Faculdade de Economia

Universidade de Coimbra

Regional input-output tables and models

Interregional trade estimation and input-output modelling based on total use rectangular tables

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Coimbra, Janeiro de 2009

Dissertação para **Doutoramento em Economia**, na especialidade de **Planeamento e Economia Regional**, apresentada na Faculdade de Economia da Universidade de Coimbra, orientada por: **Professor Pedro Ramos** e **Professor Geoffrey Hewings**.

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ABSTRACT

The present research concerns the study of input-output modelling and input-output table construction, when applied at the regional level. Input-output models, at the national or regional level, are known as a fundamental tool for economic analysis. Yet, in order to apply such models, the researcher must have access to the correspondent input-output tables. National-level tables are currently published by the national statistical offices according to well-defined conventions. The same, however, cannot be said about regional tables which are not provided as a rule by official statistics organisms. Being so, a great part of input-output research is still dedicated to the study of techniques for input-output table gathering.

This dissertation is, in such context, divided into three chapters. The first one is mainly theoretical, aiming to review the basic principles underlying input-output analysis at the regional level. The second and third chapters constitute the research's practical contribution, focused on two major issues, respectively: 1) interregional trade estimation and 2) input-output modelling on the basis of total-use rectangular table at purchasers' prices.

In most countries, survey-based interregional trade data does not exist. However, even when some simplifying assumptions are used in the model, a minimum amount of data on interregional trade is always necessary, in order for the model to succeed in capturing spillover and feedback effects caused by the interregional linkages. In order to evaluate the reasonability of using indirect interregional trade flows estimates, a comparison was made between alternative methodologies (with special focus on gravitational models), assessing the sensitivity of the model results. Such comparison allowed to conclude that the results of the input-output model are not greatly affected by the insertion of different

trade flow values. Thus, the results obtained do not reject the reasonability of using indirect estimates for interregional trade, whenever survey-based data is unavailable.

The official input-output tables are published on a total-use rectangular format, which is different from the lay-out upon which traditional input-output models were developed (domestic use symmetric tables). The objective here was to demonstrate the equivalence in the results of the input-output model between two alternative procedures: 1) to convert the available input-output table into a domestic-flow symmetric table at basic prices and then implement the input-output model; 2) to perform the direct modelling of the original table (the total-flow rectangular table at purchasers' prices). It has been concluded that, when the same set of hypotheses is used, there is no advantage in making a previous transformation of the original tables into the symmetric format and a previous calculation of domestic flows, since the results of the model are exactly the same.

RESUMO

O presente trabalho de investigação incide sobre modelos de input-output, a nível regional. Os modelos de input-output, ao nível nacional ou regional, são reconhecidos como uma ferramenta fundamental de análise económica. Contudo, para que tais modelos sejam aplicáveis, é necessário dispor dos quadros de input-output correspondentes. Actualmente, os quadros nacionais de input-output são publicados de forma regular pelos organismos de estatística oficiais de cada país, de acordo com convenções internacionais bem definidas. O mesmo não pode ser afirmado sobre os quadros regionais, que não fazem parte das publicações estatísticas oficiais. Sendo assim, uma boa parte da investigação na área do input-output recai ainda sobre as técnicas para a construção de quadros a nível regional.

Neste contexto, a presente dissertação divide-se em três capítulos. O primeiro possui uma natureza substancialmente teórica, pretendendo fazer uma revisão dos princípios básicos subjacentes à análise input-output ao nível regional. O segundo e terceiro capítulos constituem o contributo prático da investigação, focando dois temas específicos: 1) estimação do comércio inter-regional e 2) modelização input-output baseada em quadros rectangulares de uso total, a preços de aquisição.

Na maioria dos países não existem dados directos sobre o comércio inter-regional. No entanto, há um mínimo de informação sobre estes fluxos que é imprescindível, mesmo que no modelo sejam usadas algumas hipóteses simplificadoras. Sem essa informação, o modelo é incapaz de captar os efeitos de *spillover* (extravazamento) e de *feedback* (realimentação) causados pelas ligações inter-regionais. Com o intuito de avaliar a razoabilidade de usar estimativas indirectas para os fluxos de comércio inter-regional, foi feita uma comparação entre metodologias alternativas (com especial enfoque nos modelos gravitacionais), medindo a sensibilidade dos resultados do modelo. Esta

comparação permitiu-nos concluir que os resultados do modelo input-output não são afectados em grande medida pela inserção de diferentes valores de fluxos de comércio. Assim, os resultados obtidos não rejeitam a razoabilidade de usar estimativas indirectas para o comércio inter-regional, sempre que não estejam disponíveis dados recolhidos directamente.

Os quadros oficiais de input-output são publicados num formato rectangular, com fluxos de uso total, sendo por isso diferentes do molde tradicional sobre o qual foram desenvolvidos os modelos de input-output tradicionais (quadros simétricos com fluxos de uso doméstico). O objectivo aqui era o de demonstrar a equivalência nos resultados do modelo de input-output quando são aplicados dois procedimentos alternativos: 1) converter o quadro input-output publicado para um quadro simétrico de fluxos domésticos a preços de base e só depois implementar o modelo de input-output; 2) desenvolver o modelo directamente a partir do quadro original (o quadro rectangular de fluxos totais a preços de aquisição). Concluiu-se que, quando se utiliza o mesmo conjunto de hipóteses, não há qualquer vantagem em proceder a uma transformação prévia dos quadros originais para o formato simétrico a fluxos domésticos, dado que os resultados do modelo são exactamente os mesmos.

INTRODUCTION

The input-output framework relies upon a very simple, yet essential notion, according to which the output is obtained through the consumption of production factors (inputs) which can be, in their turn, the output of other industries. The fundamental recognition of the underlying system of interactions and interdependencies between industries is at the core of the input-output tables and model construction.

This work is concerned with the study of the input-output analysis, when applied at the sub-national or regional level. Regional input-output analysis involves, on the one hand, the access to a very detailed statistical tool about the economy we are focusing on: the regional input-output table. These tables represent a comprehensive portrait of the region describing, among other things: the technology implicit in the production process, the inter-dependencies between industries, the regional consumption patterns and the inter-dependencies between the region and the rest of the world. They may constitute an important instrument in the production of regional accounts, namely to balance the income, expenditure and production estimates of regional GDP. The information comprised in regional input-output tables permits also the conduction of interesting studies about the regional economic structure, such as the identification of key sectors or crucial interregional linkages. Yet, the most widespread use of the regional input-output tables consists in the input-output models, allowing for the assessment of economic impacts resulting from exogenous changes in final demand, which may be caused by different regional policies, for instance: regional development policies or investment on transportation infrastructures. Such sort of regional analysis is receiving increasing interest by the research community, as a results of the growing economic integration (especially in the European Union), with the associated efforts in reducing regional disparities within each country and between members. This should imply the need for some reliable accounting system that helps the identification of regional impacts and interregional spillover effects and which may be used as an instrument to monitor regional policies.

In spite of the recognized interest in regional input-output models, the difficulties occur when it comes to obtain the necessary regional input-output tables and, more even, when the researcher intends to link several regions through a multiregional input-output system. Regional input-output tables are not provided as a rule by official statistics organisms. In the majority of the European countries, only some regional indicators are published on a regular basis, such as: regional output by industry, regional value added by industry and total intermediate consumption by industry. These data may be taken as a starting point to assemble the individual regional tables, using a combination of non-survey and survey methods and taking the national input-output table as a reference to obtain the regional counterpart. It is consensual that the more direct information is incorporated in the table, the more accurately it tends to reflect regional reality. However, the introduction of direct information implies higher costs, which forces the researcher to make this in a selective way (more or less restrictively, depending on the resources available to conduct regional surveys). The attempt of assembling regional input-output tables and multiregional input-output systems using limited direct information is often considered to be an unfeasible endeavor. We do not agree with this view. Conversely, we share the perceptible opinion of other researchers which have devoted their efforts to the construction of regional input-output models, even in developing countries, in which information is more limited. Yet, in order to obtain the best possible results, we consider that it is necessary to:

- Find an adequate equilibrium in the existing trade-off between model complexity and model construction practicability. This means that the existing models must be evaluated in face of the information availability in each particular case.
- Study and evaluate different non-survey methodologies to estimate inexistent information (such as interregional trade) and critically analyze the implicit hypotheses, testing the sensitivity of the model solutions to the techniques and hypotheses assumed.
- Get the maximum benefit of the existing information and try to adapt the proposed models in order to fit into the (sometimes more advantageous) format in which information is provided. For example, traditional input-output models were developed within the symmetric framework, meaning that the supporting input-output tables were product-by-product or industry-by-industry tables. Currently, however, most of the countries compile and publish their national input-output

tables in the rectangular or Make and Use format (introduced by the United Nations in 1960's), requiring some adaptation in the existing models.

With the present work we aim to achieve the following broad objectives, which correspond to the three Chapters included in this dissertation:

- To make a broad review of the state of knowledge regarding input-output modeling and input-output table construction at the regional level, paying special attention to the quantitative and qualitative disagreement between the data requirements implicit in the traditional input-output models and the usually available data. We intend to accomplish this objective in Chapter 1 of the Dissertation: "Introducing Input-output analysis at the regional level: basic notions and specific issues".

- To study and test methodologies to overcome the above mentioned mismatch between data requirements and data availability, focusing on two specific issues:

    - Interregional trade indirect estimation, as a viable alternative to solve the common difficulty in regional table construction – the inexistence of survey-based interregional trade data. This will be the focus of Chapter 2: "Determining interregional trade flows in a many-region system".

    - Input-output modeling based on total use rectangular input-output tables, implying the adaptation of the traditional input-output model to the format in which the input-output database (published on a regular basis for the national level) is currently provided. This issue is considered in Chapter 3: "Input-output modeling based on total use rectangular tables".

The research is guided towards the answer to the following questions (of which some involve specific concepts to be explained in the development of the dissertation):

- What is the state of the art of regional input-output table construction and modelling?

- How easy is it to apply the existing models to countries like Portugal, with limited regional and interregional information?
- What models have been used to indirectly estimate interregional trade flows?
- What is their actual applicability under a context of very limited *a priori* information?
- When applying different interregional trade estimation methodologies:
    - What is the degree of closeness of each estimated matrix to the real matrix of flows?
    - Which method generates the most accurate estimated matrix?
    - How sensitive are the values obtained in the final trade matrix to different estimating methods?
    - How sensitive is the solution of the input-output model to the insertion of different interregional trade values? In other words, how important is the choice of interregional trade estimation method to the solution of the input-output model?
- What procedures and related hypotheses may be used to perform input-output modelling when the basic data consists of a total-use rectangular table at purchasers' prices, with no available import matrix?
- Is it advantageous to perform a previous transformation of the original tables into the symmetric format and a previous calculation of domestic flows, before implementing the model?

The main motivation to the conduction of this research is a very practical one: to help the researchers in defining their strategy in future efforts of multiregional input-output table construction and modelling. We hope to give our contribution to the construction of such model to the Portuguese economy. Despite being heavily theoretical, the underneath intention of this work is evident in various parts of the dissertation.

CHAPTER 1 – INTRODUCING INPUT-OUTPUT ANALYSIS AT THE REGIONAL LEVEL: BASIC NOTIONS AND SPECIFIC ISSUES.

1.1. Introduction

The main objective of the well known input-output model, developed by Leontief in the late 1930s, is to study the interdependence among the different sectors in any economy (Miller and Blair, 1985). This tool holds upon a very simple, yet essential notion, according to which the output is obtained through the consumption of production factors (inputs) which can be, in their turn, the output of other industries. Hence, one of the principal tasks of input-output analysis is to identify the indirect demands concerning the intermediate consumptions necessary to generate the outputs.

The origins of the basic notion behind the input-output model go back to the 18th century, when Quesnay published the "*Tableau Economique*". His objective was to describe the economic transactions established between three social classes: landowners, farmers and rural workers (productive class) and the sterile class, composed by artisans and merchants (this classification reflects the physiocrats' philosophy, according to which agriculture was the only wealth generating sector).

Over more than one century, this idea of economic interdependence had a new and important contribution, with the work developed by Walras[1]. This economist introduced the general equilibrium model, aiming to determine prices and quantities of all economic markets. In this model Walras used a set of production coefficients very similar to the ones defined *a posteriori* in the Leontief's input-output model: they compared the amount of production factors used in production with the total output obtained (Miller and Blair, 1985).

The perception and depiction of the interactions among the different economic activities (besides the spatial dimension which is being considered) allows, on the one hand, the access to a very detailed statistical tool about the economy we are focusing on: the input-

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[1] *Walras, L. 1874. "Elements of pure economics". Translated by W. Jaffé. Hmoewood, Illinois: Richard Irwin, Inc., 1954.* Referred in Miller and Blair (1985).

output table. An input-output table records the "flows of products from each industrial sector considered as a producer to each of the sectors considered as consumers" (Miller and Blair, 1985, p. 2). This table gives us a quite complete picture of the economy at some specific point in time, providing estimates for an important set of macroeconomic aggregates (production, demand components, value added and trade flows) and disaggregating these among the different industries and products. Besides, the input-output table is a suitable instrument to perform structural analysis of the correspondent economy, depicting the interdependence between its different sectors and between the economy and the rest of the world (ISEG/CIRU, 2004). On the other hand, the input-output table provides an important database to the construction of input-output models which may be used, for example, to evaluate the economic impact caused by exogenous changes in final demand (Miller, 1998).

The original applications of the input-output model were made at a nation-wide level[2]. However, the interest in extending the application of the same framework to spatial units different from the country (usually, sub-national regions) led to some modifications in the national model, originating a set of regional input-output models. According to Miller and Blair (1985), there are two specific characteristics referring to the regional dimension which make evident and necessary the distinction between national and regional input-output models. First, the productive structure of each region is specific, probably being very different from the national one; second, the smaller the focusing economy, the more it depends on the exterior world (this including the other regions of the same country and other countries), making exports and imports to become more important in determining the region's demand and supply.

Since the 1950's, different regional input-output models were developed, being distinguished through the following criteria: (1) the number of regions taken into account; (2) the recognition (or not) of interregional linkages; (3) the degree of detail implicit in interregional trade flows (which is related to the degree of detail demanded for the input-

[2] An example of this is the pioneering application of Leontief to the United States that became public through the book "The Structure of the American Economy, 1919-1929", published for the first time in 1941.

output data) and (4) the kind of hypotheses assumed to estimate trade coefficients. The first criterion is used to distinguish the single-region model from the several types of models designed to systems with more than one region. The single-region model seeks to capture intra-regional effects alone. So, its crucial limitation consists of the fact that it ignores the effects caused by the linkages between this region and the others. But in reality, when one region increases its production, as a reaction to some exogenous change in its final demand for example, some of the inputs needed to answer the production augment will come from the remaining regions, originating an increase of production in these regions – these are the spillover effects. The remaining regions, in turn, may need to import inputs from other regions (probably including the first region) to use in their own production. These involve the concept of interregional feedback effects: those which are caused by the first region in itself, through the interactions it performs with the remaining regions (Miller, 1998). The seminal applications of input-output analysis to systems with more that one region, capturing the effects caused by the interconnections between the different regions (which corresponds to the second criterion previously referred), had the fundamental contributions of Walter Isard (Glasmeier, 2004). These contributions originated the interregional model also known as Isard's model. Practical difficulties in implementing the interregional model, mainly due to its high requirements in terms of interregional trade data, motivated the emergence of multi-regional models (of which the Chenery-Moses model is the most popular). As we shall see latter on this Chapter, the different many-region models are distinguishable through the third and fourth criteria mentioned above.

This brief introduction to regional input-output models makes clear that their implementation requires the access to some data on interregional trade flows (more or less detailed, depending on the specific type of regional input-output model). But how relevant are actually interregional trade flows to regional economies? Some regional studies have proved that trade flows established between one region and the remaining regions tend to be more significant than trade flows established between the same region and foreign countries (Munroe and Hewings, 1999). Moreover, interregional trade is indeed growing faster than intra-regional and international trade (Jackson *et al*., 2004).

One of the reasons for the rapid growth of interregional trade is the fact that it is currently replacing much of the intra-regional transactions, in a process called "hollowing-out": it implies that the density of relations within the regional economy tends to diminish, in favour of interregional linkages (Polenske and Hewings, 2004). Given its relative importance in the region's external trade, the knowledge of the volume and nature of interregional trade flows constitutes a critical issue for regional analysis. For example, a deficit in the region's trade balance means that the region relies on income transfer and/or granting of savings from other regions, within the country or from the rest of the world (Ramos and Sargento, 2003). In a more detailed perspective, knowledge about regional external trade, segmented by commodities, allows us to characterize productive specialization, foresee eventual productive weaknesses as well as determine the region's dependency on the exterior (or in some cases the exterior's dependency on the region) regarding to the supply of different commodities. In spite of its recognized importance, interregional trade flows established between regions of the same country constitute precisely the hardest data to find among the set of data necessary to implement the input-output model.

The previous paragraph leads us to the first of the fundamental issues underlying the present work, which also constitutes one of the main challenges of regional input-output researchers: obtaining the regional data necessary to implement input-output models, with special concern in interregional trade. The existing regional data, provided by the official organisms of statistics, is usually "less than perfect", meaning that it is more or less distant from the ideal set of data required by each type of regional input-output model. Facing this problem, the researcher may follow two alternatives (or do both): adapt the model to the existing data and / or estimate (or directly collect) the inexistent data. Even when some adaptation is made, through the use of some assumptions, a minimum amount of data on interregional trade (besides other input-output table components) is always necessary, so that the model succeeds in capturing spillover and feedback effects caused by the interregional linkages. Being so, some techniques must be adopted to assess those data. These techniques can be classified according to the degree of incorporation of direct regional information. Most of the researchers use hybrid methods, combining some survey information with non-survey techniques, in which specific regional indicators are applied to convert national values into regional ones. It is consensual that the more direct information is incorporated in the table, the more accurately it tends to reflect regional reality. However, the introduction of direct information implies higher costs, which forces the researcher to make this in a selective way (more or less restrictively, depending on the resources available to conduct regional surveys). Besides, even if the research team doesn't face any restrictions in terms of money, time, manpower or logistic resources, this doesn't guarantee that a pure survey-based table is completely exempt of errors. In fact, according to Jensen (1980), errors in survey tables can result from errors in the process of gathering the data (for example: errors arising from incorrect definition of the sample, hiding of information or lack of concern in answering the questionnaires by the respondents) or errors in compilation procedures. Besides, other problems may arise whenever the questions included in the questionnaires require very detailed information to which some respondents may not be able to answer. In this context, Jensen (1980) argues that the concept of holistic accuracy must be privileged, meaning that the assembly of direct information should be directed only towards the larger or most important elements of the economy being studied, thus ensuring a correct representation of the structure of the economy, in general terms (Hewings, 1983). In other words, hybrid methods assure the best compromise between accuracy and required resources. The theoretical review of the main techniques used to generate undisclosed data will be made on this Chapter. In what concerns specifically to the problem interregional trade estimation, the main existing techniques will be discussed on Chapter 2.

An additional important challenge faced by input-output researchers consists in adapting the traditional input-output models in order to fit them into the specific format in which information is available. The fact is that, sometimes, input-output rough data exists, but it is provided in a different way from that underlying the traditional input-output models. For example, traditional input-output models were developed within the symmetric framework, meaning that the supporting input-output tables were product-by-product or industry-by-industry tables. Product-by-product tables have products as the dimension of both rows and columns, showing the amounts of each product used in the production of which other products. In turn, industry-by-industry tables have industries as the dimension of both rows and columns, showing the amounts of output of each industry used in the production of which other industries (UN, 1993). Currently, however, most of the countries compile and publish their national input-output tables in the rectangular or Make and Use format (introduced by the United Nations in 1960's). In this framework, two dimensions are simultaneously considered (industries and products) and two tables are essential: the Use table, which describes the consumption of products *j* by the several industries *i*, and the Make table that represents the distribution of the industries' output by the several products. In conjunction, these tables depict how supplies of different products originate from domestic industries and imports and how those products are used by the different intermediate or final users, including exports (UN, 1993). The procedures and hypotheses adopted in input-output table construction as well as in input-output modelling should be suited to fit this data format.

Another example of non-coincidence between the model's data requirements and data availability is at the intermediate transactions table: the nuclear part of an input-output table, which represents the intermediate consumption of the several products made by the different industries. In some countries, like Portugal, the national intermediate transactions table is provided in a total use basis, meaning that the amount of products recorded as inputs in the intermediate consumption of the different industries comprise either nationally produced or imported products. However, some input-output models involve the determination of impacts within the region (or within the nation, depending on the spatial dimension being considered), implying that the computed effects should be cleaned from effects on imports. In such case, the model should be adapted, under some hypotheses, to fit the available total use data.

The choice of the proper hypotheses to develop national and regional input-output models when input-output data is not available in the traditional format is the second fundamental issue underlying the present work. Being so, we aim to provide in this Chapter some fundamental concepts on the accounting systems in which input-output data are currently provided, so that latter on this work (in Chapter 3) the above referred hypotheses can be discussed in a clearer way. Obviously, instead of adapting the models to fit the existing data, an alternative consists in transforming the data in order to match the hypotheses beneath the traditional models. In the above mentioned situations this would imply: (1) converting the Make and Use format into a symmetric format previously to the development of the model and (2) subtract imports from the total flow intermediate transactions table previously to the development of the model. The pertinence and feasibility of this alternative will also be discussed in Chapter 3 of this work.

This Chapter aims to achieve two fundamental objectives, which became clearer with the previous introduction:

- Make a comprehensive review of the state of the art concerning input-output modelling (mainly at the regional level) and techniques for regional input-output table construction, providing consistency to the accomplishment of the practical objectives of the present work.
- Make a critical appraisal of the proposed input-output models and techniques of regional input-output table construction, focusing specially on the quantitative and qualitative disagreement between the required and the available data. In this context, two issues will receive special attention: interregional trade estimation and input-output modelling based on total use rectangular input-output tables. The critical evaluation of the existing methodologies will provide justification for the options made on the empirical applications of the present work (developed in Chapters 2 and 3).

This Chapter is organized in seven sections, including this Introduction. The second section aims to introduce the foundations of the input-output model, presenting the basic structure of a (national) input-output table and the deduction of the Leontief's input-output model from that table. In section 1.3, we will review the most important regional input-output models, involving one or more regions, discussing the theoretical and practical implications of each one. Section 1.4 brings in the problem of table construction, at the regional level, discussing the advantages and drawbacks of survey, hybrid and nonsurvey approaches. The issue of accuracy assessment of the constructed tables will also be dealt with in this section. Next, in section 1.5, we turn to the specific features of the accounting systems implicit in the official national tables, which necessarily have an influence on the techniques used for regional table construction and on the hypotheses assumed in national and regional input-output modelling. Of these specific features, we will focus our attention on the Make and Use format (contrasting to the symmetric format) and on total intermediate transactions flows (as opposed to intra-regional or domestic flows). Section 1.6 provides some insight into the problem of estimating interregional trade data, which will be further developed in Chapter 2. Finally, section 1.7 presents a summary of the main conclusions of this Chapter.

1.2. Foundations of input-output: basic input-output table and deduction of the Leontief model.

The several input-output interconnections existing in any economy (of any geographic dimension: a city, a region, a country, an integrated bloc of countries, etc), may be traced in a very simple but elucidating way through an input-output table. An input-output table records the "flows of products from each industrial sector considered as a producer to each of the sectors considered as consumers" (Miller and Blair, 1985, p. 2). Let's illustrate this with the example of one hypothetical national economy that has *n* industries and, for simplicity, let's assume a one-to-one relationship between industries and products: *i.e.*, each product is produced by only one industry and each industry produces only one product[3]. In the production process, each of these industries uses products that were produced by other industries and produces outputs that will be consumed by final users (for private consumption, government consumption, investment and exports) and

[3] This is called the homogeneity assumption, used as a simplification in the traditional input-output analysis. This assumption will be discussed later on in this chapter and it will be relaxed in the models developed in Chapter 3.

also by other industries, as inputs for intermediate consumption[4]. These transactions may be arrayed in an input-output table, as illustrated in Figure 1. 1:

Figure 1. 1 – Simplified structure of a national IO table, with total use flows.

Products	1 ... n	Total Final Demand	Total Demand
1 ... n	Total interindustry transactions		
Total Intermediate Consumption			
Value Added			
Total Supply of domestic products			
Imported products			
Total Supply			

Looking across the rows in this table, we can observe how the output of each product is used throughout the several consumers of this economy: the total output of each product *i* (x_i) is used for intermediate consumption by the various industries *j* and for the diverse final demand purposes. As mentioned in the Figure's label, this is a total flow table, meaning that the flows recorded as intermediate and final demand refer not only to domestically produced input, but also to imported input. The columns of Figure 1. 1 provide information on the input composition of the total supply of each product *j* (x_j): this is comprised by the national production and also by imported products. The value of domestic production consists of intermediate consumption of several industrial inputs *i* plus value added[5]. The interindustry transactions table is a nuclear part of this table, in the sense that it provides a detailed portrait of how the different economic activities are interrelated. Since, in this table, intermediate consumption is of the total-flow type, this implies that true technological relationships are being accounted for. In fact, each column of the intermediate consumption table describes the total amount of each input *i*

[4] Intermediate consumption consists in the value of products "which are used or consumed as inputs in the process of production during a specific accounting period." (Jackson, 2000, p.110)
[5] Value added is measured by the payments made for other production factors, like labour and capital (thus including compensation of employees, profits and capital consumption allowances).
In this simplified structure, for the moment, we are neglecting some elements of the table, such as trade and transport margins and taxes (less subsidies) on products.

consumed in the production of output *j*, regardless of the geographical origin of that input.

Figure 1. 2 - Simplified structure of a national IO table, with domestic flows.

Products	1 ... n	Final Demand	Total Demand of domestic products
1 ... n	interindustry transactions of domestically produced inputs		
Imports			
Total Intermediate Consumption			
Value Added			
Total Supply of domestic products			

In alternative, input-output interconnections can be presented considering only domestically produced products in the inputs to be used in intermediate and final consumption. In such case, the table will have a different structure, illustrated in Figure 1. 2 above.

Three major differences exist between this table and the former:

1) The amounts of products used in intermediate consumption by the several industries and by the various final users comprise only domestically produced inputs. In this case, the interindustry transactions table is no longer representative of a technological matrix. It rather represents the intra-national interindustry transactions, which are determined not only by technological factors, but also by trade factors.

2) The row referring to imports has a different arrangement in the table and also a different meaning. Instead of being disaggregated by products and included in the intermediate and final demand flows (as they were in the total-flow table), the imported inputs are now lumped together in a single row, which must be added to the total intermediate consumption of domestic inputs (and to the total final

demand of domestic products), in order to get the total amount of intermediate consumption made by each industry (and the total amount of each component of final demand). Thus, each element of this row gives us the aggregate amount of imports used by each industry and by each kind of final user. Conversely, the row of imported products in the total-flow table depicts the total amount of imports of each product *j* ($j = 1, \cdots, n$). These are added to domestic production, in order to obtain the value of total supply by product. So, in the total-flow table, the row of imported products depicts imports disaggregated by products, whereas in the domestic-flow table, it represents imports disaggregated by destination industry.

3) As a consequence, the balance between supply and demand in the total-flow table includes imported products, whereas in the domestic-flow table this balance is made considering only domestic production.

The dichotomy between total use and domestic flows will be a recurring issue in the following sections and it will be analysed with further detail in section 1.5.3. In the following nation-level input-output model deduction, we will assume a total-use table as the starting point. The comparison between the total-use model and the model correspondent to Figure 1. 2 is left to section 1.3.1, in which a single-region case is considered. In fact, the structure of single-region models is very similar to the structure of single-nation models, as we will see in section 1.3.1.

The input-output interconnections illustrated in Figure 1. 1 can be translated analytically into accounting identities. On the demand perspective, if we let z_{ij} denote the intermediate use of product *i* by industry *j* and y_i denote the final use of product *i*, we may write, to each of the *n* products:

$$x_i = z_{i1} + z_{i2} + \ldots + z_{ii} + \ldots + z_{in} + y_i \tag{1. 1}$$

At the supply side, we know that:

$$x_j = z_{1j} + z_{2j} + ... + z_{jj} + ... + z_{nj} + w_j + \mathrm{m}_j$$

(1. 2)

in which w_j stands for value added in the production of *j* and m_j for total imports of product *j*. Of course, it is required that, for $i = j$, $x_i = x_j$, *i.e.*, for one specific product, the total output obtained in the use or demand perspective must equal the total output achieved by the supply perspective.

These two equations can be easily related to the National Accounts identities. Let's use the following notation for the macroeconomic variables: C represents private consumption; F represents gross capital formation; G stands for government consumption; E and M denote exports and imports, respectively and VA means value added. All these variables represent aggregate values. Let's consider also the following sums:

$z_i = z_{i1} + z_{i2} + ... + z_{ii} + ... + z_{in} = \sum_{j=1}^{n} z_{ij}$ and $z_j = z_{1j} + z_{2j} + ... + z_{jj} + ... + z_{nj} = \sum_{i=1}^{n} z_{ij}$.

Then, if we sum up all the equations (1. 1), we get the total value of all economic activity in this economy (Miller, 1998):

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} z_i + \sum_{i=1}^{n} y_i$$

(1. 3)

Given that $\sum_{i=1}^{n} y_i = C + F + G + E$, the previous equation becomes:

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} z_i + C + F + G + E$$

(1. 4)

Similarly, if we sum up all the equations (1. 2), we must achieve the same value. This corresponds to:

$$\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} z_j + VA + M$$

(1. 5)

Given that $\sum_{i=1}^{n} z_i$ and $\sum_{j=1}^{n} z_j$ are equal, since both represent the sum of all elements of the intermediate consumption matrix ($\sum_{i=1}^{n} z_i = \sum_{j=1}^{n} z_j = \sum_{i=1}^{n}\sum_{j=1}^{n} z_{ij}$), we may write:

$$\begin{aligned} & VA + M = C + F + G + E \Leftrightarrow \\ & VA = C + F + G + (E - M) \end{aligned}$$

(1. 6)

Since VA represents the sum of the value added generated by all producers in the economy, it corresponds to the economy's gross domestic product (GDP)[6] accounted by a product approach. So, equation (1. 6) is precisely the well known macroeconomic identity between GDP when it is defined by a product approach and the same concept, defined according to the expenditure perspective:

$$GDP = C + F + G + (E - M)$$

(1. 7)

Let's refer back to the disaggregate level, embodied in equations (1. 1) and (1. 2). These are merely the mathematical representation of the information displayed in any input-output table, for a certain base-year. In order to introduce the input-output model we need to consider the fundamental concept of technical coefficient (Miller, 1998): $\frac{z_{ij}}{x_j} = a_{ij}$,

[6] In National Accounts, the aggregate value added differs from GDP, by the amount of the aggregate taxes (less subsidies) on products; i.e., it is valued at basic prices, while GDP by default, as it is defined on the expenditure side, is at purchasers prices. However, in this introductory analysis, these elements are being ignored, as it has been referred before.

which gives us the total amount of product i (domestically produced and imported) used as input in the production of one monetary unit of industry j's output. Using this definition, equation (1. 1) may be substituted for:

$$x_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ii}x_i + \ldots + a_{in}x_n + y_i$$

(1. 8)

Replicating this to each of the n products under consideration and rearranging terms in the equation, we have:

$$\begin{aligned}
&(1-a_{11})x_1 - a_{12}x_2 - \ldots - a_{1i}x_i - \ldots - a_{1n}x_n = y_1 \\
&-a_{21}x_1 + (1-a_{22})x_2 - \ldots - a_{2i}x_i - \ldots - a_{2n}x_n = y_2 \\
&\ldots \\
&\ldots \\
&-a_{i1}x_1 - a_{i2}x_2 - \ldots + (1-a_{ii})x_i - \ldots - a_{in}x_n = y_i \\
&\ldots \\
&\ldots \\
&-a_{n1}x_1 - a_{n2}x_2 - \ldots - a_{ni}x_i - \ldots + (1-a_{nn})x_n = y_n
\end{aligned}$$

(1. 9)

or, in matrix terms:

$$\begin{bmatrix} (1-a_{11}) & -a_{12} & \ldots & -a_{1i} & \ldots & -a_{1n} \\ -a_{21} & (1-a_{22}) & \ldots & -a_{2i} & \ldots & -a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{i1} & -a_{i2} & \ldots & (1-a_{ii}) & \ldots & -a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n1} & -a_{n2} & \ldots & -a_{ni} & \ldots & -a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{bmatrix}$$

(1. 10)

which may be translated into a compact form:

$$\begin{aligned}
&(\mathbf{I} - \mathbf{A}) \cdot \mathbf{x} = \mathbf{y} \Leftrightarrow \\
&\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}
\end{aligned}$$

(1. 11)

In this equation, $\mathbf{A}$ is the technical coefficient matrix (total use flows); $\mathbf{x}$ is the total output column vector and $\mathbf{y}$ is the final use column vector. From equation (1. 11), the popular input-output impact analysis can be carried out straightforwardly. Assuming a small exogenous change in the final use vector (by the amount $\Delta\mathbf{y}$), the correspondent change in the output vector ($\Delta\mathbf{x}$) can be obtained as follows:

$$\begin{aligned}\Delta\mathbf{x} &= (\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{y}\\ \Delta\mathbf{x} &= \mathbf{B}\Delta\mathbf{y}\end{aligned} \tag{1. 12}$$

There is a proportionality hypothesis embodied in this equation. It is assumed that the change occurred in the output vector is a constant proportion (given by $(\mathbf{I}-\mathbf{A})^{-1}$) of the change in the final demand vector. This fixed proportion implies that the technical coefficients comprised in matrix $\mathbf{A}$ do not change with the exogenous impact in final demand, which is a reasonable hypothesis if we consider a small impact $\Delta\mathbf{y}$. $(\mathbf{I}-\mathbf{A})^{-1}$ or $\mathbf{B}$ is the so-called Leontief inverse. Each of its elements b_{ij} traduces the value of output *i* required directly and indirectly to deliver one additional monetary unit to *j*'s demand (Miller, 1998)[7]. In analytical terms, $b_{ij} = \dfrac{\partial x_i}{\partial y_j}$. If we sum up each column of this inverse matrix, we obtain $b_{\bullet j} = \sum_{i=1}^{n} b_{ij}$, which are the output multipliers[8]. These represent the value of the economy-wide output required directly and indirectly to deliver one additional monetary unit to *j*'s demand. In other words, they measure the impact over all the economy caused by a change in the final demand for output *j*.

[7] It should be noted that output *i* may either be domestically produced or imported. Also, the initial impact over product *j* is not exclusively directed to domestic production, but is indifferent to the geographic origin of the product. Further on this Chapter, we will present alternative multipliers which measure specifically the impact over domestic production.

[8] Other types of multipliers can be computed. For example, if we weight the elements of the inverse matrix by appropriate employment coefficients, we can deduce employment multipliers, measuring the impact on employment, created by exogenous changes in final demand. For further details see, for example, Miller (1998), pp. 61-64.

The Leontief inverse can also be approximated through a mathematical series expansion. Given that the technical coefficients matrix verifies the conditions of being "(…) a square matrix $\mathbf{A}$ in which all elements are nonnegative and less than one, and in which all column sums are less than one" (Miller, 1998, p. 53), the inverse $(\mathbf{I}-\mathbf{A})^{-1}$ can be expanded using the following power series expression[9]:

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^{2} + \mathbf{A}^{3} + \cdots + \mathbf{A}^{k} + \cdots \tag{1. 13}$$

This expression highlights the presence of different types of effects (initial, direct and indirect effects)[10] caused by an exogenous change in final demand (Miller, 1998). Inserting equation (1. 13) into (1. 12), we get:

$$\begin{aligned}\Delta\mathbf{x} &= \left(\mathbf{I} + \mathbf{A} + \mathbf{A}^{2} + \mathbf{A}^{3} + \cdots + \mathbf{A}^{k} + \cdots\right)\Delta\mathbf{y} \\ \Delta\mathbf{x} &= \Delta\mathbf{y} + \mathbf{A}\Delta\mathbf{y} + \mathbf{A}^{2}\Delta\mathbf{y} + \mathbf{A}^{3}\Delta\mathbf{y} + \cdots + \mathbf{A}^{k}\Delta\mathbf{y}\end{aligned} \tag{1. 14}$$

From this equation we can see that, when the vector of final demand changes, this causes an initial effect of the same amount on the output vector, given by the first term: $\Delta\mathbf{y}$. To satisfy these new productions, industries will have to buy some new inputs, given by $\mathbf{A}\Delta\mathbf{y}$ - these are the direct effects. The remaining terms capture the indirect effects caused by the fact that the production of those new inputs also requires intermediate consumption of additional inputs. Of course, as it happens in any power series expansion, as the exponent increases, the correspondent effect decreases, implying that the latter indirect effects will be necessarily smaller than the former indirect effects and than the direct effects.

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[9] This results is similar to that of ordinary algebra, according to which a power series of infinite terms like $1 + r + r^{2} + r^{3} + \cdots + r^{k} + \cdots$ is equal to $(1-r)^{-1}$, given that $0 \leq r \leq 1$.

[10] Sometimes, initial and direct effects are lumped together and considered both as direct effects. This is why Leontief inverse is also known by the matrix of direct and indirect requirements.

Some hypotheses are implicit in the kind of reasoning exposed above, frequently pointed out as limitations of input-output (mainly when it is used as a forecast model): firstly, the elements of matrix $\mathbf{A}$ are assumed to be time-invariant, meaning that the underlying technology is constant – obviously, this is a restrictive assumption in long-run forecast applications of the model, making it more suitable to short-run uses; second, it is assumed that the a_{ij} are the same, irrespective of the scale of production (constant returns to scale), which implies that scale economies are not taken into account; thirdly, the assumption of constant a_{ij} also implies that we are dealing with a fixed proportion technology; in fact, if we consider two inputs, i and k, to produce output j, the proportion in which they are used is given by $\frac{z_{ij}}{z_{kj}} = \frac{\frac{a_{ij}}{x_j}}{\frac{a_{kj}}{x_j}} = \frac{a_{ij}}{a_{kj}}$, which is constant, since the technical coefficients are also constant (Miller e Blair, 1985); finally, the production capacity is supposed to be unlimited: when the final use of some product increases it is supposed that the output of this product and the others will be able to answer the additional direct and indirect requirements, without any capacity restrictions. With the aim of overcoming these shortcomings of the model, several developments have been introduced into the basic formulation: for instance, dynamic models that consider varying technical coefficients and models that include capacity restrictions. Yet, in the present work, the basic formulation will be used, since forecasting impacts is not our primary objective.

1.3. Regional input-output models.

In spite of having been originally conceived to national-wide applications, input-output model has been applied to sub-national geographic units since the second half of the last century. According to Miller and Blair (1985), there are two specific features associated to the regional dimension which make evident and necessary the distinction between national and regional input-output models. First, the technology of production of each region is specific, and it may be close or, on the contrary, very different from the one which is registered at the national input-output table; for example, the age of regional industries, the characteristics of input markets or the education level of the labour force are important factors that may influence the regional technology of production to deviate from the national one. Second, the smaller the economy under study, the more it depends on the exterior world, making more relevant the exported and imported components of demand and supply, respectively. It should be noted that these components correspond not only to international trade, but also to the trade between the region and the rest of the country to which the region belongs.

In this section we aim to review the main contributions in regional input-output modelling. The following models are distinguishable by four main criteria:

- the number of regions taken into account: single-region or many-region models.
- the recognition (or not) of interregional linkages;
- the degree of detail implicit in interregional trade flows (which is related to the degree of detail demanded for the input-output data) and
- the kind of hypotheses assumed to estimate trade coefficients.

We will begin by presenting the single-region model (section 1.3.1), which has a similar structure to the nation level input-output model presented in the previous section. Then we proceed to those models that try to capture, not only intra-regional transactions, but also the interconnections between regions. Of these, we begin by reviewing Leontief's intranational model (section 1.3.2), which consists of a very primary type of regional input-output model, since the only spatial effect it recognizes concerns to the one-way effect of national changes over regional output. As it will be seen, spillover and feedback regional effects are not considered by this model. The remaining models (sections 1.3.3. to 1.3.5) seek to account for inter-spatial effects. Yet, they differ in the degree of detail used in the specification of interregional trade flows. Besides, the two multi-regional models (Chenery-Moses and Riefler-Tiebout) are distinct by the hypotheses they assume to determine trade coefficients. One common feature of the last three regional models is the fact that trade coefficient stability is assumed. We will give special attention to this aspect on section 1.3.6.

1.3.1 Single-region model.

The aim of single-region input-output models is to evaluate the impact on regional output caused by changes in regional final demand. The starting point for a single-region model is, obviously, a single-region input-output table. Just like it happened in the nation-level table, exposed in the previous section, the single-region input-output table may be presented in two different versions: as a total-use table or as an intra-regional flow table[11]. The correspondence between the structure of these two regional tables and the national tables presented above is straightforward. If we consider that, in Figure 1. 1, the row of imported products includes also imports from other regions of the same country and the vector of final demand comprises also exports to the rest of the country, then this table represents a single-region input-output table, with total use flows. Correspondingly, if we take similar considerations over the table in Figure 1. 2 (concerning imports and exports) and consider additionally that the intermediate and final use flows include only regionally produced inputs, then Figure 1. 2 is converted into a single-region input-output table, with intra-regional flows. These two types of data arrangement originate two different single-region models: (1) total-use single-region model and (2) intra-regional single-region model.

The development of the total-use single-region model follows closely the development of the nation-level input-output model made in the previous section. Let's use the superscript *r* to denote a regional variable; thus, for example x_i^r is the amount of output *i* available in region *r* (including international and interregional imports), y_i^r represents regional final demand for product *i* (including the one that consists of imported products) and z_{ij}^r denotes the total amount of input *i* used in the production of output *j* in region *r*

[11] In the context of regional models the word "intra-regional" is used with the same meaning as the word "domestic", in the nation-level models.

(including imported inputs as well). Then, $a_{ij}^r = \frac{z_{ij}^r}{x_j^r}$ is the regional technical coefficient, defined in a similar way as the national one. This indicates the amount of input i necessary to produce one monetary unit of output j in region r. In should be stressed out that all the possible geographic origins of input i are being included in the calculation of this coefficient, meaning that z_{ij}^r comprises product i produced in region r but also produced in other regions or even abroad, since it is traded in region r. Using the regional variables instead of the national ones, we can write an equation similar to equation (1. 1):

$$x_i^r = z_{i1}^r + z_{i2}^r + \ldots + z_{ii}^r + \ldots + z_{in}^r + y_i^r$$

(1. 15)

Considering the regional technical coefficient $a_{ij}^r = \frac{z_{ij}^r}{x_j^r}$, this equation becomes:

$$x_i^r = a_{i1}^r x_1^r + a_{i2}^r x_2^r + \ldots + a_{ii}^r x_i^r + \ldots + a_{in}^r x_n^r + y_i^r$$

(1. 16)

The compact matrix representation correspondent to the previous equation (considering one equation like this to each of the n products) is:

$$\begin{aligned} &(\mathbf{I} - \mathbf{A^r}) \cdot \mathbf{x^r} = \mathbf{y^r} \Leftrightarrow \\ &\mathbf{x^r} = (\mathbf{I} - \mathbf{A^r})^{-1} \mathbf{y^r} \end{aligned}$$

(1. 17)

This solution allows us to quantify the impact over the total output available at region r caused by a change in regional final demand. Similarly to what was done at the national level, we may write:

$$\Delta\mathbf{x^r} = (\mathbf{I} - \mathbf{A^r})^{-1} \Delta\mathbf{y^r}$$

(1. 18)

It should be noted that this impact is not limited to the region itself; instead, some of the impact measured by this equation is felt outside the region, via effect on imported products (included in the values of vector $\mathbf{x^r}$). Yet, it is possible to compute the impact over regional production from the total-use model. If we pre-multiply both sides of the previous equation by $(\mathbf{I}-\hat{\mathbf{c}})$, in which $\hat{\mathbf{c}}$ is a diagonal matrix[12] of import propensities, we get the impact over regional production. The discussion of the reasonability of this procedure is not made here, being left to Chapter 3.

Let's now consider an intra-regional input-output table (similar to the one in Figure 1. 2) as the starting point to the input-output model development. The major difference relies on the type of coefficient used: instead of the regional technical coefficient, it is used a coefficient that indicates the amount of regionally produced input *i* necessary to produce one monetary unit of output *j* in region *r*. This is called an intra-regional input coefficient, being this label sometimes simplified to regional input coefficient (Miller and Blair, 1985). Let z_{ij}^{rr} denote the amount of regionally produced input *i* used in the production of output *j* in region *r*. Then, the intra-regional input coefficient may be computed as: $a_{ij}^{rr}=\frac{z_{ij}^{rr}}{e_j^r}$, in which e_j^r denotes regional production of product *j*. Considering additionally f_i^r as the region's final demand towards product *i* produced in region *r* (including regional requirements as well as exports for any other regions, national or foreign), the solution of the single-region input-output model with intra-regional flows follows the same procedures as before. In matrix terms, let's use the notation:

- $\mathbf{A^{rr}}$ - a matrix composed by intra-regional input coefficients a_{ij}^{rr};
- $\mathbf{e^r}$ - the vector of output produced in region *r*;
- $\mathbf{f^r}$ - the vector of regional final demand towards products produced in region *r*.

Then, the final equation of the single-region model with intra-regional flows is:

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[12] We will use $\hat{\mathbf{a}}$ to note a diagonal matrix composed by the elements of column vector $\mathbf{a}$.

$$\mathbf{e}^{\mathbf{r}} = \left(\mathbf{I} - \mathbf{A}^{\mathbf{rr}}\right)^{-1}\mathbf{f}^{\mathbf{r}}$$
$$\mathbf{e}^{\mathbf{r}} = \mathbf{B}^{\mathbf{rr}}\mathbf{f}^{\mathbf{r}}$$

(1. 19)

Applying impact analysis to this model, we get:

$$\mathbf{\Delta e}^{\mathbf{r}} = \mathbf{B}^{\mathbf{rr}}\mathbf{\Delta f}^{\mathbf{r}}$$

(1. 20)

The intra-regional inverse matrix $\mathbf{B}^{\mathbf{rr}}$ measures the impact of changes in final demand for regional products over regionally produced output. The fundamental differences between this equation and equation (1. 18) are: (1) the impact is quantified over regional production ($\mathbf{e}^{\mathbf{r}}$), whilst, in equation (1. 18), the impact is quantified over total output available at the region ($\mathbf{x}^{\mathbf{r}}$); (2) the initial change refers to final demand for regionally produced products, $\mathbf{\Delta f}^{\mathbf{r}}$, whereas, in equation (1. 18), the initial change refers to regional final demand (for both regional production and imports: $\mathbf{\Delta y}^{\mathbf{r}}$) and (3) the inverse matrix is obtained from intra-regional coefficients, whereas in equation (1. 18), the inverse matrix is obtained from true regional technical coefficients.

Of course, the practical application of the single-region model with intra-regional flows requires that the researcher has previous access to the vector of regional outputs, which generally occurs, and also to the matrix of intra-regional flows $\mathbf{Z}^{\mathbf{rr}}$ and to the vector of final demand $\mathbf{f}^{\mathbf{r}}$. These two latter statistics are much more difficult to obtain. As stated in Miller (1998), "To generate these kinds of data through a survey, respondents must be able to distinguish regionally supplied inputs from imported products" (p. 87). This is valid to firms, when asked about their intermediate consumption patterns, but also to final users. It is obvious that the fundamental problem in conducting such a survey is not the usually mentioned time and cost restrictions, but rather the fact that the respondents may not know the answer. In fact, most of industrial units buy their inputs in wholesale traders which, in turn, sell a mix of regional and imported products. It should be stressed that imported products, in a regional context, involve also products from other regions. Thus, it is very difficult to firms to answer whether a specific input *i* was imported or not, from other countries or other regions. For evident reasons, the problem of not knowing the origin of the products is even more manifest in what respects to final consumers. Being so, a set of hypotheses is usually applied in order to estimate $\mathbf{A}^{\mathbf{rr}}$ from a regional technical coefficients matrix $\mathbf{A}^{\mathbf{r}}$. In Chapter 3 it will be shown that, if consistent hypotheses are used in both types of single-region models (total-flow and intra-regional flow), the results provided by both are equivalent and the total-flow single-region model is capable of measuring the same kind of impacts as the intra-regional single-region model does.

Regardless of the type of flows being considered, the single-region model has a crucial limitation of theoretical nature: it consists of the fact that it ignores the effects caused by the linkages between this region and the others (in the same country and abroad). Exports are, thus, considered as exogenous variables. However, in reality, when a new final demand occurs in one specific region, the impact doesn't confine itself to its boundaries; instead, in order to satisfy the new final demand, the first region will need to import goods and services from the remaining regions, to use as intermediate consumption. This effect is indeed of growing importance, given the increasing economic integration between the different countries and regions (Van der Linden and Oosterhaven, 1995). One of two fundamental inter-spatial effects, which are neglected by the single-region model, are the spillover effects, which account for the change in the production of other regions caused by input purchases made by the first region (to answer its own additional needs). The remaining regions, in turn, may need to import inputs from other regions (probably including the first region) to use in their own production. These involve the concept of interregional feedback effects: those which are caused by the first region in itself, through the interactions it performs with the remaining regions (Miller, 1998).

1.3.2 Leontief's intranational model.

The precursor of input-output analysis developed his first spatial input-output model in 1953. This was a very simple model, both in analytical as well as in data requirements. In his intranational model, also called balanced regional model, he combined the traditional input-output analysis with the awareness that "some commodities are produced not far from where they are consumed, while the others can and do travel long distances between the place of their origin and that of their actual utilization" (Leontief, 1953, p. 93). In order to account for such spatial interaction, yet in a crude manner, he begins by distinguishing two classes of commodities: "regional" and "national". "Regional" commodities are supposed to be regionally balanced, which means that all the regional production is consumed in the same region. Examples of such goods might be: utilities, personal services and real estate (Miller and Blair, 1985). Conversely, "national" commodities are those which are "...easily transportable..." (Leontief, 1953, p. 94) and in which production-consumption balance occurs only at the national or even at the international level. Products like cars or clothes can fit into this category. This implies that one region may have production in excess in some "national" product, originating exports to the rest of the country, or instead, it may have a deficit, which leads to imports from the rest of the country. The model only computes net trade flows, rather than gross exports and gross imports, and it doesn't determine the region of origin (destination) of the imports from (exports to) the rest of the country. This is the reason why the author prefers to label this model intranational, instead of interregional.

The ultimate aim of this model is to determine the regional impact of an exogenous change in the final demand for "national" and / or "regional" products (Miller and Blair, 1985). The following set of hypotheses support the development of the model:

- There are n products, divided in "regional" (from 1 to h) and "national" (from $h+1$ to n), according to the previous definitions; this classification is known *a priori*.

- There are k regions.

