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N^* resonances in the $\pi\pi N$ system

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Abstract. We have solved the Faddeev equations for the $\pi\pi N$ system and coupled channels resulting into the dynamical generation of two N^*, $N^*(1710)$ and $N^*(2100)$, and one Δ states, $\Delta(1910)$, all of them with $J^P = 1/2^+$. In addition, signatures for a new N^* resonance with $J^P = 1/2^+$ are found around at an energy of 1920 MeV.

1 Introduction

The $\pi\pi$ and πN systems have been extensively studied [1–5] and dynamical generation of several resonances has been found. For instance, the $\sigma(600)$, the $f_0(980)$ and the $a_0(980)$ resonances appear in the $\pi\pi$, $K\bar{K}$ and $\pi\eta$ interaction [1, 2], while the $N^*(1535)$ state is observed in the πN and coupled channel systems [3, 4]. All these states have an important hadron-hadron component in their respective wave functions, which play an important role in describing their properties. Since the interaction between $\pi\pi$ and πN is attractive in some energy range, it is plausible to think that the addition of one more particle, for example, a pion, could lead to new states with the three-body components of its wave function playing a dominant role in understanding the experimental data. There are in fact several resonances listed by the Particle Data Group (PDG) [6] which can be expected to belong to this group, like, for example, the $N^*(1710)$, which has a large branching ratio to $\pi\pi N$ $(40-90\%)$. Therefore, a study of the $\pi\pi N$ system and coupled channels could lead to dynamical generation of the $N^*(1710)$ and, hence, a better understanding of its characteristics.

With this motivation, we have studied the $\pi\pi N$ system and coupled channels using the formalism developed in [7] which leads to dynamical generation of all the low-lying $1/2^+$, $S = -1$ resonances listed in the PDG in the energy region 1500-1800 MeV. As we shall see in the next sections, three resonances corresponding to the $N^*(1710)$, $N^*(2100)$ and $\Delta(1910)$ are found and a new N^* resonance is found around 1920 MeV, supporting the prediction of [8].

2 Framework

In order to investigate the possibility of generating states which could have a strong coupling to $\pi\pi N$, we calculate the three-body T-matrix for the $\pi\pi N$ system and its coupled channels and search for peaks which could be identified with N^* resonances. In order to get the T matrix for the three-body system we adopt the formalism developed in [7, 9–11] which is based on the Faddeev equations [12]. In terms of the Faddeev partitions, T^1, T^2 and T^3, the three-body T-matrix is written as

$$T = T^1 + T^2 + T^3. \tag{1}$$

In our formalism we rewrite these partitions as [7, 9]

$$T^i = t^i\delta^3(\mathbf{k}_i' - \mathbf{k}_i) + \sum_{j\neq i=1}^{3} T_R^{ij}, \quad i = 1, 2, 3 \tag{2}$$

with $\mathbf{k}_i$ ($\mathbf{k}_i'$) being the initial (final) momentum of the particle i. The T_R^{ij} partitions include all the different contributions to the three-body T- matrix in which the last two interactions are given in terms of the two-body t-matrices t^j and t^i, respectively and satisfy the equations

$$\begin{aligned}
T_R^{12} &= t^1 g^{12} t^2 + t^1\left[G^{121}\,T_R^{21} + G^{123}\,T_R^{23}\right]\\
T_R^{13} &= t^1 g^{13} t^3 + t^1\left[G^{131}\,T_R^{31} + G^{132}\,T_R^{32}\right]\\
T_R^{21} &= t^2 g^{21} t^1 + t^2\left[G^{212}\,T_R^{12} + G^{213}\,T_R^{13}\right]\\
T_R^{23} &= t^2 g^{23} t^3 + t^2\left[G^{231}\,T_R^{31} + G^{232}\,T_R^{32}\right]\\
T_R^{31} &= t^3 g^{31} t^1 + t^3\left[G^{312}\,T_R^{12} + G^{313}\,T_R^{13}\right]\\
T_R^{32} &= t^3 g^{32} t^2 + t^3\left[G^{321}\,T_R^{21} + G^{323}\,T_R^{23}\right].
\end{aligned} \tag{3}$$

