle) and L_{ij} is a sufficiently large number computed as $L_{ij} = \max\{0, b_j - t_j + \max_i\{d_{ij}\}/\bar{v}^r\}$. The constraints (4.3.13) ensure that only one speed level is chosen per arc. The constraints (4.3.14) ensure that the load carried for an arc cannot be negative. The constraints (4.3.15) ensure that the service time start cannot be negative. Finally, the constraints (4.3.16)-(4.3.17) are integrality constraints.

4.4 The Bi-objective Pollution-Routing Problem with Mixed Linehauls and Backhauls

In this section, the bi-objective PRPMB is defined and formulated. The construction of the model was performed by subsuming and adapting the models of Belmecheri *et al.* [158] and Koç *et al.* [57] in the same optimization model. The assumptions made in the bi-objective PRPMB model are presented below.

Assumptions:

- (i) All the routes start and end at a single depot;
- (ii) A route is performed by one vehicle only;
- (iii) All the customers must be served and have their demands fulfilled;
- (iv) All the deliveries are supplied by the depot;
- (v) All the pickups must return to the depot;

- (vi) Split deliveries or pickups are not allowed;
- (vii) Any customer visit sequence is allowed;
- (viii) Routes with backhauls customers only are allowed;
- (ix) The capacity of the vehicles cannot be exceeded;
- (x) The summation of all the deliveries cannot exceed the capacity of the vehicle(s) with the highest payload;
- (xi) The summation of all the pickups cannot exceed the capacity of the vehicle(s) with the highest payload;
- (xii) The time windows must be complied with;
- (xiii) Early arrivals are allowed and no penalty cost is incurred in such a situation;
- (xiv) All the relevant data for the problem is deterministic and known *a priori*;
- (xv) The planning horizon is at most one day (24h).

The bi-objective PRPMB consists of routing a set of capacitated and heterogeneous vehicles to serve a set of customers within their respective time windows. The problem calls for the determination of the optimal speed and load on each arc of a route. The set of optimal routes that minimize two objective functions is to be found.

Similarly to Demir *et al.* [139], the minimization of the total fuel consumption/emissions and of the total driving time is considered. The two objectives are conflicting due to the effect of the speed. The total driving time is minimized the faster the vehicles are driven. Higher speeds translate into a higher fuel consumption/emissions.

The bi-objective formulation for the PRPMB does not monetize the fuel consumption/emissions. The total amount of the emissions, in kilograms, is measured. In a bi-objective formulation, it is more natural for incommensurability to exist between the two objectives. If both criteria were converted into the same unit measure, they could be aggregated in a single objective.

Also, it can be argued that the emissions should not be translated into a cost. Their monetization obfuscates what we consider ought to be the aim behind an environmentally conscious model: the minimization of the total amount of the emissions, not of the cost incurred due to them.

Finally, as discussed in the subsection 2.4.1, no consensus exists on a single price to put on them. The emissions are thus measured in kilograms and the total driving time

in seconds (no driver payment is considered). Similarly to Bektaş and Laporte [18], a conversion factor $q_g = 2.32kg$ of CO₂/liter of diesel is considered.

The bi-objective PRPMB is defined on a complete and directed graph $G = (N, A)$ and uses a copy of the depot. A single depot, a heterogeneous fleet, and time windows are considered. The indices, parameters, sets, and decision variables are described and summarized below.

Indexation:

$i, j \rightarrow$ Indices for the customers/arcs;

$k \rightarrow$ Index for the vehicles;

$r \rightarrow$ Index for the speed levels.

Parameters:

$n \rightarrow$ number customers;

$n_d \rightarrow$ number of linehaul customers;

$a_i \rightarrow$ time window lower bound of the customer i ;

$b_i \rightarrow$ time window upper bound of the customer i ;

$t_i \rightarrow$ service time of the customer i ;

$d_i \rightarrow$ demand of the linehaul customer i : $d_i = 0, \forall i \in \{0\} \cup P_c \cup \{n + 1\}$;

$p_i \rightarrow$ supply of the backhaul customer i : $p_i = 0, \forall i \in \{0\} \cup D_c \cup \{n + 1\}$;

$d_{ij} \rightarrow$ distance between the node i and the node j ;

$Q^k \rightarrow$ maximum payload of the vehicle k .

Sets:

$D_c \rightarrow$ Set of linehaul customers: $D_c = \{1, \dots, n_d\}$;

$P_c \rightarrow$ Set of backhaul customers: $P_c = \{n_d + 1, \dots, n\}$;

$N_0 \rightarrow$ Set of customers: $N_0 = D_c \cup P_c$;

$N \rightarrow$ Set of nodes: $N = \{0\} \cup N_0 \cup \{n + 1\}$. The nodes 0 and $n + 1$ are copies of the depot;

$A \rightarrow$ Set of arcs: $A = \{(i, j) : i, j \in N, i \neq n + 1, j \neq 0, i \neq j\}$;

$K \rightarrow$ Set of heterogeneous vehicles: $K = \{1, \dots, |K|\}$;

$R \rightarrow$ Set of speed levels: $R = \{1, \dots, |R|\}$.

Decision Variables:

$x_{ij}^k \rightarrow$ Binary variable equal to 1 if the vehicle k traverses the arc (i, j) . Equal to zero otherwise;

z_{ij}^{rk} → Binary variable equal to 1 if the vehicle k traverses the arc (i, j) at the speed level r . Equal to zero otherwise;

f_{ij}^k → Continuous variable representing the load carried in the vehicle k for the arc (i, j) ;

\bar{v}^r → Continuous variable representing the *average speed* of the speed level r ;

y_i → Continuous variable representing the service time start at the customer i ;

s_j → Continuous variable representing the total driving time for a route that has the customer j as the last customer.

Similarly to Bektaş and Laporte [18] and Koç *et al.* [134], it is assumed that the speed limitations are the same for every arc. The calculation of the *average speed* for each speed level $r \in R$ has been previously explained (see subsection 2.4.3). An extra term in the objective function to measure the contribution of $k^k N^k V^k$ (the vehicles' engine module) is included. The MILP formulation for the bi-objective PRPMB is presented in the next page.

The objective function \mathbf{z}_1 (4.4.1)-(4.4.4) comprises four terms and seeks the minimization of the total emissions. The term (4.4.1) measures the total emissions due to the vehicles' engine module. The term (4.4.2) measures the total emissions due to the vehicles' curb weight. The term (4.4.3) measures the total emissions due to the load carried. The term (4.4.4) measures the total emissions due to the speed variations. The objective function \mathbf{z}_1 directly measures the total emissions. Its terms are multiplied by a conversion factor q_g , so that \mathbf{z}_1 yields the total amount of emitted CO₂ in kilograms. The objective function \mathbf{z}_2 (4.4.5) measures the total driving time in seconds for all the routes.

The constraints (4.4.6) ensure that each vehicle departs from the depot at most once. These constraints are formulated as \leq so that the model chooses how many vehicles to deploy. Should it be required that all the vehicles be routed, these constraints can be adapted to $= 1$. The constraints (4.4.7) are degree constraints and ensure that each customer is visited exactly once and by one vehicle only. The constraints (4.4.8) are used to model the connectivity of the routes and state that if a vehicle arrives at a node, it must also depart from it.

The constraints (4.4.9)-(4.4.10) are capacity constraints and are used to limit the weight of the load carried in the vehicles. The constraints (4.4.9) ensure that the weight carried in a vehicle after a backhaul customer is visited is at least the weight of the load picked up at that customer and cannot exceed the capacity of that vehicle. The constraints (4.4.10) ensure that the weight carried in a vehicle after a linehaul customer is visited is at most the amount of capacity of the vehicle minus the weight of the demand dropped off at that customer.

$$\text{minimize } \mathbf{z}_1 = \sum_{k \in K} \sum_{(i,j) \in A} q_g \lambda k^k N^k V^k d_{ij} \sum_{r \in R} \frac{z_{ij}^{rk}}{\bar{v}^r} \quad (4.4.1)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} q_g \lambda \gamma^k \alpha_{ij} d_{ij} w^k x_{ij}^k \quad (4.4.2)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} q_g \lambda \gamma^k \alpha_{ij} d_{ij} f_{ij}^k \quad (4.4.3)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} q_g \lambda \gamma^k \beta^k d_{ij} \sum_{r \in R} (\bar{v}^r)^2 z_{ij}^{rk} \quad (4.4.4)$$

$$\text{minimize } \mathbf{z}_2 = \sum_{j \in N_0} s_j \quad (4.4.5)$$

subject to

$$\sum_{j \in N_0} x_{0j}^k \leq 1 \quad \forall k \in K, \quad (4.4.6)$$

$$\sum_{k \in K} \sum_{j \in N, j \neq \{0, i\}} x_{ij}^k = 1 \quad \forall i \in N_0, \quad (4.4.7)$$

$$\sum_{j \in N, j \neq \{n+1, i\}} x_{ji}^k = \sum_{j \in N, j \neq \{0, i\}} x_{ij}^k \quad \forall i \in N_0, \forall k \in K, \quad (4.4.8)$$

$$p_i x_{ij}^k \leq f_{ij}^k \leq Q^k x_{ij}^k \quad \forall i \in P_c, \forall j \in N, j \neq \{0, i\}, \forall k \in K, \quad (4.4.9)$$

$$0 \leq f_{ij}^k \leq (Q^k - d_i) x_{ij}^k \quad \forall i \in D_c, \forall j \in N, j \neq \{0, i\}, \forall k \in K, \quad (4.4.10)$$

$$\sum_{k \in K} \sum_{j \in N, j \neq \{n+1, i\}} f_{ji}^k - \sum_{k \in K} \sum_{j \in N, j \neq \{0, i\}} f_{ij}^k = d_i \quad \forall i \in D_c, \quad (4.4.11)$$

$$\sum_{k \in K} \sum_{j \in N, j \neq \{n+1, i\}} f_{ji}^k - \sum_{k \in K} \sum_{j \in N, j \neq \{0, i\}} f_{ij}^k = -p_i \quad \forall i \in P_c, \quad (4.4.12)$$

$$\sum_{i \in D_c} d_i \sum_{m \in N, m \neq \{0, i\}} x_{im}^k - M(1 - x_{0j}^k) \leq f_{0j}^k \leq Q^k x_{0j}^k \quad \forall j \in N_0, \forall k \in K, \quad (4.4.13)$$

$$y_i - y_j + t_i + \sum_{r \in R} \frac{d_{ij}}{\bar{v}^r} z_{ij}^{rk} \leq N_{ij}(1 - x_{ij}^k) \quad \forall i \in N, \forall j \in N_0, i \neq \{n+1, j\}, \forall k \in K, \quad (4.4.14)$$

$$a_i \leq y_i \leq b_i \quad \forall i \in N_0, \quad (4.4.15)$$

$$y_j + t_j - s_j + \sum_{r \in R} \frac{d_{jn+1}}{\bar{v}^r} z_{jn+1}^{rk} \leq P_{jn+1}(1 - x_{jn+1}^k) \quad \forall j \in N_0, \forall k \in K, \quad (4.4.16)$$

$$\sum_{r \in R} z_{ij}^{rk} = x_{ij}^k \quad \forall (i, j) \in A, \forall k \in K, \quad (4.4.17)$$

$$f_{ij}^k \geq 0 \quad \forall (i, j) \in A, \forall k \in K, \quad (4.4.18)$$

$$y_i \geq 0 \quad \forall i \in N_0, \quad (4.4.19)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall (i, j) \in A, \forall k \in K, \quad (4.4.20)$$

$$z_{ij}^{rk} \in \{0, 1\} \quad \forall (i, j) \in A, r \in R, \forall k \in K. \quad (4.4.21)$$

The constraints (4.4.11)-(4.4.12) ensure the balance of flow when a linehaul or a backhaul customer is visited, respectively. The constraints (4.4.11) state that the difference in the loads for all the in-bound and out-bound arcs for the linehaul customer i must be equal to the amount dropped off at that customer. Similarly, the constraints (4.4.12) state that the difference in the loads for all the in-bound and out-bound arcs for the backhaul customer i must be equal to the amount picked up at that customer.

The constraints (4.4.13) ensure that, in a route, a vehicle departs from the depot carrying the total demand of its linehaul customers for that route and M is a sufficiently large and positive constant. These constraints only enforce that the load should be between two values. The amount of load carried when leaving the depot should be fixed to the LHS of these constraints. This is guaranteed due to the interaction between these constraints and the capacity (4.4.9)-(4.4.10) and balance of flow (4.4.11)-(4.4.12) constraints. For the balance of flow to be correctly calculated for all the individual nodes (customers) along a route, while also satisfying the capacity constraints, the amount of load carried when leaving the depot must be correctly calculated.

The constraints (4.4.14)-(4.4.15) enforce the time windows and N_{ij} is a sufficiently large number computed as $N_{ij} = \max \{0, b_i + s_i + d_{ij}/l_{ij} - a_j\}$. The constraints (4.4.16) are used to calculate the total driving time s_j for each route and P_{jn+1} is a sufficiently large number computed as $P_{jn+1} = \max \{0, b_{n+1} - t_{n+1} + \max_{n+1} \{d_{jn+1}\}/\bar{v}^r\}$.

The constraints (4.4.17) ensure that only one speed level is chosen per arc. The constraints (4.4.18) ensure that the load carried for an arc cannot be negative. The constraints (4.4.19) ensure that the service start time cannot be negative. Finally, the constraints (4.4.20)-(4.4.21) are integrality constraints.

4.4.1 Strengthening of the Formulation

Bektaş and Laporte [18] suggest a lifting on the bounding constraints (4.4.15) to reduce the computation time. This relaxation is performed through a set of inequalities, as proposed by Cordeau *et al.* [159], following Desrochers and Laporte [160]. These can be adapted to the bi-objective PRPMB as follows:

$$y_i - \sum_{k \in K} \sum_{r \in R} \max \{0, a_j - a_i + t_j + d_{ji}/\bar{v}^r\} z_{ji}^{rk} \geq a_i \quad \forall i \in N_0, j \in N, j \neq \{n+1, i\}, \quad (4.4.22)$$

$$y_i - \sum_{k \in K} \sum_{r \in R} \max \{0, a_j - a_i + t_j + d_{ji}/\bar{v}^r\} z_{ji}^{rk} \leq b_i \quad \forall i \in N_0, j \in N, j \neq \{0, i\}. \quad (4.4.23)$$

The constraints (4.4.22)-(4.4.23) were implemented in our computational experiments. The authors also suggest supplementing the formulation with two-node sub-tour breaking constraints. Our preliminary computational experiments showed no need for the use of such constraints as sub-tours were not being created. Nonetheless, these can be adapted to the bi-objective PRPMB as follows:

$$\sum_{k \in K} x_{ij}^k + \sum_{k \in K} x_{ji}^k \leq 1 \quad \forall i, j \in N_0, i \neq j. \quad (4.4.24)$$

The constraints (4.4.24) ensure explicitly that sub-tours are not created. These constraints are applied to every pair of nodes and operate by enforcing that if the vehicle k travels over the arc (i, j) , no other vehicle can do so nor the same vehicle or any other can travel over the inverse arc (j, i) .

4.5 Conclusion

In this chapter, a MILP model for the bi-objective PRPMB was proposed. A heterogeneous fleet and mixed linehauls and backhauls, a previous gap in the literature, were accounted for. All the assumptions made were presented and the works on which the model is based were introduced and described. The problem statement was defined and the model explained. In the next chapter, the computational results are presented.

Chapter 5

Results and Discussion

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5.1 Introduction

In this chapter, the results of the computational experiments are reported. Since mixed linehauls and backhauls have not been incorporated into the PRP, we provide an investigation of the effects of this backhauling strategy in comparison to the standard version of the problem.

For this purpose, two different models were tested. The first model, hereafter denoted by M1, is the single-objective PRPMB. With M1, we wish to quantify the economic and environmental impacts of the incorporation of mixed linehauls and backhauls. The second model, hereafter denoted by M2, is the bi-objective PRPMB. With M2, we wish to explore the trade-offs between the emissions and the total driving time. For comparison purposes with other studies, a homogeneous fleet was employed in both models.