- The technical coefficients, $a_{ij} = \frac{z_{ij}}{x_j}$, are known and the same technological matrix is used for all regions and for the nation as a whole.

- The national and regional outputs, as well as the national and regional final demands, are known *a priori* (for both "national" and "regional" commodities);

"national" commodities are marked with the subscript $_N$ and "regional" commodities are marked with the subscript $_R$; let the national outputs and final demand be represented by the following vectors. It should be emphasized that these subscripts refer to types of products and not to geographic locations:

$$\mathbf{x_R} = \begin{bmatrix} x_1 \\ x_1 \\ \vdots \\ x_h \end{bmatrix} \qquad \mathbf{x_N} = \begin{bmatrix} x_{h+1} \\ x_{h+2} \\ \vdots \\ x_n \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} \mathbf{x_R} \\ \mathbf{x_N} \end{bmatrix}$$

$$\mathbf{y_R} = \begin{bmatrix} y_1 \\ y_1 \\ \vdots \\ y_h \end{bmatrix} \qquad \mathbf{y_N} = \begin{bmatrix} y_{h+1} \\ y_{h+2} \\ \vdots \\ y_n \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \mathbf{y_R} \\ \mathbf{y_N} \end{bmatrix}$$

At the regional level we have precisely the same set of variables; as usual, a superscript ^{r} is used to denote a regional variable.

- The market share of each region in providing each of the "national" products, $\tau_N^r = \frac{x_N^r}{x_N}$, is also given *a priori* and it is assumed to be constant, i.e., "the regional output of these commodities is assumed to expand and contract proportionally with the change in national demand" (Polenske, 1995).

Using these hypotheses, equation $(\mathbf{I} - \mathbf{A}) \cdot \mathbf{x} = \mathbf{y}$, deduced previously, yields for the economy as a whole. Only, in this case, matrix $\mathbf{A}$ may be looked as a composition of four different matrices, taking into account the classification of commodities into "regional" and "national":

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{RR}} & \mathbf{A_{RN}} \\ \mathbf{A_{NR}} & \mathbf{A_{NN}} \end{bmatrix} \tag{1. 21}$$

Making use of the previously defined composed vectors $\mathbf{x}$ and $\mathbf{y}$, the solution of the model can be expressed, in this case as (Miller and Blair, 1985):

$$\begin{bmatrix} (\mathbf{I}-\mathbf{A_{RR}}) & -\mathbf{A_{RN}} \\ -\mathbf{A_{NR}} & (\mathbf{I}-\mathbf{A_{NN}}) \end{bmatrix} \begin{bmatrix} \mathbf{x_R} \\ \mathbf{x_N} \end{bmatrix} = \begin{bmatrix} \mathbf{y_R} \\ \mathbf{y_N} \end{bmatrix} \Leftrightarrow$$
$$\begin{bmatrix} \mathbf{x_R} \\ \mathbf{x_N} \end{bmatrix} = \begin{bmatrix} (\mathbf{I}-\mathbf{A_{RR}}) & -\mathbf{A_{RN}} \\ -\mathbf{A_{NR}} & (\mathbf{I}-\mathbf{A_{NN}}) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{y_R} \\ \mathbf{y_N} \end{bmatrix}$$

(1. 22)

This equation quantifies the nation-wide impact on the total output of each type of products, caused by an exogenous change in the demand for "the outputs of one or more national sectors and/or one or more regional sectors" (Miller and Blair, 1985, p. 87). The Leontief inverse may also be seen as decomposed in two, in which the upper part represents the direct and indirect requirements of "regional" products and the lower part represents the direct and indirect requirements of "national" products (Leontief, 1953):

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1h} & b_{1\,h+1} & \cdots & b_{1n} \\ b_{21} & b_{21} & \cdots & b_{2h} & b_{2\,h+1} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{h1} & b_{h2} & \cdots & b_{hh} & b_{h\,h+1} & \cdots & b_{hn} \\ b_{h+1\,1} & b_{h+1\,2} & \cdots & b_{h+1\,h} & b_{h+1\,h+1} & \cdots & b_{h+1\,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nh} & b_{n\,h+1} & \cdots & b_{nn} \end{bmatrix} = \begin{bmatrix} \mathbf{B_R} \\ \mathbf{B_N} \end{bmatrix}$$

(1. 23)

Until now, no spatial dimension was included in the model. To do so, we need to consider the market share of each region in providing each of the "national" products, $\tau_N^r = \dfrac{x_N^r}{x_N}$. From this, it follows that, for each region r, the output of the "national" commodities is a function of the national output of the same commodities:

$$x_N^r = \tau_N^r x_N$$

(1. 24)

or, in matrix terms, $\mathbf{x_N^r} = \hat{\boldsymbol{\tau}}\mathbf{x_N}$, in which $\hat{\boldsymbol{\tau}}$ represents a diagonal matrix with the market shares for each "national" product in the main diagonal.

In what concerns to "regional" commodities, we can define the regional output through the demand perspective. But we must be aware that "regional" inputs may be required to the production of both "regional" and "national" industries operating in that region. Being so, the regional output of "regional" commodities is given by:

$$x_R^r = \sum_{j=1}^{h} a_{Rj} x_j^r + \sum_{j=h+1}^{m} a_{Rj} x_j^r + y_R^r$$

(1. 25)

which, using the relevant sub-matrices defined in (1. 21), and considering also equation (1. 24), corresponds to:

$$\begin{aligned}
&\mathbf{x_R^r} = \mathbf{A_{RR}}\mathbf{x_R^r} + \mathbf{A_{RN}}\mathbf{x_N^r} + \mathbf{y_R^r}\\
&(\mathbf{I} - \mathbf{A_{RR}})\mathbf{x_R^r} = \mathbf{A_{RN}}\mathbf{x_N^r} + \mathbf{y_R^r}\\
&\mathbf{x_R^r} = (\mathbf{I} - \mathbf{A_{RR}})^{-1}\mathbf{A_{RN}}\mathbf{x_N^r} + (\mathbf{I} - \mathbf{A_{RR}})^{-1}\mathbf{y_R^r}\\
&\mathbf{x_R^r} = (\mathbf{I} - \mathbf{A_{RR}})^{-1}\mathbf{A_{RN}}\hat{\boldsymbol{\tau}}\mathbf{x_N} + (\mathbf{I} - \mathbf{A_{RR}})^{-1}\mathbf{y_R^r}
\end{aligned}$$

(1. 26)

This equation means that the regional output of "regional" commodities is a function of the regional demand for these commodities, but also of the national output for "national commodities". However, according to equation (1. 22), national output for "national commodities" is, in turn, a function of national demand for both types of products. Thus, ultimately, this model quantifies the regional impact caused by changes in the national demands for both products, allocating "the impacts of new $\mathbf{y_R}$ and $\mathbf{y_N}$ demand to the various sectors in each region" (Miller and Blair, 1985, p. 88).

In spite of its pioneering character in trying to capture spatial interactions and its ease application, the results from the empirical applications of this model were not satisfactory, because it relies on the use of net trade flows leading to the underestimation of the interregional feedback effects previously referred (Polenske and Hewings, 2004). The importance of these effects will be discussed later on.

1.3.3 Isard's IRIO (interregional input-output model).

An interregional input-output model was proposed by Isard in a paper published in 1951 (Isard, 1951). The review of this model will be presented following Miller (1998)'s example for a two-region system: region *r* and region *s*. Let's consider that each region has *n* industries and each industry produces only one product (and *vice-versa*). Then, the domestic production of product *i* in region *r*, may be written in the demand perspective, as:

$$e_i^r = \left(z_{i1}^{rr} + z_{i2}^{rr} + \ldots + z_{in}^{rr}\right) + \left(z_{i1}^{rs} + z_{i2}^{rs} + \ldots + z_{in}^{rs}\right) + f_i^r \tag{1. 27}$$

This equation embraces intra-regional intermediate uses of input *i* and also inter-regional sales of the same input for intermediate consumption, as well as for final uses (these are included in the aggregate f_i^r which contains: private and government consumption, and investment in the region, exports for other regions for final uses and total exports to foreign countries). It should be noted that only the demand addressing regional production is included in the amount f_i^r.

Using the supply perspective, the production of product *j* in region *r* is given by:

$$e_j^r = \left(z_{1j}^{rr} + z_{2j}^{rr} + \ldots + z_{nj}^{rr}\right) + \left(z_{1j}^{sr} + z_{2j}^{sr} + \ldots + z_{nj}^{sr}\right) + m_j^r + w_j^r \tag{1. 28}$$

In this equation, m_j^r represents international imports used as intermediate consumption in the production of *j*.

The two preceding equations make clear that the interregional input-output model is inherently an intra-regional flow input-output model; the fundamental difference between this model and the intra-regional single-region input-output model described before consists of the fact that the former model takes into account the spillover and feedback effects, through the inclusion of one (or more) additional region in the system.

The next step consists in developing the model, which requires the use of the following coefficients:

- $a_{ij}^{rr} = \frac{z_{ij}^{rr}}{e_j^r}$, for region *r* and $a_{ij}^{ss} = \frac{z_{ij}^{ss}}{e_j^s}$, for region *s*, as intra-regional input coefficients;

- $a_{ij}^{rs} = \frac{z_{ij}^{rs}}{e_j^s}$ and $a_{ij}^{sr} = \frac{z_{ij}^{sr}}{e_j^r}$, as interregional trade coefficients. For example, a_{ij}^{rs} represents the amount of input *i* from region *r* necessary per monetary unit of product *j* produced in region *s*.

Using these coefficients, equation (1. 27) can be written as:

$$e_i^r = a_{i1}^{rr} e_1^r + a_{i2}^{rr} e_2^r + \ldots + a_{in}^{rr} e_n^r + a_{i1}^{rs} e_1^s + a_{i2}^{rs} e_2^s + \ldots + a_{in}^{rs} e_n^s + f_i^r$$

(1. 29)

This equation may be expressed in matrix terms, given:

- $\mathbf{A^{rr}}$, as the intra-regional input coefficient matrix for region *r* (generic element: a_{ij}^{rr});

- $\mathbf{A}^{\mathbf{ss}}$, as the intra-regional input coefficient matrix for region *s* (generic element: a_{ij}^{ss});
- $\mathbf{A}^{\mathbf{rs}}$, as the interregional trade coefficient matrix with generic element a_{ij}^{rs};
- $\mathbf{A}^{\mathbf{sr}}$, as the interregional trade coefficient matrix with generic element a_{ij}^{sr};
- $\mathbf{f}^{\mathbf{r}}$ and $\mathbf{f}^{\mathbf{s}}$, as the final demand vectors for production of region *r* and *s*, respectively.
- $\mathbf{e}^{\mathbf{r}}$ and $\mathbf{e}^{\mathbf{s}}$, as the output vectors for region *r* and *s*, respectively.

Hence, the following system of equations yields for the two regions:

$$\begin{aligned}
&(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})\mathbf{e}^{\mathbf{r}} - \mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}} = \mathbf{f}^{\mathbf{r}} \\
&-\mathbf{A}^{\mathbf{sr}}\mathbf{e}^{\mathbf{r}} + (\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\mathbf{e}^{\mathbf{s}} = \mathbf{f}^{\mathbf{s}}
\end{aligned} \tag{1. 30}$$

If we define a matrix $\mathbf{A}^{\mathbf{IS}}$ as a partitioned matrix composed of four sub-matrices defined previously in this model[13]:

$$\mathbf{A}^{\mathbf{IS}} = \begin{bmatrix} \mathbf{A}^{\mathbf{rr}} & \mathbf{A}^{\mathbf{rs}} \\ \mathbf{A}^{\mathbf{sr}} & \mathbf{A}^{\mathbf{ss}} \end{bmatrix};$$

if, additionally, we aggregate the output and final demand vectors like:

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}^{\mathbf{r}} \\ \mathbf{e}^{\mathbf{s}} \end{bmatrix} \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^{\mathbf{r}} \\ \mathbf{f}^{\mathbf{s}} \end{bmatrix},$$

the matrix system for the two-region interregional model assumes the following expression:

$$(\mathbf{I}-\mathbf{A}^{\mathbf{IS}}) \cdot \mathbf{e} = \mathbf{f} \tag{1. 31}$$

Thus, the solution to this model is given by:

[13] superscript ^{IS} stands for Isard's model

$$\mathbf{e} = (\mathbf{I} - \mathbf{A}^{\mathbf{IS}})^{-1} \cdot \mathbf{f} \Leftrightarrow \mathbf{e} = \mathbf{B}^{\mathbf{IS}}\mathbf{f}\ .$$

(1. 32)

From this equation, one can perform economic impact analysis, making:

$$\Delta\mathbf{e} = (\mathbf{I} - \mathbf{A}^{\mathbf{IS}})^{-1} \cdot \Delta\mathbf{f} \Leftrightarrow \Delta\mathbf{e} = \mathbf{B}^{\mathbf{IS}}\Delta\mathbf{f}$$

(1. 33)

This final equation is similar to the one found in the single-region model with intra-regional flows (equation (1. 20)), but the similitude is misleading, since the degree of detail and complexity in this model is much higher. Now the economic impact is determined in terms of the different regions, but also in terms of the different industries, because the interregional trade flows comprised in the model not only specify the region of origin and the region of destination, but also the industry of origin and of destination (Isard, 1951). In other words, the model assumes that "(…) any given commodity produced in a region is distinct from the same good produced in any other region" (Toyomane, 1988, p. 16). Besides, the previously referred spillover and feedback effects are now accounted for: any change in the final demand of one region causes effects on the others and these return to the first region, through the interregional linkages specified in the model. The magnitude of the interregional feedback effects may be isolated. Following Miller (1966), and going back to equation (1. 30), the outputs of each region may be written in terms of the final demands $\mathbf{f}^{\mathbf{r}}$ and $\mathbf{f}^{\mathbf{s}}$, as follows:

$$\begin{cases}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})\mathbf{e}^{\mathbf{r}}-\mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}}=\mathbf{f}^{\mathbf{r}}\\ -\mathbf{A}^{\mathbf{sr}}\mathbf{e}^{\mathbf{r}}+(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\mathbf{e}^{\mathbf{s}}=\mathbf{f}^{\mathbf{s}}\end{cases}$$

$$\begin{cases}\mathbf{e}^{\mathbf{r}}=(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}}+(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}}\\ -\mathbf{A}^{\mathbf{sr}}\left[(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}}+(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}}\right]+(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\mathbf{e}^{\mathbf{s}}=\mathbf{f}^{\mathbf{s}}\end{cases}$$

$$\begin{cases}---\\ -\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}}-\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}}+(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\mathbf{e}^{\mathbf{s}}=\mathbf{f}^{\mathbf{s}}\end{cases}$$

$$\begin{cases}---\\ \left[-\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}+(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\right]\mathbf{e}^{\mathbf{s}}=\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}}+\mathbf{f}^{\mathbf{s}}\end{cases}$$

$$\begin{cases}\mathbf{e}^{\mathbf{r}}=\left[(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})-\mathbf{A}^{\mathbf{rs}}(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})^{-1}\mathbf{A}^{\mathbf{rs}}\right]^{-1}\mathbf{f}^{\mathbf{r}}+\\ \left[(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})-\mathbf{A}^{\mathbf{rs}}(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})^{-1}\mathbf{A}^{\mathbf{sr}}\right]^{-1}\mathbf{A}^{\mathbf{rs}}(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})^{-1}\mathbf{f}^{\mathbf{s}}\\ \\ \mathbf{e}^{\mathbf{s}}=\left[(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})-\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\right]^{-1}\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}}+\\ \left[(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})-\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\right]^{-1}\mathbf{f}^{\mathbf{s}}\end{cases}$$

(1. 34)

Let's analyze the economic significance of this equation, taking for example, the expression of region *s*'s output: this is determined by the total requirements needed to satisfy the final demand within the region and also by the total requirements needed to satisfy the final demand in region *r*. Let's look at these two components with greater detail, starting with the requirements to provide $\mathbf{f}^{\mathbf{s}}$. In an intra-regional single-region model with no interregional linkages, the total effect on region *s* would be given by the traditional inverse $\left(\mathbf{I}-\mathbf{A}^{\mathbf{ss}}\right)^{-1}$. However, this is not the case here. Therefore, we must consider also the intrerregional feedback effects. First, region *s* will require inputs from region *r*; this link is expressed by the interregional trade matrix $\mathbf{A}^{\mathbf{rs}}$. In turn, region *r* will answer this new demand through its total requirements matrix: $\left(\mathbf{I}-\mathbf{A}^{\mathbf{rr}}\right)^{-1}\mathbf{A}^{\mathbf{rs}}$. But we are seeking for the effects on region *s*'s output. The additional production in *r* will be reflected in *s*, through the demand for inputs expressed by $\mathbf{A}^{\mathbf{sr}}$. Thus, the interregional feedback effect felt in region *s* is given by $\mathbf{A}^{\mathbf{sr}}\left(\mathbf{I}-\mathbf{A}^{\mathbf{rr}}\right)\mathbf{A}^{\mathbf{rs}}$. The other component of $\mathbf{e}^{\mathrm{s}}$ implicit in equation (1. 34) is resultant from the requirements necessary to provide $\mathbf{f}^{\mathbf{r}}$. First, region *r* will suffer an intra-regional effect given by $\left(\mathbf{I}-\mathbf{A}^{\mathbf{rr}}\right)^{-1}\mathbf{f}^{\mathbf{r}}$. Because of this, region *r* will import some inputs from region *s*; this effect is felt on region *s* by the amount $\mathbf{A}^{\mathbf{sr}}\left(\mathbf{I}-\mathbf{A}^{\mathbf{rr}}\right)^{-1}\mathbf{f}^{\mathbf{r}}$. This can be seen as a new demand in *s*, which causes effects similar to those explained before, given by the direct and indirect requirements matrix: $\left[(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})-\mathbf{A}^{\mathbf{sr}}(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\right]^{-1}$.

From the previous exposition, the following question may arise: what is the magnitude of the interregional effects, or, in other words, what is the amount of error caused by neglecting these effects? To answer this question, let's suppose that it has occurred a change in final demand for regional products, either originated in region *r* or in region *s*: $\Delta\mathbf{f}^{\mathbf{r}}$. The effect of this in region *r* is given by: $\Delta\mathbf{e}^{\mathbf{r}}=\left[(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})-\mathbf{A}^{\mathbf{rs}}(\mathbf{I}-\mathbf{A}^{\mathbf{ss}})^{-1}\mathbf{A}^{\mathbf{sr}}\right]^{-1}\Delta\mathbf{f}^{\mathbf{r}}$. If no interregional feedback effects were taken into account, the correspondent effect would be: $\Delta\mathbf{e}^{\mathbf{r}}=\left(\mathbf{I}-\mathbf{A}^{\mathbf{rr}}\right)^{-1}\Delta\mathbf{f}^{\mathbf{r}}$. Then, the difference between these two effects reflects the amount due to interregional feedback effects. This is, still, an empirical issue. In some cases, the error may be small, as in the tests made in Miller (1966). In others, the error is quite significant, as it happened in the empirical application for the interregional model with eight-region and twenty-three industries, made by Greytrack (1970): this author concluded that, when feedback effects are taken into consideration, the obtained multipliers are about 14% larger than when these effects are neglected. Anyway, these two empirical results depend heavily on the data, in particular, on the degree of auto-sufficiency and on the dimension of the regions under study.

Of course, the degree of complexity involved in interregional input-output model has a reflection on the demand of data to implement it: it's extremely data demanding, especially in what concerns to interregional trade flows. In fact, if it is difficult to gather data on trade flows from one region to the others, it is even more difficult to collect these data specifying the industry of origin and the industry of destination of those flows.

It should be also noted that, in the example used to expose this model, only two regions were considered. In this case, the compact matrix $\mathbf{A^{IS}} = \begin{bmatrix} \mathbf{A^{rr}} & \mathbf{A^{rs}} \\ \mathbf{A^{sr}} & \mathbf{A^{ss}} \end{bmatrix}$ is composed of 4 matrices, each with dimension $n \times n$ (being n the number of industries and of products). If three regions were considered, then matrix $\mathbf{A^{IS}}$ would be composed of 9 $n \times n$ matrices. Generalizing, if k regions are considered, matrix $\mathbf{A^{IS}}$ is a composition of k^2 $n \times n$ matrices. Then, it is clear that the amount of data required to implement such a model increases quickly with the number of regions being studied (Miller and Blair, 1985).

Finally, it should be emphasized that the use of equation (1. 33) implies the supposition of constant elements in matrix $\mathbf{A}$ (Isard,1951). But now these elements comprise two kinds of coefficients: intra-regional input coefficients and trade coefficients (Oosterhaven, 1984). The stability supposition is, therefore, extended to the trade coefficients, which is a very restrictive assumption. The implications of the interregional trade stability supposition will be focused with more detail in section 1.3.6.

1.3.4 Chenery-Moses's MRIO (multiregional input-output model).

Given the difficulty in gathering the data required to implement the Isard's model, it has seldom been applied. With the aim of overcoming this drawback, Chenery (1953) and Moses (1955) developed the first version of a multi-regional input-output model, which used the following simplification: interregional trade flows are only specified by region of origin and region of destination, being ignored the specific industry (or final consumer) of destination.

The data requirements to this model imply that the researcher has previous access to four sets of data. The first consists of an Origin-Destination (O-D) matrix for each and every product, depicting intra and interregional shipments of the outputs of that product. Such matrix can be illustrated by:

Figure 1. 3 – Intra and interregional shipments of product *i*.

		DESTINATION	
		r	***s***
ORIGIN	***r***	x_i^{rr}	x_i^{rs}
	s	x_i^{sr}	x_i^{ss}
	SUM	R_i^r	R_i^s

In this matrix, x_i^{rr}, for example, represents the amount of product *i* produced and consumed in region *r* and x_i^{sr} represents the amount of product *i* shipped by region *s* to region *r*, without specifying the type of buyer in the region of destination (it may be used by any industry or even to final users). The column total of this matrix will be denoted by: R_i^r, for the first column, representing the total amount of product *i* available in region *r*, except for foreign imports; R_i^s, for the second column, representing the total amount of product *i* available in region *s*, except for foreign imports.

The second set of information consists of an interindustry flow matrix for each region. For example, for region s, we will need a matrix $Z^{\bullet s}$, in which each element $z_{ij}^{\bullet s}$ describes "the value of purchases by each industry in a region from each industry in the nation as a whole during some base period" (Moses, 1955, p. 805). In other words, all geographic origins of input *i* are being considered, except for foreign countries.

Finally, it is necessary to know, in advance, the vectors of regional final demand for each region (in which, for example, y_i^r denotes final demand for product *i* in region *r*, including all regional sources of *i*) and also the vectors of regional production in each region (e_i^r stand for regional production of product *i* in region *r*).

The starting equations to the development of this model are similar to those presented in the previous section. The balance equations state “that the output of each industry in each region is equal to its sales to all industries and final demand sectors in all regions” (Moses, 1955, p. 804). Considering, as before, a system of two regions (r and s) with n industries and n products each, we may write:

$$
\begin{aligned}
e_i^r &= \left(z_{i1}^{rr} + z_{i2}^{rr} + \ldots + z_{in}^{rr} + f_i^{rr}\right) + \left(z_{i1}^{rs} + z_{i2}^{rs} + \ldots + z_{in}^{rs} + f_i^{rs}\right) \\
e_i^s &= \left(z_{i1}^{ss} + z_{i2}^{ss} + \ldots + z_{in}^{ss} + f_i^{ss}\right) + \left(z_{i1}^{sr} + z_{i2}^{sr} + \ldots + z_{in}^{sr} + f_i^{sr}\right),
\end{aligned}
\tag{1. 35}
$$

for all $i = 1, \cdots, n.$[14]

Given that intra-regional flows such as z_{ij}^{rr} or f_i^{rr} are not easy to obtain through direct observation, some hypotheses are considered in order to use more accessible data. The first fundamental hypothesis consists of the introduction of the so-called trade coefficients. Using the information of the O-D matrix depicted before, and dividing each element of the first column by its column total, we obtain the proportions of the product available in region r that is provided by the region itself and by the other region. Analytically, we make: $t_i^{rr} = \frac{x_i^{rr}}{R_i^r}$ and $t_i^{sr} = \frac{x_i^{sr}}{R_i^r}$. Because these coefficients are computed dividing each element of the O-D matrix by the column total, the MRIO model is sometimes called a column-coefficient model (Polenske, 1995)[15].

[14] As it happens in the original presentation of the model, it is assumed that the two-region system is closed, thus having no interaction with the rest of the world, either through international imports or exports. This is clear in Moses (1955), in which the author states: “Let us assume a closed economy divided into r regions which are open to one another for trade in n homogeneous commodities” (p. 827). For this reason, international exports are not included in equation (1. 35).

[15] The column-coefficient multi-regional model, developed by Chenery and Moses, is the most popular MRIO and the pioneering one; yet, other MRIO models, that computed trade coefficients in a different manner, were proposed thereafter: namely the row-coefficient model (in which trade coefficients are obtained dividing each element of the O-D matrix by the row total) and the gravity-trade model, also called Leontief-Strout Gravity model (in which trade coefficients are computed on the basis of a gravitational formula to calculate trade flows, being these a function of total regional outflows, total regional inflows and the cost of transferring the goods from one region to another). Details on these alternative multi-regional models can be found in Polenske (1970a), Polenske (1970b), Polenske (1995) and Toyomane (1988). Empirical tests applied to these models, with the aim of assessing their capacity in estimating regional

The essential hypotheses underlying MRIO is the assumption of the same trade coefficient to all the different uses in the destination region. In other words, this means that if, for example, $t_i^{rs} = 0{,}4$, meaning that 40% of all the product *i* available in region *s* comes from region *r*, the assumption implies that for all intermediate and final uses of product *i* in region *s*, 40% comes from region *r* and only 60% is provided by region *s* itself[16] (Toyomane, 1988). This is often called the import proportionality assumption (Riefler and Tiebout, 1970). Moses (1955) recognizes that it is an imperfect assumption; yet, it is adopted, given the fact "that it is impossible to implement statistically a model which applies separate trading patterns to each industry" (Moses, 1955, p.810).

Introducing these trade coefficients into equations (1. 35), and making use of the known information on the matrices $Z^{\bullet s}$ and $Z^{\bullet r}$ and on the vectors of regional final demand, we get:

$$
\begin{aligned}
e_i^r &= \left(t_i^{rr} z_{i1}^{\bullet r} + t_i^{rr} z_{i2}^{\bullet r} + \ldots + t_i^{rr} z_{in}^{\bullet r} + t_i^{rr} y_i^r\right) + \left(t_i^{rs} z_{i1}^{\bullet s} + t_i^{rs} z_{i2}^{\bullet s} + \ldots + t_i^{rs} z_{in}^{\bullet s} + t_i^{rs} y_i^s\right) \\
e_i^s &= \left(t_i^{ss} z_{i1}^{\bullet s} + t_i^{ss} z_{i2}^{\bullet s} + \ldots + t_i^{ss} z_{in}^{\bullet s} + t_i^{ss} y_i^s\right) + \left(t_i^{sr} z_{i1}^{\bullet r} + t_i^{sr} z_{i2}^{\bullet r} + \ldots + t_i^{sr} z_{in}^{\bullet r} + t_i^{sr} y_i^r\right)
\end{aligned}
\tag{1. 36}
$$

for all $i = 1, \cdots, n$.

The other type of coefficient used in this model consists of technical coefficients. In this particular case, the inter-industry flows (described in matrices $Z^{\bullet s}$ and $Z^{\bullet r}$), record the total inputs used by each industry in each region, regardless of the regional provenience of those inputs (assuming a closed economy, thus with no imports from abroad)[17].

---

outputs and interregional trade flows, revealed that, concerning the row coefficient, the results are not satisfactory (Polenske, 1970b); in the case of the gravity-trade model, this was only applicable when access to previous interregional trade flows existed; otherwise, the model had to be solved iteratively, and it didn't converge (Polenske, 1995).

[16] Once again, we stress the fact that, in this model, foreign imports are excluded from this assumption.

[17] Instead of considering that the known information on inter-industry flows concerns to flows coming from the whole nation, as it is done in Miller (1985), Miller (1998) and Moses (1955) we might consider that the researcher has previous access to a technological matrix, i.e., a matrix with total flows (inputs coming from the whole nation and also from abroad). In this case, the trade coefficients would have to be computed in a

Let $z_{ij}^{\bullet s}$ represent the total amount of product *i* used as an input by industry *j* in region *s*. Symbol $\bullet$ is used to represent the summation of all the geographical origins of input *i*, except for foreign countries. Then, $a_{ij}^{\bullet s} = \dfrac{z_{ij}^{\bullet s}}{e_j^s}$ is the technical coefficient for region *s* and it represents the amount of product *i* necessary to produce one unit of industry *j*'s output in region *s*, considering the inputs provided by all the regions in the system (Moses, 1955). Using these coefficients, the system of equations (1. 36) becomes:

$$
\begin{aligned}
e_i^r &= \left(t_i^{rr} a_{i1}^{\bullet r} e_1^r + t_i^{rr} a_{i2}^{\bullet r} e_2^r + \ldots + t_i^{rr} a_{in}^{\bullet r} e_n^r + t_i^{rr} y_i^r\right) + \left(t_i^{rs} a_{i1}^{\bullet s} e_1^s + t_i^{rs} a_{i2}^{\bullet s} e_2^s + \ldots + t_i^{rs} a_{in}^{\bullet s} e_n^s + t_i^{rs} y_i^s\right) \\
e_i^s &= \left(t_i^{ss} a_{i1}^{s} e_1^s + t_i^{ss} a_{i2}^{\bullet s} e_2^s + \ldots + t_i^{ss} a_{in}^{\bullet s} e_n^s + t_i^{ss} y_i^s\right) + \left(t_i^{sr} a_{i1}^{\bullet r} e_1^r + t_i^{sr} a_{i2}^{\bullet r} e_2^r + \ldots + t_i^{sr} a_{in}^{\bullet r} e_n^r + t_i^{sr} y_i^r\right)
\end{aligned}
$$

for all $i = 1, \cdots, n.$

(1. 37)

In this equation, technology and trade are treated as separate factors, thus representing an advantage of this model over the IRIO model (Toyomane, 1988). In fact, given that the factors that influence technology and trade are most likely different, is seems more adequate to treat the two components separately.

Generalizing for all regions and products, we may write the structural form of the model in matrix terms. To do so, let's consider:

- the input matrices: $\mathbf{A^{\bullet r}}$, of generic element $a_{ij}^{\bullet r}$; $\mathbf{A^{\bullet s}}$, of generic element $a_{ij}^{\bullet s}$;

---

consistent manner, dividing each element of the O-D matrix, not by its column total, but by the total output available at the correspondent region (which includes foreign imports).

- the trade matrices $\hat{\mathbf{T}}$, for each pair of origin and destination, with all the products being traded represented in the main diagonal; for example: $\hat{\mathbf{T}}^{\mathbf{rs}} = \begin{bmatrix} t_1^{rs} & 0 & 0 & 0 \\ 0 & t_2^{rs} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & t_n^{rs} \end{bmatrix}$;

- the vectors of final demand in region r ($\mathbf{y}^{\mathbf{r}}$) and in region s ($\mathbf{y}^{\mathbf{s}}$). Since, in this model, the trade flows don't specify the type of user at the destination, this implies that final demand of each region may as well be partially supplied by imports from the other region.

Joining these data together, the structural form of this two-region model, in the demand perspective, is given by:

$$\begin{aligned} \mathbf{e}^{\mathbf{r}} &= \mathbf{T}^{\mathbf{rr}}\mathbf{A}^{\bullet\mathbf{r}}\mathbf{e}^{\mathbf{r}} + \mathbf{T}^{\mathbf{rs}}\mathbf{A}^{\bullet\mathbf{s}}\mathbf{e}^{\mathbf{s}} + \mathbf{T}^{\mathbf{rr}}\mathbf{y}^{\mathbf{r}} + \mathbf{T}^{\mathbf{rs}}\mathbf{y}^{\mathbf{s}} \\ \mathbf{e}^{\mathbf{s}} &= \mathbf{T}^{\mathbf{sr}}\mathbf{A}^{\bullet\mathbf{r}}\mathbf{e}^{\mathbf{r}} + \mathbf{T}^{\mathbf{ss}}\mathbf{A}^{\bullet\mathbf{s}}\mathbf{e}^{\mathbf{s}} + \mathbf{T}^{\mathbf{sr}}\mathbf{y}^{\mathbf{r}} + \mathbf{T}^{\mathbf{ss}}\mathbf{y}^{\mathbf{s}} \end{aligned}$$

(1. 38)

If we take the following partitioned matrices[18] and vectors: $\mathbf{A}^{\mathbf{CM}} = \begin{bmatrix} \mathbf{A}^{\bullet\mathbf{r}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{\bullet\mathbf{s}} \end{bmatrix}$; $\mathbf{T} = \begin{bmatrix} \mathbf{T}^{\mathbf{rr}} & \mathbf{T}^{\mathbf{rs}} \\ \mathbf{T}^{\mathbf{sr}} & \mathbf{T}^{\mathbf{ss}} \end{bmatrix}$; $\mathbf{e} = \begin{bmatrix} \mathbf{e}^{\mathbf{r}} \\ \mathbf{e}^{\mathbf{r}} \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} \mathbf{y}^{\mathbf{r}} \\ \mathbf{y}^{\mathbf{s}} \end{bmatrix}$, the final equation, in a compact form is:

$$\mathbf{e} = \mathbf{T}\mathbf{A}^{\mathbf{CM}}\mathbf{e} + \mathbf{T}\mathbf{y}$$

(1. 39)

[18] superscript $^{\text{CM}}$ stands for Chenery-Moses's model.

This equation can be used to perform economic impact analysis, similarly to equation (1. 33), in IRIO. Given any change in $\mathbf{y}$, caused either by changes in one or more product final demands of region *r* or region *s*, the new output vector is

$$\mathbf{e} = \mathbf{TA}^{\mathbf{CM}}\mathbf{e} + \mathbf{Ty} \Leftrightarrow (\mathbf{I} - \mathbf{TA}^{\mathbf{CM}})\mathbf{e} = \mathbf{Ty} \Leftrightarrow \Delta\mathbf{e} = (\mathbf{I} - \mathbf{TA}^{\mathbf{CM}})^{-1}\mathbf{T}\Delta y \tag{1. 40}$$

It should be noted that the inverse matrix relating the new final demand with the new output vector is now $(\mathbf{I} - \mathbf{TA}^{\mathbf{CM}})^{-1}\mathbf{T}$. This implies that the multiplier effect, quantified by the column sum of this matrix, can be understood as a two-stage effect (Miller, 1998): first, matrix $\mathbf{T}$, with the trade coefficients, operates the distribution of the new final demands in each region by the suppliers in each region; then, multiplying this by $(\mathbf{I} - \mathbf{TA}^{\mathbf{CM}})^{-1}$, it gives us the total impacts (direct and indirect) in the regional industries.

1.3.5 Riefler-Tiebout's bi-regional input-output model.

Riefler and Tiebout (1970) also made an important contribution to regional input-output models, proposing a specific formulation of an interregional input-output model, suited for the particular situation in which the system is composed by two-regions (plus the rest of the world) and the researcher has previous access to an imports matrix and an exports matrix for one of those regions. It is clear by now that this model requires more survey-based data than the Chenery-Moses multi-regional model. In Riefler and Tiebout (1970), these authors used the example of a bi-regional system composed of California and Washington, since there was an exports matrix and an imports matrix for Washington. These two matrices depicted, for each input and for each consuming industry and final demand sector, the percentage of the input that came from / went to abroad (including here the other region and foreign suppliers / receivers). In this case, the fundamental problem would be confined to partitioning this matrix into imports from (exports to) California and imports from (exports to) the rest of the world (Harrigan, *et al*. 1981).

The structural equations of the model are similar to those presented in Isard's IRIO model. Considering again two regions, r and s, and using the previously defined matrices of coefficients, the structural equations could be written as in equation (1. 30):

$$
\begin{aligned}
&(\mathbf{I}-\mathbf{A}^{\mathbf{rr}})\mathbf{e}^{\mathbf{r}} - \mathbf{A}^{\mathbf{rs}}\mathbf{e}^{\mathbf{s}} = \mathbf{f}^{\mathbf{r}} \\
&-\mathbf{A}^{\mathbf{sr}}\mathbf{e}^{\mathbf{r}} + (\mathbf{I}-\mathbf{A}^{\mathbf{ss}})\mathbf{e}^{\mathbf{s}} = \mathbf{f}^{\mathbf{s}}
\end{aligned}
\qquad \textbf{(1. 41)}
$$

The contribution of Riefler and Tiebout's model concerns to the way in that they specify the interregional trade coefficients a_{ij}^{rs} and a_{ij}^{sr}. Making use of the existent data, they assume a compromise between the ideal interregional trade coefficients proposed by Isard and the simplified trade coefficients used by Chenery and Moses. In the Isard's IRIO model, a_{ij}^{rs} was obtained dividing the observed flow z_{ij}^{rs} by the observed value e_j^s. However, as referred before, this demands a degree of detail in the data that is seldom available. In the Chenery-Moses model, a_{ij}^{rs} was surrogated by the multiplication of the trade coefficient by the technical coefficient for region M: $a_{ij}^{rs} = t_i^{rs} \cdot a_{ij}^{\bullet s}$; in this case, $t_i^{rs} = \dfrac{x_i^{rs}}{R_i^s}$, which implies the use of the imports proportionality assumption. Using the imports matrix for Washington, Riefler and Tiebout demonstrate that this assumption is far from being verified in reality; in fact, different industries present a different input import's propensity. One of the main reasons behind this is the fact that inputs and outputs are classified under non-homogeneous groups of products. Thus, they propose a procedure in which this assumption is avoided.

From the import matrix, available for Washington (region r), they were able to compute the percentage of the total imports of input i (coming from California - region s - and from abroad) which was used in the intermediate consumption of industry j. In order to obtain the interregional trade coefficient a_{ij}^{sr}, the only hypotheses to assume was that a constant share of those imports came from region s – this share was computed for each input i making use of a set of trade related statistics, including the US Census of

Transportation[19]. Similarly, from the export matrix, they could calculate the percentage of the total exports of input *i* which was destined to the consumption of industry *j* (both in California and in foreign countries). Given this, a_{ij}^{rs} could then be computed assuming that a constant share of these exports was destined to region *s* (California). This export share was estimated using the same set of statistical sources as for the import share. These interregional trade coefficients were then introduced in equation (1. 41), allowing the deduction of the inverse matrix expression, as presented in IRIO[20]:

$$\begin{aligned}&(\mathbf{I}-\mathbf{A}^{\mathbf{RT}})\cdot\mathbf{e}=\mathbf{f}\\&\mathbf{e}=(\mathbf{I}-\mathbf{A}^{\mathbf{RT}})^{-1}\cdot\mathbf{f}\Leftrightarrow\mathbf{e}=\mathbf{B}^{\mathbf{RT}}\mathbf{f}\end{aligned}\tag{1. 42}$$

As it became clear from the previous exposition, the practical utility of this model is limited to situations in which exist both an import matrix and an export matrix to one of the regions under study. Besides, the model implies that one region sponges the information existent for the other region. Thus, even when such information is available, the first concern of the researcher should be to question the accuracy of the pre-existent matrices. The difficulties in conducting surveys to assess the proportion of imported products used as intermediate consumption have already been mentioned in the exposition of the single-region model. Mainly, they have to do with the fact that, usually, the sources of information – respondent enterprises – cannot distinguish their inputs into imported and regional ones. Yet, is should be recognized that it is easier to obtain an answer to the question “Are the inputs imported or regional?” than to the question “Are the inputs imported or regional and where do the imported inputs come from?”. The latter question is implicit in the trade data required by the Isard model, whereas the former is implicit in the trade data demanded by the Riefler-Tiebout model. Hence, this model seeks for an estimative of the Isard’s trade coefficients, using less demanding survey information and complementing it by the use of alternative sources as the Census of Transportation. In what concerns to the export matrix, the problems in gathering such

[19] See Riefler and Tiebout (1970) for further details.
[20] In equation (1.42), RT stands for Riefler and Tiebout.

information are similar, or even more serious. The fact is that firms know the proportion of output they export, but they usually cannot distinguish the specific destination user of their exports, in terms of industries or final users. As a conclusion, while theoretically interesting, this model has a considerable practical disadvantage over the Chenery-Moses model.

1.3.6 Trade coefficient stability.

As we have seen, the previously described interregional and multiregional input-output models may be used as predictive models, aiming to quantify the economic impacts over the different sectors of the different regions, caused by changes in regional final demands. In this kind of applications, besides the usual assumption of constant technical coefficients, an additional supposition is implicit: trade coefficients are stable[21] (Riefler and Tiebout, 1970). This means that, when a shift in final demand occurs, the trading patterns remain unaltered. This is, in fact, a much stronger assumption than the classical assumption of constant technical coefficients (Batten and Boyce, 1986). Moses (1955) performs an ample scrutiny of this supposition, analyzing the economic forces behind the trading patterns and the conditions that must hold for their stability. Trade flows are influenced essentially by "cost-price relationships and regional capacities for production and distribution" (Moses, 1955, p. 810). Being so, trade coefficients are stable if the following conditions hold: regional costs of production are constant, unitary costs of transportation are fixed and the capacity of production can be easily increased. As to the first two conditions, these are very restrictive, but they are generally adopted in input-output models, thus being a general limitation of these models and not a specific problem of MRIO models. As to the third condition, its reasonability depends essentially on the elasticity of production factors, in particular, labour. From this point of view, it would be preferable to apply this model to long-run periods, since it would facilitate the adjustment of production capacities. However, this would enhance the probability of regional technological changes, affecting not only the stability of trade coefficients, due to

[21] In Isard's model, trade coefficients are not explicit in the model. However, assuming intra-regional and interregional input coefficients as constant corresponds to the assumption that, not only the input structure is constant, but also the percentage that comes from the region itself and from other regions.

changes in relative costs of production, but also the typically assumed stability of technical coefficients. Moses (1955) concludes arguing that the MRIO model is best suited to short-run impact analysis, given that the factors of production are below full employment situation.

1.4. Obtaining the data for regional input-output models: table construction.

As any model requires its own database, the regional input-output models implementation also implies the previous existence of the correspondent input-output tables. Yet, whereas at the national level the input-output tables are regularly provided by the official statistics, according to standardized rules, the same does not apply to the regional dimension. For that reason, the construction of regional input-output tables has been, by itself, one of the most debated themes in regional literature. The researchers seek for a compromise on the adoption of common rules which allow for the comparability of regional economic structures in space and time (Hewings and Jensen, 1986).

According to Jensen (1990), it is possible to distinguish four stages in what is concerned to the history of regional input-output table construction: (1) in the first stage, national coefficients were used directly, without any adjustments, in the regional input-output table; (2) in the second stage, those national coefficients were adjusted in order to reflect some specific regional characteristics; (3) the third stage, named the "classical era of regional input-output" (Jensen, 1990, p.11) was dominated by the supporters of survey-based tables, which were elaborated by research teams and implied a vast field work; in this era, "the achievement of the highest quality table was regarded in itself as an end" (p. 11); (4) the end of the classical era was determined by the high requirements in terms of human labour, logistics, money, etc, demanded by survey tables. Today we are at the fourth stage, in which hybrid tables, that combine direct information with values obtained by non-survey techniques, are seen as the most adequate alternative. Survey, non-survey and hybrid techniques will be discussed with further detail in the subsequent sections.

1.4.1 Survey table

As the name suggests, survey tables are assembled on the basis of the direct surveys made to firms, consumers and government institutions, and also on the basis of experts' judgments about each sector. These are commonly seen as the most accurate tables, since they attempt to reflect all the specific characteristics of the regional economy. However, besides the already mentioned problem of being very time and cost demanding, survey techniques involve other pitfalls. Several types of errors can emerge immediately in the process of gathering the data: for example, errors arising from incorrect definition of the sample, poor design of questionnaires, hiding of information or lack of concern in answering the questionnaires (Jensen, 1980). Even with exactly the same set of data, different research teams can achieve different input-output tables, because compilation procedures are not unique (for example, there are different methods of making the reconciliation between sales and purchases data)[22]. Moreover, when non coincident data are provided by different sources – by statistical methods based on surveys on the one hand, and by experts' judgments, on the other hand – the difficulty is to decide which of these sources is more reliable (Jensen, 1990). Besides these problems, survey methods involve also other difficulty: as referred before, sometimes the questions included in the questionnaires require very detailed information to which some respondents may not be able to answer (for example, if the inputs used are imported or not and from where they are imported). Being so, even official organisms of statistics, when they compute regional input-output tables, are often forced to use some hypotheses in order to complement the information they cannot obtain directly from surveys, thus using a hybrid method. We will get back to this issue in Chapter 3.

[22] Reconciliation problems occur whenever there is no coincidence with the values obtained by the seller perspective (values at the rows) and the ones assembled through a user perspective (at the columns) (Lahr, 1993).

1.4.2 Non-survey and hybrid techniques.

Non-survey techniques applied to regional input-output tables can be generally defined as a set of procedures that aim to fill the components of a regional table on the basis of values comprised in a similarly structured national table (Jensen, 1990). These techniques are also called top-down methods, since they use the values of the whole nation as a starting point and then apply specific regional indicators to regionalize them. The indicators used depend on the available data at the regional level, but usually they embrace employment and income data.

Obviously, non-survey techniques include a vast set of methods, which is not homogeneous. Yet, regardless of the specific method to be used, the accuracy of non-survey regional input-output tables is highly determined by the following issues (Lahr, 1993): industrial mix, technology and external trade. Each of these elements assumes a different influence in the table's accuracy. For example Park, *et al.* (1981) have made some tests in order to investigate the effect on the input-output table created by errors in technology and in trade estimation. They conclude that errors in estimating the regional inputs coefficients are more determined by errors in regional trade estimation than by errors in technical coefficients. Let's explain each of these elements with greater detail.

Differences between the regional industrial mix and the national one may create errors in the tables derived by non-survey methods, since national structures are applied to regional industries in which the proportion of each sub-industry is different from the national one (some sub-industries that exist at the national level may even be inexistent at the some specific region). However, this is a problem that may be partially solved if the researcher uses a high degree of disaggregation when regionalizing the national table (Park, *et al.*, 1981; Goldman, 1969). Today this doesn't constitute an operational problem, given the high computational capacities to deal with highly detailed tables.

Technology and external trade issues have been treated by the literature in a very confusing manner. As an example, Czamanski and Malizia (1969) state that one of the most important sources of error caused by the use of the national coefficients is the "relative importance and structure of foreign trade" (p. 65). This makes clear that technology and trade are mixed topics. We recognize that, in regional input-output models, these two issues are always interconnected; yet, we'll try to treat them as two separate items in what concerns to input-output table assemblage.

In non-survey regional input-output tables, the technical coefficients matrix is sometimes set equal to its national counterpart. This is called the "national technology assumption". It is convenient to recall what these technical coefficients mean: they express the amount of input *i* per unit of output *j*, regardless of the geographic origin of input *i*. This means that the national interindustry transactions matrix which is used as a starting point has to be a total flow matrix[23], thus including both nationally produced and imported inputs. So, the implicit hypothesis is that technology, in the production function sense, is spatially invariant within the same country (Lahr, 1993). Given that a high disaggregation is in fact used in the industry classification, this hypothesis is not very restrictive (Madsen and Jensen-Butler, 1999). It is rather acceptable to assume that some specific industry (taken at a very refined level of disaggregation) uses the same productive recipe in region A or in any other region of the same country. Moreover, some empirical exercises have concluded that this assumption doesn't cause major errors in the table. As an example, Boomsma and Oosterhaven (1992) compared the regional technical coefficients obtained through this national technology assumption with regional technical coefficients obtained via direct information and concluded that "the national technology assumption produces a close approximation, even for subsectors that are specific for the region at hand" (p. 278). This is an argument in favour of non-survey techniques, in what technology is concerned. Of course, this is an empirical issue, which depends greatly on the specific national and regional economies under study. Harrigan *et al.* (1980), for example, performed an empirical comparison between a survey-based regional technological matrix (existing for Scotland) and the correspondent non-survey matrix, computed using the technology of

[23] In the Portuguese National Accounts, the national matrix is, in fact, a total flow matrix. But this is not the rule; in some countries, as in the USA, the published national matrices exclude foreign imports. This implies a previous adjustment to the national matrix before regionalizing it. This will be further discussed in sections 5.2 and 5.3.

the United Kingdom. Contrarily to the case mentioned before, the results of this study allow to conclude for large differences between these two matrices; thus, in this case, the use of the national technical coefficients leads to substantial biases in regional impact analysis. Still, the national technology assumption continues to be a crucial hypothesis to assume in regional table construction, since the alternative survey regional tables seldom exist.

Using the national technology assumption, the regionalization of the interindustry transactions table is usually made using the industry's total intermediate consumption in each region as the key regionalizing indicator. This means that only the purchases are regionalized. In other words, each column of the national interindustry transactions table is divided in as much columns as the number of regions. This is called a columns-only or purchases-only regionalization (Oosterhaven, 1984). Assuming two regions, *r* and *s*, the resulting regional columns depict how much of the intermediate consumption of industry *j* is used in region *r* and in region *s* (being both regional columns decomposed in several inputs). The spatial origin of the inputs is not specified. An alternative way of regionalizing the national interindustry transactions table, presented by Oosterhaven (1984), would be rows-only: each national row, that describes the intermediate sales of inputs *i*, would be divided by regions. In this case, the resulting regional rows would specify how much of the intermediate sales of industry *i* would be made by region *r* and by region *s* (being both decomposed by the several consuming industries). The spatial destination of these sales would not be specified. This alternative is less used than the first, mainly because the available regional data that serve as an indicator to regionalize are frequently of the "purchase" kind: usually the researcher has access to regional values of intermediate consumption by industry, but not to regional values of intermediate sales by industry[24].

[24] In Oosterhaven (1984) another alternative is presented, that combines the information of the columns-only with the rows-only method of regionalization. The regional table obtained in this way is called a Full Information table. Besides being more data demanding, this method implies the reconciliation between the two types of information.

Last, but not least, the accuracy of non-survey tables is determined by the methods used to estimate region's external trade. We have to distinguish the two types of external trade concerning a regional economy: (1) imports and exports between it and other countries and (2) imports and exports between it and other regions (Isserman, 1980). The first kind of external trade is not really a problem, because this is usually available from official statistical sources. The difficulty is in estimating interregional trade flows. This is an old and remaining problem, as suggested by Czamanski and Malizia (1969): "Foreign trade is an especially sensitive issue at the regional level because of the notorious lack of reliable data on interregional flows" (p. 65). Despite the non-survey techniques chosen to estimate interregional trade, this problem comprises two distinct questions:

(1) How to estimate the interregional trade flows necessary to fulfill an Origin-Destination matrix to each of the products being considered? In other words, only the commodity shipments from and to each region are computed. This is an unavoidable concern to anyone who pretends to assemble a multi-regional input-output table. In fact, we have seen in section 1.3 that, among the three many-region models (Isard, Chenery-Moses and Riefler-Tiebout models), the Chenery-Moses model was the one that implied the minimum amount and detail of data, concerning interregional trade. Such data consisted precisely of an Origin-Destination matrix to each of the products being considered, depicting the shipments from the region of origin to the region of destination. Even if the investigation falls upon a single-region table, it is still necessary to estimate exports from the region to the rest of the country, as part of the demand directed to the region, and imports coming from other regions, given that these are part of the supply available at the region[25].