This means that the full three-body T-matrix can be related to T_R^{ij} through

$$\begin{aligned}
T &= \sum_{i=1}^{3} T^i = \sum_{i=1}^{3} t^i\delta^3(\mathbf{k}_i' - \mathbf{k}_i) + T_R\\
T_R &\equiv \sum_{i=1}^{3}\sum_{j\neq i=1}^{3} T_R^{ij}
\end{aligned} \tag{4}$$

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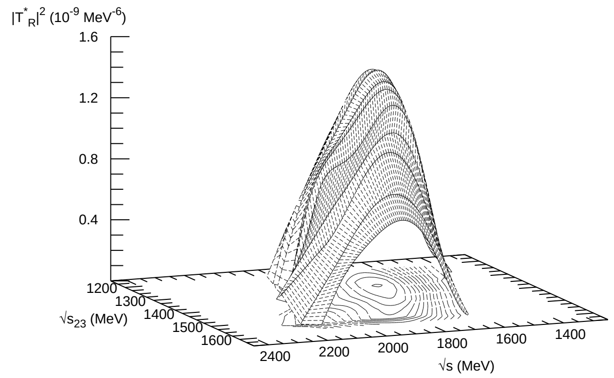

Fig. 1. The squared amplitude for the $\pi\pi N$ system in isospin 1/2 configuration as a function of $\sqrt{s}$ and $\sqrt{s_{23}}$.

In Eq.(3), g^{ij}'s correspond to the three-body Green's function of the system and G^{ijk} to a loop function of three-particles (for their definitions see [7, 9–11]). We always work with S wave interactions, therefore, all the matrices in Eq.(3) are projected in S -wave, thus giving total $J^\pi = 1/2^+$. In order to identify possible three-body states, we have to project the T_R-matrix (Eq. (4)) on the isospin base. We choose the base in which the states are defined in terms of the total isospin of the three body system, I, and the total isospin of two pions, $I_{\pi\pi}$, i.e., $|I, I_{\pi\pi}\rangle$ [9]. Since neither $\sum_{i=1}^{3} t^i \delta^3(\mathbf{k}'_i - \mathbf{k}_i)$ nor $t^i g^{ij} t^j$ can give any three-body resonance structure because there are no three-body loops included, we can restrict ourself to the study of the properties of

$$T_R^* = T_R - \sum_{i=1}^{3} \sum_{j\neq i=1}^{3} t^i g^{ij} t^j \tag{5}$$

3 The $N^*(1710)$ as a $\pi\pi N$ resonance

We solve Eq. (4) considering $\pi\pi N$, $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ as coupled channels for total charge zero and using the unitary chiral approach of [1–3] to calculate the $\pi\pi$ and πN t-matrices. The effect of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ channels around 1700 MeV is found to be negligible and, thus, we present the formalism in the simplest way, i.e., taking only the $\pi\pi N$ channels in total charge zero into account: $\pi^0\pi^0 n$, $\pi^+\pi^- n$, $\pi^-\pi^+ n$, $\pi^0\pi^- p$, $\pi^-\pi^0 p$. We label these particles as 1, 2, 3 according to the order in which they are written.

We show the s-wave squared T_R^* amplitude for the $\pi\pi N$ system in Fig. 1, projected on the isospin base $| I, I_{\pi\pi}\rangle = | 1/2, 0\rangle$ [9]. We obtain a peak at $\sqrt{s} = 1704$ MeV, with a full width at half maximum of 375 MeV which is in good agreement with the characteristics of the $N^*(1710)$ [6]. Hence, we relate this peak with the $N^*(1710)$. It is interesting to note that if instead of using the invariant mass of one of the πN pairs ($\sqrt{s_{23}}$) to make the plot we consider the invariant mass associated to the $\pi\pi$ pair, the peak appears when the $\sigma(600)$ resonance gets dynamically generated in the $\pi\pi$ interaction [1, 2]. Therefore, we can conclude that the $N^*(1710)$ has a large $\pi\pi N$ component with the $\pi\pi$ subsystem generating the σ resonance [9].

Although there is a clear evidence for the $N^*(1710)$, this work fails to get any trace of the Roper resonance, which means that considering the $\pi\pi N$ system in s-wave interaction is not sufficient to generate the Roper resonance, which is not surprising. Other works such as the Juelich model [13], which successfully describes the dynamical generation of the Roper resonance, contains additional information on the πN, $\pi\Delta$, ρN coupled channels and σN forces. Also, in [14] an important contribution of the $\pi\Delta$ channel and $\pi\pi$ final state interaction (with one of the pions coming from the decay of the Δ resonance) to the Roper resonance has been claimed. Such an information is not present in our formalism. The situation is different for the case of the $N^*(1710)$ with its large empirical coupling to $\pi\pi N$ and weaker to πN and other coupled channels.