The two models were coded in Python 3.8 and solved using CPLEX 20.1 with default settings and using up to 32 threads. All the tests were run on a server with an Intel® Xeon® Gold 6138 CPU with 2.0 GHz (max. 3.7 GHz, 20 cores, 40 threads and 27.5 MB cache) and 320 GB of RAM. Similarly to the previous studies, a common time limit of 3 hours was imposed on each of the instances. This chapter starts by describing the PRP benchmark instances.

5.2 Instances

The two models were tested on instances adapted from the PRP benchmark instances. The first set is the PRPLIB. This set was generated by Demir *et al.* [129]. It is comprised of 9 sets of 20 instances each, ranging from 10 customers to 200 customers. All the instances use real geographic distances and the customers represent randomly selected cities of the United Kingdom. On these instances, the customers' demands, time windows, and service times were randomly generated. The set is hereafter denoted by $UKn_m\text{-A}$ where n is the number of customers and m is the number of the instance in each of the sets.

Vehicles with a curb weight $w = 6350kg$ and a maximum payload $Q = 3650kg$ are considered. The minimum traveling speed for a vehicle is $20km/h$ while the maximum one is $90km/h$. No speed discretization is explicitly given. In our implementation, a 10-point discretization was used, following Bektaş and Laporte [18]. The planning horizon is 9 hours ($32400s$). The PRPLIB set is publicly available for download at [162].

The second set of instances was generated by Kramer *et al.* [149]. This set is the same as the PRPLIB but narrower time windows are considered. All the remaining data is unchanged. This set is hereafter denoted by $UKn_m\text{-B}$ and $UKn_m\text{-C}$. In the $UKn_m\text{-B}$ instances, each customer's time window was randomly generated with a

uniform probability between $[2000s, 5000s]$. In the UK n_m -C instances, it was randomly generated between $[2000s, 15000s]$. The set is publicly available for download at [163].

The instances with 10, 15, and 20 customers were used in our computational tests. They do not, however, consider backhaul customers. There is a need to adapt them. For this purpose, each PRP instance was subdivided into three instances. For each one of these instances, the backhauls customers are present in different proportions in relation to the total number of customers. Three different groups, hereafter denoted by Group A, Group B, and Group C were created. The Group A has 50% of backhaul customers. The Groups B and C have 30% and 10%, respectively.

Considering the order in which the customers appeared in a given instance, they were converted into backhaul customers taking into account their ordinal number. In the Group C, the backhaul customers are those whose ordinal number is a multiple of 10. In the Group B, the backhaul customers are those whose ordinal number is a multiple of 10 or 4. The backhaul customers for the Group A are those whose ordinal number is a multiple of 10, 4, or 2. Whenever a customer was flagged as being a backhaul, its demand was converted into a supply.

5.3 Computational Results for M1

5.3.1 Implementation of the Pollution-Routing Problem

The PRP model of Demir *et al.* [129] was implemented to assess whether the optimal solutions reported by the authors for the 10-node instances could be obtained. This was done to establish a common and accurate basis for comparison for M1.

The Table B.1, in the Appendix B, provides the parameters and their values used in the model of Demir *et al.* [129]. The same values for the parameters were used in our implementation. The Table B.3, in the Appendix B, compares the results obtained by our implementation and those of Demir *et al.* [129], for the UK10_ m -A set, and Kramer *et al.* [149], for the UK10_ m -B and UK10_ m -C sets. The PRP model, the parameters, and their values are the same for both authors.

The Table B.3 shows that our implementation of the PRP derived solutions with a maximum total cost deviation of 0.11% and 0.36% from those reported by Demir *et al.* [129] and Kramer *et al.* [149], respectively. On five instances (UK10_07-A, UK10_16-A, UK10_19-A, UK10_08-C and UK10_20-C), the exact same value was obtained. The deviations were at most 0.25% on the remaining instances.

The deviations across the values are attributed to the discretization of the speed. In both works, no speed discretization is explicitly mentioned. In practice, given that the deviations are very small, our implementation is accurate. Since most values are not an exact match, the ones obtained by our implementation are used hereafter for comparison purposes.

The Table B.3 also shows that the CPU times obtained by our implementation on the UK10_ m -A set were substantially lower than those reported by Demir *et al.* [129]. The authors had less computational power than the one in this study. Demir *et al.* [129] conducted their experiments on a server with 3 GHz and 1 GB of RAM.

5.3.2 The effects of the incorporation of mixed linehauls and backhauls

To assess the economic and environmental impacts of the incorporation of mixed linehauls and backhauls, a comparison is performed between the solutions' total costs and emissions reported for the standard version of the problem (PRP) and those for the single-objective PRPMB using a homogeneous fleet (M1). For an accurate comparison, the monetization of the fuel consumption/emissions and the drivers' wages were included in the objective function. The objective function for M1 is shown below. Only the objective function is altered. The constraints (4.4.6)-(4.4.21) (see section 4.4) for the PRPMB are unchanged.

$$\text{minimize } \sum_{k \in K} \sum_{(i,j) \in A} (c_f + e) \lambda k^k N^k V^k d_{ij} \sum_{r \in R} \frac{z_{ij}^{rk}}{\bar{v}^r} \quad (5.3.1)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} (c_f + e) \lambda \gamma^k \alpha_{ij} d_{ij} w^k x_{ij}^k \quad (5.3.2)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} (c_f + e) \lambda \gamma^k \alpha_{ij} d_{ij} f_{ij}^k \quad (5.3.3)$$

$$+ \sum_{k \in K} \sum_{(i,j) \in A} (c_f + e) \lambda \gamma^k \beta^k d_{ij} \sum_{r \in R} (\bar{v}^r)^2 z_{ij}^{rk} \quad (5.3.4)$$

$$+ \sum_{j \in N_0} f d s_j \quad (5.3.5)$$

subject to

$$(4.4.6) - (4.4.21) \quad (5.3.6)$$

Our formulation uses a set K of vehicles. This set needs to be defined prior to the optimization procedure. The number of vehicles in the set for each M1 instance became the one reported in the optimal solution for the corresponding PRP instance. If the instance UK n _ m used, for example, 2 vehicles in the PRP, the set K for M1 and for this instance became $K = \{1, 2\}$ where 1 and 2 are two identical vehicles. M1 then chose

how many to deploy. Since backhaul customers are considered, the number of vehicles deployed in a PRP solution can be used as an upper bound for the number of vehicles to deploy in M1. This holds because the load inside the vehicles increases and decreases in M1.

The Tables C.2-C.10, in the Appendix C, present the detailed results for the 10-, 15-, and 20-customers sets for all groups of backhaul customers. The Tables 5.3.1-5.3.3 summarize the average results for each set and group of backhaul customers. In the Tables 5.3.1-5.3.3, the following notation is used and results are summarized:

Set: set of instances, which includes all the 20 instances of a given set;

G.: group of backhaul customers (A, B or C);

TC: total cost;

CO₂: CO₂ emissions;

avg. Dif.: the average difference, in percentage, considering both the improvements (reductions) and the deteriorations (increases) in the total cost/emissions, which is computed as $\left[\sum_{i=1}^{20} 100 \frac{TC_{i,M1} - TC_{i,PRP}}{TC_{i,PRP}} \right] / 20$ for the total costs. The same applies to the emissions;

max. Imp.: the maximum improvement (the most negative difference), in percentage, in the total cost/emissions, which is computed as $\min_i \left\{ 100 \frac{TC_{i,M1} - TC_{i,PRP}}{TC_{i,PRP}} \right\}$ for the total costs. The same applies to the emissions;

max. Det.: the maximum deterioration (the highest positive difference), in percentage, in the total cost/emissions, which is computed as $\max_i \left\{ 100 \frac{TC_{i,M1} - TC_{i,PRP}}{TC_{i,PRP}} \right\}$ for the total costs. The same applies to the emissions;

avg. Imp.: the average of all the negative differences, in percentage, in the total cost/emissions, which is computed as $\left[\sum_{i=1}^n 100 \frac{TC_{i,M1} - TC_{i,PRP}}{TC_{i,PRP}} \right] / n$, if $TC_{i,M1} - TC_{i,PRP} < 0$, n being the number of negative differences, for the total costs. The same applies to the emissions;

avg. Det.: the average of all the positive differences, in percentage, in the total cost/emissions, which is computed as $\left[\sum_{i=1}^n 100 \frac{TC_{i,M1} - TC_{i,PRP}}{TC_{i,PRP}} \right] / n$, if $TC_{i,M1} - TC_{i,PRP} \geq 0$, n being the number of positive differences, for the total costs. The same applies to the emissions;

avg. CPU: the average CPU time, in seconds, obtained over the 20 instances of a set, which is computed as $\left[\sum_{i=1}^{20} CPU_i \right] / 20$.

Table 5.3.1: The average results for the 10-node instances for M1.

Set	G.		<i>avg. Dif.</i> (%)	<i>max. Imp.</i> (%)	<i>max. Det.</i> (%)	<i>avg. Imp.</i> (%)	<i>avg. Det.</i> (%)	<i>avg. CPU</i> (s)		
UK10_m-A	C	TC	-1.52	-8.61	0.35	-2.24	0.15	490.73		
		CO2	-1.05	-7.82	4.91	-2.20	1.09			
	B	TC	-3.85	-10.29	0.21	-4.55	0.15		966.28	
		CO2	-1.79	-14.75	5.31	-5.05	3.10			
	A	TC	-4.45	-13.89	0.36	-4.97	0.21			1195.11
		CO2	-1.76	-14.06	8.62	-4.70	3.71			
UK10_m-B	C	TC	0.03	-0.56	0.66	-0.26	0.22	0.22		
		CO2	0.40	-0.99	5.74	-0.49	0.99			
	B	TC	0.01	-1.20	0.75	-0.35	0.30		0.23	
		CO2	0.03	-2.12	1.51	-0.73	0.53			
	A	TC	-0.04	-1.18	0.89	-0.54	0.45			0.23
		CO2	0.34	-2.23	8.98	-1.04	1.73			
UK10_m-C	C	TC	-0.49	-6.42	0.83	-1.13	0.28	10.80		
		CO2	-1.26	-19.83	1.54	-2.84	0.67			
	B	TC	-0.92	-6.71	1.00	-1.29	0.55		13.79	
		CO2	0.73	-20.30	42.50	-3.25	6.70			
	A	TC	-1.16	-7.16	0.95	-1.55	0.38			16.14
		CO2	0.98	-21.04	44.30	-2.92	8.23			

Table 5.3.2: The average results for the 15-node instances for M1.

Set	G.		<i>avg. Dif.</i> (%)	<i>max. Imp.</i> (%)	<i>max. Det.</i> (%)	<i>avg. Imp.</i> (%)	<i>avg. Det.</i> (%)	<i>avg. CPU</i> (s)		
UK15_m-A	C	TC	-1.84	-9.37	0.04	-2.17	0.03	10800*		
		CO2	-1.39	-9.71	0.16	-1.89	0.10			
	B	TC	-4.60	-12.00	0.00	-4.60	0.00		10800*	
		CO2	-4.45	-12.01	2.41	-5.11	1.43			
	A	TC	-5.89	-11.88	0.00	-5.89	0.00			10800*
		CO2	-5.76	-12.07	1.45	-6.52	1.06			
UK15_m-B	C	TC	-0.89	-14.60	0.54	-2.51	0.18	4.51		
		CO2	0.66	-3.24	13.95	-0.86	1.32			
	B	TC	-1.22	-8.59	1.38	-2.10	0.41		3.12	
		CO2	1.54	-12.20	15.24	-2.07	3.94			
	A	TC	-1.63	-9.23	0.48	-2.28	0.30			4.71
		CO2	1.27	-13.24	11.79	-2.30	4.20			
UK15_m-C	C	TC	-1.52	-14.67	0.46	-2.92	0.20	2678.83		
		CO2	-0.59	-4.43	0.81	-1.35	0.35			
	B	TC	-2.89	-21.77	1.01	-3.98	0.39		2018.09	
		CO2	0.20	-10.96	7.45	-2.65	2.53			
	A	TC	-3.34	-22.15	1.05	-4.62	0.51			2343.45
		CO2	0.49	-11.69	11.53	-2.57	4.24			

* Not solved to optimality in 3 hours.

Table 5.3.3: The average results for the 20-node instances for M1.

Set	G.		<i>avg. Dif.</i> (%)	<i>max. Imp.</i> (%)	<i>max. Det.</i> (%)	<i>avg. Imp.</i> (%)	<i>avg. Det.</i> (%)	<i>avg. CPU</i> (s)
UK20_m-A	C	TC	-1.76	-5.38	1.93	-2.40	0.78	10800*
		CO2	-2.51	-10.05	0.88	-3.06	0.60	
	B	TC	-4.67	-13.01	1.95	-5.68	1.08	
		CO2	-4.36	-15.43	3.18	-5.35	1.25	
	A	TC	-5.49	-10.86	1.79	-6.21	0.96	
		CO2	-5.26	-11.08	0.78	-6.26	0.42	
UK20_m-B	C	TC	-0.13	-0.98	0.52	-0.39	0.17	123.57
		CO2	-0.22	-2.44	1.20	-0.80	0.37	
	B	TC	-0.17	-2.36	1.14	-0.82	0.35	
		CO2	-0.20	-3.90	2.00	-1.53	0.68	
	A	TC	-0.29	-1.79	0.74	-0.87	0.42	
		CO2	-0.37	-2.91	1.42	-1.49	0.84	
UK20_m-C	C	TC	-0.98	-4.65	1.63	-1.26	0.60	10800*
		CO2	-1.00	-3.45	0.24	-1.13	0.16	
	B	TC	-2.38	-8.80	0.08	-2.51	0.08	
		CO2	-1.79	-8.99	6.68	-2.96	1.71	
	A	TC	-2.64	-9.82	0.20	-2.95	0.11	
		CO2	-2.13	-7.02	2.67	-2.52	1.35	

* Not solved to optimality in 3 hours.

The analysis of the Tables C.2-C.10 and Tables 5.3.1-5.3.3 allows for deriving some insights. In general, the results suggest that the incorporation of mixed linehauls and backhauls generates improvements and deteriorations, irrespective of whether a total cost perspective is adopted or if the solutions are analyzed from an emissions standpoint. The improvements outweighed the deteriorations both in number and in scale.

A total of 540 instances were run. Around 72% showed some degree of improvement (reduction) in the solution's total cost. An improvement $\geq 1\%$ or $\geq 5\%$ was obtained on around 37% and 17% of the instances, respectively. The improvements could be as high as 22.15%, which was registered in the UK15_m-C set with 50% of backhaul customers (Group A).

Around 28% of the instances showed some degree of deterioration (increase), or no deterioration at all, in the solution's total cost. However, only around 1% showed a deterioration $\geq 1\%$. No deterioration $\geq 5\%$ was registered. The deteriorations could be as high as 1.95%¹, which was registered in the UK20_m-A set with 30% of backhaul customers (Group B).

¹Obtained on an instance not solved to optimality. Cf. 1.38% in the UK15_m-B set with 30% of backhaul customers (Group B).

From an emissions standpoint, around 63% of the instances showed some degree of improvement in the solution's total CO₂ emissions. An improvement $\geq 1\%$ or $\geq 5\%$ was obtained on around 39% and 13% of the instances, respectively. The improvements could be as high as 21.04%, which was registered in the UK10_ *m*-C set with 50% of backhaul customers (Group A).

Around 37% of the instances showed some degree of deterioration, or no deterioration at all, in the solution's total CO₂ emissions. A deterioration $\geq 1\%$ or $\geq 5\%$ was obtained on around 14% and 5% of the instances, respectively. The deteriorations could be as high as 44.30%, which was also registered in the UK10_ *m*-C set with 50% of backhaul customers (Group A).

The Tables 5.3.1-5.3.3 show that the average improvements in the total costs were always greater than the corresponding average deteriorations, regardless of the instance set or group of backhaul customers. This, in turn, translated into average negative differences for most sets and groups. On average, the solutions' total costs were greater reduced than increased.