(2) How to estimate the interregional (and international) imports comprised in the regional technological matrix, in order to achieve intra-regional input coefficients? In other words, this means: how to determine the proportion of the inputs used that comes from the region itself? This may be seen as an optional

[25] As it will be seen latter, in single-region tables, usually net interregional flows are estimated, instead of gross exports to the rest of the country and gross imports from the rest of the country.

task in the table construction stage. Given that non-survey techniques are used, the same hypotheses that can be used to estimate intra-regional input coefficients in the table construction stage may, as an alternative, be applied in the model phase, starting from a total use technological matrix. This will be demonstrated in the third Chapter of this work.

Non-survey techniques used in interregional trade estimation are one of the core subjects in this work. Being so, they will not be exposed in this section. Instead, they will be treated with further detail in the following sections: question (1) will be addressed in section 1.6 of this Chapter and in Chapter 2; question (2) will be treated in section 1.5.3 and also in Chapter 3.

Between survey and non-survey methods there is a wide range of techniques that combine direct information with estimated values: these are generically labeled as hybrid techniques (Lahr, 1998). In reality, it is very difficult to find tables which are exclusively survey or non-survey. This is because purely survey tables are too expensive to construct and purely non-survey tables are too inaccurate for conducting input-output analysis (Dewhurst, 1990). According to Round (1983), "The terms non-survey and survey suggest the existence of two well defined and mutually exclusive groups, but in practice virtually all input-output tables are hybrid tables constructed by semi-survey techniques, employing primary and secondary sources to a greater or lesser extent" (p. 190). Anyway, it is consensual that the more direct information is incorporated in the table, the more accurate it tends to be. Of course, direct information implies higher costs, which leads to a cost-benefit analysis; according to West (1990), the equilibrium occurs when the marginal benefit of substituting estimated for direct information in the table equals the marginal cost of obtaining this direct information. Being so, the introduction of direct information must be selective. For example, Lahr (1993) considers that this direct information should always be obtained, at least, in sectors in which the region is highly specialized or in sectors in which technological differentiation is more probable, namely, resource-based industries and residual categories, because these are more likely to have a regionally differentiated industrial mix[26] (example: "Manufacture of other non-metallic mineral products").

1.4.3 Matrix adjustment methods: the particular case of RAS.

Belonging to the vast family of hybrid methods are matrix adjustment methods. Matrix adjustment methods are applied whenever the researcher wants to find the values to fill in a specific matrix on the basis of another matrix, which can be considered as a good indicator to the first one, and with regard to specific prior restrictions (Harrigan, 1990). For example, these methods can be applied to estimate a regional technical coefficients matrix, which will be the target matrix, on the basis of: (1) the national technical coefficients matrix or a regional technical coefficients matrix for a comparable region and (2) some partial information about the target matrix, usually concerning its column and row totals. In this case, the adjustment is made across space. But we can similarly apply a matrix adjustment method to update any technical coefficient matrix (regional or national) existing for an earlier year to a more recent year – here, the adjustment is made across time (Miller and Blair, 1985). Either the adjustment is made across space or across time, the general principle behind matrix adjustment methods "consists into finding what is the matrix which is closest to an initial matrix but with respect of the column and row sum totals of the second matrix" (de Mesnard, 2003, p. 1).

Matrix adjustment methods are intensively used in several types of applications, ranging from the assemblage of National Accounts to many other fields in which the missing data can be presented in a matrix form: international and interregional trade, migration flows, transportation flows, and so on (Lahr and de Mesnard, 2004; Jackson and Murray, 2004). Additionally, matrix adjustment methods comprise a vast set of algorithms. An exhaustive review of these algorithms is beyond the objectives of the present work[27]. We

[26] Yet, it should be noted that, when the model is based on a fine product disaggregation, these residual categories tend to disappear or to become insignificant, such that their influence to the global accuracy of the model is almost null.
[27] Recently, matrix adjustment methods have gained a renewed interest. Thus, there are some recent papers that make a quite complete review and discussion of the alternative matrix adjustment algorithms. Some

will focus only on the most popular matrix adjustment method: RAS. Among other attributes, which will be mentioned below, RAS presents two main practical advantages over competitive algorithms: it is a very simple technique and it requires a minimum amount of data (Lahr and de Mesnard, 2004; Mohr, Crown and Polenske, 1987). Besides, a number of the empirical studies (for example: de Mesnard (2003), Oosterhaven, Piek and Stelder (1986) and Jackson and Murray (2004)), that intend to assess the relative performance of alternative matrix adjustment methods conclude that, most of the times, RAS produces the best results, measured by the proximity between the estimated matrix and a known target matrix[28].

Let's review the application of RAS, through the following example of adjustment across space: the researcher wants to find a matrix of regional technical coefficients for region 1, $\mathbf{A}^1$, through the adjustment of a previously known matrix of regional technical coefficients $\mathbf{A}^0$ existing for the same year, for a comparable region[29]: region 0. To do so, the researcher has previous access to four pieces of information:

- The matrix of regional technical coefficients $\mathbf{A}^0$, which is the starting matrix;
- Total output by industry for region 1: x_j^1;
- Total intermediate sales of each commodity *i*, for region 1: $z_{i\bullet}^1 = \sum_j z_{ij}^1$; this correspond to the sum of all elements of row *i* in the matrix of intermediate transactions $\mathbf{Z}$.

examples are: Lahr and de Mesnard (2004), de Mesnard (2003), de Mesnard (2006) and Jackson and Murray (2004).

[28] Yet, these authors also stress the fact that the empirical results cannot be definitively generalized, because they depend heavily on the particular features of the matrices to be adjusted and also on the measures used as matrix comparison methods.

[29] We will get back to RAS technique, in Chapter 2, where it will be applied in a different context: in the process of trade flows estimating.

- Total intermediate consumption for each industry *j*, for region 1: $z_{\bullet j}^{1} = \sum_{i} z_{ij}$; this correspond to the sum of all elements of column *j* in the intermediate transactions $\mathbf{Z}$.

The RAS procedure is carried out iteratively, in several steps. Firstly, it is assumed that technical coefficients are equal in both regions: $\mathbf{A}^{0} = \mathbf{A}^{1}$. Under this hypothesis, the intermediate transactions matrix for region 1 would be obtained multiplying these technical coefficients by the correspondent industry outputs. In matrix terms, this can be written as:

$$\mathbf{Z}^{I} = \mathbf{A}^{0}\hat{\mathbf{x}}^{1} \tag{1. 43}$$

in which $\mathbf{Z}^{I}$ represents a first estimate (denoted by index *I*) of the intermediate transactions matrix for region 1, with generic element $\left(z_{ij}^{1}\right)^{I}$, and $\hat{\mathbf{x}}^{1}$ represents a diagonal matrix with total industry output x_{j}^{1} for region 1 in the main diagonal. $\mathbf{Z}^{I}$ illustrates the intermediate transactions that would be observed in region 1 if there were no differences between technological structure of region 1 and region 0 (Jackson and Murray, 2004). The row sums of this matrix, $\left(z_{i\bullet}^{1}\right)^{I} = \sum_{j}\left(z_{ij}^{1}\right)^{I}$, must be compared with the actual, known, row sums $z_{i\bullet}^{1}$. To do so, let's take quotient $r_{i}^{I} = \frac{z_{i\bullet}^{1}}{\left(z_{i\bullet}^{1}\right)^{I}}$. The numerator of this quotient comprises known information, whereas the denominator is a result of the admitted hypothesis about the regional technical coefficients. Hence, quotient r_{i}^{I} is an indicator of the sign and value of the error implicit in the hypothesis of equal regional technical coefficients. If, for example, $r_{i}^{I} < 1$, this means that all the elements of row *i* were assumed greater than they should be, since $\left(z_{i\bullet}^{1}\right)^{I} > z_{i\bullet}^{1}$. So, if we multiply all these elements by r_{i}^{I}, we will obtain a new set of technical coefficients which, after being multiplied by industry outputs, will sum exactly $z^1_{i\bullet}$. In matrix terms, and making the same to each row, this corresponds to generating a new technical coefficients matrix as:

$$(\mathbf{A})^I = \hat{\mathbf{r}}^I \mathbf{A}^0$$

(1. 44)

in which $\hat{\mathbf{r}}^I$ represents a diagonal matrix with quotients r_i^I in the main diagonal. From technical coefficients matrix $(\mathbf{A})^I$, we can now compute a new intermediate transactions matrix:

$$(\mathbf{Z^1})^{II} = (\mathbf{A})^I \hat{\mathbf{x}}^\mathbf{1}$$

(1. 45)

Given the way in which it was obtained, the row sums of this new intermediate transactions matrix must now equal the known values $z^1_{i\bullet}$. However, the column sums $\left(z^1_{\bullet j}\right)^{II} = \sum_i \left(z^1_{ij}\right)^{II}$ must be also compared with the known values $z^1_{\bullet j}$. This is why RAS is called a bi-proportional adjustment method: both row sums and column sums must be respected. Let's define quotient $s_j^{\ II} = \dfrac{z^1_{\bullet j}}{\left(z^1_{\bullet j}\right)^{II}}$. This quotient is used to uniformly adjust all the elements of column *j*, in order to obtain a new set of technical coefficients which, after being multiplied by industry outputs will sum exactly $z^1_{\bullet j}$. In matrix terms, this corresponds to:

$$(\mathbf{A})^{II} = (\mathbf{A})^I \hat{\mathbf{s}}^I = \hat{\mathbf{r}}^I \mathbf{A}^0 \hat{\mathbf{s}}^{II}$$

(1. 46)

After computing the new intermediate transactions matrix[30], $\left(\mathbf{Z}^{\mathbf{1}}\right)^{III} = \left(\mathbf{A}\right)^{II} \hat{\mathbf{x}}^{\mathbf{1}}$, the consistency between the correspondent row and column totals and the known values has to be checked again. New quotients r_i and s_j are computed and the process is repeated iteratively until convergence between the obtained and known totals is achieved. In the several applications of RAS, it has been verified that the process usually converges meaning that quotients r_i and s_j in iteration $t+1$ are closer to one that in iteration t. The iterative process should stop when the difference between the known margins and the estimated ones is very small. One concrete reference number is presented by Miller and Blair (1985) in the example used by these authors: the difference should not exceed 0,005 (Miller and Blair, 1985, p. 286).

Although quotients r_i and s_j have been presented in the context of the algebraic exposition of the RAS technique, some authors consider that they may be interpreted as economic effects (Miller and Blair, 1985). This is the case of Richard Stone[31], the precursor of this technique which considered that the uniform row adjustments (through quotient r_i) are a result of a substitution effect, while the uniform column adjustments (through quotient s_j) are a result of a productivity effect. Substitution effects may occur due to changes in the relative prices of inputs or merely because of the emergence of new substitute inputs. As an example, if plastic materials substitute metallic materials, this will reflect on the increase of technical coefficients in the row correspondent to plastic inputs (which will have a quotient r_i greater than one) and on an inverse effect in the row correspondent to metallic inputs (with r_i less than one). In what concerns to the productivity effect, this results from changes in productivity of industries, due to technological progress or change in labour skills, which originate a reduction in intermediate consumption, compensated by an increase in value added. In this case, there

[30] In the above exposition the iterative procedure is applied to the technical coefficients matrix (A), whilst quotients *r* and *s* are computed from the intermediate consumption flow matrix (Z). This is done in order to facilitate the understanding of the meaning of quotients *r* and *s* and it is correct since the multiplication of diagonal matrices is commutative, thus: $\hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{x}}\hat{\mathbf{s}} = \hat{\mathbf{r}}\mathbf{Z}\hat{\mathbf{s}}$.

[31] *Stone, R. 1961. "Input-Output and National Accounts". Paris: Organization for European Economic* Cooperation. Referred in Miller and Blair (1985).

is a reduction in all the technical coefficients of the column corresponding to the industry which productivity has changed (this is reflected by a quotient s_j less than unitary) (Miller and Blair, 1985).

This interpretation of RAS, though interesting, is questioned by several authors[32]. Conversely, it is commonly argued that RAS can be seen as a merely mathematical formula, since it is proven to correspond to the solution of a problem of minimization of information bias (de Mesnard, 2003; Miller and Blair, 1985; Oosterhaven, Piek and Stelder, 1986; Harrigan, 1990), such as[33]:

$$
\begin{aligned}
&\underset{a_{ij}^1}{Min} \quad I = \sum_i \sum_j a_{ij}^1 \ln\left(\frac{a_{ij}^1}{a_{ij}^0}\right) \\
&s.t. \\
&\sum_j z_{ij}^1 = \sum_j a_{ij}^1 x_j = z_{i\bullet}^1 \\
&\sum_i z_{ij}^1 = \sum_i a_{ij}^1 x_j = z_{\bullet j}^1
\end{aligned}
\tag{1. 47}
$$

This means that the target matrix is generated in order to be as close as possible to the prior matrix and, at the same time, respect the row and column sum constraints (Jackson and Murray, 2004)[34]. Being so, RAS tends to preserve, as much as possible, the structure of the initial matrix.

Regardless of the way in which RAS can be understood, the fact is that it has been widely applied, with quite good results, with special emphasis in input-output studies. The main theoretical advantages of RAS are listed by Oosterhaven, Piek and Stelder (1986): first,

---

[32] For example, Lahr and de Mesnard (2004), have shown that the absolute values of r_i and s_j cannot be interpreted, but rather their relative values. For further details, please refer to the paper.
[33] The analytical demonstration that RAS is a solution of a constrained problem of minimization of information bias can be found, for example, in Miller and Blair (1985), pp. 309-310 or in de Mesnard (2003), p. 7.
[34] The principle of minimization of information bias will be further developed in Section 6 of this Chapter, in the context of Spatial Interaction Models.

unlike other adjustment methods (example: minimization of square differences), RAS doesn't give a major weight to few big differences, neglecting many small differences; second, differences are weighted by their significance in the updated matrix (as it can be seen by coefficient a_{ij}^{1} in problem (1. 47)), while other methods use no weight value or instead use the value of the initial matrix; finally, RAS produces positive values at all the cells of the matrix to be adjusted, where there is positive value in the starting matrix, which is an advantage over other methods that generate surprising and economically meaningless negative coefficients (this occurs in non-biproportional approaches, for example, the minimization of square differences) (Lahr and de Mesnard, 2004). However, some disadvantages are also commonly pointed out to RAS, namely: (1) it is not capable of dealing with negative values in the matrix to be adjusted (this can be seen in equation (1. 47), which, due to the presence of the logarithmic function, is not defined for negative values); (2) every null cell in the initial matrix continues to be null in the final matrix (Oosterhaven, Piek and Stelder, 1986); this can be seen as a negative aspect in some cases. For example, if industry *j* in region 0 uses no input *i* as intermediate consumption, thus presenting a null technical coefficient, RAS forces the same to happen in region 1, which sometimes may not be desirable.

The application of RAS (or other biproportional adjustment methods) as part of a hybrid method in input-output table assemblage often involves the use of some known *a priori* information on the matrix to be found. For example, if the researcher knows, in advance, that for some industry *j* in region 1 the consumption of input *i* is null (conversely to what happens in region 0) this information should be incorporated in the adjustment method. In this case, this could be done setting the specific technical coefficient to zero in the initial matrix; as it was mentioned before, this will remain zero until the process converges. If the researcher has previous knowledge of the specific value (different from zero) of intermediate consumption flow for industry *j* and some input *i* then he/she should set that specific cell to zero and subtract the known value from the corresponding row and column totals. At the end of the RAS procedure, that known value is placed back in the appropriate cell (Lahr and de Mesnard, 2004). Dewhurst (1990) examined the performance of RAS technique, with and without the introduction of known direct information about the target matrix. His empirical exercise consisted in updating a 1973 regional matrix (for Scotland) to year 1979, for which a survey table existed. Comparing the output multipliers obtained from the RAS adjusted table (with no incorporation of known *a priori* information) with the actual output multipliers, this author concluded that the percentage errors are quite small. The same exercise was repeated, increasingly inserting more known interior information in each step. The results showed that the introduction of superior information "does on average improve the multipliers derived from the estimated table" and that "there appears to be decreasing returns to additional superior information" (Dewhurst, 1990, p. 85).

In this section special attention was dedicated to the study of a specific matrix adjustment method: RAS. This methodology was explained and its practical advantages were emphasized, reinforced by its good performance in empirical studies. This technique will be revisited Chapter 2, being used in the context of interregional trade estimation.

1.4.4 Evaluating the accuracy of estimated input-output tables.

Regardless of the specific technique (non-survey or hybrid) that has been used in the construction of an input-output table, it is important to assess, whenever possible, the accuracy or degree of exactness possessed by the obtained table (Jensen, 1980). But when the researcher arrives to such stage, three questions immediately arise:

(1) Is there any objective standard to which the estimated table can be compared?

(2) What is the concept of accuracy that should be privileged in such comparison?

(3) Which quantitative measures should be used to make such comparison, in practice?

The first question comprises two different issues: first, if for example the estimated input-output table is for a specific region, there may not be another input-output table for the same region; second, provided that such comparative table exists, it is necessary to evaluate if it is sufficiently reliable to serve as a benchmark. In fact, the true input-output table for a specific economy may never be known. As we have mentioned before (in section 1.4.2), even in the so-called survey tables, assembled by direct observations, several types of errors can emerge. Thus, it is very difficult to define a consensual standard to which constructed tables should be compared. As an alternative, Jensen (1980) proposes an indirect evaluation of the accuracy of the table, assessing its ability in generating known variables, for example, using the constructed table for year t to forecast known values of industry outputs for year t+1.

When assessing accuracy in the context of input-output studies, we should be aware of the different perceptions it can involve. Jensen (1980) has given a major contribution to the clarification of the concept of accuracy in regional input-output studies. First, it is important to distinguish the accuracy of the input-output table (called A-type accuracy) from the accuracy of the input-output model (B-type accuracy). "A-type accuracy refers to the degree to which an input-output table represents the 'true table' for the economy" (Jensen, 1980, p. 140). B-type accuracy involves a broader concept, referring "to the exactness with which the input-output model reflects the realism of the operation of the regional economy" (Jensen, 1980, p. 141). To implement an input-output model some assumptions must be added to the data comprised in the base input-output table. For example, the use of fixed technical coefficients implies the assumption that no economies of scale exist. Thus, it is important to be aware of these assumptions when applying an input-output model and check if they are acceptable in the concrete situation at hand. Probably, for a great increase in output levels, there may be considerable economies of scale, which may cause errors in the model. B-type accuracy is, hence, determined by the degree to which such assumptions are met by the real economy under study. The problem is that, generally, the researcher doesn't have such a complete knowledge about the operating of the regional economy, implying that information on B-type accuracy is difficult to obtain (Hewings and Jensen, 1986).

A-type accuracy, in turn, may be interpreted in two ways: partitive and holistic accuracy (Jensen, 1980). Partitive accuracy refers to a cell-by-cell accuracy. In this sense, the input-output table is seen as a number of separate components. Thus, an input-output table is accurate in a partitive sense, if each and every cell accurately approximates the correspondent cell in the standard table. Jensen (1980) argues very clearly that partitive accuracy is impossible to achieve in constructed regional input-output tables, given the common situation of regional data availability; moreover, it is not cost effective, because a great part of the cells of the table are not significant to the integrity of the table as a whole. Some empirical results reinforce the previous idea. Jensen (1980) refers to the conclusions of experimental work which proved "that more than fifty percent of the smaller coefficients of a table can be removed (set equal to zero) before a ten percent error appears in input-output multipliers" (p. 147). Additionally, this author stresses the fact that, even when the researcher establishes partitive accuracy as the ultimate goal, the difficulty emerges when it is necessary to assess such accuracy. The fact is that tests often made to infer partitive accuracy in non-survey regional input-output tables are erroneous, since the constructed tables are compared against survey tables which, as explained before, are not error free in a partitive sense.

Instead of partitive accuracy, Jensen (1980) advocates the use of holistic accuracy as a criterion to assess A-type accuracy. He defines holistic accuracy of an input-output table as "the ability to represent the size and structure of the economy in general terms" (Jensen, 1980, p. 143). This means that in holistic accuracy, more important than the absolute values of each cell is their relative magnitudes in relation with each other; for example, more important than accurately assess the value of each intermediate consumption flow z_{ij}^{r} is the correct assessment of the cost structure of industry j in region r, which implies an accurate assessment of the relative values of each intermediate consumption flow in column j. It is true that, if an input-output table verifies partitive accuracy, then it follows that "the table as a whole will reflect the true table with a high degree of accuracy" (Jensen, 1980, p. 142). This means that partitive accuracy in the table implies holistic accuracy. Conversely, the presence of holistic accuracy doesn't guarantee that all the cells of the table are accurate in a partitive sense; particularly it doesn't imply partitive accuracy in those cells that are less significant to the economy in study. Thus, the accuracy of the table as a whole is usually greater than the accuracy of each of its cells. This is because holistic accuracy privileges the attention concerning the larger or most important elements of the economy being studied (Hewings, 1983). This concept of accuracy has guided some projects of table assemblage through hybrid methods (for example: West (1990) and Lahr (1998)). In these projects, direct information should be used in certain cells, identified as critical to the model accuracy. If the model is directed towards a specific industry, for example, the table assembling team should pay special attention to the cells that determine the accurate representation of that industry: in general terms, these are the ones located at that column's industry and the ones which maintain strong inter-industrial relationships with it (West, 1990). Several methods have been developed to identify important sectors in the model construction stage. An important sector, in the table construction sense, is defined by Lahr (1998) as "a sector for which superior data will significantly improve nonsurvey model accuracy" (p. 4). This author's paper (jointly with other references within it) provides a comprehensive review of these methods, which are beyond the scope of the present work.

Miller and Blair (1985) illustrate the distinction of partitive and holistic accuracy through the following example (pp. 286-288): an hypothetical technical coefficients matrix for a specific target year is estimated on the basis of a similar technical coefficients matrix existing for a base year, through the application of the RAS technique. At the end of the iterative adjustment process, the estimated matrix is compared with the actual, known matrix, in two distinct ways: (1) comparing each estimated technical coefficient with the corresponding real value – which means that partitive accuracy is being evaluated and (2) comparing the output multipliers obtained from the estimated technical coefficient matrix with the output multipliers obtained from the real technical coefficient matrix – in this case, what is being evaluated is how well the technical coefficients perform in practice, which depends on how accurately they depict the structure of the economy (accuracy in the holistic sense). Using specific quantitative measures to make such comparisons (discussed later on this section), the results show that: the RAS adjusted table is not accurate in a partitive sense, since the average percentage error is around 64%; however, the correspondent output multipliers are very close to the real ones (the maximum percentage error is 1,27%), which reflects a high holistic accuracy of the estimated table.

Obviously, in what concerns to the typical applications of input-output tables, the researcher should be more concerned with the accuracy of the output multipliers, than with the accuracy of each individual technical coefficient.

It must be noted that the choice of criterion about A-type accuracy has implications on B-type accuracy. Given that the input-output model is based on the input-output table, part of the accuracy of the model is obviously determined by the accuracy of the table. Thus, beyond Type B errors, an input-output model based on a table that verifies accuracy in a holistic sense will be better suited to illustrate the functioning of the most important sectors of the economy (Jensen, 1980). Thus, the results provided by such input-output model should be cautiously interpreted, "within the unknown but probably generous limits of accuracy and precision suggested by the concept of holistic accuracy" (Hewings and Jensen, 1986, p. 317). Even if the researcher aspires to achieve partitive accuracy in the input-output table, there is no way of assuring that the same type of accuracy is achieved in the applications of correspondent model, because some assumptions are not verified: for example, temporal stability of technical coefficients. The previous issues, concerning the implications of A-type accuracy on the accuracy of the model, should also be accounted for when these input-output tables are to be nested within a broader framework (examples: social accounting systems or general equilibrium models) (Hewings and Jensen, 1986). An empirical application conducted in Israilevich *et al.* (1996) demonstrated that the choice of three alternative input-output tables used as a module of a regional econometric input-output model (one constructed with observed regional data, the second based on the national table adjusted using location quotients and the third consisting of randomly generated input-output coefficients) produced significantly different results, in both forecast and impact analysis. Therefore, the accuracy of the input-output table has a great influence on the accuracy of the model, even when the input-output table is only a component of the model.

Finally, the third question mentioned at the beginning of this section concerns to the choice among several quantitative measures of matrix comparison. Once the researcher has decided which values to use as benchmark for comparison and after having decided over the criterion of partitive or holistic accuracy, he/she must choose some specific formula to quantify the distance between the estimated table and the benchmark. Many different measures have been used in determining the accuracy of input-output tables. However, most of the times, the researcher makes no previous investigation of the properties of the chosen measure and does not seriously evaluate all the existing alternatives (Lahr, 1998). Of the existing measures, some have been most commonly used. Next, we review six of those measures, discussing their properties as well. Let's consider that the elements to be compared are technical coefficients[35] and denote the real and the estimated coefficients by a_{ij} and $\tilde{a}_{ij}$, respectively. Considering also that both the real and the estimated table are of dimension $n \times n$, we can define the following measures (for further details, refer to: Miller and Blair (1985), Jackson and Murray (2004) and Lahr (1998)):

(1) Mean Absolute Difference (MAD): $\dfrac{\sum_i \sum_j \left| a_{ij} - \tilde{a}_{ij} \right|}{n^2}$;

(2) Standardized Total Percent Error (STPE): $100 \dfrac{\sum_i \sum_j \left| a_{ij} - \tilde{a}_{ij} \right|}{\sum_i \sum_j a_{ij}}$;

(3) Root Mean Square Error (RMSE): $\left[\dfrac{\sum_i \sum_j \left(a_{ij} - \tilde{a}_{ij} \right)^2}{n^2} \right]^{0,5}$;

(4) Index of Inequality (Theil's U): $\left[\dfrac{\sum_i \sum_j \left(a_{ij} - \tilde{a}_{ij} \right)^2}{\sum_i \sum_j a_{ij}^2} \right]^{0,5}$;

[35] Of course, the same measures can be applied to compare, for example, the elements of the Leontief inverse or the output multipliers or even the estimated industry outputs. Only, in the latter two cases, the measures will not be comparing matrices, but rather vectors of values.

(5) Mean Absolute Percent Error (MAPE): $100\frac{1}{n^2}\sum_i\sum_j\frac{\left|a_{ij}-\tilde{a}_{ij}\right|}{a_{ij}}$;

(6) Weighted Absolute Difference (WAD): $100\frac{\sum_i\sum_j a_{ij}\left|a_{ij}-\tilde{a}_{ij}\right|}{\sum_i\sum_j\left(a_{ij}+\tilde{a}_{ij}\right)}$.

MAD represents the average absolute difference between the estimated and the real coefficient. For example, if $MAD = 0{,}1$, this means that, in average, the estimated coefficient exceeds or is below the real coefficient by an amount of 0,1. The major drawbacks of this measure are (Lahr, 1998): (1) it doesn't weight the differences by any value, meaning that errors in large cells have the same influence in the error measure as errors in small cells; (2) it does not provide any idea of the relative difference between the two tables. STPE overcomes the latter problem, in the sense that it compares the absolute difference between the estimated and the real table with the values of the real table. Its major limitation is the fact that absolute differences are not weighted by the values of the cell, preventing this measure to be "exceptionally sensitive to high-valued cells" (Lahr, 1998, p. 27). RMSE is a well known statistical measure of distance, here applied to input-output table comparison, which simply corresponds to the square root of mean square error. As it happens with MAD, this measure does not reflect the relative difference between the two tables. Index U answers this problem, substituting n^2 by $\sum_i\sum_j a_{ij}^2$. Both RMSE and Index U suffer from the already mentioned limitation of using no weight to emphasize differences in larger cells. MAPE is not subject to any of the problems referred before: on the one hand, each error is pondered by $1/a_{ij}$; on the other hand, it provides a measure of average relative difference between both tables. Though in a different manner, WAD, presented in Lahr (1998), also overcomes both previously mentioned problems: it does weight the absolute difference by the value of the correspondent cell in the target table and it provides an idea of the relative difference between the two tables. However, whereas MAPE weights the difference between the two tables in such a way that big absolute errors in large coefficients are minimized, WAD, does the opposite: an error in a large coefficients is doubly penalized, since the difference $\left|a_{ij} - \tilde{a}_{ij}\right|$ is multiplied by the large coefficient a_{ij}.

Given the great number of alternatives (which go beyond the ones presented on the above list), some authors prefer to apply a combination of several different measures, instead of making a choice for only one measure. For example, Jackson and Murray (2004) assess the accuracy of ten different matrix adjustment techniques, against a known table, applying four different measures of matrix comparison. Each of these measures generates a different ranking for the ten adjustment techniques. After applying the four measures, these authors make an average of the four rankings and achieve a final combined rank.

In this section, survey, non-survey and hybrid techniques were generally discussed. We have seen that complete survey tables are mostly unjustified, given that a high and often unavailable amount of resources are necessary, to achieve a table which for a number of reasons may still contain several types of errors. Additionally, not all the individual elements of an input-output table assume the same significance to the economy the table depicts. This leads us to the concept of holistic accuracy on input-output tables, which privileges the correct representation of the general structure of the economy and is consistent with the adoption of hybrid methods, in which the collection of survey information is targeted only to the critical cells to the economy. Of course, the degree to which a hybrid method deviates from a non-survey method depends on the available resources (non only money, but also time, manpower, etc) to conduct surveys, even if directed to only a small number of elements.

The next section is still dedicated to input-output table construction, but regarding to two specific issues: the regionalization of a national table when it is on a Make and Use format and the use of non-survey techniques to assess the amount of imported products comprised in intermediate and final use flows of each commodity.

1.5. The specific features of the national tables and their implications on the regional table construction processes by hybrid or non-survey methods.

The procedures and hypotheses adopted in regional input-output table construction and modelling are strongly connected to the type of information contained in the national table that is used as a starting point to achieve the regional table and the format in which this information is presented. Apparently small details can create errors in the model, if they are ignored and/or if no consistent assumptions are used. Of course, the accounting system used in the table construction phase, that further determines the set of hypotheses to assume in the model phase, depend on the amount and format of the available national and regional data (Oosterhaven, 1984).

So far in this work, single-region and multi-regional input-output models and tables have been presented according to the traditional symmetric input-output format. A symmetric input-output table can be of the product-by-product or industry-by-industry nature. Product-by-product tables consist of symmetric input-output tables with products as the dimension of both rows and columns; they show the amounts of each product used in the production of which other products. In turn, industry-by-industry tables consist of symmetric input-output tables with industries as the dimension of both rows and columns; they show the amounts of output of each industry used in the production of which other industries (UN, 1993). However, input-output models can be tailored to fit input-output tables displayed as a Make and Use (or commodity-by-industry) format. Make and Use (M&U) tables can be shortly defined as tables that depict how supplies of different products originate from domestic industries and imports and how those products are used by the different intermediate or final users, including exports (UN, 1993). Currently, most of the European countries, including Portugal, publish their National input-output tables in the Make and Use format.

The basic structure of M&U tables and the development of a commodity-by-industry national input-output model will be explained in section 1.5.1. The assemblage of a regional Make and Use table, on the basis of its national counterpart, and the regional model that can be derived from it, will be the subject of section 1.5.2. It is also very important to be aware of the manner in which intermediate imports are treated in the national table, before proceed with the regionalization. Besides, this has also important consequences on the regional model that can be derived from the regionalized table. The relevance of these issues will be justified in section 1.5.3.

1.5.1 "Commodity-by-industry" accounts.

Nowadays, most of the countries compile and publish their input-output tables in the commodity-by-industry (also called rectangular or Make and Use) format. This framework was set up at the 1960's, when the United Nations introduced the 1968 System of National Accounts. The M&U format is better suited to represent the diversity of products that is in effect produced by each industry. Moreover, the assemblage of the Make and Use tables is more closely connected to the way in which firms organize their own data, facilitating the collection of the necessary data (Piispala, 1998). In this framework, two dimensions are considered, industries and products and two tables are essential: the Use table, which describes the consumption of products *j* by the several industries *i*, and the Make or supply table that represents the distribution of the industries' output by the several products[36].

Since Make and Use tables involve two dimensions, industry and product, it is fundamental to clearly define and distinguish both concepts, in advance. The term "product" is used to refer all goods and services generated in the context of productive activity (EUROSTAT, 1996, paragraph 3.01). The term "industry" involves some more complexity. According to the 1993 System of National Accounts' definition, "an industry consists of a group of establishments engaged on the same, or similar, kinds of activity" (UN, 1993, paragraph 5.40). In practice, most of enterprises are engaged in more than one activity (UN, 1993): (1) the principal activity, the one that is responsible for the

[36] Because this framework involves two dimensions, product and industry, and to allow a better clarification of what dimension is being treated in each case, industries will always denoted by letter *i* and products, by letter *j*. This convention applies only to analytical equations relating to Make and Use format.

creation of the major part of Value Added in the enterprise; (2) the secondary activities, being defined as any other activities that generates goods or services and (3) the ancillary activities, those that support the main productive activities, such as: accounting, transportation, human resources management, etc. The fundamental distinction here refers to primary *versus* secondary activities. Let's consider the example of a pulp mill. In the process of producing pulp some residues are generated (called biomass and including pulping liquors, wood residues, and bark). These kinds of outputs are called by-products: they unavoidably result from the primary product production process, hence being technologically related to it. Let's suppose that a small part of these residues are sold to other enterprises and the remaining is used to produce energy for self use and also to provide energy to others. Then, this firm involves two distinct kinds of activity. A kind-of-activity unit (KAU) is another fundamental statistical term, defined as a part of an institutional unit in which only one particular type of economic activity is carried out (Jackson, 2000). This concept is at the basis of the definition of industry. In fact, in order to assign the activity of firms to an industry classification, enterprises "must be partitioned into smaller and more homogeneous units, with regard to the kind of production" (EUROSTAT, 1996, p. 35). In our example, this firm would have to be partitioned into two KAU's, and the correspondent activities would be assigned to two different industries: 1) the principal activity would be classified under the head "Manufacture of pulp, paper and paper products; publishing and printing" and 2) the secondary would be classified as "Electricity, gas and water supply". Concerning the production and sale of biomass there is no way in which this activity can be considered in another KAU, since, being a residue, its production costs can't be separated from those coming from the production of pulp and paper, the principal activity. So, the KAU corresponding to the main activity produces two products: pulp and biomass; in a different way, the product electricity is associated to its own KAU. By using this concept of KAU, National Accounts guarantee a partial refining of industrial classification, meaning that most of the secondary products produced in each firm are classified under a different industry heading, the one that produces those products as its principal activity. Thus, the number of secondary products included in the main industry heading is reduced. In fact, in our example, if the pulp firm was not partitioned into two different

KAU’s, energy products would be considered an output of the industry “Manufacture of pulp, paper and paper products; publishing and printing”. Still, industries classified in this way cannot be considered as being purely refined, because sometimes it is not possible to separate the secondary from the primary activity. This occurs whenever the available information obtained from enterprises doesn’t allow that separation (this is the case of most of the small firms, which have no accounting documents that allow the partition into different KAUs) or when the secondary product is a by-product (as the biomass in our example), hence precluding the separation of its cost structure from cost structure of the primary product.

Using both the concept of product and of industry, the basic structure of a Make and Use table, at the national level, can be illustrated as in the following Figure:

Figure 1. 4 – Basic structure of a National M&U table, with total flows.

	Products	Industries		
Products	---	**U**	**y**	**p**
Industries	**V**	---	---	**g**
	m	**w**		
	p	**g**		

This table comprises two fundamental sub-matrices: $\mathbf{U}$ and $\mathbf{V}$, used to represent the Use and the Make matrix respectively. $\mathbf{U}$ shows products in rows and industries in columns; conversely, in $\mathbf{V}$, each row corresponds to one industry and each column represents one product. Since it’s easier to get accurate information on products than on industries, the degree of disaggregation at the product level is usually much higher than at the industry level. For that reason, the number of products considered in sub-matrices $\mathbf{U}$ and $\mathbf{V}$ may differ from the number of industries, which gives the label "rectangular" to this format. These matrices are composed of elements u_{ji}, for the Use matrix, and v_{ij} for the Make matrix. u_{ji} represents the amount of product *j* used as an input in the production of industry *i*'s output. Usually, the M&U matrices provided by National Accounts are of the total-flow type[37]; being so, flows u_{ji} include both imported and domestically produced amounts of input *j*; v_{ij} stands for the domestic production of product *j* by industry *i* (elements of the Make matrix). $\mathbf{y}$ represents the vector that sums all the components of final demand: private consumption, government consumption, investment and exports. As it happens with the intermediate consumption, these values of final demand comprise both imported and domestically produced amounts of *j*. For such reason this is a total use table[38]. Summing all the columns in $\mathbf{U}$ and adding $\mathbf{y}$, we get vector $\mathbf{p}$, which accounts for total output of each product. The same vector (transposed) can be obtained summing all the lines of matrix $\mathbf{V}$ and adding the imported products, comprised in $\mathbf{m}$.

The commodity supply-demand balance, for a specific product *j*, may be written as[39]:

$$p_j = \sum_i v_{ij} + m_j = \sum_i u_{ji} + y_j$$

(1. 48)

In what concerns to the industries, a similar balance can be established. Being $\mathbf{w}$ the vector that represents value added by industry, one may write:

$$g_i = \sum_j v_{ij} = \sum_j u_{ji} + w_i$$

(1. 49)

37 Though, it would be possible to conceive a domestic-flow table using the Make and Use format. Since this is not the current layout in which these tables are published, we opt not to illustrate the basic structure of such a table.

38 In this sense, this table is comparable with the symmetric table of Figure 1, which was also a total use table.

39 As in the simplified presentation of the symmetric input-output table, in section 2, we are ignoring taxes and subsidies on products as well as margins. A complete version of Figure 3 will be presented in Chapter 3.

From these fundamental identities an input-output model may be derived, as for the traditional symmetric input-output table, and an inverse matrix is achieved. To develop such a model, at least two hypotheses have to be considered: (1) fixed technical coefficients and (2) a proposition that relates industry's output with commodity's output.

The first hypothesis is common to all input-output models and it has already been deeply discussed. Using the notation of the rectangular format, the technical coefficient[40] is defined as: $q_{ji} = \frac{u_{ji}}{g_i}$. Applying it to equation (1. 48), it yields:

$$p_j = \sum_i q_{ji} g_i + y_j \tag{1. 50}$$

In matrix terms, this looks like:

$$\mathbf{p} = \mathbf{Qg} + \mathbf{y} \tag{1. 51}$$

in which $\mathbf{Q}$ represents the technical coefficient matrix.

As to the second hypothesis, two major alternatives exist: (1) to assume that each product is produced in fixed proportions by the several industries, implying that the structure implicit in each column of V is assumed invariant; (2) to assume that each industry produces different products in fixed proportions, involving the hypothesis that the structure implicit in each row of V is invariant[41]. At this stage, we will opt to follow the first alternative, without further discussion on both. These will be analyzed in detail in Chapter 3. Accordingly, industry's output and commodity's output is linked through the

[40] We use the notation technical coefficient, which implies that the U matrix is comprised of flows that include not only domestic inputs, but also imported ones.
[41] In Chapter 3 we will show that the first alternative implies that all products produced by an industry are produced with the same input structure, meaning that there is one technology assigned to each industry, whereas the second alternative implies that a product has the same input structure in whichever industry it is produced, meaning that there is one technology assigned to each product. The first alternative is commonly named Industry Technology-based Assumption (ITA), whilst the second corresponds to the Commodity-based Technology Assumption (CTA).

use of the following ratio: $s_{ij} = \frac{v_{ij}}{p_j}$. This represents the market share of industry i in total supply of product j. We can rewrite this as: $v_{ij} = s_{ij} p_j$. Combining this equation in with equation (1. 49), we may state that $g_i = \sum_j s_{ij} p_j$, which in matrix terms is equivalent to:

$$\mathbf{g} = \mathbf{Sp}$$

(1. 52)

Finally, this can be introduced in equation (1. 51), and manipulated until the final inverse matrix is achieved:

$$\begin{aligned} &\mathbf{p} = \mathbf{Qg} + \mathbf{y} \\ &\mathbf{p} = \mathbf{QSp} + \mathbf{y} \\ &(\mathbf{I} - \mathbf{QS})\mathbf{p} = \mathbf{y} \\ &\mathbf{p} = (\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y} \end{aligned}$$

(1. 53)

Through this inverse, it's possible to determine the impacts on total product supply caused by changes in final demand. Using equation (1. 52) we can write:

$$\mathbf{g} = \mathbf{Sp} = \mathbf{S}(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y} \Leftrightarrow \mathbf{g} = \mathbf{S}(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}$$

(1. 54)

This equation allows the assessment of the impacts caused on the production of national industries by changes in final demand towards products, regardless of their geographic origin (either domestic or imported). This impact analysis (both concerning the effect on total product supply and on national industry production) implies that the elements of the inverse matrix remain unaltered in face of exogenous shocks. Thus, this involves not only the assumption of constant technical coefficients q_{ji}, but also the assumption of constant market shares s_{ij}, as it has been previously referred.

From these basic equations several others may be deducted, if additional hypotheses are included in the model. Again, we refer to Chapter 3 for further developments on rectangular input-output modelling.

1.5.2 Regionalizing a national Make and Use table.

In this section we are mainly interested in how to regionalize a national table as the one depicted in Figure 1. 4 and, moreover, in the specific hypotheses that are implicit in those procedures. The choice of the methods used to regionalize commodity-by-industry accounts depends on the specific data availability in each country. Concerning specifically the Portuguese context, we may refer to the recent work described in Martins *et al.* (2005), in which seven regional Make and Use tables were assembled in order to build the database for a multi-regional input-output model. In this case, the input-output model was used as a module of an environmental model designed to evaluate the regional impact of legal tools to control the emission of greenhouse gases. For other countries, we refer to the following papers: Jackson (1998) and Lahr (2001), for the U.S. case; Madsen and Jensen-Butler (1999), for the Danish case; Piispala (2000) and Koutaniemi and Louhela (2006) for the Finnish case and Eding *et al.* (1997), for the Dutch case.

The rectangular table for a single-region will have exactly the same aspect as the one illustrated in Figure 1. 4. Only, in this case, imports include also inflows coming from other regions and final demand is also comprised of exports to other regions (see Figure 1.5 ahead).

In a pure non-survey method, the regionalization of the Use matrix follows the same method as the one commonly used in the symmetric format: the national technology assumption is adopted (Jackson, 1998; Lahr, 2001; Madsen and Jensen-Butler, 1999). The researcher is supposed to have access to the total intermediate consumption of each industry at the regional level; in other words, the column total of the Use matrix, $\sum_j u_{ji}^r$, is known[42]. This is usually available information. Then, the elements of the Use matrix at the regional level are deduced from their national counterparts, using the total intermediate consumption proportion as the regionalizing factor. Going back to the taxonomy introduced by Oosterhaven (1984), this is a columns-only method of regionalization:

$$u_{ji}^{r} = u_{ji} \frac{\sum_{j} u_{ji}^{r}}{\sum_{j} u_{ji}} \tag{1. 55}$$

The superscript r is used to denote a regional variable. It must be emphasized that these u_{ji}^{r} do not represent intra-regional flows, since inputs i may come from other regions or even from abroad. Hence, each column of the regional Use matrix obtained in such a way illustrates the true technological recipe of the corresponding industry, which is assumed to be the same at the regional and at the national level.

In what concerns to the Make matrix, the regionalization can be carried using the regional proportion of industrial's output (Jackson, 1998). This implies that the table assembler has previous knowledge of the vector of industries' regional output, which is usually verified[43]. Then, we have:

$$v_{ij}^{r} = \frac{g_{i}^{r}}{g_{i}} v_{ij} \Leftrightarrow \frac{v_{ij}^{r}}{g_{i}^{r}} = \frac{v_{ij}}{g_{i}} \tag{1. 56}$$

---

[42] Otherwise, total output should be used, instead of total intermediate consumption and then equation (1. 55) would be: $u_{ji}^{r} = u_{ji} \frac{g_{i}^{r}}{g_{i}} \Leftrightarrow u_{ji}^{r} = q_{ji} g_{i}^{r}$. However, in this case, the researcher will be taking the implicit assumption that the relative share of industrial *versus* value added inputs is the same in the region as in the nation. The regional variances in the proportion of value added inputs has been identified by Round (1983) as the *fabrication effects*, being one of the factors that may create diverse technical coefficients between regions. Details on this issue may be found in Round (1983), as well as in Miller and Blair (1985).

[43] If this doesn't happen, regional output for each industry has to be estimated by means of the proportion of regional to national employment, which implies the assumption of productivity invariance among regions, within the same industry (Jackson, 1998).

The implicit assumption behind this regionalization procedure is that the weight of product *j* in total output of industry *i* is the same in the region and in the country (Lahr, 2001; Martins *et al.*, 2005). An alternative way of regionalizing the national Make matrix would be to regionalize its columns, instead of regionalizing its rows. In this case, we would have: $v_{ij}^r = \frac{v_j^r}{v_j} v_{ij} \Leftrightarrow \frac{v_{ij}^r}{v_j^r} = \frac{v_{ij}}{v_j}$. The implicit assumption here would be: the market share of industry *i* in total internal supply of product *j* is the same in the region and in the country. The option for the first alternative relies essentially on two reasons. First, the second alternative suffers from a problem of unavailable data: the fact is that, generally, the value of regional production by products is not known *a priori*, unlike the value of regional production by industry. Second, the assumption of space invariant market shares of each industry in the production of the several products implies that all regions have a similar productive structure. But if, for example, industry *i* does not exist in one of the regions, then it cannot contribute to the total product supply, opposing the implicit assumption of space invariant market shares. This seems to be in disagreement with the hypothesis assumed in the model development, on the previous section (equations (1. 51) to (1. 54)). However, assuming invariant market shares in sensitivity analysis merely implies that, in one specific region or country, market shares s_{ij} remain constant when some exogenous change occurs in final demand. This is much more reasonable than assuming invariant market shares across space when assembling regional tables.

In a rectangular format, the vector of regional final demand consists of demand for products, instead of demand directed to industries, as it happens in symmetric industry-by-industry tables. This facilitates the assemblage of such a vector for the regional level. For example, in what concerns to Private Consumption, there are surveys directed to families which ask about their patterns of product consumption. These data are usually used to establish the regional structure of Private Consumption (Lahr, 2001). For the remaining components of regional final demand (government consumption and investment), each country applies its own method of regionalization, according to the available data at the regional level. The estimation of regional exports, embracing both exports for other countries and exports or other regions, is a more complex issue, being discussed further in section 1.6 (the same applies to imports, at the supply side). At the moment, it is enough to assume that these four vectors of external trade (inter-regional and international exports and imports) are available, having been obtained by some survey or non-survey method.

The regional M&U matrix obtained in such a way (Figure 1. 5) is structured very similarly to the national counterpart, such as in Figure 1. 4, with the following exceptions: the final demand vector $\mathbf{y}$ includes not only exports to the rest of the world, but also to the rest of the country; imports are also divided in two rows: $\mathbf{m}^{\mathbf{roc}}$, coming from the rest of the country and $\mathbf{m}^{\mathbf{row}}$, coming from the rest of the world.

Figure 1. 5 – Regional Make and Use matrix, with total flows.

	Products	Industries		
Products	---	$\mathbf{U}^{\mathbf{r}}$	$\mathbf{y}^{\mathbf{r}}$	$\mathbf{p}^{\mathbf{r}}$
Industries	$\mathbf{V}^{\mathbf{r}}$	---	---	$\mathbf{g}^{\mathbf{r}}$
	$\mathbf{m}^{\mathbf{row}}$ $\mathbf{m}^{\mathbf{roc}}$	$\mathbf{w}^{\mathbf{r}}$		
	$\mathbf{p}^{\mathbf{r}}$	$\mathbf{g}^{\mathbf{r}}$		

From one regional M&U matrix derived as described before, it is possible to develop a total flow single regional input-output model[44], following just the same procedures used to derive the national model from the national table (equations (1. 51) to (1. 54)). One of the final equations will be:

[44] Interregional and multi-regional models as the ones described in section 2 can also be adjusted to the rectangular format. See, for example, Oosterhaven (1984) and Madsen and Jensen-Butler (1999). For this work's purposes, in what concerns to the rectangular format, it is sufficient to rely on the single-region case, as it will be explained in Chapter 3.

$$\mathbf{p^r} = \left(\mathbf{I} - \mathbf{Q^r S^r}\right)^{-1} \mathbf{y^r}$$

(1. 57)

in which $\mathbf{Q^r}$ is the matrix of regional technical coefficients: $q_{ji}^r = \frac{u_{ji}^r}{g_i^r}$ and $\mathbf{S^r}$ is the matrix composed of market shares at the region: $s_{ij}^r = \frac{v_{ij}^r}{p_j^r}$. This equation illustrates the impact over total product supply available at the region in study, $\mathbf{p^r}$ (either it has been produced regionally or not) originated by changes in final demand directed to supply existing at this region (this including imports from other regions and abroad) ($\mathbf{y^r}$). From such equation one can also compute the impact of $\mathbf{y^r}$ over $\mathbf{g^r}$, the vector of regional industry production:

$$\mathbf{S^r p^r} = \mathbf{S^r}\left(\mathbf{I} - \mathbf{Q^r S^r}\right)^{-1} \mathbf{y^r} \Leftrightarrow \mathbf{g^r} = \mathbf{S^r}\left(\mathbf{I} - \mathbf{Q^r S^r}\right)^{-1} \mathbf{y^r}$$

(1. 58)

1.5.3 Total versus intra-regional flows.

A preliminary distinction between total flows and intra-regional flows was already made on sections 1.2 (referring to the national level) and 1.3.1 (referring to a single-region analysis). In these sections we were dealing with the symmetric model. But, obviously, the same dichotomy emerges when we are using the rectangular model. The model implicit in equation $\mathbf{p^r} = \left(\mathbf{I} - \mathbf{Q^r S^r}\right)^{-1} \mathbf{y^r}$ involves the concept of total use flows. As it has been explained in section 1.2, the term “total use flow” is used to indicate the intermediate or final use flow of input *j*, comprising all the possible sources of that input – regional production, other-regions’ production or other countries’ production. In fact, the flows comprised in the use matrix from which technical coefficients (q_{ji}^r) are computed are total flows; as a consequence, the multiplier effect implicit on the inverse matrix $\left(\mathbf{I} - \mathbf{Q^r S^r}\right)^{-1}$ involves also an effect on imported products. However, the researcher may be interested in isolating the impact felt only on regionally produced output caused by changes in $\mathbf{y}$. Moreover, he/she may have an interest in inferring the impact on regionally produced output caused by changes in final demand directed to regional products. When such analysis is carried out, the researcher is interested in evaluating intra-regional impacts, *i.e.*, those which include solely regionally produced products[45]. With this aim, and given the already mentioned difficulties in obtaining intra-regional use flows by survey methods, he/she faces two alternative procedures: (1) compute an intra-regional use table from the total flow use table comprised in Figure 1.5; (2) use the total flow use table as a starting point to develop a model that allows the evaluation of intra-regional impacts. Either alternative involves the assumption of some simplifying hypotheses. In Chapter 3, it will be shown that, when the same set of hypotheses is used in both alternative procedures, the impacts measured by an intra-regional flow based model are the same as the impacts measured by a total use based model. We leave the development of the second alternative for Chapter 3. In the next section, we will review some of the techniques used when we decide to follow the first alternative, *i.e.*, to estimate intra-regional use flows from total use flows.