With the present formalism, we do not find the other $1/2^+$ states listed in the PDG [6]: $N^*(2100)$, $\Delta(1750)$ and $\Delta(1910)$. This can be due to the fact that the chiral πN amplitude which we use reproduces the $N^*(1535)$ state but not the $N^*(1650)$ resonance. Thus, if by chance any of the states mentioned above couples to $\pi N^*(1650)$ we would not find them.

One step further in the model should be to use a more realistic πN t-matrix which includes both states, the $N^*(1535)$ and the $N^*(1650)$. As it has been shown in [7,9,10], our formalism does not depend on the off-shell parts of the t-matrices, then, we can use the experimental phase space and inelasticities for πN scattering [15] in order to calculate the πN t-matrix and then use this one to solve Eq. (4) [10]. We discuss this issue in the next section.

4 Dynamically generation of the $N^*(2100)$ and the $\Delta(1910)$

In this section we use the experimental $L = 0$ phase shifts (δ) and inelasticities (η) [15] for the πN system and calculate from them the πN amplitudes [10] whenever we are above its threshold. The πN interaction below its threshold is determined using the model of [3]. In this way, we extend the calculations of section 3 to higher energies by using a πN t-matrix which contains the $N^*(1535)$ and $N^*(1650)$ resonances and search for the other three-body isospin 1/2 and 3/2 states with $J^P = 1/2^+$ listed by the PDG. For the $\pi\pi$ interaction we continue using the chiral amplitude [2], which reproduces quite well the experimental results of the $\pi\pi$ scattering.

We first check that with the new input we generate the $N^*(1710)$ exactly with the same properties as before. After that, with the same five $\pi\pi N$ channels listed in section 3 we solve Eq. (4) for higher energies. Later, we project our T_R^* -matrix in the isospin base in which the subsystem of

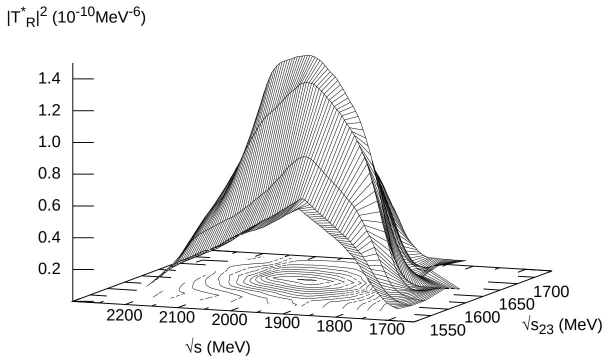

Fig. 2. The $N^*(2100)$ in the $\pi\pi N$ system with five coupled channels.

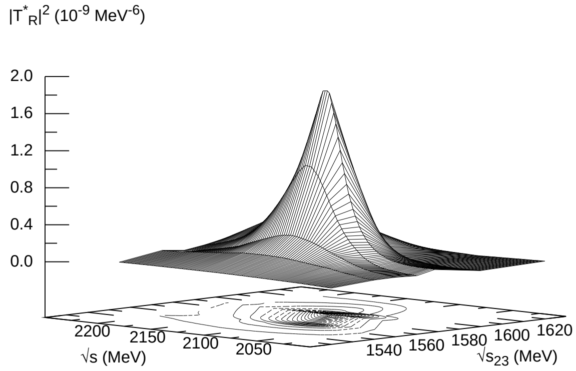

Fig. 3. The $\Delta(1910)$ in the $\pi K\Lambda$.

particles 2 and 3, i.e., pion nucleon (and its coupled channels) is $1/2$ in the initial as well as the final state and the total isospin of the system is $1/2$.

The squared T_R^* amplitude for such a case is shown In Fig. 2 as a function of the total energy of the three-body system and the invariant mass of the meson-baryon subsystem formed by the second and third particle (πN). A peak close to an energy of 2100 MeV with a width of ~ 250 MeV appears when $\sqrt{s_{23}}$ is around 1670 MeV, thus having a $\pi N^*(1650)$ structure. The peak position, its width, the spin-parity and the isospin are compatible with those of the $N^*(2100)$. Thus we identify this peak with the $N^*(2100)$.