An average negative difference of as much as 5.89%² could be obtained comparatively to the PRP. This means that, on average, as much as 5.89% could be reduced in the total costs in comparison to the standard version of the problem. Such a reduction is consistent with those found by other studies that, albeit with the application of different PRP variants, report similar figures (*e.g.*, 5% [117] and 4.5% [134]).

The Tables 5.3.1-5.3.3 also show that the average improvements and deteriorations in the total costs are closer for the UK n _ *m*-B sets, and especially for the UK10_ *m*-B set. Some positive average differences were obtained. This is explained by the narrow time windows. The model had less freedom to choose the order in which the customers were served. Fewer route configurations existed. Due to time window compliance, the chosen instantiation for the backhaul customers may have forcefully placed these customers in less beneficial places in the routes.

Conversely, the UK n _ *m*-A and UK n _ *m*-C sets, which consider looser time windows, derived the highest savings in the total costs. The results suggest that if time windows are on the narrower side, a mixed backhauling policy may yield less expressive savings in comparison to a network where only deliveries are performed.

The Tables 5.3.1-5.3.3 show that, unlike the total costs, the emissions' average improvements were not always greater than the average deteriorations. An exception are the UK n _ *m*-A sets, which is attributed to the looser time windows. Most sets and groups

²Obtained on a set not solved to optimality. Cf. 4.45% in the UK10_ *m*-A set with 50% of backhaul customers (Group A).

of backhaul customers presented average negative differences in the CO₂ emissions, despite the fact that some positive ones were registered. On average, the solutions' emissions were greater reduced than increased.

An average negative difference of as much as 5.76%³ could be obtained comparatively to the PRP. This means that, on average, as much as 5.76% could be reduced in the total emissions in comparison to the standard version of the problem. Such a reduction is consistent with those found by other studies (*e.g.*, 5% [80, 104]).

Most sets of instances with varying percentages of backhaul customers show that an average negative difference in the total costs is accompanied by an average negative difference in the emissions. This is not always the case. An average negative difference in the total costs may be accompanied by an average positive difference in the emissions (*e.g.*, UK15_ *m*-C set with the Group A).

This means that, at the individual instance level, some instances show a reduction in the total costs but an increase in the emissions. This behavior is attributed to the effect of the speed. The Table 5.3.4 contrasts four PRP *vs.* M1 solutions to illustrate this. An increase in the total costs but a reduction in the emissions was registered only in instances not solved to optimality, deeming their analysis inaccurate. The following notation is used in the Table 5.3.4:

TC: Total cost of the solution, in pounds;

TD: Total distance traveled, in kilometers;

CO₂: Total amount of CO₂ emitted, in kilograms;

CD: Cost incurred due to the drivers' wages, in pounds;

CE: Cost incurred due of the vehicles' engine module, in pounds;

CC: Cost incurred due to the vehicles' curb weight, in pounds;

CS: Cost incurred due to the speed variations, in pounds;

CL: Cost incurred due to the load carried, in pounds;

$w/(A, B, C)$: with the group A, B or C of backhaul customers.

³Obtained on a set not solved to optimality. Cf. 1.79% in the UK10_ *m*-A set with 30% of backhaul customers (Group B).

Table 5.3.4: M1 solutions with lower total costs but higher emissions

Model	Instance	TC (£)	TD (km)	CO ₂ (kg)	CD (£)	CE (£)	CC (£)	CS (£)	CL (£)
PRP	UK10_05-A	175.67	446.96	179.03	67.64	31.01	33.39	37.63	6.01
	UK10_13-A	195.81	510.49	206.75	71.05	34.47	38.14	44.94	7.22
	UK15_04-B	396.46	863.45	338.95	191.92	68.77	64.50	58.88	12.39
	UK15_03-C	336.11	750.88	29.81	157.00	56.07	56.09	57.04	9.90
M1	UK10_05-A w/ B	173.03	420.83	187.63	59.80	25.90	31.44	45.24	10.65
	Dif. (%)	-1.51	-5.85	4.81	-11.59	-16.48	-5.85	20.23	77.31
M1	UK10_13-A w/ C	194.09	484.65	216.91	63.19	29.43	36.21	53.63	11.63
	Dif. (%)	-0.88	-5.06	4.91	-11.06	-14.61	-5.06	19.35	60.94
M1	UK15_04-B w/ B	388.57	974.49	386.24	155.49	74.53	72.80	69.99	15.76
	Dif. (%)	-1.99	12.86	13.95	-18.98	8.37	12.86	18.86	27.26
M1	UK15_03-C w/ A	329.00	771.44	322.51	134.38	56.27	57.63	61.02	19.70
	Dif. (%)	-2.12	2.74	8.66	-14.41	0.36	2.74	6.97	98.95

The Table 5.3.4 shows that, in M1, increases were registered in CL in comparison to the PRP. The average load carried along the arcs is generally higher when backhaul customers are present in a network since pickups are also performed.

At the same time, on all of the M1 solutions, CD, and hence the total driving time, decreased. Note that CD is the predominant cost in the objective function. To decrease the driving time, the speeds were increased. This translates into a higher CS. CD decreased irrespectively of whether the vehicles traveled a greater TD or not, and of whether CE was lower or not.

The results for these instances indicate that the model compensated the increase in CL by yielding solutions where CD is lower. However, increasing CS to bring down CD proved to be detrimental to the emissions. In the first two M1 solutions, TD was lower than in the PRP, whereas in the latter two it was higher. CC, which can be seen as a proxy for TD, increases/decreases by the same proportion of TD.

CE, which is higher for lower speeds, but higher for a greater distance, may increase or decrease depending on the trade-off between CS and TD (CC). In the first two M1 solutions, CS was higher and TD (CC) was lower. CE decreased. In the latter two solutions, both CS and TD (CC) were higher. CE was increased because the increase in CS was countered with an increase in TD (CC).

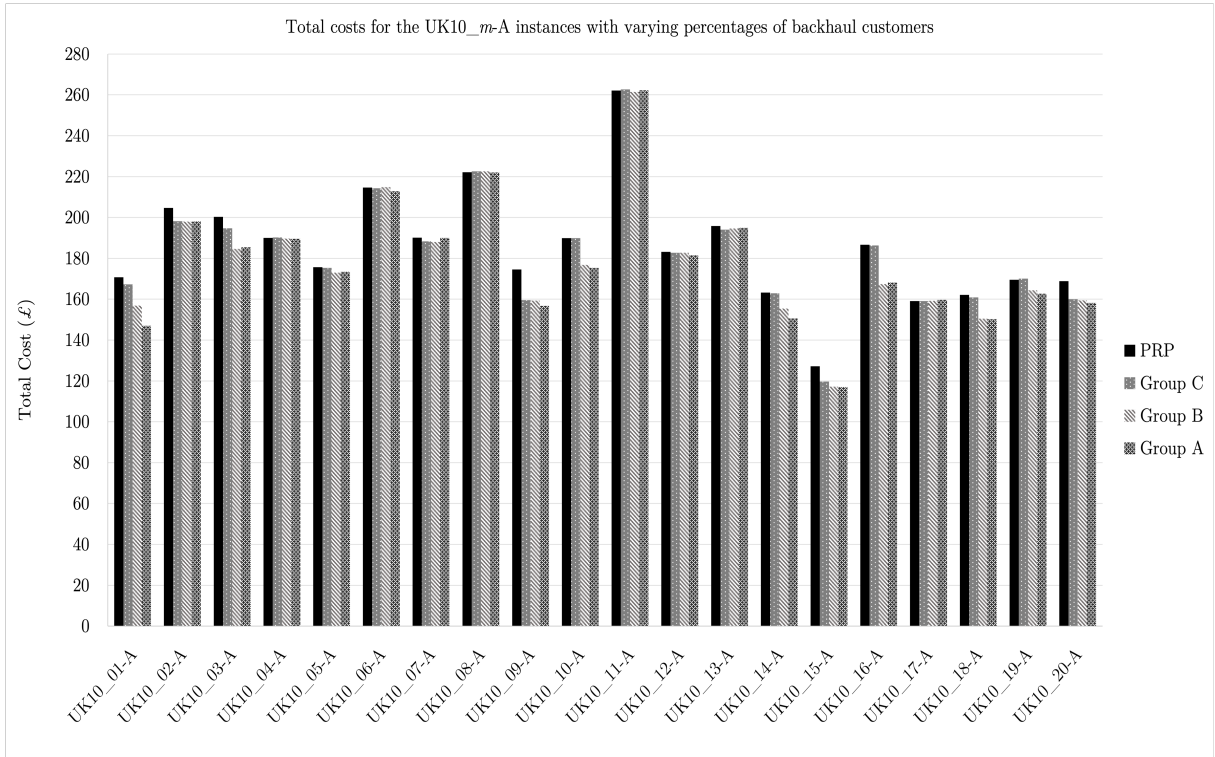


Figure 5.3.1: Total costs for the UK10_m-A instances with varying percentages of backhaul customers

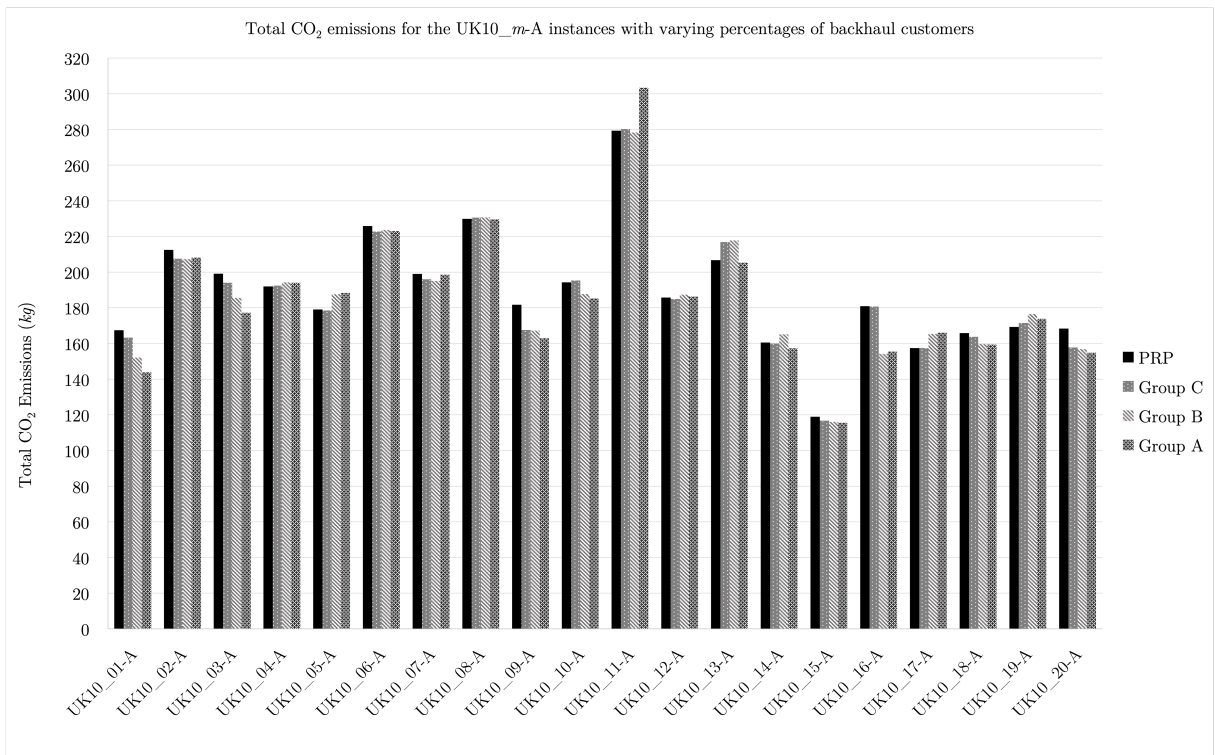


Figure 5.3.2: Total CO₂ emissions for the UK10_m-A instances with varying percentages of backhaul customers

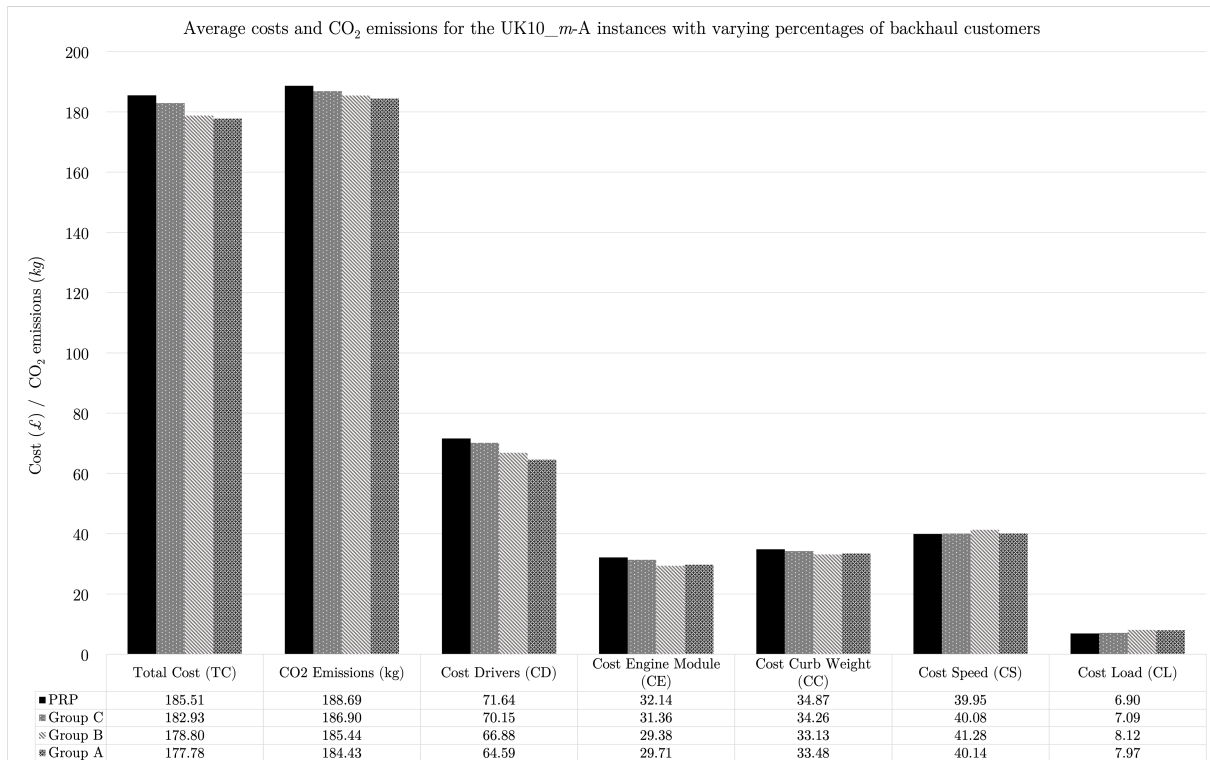


Figure 5.3.3: Average costs and CO₂ emissions for the UK10_ *m*-A instances with varying percentages of backhaul customers