1.5.4 Techniques used to estimate intra-regional use flows from total use flows.

Each flow of the table comprised in Figure 1.5, concerning intermediate or final use, is composed of imported and regionally produced products. In fact, each use flow can be seen as the sum of three components. Let's consider u_{ji}^{r}, as the intermediate use of input *j* by industry *i* in region *r* (irrespective of the geographic origin of *j*), and y_{j}^{r} as the final use of *j* in region *r* (irrespective of the its geographic origin). Then, we have:

$$u_{ji}^{r} = u_{ji}^{rr} + u_{ji}^{roc\,r} + u_{ji}^{row\,r} \tag{1. 59}$$

and

[45] In models applied to the nation level, the designation "intra-regional flows" is substituted by "domestic flows", since in this case imports include only those coming from foreign countries. We will get back to the issue of total use flows versus intra-regional / domestic flows in Chapter 3.

$$y_j^r = y_j^{rr} + y_j^{roc\,r} + y_j^{row\,r} \tag{1. 60}$$

in which u_{ji}^{rr} represents the amount of regionally produced input *j* used as intermediate consumption by industry *i* in the same region *r*; $u_{ji}^{roc\,r}$ represents the amount of input *j* imported from the rest of the country, used as intermediate consumption by industry *i* of region *r* and $u_{ji}^{row\,r}$ represents the amount of input *j* imported from the rest of the world, used as intermediate consumption by industry *i*. Similar notation is used for final consumption.

Hence, the problem consists in obtaining u_{ji}^{rr} and y_j^{rr} from u_{ji}^{r} and y_j^{r}, respectively. This implies the estimation of one imports matrix (or two, considering that interregional and international imports are estimated separately), depicting the intermediate and final use of each imported product.

This problem receives a different solution depending on the context of data availability. In what data availability is concerned, we must distinguish international imports from interregional imports. The fact is that, usually, there is some partial information on international imports made by regions, provided by the official organisms of statistics: the total amount of regional imports, decomposed by products is currently available data. The same cannot be said about interregional imports. Excepting some few countries (Canada, for instance) in which surveys are regularly conducted to estimate interregional trade flows, the most common situation is that official organisms of statistics don't provide any information on these flows, not even the total amounts of imports destined to (exports originating from) each region. In this context, we can usually identify one of the three following circumstances concerning trade data information to perform the estimation of the import matrix, ordered by increasing survey-based data availability:
1) The researcher has no information relating total supply of the different products separated by origin (regional production and imports coming from the rest of the country and rest of the world).

2) The researcher has access to total product supply, separated by origin – supply-side information; then, the problem is limited to the estimation of the proportions in which those imports are used by different final or intermediate users (demand-side information).
3) The researcher has access to some additional information besides the total product supply separated by origin. For instance sometimes there is full information concerning regional imports, meaning that "a matrix of intermediate imports is available" (Harrigan *et al.*, 1981, p. 70) for region *r*. In this event, the problem consists merely in decomposing that matrix into two import matrices: one for rest-of-the-country products and another for the rest-of-the-world products.

Depending on the precise situation of data availability, different solutions have been proposed in the input-output literature, which will be next reviewed.

Whenever there is no *a priori* information on imports, not even the total amount of imports by product, a set of so-called "purely non-survey techniques" may be applied (Miller and Blair, 1985). Non-survey techniques used to derive intra-regional flows from total use flows can be generally divided into location quotient (*LQ*) and commodity balance (*CB*) techniques.

Location Quotients (*LQ*) are a measure of regional specialization. In its simplest form, location quotient is usually defined for product *j* in region *r*, by (Miller and Blair, 1985):

$$LQ_j^r = \frac{v_j^r / v^r}{v_j / v} \tag{1. 61}$$

in which: v_j^r denotes production of *j* in region *r*; v^r represents total production in region *r*; v_j and v represent similar variables for the nation level. If data on output are not available, then other variables can be used to measure relative concentration: employment[46], value added, and so on.

LQ_j^r measures the relative specialization of region *r* in producing product *i*, "comparing the relative importance of an industry in a region to its relative importance in the nation or some other base economy" (Schaffer and Chu, 1969, p. 85). In fact, the numerator of (1. 61) represents the weight of product *j* in total regional production; this is compared with the weight of product *j* in total national production (in the denominator). If $LQ_j^r > 1$, then the production of *j* is more localized, or concentrated, in region *r* than in the nation as a whole (Miller and Blair, 1985). The opposite can be stated if $LQ_j^r < 1$: region *r* is relatively less specialized in the production of *j* than the nation.

This simple measure has been used in estimating the intra-regional flows from total use flows. The reasoning is as follows: if region *r* is relatively more specialized in the production of *j* than the nation ($LQ_j^r > 1$), then it is assumed that all the requirements of *j* to met intermediate and final consumption are provided by the region itself; once regional requirements are satisfied, the regional surplus, given by the difference between regional output and regional requirements, is considered as an export from region *r*. The implicit reasoning behind this is that the weight of *j* in national production is an indicator of the weight that *j* has on regional demand. Conversely, if region *r* is relatively less specialized in *j* than the nation ($LQ_j^r < 1$), then it is assumed that some of the regional requirements of *j* have to be imported; the capacity of the region in self providing product *j* is given by its relative specialization in *j*, i.e., by LQ_j^r, originating the following equations for intra-regional intermediate and final consumption flows:

$$u_{ji}^{rr} = LQ_j^r u_{ji}^r$$
$$y_j^{rr} = LQ_j^r y_j^r$$

[46] The use of employment as a *proxy* for output involves the assumption of identical regional and national industry productivity.

(1. 62)

The remaining proportion, $\left(1-LQ_j^r\right)$ is accounted as an import of input j (Miller and Blair, 1985).

In spite of being an easy to handle measure, which requires a small amount of data, this method suffers from three quite restrictive simplifying assumptions:

1. The major problem of this non-survey technique is the fact that it considers that, if region r presents a relative high weight of commodity j in the total available output, it is capable of self-providing its own requirements of the same product; by doing this, *LQ* technique ignores the specific structure of regional demand, assuming that it is equal to the structure of national demand (Greytak, 1969). In fact, the structure of regional demand may determine, for example, a high level of demand for commodity j, implying that, even with a high relative weight on production, regional production is not enough to satisfy regional demand, which may have to be fulfilled by imports. This happens, for example, when commodity j is intensively used as an input for intermediate consumption by a certain industry i in which region r is specialized (Sargento, 2002). This is also emphasized in Schaffer and Chu (1969), stating that "To ensure success in using the simple location quotient, the local industry structure must closely resemble the national structure: this requirement is seldom met" (p. 86). In other words, given that regions have quite different productive structures, the simplifying assumption of spatially invariant demand structure is unacceptable.

2. Another limiting consequence arises from the hypothesis under which, when *LQ* is greater that one, the region is capable of self-providing its own requirements of the same product, exporting the surplus to the rest of the nation: interregional imports of j are assumed to be null ($u_{ji}^{roc\,r}=0$, for all industries i and $y_j^{roc\,r}=0$). Thus, this method relies on the principle of maximum local trade (Morrison and Smith, 1974), meaning that if the commodity "is available at a local source, it will be purchased from that source" (Harrigan *et al.*, 1981, p.71). By doing this, it doesn't account for the high probability of existing simultaneous import and

export of the same product (crosshauling). In fact, even if the researcher works with a highly disaggregated classification of products, these cannot be assumed as homogeneous, making interregional (as well as international) trade to be composed, to a great extent, by simultaneous import and export of the same product.

3. The *LQ* measure is asymmetric. In fact, for any row for which the LQ_j^r is less than one, the correspondent import coefficients will vary with the size of LQ_j^r: for smaller LQ_j^r's, the correspondent import coefficients will be larger; but if LQ_j^r is greater than one, the correspondent row in the import matrix will be arbitrarily filled with zeros, irrespective of the size of LQ_j^r (Miller and Blair, 1985; Harrigan *et al.*, 1981).

These significant drawbacks make the *LQ* method too simplistic to serve the purposes of regional input-output analysis. As stated in Round (1978a), referring to *LQ*, "(…) it is difficult to be optimistic about the possibility of estimating trade flows (which inevitably result from a complex set of regional relationships) using such simple constructs" (p. 290). The unsuitability of the assumed hypotheses is immediately reflected on the fact that the *LQ* method provides estimates for the use of the regional production of *j* (final and intermediate) that are usually inconsistent with the previously known value of the regional production of *j*. In fact, the *LQ*-based estimated regional production of product *j*, $\tilde{v}_j^r$, will be (Miller and Blair, 1985):

$$\tilde{v}_j^r = \sum_i u_{ji}^{rr} + y_j^{rr} = \begin{cases} \sum_i LQ_j^r u_{ji}^r + LQ_j^r y_i^r & \text{if} \quad LQ_j^r < 1 \\ \sum_i u_{ji}^r + y_j^r & \text{if} \quad LQ_j^r \geq 1 \end{cases} \tag{1. 63}$$

The arbitrariness of the hypotheses assumed implies that there is no guarantee that the estimated regional output, $\tilde{v}_j^r$, is compatible with the actual, known, regional output v_j^r. So, if $\tilde{v}_j^r \leq v_j^r$, the required adjustment is made allocating the residual to exports from the region to the rest of the nation. Conversely, if $\tilde{v}_j^r > v_j^r$, the intra-regional flows correspondent to row *j* are all adjusted downward, being multiplied by $\frac{v_j^r}{\tilde{v}_j^r}$. Both types of adjustments may be required in either case of *LQ* value: greater or lower than one. As stated in Round (1978a), concerning the *LQ* method, "Commodity exports from the region are invariably ascertained as a residual after the final output and total intermediate sales have been deducted from gross sales. As a consequence, even in the situation where the [LQ]-values indicate export orientation, there is no guarantee that these residuals are positive" (p. 291).

Commodity Balance technique differs from *LQ* technique, since it is not based in any measure of regional specialization, but rather on the regional balance of trade for each commodity (Harrigan *et al.,* 1981). Let's define regional requirements of *j* by D_j^r. For any region, the following balance must hold:

$$D_j^r = v_j^r + m_j^{row\,r} + m_j^{roc\,r} - d_j^{r\,row} - d_j^{r\,roc}$$

(1. 64)

This means that regional requirements of *j* are provided by regional production added by regional imports and subtracted of regional exports. Based on this balance, the *CB* technique is applied as follows: if $v_j^r \geq D_j^r$, then it is assumed that the region has the capacity to provide all the requirements of *j* in region *r*. Conversely, if $v_j^r < D_j^r$, it is assumed that the self sufficiency of the region is limited to the proportion $\frac{v_j^r}{D_j^r}$. The remaining regional requirements will have to be fulfilled by imports coming from outside the region (both from the remaining regions and from abroad).

The basic principle underlying *CB* technique is, thus, much similar to the one underlying *LQ*. The region's ability to supply its own needs of a certain input is determined by the value of one specific quotient (LQ_j^r in *LQ* technique and $\frac{v_j^r}{D_j^r}$ in *CB* technique): when such quotient is greater than one, the region's needs are totally provided by regional production; when it is less than one, the ability of the region in self-providing inputs is reduced to the value given by the quotient. Thus, these techniques are also comparable in their restrictive hypotheses. In fact, problems 2 and 3 pointed out to *LQ* are shared by *CB*. Just like *LQ*, it assumes null crosshauling and it makes an asymmetric interpretation of the value of the quotient ($\frac{v_j^r}{D_j^r}$, in this case). Nevertheless, *CB* still can be understood as theoretical superior to *LQ*, in the sense that it doesn't make any assumption about the structure of regional demand; instead, it uses the observed value of regional requirements D_j^r.

Let's now assume that we have some partial information on trade flows: total imports (by products) coming from international sources and from other regions to region *r* are known.[47] This means that we are assuming that the sum $m_j = m_j^{roc\,r} + m_j^{row\,r}$ is available information. Thus, our purpose is to estimate the imports by destination; that means that for each imported product we are searching for which part is used as intermediate consumption (in each industry) and as final use (of each kind).

One of the partially survey techniques is the so-called Moses Technique (*MT*) (Harrigan *et al.,* 1981). Observing Figure 1.5 again, we can see that total supply of *j* in region *r* is given by[48]:

[47] While this is usually verified for international imports, this assumption over interregional imports implies as a rule that the total amount of imports is previously estimated by some other method (again, concerning the discussion of these methods, we refer no section 6.2)
[48] In all section 5.4 we will be using the notation of the rectangular model. However, all the proposed techniques can also be applied to symmetric tables, to compute symmetric intermediate consumption tables and final demand vector with intra-regional flows from the correspondent total-flow tables.

$$p_j^r = v_j^r + m_j^{row} + m_j^{roc}$$

(1. 65)

in which $v_j^r = \sum_i v_{ij}^r$. Dividing all the elements of this equation by p_j^r, we may compute the corresponding coefficients:

$$1 = \frac{v_j^r}{p_j^r} + \frac{m_j^{row\,r}}{p_j^r} + \frac{m_j^{roc\,r}}{p_j^r}$$

(1. 66)

Each of these coefficients express the proportion in which each origin (the region itself, the rest of the country and the rest of the world) contributes to total supply of product *j* in region *r*. More precisely, $\frac{m_j^{row\,r}}{p_j^r}$ and $\frac{m_j^{roc\,r}}{p_j^r}$ represent the average import propensity of product *j* (from other countries and from other regions, respectively). The essential assumption of *MT* is that this average import propensity, computed on the supply side, is applicable to all the demand flows for product *j*. In other words, if for example $\frac{m_j^{roc\,r}}{p_j^r} = 0{,}3$, $\frac{m_j^{row\,r}}{p_j^r} = 0{,}4$ and $\frac{v_j^r}{p_j^r} = 0{,}3$, this assumption means that, for all the possible uses of product *j* (intermediate or final), 30% of those uses will be satisfied by regionally produced output, 40% will come from other countries and 30%, from the rest of the country. Then, we may compute intra-regional intermediate and final use flows as follows:

$$u_{ji}^{rr} = u_{ji}^r - u_{ji}^{roc\,r} - u_{ji}^{row\,r}$$

(1. 67)

and

$$y_j^{rr} = y_j^r - y_j^{roc\,r} - y_j^{row\,r}$$

(1. 68)

which, including the previous assumption on the average import propensity[49], leads to:

$$u_{ji}^{rr} = u_{ji}^{r} - \frac{m_j^{roc\,r}}{p_j^r} u_{ji}^{r} - \frac{m_j^{row\,r}}{p_j^r} u_{ji}^{r} \Leftrightarrow$$
$$u_{ji}^{rr} = u_{ji}^{r}\left(1 - \frac{m_j^{roc\,r}}{p_j^r} - \frac{m_j^{row\,r}}{p_j^r}\right) \tag{1. 69}$$

and

$$y_j^{rr} = y_j^{r} - \frac{m_j^{roc\,r}}{p_j^r} y_j^{r} - \frac{m_j^{row\,r}}{p_j^r} y_j^{r} \Leftrightarrow$$
$$y_j^{rr} = y_j^{r}\left(1 - \frac{m_j^{roc\,r}}{p_j^r} - \frac{m_j^{row\,r}}{p_j^r}\right) \tag{1. 70}$$

The use of this import proportionality assumption will be examined thoroughly in Chapter 3, in the context of a national input-output table. Its reasonability will be further discussed. Also, starting from a total flow M&U, it will be shown that such an assumption may be used in two different approaches: (1) to construct a domestic or intra-regional flow table, subtracting the flows of imported products, and then develop an input-output model from it, which allows the assessment of the impact on domestically produced output caused by changes in final demand directed to domestic products; (2) to be directly incorporated into a model developed on the basis of a total flow table, in order to convert it on a model that measures domestic or intra-regional impacts. Moreover, it will be demonstrated that the result of these two procedures, in quantifying domestic impacts, is exactly the same.

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[49] The same kind of assumption had already been used when presenting the Chenery-Moses model. However, there is a crucial difference between the trade coefficients defined there and the import coefficients $\frac{m_j^{row\,r}}{p_j^r}$ and $\frac{m_j^{roc\,r}}{p_j^r}$ used here: in this case, the shipments of product *i* are divided by total supply, including foreign imports, whereas in the Chenery-Moses model the trade coefficients are obtained dividing the shipments of product *i* by total supply of *i* in the region, except for foreign imports.

The other partially survey technique is the Tiebout (*TB*) method (Harrigan *et al.*, 1981). As it was explained when presenting the Riefler-Tiebout bi-regional model (in section 1.3.5), this method assumes that there is already an imports matrix for region *r*, describing the intermediate use of all imported products *j* by all producing industries in region *r* and also the several final uses of imported products *j*. These regional imports comprise inflows from all possible origins to region *r*. This means that the aggregates $\left(u_{ji}^{roc\ r}+u_{ji}^{row\ r}\right)$ and $\left(y_{j}^{roc\ r}+y_{j}^{row\ r}\right)$ are known *a priori*. Only the individual components $u_{ji}^{roc\ r}$, $u_{ji}^{row\ r}$, $y_{j}^{roc\ r}$ and $y_{j}^{row\ r}$ are unknown. Thus, the intra-regional input flows can be immediately obtained by subtraction; considering $\left(u_{ji}^{roc\ r}+u_{ji}^{row\ r}\right)=u_{ji}^{or}$ and $\left(y_{j}^{roc\ r}+y_{j}^{row\ r}\right)=y_{j}^{or}$, we have:

$$u_{ji}^{rr}=u_{ji}^{r}-u_{ji}^{or}$$

(1. 71)

and

$$y_{j}^{rr}=y_{j}^{r}-y_{j}^{or}$$

(1. 72)

Hence, the Tiebout Method is used merely to decompose the imports matrix into imports from other regions and imports from other countries. To do so, it considers the following coefficients: $\frac{m_{j}^{roc\ r}}{m_{j}^{or}}$ and $\frac{m_{j}^{row\ r}}{m_{j}^{or}}$, in which $m_{j}^{or}=m_{j}^{roc\ r}+m_{j}^{row\ r}$. These coefficients express the percentage of imports that comes from the rest of the country and from the rest of the world, respectively. The hypothesis used here consists in assuming that these percentages apply uniformly to all possible uses of *j*; thus, it is equivalent to what is done in the Moses technique, except for the fact that it relies on a higher degree of *a priori* information. Then, we have:

$$u_{ji}^{roc\ r} = u_{ji}^{or}\left(\frac{m_j^{roc\ r}}{m_j^{or}}\right)$$

(1. 73)

and

$$u_{ji}^{row\ r} = u_{ji}^{or}\left(\frac{m_j^{row\ r}}{m_j^{or}}\right),$$

(1. 74)

for intermediate uses and:

$$y_j^{roc\ r} = y_j^{or}\left(\frac{m_j^{roc\ r}}{m_j^{or}}\right)$$

(1. 75)

and

$$y_j^{row\ r} = y_j^{or}\left(\frac{m_j^{row\ r}}{m_j^{or}}\right),$$

(1. 76)

for final uses. This technique has been applied, for example, in Oosterhaven and Stelder (2007), in their comparison between four alternative non-survey intercountry input-output table construction methods, for nine Asian countries and the USA. More precisely, given that the import matrix was previously known (considering all possible origins of flows), the Tiebout method has been used, for each country, to make the split between imports.

The four techniques presented before involve diverse data requirements. Thus, it is expected that the more survey-based information is used, the more accurate are the results generated by them. In Harrigan *et al.* (1981), the results obtained from each of these techniques in the estimation of an imports matrix were compared with a survey based import matrix, existing for Scotland. The simulation results showed that, as expected, the techniques which involve the use of some survey information on trade flows are more accurate than any of the non-survey methods, which originate very unreliable results.

Given the evident problems in using these non-survey techniques, especially in what concerns to LQ, the researcher should restrict their use to situations in which there is no information at all on the total amount of imports. Provided that the total amount of international imports is usually available for the researcher (conversely to the total amount of interregional imports), these non-survey methods should be applied referring only to interregional imports. It should be noted, however, that the partially survey methods are not exempt of limitations. In fact, the imported share of each total use flow is assumed invariant with the type of use, as it happens also in both non-survey techniques. But, in reality, sometimes intermediate uses tend to reflect a greater import propensity than final uses (as it happened in the empirical application in Harrigan *et al.,* 1981) and some final uses tend to show a lower import propensity than others (for example, exports tend to comprise a lower share of imported products than household consumption)[50].

1.6. Models to assess interregional trade data.

1.6.1 The relevance and nature of external trade in regional economies.

External trade holds an extreme importance in regional economies, in particular in small areas. It can be divided into trade with other regions of the same country and international trade. Today, regional scientists fully recognize the importance of knowing the magnitude and nature of the economic interdependence between each region and the rest of the world, in order to better identify the whole implications of regional policies. According to Munroe and Hewings (1999), "If international trade has significant impacts

[50] In some applications, this is incorporated assuming that exports do not comprise any imported products, i.e., countries (or regions) do not import to export. This is done, for example, in Miller and Blair (1985), p. 295, when these authors define the proportion of regional needs self provided by the region, through the following quotient, named Regional Purchase Coefficient (RPC): $RPC_j^r = \frac{v_j^r - d_j^r}{v_j^r - d_j^r + m_j^r}$. In this case, exports are being excluded meaning that imports will be allocated to all uses, except for exports. In other words, there is no re-exporting of imported products (Lahr, 2001). Conversely, in Moses technique, for example, the correspondent coefficient is: $RPC_j^{r\,\prime} = \frac{v_j^r}{v_j^r + m_j^r}$, meaning that the assumption of invariant import propensity is extended to all uses, including exports.

on economic growth and welfare concerns (employment, income, etc), it should follow that trade within countries may also merit much further consideration" (p. 2). For example, a deficit in the region's trade balance means that the region relies on income transfer and/or granting of savings from other regions (Ramos and Sargento, 2003). In a more detailed perspective, knowledge about regional external trade, segmented by commodities, allows us to characterize productive specialization, foresee eventual productive weaknesses as well as determine the region's dependency on the exterior (or in some cases the exterior's dependency on the region) regarding to the supply of different commodities. In what concerns to the application of input-output models, the knowledge of interregional trade flows, at least the pooled volume of exports and imports by commodity, is an essential requirement to allow the consideration of spillover and feedback effects, as it has been explained before.

Recent studies applied to interregional systems in USA and Japan have demonstrated that interregional trade is growing faster than international trade (Jackson *et al*., 2004). Reinforcing this idea, Munroe and Hewings (1999) present the example of U.S. Midwest region, in which the volume of trade among the five states that compose this region exceeds the volume of trade between these states and the main foreign trading partners of USA. The reasons behind the increasing importance of interregional trade are determined, to a great extent, by the significant transportation costs reduction and the deepening integration of regions in the global economy, that have occurred in the recent decades. These factors led, not only to an increase in trade between different regions, but also to the emergence of new features of interregional trade. Polenske and Hewings (2004) focus on three new issues concerning interregional trade: increasing complexity in production processes, intra-industry trade and "hollowing-out" tendency. Trade linkages among regions involve a growing sophistication, since now firms look across the whole country or even across different countries in order to find the most cost competitive locations to produce in each different stage of the production chain. This is one of the reasons why intra-industry trade or crosshauling, i.e., trade characterized by imports and exports of the same product, is becoming more important (Wixted, Yamano and Webb, 2006). Another important factor determining the increase of crosshauling is the growing product differentiation, making consumers to look for differentiated products in other regions, instead of the ones that are produced regionally. The structural change in interregional trade is also determined by a decrease in intra-regional transactions, in favor of an increase in interregional trade: this is called a “hollowing-out” process (Polenske and Hewings, 2004), since this implies that the density of relations within the regional economy tends to diminish.

In spite of this recognized importance, the available studies on the specific issue of interregional trade are rare, especially due to the difficulty in obtaining the necessary data. Besides that, the techniques often used to assess this required data sometimes fail to capture the real amount of exports and imports within regions. Non-survey methods that estimate net trade flows, for example, are clearly not suited, given the growing importance of crosshauling. Net values will always be small in comparison with the gross flows of exports and imports, underestimating the real relevance of interregional trade in the formation of regional GDP (Harris and Liu, 1998).

As it has been mentioned before in this essay, one of the main problems in regional table assembly is in obtaining interregional commodity flows. In input-output practical applications, the knowledge of this data is of fundamental importance, in two perceptions: (1) in a statistical perspective, since they constitute an essential part of regional supply and demand, necessary to ensure consistency in the system of regional input-output tables (2) in the modeller perspective, because of the already mentioned importance of interregional feedback effects, that can only be accounted for when interregional trade flows are known. Given the known difficulties in collecting such information directly, the debate is focused on non-survey techniques. The objective of section 1.6 is to expose clearly the problem of interregional trade estimation and critically review the major non-survey techniques which have been used to estimate interregional commodity flows.

The problem of interregional trade estimation can be illustrated as follows. Let’s assume a system with k regions of origin (denoted by a superscript r) and k regions of destination (denoted by a superscript s). Then the problem consists in estimating the interregional shipments of j, x_j^{rs}, $r, s = 1, \cdots, k$, as illustrated by the following matrix[51]:

Figure 1. 6 – Interregional trade flows of commodity *j* from Region *r* to Region *s*: x_j^{rs}.

Destination / Origin	**Region 1**	**Region 2**	**…**	**Region k**	**Sum**
Region 1	0	x_j^{12}	…	x_j^{1k}	$d_j^{1\,roc}$
Region 2	x_j^{21}	0	…	x_j^{2k}	$d_j^{2\,roc}$
…	…	…	0	…	
Region k	x_j^{k1}	x_j^{k2}	…	0	$d_j^{k\,roc}$
Sum	$m_j^{roc\,1}$	$m_j^{roc\,2}$	…	$m_j^{roc\,k}$	$d_j = m_j$

This problem may be addressed in three steps, of increasing complexity: (1) determining the net trade between each region of the system and the rest of the country; (2) determining gross exports and imports from net flows, which means, solving the problem of crosshauling; (3) determining interregional trade flows, for every product, established between each region of origin and each region of destination, i.e., fulfill a complete O-D matrix for each and every product being traded. We will deal with problems (1) and (2) in sections: 1.6.2 and 1.6.3, respectively. The problem of fulfilling the whole Origin-Destination matrix for each commodity being traded is rather complex and, in such context, specific models are required. These models belong to the generic heading of spatial interaction models, being the core subject of Chapter 2.

1.6.2 Determining net exports in single-region input-output models.

When assembling a single-region input-output table from the correspondent national table, all the components of the table can be obtained for the region on the basis of the

[51] We are assuming, at the moment, that only interregional trade is being estimated and not intra-regional trade. We will get back to this issue in section 6.3.

national values (or from direct regional sources), except for interregional trade. In this case, obviously, there is no counterpart at the national level. Thus, the methodology usually consists in estimating interregional trade only when the rest of the table is already assembled. In order to achieve the estimation of interregional trade in a multi-regional system, the first step consists in estimating the trade balance, for each product, between each region and the remaining regions of the system.

In single-region tables, exports from (and imports to) the region to (from) the rest of the country are determined as outflows (and inflows) without specifying the region of destination (origin). Moreover, there are no consistency constraints regarding these trade flows, since the other regions' inflows and outflows are not being estimated[52]. Finally, exports flows are treated as exogenous components of regional final demand in the correspondent single-region input-output model.

The non-survey techniques exposed in section 1.5.4, as techniques used to estimate intra-regional flows from total use flows, have also been used to estimate trade flows between each region and the rest of the country (Jackson, 1998). In this context, *LQ* and *CB* techniques are used in the table assemblage stage, in order to assess the values comprised in the vector of imports from the rest of the country, $\mathbf{m}^{\mathbf{roc\,r}}$, and in the vector of exports to the rest of the country, $\mathbf{d}^{\mathbf{r\,roc}}$, included in final demand. As it will be explained, these methods don't provide exactly the estimates of $\mathbf{m}^{\mathbf{roc\,r}}$ and $\mathbf{d}^{\mathbf{r\,roc}}$, but rather one vector of net exports, given by the difference $\mathbf{d}^{\mathbf{r\,roc}} - \mathbf{m}^{\mathbf{roc\,r}}$ (or equivalently, one vector of net imports given by $\mathbf{m}^{\mathbf{roc\,r}} - \mathbf{d}^{\mathbf{r\,roc}}$).

Starting from the Location Quotient, let´s observe again equations (1. 62). These equations are applied when region *r* is relatively less specialized in *j* than the nation ($LQ_j^r < 1$). This means that the capacity of the region in self providing product *j* is given by its relative specialization in *j*, i.e., by LQ_j^r and some of the regional requirements of *j*

[52] Of course, if interregional trade is being estimated for each single-region table within a more complex multi-regional system, one restriction must be observed: the sum of all interregional exports must equal the sum of all interregional imports.

have to be imported (in this case, there are supposedly no exports of product *j* – it is the "no crosshauling" assumption). Then, the value of regional production of product *j*, is given by:

$$v_j^r = \sum_i u_{ji}^{rr} + y_j^{rr} = \sum_i LQ_j^r u_{ji}^r + LQ_j^r y_j^r = LQ_j^r \left(\sum_i u_{ji}^r + y_j^r \right) \Leftrightarrow$$
$$v_j^r = LQ_j^r \left(\sum_i u_{ji}^r + y_j^r \right)$$

(1. 77)

Considering that regional requirements of product j (given by $\sum_i u_{ji}^r + y_j^r$) are satisfied by regional production and imports, we may derive an equation for regional imports:

$$v_j^r = LQ_j^r \left(v_j^r + m_j^r \right) \Leftrightarrow$$
$$v_j^r \left(1 - LQ_j^r \right) = LQ_j^r m_j^r \Leftrightarrow$$
$$m_j^r = \left(\frac{1}{LQ_j^r} - 1 \right) v_j^r$$

(1. 78)

Moreover, if we assume that crosshauling does not exist, this equation provides an estimation of net regional imports of product *j*. Isserman (1980) presents an equation for net regional exports of product *j* (NEX_j^r) which corresponds exactly to the symmetric of equation (1. 78) (except for the fact that regional employment is used by this author instead of regional production):

$$NEX_j^r = (1 - \frac{1}{LQ_j^r}) v_j^r, \text{ if } LQ_j^i > 1$$

(1. 79)

This implies that, in Isserman (1980), the *LQ* method is being used in a symmetric manner. This represents a slight difference comparing to the way in which LQ method was applied in intra-regional flow estimation (section 1.5.4). In fact, in equation (1. 79), the value of *LQ* is inserted, whether it is greater or smaller than one. Conversely, in section 1.5.4, it became clear that there was an asymmetric treatment of the *LQ* value: when it was greater than one, all regional requirements were assumed to be provided by regional production and gross exports were computed as a residual; this was done to every value above one, regardless of its magnitude.

Equation (1. 79) may be presented as follows:

$$NEX_j^r = (1-\frac{1}{LQ_j^r})v_j^r \Leftrightarrow NEX_j^r = \left(1-\frac{1}{\frac{\frac{v_j^r}{v^r}}{\frac{v_j}{v}}}\right)v_j^r$$

$$NEX_j^r = \left(1-\frac{\frac{v_j}{v}}{\frac{v_j^r}{v^r}}\right)v_j^r \Leftrightarrow NEX_j^r = \left(1-\frac{v^r}{v_j^r}\cdot\frac{v_j}{v}\right)v_j^r \Leftrightarrow$$

$$NEX_j^r = \left(\frac{v_j^r}{v^r}-\frac{v_j}{v}\right)\cdot v^r$$

(1. 80)

This version of the equation "is most useful for identifying the theoretical rationale behind the location quotient approach" (Isserman, 1980, p. 157). In equation (1. 80), net exports are estimated as a result of the difference between the relative weight of product *j* in total regional production and an estimate of regional demand of product *j*, assuming that this is proportional to the weight product *j* in total national production. Thus, the *LQ* method "tends to assume away the very regional differences a regional input-output model is designed to highlight" (Round, 1983, p. 197). The structure of regional demand for each product *j* is then assumed to be spatially invariant, being this the major limitation of *LQ*, as already mention in section 1.5.4.

Three additional quite restrictive assumptions are generally pointed out to the use of *LQ* in assessing regional trade flows for each commodity[53] (Harris and Liu, 1998). These will be presented in a critical way.

1. It is commonly argued that "There must be no cross-hauling between regions of products belonging to the same industrial category, so if a region is an exporter of *i*, its consumption of *i* is entirely from the region's production" (Harris and Liu, 1998, p. 853). In fact, there is some imprecision in this statement. The fact that *LQ* gives an estimative of net exports doesn't imply that cross-hauling doesn't exist, but rather that *LQ* is designed to estimate net flows instead of separate gross flows. This distinction is assumed away, by asserting that there is no cross-hauling (Isserman, 1980). As stated in Jackson (1998), "If there was no cross-hauling, then the estimate of rest-of-nation exports would be gross rather than net (…)" (p. 234).

2. The country, as the sum of *k* regions, is neither a net exporter nor importer of *j* (Isserman, 1980). The demonstration is based on a transformation of equation (1. 80): $NEX_j^r = \left(\frac{v_j^r}{v^r} - \frac{v_j}{v}\right) \cdot v^r \Leftrightarrow NEX_j^r = v_j^r - \frac{v_j}{v} \cdot v^r$. Summing for all the *k* regions of the system, we get a null sum:

[53] Besides these, another assumption, considered as an additional limitation to the use of *LQ*, is mentioned in some papers (for example, in Isserman (1980), Harris and Liu (1998) and Harrigan *et al.*, (1981)), referring to equal regional and national productivity per employee. But this supposition is only required when the share of regional to national employment is used as a proxy to regional contribution to national production. However, such assumption is avoidable if alternative variables, based on currently available data on production, value added, among others, are used in the definition of *LQ*.

$$\sum_{r=1}^{k} NEX_j^r = \left(v_j^1 - \frac{v_j}{v}\cdot v^1\right) + \left(v_j^2 - \frac{v_j}{v}\cdot v^2\right) + \cdots + \left(v_j^k - \frac{v_j}{v}\cdot v^k\right)$$

$$\sum_{r=1}^{k} NEX_j^r = \left(v_j^1 + v_j^2 + \cdots + v_j^k\right) - \left(v^1 + v^2 + \cdots + v^k\right)\frac{v_j}{v}$$

$$\sum_{r=1}^{k} NEX_j^r = v_j - v\frac{v_j}{v} = 0$$

(1. 81)

3. The region as a whole is neither a net exporter nor a net importer. In fact, if we make the sum of the net exports for all *n* products of region *r*, we get a null sum:

$$\sum_{j=1}^{n} NEX_j^r = \sum_{j=1}^{n} v_j^r - \frac{\sum_{j=1}^{n} v_j}{v}\cdot v^r$$

$$\sum_{j=1}^{n} NEX_j^r = v^r - \frac{v}{v}v^r = 0$$

(1. 82)

This restriction is very limiting, since there is no theoretical reason to force the regional trade balance with the rest of the nation to be null.

The shortfall related to the second assumption can be prevented if the method is restricted to the estimation of trade flows between region *r* and the rest of the country instead of using it to estimate both types of trade flows (interregional and international) in conjunction. In other words, commonly accessible data on international trade should be used, in order to avoid such restrictive assumption as the inexistence of surplus or deficit at the nation level. However, some adaptation must be made to the LQ method when the objective is to deal solely with interregional trade. An appropriate variable must be used in the definition of *LQ*, which is exempt of the effects created by international flows. Let's define the variable AO_j^r, representing available output in region *r* to satisfy domestic demand (demand directed to region *r* and also to the remaining regions of the country) (Sargento, 2002):

$$AO_j^r = v_j^r + m_j^{row\,r} - d_j^{r\,row},$$

(1. 83)

in which $d_j^{r\,row}$ denotes exports of *j* from region *r* to foreign countries. Defining the *LQ* on the basis of this new variable ($\left(LQ_j^r\right)^*$), we get:

$$\left(LQ_j^r\right)^* = \frac{AO_j^r / AO^r}{AO_j / AO}$$

(1. 84)

In this case, using an equation equivalent to (1. 80), net exports from the rest of the country (or net imports, if the quotient is below unity) are estimated as: the difference between the available output in region *r* to satisfy domestic demand of product *j* and the estimated regional requirements of product j, assuming that it is a proportion of total available output in the region r, given by the weight of product *j* in domestic demand at the national level (which corresponds to the assumption of identical regional demand structure in the region and in the country):

$$NEX_j^r = AO_j^r - \frac{AO_j}{AO} \cdot AO^r$$

(1. 85)

If the researcher uses such version of *LQ*, obtaining international trade data from an independent source, the previously referred assumption 2 is not only appropriate, but rather a necessary constraint. Obviously, for interregional trade flows of each commodity, it is required that one region’s exports are equal to the imports of the rest of the regions.

In short, the *LQ* method suffers from the fact that it relies on two erroneous assumptions: 1) the assumption of spatially invariant demand structure and 2) the obligation of null balance of trade between the region and the rest of the nation.

Commodity Balance is an alternative method to assess net commodity flows between one region and the rest of the country. This method relies upon the balance that must hold between total supply and total demand for each commodity. Total regional supply is equal to the sum of regional output with regional imports; total demand is given by regional requirements plus regional exports. Defining, as before, regional requirements of j by D_j^r, the following balance must hold, for any region:

$$D_j^r + d_j^{r\,row} + d_j^{r\,roc} = v_j^r + m_j^{row\,r} + m_j^{roc\,r}$$

(1. 86)

Using the previously defined variable $AO_j^r = v_j^r + m_j^{row\,r} - d_j^{r\,row}$, we get:

$$D_j^r + d_j^{r\,roc} - m_j^{roc\,r} = AO_j^r$$
$$NEX_j^r = AO_j^r - D_j^r$$

(1. 87)

When $NEX_j^r < 0$, the balance constitutes the value of net imports; when it is positive, then it corresponds to the value of net exports.

Let's compare this equation with equation (1. 85). Both equations attempt to estimate net exports through the difference between available regional output and regional requirements. However, while *LQ* uses a strong and unlikely assumption to provide an estimate of regional requirements for product *j* (given by $\frac{AO_j}{AO} \cdot AO$), *CB* uses the actual value of regional requirements directly. This suggests that, in practice, the direct use of *CB* should be preferred over the use of *LQ*. The remark made by Stevens and Treyz (1989) provides additional support to this argument: "(…) the alternative methods are based on the reasonable assumption that the greater the ratio of regional supply to regional demand, the more a region is likely to buy from itself; however, LQE and LQS measures are only proxies for this ratio, whereas the SRD is the ratio itself" (p. 252)[54]. But there is also a practical problem affecting *CB* method: it calculates NEX_j^r as a residue, whose value guarantees the verification of the equilibrium between supply and demand; thus, the mistakes made in estimating the remaining components of the regional table are included in this value. Still, it has a significant theoretical advantage over *LQ*: it takes into account the specific structures of demand estimated to the region under study[55]. Jackson (1998) reinforces this idea stating that "the supply-demand pool approach can be argued to be theoretically superior to methods based on location quotients, which do not account for variations in the final demand structure" (p. 233). This author suggests the application of *CB* technique in his description of how to regionalize commodity-by-industry accounts. Commodity-balance was also applied in Jensen-Butler and Madsen (2003) as a first step in interregional trade estimation, to obtain the net exports of each product made by each region.

The empirical application conducted in Sargento (2002), which aimed to compare the results provided by *LQ* and by *CB*, suggested as well that Commodity Balance was the most adequate method to estimate trade flows between the Portuguese region under study (*Região Centro*) and the rest of the country. Even with no survey data on interregional trade flows to make an objective evaluation of each method's accuracy, the knowledge about the region under study allowed the author to consider the results provided by *CB*, more adequate than the ones generated by *LQ*. For example, the structure of net imports suggested by *CB* reflects the specific structure of intermediate consumption of the region in study which is clearly associated to some important industries in the region. One of the paradigmatic cases involved forestry products: the region was found to be a net exporter of these products according to the *LQ* method (which may seem, at first glance, more

[54] LQE and LQS stand for *LQ* based on employment and on supply variables, respectively; SDR is for supply-demand ratio, the author's label for commodity-balance.

[55] This is a theoretical advantage only when the technique used to estimate the vectors of regional intermediate and final demand, in assembling the regional input-output table is a suitable one. If we resort to non-survey methods applying the national structures to the region under study, without incorporating any superior information, then this advantage looses a lot of its significance. Conversely, if for example one uses surveys directed to families to estimate patterns of regional private consumption, some specific features of regional demand's structure are being introduced, thus influencing the values of net exports estimated by *CB*.

suited to the reality of the region, which is known by its forest extension) and a net importer, according to *CB* estimate. However, taking into account the specific structure of forest products demand in the region, we realize that intermediate demand to provide wood and cork as well as paper industries, with a great importance in *Região Centro*, is by itself above the available regional output. Thus, the negative sign for net trade flows given by *CB* method is probably a better estimate than the one provided by *LQ*. Other examples concerning the empirical comparison of the results provided by both methods can be found in Sargento (2002) and Sargento and Ramos (2003).

The option for *CB* technique to estimate net trade flows between one region and the rest of the country is also patent in other empirical works carried out by Portuguese research teams, which had the objective of assembling single-region input-output tables for other Portuguese regions. This is the case in CCRN/MPAT (1995) applied to *Região Norte* and CIDER/CCRA (2001) applied to *Região do Algarve*. In the construction of the input-output table for *Região Autónoma dos Açores* (Azores Islands), this method was applied only partially, to services and some residual goods, since a great part of inter-regional trade is established between the islands and the mainland, by air and sea, for which the information provided by the Statistics of Transports and Communications is quite complete (ISEG/CIRU, 2004).

1.6.3 From net to gross trade flows: the problem of crosshauling.

Both methodologies presented in the previous sections lead to net trade flows (net exports, when positive, or net imports, when negative) between the region and the rest of the country. From *LQ* or *CB* technique, we obtain:

$$NEX_j^r = d_j^{r\,roc} - m_j^{roc\,r} \tag{1. 88}$$

The difficulty here is that this net value is compatible with an infinite number of values for gross trade flows. Yet, the knowledge of the values of total gross exports and gross imports for each region and each products is usually required to proceed with the estimation of interregional trade flows, that is, to fulfill the complete O-D matrix for each product being traded. The problem of obtaining gross exports and gross imports from the net trade balance is termed the crosshauling problem. One possible attitude consists in ignoring crosshauling. This corresponds to assume that: when the region is a net exporter of some product, there are no imports of the same product (and, in this case, net exports will equal gross exports); when the region is a net importer, there are no exports of the same product (in this case, gross imports will be set equal to net imports). But this is an extremely simplistic approach, given that net flows are usually very small when compared to gross values of exports and imports. This is demonstrated, for example, by Susiluoto (1997), in which trade between three Finish regions was estimated both through an inquiry and using the commodity balance method. The values of interregional trade provided by the commodity balance method were systematically lower than the ones obtained from the inquiry, which is expected, given that the first method accounts only for net trade flows.

Crosshauling consists in simultaneous import and export of products under the same classification. In section 1.6.1 we have already addressed this issue, explaining some of the factors that have led to an increase in crosshauling (also named intra-industry trade). What matters here is that this is an unavoidable issue, even if a high degree of disaggregation is used in product's classification and in regions' definition. It is true that, "the principle of applying a high level of disaggregation, both in terms of commodity and geography, reduces the problem somewhat" (Madsen and Jensen-Butler, 1999, p. 297). However, the problem would only be totally solved if commodities were completely homogeneous or if an infinitely thin disaggregation concerning products and regions were used (Toyomane, 1988).

One of the possible solutions to the crosshauling problem is to set arbitrarily a crosshauling rate. This is done for example in Madsen and Jensen-Butler (1999), in which the share is set equal to 10%. A crosshauling share χ can be defined as:

$$\chi = \frac{\left|NEX_j^r\right|}{d_j^{r\,roc}}$$

(1. 89)

This share represents the weight of net exports (in absolute value) over gross exports. From this crosshauling share, it is possible to achieve the value of gross exports and gross imports on the basis of the known value of net exports. This is done as follows:

$$\begin{cases} When\ NEX_j^r > 0 \Rightarrow d_j^{roc\,r} = \dfrac{NEX_j^r}{\chi}\ \ and\ m_j^{r\,roc} = d_j^{roc\,r} - NEX_j^r \\ When\ NEX_j^r < 0 \Rightarrow d_j^{r\,roc} = \dfrac{\left|NEX_j^r\right|}{\chi}\ and\ m_j^{roc\,r} = d_j^{r\,roc} + \left|NEX_j^r\right| \end{cases}$$

(1. 90)

Sometimes this crosshauling share is not settled in a complete ad-hoc basis, but it is rather based on some information on interregional trade flows available from transport statistics. This was done, for example, in Ramos and Sargento (2003). The crosshauling shares were settled through the comparison between the net trade balance and total regional outflows (in physical quantities), recorded in transport statistics. However, this is not a straightforward solution, given the known drawbacks of transport statistics. Ramos (2001) refers five problems related to transport statistics provided by the Portuguese national institute of statistics. First, these statistics are only adequate to provide information on flows of goods and not on flows of services, since interregional flows of services occur due to movements of persons and not due to movements of products. Second, all trade flows are expressed in physical units, which, on the one hand prevents the sum of flows of different products and, on the other hand, tends to emphasize heavier products, neglecting others that may have a higher value (yet less heavy). This last problem is reinforced by the fact that, in what concerns to road traffic, all vehicles below some weight are excluded from the population of which samples are collected. Another important difficulty is related to the classification of goods used by transport statistics, which is not coincident with the National Accounts classification, used in input-output table assemblage. Finally, some flows recorded by transport statistics are not true interregional trade, but rather trade between transport platforms which serve merely as points of departure (or entry) for international exports (or imports). Thus, it is not easy to separate interregional from international trade.

Given the difficulties in dealing with crosshauling, probably the best procedure consists in adopting a methodology which does not require the direct estimation of crosshauling. This can be done if the researcher opts by estimating the content of an O-D matrix comprising both intra and interregional trade (represented in Figure 1. 7), instead of estimating the content of an O-D interregional matrix as the one depicted in Figure 1. 6.

Figure 1. 7 – Intra and Interregional trade flows of commodity *j* from Region *r* to Region *s*: x_j^{rs}.

Destination / Origin	**Region 1**	**Region 2**	**…**	**Region k**	**Sum**
Region 1	x_j^{11}	x_j^{12}	$\dots$	x_j^{1k}	$\left(v_j^1 + m_j^{row\,1}\right)$
Region 2	x_j^{21}	x_j^{22}	$\dots$	x_j^{2k}	$\left(v_j^2 + m_j^{row\,2}\right)$
…	$\dots$	$\dots$	$\ddots$	$\dots$	$\dots$
Region k	x_j^{k1}	x_j^{k2}	$\dots$	x_j^{kk}	$\left(v_j^k + m_j^{row\,k}\right)$
Sum	$\left(D_j^1 + d_j^{1\,row}\right)$	$\left(D_j^2 + d_j^{2\,row}\right)$	$\dots$	$\left(D_j^k + d_j^{k\,row}\right)$	$\sum_r \left(v_j^r + m_j^{row\,r}\right) = \sum_s \left(D_j^s + d_j^{s\,row}\right)$

The elements of the matrix in Figure 1. 6 are the same as the ones in Figure 1. 7, except in what concerns to the main diagonal and the row and column totals. The main diagonal in Figure 1. 7 comprises intra-regional trade of product *j*. The row sums represent total supply of product *j* in region *r*, before accounting for interregional imports. The column sums represent total use of product *j* of region *s*, before accounting for interregional exports. These totals correspond exactly to the information that the researcher usually gets when the regional table is assembled, before proceeding to the estimation of interregional trade. In fact, as explained in section 1.6.2, net exports are obtained by the difference between these two totals (see equation (1. 87)). It is clear that no values of gross exports or gross imports are needed *a priori* (at the interregional level) to apply such methodology; thus, the problem of crosshauling is avoided. Using this approach, the problem of estimating interregional trade is solved in only two steps: (1) regional input-output table assemblage, without accounting for interregional trade and (2) estimation of intra and interregional trade flows (fulfilling the matrix of Figure 1. 7). When this matrix is fulfilled, the researcher has access to the *a posteriori* values of gross imports and gross exports of product *j*, for each region: they correspond to the column sums and row sums (respectively) of the off-diagonal values of the matrix. Though, the major problem with this approach consists in finding the adequate model to fulfill the inner part of the matrix in Figure 1. 7. As it will be seen in Chapter 2, interregional trade estimation is already a very complex problem when the objective is to estimate a matrix such as in Figure 1. 6. When it comes to estimate a matrix like the one in Figure 1. 7 additional difficulties arise; for example, in models that use distance between regions as one of the explaining factors of interregional trade, one of the problems relies in finding a proper way to compute intra-regional distance, in order to estimate the main diagonal elements of the matrix.

From the previous paragraph, it seems that the non-survey techniques presented on section 1.6.2 are dispensable. This is true, if we are dealing with a multi-regional system of input-output tables, to be used in multi-regional input-output analysis. In this case, the researcher can follow directly from the table assemblage stage, where $v_j^1, \cdots, v_j^k$ and $D_j^1, \cdots, D_j^k$ were estimated, to the fulfillment of the intra and interregional trade matrices. However, if the researcher is interested in performing input-output analysis over one single-region, for instance for region 1, then only v_j^1 and D_j^1 are achieved in the first step of assemblage. Thus, he/she will need to go through the first two steps mentioned in section 1.6.1: after assembling the regional table, the net balance of trade for each product is computed and then crosshauling must be estimated, in order to get gross values of exports and imports.

Turning back to the many-region case, we still have to deal with the problem of fulfilling the O-D matrix (either comprised only of interregional trade or of intra and interregional trade), using the known margin totals as restrictions of the model. The presentation, discussion and empirical assessment of the proposed models to solve this problem, is left to Chapter 2. Such models are generally termed spatial interaction models.

1.7. Conclusions.

In this Chapter we had the main objective of making a broad and critical review of the state of knowledge regarding input-output modelling and input-output table construction at the regional level. In all sections of this Chapter we were concerned with the practical applicability of the models and techniques proposed by the literature, having in mind the quantitative and qualitative disagreement that usually exists between the required and the available data. Such review was essential to systematize ideas and present the primary concepts which will be dealt with in the subsequent Chapters. Being so, we tried to be parsimonious in this theoretical review (leaving out some important topics of input-output analysis – as for example, closing the model with respect to households, supply-side models, dynamic models, and so on), so that it could be concise and practical oriented.