Since this peak appears when one of the πN pairs rearranges itself as the N^* (1650) and has been obtained only by using a πN *t*-matrix which contains the information on the $N^*(1650)$, as well as on the $N^*(1535)$, we conclude that the inclusion of the $N^*(1650)$ in the πN subsystem is essential to generate this resonance .

Considering only $\pi\pi N$ channels and using the new πN input, we do not find evidence for any resonance in the isospin 3/2 configuration. However, the situation changes when we introduce the coupled channels.

We now take into account 14 coupled channels: the five $\pi\pi N$ channels with total charge zero, i.e., $\pi^0\pi^0 n$, $\pi^0\pi^- p$, $\pi^-\pi^+ n$, $\pi^-\pi^0 p$, $\pi^+\pi^- n$ and the corresponding $\pi K\Sigma$, $\pi K\Lambda$ and $\pi K\eta$ channels, $\pi^0 K^+\Sigma^-$, $\pi^0 K^0\Sigma^0$, $\pi^0 K^0\Lambda$, $\pi^0\eta n$, $\pi^+ K^0\Sigma^-$, , $\pi^- K^+\Sigma^0$, $\pi^- K^0\Sigma^+$, $\pi^- K^+\Lambda$ and $\pi^-\eta p$. Again, we label them as particles 1, 2 and 3 in the order in which they are written above and solve Eqs. (4). As there are no data for $K\Sigma \to K\Sigma$, $K\Lambda \to K\Lambda$, etc., we take the model of [3] to calculate the corresponding amplitudes.

The introduction of the coupled channels modifies the peak in Fig. (2), which now appears at an energy of 2080 MeV with a width of 54 MeV for a $\sqrt{s_{23}}$ near 1570 MeV and with a bigger magnitude [10] . Therefore, the inclusion of the $\pi K\Sigma$, $\pi K\Lambda$ and $\pi\eta N$ channels makes the resonance shown in Fig. (2) more pronounced (by an order of magnitude in the squared T_R^*-matrix) and much narrower. These changes in the results can be easily understood by noticing that after adding the rest of coupled channels the wave function of the resonance contains extra components which have smaller phase space in the decay of the resonance. At the same time, the $\pi\pi N$ component becomes smaller due to the normalization of the wave function and, hence, the decay into $\pi\pi N$ is also reduced.

We also find a peak in the $\pi K\Lambda$ amplitude for total isospin I=3/2 at an energy of ~ 2126 MeV with a width of ~ 42 MeV with isospin $I_{K\Lambda} = 1/2$ in the initial as well as in the final state [10]. This peak can be identified with the $\Delta(1910)$ listed in [6], whose position, given by different partial wave analyses, ranges up to 2070 MeV and the width varies from 190-500 MeV.

It is rewarding to note that we get smaller widths than the experimental ones. In our approach this fact is normal since we do not consider the πN decay channels and they should contribute to increase the widths. Note that this can be done even with a small coupling to the πN components, as implicitly assumed here, since there is more phase space for decay into the πN channel (see [7, 9, 10] for more discussion).

We do not find any evidence of the $\Delta(1750)$, which could indicate a different structure for this state than the one studied in this work.

5 Signatures for a new N^* state around 1920 MeV

So far, we have investigated possible resonant states in the $\pi\pi N$ system and its coupled channels which have been obtained adding a pion to those pseudoscalar-baryon systems which couple strongly in $J^\pi = 1/2^-$ and isospin $1/2$ configuration, i.e., πN, $K\Sigma$, $K\Lambda$ and ηN . The invariant mass of this pseudoscalar-baryon subsystem has been varied around 1530-1650 MeV, thus, treating the three-body system as a πN^* system, although within the three-body

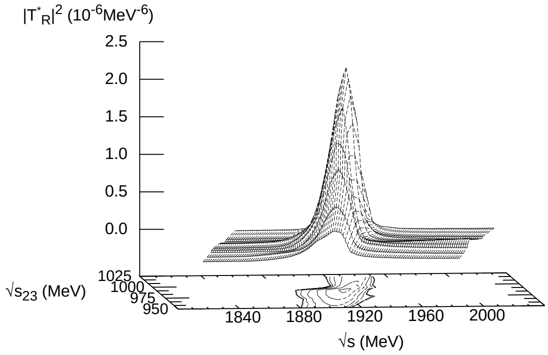

Fig. 4. A possible N^* in the $NK\bar{K}$ channel around 1920 MeV.