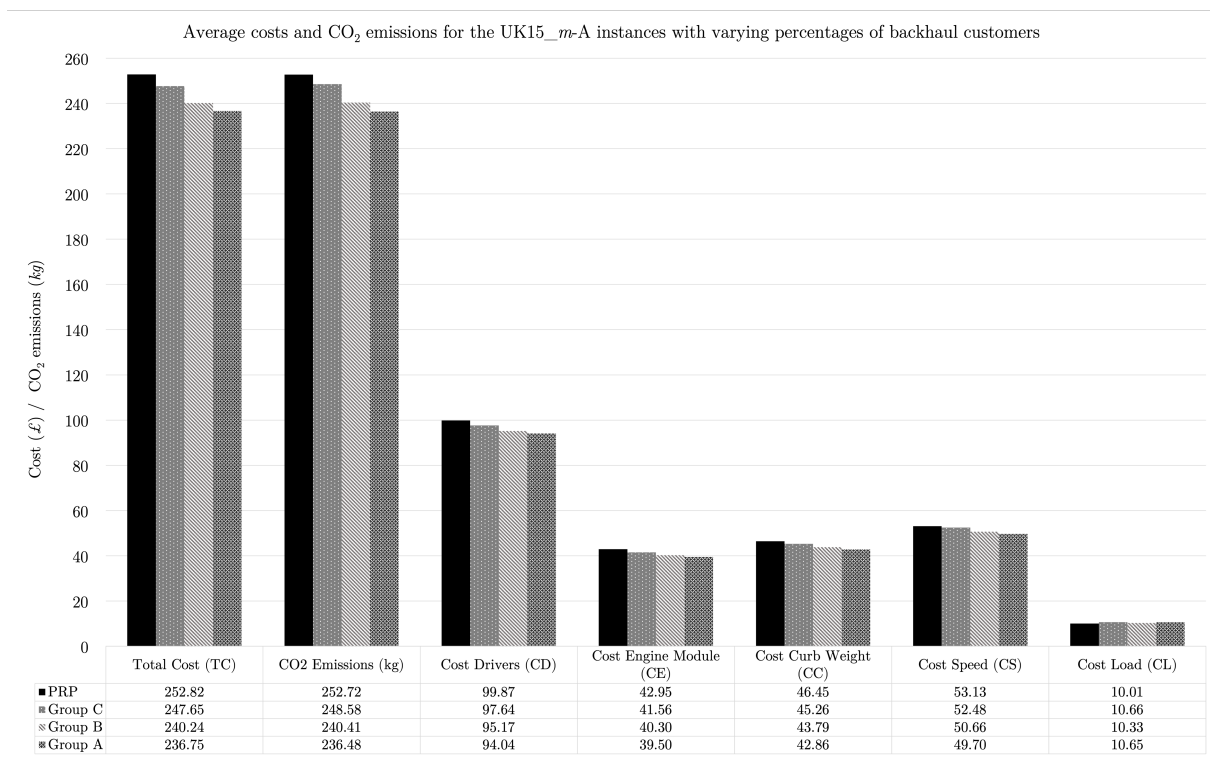


Figure 5.3.4: Average costs for the UK15_ *m*-A instances with varying percentages of backhaul customers

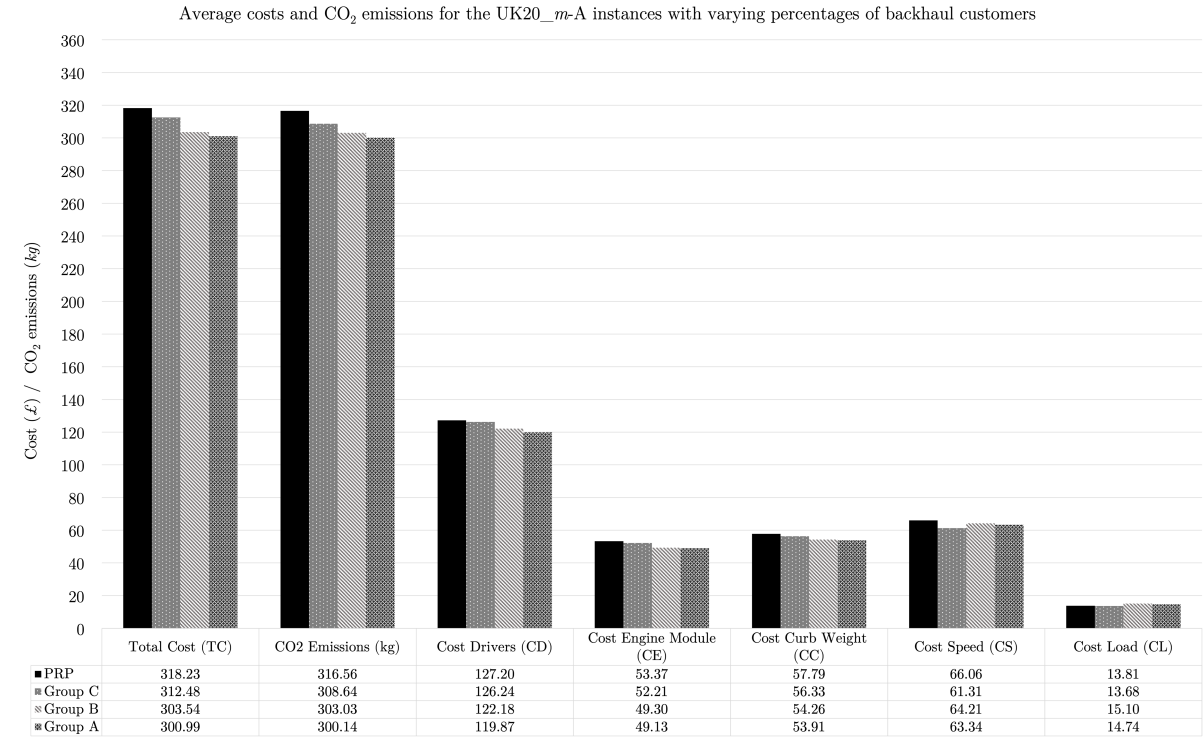


Figure 5.3.5: Average costs for the UK20_*m*-A instances with varying percentages of backhaul customers

The Figures 5.3.1 and 5.3.2 exemplify, for each instance of the UK10_*m*-A set, how the total costs and emissions varied with varying percentages of backhaul customers, respectively. The Figures 5.3.3-5.3.5 show how the average costs and emissions varied with varying percentages of backhaul customers for the UK n _m-A sets. These sets are used in the figures since they derived the best average results.

The Figures 5.3.1 and 5.3.2 show a heterogeneous behavior in terms of total cost and emissions increase or decrease for the UK10_*m*-A set with varying percentages of backhaul customers. Although not shown herein, a similar behavior is registered in the remaining sets.

On some instances, expressive reductions were obtained with the increase in the percentage of backhaul customers (*e.g.*, UK10_01-A). This is far, however, from being a predominant behavior. Increasing the percentage of backhaul customers could, for example, allow for: (*i*) inexpressive savings (*e.g.*, UK10_04-A for the total costs and UK10_15-A for the emissions); (*ii*) a reduction but no (or only very slight) further reductions (*e.g.*, UK10_02-A for the total costs and UK10_20-A for the emissions); (*iii*) reductions and increases and vice-versa.

This heterogeneous behavior makes the choice of an *optimal* percentage of backhaul customers dependent on the overall instance structure, and thus on what customers

are selected to become backhauls. Nonetheless, the Figures 5.3.3-5.3.5, conjointly with the Tables 5.3.1-5.3.3, show that the Group A, which has 50% of backhaul customers, consistently yielded the highest average savings comparatively to the PRP.

The explanation for this is based on the fact that the network becomes more balanced and the vehicles can more efficiently pick up and deliver during the routes when the number of linehaul and backhaul customers is similar. Less loading up at the depot is needed since fewer linehaul customers are present. The number of vehicles used is also reduced. Another study found a similar conclusion, reporting the lowest total cost on a network comprising 5 backhaul customers and 4 linehaul customers [126].

The Figures 5.3.3-5.3.5 show that CL is the only cost that is generally higher in M1 in comparison to the PRP. This owes to the fact that backhaul customers are present in the network. Given that pickups are also performed, the vehicles will generally travel with higher average loads along the arcs. In fact, the average CC (TD) is simultaneously lower. Recall that CC is accounted for in the objective function as $(c_f + e)\lambda\gamma^k\alpha_{ij}d_{ij}w^kx_{ij}^k$ and CL as $(c_f + e)\lambda\gamma^k\alpha_{ij}d_{ij}f_{ij}^k$. Covering less distance (d_{ij}) with a higher average CL leads to a higher average load (f_{ij}^k) carried.

To compensate the increase in CL, one might expect that the vehicles would travel at higher speeds to drop some of the loads off more quickly. This may occur in individual instances. However, increasing the speeds aggravates the vehicles' fuel consumption/emissions. Increasing the speeds, however, reduces the total driving time, and thus CD (the predominant cost), and also CE, which is higher for lower speeds. Interestingly, CS is generally lower in M1, or slightly elevated, comparatively to the PRP. At the same time, CD, and thus the total driving time, is also lower, as is CE.

The results suggest that the overall increase in CL is mainly compensated not by an increase/decrease in the speeds but by a decrease in the total distance traveled. The total distance traveled seems to be the pivotal issue which most influences how much better or worse a M1 solution is comparatively to the PRP, for the considered sets of instances. That is, the vehicles may either travel at higher or lower speeds while also being able to reduce the total driving time because the total distance traveled is also lower. This seems to be the main trade-off in M1 *vs.* PRP. To cope with the increase in the load, the model chose to cover less distance (also reducing the number of vehicles). This allowed for a overall reduction in all the remaining costs and in the emissions.

Depending on how much load the vehicles carry, how large a distance is covered with such load, and how tight a customer's time window is, the model may choose to increase or decrease the speeds on certain arcs. Since TD is generally lower in M1, shorter distances tend to be covered, even if at higher speeds and/or with higher loads.

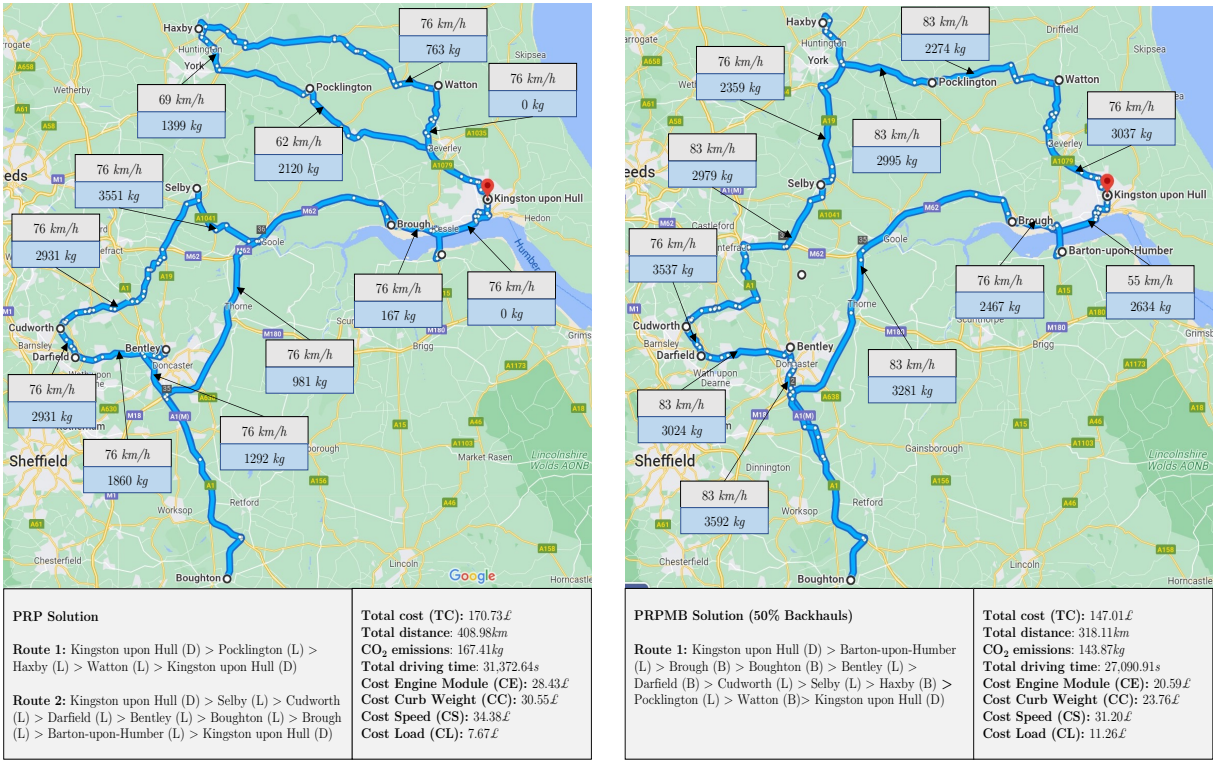


Figure 5.3.6: PRP vs. M1 solution with 50% backhauls for the UK10_01-A instance

The Figures 5.3.6 and 5.3.7 contrast two M1 vs. PRP solutions. In the captions, each customer is identified as (L), a linehaul customer, or (B), a backhaul customer. (D) stands for the depot.

The Figure 5.3.6 shows that a reduction of around 22% in the total distance traveled is obtained in M1 comparatively to the PRP. CC is decreased by the same proportion. CL is increased in around 47%. The model chose to increase the speeds on certain arcs to bring down the total driving time. The total driving time is reduced in around 14%, which translates into a saving of around 9.51£. CE decreased in around 28%.

It can be seen that the increase in the speeds was not detrimental to CS nor to the emissions. Both were reduced. The increase in CL was thus compensated by decreasing the total distance traveled. Traveling less distance allowed the vehicles to travel at higher speeds to reduce CD and CE. The decrease in CD and CE largely offsets the increase in CL, without increasing CS, because the total distance traveled is also lower.

The Figure 5.3.7 shows that a reduction of around 8% in the total distance traveled is obtained in M1 comparatively to the PRP. CC is decreased by the same proportion. In this particular case, CL is decreased in around 28%. The instantiation for the backhaul customers is a rather beneficial one, whereby the arcs with greater distances were traversed with lower loads. Nonetheless, the model increased the overall speeds to bring down the total driving time (thus CD) and CE.

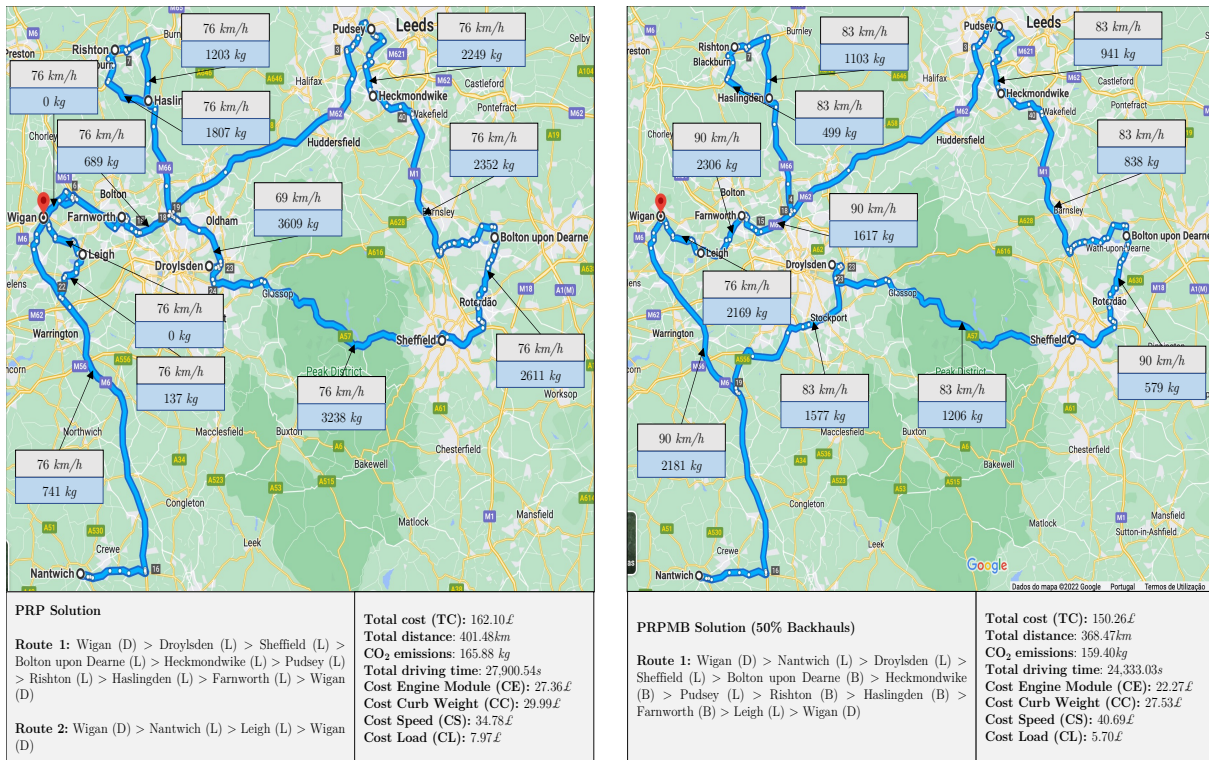


Figure 5.3.7: PRP vs. M1 solution with 50% backhauls for the UK10_18-A instance

The increase in the speeds is done at the expense of a higher CS because the total distance traveled could not be reduced so substantially. But as it can be seen, the increase in CS is largely offset by the decrease in CD and CE. Since the total distance traveled could only be reduced so much, and given a beneficial instantiation of the backhaul customers, the speeds were increased to reduce the overall total cost figure. Naturally, due care should be taken into consideration when interpreting individual instances as a general pattern cannot be derived from them.