From this review, seven main conclusions may be drawn, presented in the following paragraphs.

(1) First, it is evident that the input-output framework continues to be intensively studied and empirically applied, in spite of its limitations, related to the set of hypotheses underpinning the model. This means that the limitations of this framework are transcended by its two main strengths: it is a fundamental tool for economic analysis (concerning the input-output model) and it comprises a considerably detailed statistical instrument (the input-output table).

(2) Second, the adaptation of the input-output framework to the regional level is extremely important, since regional features are specific and regional problems

may differ considerably from national problems (Miller and Blair, 1985). For example, the dependence on imports (to provide regional needs for commodity supply) and on exports (to drain regional production) tends to be much more relevant at the smaller regional level than at the national level (Munroe and Hewings, 1999). For this reason, one of the most important tasks in the construction of regional input-output tables consists precisely in the assessment of the region's exports and imports, which comprise trade flows established with the remaining regions of the same country and also with other countries. In this context, the problem relies on the estimation of interregional trade flows, since international exports and imports are usually provided by official statistical sources.

(3) Third, whenever the economic system under study includes more than one region, the adequate input-output model to apply in this context must be capable of accounting for the effects caused by interregional linkages – spillover and feedback effects (Miller, 1998). The fundamental contributions of the regional field of input-output analysis, which emerged in the 1950's, have been directed to the accomplishment of this objective, through the proposal of different versions of many-region models. These different many-region models (of which the most important were reviewed in section 1.3) reflect different attitudes concerning the trade off between the detail degree in describing interregional linkages and the demand of trade data. In this context, the most data demanding many-region model is the Isard's interregional input-output model, which is also the one that attempts to describe interregional trade flows with a higher detail. In opposition, the Chenery-Moses multi-regional model applies certain hypotheses in order to avoid such a high demand for observed interregional trade data. More precisely, it uses the import proportionality assumption, which states that the percentage of imports comprised in the regional demand for some specific product is the same, regardless of the type of intermediate or final use of that product, being given by the share of imports in the total supply of that same product. Still, a certain amount of interregional trade data is always required to the implementation of such model: more precisely, it requires a complete origin-destination matrix for

each commodity, comprised of shipments from all possible origins to all possible destinations (without specifying the type of user in the destination region).

(4) The database for input-output model implementation consists of the correspondent input-output matrix (or system of matrices, in the case of many-region models). Yet, whereas at the national level the input-output tables are regularly provided by the official statistics, according to standardized rules, the same does not apply to the regional dimension. For that reason, the construction of regional input-output tables continues to be, by itself, one of the most debated themes in regional literature. In the review of the proposed survey, non-survey and hybrid techniques of input-output table construction, we found that, currently, it is very difficult to find tables which are exclusively survey or non-survey (Dewhurst, 1990). In fact, on the one hand, pure non-survey tables are criticized for being extremely mechanical, neglecting all the specific regional features that the regional input-output table intends to capture. Besides, there is a minimum of survey regional data (concerning, for example, regional output and regional value added by industry) which is usually available from official organisms of statistics, making it possible to incorporate such direct data, even for a single-person team research, with very low budget to table construction. On the other hand, pure survey methods involve several difficulties; the most often mentioned are its high requirements in time, money, human and logistic resources. But other problems must be taken into account when evaluating the possibility of survey gathering of input-output data (Jensen, 1980; Jensen, 1990). Besides some errors that may occur in the process of gathering the data, there is a specific problem that cannot be surpassed by the allocation of more money or other resources to the survey task: it consists simply of the fact that some questions that must be included in the questionnaires require very detailed information to which some respondents may not be able to answer. This problem was illustrated in the special context of the assessment of the proportion of imported products comprised in the intermediate and final use flows. This practical difficulty, sometimes, forces the official organisms of statistics

themselves to adopt some hypotheses, as surrogates of the information they cannot obtain from surveys.

(5) The accuracy assessment of the constructed regional input-output table (or of any component of it) is a quite controversial matter. Firstly, because the benchmark for comparison (usually a survey table for the same economy) may be either inexistent or it may suffer from its own accuracy problems. Secondly, in spite of the important contribution of Jensen (1980), the adequate concept of accuracy to consider in each situation is still not consensual. Finally, there are multiple measures of comparison between two tables, being the choice upon one of them one more subject of debate.

(6) Besides the difficulties created by inexistent data (as in the case of interregional data), another practical challenge faced by input-output researchers is qualitative mismatch between the existing data and the model requirements. The fact is that, sometimes, input-output data is provided in a different way from that underlying the pioneering input-output models. Thus, input-output models must be adapted in order to fit into the specific format in which information is available. We addressed this issue, even in a preliminary approach, focusing on two specific topics: (1) the adaptation of the input-output model and of the techniques for regional input-output construction to the Make and Use format and (2) the use of techniques to estimate intra-regional flows from total use flows (those which include imported and regionally produced products), when no import matrices exist *a priori*. We were able to exemplify, for the national and the single-region case, how the input-output model can, under some hypotheses, be adapted to fit the Make and Use format. Concerning the second topic, we concluded that different non-survey or partial survey techniques can be used to convert total flows into intra-regional flows. The adaptation of the input-output model to the Make and Use format and the adoption of hypotheses to deal with total flow tables consist of some of the core subjects to be further developed in Chapter 3 of this work.

(7) The problem of estimating interregional trade comprises different stages, which differ according to the number of regions under study. When dealing with single-region tables, the first step consists in estimating net interregional trade flows. The analysis made in section 1.6.2 of the different techniques to achieve this goal suggests that Commodity Balance should be preferred over Location Quotient, since: (1) it takes into account the specific structure of demand estimated to the region under study, while *LQ* assumes that such structure is spatially invariant and (2) conversely to *LQ*, it doesn't force the sum of interregional net exports – that means, the regional trade balance – to be null. After having estimated net trade flows, crosshauling must be accounted for and gross exports and gross imports must be estimated. Nevertheless, the estimation of crosshauling rates remains a problem to which no direct answers exist. When we are dealing with a multi-regional system, the problem of estimating interregional trade may be addressed in a different manner. In order to avoid the crosshauling problem, the researcher should focus on the estimation of the elements of an intra and interregional trade matrix, for which the margin totals are known. Anyway, the fulfillment of an O-D matrix, using the known margin totals as restrictions of the model, still represents a problem that needs to be solved. Spatial interaction models are targeted to the solution of this kind of problems. The study and empirical evaluation of such models is left to Chapter 2.

1.8. Notation.

Variables:

x_i - output of product *i*;

z_{ij} - Amount of product *i* used as an intermediate input in the production of industry *j*;

w_j - value added in industry *j*;

m_j - total imports of product *j*;

y_i - Final demand for product i (it includes: final consumption, gross capital formation and exports);

x_i^r - output of product i in region r;

e_i^r - regional production of product i;

z_{ij}^r - total amount of product i (regionally produced and imported) used as an intermediate input in the production of industry j, in region r;

z_{ij}^{rr} - amount of regionally produced product i used as an intermediate input in the production of industry j, in region r;

f_i^r - region's final demand for product i produced in region r (including regional requirements as well as exports for any other regions, national or foreign);

y_i^r - regional final demand for product i;

z_{ij}^{rs} - amount of product i coming from region r that is used as an intermediate input by industry j in region s;

x_i^{sr} - amount of product i shipped by region s to region r, without specifying the type of buyer in the region of destination.

R_i^r - total amount of product i available in region r, except for foreign imports;

f_i^{rs} - amount of product i produced in region r and shipped to region s.

$z_{ij}^{\bullet s}$ - total amount of product i (produced in region s and in the other regions of the same country) used as an input by industry j in region s;

v_{ij} - domestic production of product j by industry i (elements of the Make matrix – rectangular model);

u_{ji} - the amount of product j used as an input in the production of industry i's output (elements of the Use matrix – rectangular model);

p_j - total supply of product j (rectangular model);

g_i - domestic production of industry i (sum of the rows of the Make matrix);

AO_j^r - available output in region r to satisfy domestic demand (demand directed to region r and also to the remaining regions of the country).

D_j^r - total requirements of i in region r.

$d_j^{r\,roc}$ - exports from region r to the rest of the country.

$m_j^{roc\,r}$ - imports from the rest of the country to region r.

$NEX_j^r = e_j^{r\,roc} - d_j^{roc\,r}$ - net exports of product j by region r.

i - column vector appropriately dimensioned, composed by 1's.

^ - diagonal matrix.

Superscript ^{row} – coming from (or going to) the rest of the world.

Superscript ^{roc} – coming from (or going to) the rest of the country.

Coefficients:

a_{ij} - technical coefficient (at national level);

b_{ij} - generic element of the Leontief inverse matrix;

$b_{\bullet j}$ - output multiplier ($b_{\bullet j} = \sum_i b_{ij}$);

a_{ij}^r - regional technical coefficient; $a_{ij}^r = \dfrac{z_{ij}^r}{x_j^r}$;

a_{ij}^{rr} - intra-regional input coefficient; $a_{ij}^{rr} = \dfrac{z_{ij}^{rr}}{e_j^r}$;

a_{ij}^{rs} - interregional trade coefficient, representing the amount of input i from region r necessary per monetary unit of product j produced in region s; $a_{ij}^{rs} = \dfrac{z_{ij}^{rs}}{e_j^s}$;

t_i^{sr} - trade coefficient, representing the proportion of product i available in region r that comes from region s; $t_i^{sr} = \dfrac{x_i^{sr}}{R_i^r}$;

$a_{ij}^{\bullet s} = \dfrac{z_{ij}^{\bullet s}}{e_j^s}$ - technical coefficient for region s: it represents the amount of product i necessary to produce one unit of industry j's output in region s, considering the inputs provided by all the regions in the system.

$q_{ji} = \dfrac{u_{ji}}{g_i}$ - Technical coefficient in the rectangular model (amount of product *j* used as input in the production of one unit of industry *i*'s output);

$s_{ij} = \dfrac{v_{ij}}{p_j}$ - industry *i*'s market share in product *j*'s total supply.

Matrices and vectors:

$\mathbf{I}$ - identity matrix;

$\mathbf{x}$ - output vector;

$\mathbf{y}$ - final use vector;

$\mathbf{A}$ - technical coefficients matrix;

$\mathbf{B}$ - Leontief's inverse;

$\mathbf{A^r}$ - regional technical coefficients matrix ;

$\mathbf{y^r}$ - regional final demand vector;

$\mathbf{x^r}$ - regional output vector;

$\mathbf{e^r}$ - vector of output produced in region *r*;

$\mathbf{Z^{rr}}$ - matrix if intra-regional intermediate use flows;

$\mathbf{A^{rr}}$ - intra-regional input coefficients matrix;

$\mathbf{f^r}$ - vector of regional final demand for products produced in region *r*.

$\mathbf{A^{rs}}$ - interregional trade coefficient matrix;

$\mathbf{T^{rs}}$ - matrix of trade coefficients t_i^{rs} in the main diagonal;

$\mathbf{Q}$ - technical coefficient matrix (rectangular model);

$\mathbf{g}$ - vector of industries' internal production (rectangular model);

$\mathbf{U}$ - intermediate consumption matrix (rectangular model);

$\mathbf{V}$ - Make matrix (rectangular model);

$\mathbf{S}$ - matrix of market shares s_{ij}; (industry-based technology assumption on the rectangular model);

$\mathbf{p}$ - Vector of products' total supply (rectangular model);

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CHAPTER 2 – DETERMINING INTERREGIONAL TRADE FLOWS IN A MANY-REGION SYSTEM.

2.1 Introduction.

Interregional trade estimation has been extensively pointed out as a crucial problem to overcome when constructing any many-region or many-country input-output table. As it has been explained in the first Chapter of this work, the knowledge of interregional trade flows, at least the pooled volume of exports and imports by commodity, is an essential requirement to allow the consideration of the important spillover and feedback effects caused by interregional linkages. Besides, it is important, for regional analysis purposes, to be acquainted with the magnitude and nature of the economic interdependence between each region and the rest of the world.

In spite of its recognized importance, the fact is that, in most countries, there are no completely reliable survey-based statistics on interregional trade. The most approximate available source of data on interregional trade consists of transport statistics, which, however, are not suited for the needs of input-output table construction, for several reasons (Ramos, 2001; Verduras, 2004; Alward, Olson and Lindall, 1998): 1) they do not cover service trading; 2) transport flows are expressed in physical units, requiring the access to some value / volume relation; 3) flows shipped by manufacturers are not distinguished from flows shipped by resellers, leading to problems of double-counting; 4) regions with transport platforms appear with an over-estimation of trade flows. Because of these problems, transport statistics are only used as an indirect source of data in interregional trade estimation (as, for example, in Schwarm, Jackson and Okuyama, 2006 and in Ferreira, 2008). Given the already mentioned problems associated with the conduction of surveys (see section 1.4.2 of Chapter 1), the alternative consists in using non-survey methods to generate the undisclosed values of interregional trade or, at least, to complement some partial information existing on those flows.

The problem of interregional trade estimation using non-survey methods can be studied under the context of spatial interaction models: the aim is to estimate a set of flows between several origins and several destinations, separated in space. Belonging to the family of spatial interaction models, there are different methodologies which differ not only on the theoretical foundations, but also on their practical applicability, especially determined by data demand issues. It is, therefore, important to make a deep analysis of each of the alternatives, keeping in mind the objective of the model and its adequacy to the problem under study: trade flow estimation.

The use of spatial interaction models (with special emphasis on the gravity model) in the context of interregional trade estimation is not new. Such type of application of spatial interaction models can be found in the literature since Wilson (1970), Kim, Boyce and Hewings (1983), Alward, Olson and Lindall (1998) and, more recently, Schwarm, Jackson and Okuyama (2006) and Ferreira (2008), mentioning only some examples. However, usually each researcher makes an option for some specific type of model, without making a comparison between the results provided by that model and the existing alternatives. This is due to the fact that the evaluation of the model results requires the access to some benchmark values, which are typically unavailable (Hewings and Jensen, 1986; Canning and Wang, 2006) – the inexistence of that data is precisely the motivation for the application of the model in the first place. Yet, we consider that it is paramount to investigate the relative accuracy of the several proposals, in order to evaluate the reasonability of using those non-survey methods as a viable alternative to survey methods, in input-output table construction. Hence, in the present Chapter, the empirical comparison between different interregional trade estimation methods will be made using European countries instead of regions and officially known inter-country trade flows as the benchmark for model accuracy evaluation.

In the present work we will assume the context of very limited information as the data scenario faced by the researcher: it is considered that, concerning trade flows, the only previously known information consists of the total value, by product, that enters into each destination region and the total value also by product that is shipped from each origin region. The motivation to do so relies on the fact that the conclusions of this study are intended to be used by regional input-output assemblers, which in most cases face the obstacle of a complete lack of data on interregional trade flows, except from those values of total exports and imports – usually obtained previously along with the remaining components of the regional input-output table (as explained in section 1.6 of Chapter 1).

The main objectives of the present Chapter are:

- To make a comprehensive review of the proposed models for interregional trade estimation and evaluate the practical applicability of each of the models under a context of very limited *a priori* information.
- To make an empirical comparison between different methodologies of interregional trade estimation. This comparison is guided towards the following research questions:
  - What is the degree of closeness of each estimated matrix to the real matrix of flows?
  - Which method generates the most accurate estimated matrix?
  - How sensitive are the values obtained in the final trade matrix to different estimating methods?
  - How sensitive is the solution of the input-output model to the insertion of different interregional trade values? In other words, how important is the choice of interregional trade estimation method to the solution of the input-output model?

The answers found to these questions are of extreme importance to any researcher who intends to use a non-survey method to estimate interregional trade. For example, if the solution of the input-output model is found to have a low sensitivity to the choice of trade estimating method, then, this can be used as an argument to opt for a simple non-survey method of trade estimation. Conversely, if the choice of method reflects highly on the trade values and also on the results of the input-output model, then, that choice should be made carefully and a pure non-survey method may not constitute a viable alternative.

This Chapter is organized in six sections, including this Introduction. In the second section, the problem under study is presented and framed within the theoretical reasoning of spatial interaction problems. Section 2.3 comprises a review of the most important models belonging to the family of spatial interaction models, discussing their theoretical foundations and also their practical applicability in the context of interregional trade estimation. Section 2.4 is dedicated to a more detailed study of the most frequently used spatial interaction model, namely the gravity model. The often advocated good performance of this model as an explanatory framework is tested, through an extensive econometric application. The fifth section contains the nuclear part of the present Chapter's empirical application: the comparison between different methods of interregional trade estimation. Finally, section 2.6 presents a summary of the main conclusions of this Chapter.

2.2 Interregional trade flow estimation as a spatial interaction problem.

As stated in Hewings, Nazara and Dridi (2004), "interregional interaction is not only inevitable, it is often the key to enhancing the success of a region" (p. 13). Among the different types of interaction that may exist between different regions, our interest is focused on interregional trade. This Chapter is concerned with the specific problem of estimating the values to be inserted into an Origin-Destination (O-D) matrix, depicting – for each product – interregional trade flows from each region of origin to each region of destination.

It is assumed that the column and row totals are previously known. Moreover, since only interregional trade flows are being estimated (and not intra-regional ones), the main diagonal values are previously set equal to zero. Let's assume a system with *k* regions of origin (denoted by a superscript *r*) and *k* regions of destination (denoted by a superscript *s*). Then the problem consists in estimating the interregional shipments of *j*, x_j^{rs}, $r, s = 1, \cdots, k$, as illustrated by Figure 2. 1.

Figure 2. 1 – Interregional trade flows of commodity *j* from Region *r* to Region *s*: x_j^{rs}.

Origin \ Destination	Region 1	Region 2	...	Region k	Sum
Region 1	0	x_j^{12}	$\dots$	x_j^{1k}	$d_j^{1\,roc}$
Region 2	x_j^{21}	0	$\dots$	x_j^{2k}	$d_j^{2\,roc}$
...	$\dots$	$\dots$	0	$\dots$	$\dots$
Region k	x_j^{k1}	x_j^{k2}	$\dots$	0	$d_j^{k\,roc}$
Sum	$m_j^{roc\,1}$	$m_j^{roc\,2}$	$\dots$	$m_j^{roc\,k}$	$d_j = m_j$

This is a typical problem to be addressed by models generally called as spatial interaction models (Alward, Olson and Lindall, 1998). Broadly speaking, the goal of these models is to explain and/or estimate spatial interaction flows, defined as the movement or communication between different spaces. This interaction implies a decision taken after a cost-benefit analysis, in which the individual evaluates the trade-off between the benefit from the movement (related to the motivation that causes it) and the cost of that same movement (which corresponds to the traveling across the spatial separation between his / her origin and the several destinations) (Fotherigham and O'Kelly, 1989). Besides trade flows, in which we are specifically interested, spatial interaction models deal with a vast collection of flows, such as: migration, information flows, traffic flows and commuting movements, among others. In presence of such a variety of applications, there is no specific type of model which is superior to all the remaining, whatever the topic that it is applied to. Being so, in each particular circumstance, the researcher must decide which is the most adequate, among all the proposed models (Isard, 1998). Yet, one of the conditions which may restrict the choice of the researcher is the context of information availability. We can distinguish two possible information contexts, which we have decided to name as Type (a) and Type (b):

(a) *Type (a) information context*: spatial interaction flows are **known** *a priori*, at least for some time period previous to the one that the researcher wants to study.

(b) *Type (b) information context*: Spatial interaction flows are totally **unknown** *a priori*.

As it will be made clear in the following section, some of the spatial interaction models, namely the gravity model, may be applied in any of these two different contexts, yet with different research objectives, while others, like information theory models, require the existence of some previous information on the spatial interaction flows.

Regardless of the specific goal of the investigation, there are four fundamental elements that are common to every spatial interaction model (Fotherigham and O'Kelly, 1989):

- One interaction matrix, **T**, structured as in Figure 2. 1, consisting of spatial interaction flows to be determined, between region *r* and region *s*, T^{rs}, with *k* rows (origins) and *k* columns (destinations)[56];
- One matrix **C**, of the same dimension as **T**, of which elements δ^{rs} represent spatial separation between each origin *r* and each destination *s* (this separation can be measured in several ways, like physical distance, transportation costs, etc).
- One or more ϖ^{r} variables for each origin *r* ($r = 1, \cdots, k$), called propulsion measures of origins; these are measures that influence the volume of flows that leaves each

[56] Since, in this Chapter, spatial interaction flows will always refer to flows of some commodity *j,* we will simplify the notation, avoiding the use of subscript *j;* in this case, we write T^{rs} as a simplification of T_j^{rs}.

origin; for example, if one is dealing with migration movements, one of the variables to be considered here is, necessarily, the unemployment rate at each origin.

- One or more ϖ^s variables for each destination s ($s = 1, \cdots, k$), which are attraction measures for destinations. Again, if migration is the subject under study, the unemployment rate at destination *s* will also be one of the variables to account for, yet having here a negative influence on the attractiveness of that destination. In spite of this example, generally the variables used to measure the potential of attraction in the destinations may be different from the ones used to capture the propulsion force of origins.

The function of any spatial interaction model is to relate the values on matrix *T* (the dependent variable) with variables ϖ^r, ϖ^s and δ^{rs}. If one admits, for simplicity sake, that there is only one propulsion variable for each origin and one attraction variable for each destination, the purpose consists in finding the appropriate form to the following function:

$$T^{rs} = f\left(\varpi^r; \varpi^s; \delta^{rs}\right), \tag{2. 1}$$

in which T^{rs} represents the spatial interaction flow between *r* and *s*.

In the development path of investigation in spatial interaction models, several specific forms have been proposed to the general function in (2. 1), which can be influenced by practical application issues or by the theoretical foundations in which they rely. On the following sections, the intention is to review the most important models belonging to the family of spatial interaction models. We present them in a stepwise fashion, beginning by the pioneering approach, that is, by the gravity model. This model will deserve a good deal of our attention, since it is still one of the most attractive among spatial interaction models, especially in empirical applications to trade. Then, we proceed with probabilistic models, established on the concepts of entropy and information theory and, at last, with a brief reference to behavioural models. In each model, we will present the underlying theoretical concepts and discuss their applicability in the specific study of trade flows.

2.3 Theoretical review of the spatial interaction models proposed to estimate trade flows.

2.3.1 Gravity models.

The original spatial interaction models to be applied to human interactions were based on the analogy with physical interaction among particles. In the 1930's, Newton's gravitational law[57] was applied by Reilly, for the first time, to the study of human interactions, more precisely, to trade flows, establishing that "two cities attract trade from an intermediate town in the vicinity of the breaking point, approximately in direct proportion to the population of the two cities, and in inverse proportion to the squares of the distances of the intermediate town"[58]. When we apply gravitational law to trade, we meet:

$$x^{rs} = G\frac{(P^r)(P^s)}{(\delta^{rs})^2}$$

(2. 2)

in which: x^{rs} represents exports from origin r to destination s, G is a constant of proportionality, P^r and P^s express the populations of origin r and destination s and δ^{rs} represents spatial separation – the distance – between each origin r and each destination s.

[57] According to which the attraction between to bodies, *r* and *s* is directly proportional to their masses and inversely proportional to the distance between them, in the specific form: $F_{rs} = G\frac{m_r m_s}{\delta_{rs}^{\ 2}}$.

[58] *Reilly, W. 1931. "The law of retail gravitation". New York: Pilsbury, republished in 1953,* p. 9. Referred in Batten and Boyce (1986), p.360.

We can also see that this equation consists of a very particular case of equation (2. 1); in this case, the propulsion and attraction variables are represented by population at the origin and at the destination, respectively, and spatial separation is measured by the distance between the origin and the destination.

In the 1940's, this model knew a great development explaining and forecasting spatial interaction among populations. The mathematical simplicity and intuitive nature of the gravitational hypothesis, as well as the reasonability of the results produced by its application were at the basis of the success this model had among researchers (Sen and Smith, 1995). The growing application of gravitational model to different kinds of spatial interaction led investigators to seek for the specific formulation that generated the best results to the matter under scrutiny, moving away from the rigidity associated to Newton's formula (Roy and Thill, 2004). The first step that was taken in order to enhance flexibility of gravity model was to consider that the exponent of distance between each origin r and each destination s should not be fixed at any specific value. This exponent expresses the distance friction effect, i.e, the flow's sensibility to spatial separation. Though Newton's gravitational formula indicates 2 as the proper exponent, there is no reason to transfer this value to other kinds of interaction. Thus, it is consensual that such exponent should be estimated in each particular case. Isard and Bramhall (1960), for example, reflect this generalization in the gravitational formula that they propose:

$$x^{rs} = G\frac{\left(P^{r}\right)\left(P^{s}\right)}{\left(\delta^{rs}\right)^{\alpha}}$$

(2. 3)

Besides this generalization, other adjustments were subsequently made to equation (2. 3), involving the debate on several issues, such as: the use of alternative variables to measure each region's mass, instead of population; the use of alternative measures of distance; the use of specific weights related to the origin and destination masses, instead of considering them to be unitary, *a priori* and, finally, the use of alternative formulations to express the influence of spatial separation on spatial interaction – the so-called distance deterrence function (Batten and Boyce, 1986). The discussion of each of these issues is presented in the next paragraphs.

One of the problems is the selection of the specific measure to the mass variable: it must be chosen in such a way that assures it is the proper one to the actual problem under study. For example, when dealing with migration studies, the relevant measures should be regional income or regional unemployment, for instance, and not regional population (Isard, 1998).

The researcher should also adapt the choice of spatial separation measures to the interaction phenomenon at hand. For example, traveling time is a more adequate spatial interaction deterrence measure to urban traffic studies than physical distance. In other contexts, broader definitions of distance are used; this is the case of social distance (Isard and Bramhall, 1960), a concept that intends to evaluate the influence of several factors, such as cultural, political, religious (or other) patterns on certain kinds of social interactions, like migration or even marriages. In this case, distance may be measured in ordinal terms and not in cardinal terms, i.e., the number defining the distance measure simply represents the degree of proximity between two spaces or two individuals, having no meaning when taken in an isolated manner (Sen and Smith, 1995).

The issue of finding the adequate measures to the variables of the model is not the only one involved in this debate. The functional form in which these variables should be presented continues to be a matter of great discussion (Dentinho, 2002). For example, instead of assuming implicitly that both masses receive a similar weight (equal to one), as in equation (2. 3), it may be proved empirically that non-unitary weights are more suitable in explaining the specific interaction flows under study. Considering this generalization into equation (2. 3), we get:

$$x^{rs} = G\frac{\left(P^{r}\right)^{\alpha_1}\left(P^{s}\right)^{\alpha_2}}{\left(\delta^{rs}\right)^{\alpha_3}} \tag{2. 4}$$

in which weights α_1 and α_2 are assigned to the masses of origin and destination, respectively, δ^{rs} represents spatial separation between each origin *r* and each destination *s* and α_3 is the distance decay parameter.

The last issue raised by gravitational model relates to the functional form to represent spatial separation. There are two predominant forms that can be found in the literature on this matter: the power function and the exponential function (Fotherigham and O'Kelly, 1989). The power function is the one that has been used in the previous presentation of the gravity model; in this case, distance (or other spatial separation variable) is raised to an exponent: $(\delta^{rs})^{\alpha_3}$. If the exponential functional form was used to represent spatial separation, instead of the power functional form, equation (2. 4) would be:

$$x^{rs} = G\frac{(P^r)^{\alpha_1}(P^s)^{\alpha_2}}{\exp(\alpha_3\delta^{rs})} \tag{2. 5}$$

The choice between these two functional forms of spatial separation is mostly an empirical issue. The researcher must analyze, in each situation, whether there is an exponential or a power functional form for the negative relationship between spatial separation and spatial interaction. However, there are two other features that should be borne in mind when choosing one or another functional form (Isard, 1998): 1) the economic meaning of parameter α_3 and 2) the behavior of each function when distances are very small.

As to the first item, we must realize that α_3 can be interpreted as elasticity in power functions, but not in exponential functions. In fact, if we take the logarithmic form[59] of equation (2. 4) and (2. 5), we get, respectively:

[59] We use notation $\ln$ to represent $\log_e$.

$$\ln x^{rs} = \ln G + \alpha_1 \ln P^r + \alpha_2 \ln P^s - \alpha_3 \ln \delta^{rs}$$

(2. 6)

and

$$\ln x^{rs} = \ln G + \alpha_1 \ln P^r + \alpha_2 \ln P^s - \alpha_3 \delta^{rs}$$

(2. 7)

Being so, in equation (2. 6), $\alpha_3 = \frac{\partial \ln x^{rs}}{\partial \ln \delta^{rs}}$, meaning spatial interaction elasticity to distance. This is not the case in equation (2. 7), where distance is not in logarithm form. This feature constitutes an advantage of power functional form over exponential form, in particular in what concerns to trade applications. In this case, it is interesting to infer the trade flow elasticity to distance, especially when different commodities are being studied. We will get back to this issue on Section 2.4.

The second item referred above is related to the fact that power functional form has the disadvantage of overestimating spatial interaction value when distance between origin and destination tends to zero (since $\frac{1}{(\delta^{rs})^{\alpha_3}}$ tends to infinity as δ^{rs} tends to zero). Exponential function does not face this problem, because, when δ^{rs} tends to zero, $\frac{1}{\exp(\alpha_3 \delta^{rs})}$ tends to 1. This feature of the power functional form may create problems whenever the objective involves the estimation of spatial interaction flows between different spaces, but also within the same space. This occurs, for instance in problems of intra and interregional trade estimation (like the one illustrated in section 1.6.3 of Chapter 1). But, even in these cases, intra-regional distance may be calculated, instead of being assumed a null distance.

In spite of the importance of the theoretical debate, the option for the suitable gravity equation is more an empirical matter than a theoretical one. In what concerns to trade flow empirical studies, equations such (2. 4) are the most commonly applied, using GDP or related variables to express the size of the origin and of the destination. The specific way in which this basic concept is applied in the study of trade flows depends, however, on the information context that is faced by the researcher (previously mentioned as type (a) and type (b) information contexts). When trade flows are known *a priori* (type (a) information context) the model is usually used to explain trade flows' behaviour, through econometric modelling; conversely, in type (b) information context, the model is applied in order to assess the unknown flows.

In type (a) information contexts, equation (2. 6) can be the starting point to a regression equation like:

$$\ln x^{rs} = \ln G + \alpha_1 \ln Y^r + \alpha_2 \ln Y^s - \alpha_3 \ln \delta^{rs} + \varepsilon^{rs}$$

(2. 8)

in which Y^r and Y^s represent the gross domestic product at the region of origin and of destination, respectively, ε^{rs} stands for the error term and the remaining variables are defined as above referred[60].

In type (b) information contexts, gravity model is used with the aim of generating the unknowns x^{rs} of an Origin-Destination matrix like the one in Figure 2. 1. Using an equation similar to equation (2. 4), interregional trade flows first estimative, denoted by $\tilde{x}^{rs}$, could be obtained by[61]:

$$\tilde{x}^{rs} = G \frac{\left(Y^r\right)^{\alpha_1}\left(Y^s\right)^{\alpha_2}}{\left(\delta^{rs}\right)^{\alpha_3}}.$$

(2. 9)

[60] As it will be seen in section 4, this basic equation can be modified, either by the addition of other explicative variables, or by using a specific formulation which allows the incorporation of spatial dependence effects.

[61] By now, we are assuming that parameters G, α_1, α_2 and α_3 are known values.

However, in most cases, the column and row totals of Figure 2. 1 (meaning: the sum of all inflows into each destination and of all outflows from each origin) are previously known. In this case, the estimated values must verify the following additivity constraints: $\sum_s \tilde{x}^{rs} = d^r$ and $\sum_r \tilde{x}^{rs} = m^s$, and the model is doubly-constrained (Isard, 1998)[62]. What happens, yet, is that the substitution of the known values of variables Y^r, Y^s and δ^{rs} will most certainly produce a matrix of flows in which the row and column totals do not match with the *ex ante* values. The agreement with the additivity constraints can, then, be assured through an iterative procedure of bi-proportional adjustment, alike to RAS technique[63] (Batten and Boyce, 1986). After the iterative procedure convergence, the adjusted flows can be expressed by:

$$\left(\tilde{x}^{rs}\right)_{RAS} = J^r \tilde{x}^{rs} L^s = J^r G \frac{\left(Y^r\right)^{\alpha_1}\left(Y^s\right)^{\alpha_2}}{\left(\delta^{rs}\right)^{\alpha_3}} L^s$$

(2. 10)

in which J^r and L^s are the product of different values that, in each iteration, aim to adjust the row and column results, respectively, to the previously known totals. As we shall see, parameters like J^r and L^s, called balancing parameters, always appear in the solution of any doubly-constrained spatial interaction model, either it is a gravitational model or not. These parameters have the mission of assuring the verification of additivity constraints.

Obviously, the precise results obtained by the RAS-type iterative procedure are determined by the values considered in the starting matrix. In fact, taking the general case of *k* origins (and destinations), the problem corresponds to a system with *2k* additivity

[62] Other types of problems may be derived: (1) production-constrained, when only the row additivity constrain is considered, $\sum_r \tilde{x}^{rs} = m^s$; (2) attraction-constrained, when only the column additivity constrain is considered: $\sum_s \tilde{x}^{rs} = d^r$; and unconstrained, which obviously results when no restrictions are taken into account (Fotherigham and O'Kelly, 1989).

[63] Please see Chapter 1, section 4.3.1, for details on the RAS technique.

restrictions and k^2 elements x^{rs} to be determined[64]. Whenever $k > 2$, this system admits several solutions (Lahr and de Mesnard, 2004). Yet, the solution provided by RAS iterative procedure tends to preserve, as much as possible, the structure of the initial matrix, changing it only in a minimum amount necessary to respect the row and column sum constraints (Jackson and Murray, 2004). In fact, as it was referred in section 1.4.3.1 of Chapter 1, the solution of RAS corresponds to the solution of a problem of minimization of information bias, meaning that the target matrix is generated in order to be as close as possible to the prior matrix and, at the same time, respect the row and column sum constraints[65]. This makes evident that the starting matrix is determinant to the final solution, leading us back to the fundamental issue: finding the adequate model to accurately generate the first estimate of interregional flows, i.e., finding the initial matrix.

The potential extensions to the basic gravity equation used in Type (a) studies as well as the difficulties faced by the researcher when trying to apply the gravity model in Type (b) information context will be deeply analyzed in section 2.4.

2.3.2 Entropy-based models.

In the beginnings of 1970's there was a new line of research in spatial interaction, in which we must emphasize the work of Wilson (1970), for his pioneering contribute. The use of entropy maximizing principles originated the development of the so-called probabilistic family of spatial interaction models (O'Kelly, 2004).

Entropy-based models are suitable to be used as type (b) spatial interaction models, that means, whenever the objective is to estimate interaction flows, having no previous information on their value.

[64] If the main diagonal elements are *a priori* considered to be null, then there will be only $k^2 - k$ elements to be determined.
[65] We will analyze problems of minimization of information bias with more detail in section 3.3.

To better understand the notion of entropy when applied in the context of spatial interaction models, it is useful to introduce the concepts of micro state and macro state of a spatial interaction system (Snickars and Weibull, 1977). Let us use an example. Suppose that there are $T = 4$ individuals performing commuting movements between two regions, *A* and *B*. These movements can be represented either at a micro level or at a macro level. A micro level description of the movements (micro state) consists of describing the exact movement taken by each of the four individuals. An example of a micro state in this system could be the one illustrated in Figure 2. 2: individual 1, 2, 3 and 4 are making the same movement, from *A* to *B* (let this be micro state *a*). Another possible micro state could be micro state *b*, in Figure 2. 3.

Figure 2. 2 – Micro state *a*.

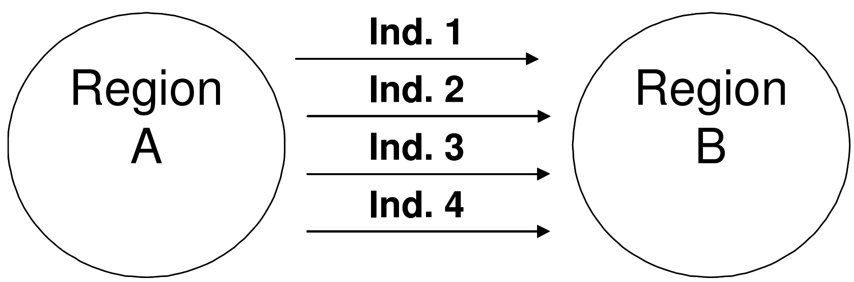
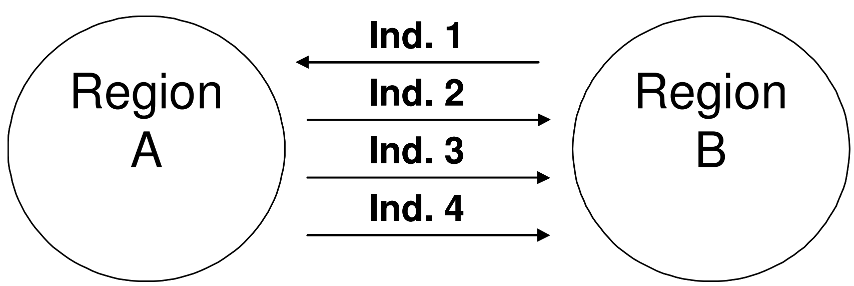


Figure 2. 3 – Micro state *b*.



At all, we could find 16 different micro states like these in our example. The macro state correspondent to micro state *a* can be expressed by the following matrix $\mathbf{R}$, with elements T^{rs}, $r = A, B;\ s = A, B$: $\mathbf{R} = \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix} \end{array}$, where $\sum_r \sum_s T^{rs} = T = 4$.

So, the macro state is represented by a matrix $\mathbf{R}$ of spatial interactions T^{rs}, which counts the number of individuals in each cell. It is clear that every macro state is a "reflection of individual actions taken on a disaggregate or micro level" (Snickars and Weibull, 1977, p.137). Further, in principle, every micro state that is compatible with the information on the number of individuals (T) is equally probable. Being so, the probability of an arbitrary macro state is proportional "to the number of micro states that yield that very macro state on aggregation" (Snickars and Weibull, 1977, p.138).

It should be stressed that spatial interaction entropy models are applied in a context where there is no information about the micro level events and, additionally, there is no *a priori* observations available at the macro level. The goal of these models is precisely to obtain this aggregate information.

Following with our example, the number of micro states consistent with macro state $\mathbf{R}$ is equal to 1. In other words, there is only one situation in which every cell is null except from T^{AB} and that occurs when all the four individuals travel from A to B. Let $w(\mathbf{R})$ be the number of micro states consistent with some $\mathbf{R}$ macro state. Using combinatorial analysis to obtain a general formula, we can state that:

$$w(\mathbf{R}) = \frac{T!}{\prod_r \prod_s T^{rs}!}$$

(2. 11)

Applying (2. 11) to the example above, we have $w(\mathbf{R}) = \frac{4!}{4!} = 1$.

The entropy maximizing principle consists in choosing the most probable distribution (or macro state) $\mathbf{R}$, which is the one that can be replicated by the maximum number of micro states, i.e.: $Max \quad w(\mathbf{R})$ (Batten and Boyce, 1986; Nijkamp and Paelink, 1974).

For this reason, entropy maximizing model is included in the probabilistic class of spatial interaction models (Nijkamp and Paelink, 1974).

Looking back to our commuting example, the most probable distribution of the commuting movements would be: $\mathbf{R}^* = \begin{array}{c} \\ A \\ B \end{array}\!\!\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \end{array}$, with $w(\mathbf{R}^*) = \frac{4!}{2! \times 2!} = 6$. In fact, this solution implicitly involves two constraints, associated to the information comprised in our concrete example: (1) the total number of commuting individuals is known and the solution must respect that previous knowledge: $\sum_r \sum_s T^{rs} = T = 4$ and (2) there are no commuting movements from a region to itself, i.e., $T^{AA} = T^{BB} = 0$. Otherwise, the unconstrained maximization of $w(\mathbf{R})$ would lead to: $\mathbf{R}^{**} = \begin{array}{c} \\ A \\ B \end{array}\!\!\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$, with $w(\mathbf{R}^{**}) = \frac{4!}{1! \times 1! \times 1! \times 1!} = 24$. From this example, it is clear that, in principle, the free maximization of the entropy of any spatial interaction matrix leads to an even distribution of the individuals among the cells of the matrix. It is the introduction of some constraints that makes the solution to deviate from a uniform distribution. The solution will, then, maximize entropy or dispersion, given the known restrictions. The resulting maximum entropy is obviously less than the maximum entropy obtained from an unconstrained maximization (in our simple example, we get an entropy of 6, compared to 24, respectively).

In practice, actually, the maximized equation is the logarithmic transformation of equation (2. 11), that is:

$$\ln w(\mathbf{R}) = \ln T! - \sum_r \sum_s \ln T^{rs}!$$

(2. 12)

Using Stirling[66] approximation for factorials and dropping $\ln T!$ (since that, being a constant, it will not interfere on the maximization process), the final version of the objective function is:

$$\underset{T^{rs}}{Max} \quad \ln w(\mathbf{R}) = -\sum_r \sum_s \left(T^{rs} \ln T^{rs} - T^{rs} \right)$$

(2. 13)

If we are dealing with a doubly-constrained problem, in which total flows leaving each origin (O^r) and total flows arriving each destination (D^s) are known, two additivity constraints must be considered. Besides that, as long as the data allow it, we must also take into account a restriction which is related to the behaviour of the individuals that we are studying (Roy and Thill, 2004); for instance, if we are dealing with commuting or trade, one relevant restriction to take into the model is transportation cost restriction, given by $\sum_r \sum_s \pi^{rs} T^{rs} = C$, in which π^{rs} represents the (known) unitary transportation cost of traveling from r to s and C represents total transportation budget. Thus, the entropy maximizing problem may be stated as follows:

$$\underset{T^{rs}}{Max} \quad \ln w(\mathbf{R}) = -\sum_r \sum_s \left(T^{rs} \ln T^{rs} - T^{rs} \right) \quad subject \quad to$$
$$\sum_s T^{rs} = O^r$$
$$\sum_r T^{rs} = D^s$$
$$\sum_r \sum_s \pi^{rs} T^{rs} = C$$

(2. 14)

The correspondent Lagrangean function is:

[66] According to which $\ln x! = x(\ln x - 1)$ - see, for example, Wilson (1970), p. 271.

$$L\left(T^{rs},\lambda^r,\mu^s,\beta\right)=-\sum_r\sum_s\left(T^{rs}\ln T^{rs}-T^{rs}\right)+\lambda^r\left(O^r-\sum_s T^{rs}\right)+\mu^s\left(D^s-\sum_r T^{rs}\right)+$$
$$+\beta\left(C-\sum_r\sum_s\pi^{rs}T^{rs}\right)$$

(2. 15)

in which λ^r stands for the Lagrangean multipliers related to constraints of origin *r*, μ^s, are the Lagrangean multipliers related to constraints of destination *s*, and β is the Lagrangean multiplier related to cost constraint.

Working out the first-order partial derivates (for the unknowns λ^r, μ^s, β and T^{rs}) and equating these to zero, we obtain a system of $\left(k^2+2k+1\right)$ equations, being k the number of origins and of destinations. These equations are divided by: k equations of additivity constraint at the origin, k equations of additivity constraint at the destination, 1 equation concerning the cost constraint and, finally, k^2 equations[67] concerning the first-order partial derivate with respect to the spatial interaction flows T^{rs}, expressed as:

$$\frac{\partial L}{\partial T^{rs}}=0\Leftrightarrow-\left(\ln T^{rs}+T^{rs}\frac{1}{T^{rs}}-1\right)-\lambda^r-\mu^s-\beta\pi^{rs}=0\Leftrightarrow$$
$$\ln T^{rs}=-\left(\lambda^r+\mu^s+\beta\pi^{rs}\right)\Leftrightarrow$$
$$T^{rs}=\exp\left[-\left(\lambda^r+\mu^s+\beta\pi^{rs}\right)\right]$$

(2. 16)

If we multiply and divide the right member of the solution in (2. 16) by O^r and D^s, we may re-write the previous solution in the following way:

$$T^{rs}=J^rO^rL^sD^s\exp\left(-\beta\pi^{rs}\right)$$

(2. 17)

[67] As it was illustrated in Figure 1, in many spatial interaction problems, it is *a priori* assumed that there are no spatial interaction flows from the region to itself. In such case, there will be only k^2-k equations like equation (2. 16).

in which $J^r = \dfrac{\exp(-\lambda^r)}{O^r}$ e $L^s = \dfrac{\exp(-\mu^s)}{D^s}$.

Finally, replacing this solution in the additivity constraints at origin and destination, we may re-write parameters J^r and L^s as:

$$J^r = \frac{1}{\sum_s L^s D^s \exp(-\beta\pi^{rs})} \tag{2.18}$$

and

$$L^s = \frac{1}{\sum_r J^r O^r \exp(-\beta\pi^{rs})}. \tag{2.19}$$

Let's analyze this solution carefully. First, we can observe that equation (2. 17) fits into the general specification of spatial interaction models expressed by equation (2. 1): it defines spatial interactions as a function of origin-related and destination-related variables and also of spatial separation. Second, let's compare it with equation (2. 10). Equation (2. 10) corresponds to the solution of the doubly constrained gravitational model, presented in the previous section. In fact, if we take into account some common structural features between those two equations, solution (2. 17) resembles a gravitational specification: spatial separation function (here expressed in exponential form, instead of being expressed through a power function) is multiplied by the masses of origin and of destination and by the balancing parameters J^r and L^s, which assure that the model verifies the additivity constraints (just like they did in equation (2. 10)). This allows us to conclude that entropy maximizing method provides a theoretical framework supporting the use of the gravitational formula (Dentinho, 2002; Batten and Boyce, 1986; Wilson, 1970).

Based on this entropy maximizing model, Wilson has produced a family of solutions, in which each one differs by the number and type of considered restrictions (Wilson, 1970): doubly-constrained, which corresponds to the solution previously derived, production-constrained (considering only the origin constraint), attraction-constrained (considering only the destination constraint) and unconstrained (in which no restriction is accounted for). The solving method for each of these types of models is similar to the one used to model (2. 14): each one can be solved by means of the adequate Lagrangean function.

The entropy model described in (2. 14) can be represented as well in an alternative way, using the concept of probability (Batten and Boyce, 1986). If we define $p^{rs} = \frac{T^{rs}}{T}$ as the probability of occurring a spatial interaction movement between r and s (in which, obviously, $\sum_r \sum_s p^{rs} = 1$), we can re-write the objective function in the following way:

$$\ln w(\mathbf{R}) = -\sum_r \sum_s \left\{ p^{rs} T \ln(p^{rs} T) - p^{rs} T \right\}$$

(2.20)

This equation can be simplified in the following way:

$$\begin{aligned}
&-\sum_r \sum_s \left\{ p^{rs} T \ln(p^{rs} T) - p^{rs} T \right\} = -T \sum_r \sum_s \left\{ p^{rs} \ln(p^{rs} T) - p^{rs} \right\} = \\
&-T \sum_r \sum_s \left\{ p^{rs} \left(\ln p^{rs} + \ln T \right) - p^{rs} \right\} = \\
&-T \sum_r \sum_s \left\{ p^{rs} \ln p^{rs} + p^{rs} \ln T - p^{rs} \right\} = \\
&-T \left\{ \sum_r \sum_s p^{rs} \ln p^{rs} + \sum_r \sum_s p^{rs} \ln T - \sum_r \sum_s p^{rs} \right\} = \\
&-T \sum_r \sum_s p^{rs} \ln p^{rs} - T(\ln T - 1) \sum_r \sum_s p^{rs}
\end{aligned}$$

Since $\sum_r \sum_s p^{rs} = 1$, we get:

$$-T\sum_r\sum_s p^{rs}\ln p^{rs} - T(\ln T - 1)$$

(2.21)

Equation (2.21) can still be simplified, considering that: a) $-T(\ln T - 1)$ is a constant, thus being innocuous to the result of the maximization procedure; b) the objective function is "invariant with a monotonically increasing transformation" (Nijkamp and Paelink, 1974, p.20) as multiplying by *T*. Thus, the final expression for the objective function will be:

$$\Omega = \ln w(\mathbf{R}) = -\sum_r\sum_s p^{rs}\ln p^{rs}$$

(2.22)

In order to reformulate the entire model (2. 14) to the probability version, it is also necessary to transform the restrictions, as follows: $\sum_s p^{rs}T = O^r$; $\sum_r p^{rs}T = D^s$ and $\sum_r\sum_s \pi^{rs}p^{rs}T = C$. So, the final version of the entropy maximizing problem is (Nijkamp and Paelink, 1974):

$$\begin{aligned}
&\underset{p^{rs}}{Max} \quad \Omega = \ln w(\mathbf{R}) = -\sum_r\sum_s p^{rs}\ln p^{rs} \quad \textit{subject} \quad \textit{to} \\
&\sum_r\sum_s p^{rs} = 1 \\
&\sum_s p^{rs} - \frac{O^r}{T} = 0 \qquad\qquad\qquad\qquad . \\
&\sum_s p^{rs} - \frac{D^s}{T} = 0 \\
&-\sum_r\sum_s \pi^{rs}p^{rs} + \frac{C}{T} = 0
\end{aligned}$$

(2.23)

This specification of the entropy maximizing model will be useful to compare entropy models with information theory based models, which will be discussed in the next section.

In spite of being theoretically appealing, entropy based model faces serious problems of applicability. Let us observe again model (2. 14); the empirical use of this model requires the introduction of a cost constraint, which, in practice, implies that the researcher has previous access to: the unitary transportation cost of traveling from *r* to *s*, π^{rs}, as well as C, the total transportation budget. But, on the one hand, these data are seldom available and are not easily substituted for proxy variables. One could think of using physical distances instead of economic cost, for example; although, to apply the equivalent constraint, it would be necessary to know the maximum number of kilometers traveled by the flow in question. This is obviously very difficult (if not impossible) to obtain. On the other hand, such constraint is somewhat unrealistic, since there is no effective restriction which submits a transportation system to a maximum amount of transportation cost – it is always possible to raise the transportation budget, in detriment of other activities such as consumption, investment and so on. Thus, another option would be to disregard the cost constraint and consider only the additivity constraints. But this alternative would result in a solution in which the spatial interaction flow would be completely independent from spatial dimension. As we have seen, in an intuitive manner, from the example above, the free maximization of entropy tends to yield a solution with equal values in all the cells of the spatial interaction matrix. Adding additivity constraints makes the solution to deviate from that uniform distribution, but only by a minimum amount necessary to respect those constraints. In fact, it is the cost restriction that provides a true spatial dimension into the model: inserting cost constraints makes the spatial interaction flows to be no longer equally probable. Instead, spatial interaction flows established between closer regions will be more probable than those established between distant regions. If we consider an entropy maximization problem in which restrictions refer only to the additivity constrains, the solution of such problem would be:

$$T^{rs} = J^{r} O^{r} D^{s} L^{s}$$

(2.24)

There is no mention to spatial reality in this model, which makes it unacceptable when the goal is to estimate spatial interaction flows.