Faddeev equations. However, there are other possible configurations of this three-body system, like , $Nf_0(980)$ or $Na_0(980)$ which until now we have not discussed.

In the chiral models, the $f_0(980)$ and the $a_0(980)$ resonances get dynamically generated in the $\pi\pi$, $K\bar{K}$ and $\pi\eta$, $K\bar{K}$ systems [1,2], respectively. Thus, in order to study systems made of a nucleon and one of these resonances, we consider, for total charge zero, the following set of coupled channels: $n\pi^0\pi^0$, $p\pi^0\pi^-$, $n\pi^0\eta$, $n\pi^+\pi^-$, $n\pi^-\pi^+$, $p\pi^-\pi^0$, $p\pi^-\eta$, nK^+K^-, $nK^0\bar{K}^0$, pK^0K^-. We label the particles as 1, 2, 3 in the order in which they are written above. We solve Eq. (4) considering these channels and make isospin combinations of the $NK\bar{K}$ channels in order to identify possible states in the corresponding T-matrix.

The $NK\bar{K}$ amplitude shows a peak around 2080 MeV, with a width of 51 MeV (which we do not show here) for the configuration in which the total isospin of the $NK\bar{K}$ system is equal to 1/2 with the isospin of the $K\bar{K}$ subsystem equal to one. This peak can be identified as the $Na_0(980)$ partner of the peak shown in Fig. 2. Therefore the peak corresponding to the $N^*(2100)$ has been seen in $\pi\pi N$ system as well as in the $NK\bar{K}$ system.

Interestingly, along with this $N^*(2100)$ state, the T matrix shows another peak at $\sqrt{s} = 1924$ MeV with a width of 20 MeV, as can be seen in Fig. 4.

Assuming an average mass for the kaons of 496 MeV and 939 MeV for the nucleon, this state is about 7 MeV below the $NK\bar{K}$ threshold. Suggestions about the existence of such a resonance were made by the authors in [8], in which they studied the $NK\bar{K}$ channel through effective two-body potentials for the $\bar{K}N$, $\bar{K}K$, KN interactions. Interestingly, the existence of a $1/2^+$ N^* resonance around 1935 MeV has also been proposed earlier [16] on the basis of a study of the data on the $\gamma p \to K^+\Lambda$ reaction in an isobar model. Recently, a theoretical study of the $\gamma p \to K^+\Lambda$ and $\gamma p \to K^+\Sigma$ reactions has been made investigating the possibility that the enhancement observed close to threshold in the $\gamma p \to K^+\Lambda$ reaction could be an indication of the presence of a N^* around 1920 MeV and further tests are proposed in order to clarify the J^P assignment for this state [11].

The peak at 1920 MeV is also observed if instead of considering the $K\bar{K}$ subsystem in isospin one we put it in isospin zero, i.e., considering the $Nf_0(980)$ component of the $NK\bar{K}$ channel [10].

6 Conclusions

We have studied the $\pi\pi N$ system and coupled channels, first, using the chiral amplitude to describe the πN interaction, and later, using experimental phase shifts and inelasticities in order to calculate the πN t-matrix. In this way, for the latter case, we ensure the presence of the $N^*(1535)$ as well as the $N^*(1650)$, which is not generated in the chiral model of [3], in the πN t-matrix used as input to solve the Faddeev equations. We find evidence for the dynamical generation of the $N^*(1710)$, the $N^*(2100)$, and also for the $1/2^+$ $\Delta(1910)$ resonance. We have also investigated the $NK\bar{K}$, $N\pi\pi$ and $N\pi\eta$ channels where the $K\bar{K} - \pi\pi$ subsystem rearranges itself as a $f_0(980)$ resonance, while $K\bar{K}-\pi\eta$ acts like the $a_0(980)$. We obtain a new peak at ~ 1924 MeV with a strong coupling to $Na_0(980)$ and $Nf_0(980)$.

Acknowledgements

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