Regarding the processing times, no instance in the UK15_m-A and UK20_m-A sets was solved to optimality within 3h. Nevertheless, these sets consistently derived solutions with significant total cost savings in comparison to the PRP, even though much higher MIP gaps were always registered.

M1 proved to be a much more challenging problem to solve in comparison to the PRP. For example, whereas the UK10_m-A set took an average of 39.73s to solve to optimality in the PRP, this set took an average of 490.73s in M1 with the Group C (a 1135% increase), 966.28s with the Group B (a 2332% increase) and 1195.11s with the Group A (a 2908% increase). In general, increasing the percentage of backhaul customers made the problems more challenging to solve.

This opens up room for a two-index formulation for the PRPMB, if only a homogeneous fleet is accounted for. The set K needs not to be utilized in this case. A two-index formu-

lation is likely to help to curb the overall computation times. A three-index formulation significantly rises the number of binary variables.

If a heterogeneous fleet is accounted for, an indexation on the vehicle type (rather on the vehicle k itself, regardless of type) may prove to be advantageous, namely when a larger number of customers or fleet size must be considered. The challenge lies in reformulating the constraints for the vehicle load when leaving the depot.

According to these results, the incorporation of mixed linehauls and backhauls is mainly beneficial from both a total cost perspective and emissions standpoint. Around 28% and 37% of the instances showed some deterioration in the total costs/emissions, respectively. However, these deteriorations were largely inexpressive when compared to the magnitude of the improvements.

Improvements $\leq 1\%$ in the total costs/emissions were registered in around 35% and 24% of the instances, respectively. However, the model enabled to obtain solutions with significant savings, especially when looser time windows were considered.

Another possible way to look at these results is to realize that a total cost deterioration $\leq 1\%$ was obtained in around 95% of the instances that showed a deterioration. Respectively, an emissions deterioration $\leq 1\%$ was obtained in around 65% of the instances that showed a deterioration.

These results can be of interest for the practitioners who may be considering the simultaneous implementation of a mixed backhauling policy and of more environmentally conscious models for vehicle routing. They suggest that by using the PRP under such a policy, the total costs and emissions will only marginally increase, if an increase is registered in the first place. In other words, the potential for cost/emissions reduction seems likely to outweigh the potential for cost/emissions increase.

5.4 Computational Results for M2

In order to solve the bi-objective PRPMB model, MOMP methods must be applied. The focus of this dissertation is to try to generate as many efficient solutions as possible so that comprehensive representations of the Pareto fronts are provided. For this purpose, the ε -constraint technique (EC) was chosen both for its effectiveness and simplicity of implementation. Firstly, an explanation of the method is provided. Its main advantage and challenge are underlined.

In MOMP problems, and considering a real-life decision-making setting, the DM must usually select a single efficient solution from the Pareto front to implement. To offer further information, the efficient solution that minimizes the Chebyshev distance to a

reference point was computed for each individual instance. As explained in the subsection 5.4.2, the calculation of such a solution according to this metric assumes a risk-averse attitude from the DM.

5.4.1 The ε -constraint technique

The EC is a well-known *a posteriori* generating technique for solving MOILP and MOMILP problems. This technique uses a scalarization of the objective functions transforming the multi-objective problem into a series of single-objective ones.

In the EC, a so-called *surrogate scalar function* is selected to be optimized while all the remaining objective functions become part of the constraint set. In general, this surrogate function is designed to temporarily aggregate the various objective functions, and no particular concern is given as to whether or not it represents the actual preferences of the DM [161]. Naturally, the DM can always choose as the surrogate function the objective function he/she assigns the most importance to. Without loss of generality, let us assume that one has the following maximization problem:

$$\text{maximize } \mathbf{z}_k = \underline{\mathbf{c}}_k \mathbf{x} \quad (5.4.1)$$

subject to

$$\underline{\mathbf{x}} \in S \quad (5.4.2)$$

where $\mathbf{z}_k = \underline{\mathbf{c}}_k \mathbf{x}$ denotes the $k \geq 2$ objective functions to be simultaneously maximized, $\underline{\mathbf{c}}_p = (c_1, c_2, \dots, c_n)$, $p = 1, \dots, k$, denotes the vector of the objective function coefficients, and $\underline{\mathbf{x}} = (x_1, x_2, \dots, x_n)^\top$ denotes the vector of decision variables, which belong to a non-empty feasible region $S \subseteq \mathbb{R}^n$. Let us assume that $\mathbf{z}_i = \underline{\mathbf{c}}_i \mathbf{x}$ is selected to be optimized. This assumption is non-restrictive and any other function can be chosen. The problem takes the form:

$$\text{maximize } \mathbf{z}_i = \underline{\mathbf{c}}_i \mathbf{x} \quad (5.4.3)$$

subject to

$$\mathbf{z}_p = \underline{\mathbf{c}}_p \mathbf{x} \geq \varepsilon_p \quad \forall p = 1, \dots, k, p \neq i, \quad (5.4.4)$$

$$\underline{\mathbf{x}} \in S \quad (5.4.5)$$

where $\mathbf{z}_p = \underline{\mathbf{c}}_p \mathbf{x}$, $p = 1, \dots, k, p \neq i$, are the $k - 1$ objective functions which become part of the constraint set and on which the DM imposes a lower bound ε_p . Should the minimization of any of the p objective functions be intended, then an upper bound, such that $\mathbf{z}_p = \underline{\mathbf{c}}_p \mathbf{x} \leq \varepsilon_p$, would be imposed.

In each run of the technique, the right-hand side of the constraints (5.4.4) is increased (or decreased, if a minimization is intended) by a given amount δ and the problem is solved again. This is repeated until the problem becomes infeasible and no new solution can be obtained. Thus, via the parametric variation of δ units of the right-hand-side of the constraints (5.4.4), and after a certain number of runs, one eventually obtains all solutions for the problem. For MOILP problems, $\delta = 1$, since the variables, and thus the objective functions, can only take on integer (discrete) values, provided that $\underline{\mathbf{c}}_p$ are integers.

The main challenge in the EC is to guarantee the strict efficiency of the solutions obtained [140]. To ensure this, the EC can be adapted to include a small perturbation ω of the remaining objective functions which became part of the constraint set [161]. The perturbed objective functions are then added to the objective function which was selected to be optimized. The problem takes the form:

$$\text{maximize } \mathbf{z}_i = \underline{\mathbf{c}}_i \underline{\mathbf{x}} + \omega \sum_{p=1, p \neq i}^k \mathbf{z}_p = \underline{\mathbf{c}}_p \underline{\mathbf{x}} \quad (5.4.6)$$

subject to

$$\mathbf{z}_p = \underline{\mathbf{c}}_p \underline{\mathbf{x}} \geq \varepsilon_p \quad \forall p = 1, \dots, k, k \neq i. \quad (5.4.7)$$

$$\underline{\mathbf{x}} \in S \quad (5.4.8)$$

As referred to in the subsection 3.4.1, the EC offers several advantages over the remaining *a posteriori* generating methods which use a scalarization approach. The main advantage of the EC is that it can generate the entire efficient set even for relatively large problems [140]. This reinforces the confidence of the DM in choosing a most-preferred solution since numerous trade-offs can be accounted for and evaluated.

5.4.2 The minimization of the Chebyshev distance to a reference point

In MOMP problems, and considering a real-life decision-making setting, the DM must usually select a single efficient solution to implement. The computation of the efficient solution that minimizes the distance to a reference point is interesting to find the solution that is closest, according to a given metric, to the DM's aspiration levels.

The ideal solution, which is comprised of the individual optima ($\mathbf{z}_1^*, \dots, \mathbf{z}_k^*$) for each objective function, is usually considered as the reference point because it provides full satisfaction across all objective functions. However, in non-trivial MOMP problems, the ideal solution is unattainable. One is then interested in finding the efficient solution that

minimizes the distance, or *discomfort*, of not being able to reach it. Considering the ideal solution as the reference point, the problem of minimizing the distance to it according to a given L_m metric, where $m \geq 1$ is a real number, is given as:

$$\min_{i=1,\dots,k} \|\mathbf{z}_i^* - \mathbf{z}_i\|_m \quad (5.4.9)$$

subject to

$$\underline{\mathbf{x}} \in S \quad (5.4.10)$$

where \mathbf{z}_i^* , $i = 1, \dots, k$, denotes the individual optimum for the \mathbf{z}_i objective function. Depending on the considered value for m , the minimization of the distance to the reference point is computed according to different metrics.

When $m = 1$, one has the L_1 metric, commonly referred to as the *Manhattan distance*. All the distances, whether large or small, from the reference point are taken into account in direct proportion to their magnitude. Since all the distances accounted for, there is a compensatory effect. This can be interpreted as a more lax attitude from the DM. It is expected that the various distances will eventually balance one another and the process converges to a good compromise solution. The minimization of the Manhattan distance to a reference point is computed as:

$$\min_{i=1,\dots,k} \|\mathbf{z}_i^* - \mathbf{z}_i\|_1 = \sum_{i=1}^k |\mathbf{z}_i^* - \mathbf{z}_i| \quad (5.4.11)$$

subject to

$$\underline{\mathbf{x}} \in S \quad (5.4.12)$$

The minimization of the Manhattan distance to a reference point always leads to an extreme efficient solution (a vertex) [161]. Note that S is convex (composed of linear constraints).

When $2 \leq m \leq \infty$, one has a L_m metric. As the value for m increases, the larger distances from the reference point gain more relevance than the smaller ones. In other words, the compensatory effects dims. The minimization of the distance to a reference point according to a L_m metric is computed as:

$$\min_{i=1,\dots,k} \|\mathbf{z}_i^* - \mathbf{z}_i\|_p = \sqrt[p]{\sum_{i=1}^k |\mathbf{z}_i^* - \mathbf{z}_i|^p} \quad (5.4.13)$$

subject to

$$\underline{\mathbf{x}} \in S \quad (5.4.14)$$

The *Chebyshev distance* to the reference point, or L_∞ metric, is a special case of the L_m metrics when $m \rightarrow \infty$. The L_∞ metric assumes a risk-averse attitude from the DM because only the largest absolute distance to the reference point is accounted for. No compensatory effect exists. The minimization of the Chebyshev distance to the reference point is computed as:

$$\min_{i=1,\dots,k} \|\mathbf{z}_i^* - \mathbf{z}_i\|_\infty = \max_{i=1,\dots,k} |\mathbf{z}_i^* - \mathbf{z}_i| \quad (5.4.15)$$

subject to

$$\underline{\mathbf{x}} \in S \quad (5.4.16)$$

The minimization of the Chebyshev distance to a reference point allows for both supported and unsupported (interior to the convex hull) efficient solutions to be obtained [161]. The formulation for the L_∞ metric (5.4.15)-(5.4.16) is non-linear since it is a *min max* problem. The formulation can be linearized considering an additional non-negative variable v . The problem takes the form:

$$\min v \quad (5.4.17)$$

subject to

$$v \geq \mathbf{z}_i^* - \mathbf{z}_i \quad \forall i = 1, \dots, k, \quad (5.4.18)$$

$$v \geq 0 \quad (5.4.19)$$

$$\underline{\mathbf{x}} \in S \quad (5.4.20)$$

Similarly to the EC, the strict efficiency of the solutions must be ensured. A small perturbation w , $w > 0$, of the L_1 metric can be added to the L_∞ metric to ensure this [161]. The augmented L_∞ metric (L_∞^w) takes the form:

$$\min v + w \sum_{i=1}^k |\mathbf{z}_i^* - \mathbf{z}_i| \quad (5.4.21)$$

subject to

$$v \geq \mathbf{z}_i^* - \mathbf{z}_i \quad \forall i = 1, \dots, k, \quad (5.4.22)$$

$$v \geq 0 \quad (5.4.23)$$

$$\underline{\mathbf{x}} \in S \quad (5.4.24)$$

Still, if the k objective functions are expressed in different magnitude orders, the L_∞^w metric can be expanded to include λ_i weights. These weights are not preference-based coefficients but analytical tools to ensure that the k objective functions are expressed in

the same magnitude order. The weighted and augmented L_∞ metric ($L_\infty^{\lambda:w}$) takes the form:

$$\min v + w \sum_{i=1}^k |\mathbf{z}_i^* - \mathbf{z}_i| \quad (5.4.25)$$

subject to

$$v \geq \lambda_i(\mathbf{z}_i^* - \mathbf{z}_i) \quad \forall i = 1, \dots, k, \quad (5.4.26)$$

$$v \geq 0 \quad (5.4.27)$$

$$\underline{\mathbf{x}} \in S \quad (5.4.28)$$

In the next subsection, the computational results for M2 are presented.

5.4.3 Bi-objective solutions

In this subsection, the computational results for the bi-objective PRPMB using a homogeneous fleet (M2) are reported. The same values for the parameters as in M1 are considered (see Appendix B). The number of vehicles in the set K was defined similarly. The perturbed EC technique and $L_\infty^{\lambda:w}$ metric were applied.

M2 was transformed from a MILP model into an ILP one. Both types of models allow the EC to generate a new solution in each run. However, in MILP models these solutions can be as close to one another as intended, depending on the computational effort one is willing to expend. This is because the solution space is continuous. In ILP, the solution space is discrete and the EC generates solutions further apart from one another, comparatively. This is more interesting for trade-off analysis.

All M2 coefficients were thus converted into integers and all the continuous decision variables were converted into integer ones. This is done to decrease by $\delta = 1$ unit the RHS of the constraint in the EC. The minimization of the total CO₂ emissions (\mathbf{z}_1) was considered. The total driving time (\mathbf{z}_2) became part of the constraint set.

Each term of \mathbf{z}_1 was multiplied by a 10^5 scaling factor and the coefficients were rounded into integers. Such a scaling factor was used to preserve 5 decimal places in the solutions' emissions. No claim is made that this is the best value to use. As for \mathbf{z}_2 , the divisions (d_{ij}/v^r) in the time window constraints were rounded into integers.

As an example, an efficient solution for M2 took the general form (12345678, 91011). This means that $\mathbf{z}_1 = 123.45678kg$ of emitted CO₂ and $\mathbf{z}_2 = 91011s$ of total driving time. Naturally, the ILP version of M2 is not the exact same as the MILP one. Nonetheless, we consider it to be a close enough one to render the analysis accurate.

The Tables D.2-D.4, in the Appendix D, present the detailed results for some of the 10- and 15-customers sets for all groups of backhaul customers. The results for the UK10_*m*-B, UK10_*m*-C, and UK15_*m*-B sets are reported. The preliminary experiments with the remaining sets showed that exceedingly long CPU times were being obtained to solve a single instance. This is due to overall hardness of the problems, or otherwise a high number of solutions found.

The Table 5.4.1 summarizes the average results for each instance set and group of backhaul customers. In the Table 5.4.1, the following notation is used and results are summarized:

Set: set of instances, which includes all the 20 instances of a given set;

G.: group of backhaul customers (A, B or C);

Sol.: the number of efficient solutions found by the EC;

CO₂:

max.: the maximum amount, in kilograms, of emitted CO₂⁴;

min.: the minimum amount, in kilograms, of emitted CO₂⁵;

Dif.: the difference, in kilograms, between the maximum and minimum amount of emitted CO₂.