Batten and Boyce (1986) refer to an alternative version of the entropy-maximizing model: the entropy constrained model. This was proposed earlier by Erlander (1977) and consisted of minimizing transportation cost, subject to the additivity constraints and to a minimum degree of dispersion or entropy ($\overline{E}$) of flows:

$$\begin{aligned}
&\underset{T^{rs}}{Min} \quad C = \sum_r \sum_s \pi^{rs} T^{rs} \quad subject \quad to \\
&\sum_s T^{rs} = O^r \\
&\sum_r T^{rs} = D^s \\
&-\sum_r \sum_s \left(T^{rs} \ln T^{rs} - T^{rs}\right) \geq \overline{E}
\end{aligned}$$

(2. 25)

In this model, the efficiency of the spatial interaction system is maximized (using the minimization of costs as the objective function), maintaining a minimum degree of entropy in the system, which can be understood as an expression of interactivity (Erlander, 1980) or interdependence between the various regions under study. Such model has been also suggested by Kim, Boyce and Hewings (1983), being the entropy constraint interpreted as a condition which would ensure a minimum level of crosshauling.

Of course, the practical problem in applying such a model is in the difficulty of setting an *a priori* fixed value of entropy. Should some previous matrix of flows exist, the problem may be addressed simply by assuming the degree of entropy of that matrix. In such event, the researcher would be imposing to the estimated trade flows the same degree of dispersion that they reflected in previously known matrix (of some previous year, for example, as it was done in Kim, Boyce and Hewings (1983)). But, once again, this demands the existence of some previous information, which is a seldom verified requirement. So, an alternative consists in eliminating this constraint. In this case, the model becomes a classical transportation model of linear programming (Batten and Boyce, 1986; Erlander, 1977). But then, if we are dealing with the estimation of interregional trade flows, for example, the solution will tend to exclude the existence of cross-hauling (the simultaneous flow of the same commodity in both directions between r and s), since it will always be less expensive to maximize the consumption of regionally produced goods (Kim, Boyce and Hewings, 1983; Ferreira, 2008). Conversely to the entropy maximization formulation, in which the solution tends towards an equal distribution among the various pairs of regions, the solution of the classical transportation model tends to place the maximum flows at the diagonal of the O-D matrix. This occurs because "The transportation model of linear programming as an explicative approach depends heavily on the assumption of perfect competition" (Batten and Boyce, 1986, p. 385). But, in real world, cross-hauling does exist because products are not homogeneous, which leads to an imperfect competition behaviour.

This allows us to conclude that the proposed alternatives, namely the entropy-constrained model, or even the related transportation model of linear programming, do not overcome the difficulties of practical applicability affecting entropy models, especially in what concerns to the estimation of interregional trade flows. Moreover, given that the general solution of the entropy model is not considerably different from the general solution of the gravitational model, as it was mentioned above, this model is often omitted in empirical applications, in favor of the gravitational model, which do not suffer from the same drawbacks.

2.3.3 Information theory models.

Information theory is a "statistical inference technique evaluating the change in a probability distribution due to the supply of certain new information" (Roy and Thill, 2004, p. 347). To better understand the application of information theory in spatial interaction models, we begin by introducing the concept of information content of a given message. Let's suppose we are about to receive a completely reliable message about the occurrence of some event X and let q be the probability that such event will occur ($0 \leq q \leq 1$). When we later receive the message saying that event X has indeed occurred, there is some information content in that message (Theil, 1967). If q was previously 0,99, there is no surprise when we know that X occurred, because it was practically certain. Then, the information content of the received message is small (Theil, 1967). Conversely, if q was previously 0,01, the received message saying that X occurred comprises a large information content (or surprise). Given this example, the issue consists in finding the best function to express the information content of a given message. Denoting this function by $h(q)$, the problem is: "What is the adequate functional form to $h(q)$?". There is no definitive answer to this question, but it is incontestable that it should be some decreasing function of the *ex ante* probability q. In fact, "the more unlikely the event before the message on its realization, the larger the information content of this message" (Theil, 1967, p. 3). From all the possible decreasing functions, the logarithm of the inverse of the *ex ante* probability q is generally adopted, because of its particular properties[68]. Thus, we have:

$$h(q) = \ln\left(\frac{1}{q}\right) \tag{2. 26}$$

We can now generalize this concept of information content of a message to situations in which the received message is not completely reliable (for example, in weather forecast, if it is predicted that the day after will be raining, there is not 100% reliability in this message, but it should rather be verified with some lower probability, let's say, 90%). In this case, the information content of the message measures the information bias caused by a message which has associated some probability p of occurrence of event X, in relation to an *ex ante* probability q (in the weather forecast example, this *ex ante*

[68] Namely, because it verifies additivity in the presence of many independent events; for further details on this, please refer to Theil (1967).

probability could be, for instance, $q = 0{,}5$, meaning that there is a 50% chance of raining in any random day of the year). When the message is not completely reliable, instead of considering a numerator 1 as in expression (2. 26), we consider the numerator p, the so-called *ex post* probability (Theil, 1967). Hence, the information bias caused by the message can be expressed by an indicator as:

$$h(p,q) = \ln\left(\frac{p}{q}\right)$$

(2. 27)

This indicator measures the change of information obtained when probability changes from q to p (Batten, 1982; Batten and Boyce, 1986). If $p = q$, it follows that $h = 0$, meaning that the information bias is null; if $p > q$, then $h > 0$, meaning that the information bias is positive; finally, if $p < q$, then $h < 0$, meaning that the information bias is negative. The larger the difference between p and q, the larger h will be.

The application of this concept to spatial interaction models is appropriate whenever we possess some previous information on the matrix of flows that we intend to estimate. Let us consider, again, the model expressed in (2.23) (doubly-constrained Wilson model, in the probability version with $p^{rs} = \dfrac{T^{rs}}{T}$). The previously known information may be, for example, the knowledge of some matrix of flows Q^{rs}, with a total of flows that reaches Q, for a period of time t_0, earlier than t_1 (the one to which we intend to obtain the estimation). Thus, for every origin/destination pair *rs*, it is possible to determine the observed $q^{rs} = \dfrac{Q^{rs}}{Q}$. The information bias between the p^{rs} estimated for time period t_1 and the observed q^{rs}, relative to t_0, will be given by (Snickars and Weibull, 1977):

$$I(p,q) = \sum_r \sum_s p^{rs} \ln\left(\frac{p^{rs}}{q^{rs}}\right)$$

(2.28)

The major difference between (2.28) and (2. 27) is simply the fact that, in the former expression, the relative difference between p and q is weighted by the estimated probabilities p^{rs}, meaning that this indicator tends to emphasize the difference in the most significant flows.

The information bias minimizing method consists precisely in minimizing $I(p,q)$, subject to the additivity constraints in the probability form (Roy and Thill, 2004):

$$\underset{p^{rs}}{Min} \quad I(p,q) = \sum_r \sum_s p^{rs} \ln\left(\frac{p^{rs}}{q^{rs}}\right) \quad subject \quad to$$

$$\sum_s p^{rs} - \frac{O^r}{T} = 0$$

$$\sum_r p^{rs} - \frac{D^s}{T} = 0$$

(2.29)

This principle is based on the fact that the estimated probability distribution that results from (2.29), $\hat{p}^{rs}$, is the one which contains the minimum deviation relatively to q^{rs}, among all the probability distributions that are compatible with a given information on the matrix of flows at t_1 (Batten and Martellato, 1985). This information at t_1, in this case, is no more than the knowledge of the margin totals, O^r and D^s, respectively, which are taken into account by means of the model constraints. Thus, there is no reason to select a probability distribution different from $\hat{p}^{rs}$, since any other will comprise more deviation than $\hat{p}^{rs}$ (Snickars and Weibull, 1977). In other words, the use of information minimizing principle means that the spatial interaction problem is considered as a problem of finding a matrix as close as possible to the previously known matrix, that also satisfies the problem constraints (Willekens, 1983; Harrigan, 1990).

It follows that, when the problem constraints are merely the additivity constraints, the information minimizing problem can be solved by means of an iterative procedure of bi-proportional adjustments, like RAS technique or other similar to it. This can be easily proven, as in Miller and Blair (1985), pp. 309-310. Yet, according to Batten (1982), the use of this kind of procedures to solve this problem has some disadvantages comparatively to the solution through linear programming. First, it doesn't allow the consideration of additional variables, beyond the standard additivity constraints; this is a limit to the explicative capacity of the model, since it is often convenient to consider other restrictions related, for example, to limits in productive capacity or in transportation budgets. Secondly, it doesn't permit the inclusion of restrictions in inequality form which, in spatial interaction problems, may exhibit some advantages. For example, it may be interesting to consider an upper limit to transportation costs or a minimum degree of entropy in the spatial interaction system[69]. However, these disadvantages are of minor importance in practical applications, since such specific information (like productive capacity, transportation budgets or previously known entropy level) is rarely available.

The choice of the objective function presented in (2.29), instead of objective function of problem (2.23) (entropy maximizing), must be conditional to the knowledge of some *a priori* information on the matrix we intend to estimate (Batten, 1982). Supposing that we also know and use transportation cost information, as it was assumed in the entropy maximizing problem, the final version of the information minimizing problem will be (Snickars and Weibull, 1977):

[69] We will get back to the case of using a minimum entropy constraint in section 5.3.

$$\underset{p^{rs}}{Min} \quad I(p,q) = \sum_r \sum_s p^{rs} \ln\left(\frac{p^{rs}}{q^{rs}}\right) \quad subject \quad to$$

$$\sum_s p^{rs} - \frac{O^r}{T} = 0$$

$$\sum_r p^{rs} - \frac{D^s}{T} = 0$$

$$\sum_r \sum_s p^{rs} \pi^{rs} = C$$

$$q^{rs} \quad is \quad known$$

(2.30)

The solution to this problem is found through the usual construction of Lagrangean function, as follows:

$$L(p^{rs}, \lambda^r, \mu^s, \beta) = \sum_r \sum_s p^{rs} \ln\left(\frac{p^{rs}}{q^{rs}}\right) + \lambda^r \left(\sum_s p^{rs} - \frac{O^r}{T}\right) + \mu^s \left(\sum_r p^{rs} - \frac{D^s}{T}\right) +$$
$$+ \beta\left(\sum_r \sum_s \pi^{rs} p^{rs} - C\right)$$

$$\frac{\partial L}{\partial p^{rs}} = 0 \Leftrightarrow p^{rs} \cdot \frac{\frac{1}{q^{rs}}}{\frac{p^{rs}}{q^{rs}}} + \ln\left(\frac{p^{rs}}{q^{rs}}\right) + \lambda^r + \mu^s + \beta\pi^{rs} = 0 \Leftrightarrow$$

$$\ln\left(\frac{p^{rs}}{q^{rs}}\right) = -\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right) \Leftrightarrow \ln p^{rs} - \ln q^{rs} = -\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right) \Leftrightarrow$$

$$\ln p^{rs} = -\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right) + \ln q^{rs} \Leftrightarrow p^{rs} = \exp\left[-\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right) + \ln q^{rs}\right] \Leftrightarrow$$

$$p^{rs} = \exp\left[-\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right)\right] \cdot q^{rs}$$

(2.31)

in which λ^r, μ^s and β are the Lagrangean multipliers related to the first, second and third restrictions, respectively.

Let us pay attention to the similitude between the structure of this solution and of solution (2. 16), that results from the entropy maximizing problem. The main difference arises from the fact that, in solution (2.31) there is some previous information on the spatial interaction flow pattern. However, if such information does not exist (or if it has not enough credibility to be used), it is reasonable to assume that, *a priori*, all the flows are uniformly distributed among the $m \times n$ cells of the matrix (admitting that there are m origins e n destinations). In this case, only the additivity and cost constraints information will be used. The supposition of spatial interaction flows uniformity is equivalent to the consideration that, *a priori*, each origin/destination flow is equal to $T^{rs} = \dfrac{T}{m \times n}$, to which corresponds an "observed" probability of $q^{rs} = \dfrac{T^{rs}}{T} = \dfrac{1}{m \times n}$. So, equation (2.31) becomes:

$$\hat{p}^{rs} = \frac{1}{m \times n} \exp\left[-\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right)\right] \Leftrightarrow T^{rs} = \frac{T}{m \times n} \exp\left[-\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right)\right].$$

(2.32)

The similarity between this solution and the entropy maximizing solution allows us to conclude that the entropy maximizing problem may be seen as a particular case of information minimizing problem, in which we assume that the starting matrix is a matrix with uniformly distributed spatial flows (Batten, 1982).

Solution (2.31) may be written in a more reduced way, as follows:

$$p^{rs} = \exp\left[-\left(1 + \lambda^r + \mu^s + \beta\pi^{rs}\right)\right] \cdot q^{rs} \Leftrightarrow$$
$$p^{rs} = \exp(-1) \cdot \exp\left(-\lambda^r\right) \cdot \exp\left(-\mu^s\right) \cdot \exp\left(-\beta\pi^{rs}\right) \cdot q^{rs}$$

Using variables $j^r = \dfrac{\exp(-1) \cdot \exp\left(-\lambda^r\right)}{O^r}$ and $l^s = \dfrac{\exp\left(-\mu^s\right)}{D^s}$, we can write:

$$p^{rs} = q^{rs} j^r O^r l^s D^s \exp\left(-\beta\pi^{rs}\right)$$

(2.33)

Using additivity constraints, j^r and l^s can be determined recursively in the following way:

$$\sum_s p^{rs} = \frac{O^r}{T} \Leftrightarrow \sum_s \left[q^{rs} j^r O^r l^s D^s \exp\left(-\beta\pi^{rs}\right)\right] = \frac{O^r}{T} \Leftrightarrow$$
$$j^r O^r \sum_s \left[q^{rs} l^s D^s \exp\left(-\beta\pi^{rs}\right)\right] = \frac{O^r}{T} \Leftrightarrow$$
$$j^r = \frac{1}{T\sum_j q^{rs} l^s D^s \exp\left(-\beta\pi^{rs}\right)}$$

(2.34)

and

$$\sum_r p^{rs} = \frac{D^s}{T} \Leftrightarrow \sum_r \left[q^{rs} j^r O^r l^s D^s \exp\left(-\beta\pi^{rs}\right)\right] = \frac{D^s}{T} \Leftrightarrow$$
$$l^s D^s \sum_r \left[q^{rs} j^r O^r \exp\left(-\beta\pi^{rs}\right)\right] = \frac{D^s}{T} \Leftrightarrow$$
$$l^s = \frac{1}{T\sum_r q^{rs} j^r O^r \exp\left(-\beta\pi^{rs}\right)}$$

(2.35)

Parameters j^r and l^s serve as balancing parameters just like the correspondent J^r and L^s did in gravity and entropy models.

The relative performance of different variants of model (2.30) is tested in Snickars and Weibull (1977), through an empirical application to commuting between sub-regions of Stockholm, during the period of 1965-70. The test is carried out comparing the estimative provided by each variant of the model with the real commuting flows, which are known. Four versions of the model are used: (1) independency model, in which only the additivity constraints are used; (2) classical gravity model[70], in which, besides the additivity constraints, cost constraint is also used; (3) information bias minimizing method (the one expressed by means of equation (2.29)), that uses no cost restriction, but uses a matrix of flows q^{rs} of an earlier period, as previous information and, finally, (4) New gravity model, which corresponds to the problem depicted in (2.30).

As expected, the results of the accuracy tests show that the third model is clearly superior to the first two models; besides, it is only slightly less precise than the model that uses more information, the "new gravity model". From this comparison, the authors confirm that the spatial separation effect by itself (which, in this case, was measured using traveling time) is not sufficient to determine inter-spatial flows, making gravitational model to be less effective in explaining these movements than models based in previous periods matrices do. This conclusion was predictable, since there are many other factors that may influence inter-spatial flows; if we are dealing with commuting movements, for example, the laws in force concerning territory arrangement are an important issue to consider. If it is not possible to locate industrial facilities in residential areas, then obviously workers have to commute between home and work. The influence of all the determinant factors is already comprised in the *a priori* matrix of flows and can only be taken into account if such information is used as a starting point. Once again, the problem is that, in most cases, there is no such *a priori* information, making this model impossible to use, in practice, as a generator of spatial interaction flows.

2.3.4 Behaviour models.

The models that were previously reviewed are included in a macro level perspective, i.e., they are models that search for regularities in the movements of big groups of people (or other kind of flows). But, because macro behavior is influenced by individual behavior, several authors have proposed, since the 1970's, that is should exist a better connection

[70] This denomination is the one used by the authors, to refer to a model in which distance (expressed in time, kilometres, cost or other) is considered as the determinant factor for flow distribution, as in gravitational models. However, gravitational models do not allow for the consideration of restrictions such as the cost constraint in (2.30).

between spatial interaction research and micro level models (Cesario and Smith, 1975; Sheppard, 1978).

Behavioral models fit precisely in a micro level approach, since they are based on the utility function of the individual (or group of individuals) that originates the spatial interaction flow. In these models, the mass of origin or destination is seen as an aggregate of micro level units (Isard, 1998), in which each one tries to maximize its utility, subject to a behavioral restriction (for example, a transportation budget constraint). Utility maximization is the force that lies upon the individual's choice among alternative destinations which not only respects the cost constraint, but also observes the following optimum requirement: marginal utility of destination *s* per transportation cost monetary unit is the same for any destination *s*. Batten and Boyce (1986) derive this solution starting from the supposition that "each place of origin can be conceived of as a *collective decision-unit*, which allocates a certain transportation budget among a series of shipments to alternative destinations" (Batten and Boyce, 1986, p. 372). Being so, the utility function of origin *r* is determined by the combination of deliveries to the several destinations:

$$U^r = U^r\left(T^{r1}, T^{r2}, \cdots, T^{rk}\right) \tag{2.36}$$

This function is maximized subject to a transportation budget constraint, expressed by:

$$\sum_s \pi^{rs} T^{rs} = C^r \tag{2.37}$$

in which C^r stands for the total available transportation budget of place *r*.

The lagrangean function correspondent to this optimization problem is:

$$L = U^r\left(T^{r1}, T^{r2}, \cdots, T^{rk}\right) + \lambda\left(C^r - \sum_s \pi^{rs} T^{rs}\right)$$

(2.38)

Making out the first order partial derivates, we have:

$$\begin{array}{l} \dfrac{\partial L}{\partial T^{rs}} = 0 \Leftrightarrow \dfrac{\partial U^r}{\partial T^{rs}} = \lambda \pi^{rs} \\ \dfrac{\partial L}{\partial \lambda} = 0 \Leftrightarrow \sum_s \pi^{rs} T^{rs} = C^r \end{array} \quad \text{with } s = (1, \cdots, k).$$

(2.39)

The first set of equations on system (2.39) can be solved dividing each of the first $(k-1)$ equations for the last one; this results in:

$$\frac{\partial U^r / \partial T^{rs}}{\partial U^r / \partial T^{rk}} = \frac{\lambda \pi^{rs}}{\lambda \pi^{rk}} \Leftrightarrow \frac{\partial U^r / \partial T^{rs}}{\pi^{rs}} = \frac{\partial U^r / \partial T^{rs}}{\pi^{rk}}, \text{ with } s = (1, \cdots, k).$$

(2.40)

This equation is the equilibrium condition, according to which marginal utility of destination *s* per transportation cost monetary unit is the same for any destination *s*. All the *k* unknowns T^{rs} can be solved for a given origin *r*, through the consideration of (2.40) along with the cost constraint (2.37).

Batten and Boyce (1986) go further and show that, under some presuppositions on the utility functional form, utility maximization leads to a solution that assumes a gravitational formulation. These authors start from a specific collective utility function[71] for origin r and prove that the solution of maximizing such an utility function is equivalent to a gravitational type solution. This allows us to conclude that a micro level approach may be used as one more theoretical argument to the use of gravitational model.

With a similar reasoning, Isard (1975) shows that utility maximization, subject to a limit imposed by the individual on the number of trips he/she wants to make, also leads to a gravitational type solution, in this case, of the exponential form[72]. In this case, the individual utility function is deduced from an indifference curve (consisting of indifferent combinations of number of trips and covered distance), in which it is assumed that for "for every increase in one mile, he must have c percent more trips, where c is a constant" (Isard, 1975, p.26). This simply means that the individual establishes a trade-off between the benefit from traveling and the cost related to the distance to run.

These are two examples that, under certain specific forms to utility function, the resulting demand functions for spatial interaction observe the gravitational law. In spite of this connection, the utility function approach possesses a smaller empirical potential than the preceding approaches (gravitational, entropy maximizing and information minimizing models), since it is difficult to estimate the utility functions and also because this approach requires a great set of restrictive suppositions on the individual choice behavior, which are not easy to verify (Sen and Smith, 1995; Isard, 1998; Sheppard, 1978).

For these reasons, the profound study of behavior-based models is beyond the scope of the present work.

Section 2.3 intended to review the most important members of the family of spatial interaction models, discussing their applicability in the estimation of spatial interaction flows and having in mind the specific case of interregional trade. We have seen that, even

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[71] The function they use is expressed by $U^r = \sum_{s=1}^{k} D^s f\left(\delta^{rs}\right) \ln T^{rs}$. For further details, please see Batten and Boyce, 1986, pp. 373-374.

[72] For further details, please see Isard (1975), pp. 28-29.

being sustained by different theories, the solutions of the several models show a considerable similitude. As stated in Batten and Boyce (1986), the different theoretical models lead to conclusions that "are more notable for their similarities than for their differences" (p. 357). However, these models are quite different in what concerns to their applicability, especially in interregional trade flow estimation, which is our major concern. This is because, most of the times, the estimation of interregional trade flows is made under a type (b) information context, *i.e.*, when there is no *a priory* matrix of flows. This immediately precludes the direct application of models that minimize information bias, since these require a previous matrix of flows (section 2.3.3). Problems of applicability were also identified in entropy and behaviour models (sections 2.3.2 and 2.3.4, respectively). In what respects to entropy models, such problems were related mainly to the difficulties is assessing data on transportation costs (as well as to the fact that such cost restriction may not constitute a suited interpretation of transportation systems behaviour. Concerning behaviour models, the central problem is in the proper definition of the utility function. Conversely, major strengths of the gravity model continue to be its simplicity and its good capacity to produce accurate results, especially in studies applied to trade flows (Brocker, 1989; Porojan, 2001). For this reason, gravitational formula keeps on finding numerous followers. This success is reinforced by the model's capacity to produce reasonable results, even with very aggregate starting information on spatial interaction (usually, the margin totals) and using very simple measures of spatial separation (as physical distance or mean traveling time) (Sen and Smith, 1995).

Considering the *a priori* drawbacks of entropy, information theory and behaviour models, and having recognized that the generic solutions provided by these models are similar to the generic solution of the gravity model, from now on, our attention will be focused on the latter model. Still, the concepts of entropy and information bias will be recovered in section 2.5, to be used not as generators of the initial values of interregional trade, but rather in their adjustment to the additivity constraints.

2.4 Empirical examination of the gravity model in two different information contexts: explanation and estimation.

In a previous paper (Sargento, 2007), we had already carried out a first attempt of empirically examining the gravity model, with the main objective of discussing and testing its practical applicability to the study trade flows, in both information contexts referred in section 2.3.1: with and without previous information on trade flows. The good results usually provided by gravity-based equations in trade econometric applications may suggest that the gravity model can be also successfully used in predicting trade flows. However, this is seldom subject to empirical testing. Thus, the main objectives of the present section, which is based on the above referred paper[73], consist in: 1) evaluating the explanatory power of gravity-based models in the context of interregional trade flows, identifying the relevant explanatory variables and testing different formulations of the model and 2) analyzing the performance of the gravity model as a generator of undisclosed interregional trade flows. Additionally, the empirical work described in this section also intends to fill another gap in trade flow gravity model uses: the fact that the majority of studies consider trade in an aggregate manner. Recognizing the specificity of each product, this study is applied separately to different trading products.

2.4.1 Type (a) information context.

When the researcher has previous access to a known trade matrix, the objective is to calibrate the model, i.e., to estimate the model parameters. An immediate question emerges: "if we already have the interaction matrix, why do we need to calibrate an interaction model?" (Fotheringham and O'Kelly, 1988). In fact, the calibration process is useful to forecasting purposes (admitting that the parameters remain the same in different points of time and/or space) and to draw conclusions on the behavior patterns of the

[73] The experiences and results which are described in the present section do not exactly correspond to what has been previously presented in Sargento (2007). Some improvements were introduced, namely: 1) the degree of detail in product classification (using 17 categories of manufactured products instead of 10) and 2) the way by which distance was computed, being this an issue to be explained in section 4.1.2. Yet, the main conclusions of this empirical study remain the same.

subject in study (for example: to assess the degree of elasticity of exports with respect to the distance between the trading partners and to evaluate how this varies from one product to another, or to infer about the relevance of the pre-defined explanatory variables). Thus, in this section, attention will be given to the econometric application of gravity model to explain bilateral trade flows among the "old" EU countries (before enlargement). This application was carried out in a stepwise fashion, testing several alternative equations and analyzing the results of each one. Previously to the description of the equations tested, a brief theoretical support is presented in the following section.

2.4.1.1 Gravity model extensions to trade applications in type (a) information context.

2.4.1.1.1 Augmenting the gravity equation with additional explanatory variables.

As it has been referred before, in type (a) information context, the basic gravitational equation is the starting point to an econometric model like the one in equation (2. 8). However, it has been demonstrated that the variables included in this basic equation are often not enough to explain trade flow's behavior. Numerous studies have, thus, used an augmented version of gravity model basic equation, in order to address some specific issues. One of the most important motivations to gravity model extensions is the study of preferential trade agreements effects. Some examples of this kind of exercises can be found in Martinez-Zarzoso (2003), Soloaga and Winters (1999) and Piani and Kume, (2000). The common feature of these three works is the addition of specific bloc-related dummy variables to equation (2. 8), in order to capture the effects of preferential trade agreements, especially those concerning trade creation and trade diversion. Further dummy variables are also included to isolate the effects of other determinants of trade, such as: sharing the same language or sharing a common border. The work of Blavy (2001) is another example of extending the gravity model to answer some specific trade issues. In this case, the author starts by applying the basic gravity model to trade patterns in specific region composed by six Middle East countries, reaching to the conclusion that it overestimates intranational and international trade in that region. To overcome such problem, the model is extended with specific explanatory variables, to assess the effects of: over-appreciation of exchange rates, trade barriers and political uncertainty. It is shown that the augmented model has a better performance in estimating trade flows.

2.4.1.1.2 Formal specification of spatial dependence.

One of the frequent criticisms pointed out to gravity model is the fact that it generally assumes that observations collected at different points of space are completely independent, which is not true. There are well known diffusion processes among different locations that must be taken into account, through a specific modeling of space.

One possible way to acknowledge spatial structure effects is through the inclusion in the gravity model of additional variables that, in some way, illustrate the map pattern of the observations under study. In this context, one of the fundamental contributions is the Competing Destinations model, proposed by Fotherihgam (1983). This author stressed out the fact that, sometimes, the decision of spatial interaction movement occurs in a two-stage process, in which individuals begin by choosing a broad destination region and then "choose a specific destination from the set of destinations contained within in broad region" (Fotherihgam, 1983, p. 19). In this situation, the specific destinations to be chosen in the second stage are competing for the total amount of flows that the broad region receives. When an origin is geographically accessible to many destinations, each specific destination will tend to receive less volume of flows, and *vice-versa*. Given that this spatial arrangement effect is ignored by the basic gravity model, the result is that the distance decay parameter will be misspecified, incorporating not only the behavior of distance resistance, but also the spatial structure effect. More precisely, the distance parameter will be biased upwards in more accessible origins and it will be biased downwards in inaccessible origins (Fotherihgam, 1983). The proposed solution consists in including an additional variable designed to measure accessibility of destination s to all other destinations available to origin r (Fotherihgam, 1983). Given the principle of competition between destinations, the estimated accessibility coefficient should have a negative sign: the more accessible the destination is, the less volume of flows it receives. One example of an empirical application of such model can be found in Hu and Pooler (2002). In this paper, the authors use an augmented gravity model (applied to explain international trade flows between *r* and *s*), in which spatial structure effect is captured through the inclusion of an accessibility variable, given by the weighed sum of the distances between all the origins and *s,* with each origin's mass as the relevant ponderer. They compare the performance of this model with the traditional gravity model, showing that the addition of the accessibility variable contributes to a better predictive capacity of the model. Some previously referred exercises also attempt to include in their gravity models a variable that expresses the relative locations of the different observations. That is the case, for example, in Piani and Kume, (2000) and in Soloaga and Winters (1999), which consider a relative distance (or remoteness) indicator, to control for the stronger trade intensity that usually exists between remote pairs of countries, when compared with trade between neighbors that have many other close trading partners. A simpler and more common way used to illustrate map pattern of the observations is the inclusion of a dummy variable that indicates the presence (or not) of a common border between the trading partners.

However, spatial effects are often more comprehensive, making unavoidable the use of more sophisticated modeling techniques, that fall in the spatial econometrics field. If this is the case, it is very important, not only to find the proper way to formally express the spatial effects, but also to use the adequate techniques to estimate the model. Standard regression methods (as Ordinary Least Squares) are no longer acceptable when spatial effects are definitely present[74] (Anselin and Griffith (1988)). The paper of Anselin and Griffith (1988) is crucial to systematize the nature of spatial effects. These consist of spatial dependence, on the one hand, and spatial heterogeneity, on the other hand. Spatial dependence may exist due to spillover effects across space and occurs whenever the dependent variable is "affected by the values of the dependent variable in nearby units,

[74] Yet, most of the practical applications completely ignore the possibility of spatial effects, using OLS as the single estimating method.

with nearby suitable defined" (Beck, Gleditsch and Beardsley, 2005, p.9). Moreover, it may occur merely as a side effect due to misspecifications in the data used in the model. As stated in Anselin and Bera (1998), "(…) a mismatch between the spatial unit of observation and the spatial extent of the economic phenomena under consideration will result in spatial measurement errors and spatial autocorrelation between the errors and adjoining locations" (p. 239). Spatial dependence can be discovered by the presence of autocorrelated error terms (originating the spatial error models) and/or autocorrelation in the dependent variable (resulting in the spatial lag model). Spatial heterogeneity may be due to structural instability, meaning that functional forms and/or parameters differ from one observation to another[75], or to model misspecification that leads to non-constant error term variances (heteroskedasticity). Whenever it is caused by heteroskedasticity, spatial heterogeneity can be undertaken by means of the typical solutions in traditional econometrics (*e.g.*: using Weighted Least Squares estimation or transforming the regression equation in order to generate homoskedastic errors)[76].

The attention of this study is focused on spatial dependence. To formally account for spatial dependence, however, it is necessary to introduce the concept of spatial lag. Contrarily to what happens in time-series analysis, in which the time lag is easily introduced through a backward or forward shift on the one-dimensional time axis, in cross-section analysis the problem is more complex, given the multi-dimensional character of space (Anselin and Bera, 1998). In this case, a spatial lag operator is used – a W matrix, also called connectivity matrix (Beck, Gleditsch and Beardsley, 2005). This matrix represents spatial morphology and is composed by non-stochastic w_{rs} elements, based on the geographic arrangement of observations. One of the most popular criteria to

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[75] Formally, this would mean that, for each observation *i*, there would be a function $y_i = f_i(x_i, \beta_i) + \varepsilon_i$, in which x_i is a 1**m* row of *m* explanatory variables and β_i stands for the correspondent coefficients, that are different for each *i*. The function *f* as well may be different for each *i*.

[76] Conversely, structural instability involves more complex solution procedures. In spatial analyses, structural instability may be caused by spatial nonstationarity. This means that the assumption of constant coefficients throughout space – implied in typical econometric estimations with one single equation for all spatial data – is not verified in reality (Fotheringham and Charlton, 1998). Some statistical techniques have been proposed to overcome this problem, namely the expansion method and the geographically weighted regression approach. Yet, spatial heterogeneity goes beyond the scope of the present work. Thus, the reader is referred to Fotheringham and Charlton (1998), for an explanation on these techniques.

express geographic arrangement is contiguity; following this criterion, w_{rs} assumes the value 1 if r and s are contiguous locations and the value 0, otherwise[77]. In short, the spatial lag operator can be seen as a "weighted average (with the w_{rs} being the weights) of the neighbors, or as a spatial smoother" (Anselin, 1999).

The spatial lag model, also named spatial autoregressive model, expresses the case of spatial autocorrelation in the dependent variable. Analytically, it can be written as:

$$Y = \rho WY + X\beta + \varepsilon$$

(2. 41)

in which Y represents the vector of dependent variables, X is the matrix of explanatory variables, W is the lag operator, β is the vector of parameters that reflect the influence of explanatory variables on Y and ρ illustrates the degree of the dependent variable spatial autocorrelation. This model implies the assumption of feedback effects among observations / locations: variations in the explanatory variables of location i affect the dependent variable of that location and of neighboring locations (because of the lag operator). Consequently, location i will be affected for a second time (again, because of the spatial link with its neighbors) and this process will be successively repeated as in a multiplier effect.

In the specific case of our empirical application, we will be dealing with origin-destination (O-D) data. This has implications on how to construct an adequate weight matrix W. It should be noted that, when dealing with spatially collected data, usually each observation corresponds to one region. This is not the present case: in origin-destination data, the observations vector is composed by the flows generated by every possible combination of origin and destination, in both directions (LeSage and Pace, 2005: 1). For example, if the number of origins, n, is equal to the number of destinations, the total

[77] The matrix W is row-standardized, as usual in this type of models; each row sums 1, so that there is no need to worry about the units used to measure connectivity.

number of observations will be $N = n^2$. So, the W operator must have a compatible dimension. Among the several manners in which Matrix W can be constructed, three different specifications will be considered in the present work, reflecting distinct types of spatial dependence: destination-based, origin-based, and a mixed origin-destination-based dependence. A destination-based weight matrix, named W_d, can be assembled by repeating the typical contiguity matrix n times (being n the number of spatial units included in the study) in the diagonal of an *N*N* block matrix, placing blocks of zeros in all the off-diagonal matrices (LeSage and Pace, 2005).

Figure 2. 4 – Construction of matrix W

(I)

$$w = [w_{rs}]_{14*14}; \quad w_{rs} = \begin{cases} 1, \mathit{if} \ \ r \ \mathit{and} \ \ s \ \mathit{contiguous} \\ 0, \mathit{if} \ \ r \ \mathit{and} \ \ s \ \mathit{not} \ \mathit{contiguous} \end{cases}$$

(II)

$$C = [c_{rs}]_{14*14}, \ \mathit{such} \ \ \mathit{as} \sum_s c_{rs} = 1$$

(III)

$$W = \begin{bmatrix} [C] & 0 & \cdots & 0 \\ 0 & [C] & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & [C] \end{bmatrix}_{196*196}$$

As we have previously referred, we will be using bilateral trade data between 14 European countries as the set of spatial observations. This corresponds to a 14*14 matrix of origin-destination observations. Making use of the most common concept of contiguity – sharing of a common border – the construction of the spatial lag operator involves the assemblage of a starting 14*14 w matrix, with elements w^{rs} that are given the value 1 if r

and s share a common border and the value 0, otherwise. Matrix w is then row-standardized, becoming matrix C. Finally, W_d is formed spreading out the 14*14 matrix C on a 196*196 matrix. Figure 2.4 illustrates the process of matrix W_d construction[78].

The use of spatial lag operator W_d in an autoregressive process (either a spatial autoregressive or a spatial error model) captures a "destination-based" spatial dependence (LeSage and Pace, 2005). The following equation depicts a destination-based spatial autoregressive (SAR) model for O-D flows x^{rs}, equivalent to the one presented in equation (2. 41):

$$x^{rs} = \rho_d W_d x^{rs} + \beta X + \varepsilon^{rs} \tag{2. 42}$$

Consider a specific element x^{rs} in the observations vector. The inclusion of the destination-based spatial lag in the regression equations means that flows from r to s are influenced, among other factors, by the average of flows from r to all the neighbours of s. Using a specific example, this is equivalent to say that the France to Germany value of exports is influenced by the average of exports coming from France to all the neighbours of Germany.

Another alternative consists of considering origin-based spatial dependence. In fact, "it seems plausible that forces leading to flows from any origin to a particular destination may create similar flows from neighbours to this origin to the same destination." (LeSage and Pace, 2005, p. 7). In order to capture this potential effect, an origin-based weight matrix can be computed through: $W_o = C \otimes I_{14}$. The resulting origin-based spatial autoregressive model is:

$$x^{rs} = \rho_o W_o x^{rs} + \beta X + \varepsilon^{rs}$$

[78] The W_d matrix can also be obtained by simply applying the Kronecker product between a $n*n$ identity matrix and matrix C – please see LeSage and Pace (2005) for further details.

(2. 43)

Finally, a third type of W matrix will be used in the present work[79]: $W_{od} = W_o \cdot W_d$. This matrix aims to capture an "origin-destination" mixed effect of spatial dependence. The inclusion of spatial lag $W_{od} \ln x^{rs}$ in the spatial autoregressive model means that: flows from r to s are influenced, among other factors, by the average of flows from all the neighbours of r to all the neighbours of s. This weight matrix can be computed as the *Kronecker* product: $W_{od} = C \otimes C$ (LeSage and Pace, 2005). The origin-destination-based spatial autoregressive model (SAR), becomes:

$$x^{rs} = \rho_{od} W_{od} x^{rs} + \beta X + \varepsilon^{rs}$$

(2. 44)

The other type of spatial dependence occurs through the presence of spatially autocorrelated error terms. This can be analytically reflected through a spatial error model, expressed by an equation like:

$$\begin{aligned} Y &= X\beta + u \\ u &= \lambda W u + \varepsilon \end{aligned}$$

(2. 45)

in which λ expresses the degree of spatial correlation among the model disturbances[80] and the remaining variables have the above referred meaning. In this model it is assumed that the only source of interdependence among observations is in the error formation

[79] As it will be explained in Section 2.4.1.3.2, these three alternative weight matrices will be also used in the other spatial econometric models tested on the spatial econometric application: the spatial error and the general spatial model.

[80] Another commonly used approach to specify error spatial autocorrelation is direct representation, which consists in assuming a specific functional form for the covariance matrix of the error terms, and estimate its parameters along with the regression ones. This functional form is usually some inverse function of distance, expressing the notion of spatial clustering of the disturbances (Anselin, 1999). Examples of these functions may be found in Dubin (1998). Since the empirical application of the present work falls exclusively upon the spatial lag approach, the direct representation is not subject to further development.

process, more precisely, the fact that some omitted variables are spatially correlated (Beck, Gleditsch and Beardsley, 2005).

It can also be the case that the disturbances from a spatial lag model evidence spatial autocorrelation. In such situation, it is adequate to apply a general spatial model, in which both the dependent variable and the error term are spatially correlated (LeSage, 1998). Formally, this can be expressed by:

$$\begin{aligned} Y &= \rho W_1 Y + X\beta + \mu \\ u &= \lambda W_2 u + \varepsilon \end{aligned} \qquad \textbf{(2. 46)}$$

in which W_1 and W_2 represent the lag operators used in the autoregressive processes of the dependent variable and the error term, respectively.

The emergence of new software tools and theoretical contributions to deal with spatial dependence has enhanced the empirical application of these models. Some examples can be found in Beck, Gleditsch and Beardsley, (2005) and Porojan (2001). Beck, Gleditsch and Beardsley (2005) propose an alternative connectivity measure to include in the spatial lag model. Their objective is to explain democracy level. They argue that instead of geographical notion of proximity other measures can be used. So, they propose a *W* matrix with elements given by the "volume of the dyadic trade flow between *i* and *j* as a proportion of country *i*'s total trade" (p.13); the empirical exercise proves that the spatial autocorrelation coefficient related to this connectivity matrix is statistically significant, suggesting that "countries that trade more with democracies are more likely to be democratic (…)" (p. 17). The empirical application carried out in Porojan (2001) aims to find the most proper version of gravity model to explain international trade. Several alternative equations are tested, including the gravity traditional specification, leading the author to the conclusion that the most adequate equation is the one which explicitly considers the existence of two spatial effects: spatial heterogeneity (adapting the model to account for heteroskedastic error) and spatial autocorrelation of the dependent variable.

2.4.1.2 Description of the data used.

The set of data used in this work is composed by:

- Trade flow data originated in each of 14 countries to each of the others, for year 2001, in USD and current prices; source: OECD Bilateral Trade Database 2002[81]. The 14 countries correspond to the "old" European members, in which Belgium and Luxembourg are considered jointly[82]. Trade data concerns only to manufactured products, disaggregated in seventeen groups, as illustrated in Table 2. 1. One important feature of this product classification, comprised in the OECD Bilateral Trade database, consists in the fact that it broadly matches the classification used in National Accounts and, in particular, in input-output tables (the correspondence is indicated in the third column of the table). This is important whenever the results from the trade flow estimation model are to be used in the construction of input-output tables.
- Population, year 2001, in thousands; source: OECD member countries' population 1981-2004 (thousands and indices: 2000=100). Labour Force Statistics, 2005 Edition;
- Gross Domestic Product, year 2001, in USD and current prices; source: OECD Annual National Accounts database.
- A matrix of distances between countries, based on the polygon centroid (x,y) coordinates for each NUT II comprised in the countries under study, and in weighting distances between the regions of the different countries.

[81] It should be noted that the OECD's Bilateral Trade Database provides two types of information regarding trade flows: information on exports and information on imports, to each declaring country. However, as it happens in all international trade databases, "mirror statistics often do not match between two countries (exports from the USA to France may well not agree with imports by France from the USA)." (OECD, 2002, p. 13). Beucause of that, we have opted for using the average value between the data derived from exports information provided by the country of origin and from imports information provided by the country of destination.

[82] This was done in order to maintain the same country database as in Sargento (2007). These two countries were then considered jointly because the distance data used in that work – great circle distances between capital cities – considered them as one single point.

Table 2. 1 – Product classification.

Abbrev.	**Designation**	**ISIC Rev.3**
FBT	FOOD PRODUCTS, BEVERAGES AND TOBACCO	15-16
TEX	TEXTILES, TEXTILE PRODUCTS, LEATHER AND FOOTWEAR	17-19
WOO	WOOD AND PRODUCTS OF WOOD AND CORK	20
PPP	PULP, PAPER, PAPER PRODUCTS, PRINTING AND PUBLISHING	21-22
COK	COKE, REFINED PETROLEUM PRODUCTS AND NUCLEAR FUEL	23
CHE	CHEMICALS AND CHEMICAL PRODUCTS	24
RPL	RUBBER AND PLASTICS PRODUCTS	25
ONM	OTHER NON-METALLIC MINERAL PRODUCTS	26
BMT	BASIC METALS	27
FMT	FABRICATED METAL PRODUCTS	28
MAC	MACHINERY AND EQUIPMENT, N.E.C.	29
OFF	OFFICE, ACCOUNTING AND COMPUTING MACHINERY	30
ELE	ELECTRICAL MACHINERY AND APPARATUS, NEC	31
RTV	RADIO, TELEVISION AND COMMUNICATION EQUIPMENT	32
MED	MEDICAL, PRECISION AND OPTICAL INSTRUMENTS	33
MTV	MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS	34
OTR	OTHER TRANSPORT EQUIPMENT	35

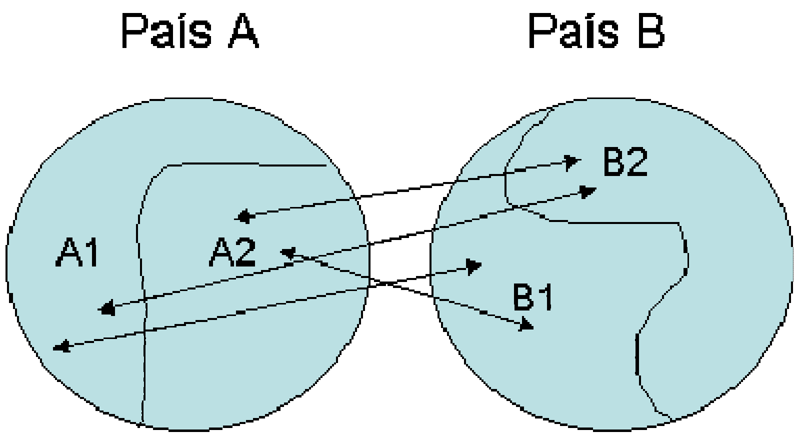
Some additional comments must be made concerning this matrix of distances. The traditional measure of distance between countries used in this kind of studies consists most of the times in using the straight line distance between capital cities[83]. However, two problems are associated with this measure: first, trade flows occur between each and every local of each economic area and not only between capital cities; besides, capital cities, sometimes are not even the central spot of the country, in geographic terms. Thus, we opted for using for each pair of countries, a weighted distance between their regions, calculated as follows. Consider two countries A and B, with two regions each, as depicted in Figure 2.5[84]. The distance between A and B was computed as:

$$d_{AB} = \frac{GDP_{A1}GDP_{B1}d_{A1B1} + GDP_{A1}GDP_{B2}d_{A1B2} + GDP_{A2}GDP_{B1}d_{A2B1} + GDP_{A2}GDP_{B2}d_{A2B2}}{GDP_{A1}GDP_{B1} + GDP_{A1}GDP_{B2} + GDP_{A2}GDP_{B1} + GDP_{A2}GDP_{B2}}$$

(2. 47)

[83] In fact, this had been the distance measure used in the experiences reported in Sargento (2006) and in Sargento (2007).
[84] NUTs II was the geographic partition used in the calculation.

Figure 2. 5 – Two countries – A and B, with two regions each.



Using such a formula, the distance between country A and country B takes into account the several distances between all regions of country A and all regions of country B, instead of considering merely the distance between two specific points of the country. The distances between pairs of NUTs II were calculated using the polygon centroid's (x,y) coordinates for each one[85], as it was done in other similar studies, like for example Dall'erba (2003).

As it was mentioned before, origin-destination flow data have specific characteristics, which must be emphasized before explaining the practical application that was carried out. First, the number of observations, N, is equal to n^2 (196, in this case, where n = 14 represents the number of countries included in the study[86]). Second, the vectors of explanatory variables have a particular feature: in the origin related variables (as, for example, GDP of origin i), the same value is repeated n times: once to each destination

[85] A polygon centroid is used to refer to its centre of mass or centre of gravity, i.e., the point about which the polygon would balance, assuming that it had a constant density. These points are widely used in Geographical Information Systems (GIS), for example, in distance calculations, just as we have done here. This distances between the NUTs II's polygon centroid came from the database provided by ArcView GIS.

[86] In practice, there are only 184 non-zero observations, since i tis assumed no intra-regional trade, making the main diagonal elements of the O-D matrix equal to zero.

country; in the destination related variables (for example, GDP of destination *j*) the same sequence of values is repeated *n* times: once to each origin country. Finally, distance and contiguity matrices are symmetric (ex.: if Germany is contiguous to France, the opposite is also true; the same reasoning applies to the distance between these two countries). The discussion of these particular features of origin-destination flow data, and its implications, is the main subject of LeSage and Pace (2005).

All the alternative equations were estimated seventeen times: once to each of the seventeen manufactured products considered in the study. The use of gravity model with individual products is less common than aggregate trade applications. However, some exceptions exist. For example, Feenstra, Markusen and Rose (1998) distinguish two groups of products: differentiated and homogeneous, expecting to find a higher value of domestic income exports elasticity in manufactured / differentiated products, when compared to the correspondent value in primary, homogeneous, resource based goods. Their results confirm the initial expectative. Srivastava and Green (1986) also include different individual product categories in their study of the determinants of trade between a large sample of nations. These authors have found that the explanatory variables are better for explaining trade intensity of manufactured goods than in nonmanufactured categories. Besides, even among the categories in which the independent variables are statistically significant, considerably varying parameters have been found (Srivastava and Green (1986)). The same conclusion was found in Jackson, Schwarm and Okuyama (2006), in their estimate of interregional trade flows to be included in an interregional Social Accounting Matrix framework for the U.S. According to the results obtained there is “considerable variation in interaction parameters across commodities”. (p. 87). The few studies that have used commodity-specific estimates of interregional trade have all concluded that there is considerable parameter variability from one product to another. This may also be an expected result in the present study. However, it should be noted that there are a priori limitations that must be taken into account when inferring the results: 1) only manufactured product categories are included in the study, preventing the comparison with non-manufactured products; 2) the level of aggregation involved in the above list of seventeen products is still very high.

2.4.1.3 Model calibration through alternative gravity equations.

2.4.1.3.1 Basic gravity equation.

Model calibration was done in a stepwise fashion, testing succeeding formulations for gravity model. The first regression, named *Model 1*, was based on the traditional specification of gravity model, expressed before in equation (2. 8) and used Ordinary Least Squares. As a matter of fact, *Model 1* acts as a benchmark for further estimates. Yet, the estimated equation was a bit different from equation (2. 8), since GDP was decomposed in two separate factors, in order to capture two distinct effects on trade: population, as a size explanatory variable, and per capita income, as an indicator of development. Being so, *Model 1* is expressed by:

$$\ln x^{rs} = \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^r + \beta_3 \ln N^s + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \varepsilon^{rs} \qquad \textbf{(2. 48)}$$

in which N stands for per capita GDP and POP is population. The remaining variables and parameters have the meaning referred before. It is expected that the coefficients associated to N and POP, have positive signs, because these are the traditional propulsion (for origins) and attraction (for destinations) variables in the gravity model. On the contrary, it is expected that the distance parameter, β_5, has a negative sign.

Equation (2. 48) was applied successively to the seventeen manufactured products. The dependent variable vector was different to each product, but the explanatory variables remained the same, since these variables are not related to any particular product. The display and analysis of the results will be targeted to some specific issues, since the complete list of results for the seventeen products would be too extensive.

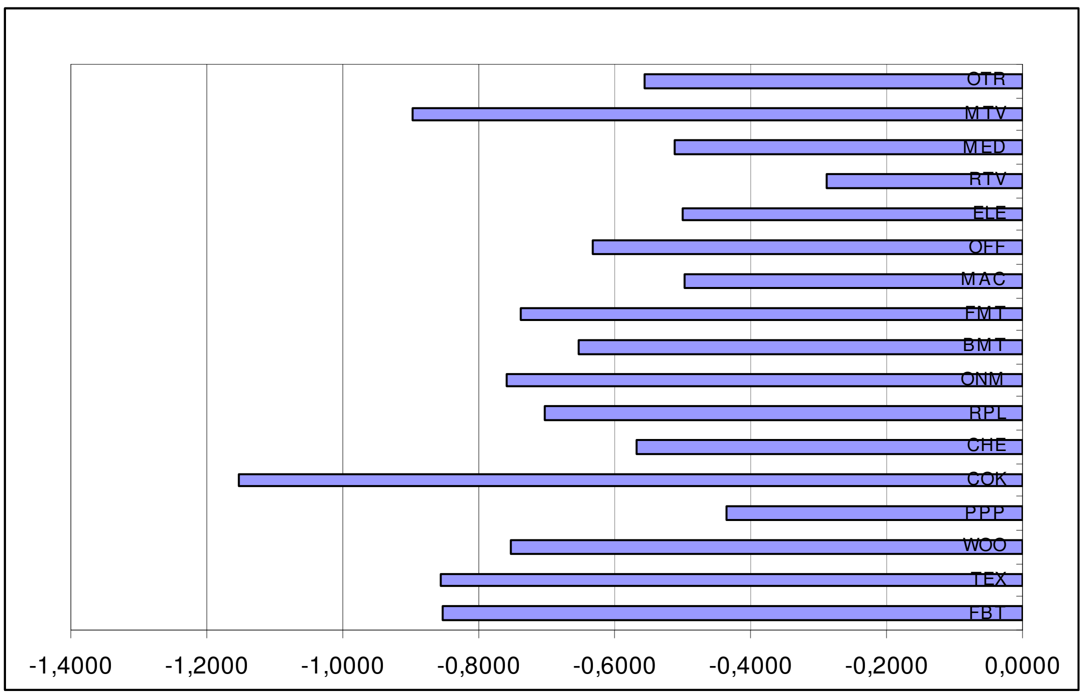
Table 2. 2 – Principal results for *Model 1*.

	R-bar squared	Statistically insignificant coefficients (5%)	Coeff. signs equal to expected?	Beta 5 (distance coeff.)
FBT	79,48%	**beta 3**	yes	-0,8537
TEX	80,05%	**beta 3**	**no: negative beta 1**	-0,8557
WOO	40,88%	**beta 3**	yes	-0,7543
PPP	56,86%	none	yes	-0,4353
COK	54,15%	**beta 3**	yes	-1,1518
CHE	75,23%	none	yes	-0,5679
RPL	83,25%	none	yes	-0,7019
ONM	77,04%	**beta 0 and beta 1**	yes	-0,7587
BMT	73,23%	**beta 3**	yes	-0,6538
FMT	80,78%	none	yes	-0,7375
MAC	81,73%	none	yes	-0,4976
OFF	66,16%	none	yes	-0,6332
ELE	75,29%	none	yes	-0,5007
RTV	66,13%	none	yes	-0,2890
MED	81,88%	none	yes	-0,5112
MTV	64,88%	**beta 3**	yes	-0,8961
OTR	81,33%	none	yes	-0,5566

Table 2. 2 sums up the more relevant results of Model 1. The first column of the table expresses the explicative power of the model, by means of its R-bar Squared. In spite of being extremely variable between the different products, it should be emphasized that this indicator assumes relatively low values for some of them, like WOO, for instance (for the full designation of the products, please see again Table 2. 1). The second column refers to the statistical relevance of the variables included in this model, through the t-statistic value (at 5% significance level). The most evident observation is that the estimated parameter associated to per capita GDP of the importing country is not significant in six of the seventeen cases. This may be a sign that size matters more than development level as an attraction measure to international trade (since the origin and destination population parameters are always statistically significant). The third column indicates the coincidence (or not) between the estimated parameters' signs and the expected ones. That coincidence is not verified in the case of TEX, in what respects to β_1. In fact, the traditional interpretation of gravity model in international trade applications is that trade tends to be greater between larger countries. However, since GDP effect was decomposed in two indicators (size and development), it could be argued that the sign of per capita

GDP is more an empirical issue, i.e., it may be positive or negative, according to the specific case. One plausible explanation to the negative sign found in TEX products is that it belongs to a class of low-technology industries, in which less developed countries are more specialized[87]. Being so, countries with a smaller per capita GDP would be expected to export more of these products and vice-versa. Finally, the last column presents the distance parameter estimated value, to each of the products. It is clear that, as expected, distance produces a negative effect on international trade flows. However, the estimated elasticity is extremely variable among the different products. Figure 2. 6 illustrates distance parameter variability.

Figure 2. 6 – Distance decay parameter variability.



[87] See, for example, the case of textiles and related products, in which Portugal, with a per capita GDP below the average, shows a great specialization.

2.4.1.3.2 Spatial econometric application.

The awareness of potential spatial dependence effects motivated us for a spatial econometric application as well. The several spatial econometric models which will be presented were all calibrated making use of the spatial functions of the Econometrics MATLAB toolbox, available at http://www.spatial-econometrics.com/.

The first step consisted in constructing the three above mentioned spatial weight matrices – destination-based, origin-based and origin-destination based[88], in order to capture the spatial structure of these particular data.

Using a stepwise approach, the first spatial model to be tested was the Spatial Error Model (equation (2. 45)). The computation of the Moran I-statistic over the least squares regression provided strong motivation to do so. This statistic[89] is designed to detect the presence of spatial autocorrelation in the residuals from a least-squares model (Le Sage, 1998). It was computed over the residuals from *Model 1*, using the correspondent function in Matlab. The values obtained for this statistic are summarized in Table 2. 3. For values above 1,96 one may reject the null hypothesis of no spatial correlation in the error terms, corresponding to a probability < 5% (Le Sage, 1998). In this Table, the bold values support the hypothesis of spatial autocorrelation in *Model 1* error disturbances. We may observe that, using weight matrix W_d, the values obtained by Moran I-statistic for each product's regression were always very high, suggesting that the least squares residuals exhibited spatial correlation.

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[88] It must be noted that there are other alternatives for mixed effects (simultaneous origin and destination based dependence), namely summing the origin-based with the destination-based W matrices. This alternative was also considered in our empirical application, but with results that did not differ from the ones using $W_o \cdot W_d$.

[89] The formula for this statistic can be seen, for example, in Le Sage (1998), p. 74.

Table 2. 3 – Moran's I statistic for *Model 1* residuals.

	Moran I stat		
	Wd	Wo	Wod
FBT	**5,80**	**4,34**	**5,21**
TEX	**6,53**	**3,93**	**3,26**
WOO	**8,46**	**4,26**	**5,13**
PPP	**8,24**	**2,41**	**2,90**
COK	**5,60**	**2,80**	1,12
CHE	**6,05**	**2,07**	0,96
RPL	**4,25**	**4,15**	**2,23**
ONM	**4,85**	**2,47**	**2,19**
BMT	**4,35**	**3,77**	**2,84**
FMT	**4,50**	**3,67**	**2,99**
MAC	**5,25**	**3,04**	1,93
OFF	**8,50**	**3,28**	**8,50**
ELE	**4,99**	**2,16**	1,37
RTV	**7,32**	1,42	1,94
MED	**4,88**	**2,72**	0,70
MTV	**8,94**	**1,97**	1,81
OTR	**2,89**	0,86	0,76

The results of this statistic were not so evident when matrices W_o or W_{od} were used instead of W_d. In these cases, the null hypothesis of no spatial correlation on the model disturbances was accepted in three of the seventeen and in eight out of seventeen regressions, respectively[90]. Yet, given the unambiguous suggestion of error autocorrelation derived from one specific representation of spatial structure, it was considered that there was enough motivation to apply a Spatial Error Model to all the manufactured products. This model, named *Model 2*, was conducted applying alternatively the three spatial weight matrices presented before, originating *Model 2A* (with weight matrix W_o), *Model 2B* (with weight matrix W_d) and *Model 2C* (with weight matrix W_{od}). The correspondent regressions can be written as:

[90] These different results occur because the formula of the Moran I-statistic makes use of matrix W (as it happens with other statistical tests for spatial autocorrelation). Thus, as stated in Dubin (1998), p. 319, "the results of this test will be conditional on the researcher's choice of W".

$$\begin{aligned}\ln x^{rs} &= \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^s + \beta_3 \ln N^r + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \mu^{rs} \\ \mu^{rs} &= \lambda_o W_o \mu^{rs} + \varepsilon^{rs}\end{aligned} \tag{2. 49}$$

$$\begin{aligned}\ln x^{rs} &= \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^s + \beta_3 \ln N^r + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \mu^{rs} \\ \mu^{rs} &= \lambda_d W_d \mu^{rs} + \varepsilon^{rs}\end{aligned} \tag{2. 50}$$

and

$$\begin{aligned}\ln x^{rs} &= \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^s + \beta_3 \ln N^r + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \mu^{rs} \\ \mu^{rs} &= \lambda_{od} W_{od} \mu^{rs} + \varepsilon^{rs}\end{aligned}. \tag{2. 51}$$

The statistical significance and value of λ will allow inferring about the presence and degree of spatial aucorrelation in the errors of the model. It is expected that, if significant, this parameter has a positive sign. As to the remaining variables, the expected signs are the same as previously mentioned. The calibration of Model 2 was done using the SEM (Spatial Error Model) function in Econometrics MATLAB toolbox (LeSage, 1998), which comprises maximum likelihood estimation method.

Spatial autoregressive model, named *Model 3,* was tested through the regression equations expressed before as equations (2. 42) to (2. 44) (respectively: *Model 3B, Model 3A* and *Model 3C).* Although the SAR model has been tested in these three versions, only the results of *Model 3C* will be shown here, since it is the one which generates higher evidence of autocorrelation in the dependent variable (see Table 2. 5). In fact, in Models 3A and 3B, the autoregressive coefficient is statistically insignificant in the majority of the regressions; moreover, it assumes a very low value when statistically significant. Conversely, in *Model 3C,* we can see that the autoregressive coefficient is statistically significant in fifteen out of seventeen cases. These results allow us to conclude that, when a mixed effect is considered in the multiplicative form, the spatial dependence hypothesis obtains a superior support. However, once again, its absolute value is not very high. In short, two main reasons made the SAR model to be considered inferior to the SEM model: (1) the low explicative power of the lagged dependent variable, even when it is statistically significant and (2) the lower fit to the sample data, reported by generally lower values obtained for the log-likelihood.

Table 2. 4 presents the principal results of this application. There are three main observations to make on these results. First, the distance resistance coefficients, though being statistically different from zero, have now much smaller values than in *Model 1*, being far from the commonly obtained values in similar gravity trade studies[91]. Also, their variability among products has also diminished. This is an interesting result, since it may be explained by the fact that, in *Model 1*, the distance coefficient could be incorporating the effect of other explanatory factors associated to the spatial structure of the observations, different from the mere distance between origin and destination. Reinforcing this idea is the fact that in *Model 2*, the autocorrelation parameter λ is now capturing some product variability, instead of β_5. The other two observations concern to the comparison that can be made between the three alternative SEM specifications. The

[91] Usually the distance coefficient is around unity.

first column of this table expresses the fit of the model by means of the maximum level obtained for the log-likelihood. According to this, and as it was previously suggested by the Moran I-statistic results, it seems that matrix W_d represents the type of spatial dependence that better reflects the way in which the errors are autocorrelated. On the one hand, in the destination-based SEM λ is statistically significant in all products, without exception. On the other hand, besides being relevant it assumes the highest values in Model 2B. The significant values found to parameter λ, suggest the existence of explanatory factors not included in the model that are spatially correlated. This last observation constituted a motivation to experiment the SAR model, in order to investigate the possibility of spatial autocorrelation in the dependent variable.

Spatial autoregressive model, named *Model 3*, was tested through the regression equations expressed before as equations (2. 42) to (2. 44) (respectively: *Model 3B*, *Model 3A* and *Model 3C*). Although the SAR model has been tested in these three versions, only the results of *Model 3C* will be shown here, since it is the one which generates higher evidence of autocorrelation in the dependent variable (see Table 2. 5). In fact, in Models 3A and 3B, the autoregressive coefficient is statistically insignificant in the majority of the regressions; moreover, it assumes a very low value when statistically significant. Conversely, in *Model 3C*, we can see that the autoregressive coefficient is statistically significant in fifteen out of seventeen cases. These results allow us to conclude that, when a mixed effect is considered in the multiplicative form, the spatial dependence hypothesis obtains a superior support. However, once again, its absolute value is not very high. In short, two main reasons made the SAR model to be considered inferior to the SEM model: (1) the low explicative power of the lagged dependent variable, even when it is statistically significant and (2) the lower fit to the sample data, reported by generally lower values obtained for the log-likelihood.

Table 2. 4 – Principal results for *Model 2*.

		Maximized Log Lik	Statistically insignificant coefficients (5%)	Coeff. signs equal to expected?	Beta 5	lambda
FBT	2A	-217,85	beta 3	yes	-0,1356	0,415
	2B	-213,82	beta 3	yes	-0,1358	0,418
	2C	-219,24	beta 3	yes	-0,1288	0,43
TEX	2A	-217,76	beta 0	no: negative beta 1	-0,1293	0,295
	2B	-202,40	beta 0	no: negative beta 1	-0,1315	0,434
	2C	-222,67	beta 0	no: negative beta 1	-0,1249	0,23
WOO	2A	-316,48	beta 3	yes	-0,1125	0,434
	2B	-280,17	beta 1	yes	-0,1182	0,602
	2C	-314,41	beta 3	yes	-0,1022	0,55
PPP	2A	-260,35	none	yes	-0,1254	0,263
	2B	-220,11	none	yes	-0,1322	0,588
	2C	-259,98	none	yes	-0,1190	0,31
COK	2A	-390,29	beta 3	yes	-0,1078	0,234
	2B	-377,19	beta 3	yes	-0,1132	0,395
	2C	-394,51	beta 3 and lambda	yes	-0,0980	not stat sig
CHE	2A	-229,32	none	yes	-0,1342	0,148
	2B	-212,15	none	yes	-0,1383	0,399
	2C	-231,18	lambda	yes	-0,1317	not stat sig
RPL	2A	-208,43	none	yes	-0,1240	0,326
	2B	-207,95	none	yes	-0,1243	0,335
	2C	-216,89	none	yes	-0,1173	0,18
ONM	2A	-239,52	beta 1	yes	-0,1140	0,225
	2B	-228,21	beta 1	yes	-0,1183	0,395
	2C	-241,44	beta 1	yes	-0,1085	0,17
BMT	2A	-256,49	beta 3	yes	-0,1270	0,305
	2B	-254,33	none	yes	-0,1275	0,331
	2C	-262,26	none	yes	-0,1194	0,21
FMT	2A	-221,87	none	yes	-0,1220	0,323
	2B	-217,47	none	yes	-0,1232	0,38
	2C	-227,15	none	yes	-0,1145	0,23
MAC	2A	-208,03	beta 3	yes	-0,1315	0,253
	2B	-197,95	none	yes	-0,1340	0,366
	2C	-212,06	lambda	yes	-0,1267	not stat sig
OFF	2A	-296,59	none	yes	-0,1190	0,27
	2B	-260,60	none	yes	-0,1242	0,548
	2C	-291,30	none	yes	-0,1242	0,548
ELE	2A	-214,31	none	yes	-0,1228	0,21
	2B	-203,67	none	yes	-0,1257	0,347
	2C	-216,50	lambda	yes	-0,1186	not stat sig
RTV	2A	-254,71	none	yes	-0,1250	0,128
	2B	-228,41	none	yes	-0,1307	0,445
	2C	-253,92	none	yes	-0,1216	0,20
MED	2A	-220,01	none	yes	-0,1160	0,213
	2B	-211,85	none	yes	-0,1182	0,33
	2C	-223,92	lambda	yes	-0,1127	not stat sig
MTV	2A	-338,46	beta 3	yes	-0,1276	0,175
	2B	-292,99	beta 3	yes	-0,1366	0,568
	2C	-339,16	beta 3	yes	-0,1236	0,18
OTR	2A	-261,86	lambda	yes	-0,1070	not stat sig
	2B	-258,40	none	yes	-0,1098	0,184
	2C	-261,96	lambda	yes	-0,1061	not stat sig

Table 2. 5 – Principal results for Model 3C.

Wod	Max Log Lik	Statistically insignificant coefficients (5%)	Coeff. signs equal to expected?	Beta 5	rho
FBT	-232,21	**beta 3 and rho**	yes	-0,1241	**not stat sig**
TEX	-223,33	**beta 0 and beta 3**	**no: negative beta 1**	-0,1198	0,052
WOO	-301,99	**beta 3**	yes	-0,0880	0,259
PPP	-245,41	**beta 3**	yes	-0,1111	0,142
COK	-392,97	**beta 3**	yes	-0,0931	0,092
CHE	-227,45	**beta 3**	yes	-0,1282	0,053
RPL	-207,62	none	yes	-0,1115	0,09
ONM	-234,99	**beta 1**	yes	-0,1023	0,092
BMT	-264,79	**rho**	yes	-0,1173	**not stat sig**
FMT	-221,11	none	yes	-0,1086	0,089
MAC	-200,21	**beta 3**	yes	-0,1209	0,088
OFF	-287,69	**beta 3**	yes	-0,1039	0,158
ELE	-199,83	none	yes	-0,1120	0,108
RTV	-234,22	**beta 3**	yes	-0,1137	0,14
MED	-210,69	**beta 3**	yes	-0,1066	0,104
MTV	-325,59	**beta 3**	yes	-0,1127	0,164
OTR	-257,59	none	yes	-0,1020	0,073

Still, as we can observe in Table 2. 5, this model confirms the results of SEM model, in what concerns to the distance decay parameters, which are lower than in *Model 1*, but always statistically significant.

In order to complete this spatial analysis, the general spatial model was also tested. The computation of the LM test to the SAR model residuals – a statistical test for spatial autocorrelation in the disturbances of the SAR model (Le Sage, 1998) – indicated that these were still autocorrelated (the LM value was always very high, irrespectively of the type of weight matrix used in the calculation). Thus, the general spatial model seems an appropriate formulation, since it also incorporates autocorrelation in the error terms. Given the results of the previous spatial model experiences (SEM and SAR), this application was made using matrix W_{od} as the spatial lag operator for the dependent variable and matrix W_d as the spatial lag operator for the error terms. The estimated regression equation, named *Model 4*, was:

$$\ln x^{rs} = \lambda_{od} W_{od} \ln x^{rs} + \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^r + \beta_3 \ln N^s + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \mu^{rs}$$
$$\mu^{rs} = \lambda_d W_d \mu^{rs} + \varepsilon^{rs}$$

(2. 52)

Three main observations are suggested by these results (see Table 2. 6). First, this model exhibits a better fit than in the two previous models. However, this new specification of the model, incorporating error autocorrelation, makes clear that the autoregressive coefficient of the dependent variable is not statistically significant in almost half of the products under study. Thus, the general spatial model seems a superior model only in ten of seventeen products; even in these cases, the ρ value is very low. Finally, even being lower than in *Model 2*, the error term autoregressive coefficient (λ) remains statistically significant in all product equations and continues to be considerably high.

Table 2. 6 – Principal results for Model 4.

Wod, Wd	Max Log Lik	Statistically insignificant coefficients (5%)	Coeff. signs equal to expected?	Beta 5	rho	lambda
FBT	-101,36	**beta 3 and rho**	yes	-0,1347	**not stat sig**	0,41
TEX	-89,73	**beta 0, beta 1 and rho**	**no: negative beta 1**	-0,1306	**not stat sig**	0,44
WOO	-164,62	**beta 1 and beta 3**	yes	-0,1099	0,10	0,54
PPP	-106,62	**beta 3 and rho**	yes	-0,1288	**not stat sig**	0,57
COK	-263,62	**beta 3 and rho**	yes	-0,1076	**not stat sig**	0,39
CHE	-99,09	**beta 3 and rho**	yes	-0,1359	**not stat sig**	0,38
RPL	-90,37	none	yes	-0,1175	0,07	0,24
ONM	-113,31	**beta 1**	yes	-0,1127	0,06	0,32
BMT	-141,33	**rho**	yes	-0,1250	**not stat sig**	0,32
FMT	-101,66	none	yes	-0,1175	0,06	0,30
MAC	-80,32	**beta 3**	yes	-0,1277	0,06	0,29
OFF	-141,91	**beta 3**	yes	-0,1158	0,10	0,51
ELE	-83,24	**beta 3**	yes	-0,1174	0,08	0,22
RTV	-107,72	**beta 3**	yes	-0,1221	0,09	0,36
MED	-94,49	**beta 3**	yes	-0,1115	0,07	0,22
MTV	-180,00	**beta 3 and rho**	yes	-0,1347	**not stat sig**	0,55
OTR	-143,34	none	yes	-0,1052	0,06	0,14

Given the high autocorrelation of errors evidenced by *Models 2* and *4*, suggesting that some relevant spatially connected explanatory variables could be missing in the model, an additional experience was carried out. In *Model 5*, a new variable was added to the traditional gravity explanatory variables. This added variable is product specific and reflects the effect of each country's specialization on the volume of exports. It should be noted that in the previous models, the vector of explanatory variables was the same, independently of the specific product in study. However, in some cases, de degree of specialization of some country in exporting a specific product *k* has an influence that may even prevail over the distance effect. Consider, for example, the product “WOOD AND PRODUCTS OF WOOD AND CORK”. The weight of this product exports on total exports of Finland is pretty above the average. More precisely, is almost 7 times the correspondent weight in the whole of countries being considered. Formally, this can be expressed by a Degree of Specialization (DS) indicator, as in equation (2. 53). The numerator of this index represents the weight of product k on origin *r*’s total exports[92]; the denominator indicates the weight of product k on all origins’ exports. Actually, this is no more than a Location Quotient, computed using exports as the variable of specialization: values above (below) 1 indicate a higher (lower) than average specialization of country *r* in exporting product k. Table 2. 7 shows the maximum values of this index obtained for each product and the correspondent highly specialized country.

:

$$DS_r^k = \frac{\dfrac{x_r^k}{\sum_{k=1}^{10} x_r^k}}{\dfrac{\sum_{i=1}^{14} x_r^k}{\sum_{i=1}^{14}\sum_{k=1}^{10} x_r^k}}$$

(2. 53)

From the previous explanation, it is clear that the expected sign for the new variable’s estimated coefficient is positive. In addition, it should be emphasized that product specialization is likely to be a spatially clustered variable, since it is often determined by natural resources allowance. Thus, it may offset some of the effects previously included in the error term autoregressive coefficient.

92 $x_r = \sum_s x^{rs}$

Table 2. 7 – Maximum value of DS, for each product, and correspondent specialized country.

	Max SD	Country
FBT	2,59	Denmark
TEX	4,84	Portugal
WOO	6,66	Finland
PPP	6,63	Finland
COK	3,11	Netherlands
CHE	2,71	Ireland
RPL	1,37	Italy
ONM	1,99	Portugal
BMT	2,40	Greece
FMT	1,64	Italy
MAC	1,89	Italy
OFF	4,48	Ireland
ELE	1,58	Portugal
RTV	3,59	Finland
MED	1,38	Denmark
MTV	1,91	Spain
OTR	2,10	France

The regression equation for *Model 5*, given below, assumes the Spatial Error Model formulation – given the low degree of autocorrelation in the dependent variable shown in *Model 3* and *Model 4*, this was not taken into account in *Model 5*.

$$\ln x^{rs} = \beta_0 + \beta_1 \ln N^r + \beta_2 \ln POP^r + \beta_3 \ln N^s + \beta_4 \ln POP^s + \beta_5 \ln \delta^{rs} + \beta_6 \ln DS^r + \mu^{rs}$$
$$\mu^{rs} = \lambda_d W_d \mu^{rs} + \varepsilon^{rs}$$

(2. 54)

The main estimation results of this model are shown in Table 2. 8. Five main comments are suggested by these achievements:

- The values of the maximized log-likelihood indicate that this formulation possesses a better overall fit to the data than *Model 2*, which corresponds to the Spatial Error Model without the additional explanatory variable.
- Statistically insignificant coefficients are fewer in this model than on the previous ones.

- All the estimated coefficient signs correspond to what was expected a priori.

- Estimated *DS* coefficients are statistically significant in all but one product and show a high sensibility of exports with respect to origin's specialization on the specific product under study. All coefficients are close to or greater than unity.

- The distance decay parameter remains statistically significant in all products, although with low absolute values.

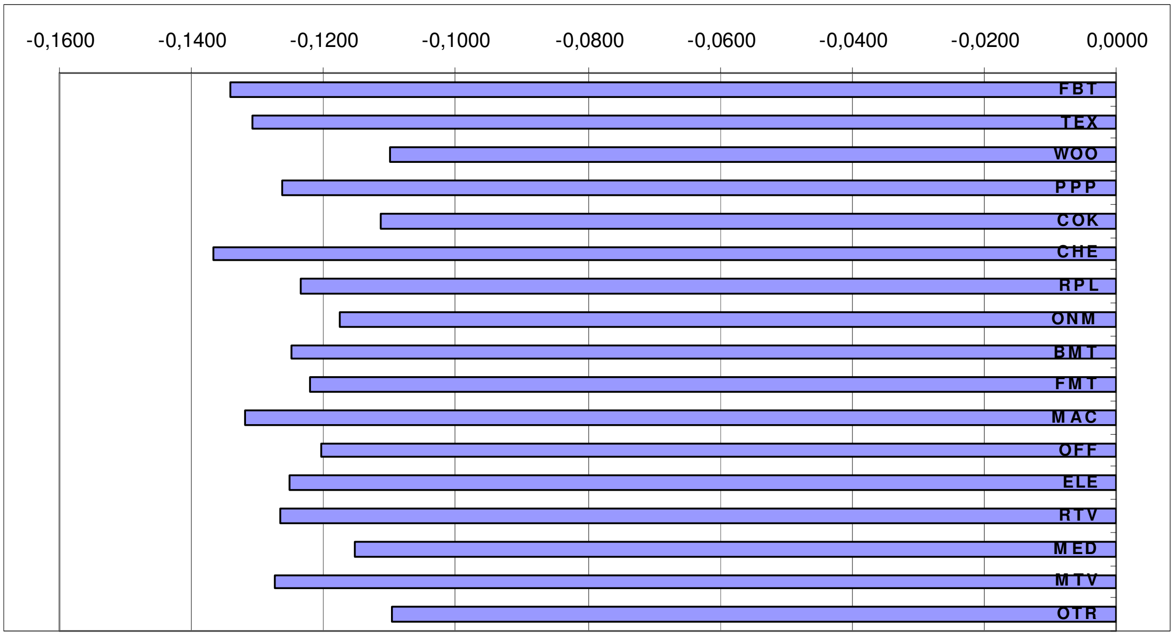
Table 2. 8 – Principal results for Model 5.

	Max Log Lik	Statistically insignificant coefficients (5%)	Coeff. signs equal to expected?	Beta 5	Beta 6	lambda
FBT	-201,14	**beta 3**	yes	-0,1341	0,74	0,32
TEX	-189,26	none	yes	-0,1306	0,81	0,36
WOO	-253,14	none	yes	-0,1099	1,19	0,29
PPP	-195,27	none	yes	-0,1262	1,16	0,28
COK	-371,22	**beta 3**	yes	-0,1115	0,72	0,33
CHE	-195,70	none	yes	-0,1366	1,17	0,28
RPL	-201,85	none	yes	-0,1234	1,04	0,29
ONM	-223,71	none	yes	-0,1175	0,91	0,35
BMT	-237,05	none	yes	-0,1248	0,94	0,21
FMT	-210,22	none	yes	-0,1219	0,90	0,31
MAC	-186,97	none	yes	-0,1319	0,85	0,26
OFF	-238,71	none	yes	-0,1203	0,98	0,33
ELE	-201,67	none	yes	-0,1252	0,68	0,32
RTV	-211,32	none	yes	-0,1266	1,13	0,21
MED	-200,34	**beta 1**	yes	-0,1153	1,77	0,16
MTV	-261,51	none	yes	-0,1272	1,22	0,18
OTR	-258,05	**beta 6**	yes	-0,1097	**not stat sig**	0,16

Additional comments on these results are pertinent. Comparing the values obtained here for the coefficient of error autocorrelation with the ones of *Model 2*, it is evident that it has diminished considerably, though remaining statistically significant. Thus, we may conclude that the introduction of the additional variable has reduced the error term autocorrelation.

Finally, it seems evident, from this and the previous models, that it is relevant to make a disaggregated analysis, instead of calibrating the model for the aggregate trade flows. In Figure 2. 7, we can observe the estimated distance coefficients, for each product, obtained in *Model 5*. Although the most distance resistant products are not the same in this Model as in *Model 1* (probably because some effects are now being captured by the *DS* variable and also because this model recognizes spatial correlation in the disturbances, unlike *Model 1*), we can still observe that some variability remains: chemical products and food, beverages and tobacco are suggested to be the most distance-sensitive, according to the results of this model.

Figure 2. 7 – Distance coefficient variability in Model 5.



The several experiences undertaken in the previous section confirm that gravity-based models generate quite good results when trade flows are known *a priori*, i.e., when the model is used with an explanatory purpose. Contrarily to what has been done in most of the gravity trade model econometric applications, some spatial econometric experiences were carried out, considering three different spatial models and three different types of spatial dependence (through the use of three distinct spatial weight matrices W). Comparing these results with the ones of *Model 1* (with no consideration of spatial effects), we have concluded that the recognition of spatial autocorrelation of errors provides unambiguous improvement in the Model, reflected by a higher statistical significance of the estimated parameters and a better consistency between expected and obtained signs. The relevance of taking error autocorrelation into account is confirmed by the fact that λ is statistically significant and assumes quite high values, especially in the destination-based version of the spatial weight matrix. Conversely, the incorporation of a spatial lag of the dependent variable (as in *Model 3*) doesn't seem critical in this sample data. Even in those cases in which the spatial autoregressive coefficient has shown statistical significance, its values were always rather low, indicating a very weak degree of spatial dependence (this result was reached both in the spatial autoregressive and in the general spatial models). Thus, the last experience relied upon the SEM formulation, yet including an additional explanatory variable, which has demonstrated to be statistically relevant: the origin's relative specialization on the exportation of product *k*. The results of this model allow for the conclusion that the augmented version of the gravity model is the best suited to explain trade flow behavior. The fact that these econometric exercises were separately applied to distinct products also made clear that each different traded product has its own specificity, originating quite variable estimated coefficients. Furthermore, the product disaggregated application is very important because, in most of the times, the gravity model is used in the construction of larger models, like input-output models, which require product specific estimated values.

2.4.2 Type (b) information context.

In the previous section we have confirmed what other studies have concluded before: that the gravity-based model is well suited to explain trade flow behaviour. The question in this section consists in discussing and exploring the potential of the model in type (b) information contexts. In this section, we intend to review the empirical work carried out in Sargento (2007), which intended to answer the above referred question. As it has been referred in the beginning of section 2.4, in type (b) information contexts, the gravity model is used with the aim of generating the undisclosed values of an Origin-Destination matrix as the one in Figure 2. 1, having usually previous access to the column and row totals. Interregional trade flows first estimative, denoted by $\tilde{x}^{rs}$, can then be obtained by:

$$\tilde{x}^{rs} = G\frac{\left(Y^{r}\right)^{\alpha_1}\left(Y^{s}\right)^{\alpha_2}}{\left(\delta^{rs}\right)^{\alpha_3}} \tag{2. 55}$$

In order to guarantee the agreement with the additivity constraints, the initial values are adjusted, through a matrix adjustment method, like RAS, for example. The underlying principle consists in finding the closest the matrix to the initial one, which also respects the known row and column totals (de Mesnard, 2003). The resulting matrix will be given by an equation like (2. 10), repeated here:

$$\left(\tilde{x}^{rs}\right)_{RAS} = J^r \tilde{x}^{rs} L^s = J^r G \frac{\left(Y^r\right)^{\alpha_1}\left(Y^s\right)^{\alpha_2}}{\left(\delta^{rs}\right)^{\alpha_3}} L^s$$

(2. 56)

being J^r and L^s defined, as before, as the balancing parameters.

As it was explained in section 2.3.1, the final estimated matrix composed by elements given by equation (2. 56) depends heavily on the initial matrix (prior to the adjustment procedure). Nevertheless, the main difficulties take place exactly when it comes to estimate the initial matrix, i.e., when applying an equation like (2. 55). In practice, we can refer to two main problems, listed below:

(1) Parameters G, α_1, α_2 and α_3 are unknown, which makes it impossible to directly apply the previous formula.

(2) Without any survey-based table to serve as a benchmark, the results provided by the gravity model cannot be not rigorously evaluated (Hewings and Jensen, 1986).

Following the work done in Sargento (2007), a first exploratory experience was carried out in which the above mentioned problems were overcome as follows:

(1) The initial matrix was obtained applying a particular version of equation (2. 55), in which almost all the unknown parameters were arbitrarily set equal to one. More precisely, we have made:

$$\tilde{x}^{rs} = G^r \frac{P^r P^s DS^r}{\delta^{rs}};$$
$$G^r = x^r \left(\sum_s \frac{P^r P^s DS^r}{\delta^{rs}} \right)^{-1}$$

(2. 57)

in which an additional variable was considered: Degree of Specialization of origin *r* (DS^r) in exporting the specific product under study, for which it was found evidence of statistical significance in the previous section (*Model 5*). The constant of proportionality G^r is a scalar that guarantees the exact observance of the *r*th row summing up constraint: $\sum_s \tilde{x}^{rs} = x^r$ [93]; it is introduced in order to make the initial matrix comparable to the real one (if no scalar was introduced, the values of both matrices would have considerably different values). As evidenced by the formula, α_1, α_2 and α_3 are assumed to be unitary. The initial matrix was iteratively adjusted, making use of the known margins. The algorithm converged after six iterations.

(2) The performance of the gravity model was tested upon international trade flows between 14 European countries – the same database that was used in the previous section[94]. The information on international trade flows is used in two stages: 1) to adjust the initial matrix to the column and row totals extracted from the international trade database; 2) to serve as a benchmark to the values generated by the model, comparing the estimated flows to the real ones[95], in order to assess

[93] The introduction of this scalar is equivalent to a first iteration of the RAS procedure, in which the row sums are the first to be adjusted.
[94] A similar solution to overcome the lack of a survey-based table to serve as a benchmark was adopted, for example, in Canning and Wang (2006).
[95] For a discussion on measures of matrix comparison, please see section 4.4 of Chapter 1.

the performance of the model – a better performance will be reflected in smaller differences between the estimated and the real matrix. In this study, the following measure of distance between matrices was used:

$$STPE = 100 \cdot \frac{\sum_s \sum_r \left| x^{rs} - \tilde{x}^{rs} \right|}{\sum_s \sum_r x^{rs}} \tag{2. 58}$$

in which *STPE* stands for: Standard Total Percentage Error.

This measure was computed in two different stages of the process: before applying RAS (indicating the distance between the initial matrix and the real one) and after applying RAS (indicating the distance between the estimated final matrix and the real one).

Table 2. 9 – STPE measured between the estimated and the real trade matrix

	BEFORE RAS	AFTER RAS
FBT	40%	30%
TEX	48%	39%
WOO	54%	42%
PPP	40%	33%
COK	61%	53%
CHE	48%	35%
RPL	41%	30%
ONM	38%	27%
BMT	46%	32%
FMT	44%	32%
MAC	44%	29%
OFF	56%	41%
ELE	47%	34%
RTV	54%	36%
MED	46%	35%
MTV	50%	31%
OTR	64%	41%
average	**48%**	**35%**

Table 2. 9 presents the obtained results for all the seventeen products. These results show that the initial matrix is quite distant from the real one (with a mean error around 50%). The iterative adjustment allows for some improvements in the matrix, yet not in a drastic way. In some products, like ONM, for example, the resulting error is rather low. However, in general, the final matrix is still quite distant from the real one, suggesting that the gravity-based formulation used here is not suitable to accurately generate the international trade flows.

From these preliminary results several questions arise, which will be addressed in the following section:

(1) How important are the initial origin-destination flows? In other words, if one applies an even simpler method to generate the initial values, or conversely, a more sophisticated one, instead of the gravity-based model, are the final results affected in a drastic way?

(2) What happens if at least some of the parameters of the model are estimated through an alternative procedure instead of being set equal to one? Does the estimated O/D matrix become closer to the real one? In fact, one of the most obvious sources of error of the previous experience relies on the fact that all coefficients were arbitrarily set equal to one, while it became clear from the previous section that the estimated parameters assume variable values from product to product and are not unitary. This is especially true in what respects to the distance coefficient, which was found to be much smaller than unity, according to the values obtained in the spatial econometric models.

(3) How sensitive is the input-output model to the insertion of different estimates for interregional trade? This means: “What is the impact on the interregional multipliers of considering different values for interregional trade?”; “Do these large errors in the O/D matrix reflect also in large errors when the model is applied, for example, to access the impact of an exogenous change in final

demand?". In fact, when the main concern relies in the application of multi-regional input-output models, as it happens in our study, this is a crucial question, since it may validate (or not) the use of indirect methods to obtain the seldom available and absolutely indispensable data on interregional trade flows.

2.5 Absolute and analytical comparison between different interregional trade estimation methods.

The objective of this section consists in making a comparison between distinct interregional trade estimation methods, both in absolute and in analytical terms. First of all, it is necessary to clarify what is meant by "absolute" and "analytical" comparison. An absolute comparison relies on the differences observed among Origin-Destination matrices of trade flows generated by different interregional trade estimation methods. An exercise of such nature was made in the previous section, in which one estimated matrix was contrasted to the real trade flow matrix. An analytical comparison goes further and involves the assessment of the impact on the multipliers obtained from the model, created by the insertion of different interregional trade values. In order to allow for this sort of comparison, it is necessary to have access to a complete multi-regional system upon which a multi-regional input-output model can be developed. Thus, the achievement of this section's purpose is conducted under 5 stages, listed below, which also correspond to the sub-sections under Section 2.5:

(1) Multi-regional input-output table assemblage[96] – this is done using the Input-output tables for each individual country and a set of bilateral trade flows as the base data;

(2) Development of the multi-regional input-output model, which implies the adoption of certain simplifying hypotheses, explained further on.

(3) Estimation of O/D trade matrices on the basis of six different methodologies.

[96] For a matter of consistency with the terms used in Chapter 1, in the presentation of the many-region models, we opt for using the designation "region" and "multi-regional" in the context of this empirical application, even when we are actually dealing with a system of countries.

(4) Comparison between the six different matrices obtained before (absolute comparison).

(5) Consecutive insertion of the different O/D matrices into the Multi-regional input-output system and model simulation in a context of final demand change, in order to evaluate the sensitivity of the model to different interregional trade estimates (analytical comparison).

2.5.1 Multi-regional input-output system assemblage.

Before explaining how the multi-regional input-output system was assembled, a previous elucidation must be made. The assemblage of the multi-regional system is not the ultimate objective of this empirical study; instead, it is merely an instrument to be used in the comparison between the different interregional trade estimation methods. Thus, the high level of aggregation considered, as well as the several simplifying hypotheses adopted, should not be overemphasized. In fact, all the subsequent experiences will be made using the assembled input-output system as the common starting point. Hence, the conclusions obtained comparing those different experiences to one another should not be affected to a great extent by the hypotheses assumed in the construction stage.

The set of countries involved in the system is the same that was considered in Section 2.4: the 14 countries belonging to the European Union before enlargement (15 minus one, since Belgium and Luxembourg are considered jointly). The reference year is 2000. The input-output data come from the ESA 95 Input-output table database, provided by the EUROSTAT, which is available on-line at the EUROSTAT webpage[97]. The bilateral trade data come from the OECD Bilateral Trade Database, Edition 2002. This trade database is used to estimate the distribution of intra-regional trade among the 14 countries included in the system. Given the discrepancies that occurred between export-based data and import-based data (commonly known as the “mirror statistics puzzle”),

[97]http://epp.eurostat.ec.europa.eu/portal/page?_pageid=2474,54156821,2474_54764840&_dad=portal&_schema=PORTAL.

meaning that the total exports reported by country A differ from the sum of total imports coming from country A reported by the other countries, a short-cut method was adopted: to consider the mean value between the export-based and the import-based OECD data. However, the totals of intra-regional imports and intra-regional exports are imposed by the values indicated by the EUROSTAT Input-output tables. In order to avoid an extremely heavy multi-regional table, the original classification embodied in the EUROSTAT Input-output tables was aggregated into 6 categories of products and industries. The following table sums up the set of data used in this study:

Table 2. 10 – Data description.

Data type	Source / Number	Description
Reference year	2000	
Countries involved in the multi-regional system	14 European countries	France (FRA), Germany (GER), Italy (ITA), Belgium + Luxembourg (BELUX), Netherlads (NLD), Denmark (DNK), Ireland (IRE), United Kingdom (GBR), Greece (GRC), Spain (ESP), Portugal (PRT), Austria (AUT), Finland (FIN) and Sweden (SWE).
Input-output data	ESA 95 Input-output tables, EUROSTAT	Tables are provided in rectangular format (Supply and Use tables); Supply table at basic prices including a transformation into purchasers' prices and Use table at purchasers' prices. Use flows include both domestically produced and imported products - total use flows.
Bilateral trade data	OECD Bilateral Trade Database, Edition 2002	Export-based data and Import-based data.
Product classification	6 categories	**A+B** – "Products of agriculture, hunting, forestry and fishing", **C** – "Mining and quarrying", **D** – "Industry", **E** – "Electricity, gas and water supply", **F** – "Construction" and **G to P** – "Services".

Some additional comments on these data are due, especially in what concerns to the Supply (or Make) and Use tables. In Section 5 of Chapter 1 (Section 1.5.1 – "Commodity-by-industry accounts"), we had the opportunity to present the rectangular or Supply and Use framework (distinguishing it from the symmetric format) and also to differentiate between total use tables and intra-regional use tables. The original tables that come from the EUROSTAT database are provided in rectangular format (thus including Make and Use tables) and the Use tables are composed by intermediate and final use flows which include not only domestically produced products, but also imported ones. Thus, they are said to be total use tables. The issue concerning the valuation of goods and services has not been addressed in Chapter 1, since we were using the simplification of ignoring taxes and subsidies on products, as well as margins. Although this is a subject to be further developed in Chapter 3, it is essential to provide here a brief explanation on the concepts of basic prices and purchasers' prices. Different concepts can be used in the valuation of input-output flows of products, ranging from the production price to the purchasers' price. In practice, however, in the input-output tables produced according to the European System of Accounts (ESA), only two price concepts are used: basic price and purchasers' price. Basic prices are similar to production prices, except for the fact that basic prices include other taxes and subsidies on production, which are not possible to allocate to specific products. Purchasers' prices, as the name indicates, represent the amount paid to obtain "a unit of a good or service at the time and place required by the purchaser" (EUROSTAT, 2002, p. 121). Basic prices (*bp*) can be obtained from purchasers' prices (*pp*), through the following calculations:

$$bp = pp - \text{taxes on products} + \text{subsidies on products - trade and transport margins}$$

In the case of the ESA tables, the balance between supply and use is made at *pp*, implying that the Supply tables include the addition of margins and taxes less subsidies, in order to convert total supply at *bp* into total supply at *pp*.

Making use of the above described database, the assembling process involved the assumption of certain hypotheses to perform the conversion of the original tables, in which use tables were composed of total use flows valuated at *pp*, into domestic Use tables, with products valuated at *bp*. This previous conversion of the Use tables corresponds to one of the options that could be taken in this stage: the other option would be to work directly with the Use tables in their original nature and then assume the same kind of hypotheses in the model development. The discussion about the choice between these two alternative approaches is the central theme of Chapter 3[98]. Thus, in this

[98] In fact, Chapter 3 will also discuss the plausibility of performing the conversion of the Supply and Use tables into the symmetric format before applying the model, *vis a vis* the direct modelling of the Supply and

empirical study, we have opted for following the usual approach adopted by other researchers and preferred by the regional input-output literature[99]: to operate first the conversion of the original tables into domestic Use tables valuated at *bp* and then develop the input-output model on the basis of the converted tables[100]. The basic structure of the final table can be observed in Figure 2. 8, which illustrates the case for 3 regions.

Use tables. The hypothesis of converting the original tables into the symmetric format was not even considered in this study. Further details on this discussion are left to Chapter 3.

[99] This procedure is advocated for example in EUROSTAT (2002) and is followed in empirical applications, for example, in ISEG/CIRU (2004) and in Oosterhaven and Stelder (2007).

[100] Still, we have opted for using the rectangular model structure, conversely to what is usually made in this kind of exercises.

		Region A			Region B			Region C			ROW	SUM
		Products	Industries	Final demand	Products	Industries	Final demand	Products	Industries	Final demand		
Region A	Products	---	$\left[u_{ji}^{AA}\right]$	y_j^{AA}	---	$\left[u_{ji}^{AB}\right]$	y_j^{AB}	---	$\left[u_{ji}^{AC}\right]$	y_j^{AC}	$y_j^{A\,ROW}$	v_j^A b.p.
	Industries	$\left[v_{ij}^{A}\right]$	---	---	---	---	---	---	---	---	---	g_i^A b.p.
Region B	Products	---	$\left[u_{ji}^{BA}\right]$	y_j^{BA}	---	$\left[u_{ji}^{BB}\right]$	y_j^{BB}	---	$\left[u_{ji}^{BC}\right]$	y_j^{BC}	$y_j^{B\,ROW}$	v_j^B b.p.
	Industries	---	---	---	$\left[v_{ij}^{B}\right]$	---	---	---	---	---	---	g_i^B b.p.
Region C	Products	---	$\left[u_{ji}^{CA}\right]$	y_j^{CA}	---	$\left[u_{ji}^{CB}\right]$	y_j^{CB}	---	$\left[u_{ji}^{CC}\right]$	y_j^{CC}	$y_j^{C\,ROW}$	v_j^C b.p
	Industries	---	---	---	---	---	---	$\left[v_{ij}^{C}\right]$	---	---	---	g_i^C b.p.
ROW		---	$\sum_j u_{ji}^{ROW\,A}$	---	---	$\sum_j u_{ji}^{ROW\,B}$	---	---	$\sum_j u_{ji}^{ROW\,C}$	---		
Trade and transp marg		---	d_i^A	---	---	d_i^B	---	---	d_i^C	---		
Taxes less sub on prod		---	l_i^A	---	---	l_i^B	---	---	l_i^C	---		
Total Int. Consumption		---	IC_i^A p.p.	---	---	IC_i^B p.p.	---	---	IC_i^C p.p.	---		
Value Added		---	VA_i^A	---	---	VA_i^B	---	---	VA_i^C	---		
SUM		v_j^A b.p.	g_i^A b.p.	---	v_j^B bp.	g_i^B b.p.	---	v_j^C bp.	g_i^C b.p.	---		

Figure 2. 8 – Basic structure of the multi-regional Make and Use system.

Notation for Figure 2. 8:

- A, B e C: indices that designate the three regions.
- u_{ji}^{AA} - generic element of the regional production Use matrix $\mathbf{U^{AA}}$, which indicates the amount of product *j* produced in region A that is used by industry *i* in region A.
- u_{ji}^{AB} - generic element of the Use matrix $\mathbf{U^{A}}^{B}$, which indicates the amount of product *j* produced in region A that is used by industry *i* in region B.
- $u_{ji}^{ROW\,B}$ - amount of product *j* produced in the rest of the world that is used by industry *i* in region B.
- y_{j}^{AA} - amount of product *j* produced in region A which is used for final demand in the region.
- y_{j}^{AB} - amount of product *j* produced in region A which is used for final demand in region B.
- $y_{j}^{A\,ROW}$ - amount of product *j* produced in region A which is exported to the rest of the world.
- v_{ij}^{A} - generic element of the Make matrix in region A. It represents the amount of product *j* produced by industry *i* in region A.
- d_{i}^{A} - total amount of margins embodied in intermediate consumption of industry *i* (it corresponds to the column sum of the matrix of margins, for industry *i*; the matrix of margins is computed under certain hypotheses, exposed further on).
- l_{i}^{A} - total amount of taxes, less subsidies, embodied in intermediate consumption of industry *i* (it corresponds to the column sum of the matrix of net taxes, for industry *i*; the computation of the matrix of net taxes will also be explained further on).
- IC_{i}^{A} - total intermediate consumption of industry *i* in region A.

- VA_i^A - value added of industry i in region A.
- v_j^A - total production of product j in region A.
- g_i^A - total production of industry i in region A.

In order to achieve such multi-regional Make and Use system, the assemblage was carried out in seven steps, namely:

(1) Aggregation of the Make and Use tables, converting the original classification of 59 product and industry categories into a classification with 6 categories (see Table 2. 10).

(2) Operate the conversion of the Use tables valuated at *pp* into Use tables valuated at *bp*. This was done making use of the proportionality assumption, commonly used in this sort of exercises[101]: for each product, the margin (net taxes) rate comprised in any type of use (intermediate or final) of that product is the same and is given by the proportion of margins (net taxes) on total supply of the same product.

(3) Eliminate the discrepancies between aggregated exports and aggregated imports among the 14 European countries involved in the multi-regional system. In principle, for each product, the aggregate value of intra-EU imports should equal the aggregate value of intra-EU exports. However, this was not verified by the values contained in the Make and Use tables obtained in the previous step, due to discrepancies originated by the different valuation prices for exports and imports. In principle, it should be possible to convert all cif prices into fob prices. Yet, only 8 out of 15 of the Make and Use tables included in the Eurostat database had a column with information for cif / fob adjustments. Being so, and given that the present exercise merely aims to provide a basis for a simulation exercise, rather

[101] Once again, we refer to Chapter 3 for a deeper discussion on the plausibility of the proportionality assumptions, considered not only with respect to margins and net taxes, but also in the treatment of imports. The analytical presentation of these hypotheses is also left to Chapter 3.

than a true multi-regional input-output system for European countries, we have applied the following solution: a) for each product, the aggregate value of intra-EU imports was made equal to the aggregate value of intra-EU exports; b) the distribution of the corrected value of intra-EU imports among the 14 countries was made assuming that the weight of each supplier country in intra-EU imports is the same that it had according to the initial value of intra-EU imports; c) the difference between the corrected column of intra-EU imports and the original one was allocated to the column of extra-EU imports, hence maintaining the total value of imports for each product.

(4) Expurgate the import content from the intermediate and final use flows, in order to get an intra-regional flow table for each region. In the Make tables, imports are split up into intra-EU and extra-EU (keeping in mind that, in the reference year of 2000, only the 15 "old" European countries are considered in such distinction). Following the same basic principle as for margins and net taxes, it was assumed a constant average import propensity, meaning that, for each product, the same rate of imports (coming from the rest of the world and from the rest of the regions involved in the system) is embodied in intermediate and final use of that product. This is a common assumption in this type of studies, which follows the suggestion implicit in the Chenery-Moses model. It has been used, for example, in Oosterhaven and Stelder (2007) and in Van Der Linden and Oosterhaven (1995). Yet, it must be noted that, in our case, the average import propensity was computed under the "no re-exports" hypothesis. Following Miller and Blair (1985) and Jackson (1998), the intention was to recognize the specificity of exports in the context of final uses, given that they involve much less incorporation of imported products than other final uses, like investment for example. In order to take this differentiation into account, the average import propensity for each product was computed dividing the corresponding amount of imports by the amount of internal demand (total production + imports – exports). Then, it was applied to all intermediate and final uses of that product, except for

exports. This means that we are assuming no import content in the export value of that product.