D.T.:

max.: the maximum driving time, in hours⁵;

min.: the minimum driving time, in hours⁴;

Dif.: the difference, in hours, between the maximum and minimum driving times.

CPU:

avg.: the average CPU time, in seconds, to find an efficient solution;

Total: the total CPU time, in seconds, to find all the efficient solutions.

Chebyshev Solution:

CO₂: the amount, in kilograms, of emitted CO₂;

D.T.: the driving time, in hours;

CPU: the CPU time, in seconds, to find the solution.

⁴In the driving time-minimizing solution.

⁵In the CO₂-minimizing solution.

Table 5.4.1: The average results for M2

Set	G.	Sol.	CO ₂ (kg)			D.T. (h)			CPU (s)		Chebyshev Solution		
			max.	min.	Dif.	max.	min.	Dif.	avg.	Total	CO ₂	D.T.	CPU
UK10_ _m -B	C	150	262.36	233.38	28.98	18.08	16.59	1.50	0.23	34.86	243.39	16.95	0.23
	B	152	261.65	233.31	28.34	18.08	16.54	1.53	0.24	41.98	243.90	16.93	0.26
	A	152	261.45	233.15	28.30	18.08	16.54	1.53	0.25	40.95	243.75	16.93	0.27
UK10_ _m -C	C	1055	260.36	189.54	70.81	14.49	11.22	3.27	7.12	8035.58	210.12	12.13	6.28
	B	1098	256.37	188.67	67.71	14.52	11.15	3.37	11.92	14,643.30	208.94	12.10	11.67
	A	1039	253.02	188.53	64.48	14.44	11.19	3.25	12.08	14,489.34	208.92	12.12	13.00
UK15_ _m -B	C	1172	388.06	308.10	79.97	23.45	19.65	3.81	10.98	13,923.67	334.55	20.77	18.10
	B	1133	374.54	306.83	67.71	23.45	19.61	3.84	12.25	18,469.50	332.51	20.62	14.96
	A	1144	372.63	306.21	66.41	23.53	19.58	3.95	13.12	21,315.07	330.02	20.53	10.54

The analysis of the Table 5.4.1 and Tables D.2-D.4 allows for deriving some insights. The results indicate that significant reductions can be achieved both in the total CO₂ emissions and total driving time. In general, the trade-off between the two objectives tends to be large enough to grant the possibility of improving one without deteriorating the other significantly.

The Table 5.4.1 shows that a minimum average difference of 28.30kg of CO₂ was registered in the UK10__m-B set with 50% of backhaul customers (Group A). A maximum average difference of 79.97kg of CO₂ was obtained in the UK15__m-B set with 10% of backhaul customers (Group C). At the individual instance level, the Tables D.2-D.4 show that the difference in the emissions could be as little as 7.94kg (UK10_₁₅-B with the Groups C, B and A) to as high as 199.39kg (UK15_₁₈-B with the Group C).

Regarding the total driving time, the Table 5.4.1 shows that a minimum average difference of 1.50h was obtained, which was registered in the UK10__m-B set with 10% of backhaul customers (Group C). A maximum average difference of 3.95h was obtained in the UK15__m-B set with 50% of backhaul customers (Group A). At the individual instance level, the Tables D.2-D.4 show that the difference in the total driving time could be as little as 0.56h (UK10_₁₅-B with the Groups C, B and A) to as high as 6.95h (UK15_₁₀-B with the Groups B and A).

On a test instance with 30 customers, Demir *et al.* [139], who introduced the bi-objective PRP, report 2h and 20l (around 46.40kg⁶ of CO₂) for the total driving time and fuel consumption differences, respectively. On instances with 100 customers, a minimum and maximum fuel consumption difference of 22.70l (around 52.55kg⁶ of CO₂) and 212.00l (around 491.84kg⁶ of CO₂) is reported by authors. For the total driving time, these figures are 2.10h and 8.80h, respectively. No backhaul customers were considered and speeds between 20km/h and 100km/h were employed. Albeit with a application of a different PRP variant, our results are in line with those reported in the literature.

The Table 5.4.1 shows that the average difference in the CO₂ emissions tended to decrease with the increase in the percentage of backhaul customers. Both the maximum and minimum average values decreased, albeit the former was reduced more substantially. Conversely, the average difference in the total driving time tended to increase with the increase in the percentage of backhaul customers. The maximum average value remained relatively stable, or slightly increased, while the minimum average value tended to decrease.

These results suggest that by varying the percentage of backhaul customers, it becomes increasingly difficult to improve one of the objectives without deteriorating the other significantly. When the number of backhaul customers was low (Group C), it was, on average, easier to improve the emissions without deteriorating the driving time so significantly. When the number of backhaul customers increased, it became, on average, easier to improve the driving time. This behavior is illustrated in the Figure 5.4.1. The Pareto fronts for the UK15_15-B instance with varying percentages of backhaul customers are depicted.

Each Pareto front in the Figure 5.4.1 shows, as expected, a conflict between the two objectives. Improving one leads to the deterioration of the other due to the effect of the speed. Looking at the Group C solutions, the emissions can be brought down from around 374.20kg, in the driving time-minimizing solution, to 343.45kg, in the emissions-minimizing solution. A difference of 30.75kg can be obtained. The driving time can be reduced from around 17.58h (63,305s) to 15.71h (56,555s), a 1.87h difference. Considering these two solutions, a reduction of around 8.21% in the emissions leads to a 10.66% increase in the driving time.

The Group B solutions allow for a 28.15kg and a 1.87h difference to be obtained in the emissions and driving time, respectively. The emissions can be decreased from around 376.61kg to 348.46kg, whereas the driving time can be decreased from around 17.58h

⁶If 2.32kg of CO₂/l of diesel is considered.

(63,305s) to 15.71h (56,555s). Hence, reducing the emissions in around 7.48% implies increasing the driving time in 10.66%.

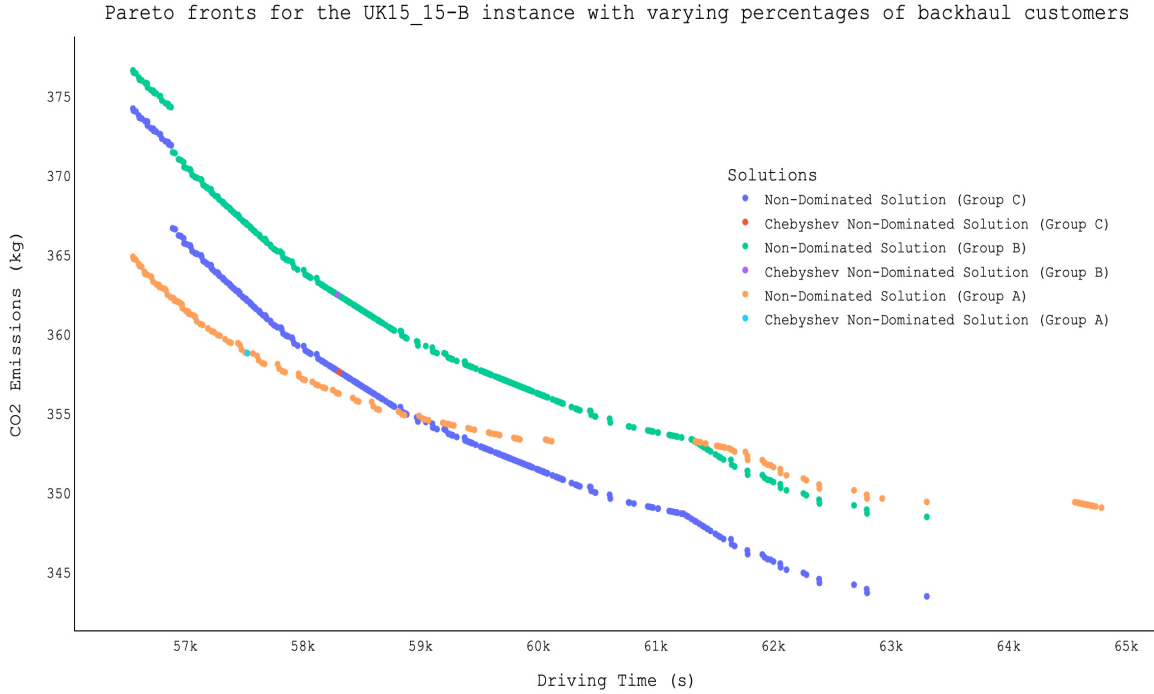


Figure 5.4.1: Pareto fronts for the UK15_15-B instance with varying percentages of backhaul customers

In the Group A solutions, the emissions and the driving time can be decreased in 15.83kg and 2.29h, respectively. The emissions can be brought down from around 364.86kg to 349.04kg, and the driving time from around 18.00h (64,790s) to 15.71h (56,555s). To reduce the emissions in around 4.34%, the driving time must be increased in 12.71%.

The Figure 5.4.1 shows that the difference in the emissions decreased and the difference in the driving time increased with the increase in the percentage of backhaul customers. This happened even though that, for this particular instance, the maximum emissions' value was obtained in the Group B and the minimum value actually increased. Also, the maximum driving time remained the same for the Groups C and B and it was increased for the Group A. The minimum driving time remained the same for all groups of backhaul customers.

The decrease in the emissions' difference is explained by the amount of load carried. When the number of backhaul customers is low (Group C), fewer savings are obtained in the emissions along a route. This is because most service requests are deliveries and the vehicles need to load up more at the depot. Although a route's average load is generally lower, the loads, especially on the first arcs, tend to be higher. The maximum amount

of emitted CO₂ tends to be higher and the minimum amount tends to be lower because the model may place these (few) customers in very beneficial or detrimental places in the routes. This is especially true in the sets tested because the time windows are narrow.

When the number of backhaul customers increases, greater emissions savings are obtained because less loading up at the depot is needed. Although a route's average load is generally higher, the loads on the arcs tend to be lower. The maximum amount of emitted CO₂ tends to be lower. Higher emissions are generally not achieved due to the aforementioned savings. The minimum amount tends, on average, to be lower, although it is higher for the UK15_15-B instance. The minimum amount may increase due to a higher average load carried.

In the Figure 5.4.1, the vertical distance between the Group C and Group B solutions is due to the fact that, for the exact same routes, speeds on the arcs, and service time starts (hence total driving times), higher average loads are carried in the Group B. This can be seen in the Tables 5.4.2-5.4.4, which show the solutions at the Pareto fronts' discontinuities for each group of backhaul customers.

The Group A solutions until around 61,100s do not follow the same behavior as the Group C and Group B solutions because the routes are not the same. For example, whereas the driving time-minimizing routes are $\langle 0, 3, 6, 13, 9, 7, 11, 1, 15, 2, 0 \rangle$ and $\langle 0, 10, 8, 12, 5, 14, 4, 0 \rangle$ for the Groups C and B, they are $\langle 0, 8, 6, 13, 9, 7, 11, 1, 15, 2, 0 \rangle$ and $\langle 0, 10, 3, 12, 5, 14, 4, 0 \rangle$ for the Group A.

The Figure 5.4.1 shows that some solutions with driving times between around 64,500s and 64,800s were found for the Group A but not for the Groups C and B. This is due to the number of backhaul customers present in the network, and thus to the load carried. Although these solutions would be capacity and time window feasible for the Groups C and B, a reduction in the emissions wouldn't be achieved. This explains the reason for the increase in the driving time difference.

In the Figure 5.4.1, the discontinuities in the Pareto fronts are caused by a variation of some x_{ij}^k variables. Some customers are swapped between routes. Some speed levels may also vary at the discontinuities. The discontinuities occur because no further combination of speeds ensures time window feasibility while also minimizing the emissions.

The Tables 5.4.2-5.4.4 show the speed level and load carried for each arc. The swapped customers are identified in bold. The routes for the Groups C and B are the same and differ only in the amount of load carried on the arcs. The routes for the Group B present higher average loads. In the Group C discontinuity, the customers 2, 3 and 15 are swapped from the Route 2 to the Route 1. The customer 8 is swapped from the Route 1 to the Route 2. Some speed levels also vary. The same rationale applies to the remaining discontinuities.

Table 5.4.2: Discontinuities in the Figure 5.4.1 for the Group A

Group	CO ₂ (kg)	
A	353,23	Route 1: < 0, 10, 3, 12, 5, 14, 4, 0 >
		Speed Levels: < 5, 5, 5, 7, 6, 6, 5 >
		Loads (kg): < 491, 838, 605, 974, 716, 1138, 1346 >
		Route 2: < 0, 8, 6, 13, 9, 7, 11, 1, 15, 2, 0 >
		Speed Levels: < 5, 6, 8, 9, 8, 8, 10, 9, 10, 9 >
		Loads (kg): < 2608, 3112, 3569, 3270, 2758, 1935, 1376, 1221, 961, 1237 >
A	353,22	Route 1: < 0, 10, 3, 12, 5, 14, 4, 2 , 15 , 0 >
		Speed Levels: < 5, 5, 5, 7, 8, 9, 8, 9, 7 >
		Loads (kg): < 751, 1098, 865, 1234, 976, 1398, 1606, 1882, 1622 >
		Route 2: < 0, 8, 6, 13, 9, 7, 11, 1, 0 >
		Speed Levels: < 5, 6, 6, 7, 6, 7, 6, 7 >
		Loads (kg): < 2348, 2852, 3309, 3010, 2498, 1675, 1116, 961 >
A	349,40	Route 1: < 0, 10, 3, 12, 5, 7, 11, 1, 0 >
		Speed Levels: < 5, 5, 5, 7, 9, 8, 8, 5 >
		Loads (kg): < 2028, 2375, 2142, 2511, 2253, 1430, 871, 716 >
		Route 2: < 0, 8, 6, 13, 9, 14, 4, 2, 15, 0 >
		Speed Levels: < 5, 6, 7, 6, 5, 5, 8, 8, 7 >
		Loads (kg): < 1071, 1575, 2032, 1733, 1221, 1643, 1851, 2127, 1867 >
A	349,39	Route 1: < 0, 10, 3, 12, 5, 14 , 4 , 2 , 0 >
		Speed Levels: < 5, 5, 5, 7, 7, 7, 6, 6 >
		Loads (kg): < 491, 838, 605, 974, 716, 1138, 1346, 1622 >
		Route 2: < 0, 8, 6, 13, 9, 7 , 11 , 1 , 15, 0 >
		Speed Levels: < 5, 6, 7, 7, 7, 7, 6, 6, 6 >
		Loads (kg): < 2608, 3112, 3569, 3270, 2758, 1935, 1376, 1221, 961 >

Table 5.4.3: Discontinuities in the Figure 5.4.1 for the Group B

Group	CO ₂ (kg)	
B	374,29	Route 1: < 0, 10, 8, 12, 5, 14, 4, 0 >
		Speed Levels: < 5, 7, 7, 8, 8, 9, 10 >
		Loads (kg): < 680, 1027, 1531, 1900, 1642, 1220, 1428 >
	↓	Route 2: < 0, 3, 6, 13, 9, 7, 11, 1, 15, 2, 0 >
		Speed Levels: < 5, 5, 8, 9, 8, 8, 9, 10, 10, 9 >
		Loads (kg): < 3574, 3341, 2884, 2585, 2073, 1250, 691, 536, 276, 0 >
371,45	Route 1: < 0, 10, 3 , 12, 5, 14, 4, 2 , 15 , 0 >	
		Speed Levels: < 5, 5, 5, 8, 9, 8, 10, 10, 10 >
		Loads (kg): < 1449, 1796, 1563, 1932, 1674, 1252, 1460, 1184, 924 >
	Route 2: < 0, 8 , 6, 13, 9, 7, 11, 1, 0 >	
		Speed Levels: < 5, 6, 9, 8, 8, 9, 10, 10 >
		Loads (kg): < 2805, 3309, 2852, 2553, 2041, 1218, 659, 504 >

Table 5.4.4: Discontinuities in the Figure 5.4.1 for the Group C

Group	CO ₂ (kg)	
C	371,89	Route 1: < 0, 10, 8, 12, 5, 14, 4, 0 >
		Speed Levels: < 5, 7, 7, 8, 8, 9, 10 >
		Loads (kg): < 1761, 2108, 1604, 1235, 977, 555, 347 >
	↓	Route 2: < 0, 3, 6, 13, 9, 7, 11, 1, 15, 2, 0 >
		Speed Levels: < 5, 5, 8, 9, 8, 8, 9, 10, 10, 9 >
		Loads (kg): < 3574, 3341, 2884, 2585, 2073, 1250, 691, 536, 276, 0 >
366,65	Route 1: < 0, 10, 3 , 12, 5, 14, 4, 2 , 15 , 0 >	
		Speed Levels: < 5, 5, 5, 8, 9, 8, 10, 10, 10 >
		Loads (kg): < 2026, 2373, 2140, 1771, 1513, 1091, 883, 607, 347 >
	Route 2: < 0, 8 , 6, 13, 9, 7, 11, 1, 0 >	
		Speed Levels: < 5, 6, 9, 8, 8, 9, 10, 10 >
		Loads (kg): < 3309, 2805, 2348, 2049, 1537, 714, 155, 0 >

Regarding the number of efficient solutions found, the Table 5.4.1 shows that, as expected, it tends to increase with the increase in the number of customers and when looser time windows are considered for the same number of customers. The explanation for this is that, when the number of customers increases, a higher number of route configurations is generally available. When looser time windows are considered, both a higher number of route configurations and speed combinations, either for the same routes or for different routes, can be tested.

As seen in the Table 5.4.1, the EC is fast especially when narrower time windows are considered. For the UK10_01-B set, it took a maximum average CPU time of around 0.25s to find an efficient solution. Since the number of solutions found is not so large, this set is solved under 42s, on average, for varying percentages of backhaul customers. Increasing the percentage of backhaul customers seems to make the problems harder to solve. This translated into higher average CPU times to find an efficient solution.

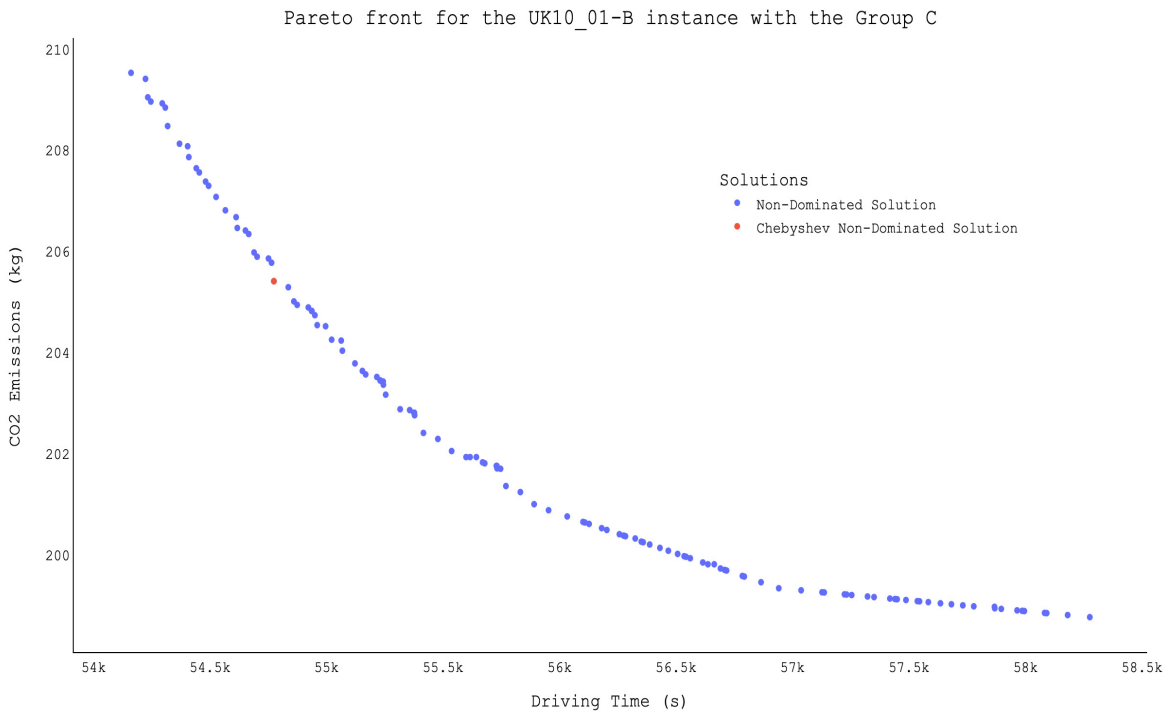


Figure 5.4.2: Pareto front for the UK10_01-B instance with the Group C

The Figure 5.4.2 shows the Pareto front for the UK10_01-B instance with 10% of backhaul customers (Group C). In this case, the trade-off between the emissions and the total driving time is not so significant.

The driving time can be brought down from around 16.19h (58,276s) to 15.04h (54,161s). The emissions can be decreased from around 209.52kg to 198.76kg. Reducing the emissions in around 5.13% implies a 7.06% increase in the driving time. In general, one objective

needs not to be greatly sacrificed in order to improve the other. This depends on the region of the Pareto front where one is situated. As discussed earlier, it can be seen that, for the Group C, it is generally easier to improve the emissions.

For example, a reduction of around 2.6% in the emissions can be achieved with an increase of around 1.5% in the total driving time, if the former are decreased from around 209.52kg, in the driving time-minimizing solution, to 204kg, with a little over of 55,000s of driving time.

Conversely, in order to slightly improve the emissions further than around 201kg, a greater sacrifice needs to be done to the driving time. For example, the driving time needs to be increased in around 3.9% in order to reduce the emissions in around 1.1%, if one were to reduce the latter from 201kg, with a little under 56,000s of driving time, to around 198.76kg, in the emissions-minimizing solution.

The distance-minimizing Chebyshev solution is situated in a region of the Pareto front which is characterized by lower driving times but higher emissions. It is an unsupported non-dominated solution. It is dominated by a convex combination of (two) extreme points: the non-dominated solutions immediately to its top-left and bottom-right, respectively.

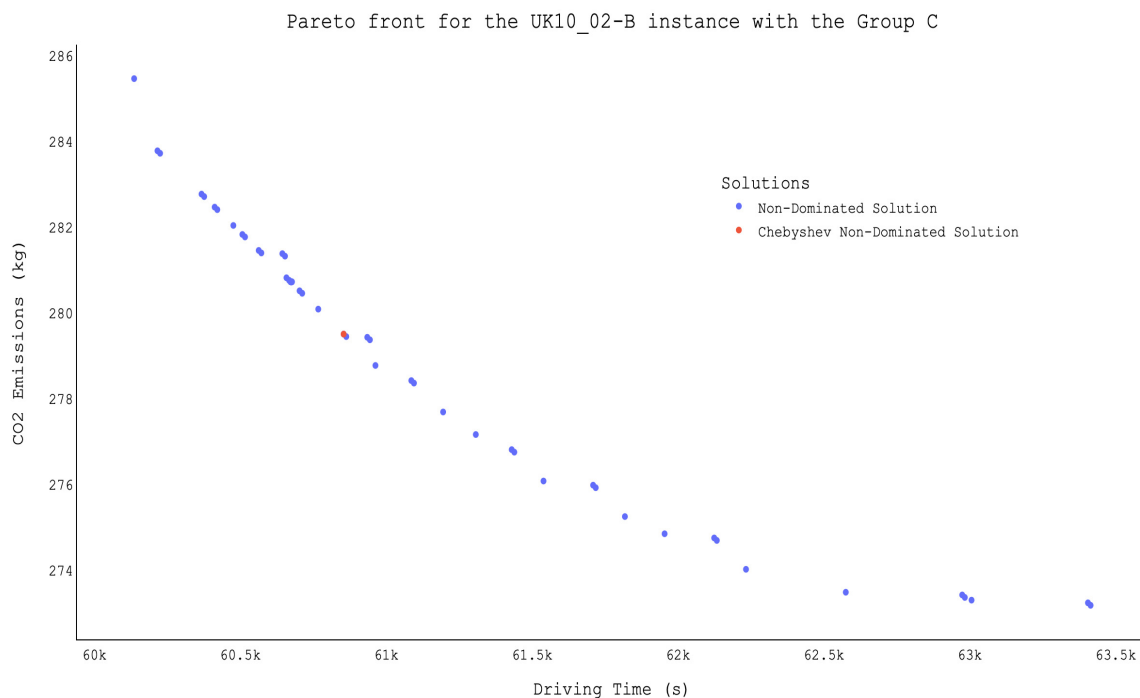


Figure 5.4.3: Pareto front for the UK10_02-B instance with the Group C

The Figure 5.4.3 shows the Pareto front for the UK10_02-B instance with 10% of backhaul customers (Group C). Fewer efficient solutions were found comparatively to the previous instance.

The emissions can be brought down from around $285.46kg$ to $273.17kg$. The driving time can be decreased from around $17.62h$ ($63,414s$) to $16.70h$ ($60,136s$). Reducing the emissions in around 4.31% implies a 5.17% increase in the driving time.

For example, the distance-minimizing Chebyshev solution is characterized by $279.50kg$ and $16.90h$ ($60,854s$). In relation to the driving time-minimizing solution, it corresponds to a 2.1% decrease in the emissions but an increase of 1.2% in the driving time.

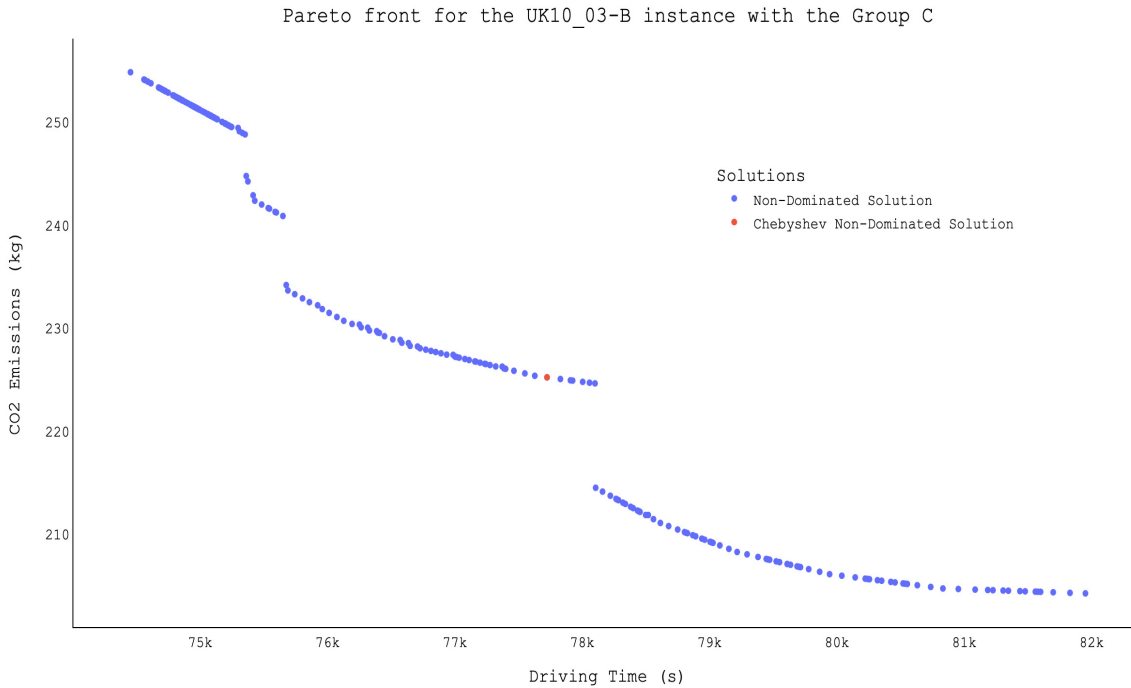


Figure 5.4.4: Pareto front for the UK10_03-B instance with the Group C

The Figure 5.4.4 shows the Pareto front for the UK10_03-B instance with 10% of backhaul customers (Group C). Unlike the previous instances, a larger trade-off between the two objectives can be obtained. The driving time can be brought down from around $22.76h$ ($81,951s$) to $20.68h$ ($74,452s$). As much as $50.62kg$ can be saved in the emissions if they are reduced from around $254.81kg$ to $204.19kg$. Reducing the emissions in around 19.87% implies a 9.15% increase in the driving time.

On this instance, a great sacrifice needs not to be done to one of the objectives in order to improve the other. A significant reduction in the emissions can be achieved if the driving time is slightly increased. The inverse also holds, whereby a significant reduction in the driving time can be obtained if the emissions are only slightly increased.

The Figure 5.4.4 shows that the Pareto front presents a *piecewise* or *staircase* behavior. The discontinuities are once more caused by the variation of a binary x_{ij}^k variable. A customer is swapped between routes. In each of the *steps* in the front, the routes are

fixed and the solutions that comprise it differ from one another due to the variation of the speeds (z_{ij}^{rk}) on the arcs. At the edge of each *step*, since no further combination of speeds ensures time window feasibility while also minimizing the emissions for the same set of routes, a customer is swapped between them, which causes the discontinuity. Some speeds may also vary at the discontinuities. The Table 5.4.5 illustrates this behavior. The swapped customers at each discontinuity are presented in bold.

Table 5.4.5: Discontinuities in the UK10_03-B instance with the Group C

CO ₂ (kg)	Routes	Speed Levels
248,77	< 0, 8, 10, 0 >	< 5, 5, 9 >
	< 0, 1, 7, 2, 9, 4, 0 >	< 5, 5, 9, 9, 9, 10 >
	< 0, 3, 5, 6, 0 >	< 5, 7, 6, 9 >
↓		
244,73	< 0, 8, 10, 0 >	< 5, 5, 10 >
	< 0, 1, 7, 2, 4, 0 >	< 5, 5, 9, 7, 10 >
	< 0, 3, 5, 6, 9 , 0 >	< 5, 7, 5, 10, 10 >
240,84	< 0, 8, 10, 0 >	< 5, 5, 9 >
	< 0, 1, 7, 2, 4, 0 >	< 5, 5, 7, 8, 9 >
	< 0, 3, 5, 6, 9, 0 >	< 5, 7, 6, 9, 10 >
↓		
234,14	< 0, 8, 10, 9 , 0 >	< 5, 5, 5, 10 >
	< 0, 1, 7, 2, 4, 0 >	< 5, 5, 9, 7, 10 >
	< 0, 3, 5, 6, 0 >	< 5, 7, 6, 10 >
224,58	< 0, 8, 10, 9, 0 >	< 5, 5, 5, 7 >
	< 0, 1, 7, 2, 4, 0 >	< 5, 5, 7, 7, 7 >
	< 0, 3, 5, 6, 0 >	< 5, 7, 6, 6 >
↓		
214,44	< 0, 8, 10, 9, 0 >	< 5, 5, 5, 10 >
	< 0, 1, 7, 2, 0 >	< 5, 5, 10, 10 >
	< 0, 3, 5, 6, 4 , 0 >	< 5, 7, 5, 5, 10 >

The Figures 5.4.5 and 5.4.6 contrast the emissions- *vs.* driving time-minimizing solutions with 50% backhauls (Group A) for the UK10_04-C and UK10_11-C instances, respectively.

In the Figure 5.4.5, it can be seen that the emissions-minimizing solution presents 181.28kg of emitted CO₂ and 56,913s (around 15.81h) of driving time. The driving time-minimizing solution presents 285.35kg of emitted CO₂ and 46,365s (around 12.88h) of

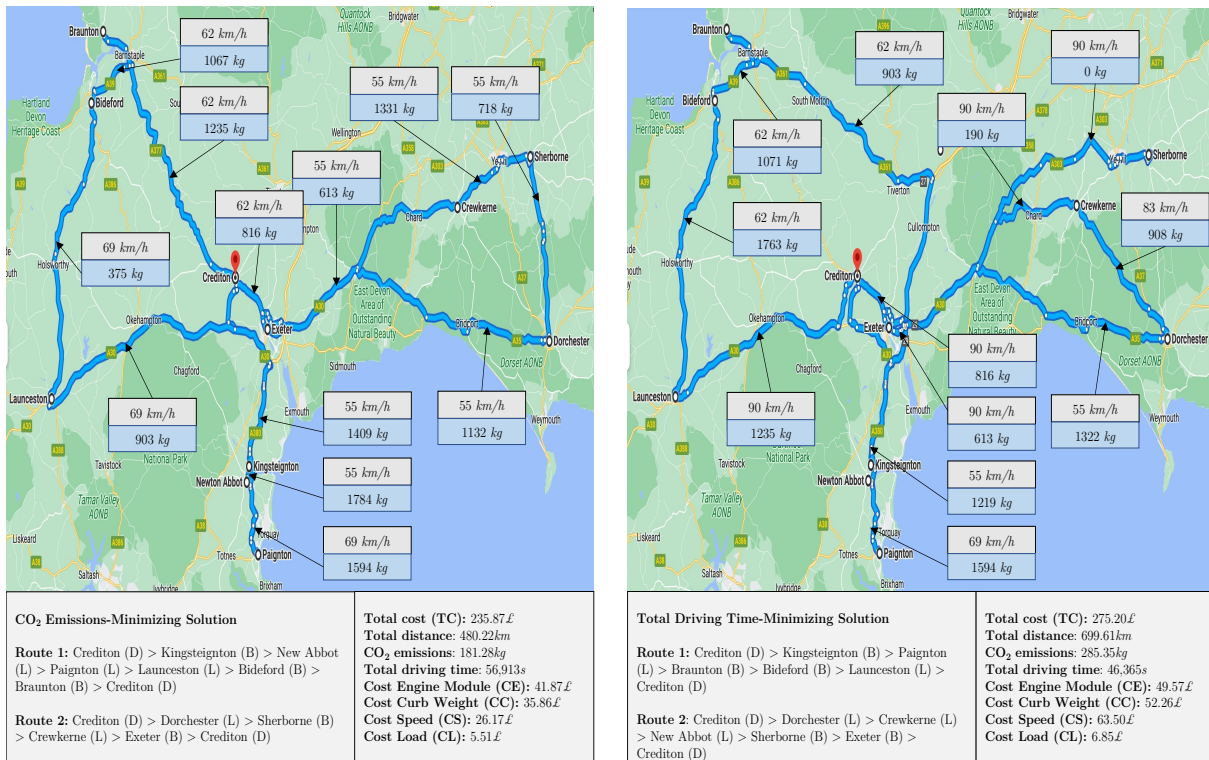


Figure 5.4.5: CO₂- vs. Total driving time-minimizing solutions with 50% backhauls for the UK10_04-C instance

driving time. If the drivers’ wages had been considered, this would translate into a CD equal to 126.46£ and 103.02£ for the emissions- and driving time-minimizing solutions, respectively. If the emissions-minimizing solution is preferred, it grants a 36.47% decrease in the emissions at the expense of a 18.53% increase in the driving time.

Interestingly, but contrarily to what one might have expected, the driving time-minimizing solution does not present the lowest total cost between the two solutions, if CD is considered. The same holds in the Figure 5.4.6.

This suggests that the minimization of the driving time may not always imply total cost minimization, due to the effect of the speed. CS is much higher (around 58.79%) in the driving time-minimizing solution comparatively to the emissions-minimizing solution. The vehicles generally traveled at higher speeds to minimize the driving time. The increase in CS largely offsets the decrease in CD. However, it should be stressed that the total cost figure highly depends on the costs considered for the drivers’ wages and emissions.

In the Figure 5.4.6, it can be seen that the emissions-minimizing solution presents 268.97kg of emitted CO₂ and 62,701s (around 17.42h) of driving time. The driving time-minimizing solution presents 372.65kg of emitted CO₂ and 48,862s (around 13.57h) of driving time. If the driving time-minimizing solution is preferred, it grants a reduction of around 22.07% in the driving time at the expense of a 27.82% increase in the emissions.

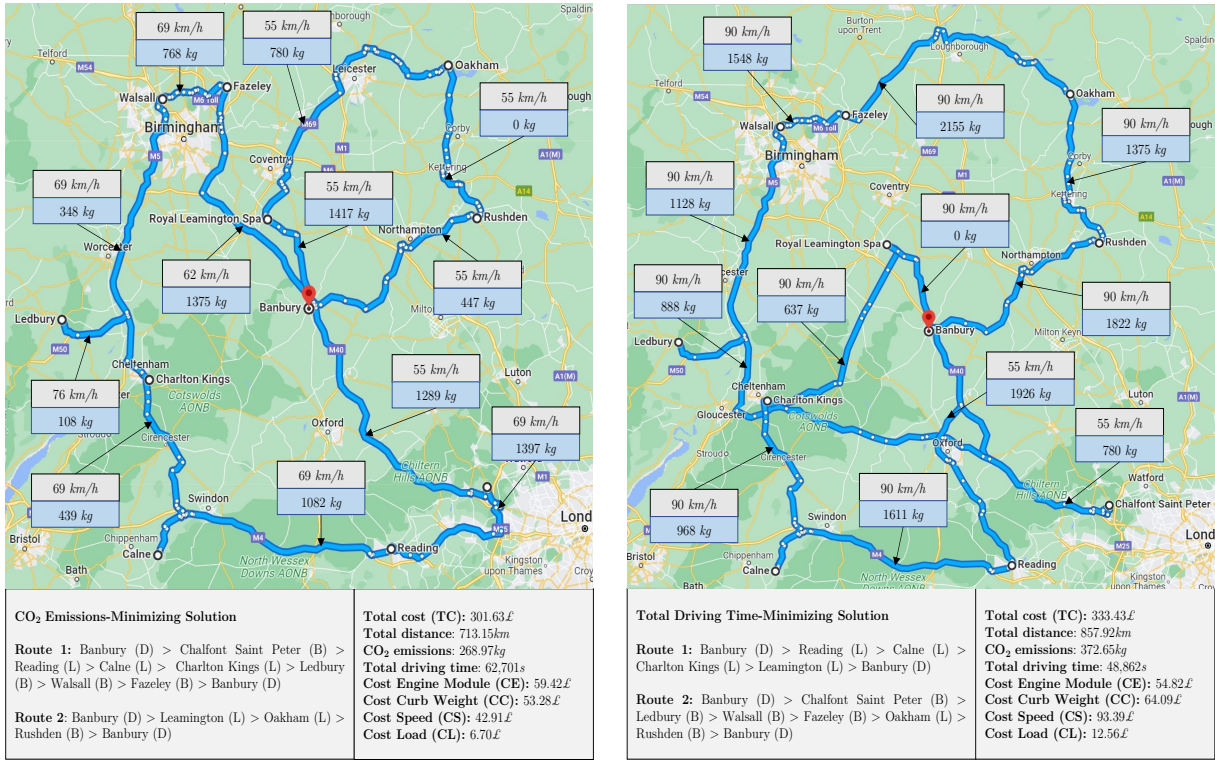


Figure 5.4.6: CO₂- vs. Total driving time-minimizing solutions with 50% backhauls for the UK10_11-C instance

5.5 Conclusion

In this chapter, the results of the computational experiments for both the single- and bi-objective PRPMB were reported.

From a total cost perspective and emissions standpoint, the single-objective PRPMB (M1) allowed for quantifying the impacts of a mixed backhauling policy in PRP. The bi-objective PRPMB (M2) allowed for the trade-offs between the emissions and the total driving time to be analyzed.

In the next chapter, the contributions of the dissertation are outlined and some final remarks are drawn.

Chapter 6

Conclusions

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6.1 Contributions and Future Research

In this dissertation, two new models were proposed for a vehicle routing problem with environmental concerns and mixed linehauls and backhauls. The contributions made herein are expected to be relevant for the scientific community and practitioners.

To the best of our knowledge, the incorporation of mixed linehauls and backhauls into the context of the Pollution-Routing Problem had heretofore not been addressed in the literature. A contribution was made with the development of a single-objective model that can account for such network characteristics. The economic and environmental impacts of such an incorporation were quantified and discussed. With the results obtained, we contribute to the discussion where a pickup and delivery network can lead to both a heightened economic and environmental performance, in contrast to a network where deliveries only are performed.

In fact, most of the instances tested showed a simultaneous reduction across all costs, as well as in the CO₂ emissions. These reductions can be expressive, depending on the instance considered. A balanced network where the number of linehaul and backhaul customers is similar proved to be the most cost-effective and emissions-minimizing one.

These results are relevant from a practical standpoint. They can contribute to effectively reduce the CO₂ emissions associated with transportation and help practitioners achieve more efficient transportation schemes at their companies, thus contributing to a higher service level differentiation and competitiveness.

Following the proposed single-objective model, another contribution was made with the proposal of its bi-objective counterpart. In fact, the Pollution-Routing Problem is characterized by several conflicting objectives. The CO₂ emissions and the total driving time are relevant ones. The bi-objective model more clearly illustrates the existing trade-offs between the two objectives. It provides the DM with a wide range of solutions he/she can choose from.

The results for the bi-objective model indicate that the trade-off between the two objectives tends to be large enough to allow for good compromise solutions to be obtained. Significant trade-offs can occur at the individual instance level. They also suggest that it becomes increasingly difficult to improve one of the objectives without deteriorating the other so significantly when the number of backhaul customers increases.

These results are also relevant from a practical standpoint. The emission of GHG to the atmosphere is a major environmental concern. It causes global warming, habitat destruction, and human health deterioration. The proposed bi-objective model can help practitioners to find good compromise solutions by capping emissions levels with minimum cost increases.

This study has its limitations. The chosen instantiation for the backhaul customers influences the results obtained. The results, therefore, cannot not be generalized. Nevertheless, we believe that the general conclusions drawn herein are likely to hold for different instantiations. Still, instances with only up to 20 customers were tested. The application of exact methods has its shortfalls when it comes to dealing with complex or large instances.

The use of metaheuristics and matheuristics for solving the PRPMB is suggested as future research direction. These methods were successfully applied to solving different PRP variants and competitive results have been reported. They are likely to derive good-quality solutions in more reasonable computation times.

In addition, the consideration of loading constraints in the PRPMB is suggested. The assortment of the items in the vehicles is a practical issue in route planing if the vehicles travel near to full capacity, which may happen when pickups are also performed. This consideration enriches the PRPMB, but may prove to be a very challenging problem to solve.

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Appendix A

Parameters used in the model of Bektas and Laporte [18]

Table A.1: Parameters used in the model of Bektas and Laporte [18]

Parameter	Meaning	Units	Value
ρ	Air density	kg/m^3	1.2041
C_d	Coefficient of aerodynamic drag	-	0.7
C_r	Coefficient of rolling resistance	-	0.01
ψ	Conversion factor (g/s to l/s)	-	737
e	Cost of CO ₂ ¹	\pounds/l	0.063
η	Diesel engine efficiency	-	0.9
U	Diesel fuel heating value	kJ/g	44
p	Driver wage	\pounds/s	0.002222 (= 8 \pounds/h)
V	Engine displacement	l	5
k	Engine friction factor	$kJ/rev/l$	0.2
N	Engine speed	rev/s	33
c_f	Fuel cost	\pounds/l	1
ξ	Fuel-to-air mass ratio	-	1
g	Gravitational constant	m/s^2	9.81
v^l	Lower speed limit	m/s	11.1 ($\approx 40km/h$)
θ	Road angle	rad	0
v^u	Upper speed limit	m/s	19.4 ($\approx 70km/h$)
a	Vehicle acceleration	m/s^2	0
w	Vehicle curb weight	kg	3000
ε	Vehicle drivetrain efficiency	-	0.37
A	Vehicle frontal surface area	m^2	5

¹Considering that 1l of diesel has 2.32kg of CO₂ and that 1kg of CO₂ costs 0.0027 \pounds .

Appendix B

Implementation of the Pollution-Routing Problem

Table B.1: Parameters used in the models of Demir et al. [124] and Kramer et al. [142].

Parameter	Meaning	Units	Value
ρ	Air density	kg/m^3	1.2041
C_d	Coefficient of aerodynamic drag	-	0.7
C_r	Coefficient of rolling resistance	-	0.01
ψ	Conversion factor (g/s to l/s)	-	737
c_f	Cost of fuel and CO ₂	\pounds/l	1.4
η	Diesel engine efficiency	-	0.9
U	Diesel fuel heating value	kJ/g	44
p	Driver wage	\pounds/s	0.002222 ($\approx 8\pounds/h$)
V	Engine displacement	l	5
k	Engine friction factor	$kJ/rev/l$	0.2
N	Engine speed	rev/s	33
ξ	Fuel-to-air mass ratio	-	1
g	Gravitational constant	m/s^2	9.81
v^l	Lower speed limit	m/s	5.5 ($\approx 20km/h$)
θ	Road angle	$^\circ$	0
v^u	Upper speed limit	m/s	25 (= $90km/h$)
a	Vehicle acceleration	m/s^2	0
w	Vehicle curb weight	kg	6350
ε	Vehicle drivetrain efficiency	-	0.4
A	Vehicle frontal surface area	m^2	3.912
Q	Vehicle maximum payload	kg	3250

Table B.2: Abbreviations used in the Table B.3 and their meaning.

Abbreviation	Meaning	Units
TC	Total cost of the solution	\pounds
TD	Total distance traveled	km
CO ₂	Total amount of CO ₂ emitted	kg
CD	Total cost incurred due to the drivers' wages	\pounds
CE	Total cost incurred due to the vehicles' engine module	\pounds
CC	Total cost incurred due to the vehicles' curb weight	\pounds
CS	Total cost incurred due to the speed variations	\pounds
CL	Total cost incurred due to the load carried	\pounds
CPU	CPU time obtained over one run of the algorithm	s
#V	Number of vehicles used	-
Dev.	Deviation from the optimal solution ¹	%

¹ Computed as $100 [(TC_{PRP(This\ Study)} - TC_{PRP})/TC_{PRP}]$.

Appendix C

Computational results for M1

Table C.1: Abbreviations used in the Tables C.2-C.10 and their meaning

Abbreviation	Meaning	Units
TC	Total cost of the solution	£
TD	Total distance traveled	km
CO ₂	Total amount of CO ₂ emitted	kg
CD	Total cost incurred due to the drivers' wages	£
CE	Total cost incurred due to the vehicles' engine module	£
CC	Total cost incurred due to the vehicles' curb weight	£
CS	Total cost incurred due to the speed variations	£
CL	Total cost incurred due to the load carried	£
CPU	CPU time obtained over one run of the algorithm	s
#V	Number of vehicles used	-
Gap	MIP gap from the optimal solution	%
Dif. TC	Increase/decrease in the solution's total cost ¹	%
Dif. CO ₂	Increase/decrease in the solution's total CO ₂ emissions ²	%

¹ Computed as $100 [(TC_{M1} - TC_{PRP}) / TC_{PRP}]$.

² Computed as $100 [(CO2_{M1} - CO2_{PRP}) / CO2_{PRP}]$.

Appendix D

Computational results for M2

Table D.1: Abbreviations used in the Tables D.2-D.4 and their meaning

Abbreviation	Meaning	Units
CO ₂ <i>max.</i>	Maximum amount of CO ₂ emitted ¹	<i>kg</i>
CO ₂ <i>min.</i>	Minimum amount of CO ₂ emitted ²	<i>kg</i>
CPU <i>avg.</i>	Average CPU time to find a non-dominated solution	<i>s</i>
CPU Total	Total CPU time to find all non-dominated solutions	<i>s</i>
D.T.	Driving time	<i>s</i>
D.T. <i>max.</i>	Maximum driving time ²	<i>s</i>
D.T. <i>min.</i>	Minimum driving time ¹	<i>s</i>
# Sol.	Number of non-dominated solutions found	-

¹ In the driving time-minimizing solution.

² In the emissions-minimizing solution.