(5) For each product, adjust the OECD-based Origin-Destination matrices to the row and column totals corresponding to the values of intra-EU exports and imports, provided by the Use and Supply (corrected) tables, respectively. This was made using the RAS procedure. It must be noted that, in the cases of products "F – Construction" and "G to P – Services", the initial O/D matrix was computed using the data on aggregate trade, since the Bilateral Trade database does not cover service trading. As it is referred in Van Der Linden and Oosterhaven (1995), this assumption is adequate, given that "trade in services is strongly related to the trade in goods, especially for trade and transportation margins" (p. 5).

(6) For each country, compute the 13 Use tables for the imported products – one to each of the remaining supplier country. This was made using the same proportionality assumption as for the aggregate intra-EU imports and for the extra-EU imports. Using an example, if France provides 1% of total supply of agricultural products in Austria (being total supply composed by Austrian production, intra-EU imports and extra-EU imports), it is assumed that, of all intermediate and final uses (except for exports) of agricultural products implicit in the total Use Austrian table, 1% corresponds to imports from France.

(7) Final assemblage of the multi-regional Make and Use table, inserting the original Make tables (respecting only to the domestic production matrix, represented by the generic element v_{ij}) and the Use tables for the domestic production and for imports, obtained in the preceding steps. The rows named in Figure 2. 8 by "*ROW*", "*Trade and transp. margins*" and "*Taxes less sub. on products*" correspond to the column sum of the matrices of extra-EU imports, trade and transport margins and net taxes, respectively, which were constructed assuming the already explained hypotheses of proportionality (steps (2) and (4)).

The assembled multi-regional Make and Use table for the 14 European countries may be observed in Annex A.2.1 (because this was too extensive to be integrated in the present text, we have opted by saving it as an excel file – named *MRMU_Table* – and making it available in the CD-rom that is attached to this dissertation).

2.5.2 Development of the multi-regional input-output model.

The development of an input-output model on the basis of the rectangular or Make and Use format has already been introduced in Chapter 1, Section 1.5.1, yet to the case of one single region (or single nation) table. Here, the same principles will be followed, using the necessary adaptations to consider the multi-regional linkages embodied in the system.

If we have a multi-regional table such as the one depicted in Figure 2. 8 as a starting point (and using the 3 regions' example), the multi-regional input-output model can be developed as follows. Let the bold notation designate the column vectors and the matrices composed by the corresponding variables introduced before. For example, $\mathbf{U^{AB}}$ stands for the matrix of intermediate consumption composed by flows u_{ji}^{AB} and $\mathbf{y^{AB}}$ represents the vector composed by flows y_{j}^{AB}. Hence, we can write the system:

$$\begin{aligned}
\mathbf{v^A} &= \mathbf{U^{AA}i} + \mathbf{y^{AA}} + \mathbf{U^{AB}i} + \mathbf{y^{AB}} + \mathbf{U^{AC}i} + \mathbf{y^{AC}} + \mathbf{y^{A\,ROW}} \\
\mathbf{v^B} &= \mathbf{U^{BA}i} + \mathbf{y^{BA}} + \mathbf{U^{BB}i} + \mathbf{y^{BB}} + \mathbf{U^{BC}i} + \mathbf{y^{BC}} + \mathbf{y}^{B\,\mathbf{ROW}} \\
\mathbf{v^C} &= \mathbf{U^{CA}i} + \mathbf{y^{BA}} + \mathbf{U^{CB}i} + \mathbf{y^{CB}} + \mathbf{U^{CC}i} + \mathbf{y^{CC}} + \mathbf{y}^{C\,\mathbf{ROW}}
\end{aligned} \tag{2. 59}$$

Additionally, making use of the intra-regional input coefficients and of the interregional trade coefficients[102], defined as:

[102] The definitions of intra-regional input coefficients and interregional trade coefficients have been presented before, in Chapter 1, when presenting the Isard's interregional model.

$$q_{ji}^{AA} = \frac{u_{ji}^{AA}}{g_i^A} \Rightarrow u_{ji}^{AA} = q_{ji}^{AA} g_i^A$$
$$q_{ji}^{AB} = \frac{u_{ji}^{AB}}{g_i^B} \Rightarrow u_{ji}^{AB} = q_{ji}^{AB} g_i^B$$

(2. 60)

The system becomes:

$$\begin{aligned}
\mathbf{v}^{\mathbf{A}} &= \mathbf{Q}^{\mathbf{AA}}\mathbf{g}^{\mathbf{A}} + \mathbf{y}^{\mathbf{AA}} + \mathbf{Q}^{\mathbf{AB}}\mathbf{g}^{\mathbf{B}} + \mathbf{y}^{\mathbf{AB}} + \mathbf{Q}^{\mathbf{AC}}\mathbf{g}^{\mathbf{C}} + \mathbf{y}^{\mathbf{AC}} + \mathbf{y}^{\mathbf{A\,ROW}} \\
\mathbf{v}^{\mathbf{B}} &= \mathbf{Q}^{\mathbf{BA}}\mathbf{g}^{\mathbf{A}} + \mathbf{y}^{\mathbf{BA}} + \mathbf{Q}^{\mathbf{BB}}\mathbf{g}^{\mathbf{B}} + \mathbf{y}^{\mathbf{BB}} + \mathbf{Q}^{\mathbf{BC}}\mathbf{g}^{\mathbf{C}} + \mathbf{y}^{\mathbf{BC}} + \mathbf{y}^{B\,\mathbf{ROW}} \\
\mathbf{v}^{\mathbf{C}} &= \mathbf{Q}^{\mathbf{CA}}\mathbf{g}^{\mathbf{A}} + \mathbf{y}^{\mathbf{CA}} + \mathbf{Q}^{\mathbf{CB}}\mathbf{g}^{\mathbf{B}} + \mathbf{y}^{\mathbf{CB}} + \mathbf{Q}^{\mathbf{CC}}\mathbf{g}^{\mathbf{C}} + \mathbf{y}^{\mathbf{CC}} + \mathbf{y}^{C\,\mathbf{ROW}}
\end{aligned}$$

(2. 61)

Besides the fixed input coefficients hypothesis, another assumption must be taken – a proposition that relates industry's output with commodity's output. To do so, we assume that each product in each region is produced in fixed proportions by the several industries, implying that the structure implicit in each column of the Make matrix is assumed invariant[103]:

$$s_{ij}^A = \frac{v_{ij}^A}{v_j^A} \Rightarrow v_{ij}^A = s_{ij}^A v_j^A$$

(2. 62)

In matrix terms, this corresponds to: $\mathbf{V}^{\mathbf{A}} = \mathbf{S}^{\mathbf{A}}\hat{\mathbf{v}}^{\mathbf{A}}$. Multiplying both sides of this equation by *i* (a column vector appropriately dimensioned, composed by 1's), we obtain:

$$\mathbf{V}^{\mathbf{A}}\mathbf{i} = \mathbf{S}^{\mathbf{A}}\hat{\mathbf{v}}^{\mathbf{A}}\mathbf{i} \Leftrightarrow \mathbf{g}^{\mathbf{A}} = \mathbf{S}^{\mathbf{A}}\mathbf{v}^{\mathbf{A}}$$

(2. 63)

[103] As we have mentioned in Chapter 1, this corresponds to the hypothesis commonly known as the Industry Technology-based Assumption (ITA), and it implies that all products produced by an industry are produced with the same input structure, meaning that there is one technology assigned to each industry. The discussion of the reasonability of this hypothesis, as well as the analysis of an alternative assumption, is left for Chapter 3.

Obviously, the same applies to regions B and C. Introducing (2. 63) into (2. 61), we get:

$$\begin{aligned}
\mathbf{v^A} &= \mathbf{Q^{AA}S^A v^A} + \mathbf{Q^{AB}S^B v^B} + \mathbf{Q^{AC}S^C v^C} + \mathbf{y^{AA}} + \mathbf{y^{AB}} + \mathbf{y^{AC}} \\
\mathbf{v^B} &= \mathbf{Q^{BA}S^A v^A} + \mathbf{Q^{BB}S^B v^B} + \mathbf{Q^{BC}S^C v^C} + \mathbf{y^{BA}} + \mathbf{y^{BB}} + \mathbf{y^{BC}} \\
\mathbf{v^C} &= \mathbf{Q^{CA}S^A v^A} + \mathbf{Q^{CB}S^B v^B} + \mathbf{Q^{CC}S^C v^C} + \mathbf{y^{CA}} + \mathbf{y^{CB}} + \mathbf{y^{CC}}
\end{aligned}$$

(2. 64)

Which can still be represented by:

$$\begin{aligned}
\mathbf{v^A} &= \mathbf{Q^{AA}S^A v^A} + \mathbf{Q^{AB}S^B v^B} + \mathbf{Q^{AC}S^C v^C} + \mathbf{y^{A\bullet}} \\
\mathbf{v^B} &= \mathbf{Q^{BA}S^A v^A} + \mathbf{Q^{BB}S^B v^B} + \mathbf{Q^{BC}S^C v^C} + \mathbf{y^{B\bullet}} \ , \\
\mathbf{v^C} &= \mathbf{Q^{CA}S^A v^A} + \mathbf{Q^{CB}S^B v^B} + \mathbf{Q^{CC}S^C v^C} + \mathbf{y^{C\bullet}}
\end{aligned}$$

(2. 65)

in which $\mathbf{y}^{A\bullet}$ represents final demand for region A's production.

If we take the following block matrices and vectors:

$$\mathbf{v} = \begin{bmatrix} \mathbf{v^A} \\ \mathbf{v^B} \\ \mathbf{v^C} \end{bmatrix};\ \mathbf{y} = \begin{bmatrix} \mathbf{y^{A\bullet}} \\ \mathbf{y^{B\bullet}} \\ \mathbf{y^{C\bullet}} \end{bmatrix};\ \mathbf{g} = \begin{bmatrix} \mathbf{g^A} \\ \mathbf{g^B} \\ \mathbf{g^C} \end{bmatrix};\ \mathbf{Q} = \begin{bmatrix} \mathbf{Q^{AA}} & \mathbf{Q^{AB}} & \mathbf{Q^{AC}} \\ \mathbf{Q^{BA}} & \mathbf{Q^{BB}} & \mathbf{Q^{BC}} \\ \mathbf{Q^{CA}} & \mathbf{Q^{CB}} & \mathbf{Q^{CC}} \end{bmatrix} \text{ and } \mathbf{S} = \begin{bmatrix} \mathbf{S^A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S^B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{S^C} \end{bmatrix},$$

we may write:

$$\begin{aligned}
\mathbf{v} &= \mathbf{QSv} + \mathbf{y} \\
(\mathbf{I} - \mathbf{QS})\mathbf{v} &= \mathbf{y} \\
\mathbf{v} &= (\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}
\end{aligned}$$

$$\mathbf{g} = \mathbf{S}(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}$$

(2. 66)

These equations allow the assessment of the impacts on the production of the several regions (concerning the effect on regional product supply as well as on regional industry production), caused by changes in the vector of final demand for regional production. Such impact analysis implies that the elements of the inverse matrix $(\mathbf{I}-\mathbf{QS})^{-1}$ remain unaltered in face of exogenous shocks. Thus, this involves not only the assumption of constant input coefficients q_{ji}, but also the assumption of constant market shares s_{ij}, as it has been previously referred.

It must be reminded that the model is developed on the basis of partial Use flows, valuated at basic prices. This implies that, in practice, the impact analysis represented in equations (2. 66), involves two stages: 1) convert the final demand data into partial flows, valuated at *bp* and 2) apply equations (2. 66) in order to obtain the impact on regional production. In fact, usually, the data on final demand is available to the researcher on a total flow basis (meaning that one knows the total value of final demand without distinguishing the origin of the products – regional or imported) and it is valuated at *pp*; thus, the same assumptions used to assemble the multi-regional table (exposed on Section 2.5.1) must be used to perform the conversion of the final demand data from *pp* to *bp* and from total flows to partial flows.

2.5.3 Alternative methodologies to estimate interregional trade.

The purpose of this section is to describe the alternative methodologies applied to estimate "interregional" trade established between the 14 European countries belonging to our database. All the six methodologies applied in this study share a common point: they all depart from the same information on the row and column totals for the O/D matrices[104]. However, as it is referred in Hulu and Hewings (1993), the fact that all the

[104] These values are extracted from the EUROSTAT tables, more precisely, the intra-EU exports and imports (after having eliminated the discrepancies, as explained in Section 5.1). Actually, we know in advance the true value of the whole trade matrix and not only the column and row totals, since we are using international trade as a benchmark for testing the non-survey estimating methods. Yet, we assume that the inner part of the matrix is not known, which corresponds to the usual information context when we are dealing with interregional and not with international trade.

estimates observe the same additivity conditions, doesn't guarantee that the final estimates are the same; in the authors' words "the bi-proportional adjustment process only guarantees accuracy at the margins (…)" (Hulu and Hewings, 1993, p. 142). Thus, it is expected that different initial matrices originate also different final matrices (after the adjustment procedure). The analysis of the results provided by the several methodologies described below will allow inferring the sensitivity of the final estimates to the different initial matrices. The different methodologies will be named as Experiences, ranging from Experience 1 to Experience 6. These Experiences can be grouped in two classes. Experiences 1 to 3 use RAS as the adjusting method to compel the row and column totals to equal the previously given values. Experiences 4 to 6 consist in repeating Experiences 1 to 3, yet making use of another adjusting method, relying on a linear programming model (which will be presented further on). As we have referred at the end of section 2.3, in practical applications, the gravity model continues to be most frequently used among the spatial interaction models. The reasons behind this choice are associated mainly to the simplicity of this model, and to the obstacles usually found to the application of the alternative spatial interaction models. This empirical exercise is not an exception. In the absence of data on transportation costs, the entropy formulation was excluded as a possibility to generate the initial matrices; similarly, the minimization of the information bias was also not considered, since the objective of this empirical exercise was to act as if there was no previous information on the inner part of the O/D matrix. Hence, in two of the three methodologies suggested to generate the initial values for the O/D matrix, the formulation is based on the gravitational formula. The remaining methodology consists of a very straightforward way of generating the initial matrix, employed with the aim of understanding the sensibility of the final matrix to completely different initial estimates.

Experience 1

Experience 1 corresponds exactly to what had been previously done in section 2.4.2, now using a different product classification. For each of the six categories of products, the initial O/D matrix was estimated through the application of a simplified gravity-based model, expressed as in equation (2. 57). Afterwards, the initial values were adjusted to the additivity restrictions, using the RAS method. Among all the six products, the maximum number of iterations needed for convergence was 13.

Experience 2

Similarly to what was done in Hulu and Hewings (1993), in their endeavour of assembling an interregional input-output table for Indonesia, we have begun by assuming that, in each country, the total amount of imports coming from the remaining 13 countries was equally divided by each of those 13 supplying countries:

$$\tilde{x}^{rs} = \frac{x^s}{13}\text{, in which } \sum_r x^{rs} = x^s \tag{2. 67}$$

Being so, the column sums of this initial matrix were necessarily equal to the reference values. However, given that the row sums did not verify the correspondent additivity constraints, the RAS method was adopted, as in Experience 1. Among all the six products, the maximum number of iterations needed for convergence was 10.

Experience 3

In Experience 3, the initial values of trade flows are determined according to the following equation:

$$\tilde{x}^{rs} = G^r \frac{P^r P^s DS^s}{\left(d^{rs}\right)^{\beta}};$$
$$G^r = x^r \left(\sum_s \frac{P^r P^s DS^r}{\left(d^{rs}\right)^{\beta}} \right)^{-1} \tag{2. 68}$$

in which β is determined in order to minimize the following indicator of error[105]:

$$I = \frac{\sum_s \left| \sum_r x^{rs} - \sum_r \tilde{x}^{rs} \right|}{\sum_r \sum_s x^{rs}}.$$

As stated in Schwarm, Jackson, and Okuyama (2006), the ideal procedure to estimate the values of the gravity parameters should be to "minimize the absolute differences between estimated and observed flows" (p. 87). Given that observed flows are supposedly unknown in our case, we have opted for constructing an indicator of error using solely the available information on row totals (of course, an equally valid alternative would be to consider the information on column totals).

With this Experience, we are trying to assess the effect on the results of estimating one of the most relevant parameters in the gravitational formula (the distance-decay parameter) through an alternative procedure, instead of considering it equal to one, *a priori*, as it is assumed in the remaining parameters. This methodology has been already applied in previous empirical exercises, such as in Ramos and Sargento (2003) and in Sargento (2007). In the first case, the performance of the methodology remained unknown, since it was applied to interregional trade flows between 7 Portuguese regions, to which there was no benchmark. In the second, it was applied to the same database that is being used here, though with a more disaggregated product classification. The results of this application in Sargento (2007) have demonstrated that, in average, the distance between the final matrix and the real one was a bit smaller than when all the parameters were being set equal to one.

[105] This particular formula for indicator *I* is the one that resembles more closely the STPE, the measure that has already been applied in section 4.2 and which will be used in the next section to compute the distance between the estimated and the real matrix. However, it is not exactly equal to SPTE. Thus, diminishing *I*, which is calculated using only the column sums, does not necessarily imply a decrease in the STPE, which involves a cell-by-cell computation of absolute difference.

Experience 4

Two major differences exist between this Experience and Experience 1. The first concerns the adjustment procedure used to make the O/D row and column totals to match with the previously known ones. Instead of using the RAS procedure, a linear programming model was applied. The reason that motivated this variation relies on the fact that, as it has been referred in Section 2.3.3, linear programming models allow for the introduction of additional constraints, beyond the standard additivity ones; moreover, the constraints can be inserted in the inequality form, which is convenient, given the objectives behind the second group of Experiences (4 to 6).

The second difference is the fact that we are now assuming some additional previous information concerning the real trade matrix. Let us suppose that, besides the information on the row and column totals, the researcher also has access to the level of Entropy embodied in the real O/D matrix, which indicates the degree of dispersion or interactivity of that matrix (Erlander, 1980). In such case, it may be desirable, in the adjustment of the initial estimates, to consider only those solutions that preserve at least the same degree of interactivity as the one implicit in the real matrix. Let S_0 be the level of Entropy embodied in the real O/D matrix; also, let $\tilde{x}_1^{rs}$ be the initial estimates of the origin-destination trade flows derived from Experience 1 and $\tilde{x}_4^{rs}$ be the final estimates obtained through the present Experience: Experience 4. Thus, Experience 4 may be expressed as follows:

$$Min \quad I\left(\tilde{x}_4^{rs}, \tilde{x}_1^{rs}\right) = \sum_r \sum_s x_4^{rs} \ln\left(\frac{x_4^{rs}}{\tilde{x}_1^{rs}}\right) \quad s.t.$$

$$\sum_s \tilde{x}_4^{rs} = x^r$$

$$\sum_r \tilde{x}_4^{rs} = x^s$$

$$-\sum_r \sum_s \tilde{x}_4^{rs} \ln \tilde{x}_4^{rs} \geq S_0$$

(2. 69)

This means that we are trying to find a matrix of flows which is as close as possible to the initial matrix, simultaneously complying with the restrictions on the row and column totals, as well as with the entropy constraint. The objective function chosen to minimize the difference between the initial and the final matrix follows the principle of minimizing the information bias. Thus, this principle was not directly applied to generate initial values, but rather in the posterior adjustment of those values.

The computation of this model (as well as the ones in Experiences 5 and 6) was made using GAMS.

Experience 5

This model replicates the previous one, with the exception of considering the initial values derived from Experience 2, instead of considering the values from Experience 1. In analytical terms, we have:

$$Min \quad I\left(\tilde{x}_5^{rs}, \tilde{x}_2^{rs}\right) = \sum_r \sum_s x_5^{rs} \ln\left(\frac{x_5^{rs}}{\tilde{x}_2^{rs}}\right) \quad s.t.$$

$$\sum_s \tilde{x}_5^{rs} = x^r$$

$$\sum_r \tilde{x}_5^{rs} = x^s$$

$$-\sum_r \sum_s \tilde{x}_5^{rs} \ln \tilde{x}_5^{rs} \geq S_0$$

(2. 70)

Experience 6

Finally, in Experience 6, the values from Experience 3 (before the RAS adjustment) are considered as the initial values to be adjusted by the linear programming:

$$
\begin{aligned}
&Min \quad I\left(\tilde{x}_6^{rs}, \tilde{x}_3^{rs}\right) = \sum_r \sum_s x_6^{rs} \ln\left(\frac{x_6^{rs}}{\tilde{x}_3^{rs}}\right) \quad s.t. \\
&\sum_s \tilde{x}_6^{rs} = x^r \\
&\sum_r \tilde{x}_6^{rs} = x^s \\
&-\sum_r \sum_s \tilde{x}_6^{rs} \ln \tilde{x}_6^{rs} \geq S_0
\end{aligned}
\tag{2. 71}
$$

The results provided by these Experiences, as well as their analysis, will be presented in the following Section.

2.5.4 Comparison between the results provided by the different estimation methods.

Before proceeding for the presentation of results, a previous note must be made. The estimation of origin-destination matrices through the Experiences described above was made only to 4 of the 6 product categories used in the multi-regional input-output system: "**A+B** – Products of agriculture, hunting, forestry and fishing", "**C** – Mining and quarrying", "**D** – Industry" and "**E** – Electricity, gas and water supply". This option was justified by the fact that, for the remaining categories, the trade data taken as "real" were already affected by some simplifying hypotheses. In fact, interregional trade for "**F** – "Construction" and "**G to P** – Services", was estimated assuming the distributions embodied in the data for aggregate trade (as explained in step (5) of multi-regional table assemblage, Section 2.5.1)

The several O/D matrices obtained through the different methodologies are compared among each other and also to the correspondent real matrices, using the already presented measure of matrix comparison: the Standard Total Percentage Error (equation (2. 58)). The results are presented in Table 2. 11. In the first column of the Table, the performance of each of the different Experiences can be evaluated: each estimated matrix is compared to the correspondent real matrix. In the remaining columns, a comparison is made between the results provided by the different estimates, meaning that an evaluation is made to the sensitivity of the final results to each methodology. A weighted average was computed for each Experience, which expresses the error associated to the corresponding methodology considering all product categories, weighting by their relative importance in aggregate trade (obviously, the errors concerning category "**D** – Industry" and "**A+B** – Products of agriculture, hunting, forestry and fishing" are the most relevant, given the higher relative weight these products have in international trade).

Table 2. 11 – Summary results from the comparison between the 6 different interregional trade estimation methods.

STPE versus…		REAL	EXP 2	EXP 3	EXP 4	EXP 5	EXP 6
EXP 1	A+B	31,9%	44,6%	19,9%	44,6%	44,6%	44,6%
	C	43,0%	34,6%	15,3%	34,6%	34,6%	34,6%
	D	28,9%	43,8%	2,0%	43,8%	43,8%	43,8%
	E	72,7%	37,4%	45,7%	37,4%	37,4%	37,4%
	aggregate	**28,4%**	**43,5%**	**2,1%**	**43,5%**	**43,5%**	**43,5%**
EXP 2	A+B	44,9%		24,9%	0,0%	0,0%	0,0%
	C	35,7%		48,9%	0,0%	0,0%	0,0%
	D	28,1%		41,9%	0,0%	0,0%	0,0%
	E	67,4%		55,8%	0,0%	0,0%	0,0%
	aggregate	**28,2%**		**41,5%**	**0,0%**	**0,0%**	**0,0%**
EXP 3	A+B	31,4%			24,9%	24,9%	24,9%
	C	50,9%			48,9%	48,9%	48,9%
	D	27,6%			42,0%	42,0%	42,0%
	E	78,0%			55,8%	55,8%	55,8%
	aggregate	**27,2%**			**41,5%**	**41,5%**	**41,5%**
EXP 4	A+B	44,9%				0,0%	0,0%
	C	35,7%				0,0%	0,0%
	D	28,1%				0,0%	0,0%
	E	67,4%				0,0%	0,0%
	aggregate	**28,2%**				**0,0%**	**0,0%**
EXP 5	A+B	44,9%					0,0%
	C	35,7%					0,0%
	D	28,1%					0,0%
	E	67,4%					0,0%
	aggregate	**28,2%**					**0,0%**
EXP 6	A+B	44,9%					
	C	35,7%					
	D	28,1%					
	E	67,4%					
	aggregate	**28,2%**					

These results suggest four main remarks:

1) The most immediate observation to be made on the results is the fact that Experiences 2, 4, 5 and 6 generate exactly the same results. As we have explained before, Experiences 4, 5 and 6 have in common the fact that they share the same Entropy constraint (for each product). The observation of these results leads us to conclude that the imposed Entropy constraint is strong enough to compel the values of the initial matrix to converge to a same final matrix, regardless of the method used to generate the initial values. The identity found between Experience 2 results and the ones resulting from Experiences 4, 5 and 6 may be explained by the fact that Experience 2 uses an even dispersion of imports for each destination country as a starting point, which makes the starting matrix to have a higher Entropy level than the one implicit in the real matrix (which corresponds to the minimum level considered in the Entropy constraint in Experiences 4, 5 and 6). This means that the introduction of an Entropy constraint through a linear programming model can be equivalent to a simpler procedure, consisting in using an exceptionally disperse matrix as a starting point and then make the adjustment through RAS. Thus, in this case, the use of such superior information about the real matrix of flows (the entropy level), doesn't seem critical to enhance the results. Given the coincidence of results provided by Experiences 2, 4, 5 and 6, our further analysis is restricted to the comparison between Experiences 1, 2 and 3.
2) Taking the first column and the aggregate error for each Experience as a reference, we conclude that Experience 3 (gravitational model in which the distance parameter is computed through the minimization of indicator I) is the one which originates the smaller aggregate error against the real values. The simple gravitational model (Experience 1) seems to generate the higher aggregate error. This result confirms what had been verified in the 10 product application used in Sargento (2007), implying that the introduction of an independent estimate for the distance-decay parameter, instead considering it to be equal to one, may represent an improvement in the results. However, the differences among aggregate errors are small, when the comparison is made against the real values.

3) The errors generated by the three Experiences are quite high for some products (achieving 78% in product “E”, Experience 3), but they are lower in the most representative products. If we limit the analysis to the “Industry” case, the most relevant in international trade, we may state that the non-survey methods proposed here produce quite reasonable results.

4) Observing the mutual differences between the several Experiences, we conclude that these are larger between Experiences 1 and 2 and between Experiences 2 and 3, than between Experiences 1 and 3. In other words, the only case in which we do not use gravitational formula as a starting point – Experience 2 – generates more outlying results, demonstrating that the way by which initial estimates are obtained is not innocuous.

In the particular case of these data, the results allow us to conclude that a gravity-based model to generate the initial values jointly with the simpler adjusting procedure originate the best results (Experience 3 provides the closer matrices). Still, some large errors observed in the first column, as well as the considerable differences existing among the first three experiences, constitute an impetus to perform an analytical comparison between the different methodologies. As explained before, the ultimate aim is to assess the extent to which these different estimates reflect themselves in different results in practical applications of the input-output model. In other words, how important is the accuracy of the interregional trade estimates to the model accuracy? Following the notation introduced by Jensen (1980) and already referred in Chapter 1 (Section 2.4.4), this would mean: how relevant is partitive accuracy implicit in the interregional trade component of the table to the holistic accuracy?

2.5.5 Input-output model sensitivity to the alternative methodologies of interregional trade estimation.

The analytical comparison between alternative interregional trade estimation methods consists of a sort of exercise that is not usually done. To the authors’ knowledge, there are two recent works which constitute the exceptions to this rule. One consists of the exercise presented in Oosterhaven and Stelder (2007), applied to the constructing the Asian-Pacific Input-output Table, given that they do compute the impact on the model multipliers of using 4 different non-survey methods of assembling the multi-regional table. Yet, the objective in Oosterhaven and Stelder (2007) is somewhat different. In their paper, what is being modified from one method to another is not the way by which the origin-destination matrices are computed (given that, in all methods, the distribution of intra-regional imports over the countries of origin is made using available trade statistics), but rather the way by which they deal with the discrepancies between intra-regional export and import data. Another relevant study to consider in this kind of sensitivity analyses is the one reported by Robinson and Liu (2006), in which an evaluation is made on the sensitivity of the multipliers obtained from a multiregional Social Accounting Matrix to different methods of estimating domestic imports and domestic exports. In their study, two distinct methods are applied to estimate the total amount of imports and exports that each individual region establishes with the remaining regions of the system – which is different to what is being estimated in our study: the distribution of the total amount of intra-EU imports by the remaining supplying countries of the system.

In our case, we intend to assess the sensitivity of the input-output model solution to the insertion of the O/D matrices derived from three of the six Experiences previously described (Experiences 1, 2 and 3, given that the remaining originated the same O/D matrices as Experience 2). Referring back to the several steps taken for the multi-regional table assemblage, this means that, in Step 6, the split up of the intra-EU imports between the 13 remaining supplying countries is made using the percentages given by the O/D matrices derived from each of Experiences, instead of the percentages derived from the OECD-based matrices. Hence, we obtain a set of three different multi-regional systems, (besides the reference one, computed on the basis of the OECD trade data) each one corresponding to a different interregional trade estimation methodology. The only difference among the obtained multi-regional tables consists precisely in the interregional Use tables, caused by the different interregional trade data inserted. In terms of Figure 2.

8, this means that the Use matrices (and also the final demand vector) located in the main diagonal are the same in all the three estimated multi-regional tables and also in the reference table. The difference relies on the off-diagonal components of the multi-regional table, given that the intra-EU imports distribution is varying from one table to another. This implies that the interregional trade coefficients embodied in the off-diagonal components of the block matrix $\mathbf{Q}$ are being substituted by different values in each different Experience. Observing equation (2. 66), it becomes clear that this has an impact on the model solution. The empirical exercise reported in this Section aims to investigate the extent to which that solution is affected.

Let us take the following simulation exercise: considering the change in the real final demand vector actually verified from year 2000 to 2001, what is the model estimate for growth in aggregate Gross Value Added (GVA) resulting from the use of the different multi-regional tables (the reference one and the other three, generated from the different trade estimation methods)? In order to answer such question, the first step consisted in collecting and aggregating the final demand vectors for each country, recorded in the Use tables for year 2001, available from the EUROSTAT database. Given that the tables are provided in current prices for the generality of the countries, we opted for using the GDP deflator (for year 2001, base 2000=100[106]) to convert the current prices vectors into vectors of final demand at 2000 prices. The objective was to obtain real growth rates. Then, these vectors were converted from *pp* total use flows into domestic flow vectors, valuated at *bp*. This was done applying the same proportionality assumptions previously presented. Afterwards, equation (2. 66) was applied in order to compute the impact of final demand change on product and industry output vectors:

$$\Delta\mathbf{v} = (\mathbf{I} - \mathbf{QS})^{-1}\,\Delta\mathbf{y}$$

$$\Delta\mathbf{g} = \mathbf{S}(\mathbf{I} - \mathbf{QS})^{-1}\,\Delta\mathbf{y}$$

(2. 72)

[106] Source: OECD, National Accounts Statistics.

Finally, a Value Added coefficient was considered to compute the vector of changes in Value Added. Taking $va_i^A = \dfrac{VA_i^A}{g_i^A}$ as the proportion of Value Added included in the output of industry i in region A, we may write: $VA_i^A = va_i^A g_i^A$. If we assume constant value added coefficients, we have, in matrix terms:

$$\begin{aligned} \mathbf{\Delta VA} &= \mathbf{v\hat{a}} \cdot \mathbf{\Delta g} \Leftrightarrow \\ \mathbf{\Delta VA} &= \mathbf{v\hat{a}} \cdot \mathbf{S}(\mathbf{I} - \mathbf{QS})^{-1} \mathbf{\Delta y} \end{aligned} \tag{2. 73}$$

in which $\mathbf{v\hat{a}}$ represents a diagonal block matrix with the country-specific diagonal matrices of value added coefficients in the main diagonal. This input-output model estimate for real GVA growth rate was first computed using the real data on interregional trade and it was subsequently calculated using the three Experiences' estimate for trade flows. Each of the three estimated GVA growth rates derived from the inclusion of the trade flow estimates is compared to the input-output model estimate for GVA growth rate, when real trade data is considered – which we designate by reference GVA growth rate. It must be noted, however, that this reference GVA growth rate is already embodied of some error, given the hypotheses implicit in the impact analysis represented by equations (2. 73), namely, the fact that input coefficients, industry market shares and value added proportions are all assumed invariant. Thus, the reference growth rate is different from the actual GVA growth rate, as it can be observed in Table 2. 12. Yet, the objective of this application is not to accurately calculate growth rates through the model, but rather to provide a reference to evaluate the deviations generated by the three interregional trade estimation methods.

Table 2. 12 – Reference growth rate, actual growth rate and model forecast error.

	Reference GVA percentual growth (with real data on IR trade)	Actual GVA percentual growth	Model forecast error
FRA	1,52%	2,23%	0,72%
GER	1,30%	1,38%	0,09%
ITA	2,29%	2,48%	0,19%
BELUX	3,80%	3,75%	-0,05%
NLD	2,17%	0,86%	-1,31%
DNK	-1,13%	-1,72%	-0,58%
IRE	7,45%	7,39%	-0,06%
GBR	3,49%	2,73%	-0,75%
GRC	5,51%	3,03%	-2,48%
ESP	3,39%	3,99%	0,60%
PRT	1,88%	2,14%	0,26%
AUT	3,18%	3,68%	0,50%
FIN	2,35%	3,23%	0,89%
SWE	3,06%	0,94%	-2,12%
WHOLE ECONOMY	**2,34%**	**2,27%**	**-0,06%**

The results of this computation are presented in Table 2.13, A and B. In part A, we may observe GVA growth rate estimated by the model, for each industry in each of the 14 countries (as well as for the aggregate economy) and for the entire system taken as a whole, using the four versions of the multi-regional table (the reference one and the other three). For each Experience, there is also a column reporting the difference (in percentage points) between the GVA growth rate obtained in that Experience and the one obtained using real interregional trade data. All the differences are computed in absolute percent points. The "average" column is calculated taking, for each country, the mean of all the differences (labelled as "diff." in the table) obtained from the three Experiences. It reflects the mean error resulting from using non-survey trade values instead of using the real values. In part B of the table the same indicators are used, but now the comparison is made among the three Experiences and not against the reference table. In both tables, differences equal to or above 0,005% are highlighted in blue.

Table 2. 13 – Differences in growth forecast as a consequence of final demand increase.

A - Comparison between the GVA growth rate using trade estimates from each individual experience and the reference GVA growth rate.

			Exp 1		Exp 2		Exp 3		
		Reference GVA growth %	growth in GVA %	diff.	growth in GVA %	diff.	growth in GVA %	diff.	average diff.
FRA	A+B	0,67%	0,67%	0,002%	0,65%	0,024%	0,67%	0,005%	0,010%
	C	1,15%	1,14%	0,004%	1,12%	0,024%	1,14%	0,006%	0,011%
	D	0,68%	0,69%	0,007%	0,66%	0,025%	0,69%	0,006%	0,013%
	E	3,96%	3,96%	0,001%	3,95%	0,008%	3,96%	0,000%	0,003%
	F	2,20%	2,20%	0,000%	2,20%	0,001%	2,20%	0,000%	0,000%
	G a P	1,66%	1,66%	0,001%	1,65%	0,003%	1,66%	0,001%	0,002%
	AGGR.	1,52%	1,52%	0,002%	1,51%	0,008%	1,52%	0,001%	0,004%
GER	A+B	0,89%	0,88%	0,005%	0,90%	0,016%	0,89%	0,001%	0,007%
	C	4,38%	4,37%	0,007%	4,39%	0,012%	4,37%	0,007%	0,009%
	D	1,18%	1,17%	0,016%	1,19%	0,011%	1,17%	0,014%	0,014%
	E	5,33%	5,32%	0,008%	5,33%	0,001%	5,33%	0,006%	0,005%
	F	-4,31%	-4,31%	0,001%	-4,31%	0,001%	-4,31%	0,001%	0,001%
	G a P	1,65%	1,64%	0,003%	1,65%	0,002%	1,64%	0,002%	0,002%
	AGGR.	1,30%	1,29%	0,006%	1,30%	0,004%	1,29%	0,005%	0,005%
ITA	A+B	0,34%	0,34%	0,000%	0,33%	0,004%	0,33%	0,004%	0,003%
	C	-0,91%	-0,92%	0,011%	-0,92%	0,005%	-0,92%	0,009%	0,008%
	D	0,92%	0,91%	0,009%	0,91%	0,001%	0,91%	0,008%	0,006%
	E	1,54%	1,53%	0,004%	1,54%	0,000%	1,53%	0,003%	0,002%
	F	3,27%	3,27%	0,001%	3,27%	0,000%	3,27%	0,001%	0,000%
	G a P	2,77%	2,77%	0,002%	2,77%	0,000%	2,77%	0,002%	0,001%
	AGGR.	2,29%	2,29%	0,003%	2,29%	0,001%	2,29%	0,003%	0,002%
BELUX	A+B	1,45%	1,35%	0,101%	1,53%	0,082%	1,42%	0,034%	0,072%
	C	-13,70%	-13,65%	0,050%	-13,89%	0,197%	-13,74%	0,047%	0,098%
	D	1,95%	1,89%	0,059%	2,01%	0,060%	1,90%	0,053%	0,057%
	E	7,34%	7,34%	0,008%	7,37%	0,030%	7,34%	0,008%	0,015%
	F	0,11%	0,10%	0,005%	0,11%	0,006%	0,10%	0,005%	0,005%
	G a P	4,53%	4,52%	0,007%	4,54%	0,007%	4,52%	0,007%	0,007%
	AGGR.	3,80%	3,78%	0,018%	3,82%	0,019%	3,79%	0,016%	0,018%
NLD	A+B	0,13%	0,14%	0,016%	0,12%	0,010%	0,14%	0,008%	0,012%
	C	7,73%	7,79%	0,068%	7,72%	0,006%	7,83%	0,100%	0,058%
	D	-1,18%	-1,16%	0,023%	-1,18%	0,005%	-1,16%	0,021%	0,017%
	E	5,38%	5,39%	0,006%	5,38%	0,002%	5,39%	0,005%	0,004%
	F	1,81%	1,81%	0,001%	1,81%	0,000%	1,81%	0,001%	0,001%
	G a P	2,79%	2,79%	0,003%	2,79%	0,001%	2,79%	0,003%	0,002%
	AGGR.	2,17%	2,18%	0,009%	2,17%	0,002%	2,18%	0,009%	0,006%
DNK	A+B	-2,16%	-2,19%	0,033%	-2,15%	0,004%	-2,19%	0,030%	0,022%
	C	-4,91%	-4,84%	0,070%	-4,78%	0,129%	-4,83%	0,083%	0,094%
	D	-1,30%	-1,36%	0,062%	-1,30%	0,004%	-1,36%	0,059%	0,042%
	E	-0,76%	-0,77%	0,004%	-0,74%	0,017%	-0,77%	0,007%	0,009%
	F	-3,55%	-3,55%	0,002%	-3,55%	0,000%	-3,55%	0,002%	0,001%
	G a P	-0,72%	-0,72%	0,004%	-0,72%	0,000%	-0,72%	0,003%	0,002%
	AGGR.	-1,13%	-1,14%	0,011%	-1,13%	0,004%	-1,14%	0,010%	0,009%
IRE	A+B	0,87%	0,93%	0,060%	0,82%	0,053%	0,91%	0,045%	0,053%
	C	10,94%	10,99%	0,051%	10,94%	0,000%	10,98%	0,043%	0,031%
	D	1,65%	1,74%	0,093%	1,59%	0,058%	1,74%	0,086%	0,079%
	E	2,28%	2,31%	0,030%	2,27%	0,019%	2,31%	0,028%	0,026%
	F	9,96%	9,96%	0,004%	9,95%	0,002%	9,96%	0,004%	0,003%
	G a P	11,15%	11,18%	0,026%	11,13%	0,016%	11,17%	0,024%	0,022%
	AGGR.	7,45%	7,50%	0,048%	7,42%	0,030%	7,49%	0,044%	0,041%

		Reference GVA growth %	Exp 1 growth in GVA %	Exp 1 diff.	Exp 2 growth in GVA %	Exp 2 diff.	Exp 3 growth in GVA %	Exp 3 diff.	average diff.
GBR	A+B	1,02%	1,02%	0,003%	1,01%	0,010%	1,02%	0,002%	0,005%
	C	-0,11%	-0,13%	0,020%	-0,13%	0,021%	-0,13%	0,023%	0,021%
	D	1,47%	1,48%	0,004%	1,44%	0,031%	1,48%	0,002%	0,012%
	E	0,13%	0,13%	0,001%	0,12%	0,008%	0,13%	0,000%	0,003%
	F	5,60%	5,60%	0,000%	5,60%	0,001%	5,60%	0,000%	0,000%
	G a P	4,12%	4,12%	0,001%	4,12%	0,003%	4,12%	0,000%	0,001%
	AGGR.	3,49%	3,49%	0,001%	3,48%	0,008%	3,49%	0,000%	0,003%
GRC	A+B	2,70%	2,71%	0,009%	2,71%	0,004%	2,71%	0,007%	0,007%
	C	0,49%	0,49%	0,003%	0,50%	0,007%	0,49%	0,001%	0,004%
	D	0,52%	0,53%	0,004%	0,53%	0,003%	0,53%	0,004%	0,004%
	E	4,77%	4,78%	0,001%	4,78%	0,001%	4,78%	0,001%	0,001%
	F	7,00%	7,00%	0,000%	7,00%	0,000%	7,00%	0,000%	0,000%
	G a P	6,64%	6,64%	0,000%	6,64%	0,000%	6,64%	0,000%	0,000%
	AGGR.	5,51%	5,51%	0,001%	5,51%	0,001%	5,51%	0,001%	0,001%
ESP	A+B	2,47%	2,48%	0,012%	2,48%	0,005%	2,48%	0,011%	0,009%
	C	1,65%	1,66%	0,019%	1,67%	0,027%	1,65%	0,009%	0,018%
	D	1,55%	1,57%	0,023%	1,56%	0,014%	1,57%	0,023%	0,020%
	E	2,45%	2,46%	0,007%	2,45%	0,005%	2,46%	0,008%	0,007%
	F	7,60%	7,60%	0,001%	7,60%	0,001%	7,60%	0,001%	0,001%
	G a P	3,48%	3,48%	0,003%	3,48%	0,002%	3,48%	0,003%	0,003%
	AGGR.	3,39%	3,40%	0,007%	3,40%	0,004%	3,40%	0,007%	0,006%
PRT	A+B	1,57%	1,64%	0,073%	1,54%	0,023%	1,63%	0,062%	0,053%
	C	-0,60%	-0,51%	0,090%	-0,62%	0,020%	-0,49%	0,107%	0,072%
	D	-0,06%	0,06%	0,119%	-0,09%	0,029%	0,05%	0,112%	0,086%
	E	3,13%	3,16%	0,027%	3,12%	0,009%	3,16%	0,028%	0,021%
	F	2,97%	2,97%	0,005%	2,97%	0,001%	2,97%	0,005%	0,004%
	G a P	2,24%	2,25%	0,012%	2,24%	0,003%	2,25%	0,011%	0,009%
	AGGR.	1,88%	1,92%	0,033%	1,88%	0,008%	1,92%	0,031%	0,024%
AUT	A+B	4,11%	4,13%	0,014%	4,15%	0,038%	4,13%	0,014%	0,022%
	C	4,13%	4,14%	0,011%	4,18%	0,049%	4,14%	0,009%	0,023%
	D	2,34%	2,36%	0,021%	2,41%	0,068%	2,36%	0,023%	0,037%
	E	9,00%	9,00%	0,006%	9,03%	0,034%	9,00%	0,001%	0,013%
	F	-1,10%	-1,10%	0,001%	-1,10%	0,005%	-1,10%	0,001%	0,002%
	G a P	3,74%	3,74%	0,002%	3,75%	0,008%	3,75%	0,003%	0,004%
	AGGR.	3,18%	3,19%	0,007%	3,20%	0,022%	3,19%	0,007%	0,012%
FIN	A+B	1,49%	1,52%	0,023%	1,50%	0,005%	1,51%	0,020%	0,016%
	C	-0,67%	-0,63%	0,038%	-0,66%	0,009%	-0,62%	0,051%	0,033%
	D	-0,94%	-0,92%	0,026%	-0,94%	0,002%	-0,92%	0,024%	0,018%
	E	2,53%	2,54%	0,010%	2,53%	0,001%	2,54%	0,010%	0,007%
	F	1,28%	1,28%	0,001%	1,28%	0,000%	1,28%	0,001%	0,001%
	G a P	3,87%	3,87%	0,004%	3,87%	0,000%	3,87%	0,004%	0,003%
	AGGR.	2,35%	2,36%	0,011%	2,35%	0,001%	2,36%	0,010%	0,007%
SWE	A+B	7,85%	7,89%	0,036%	7,85%	0,002%	7,89%	0,042%	0,027%
	C	4,68%	5,00%	0,318%	4,60%	0,087%	5,40%	0,714%	0,373%
	D	1,03%	1,07%	0,038%	0,99%	0,038%	1,07%	0,035%	0,037%
	E	11,64%	11,65%	0,010%	11,65%	0,005%	11,66%	0,016%	0,010%
	F	7,67%	7,67%	0,003%	7,67%	0,001%	7,68%	0,008%	0,004%
	G a P	3,01%	3,01%	0,002%	3,01%	0,000%	3,01%	0,003%	0,002%
	AGGR.	3,06%	3,07%	0,012%	3,05%	0,009%	3,07%	0,014%	0,011%
	average AGGR.			0,012%		0,009%		0,011%	0,011%
	WHOLE ECONOMY	2,34%	2,34%	0,000%	2,34%	0,000%	2,34%	0,000%	0,000%

B - Comparison among the different experiences.

		Exp 2 against Exp 1	Exp 3 against Exp 1	Exp 3 against Exp 2	average diff.
FRA	A+B	0,022%	0,003%	0,019%	0,015%
	C	0,020%	0,002%	0,018%	0,014%
	D	0,032%	0,001%	0,031%	0,021%
	E	0,009%	0,001%	0,008%	0,006%
	F	0,001%	0,000%	0,001%	0,001%
	G a P	0,004%	0,000%	0,004%	0,003%
	AGGREGATE	0,010%	0,000%	0,009%	0,007%
GER	A+B	0,021%	0,004%	0,017%	0,014%
	C	0,019%	0,000%	0,019%	0,013%
	D	0,027%	0,001%	0,025%	0,018%
	E	0,008%	0,002%	0,007%	0,006%
	F	0,002%	0,000%	0,002%	0,001%
	G a P	0,005%	0,000%	0,004%	0,003%
	AGGREGATE	0,010%	0,001%	0,009%	0,007%
ITA	A+B	0,004%	0,004%	0,000%	0,003%
	C	0,006%	0,002%	0,003%	0,004%
	D	0,008%	0,000%	0,007%	0,005%
	E	0,003%	0,000%	0,003%	0,002%
	F	0,000%	0,000%	0,000%	0,000%
	G a P	0,001%	0,000%	0,001%	0,001%
	AGGREGATE	0,003%	0,000%	0,003%	0,002%
BELUX	A+B	0,182%	0,066%	0,116%	0,121%
	C	0,246%	0,096%	0,150%	0,164%
	D	0,118%	0,005%	0,113%	0,079%
	E	0,038%	0,000%	0,038%	0,025%
	F	0,011%	0,000%	0,011%	0,007%
	G a P	0,015%	0,001%	0,014%	0,010%
	AGGREGATE	0,037%	0,002%	0,035%	0,025%
NLD	A+B	0,026%	0,008%	0,019%	0,018%
	C	0,075%	0,031%	0,106%	0,071%
	D	0,028%	0,002%	0,026%	0,019%
	E	0,008%	0,001%	0,007%	0,005%
	F	0,002%	0,000%	0,002%	0,001%
	G a P	0,004%	0,000%	0,004%	0,003%
	AGGREGATE	0,010%	0,000%	0,011%	0,007%
DNK	A+B	0,037%	0,003%	0,034%	0,025%
	C	0,058%	0,013%	0,045%	0,039%
	D	0,059%	0,003%	0,056%	0,039%
	E	0,021%	0,003%	0,024%	0,016%
	F	0,002%	0,000%	0,002%	0,001%
	G a P	0,004%	0,000%	0,003%	0,002%
	AGGREGATE	0,015%	0,001%	0,014%	0,010%
IRE	A+B	0,113%	0,015%	0,098%	0,075%
	C	0,051%	0,008%	0,043%	0,034%
	D	0,151%	0,007%	0,144%	0,101%
	E	0,049%	0,003%	0,046%	0,033%
	F	0,006%	0,000%	0,006%	0,004%
	G a P	0,042%	0,002%	0,040%	0,028%
	AGGREGATE	0,078%	0,004%	0,074%	0,052%

		Exp 2 against Exp 1	Exp 3 against Exp 1	Exp 3 against Exp 2	average diff.
GBR	A+B	0,013%	0,001%	0,012%	0,008%
	C	0,002%	0,003%	0,001%	0,002%
	D	0,035%	0,002%	0,033%	0,023%
	E	0,008%	0,000%	0,008%	0,005%
	F	0,001%	0,000%	0,001%	0,001%
	G a P	0,003%	0,000%	0,003%	0,002%
	AGGREGATE	0,009%	0,001%	0,008%	0,006%
GRC	A+B	0,005%	0,002%	0,003%	0,003%
	C	0,004%	0,002%	0,006%	0,004%
	D	0,001%	0,000%	0,001%	0,000%
	E	0,000%	0,000%	0,000%	0,000%
	F	0,000%	0,000%	0,000%	0,000%
	G a P	0,000%	0,000%	0,000%	0,000%
	AGGREGATE	0,001%	0,000%	0,000%	0,000%
ESP	A+B	0,007%	0,001%	0,006%	0,005%
	C	0,008%	0,010%	0,018%	0,012%
	D	0,009%	0,000%	0,010%	0,006%
	E	0,003%	0,000%	0,003%	0,002%
	F	0,000%	0,000%	0,000%	0,000%
	G a P	0,001%	0,000%	0,001%	0,001%
	AGGREGATE	0,003%	0,000%	0,003%	0,002%
PRT	A+B	0,096%	0,011%	0,085%	0,064%
	C	0,110%	0,018%	0,128%	0,085%
	D	0,147%	0,007%	0,141%	0,098%
	E	0,036%	0,001%	0,037%	0,024%
	F	0,006%	0,000%	0,006%	0,004%
	G a P	0,015%	0,001%	0,014%	0,010%
	AGGREGATE	0,041%	0,002%	0,039%	0,027%
AUT	A+B	0,023%	0,001%	0,024%	0,016%
	C	0,038%	0,002%	0,040%	0,027%
	D	0,047%	0,001%	0,045%	0,031%
	E	0,028%	0,005%	0,033%	0,022%
	F	0,003%	0,000%	0,003%	0,002%
	G a P	0,006%	0,000%	0,006%	0,004%
	AGGREGATE	0,015%	0,000%	0,015%	0,010%
FIN	A+B	0,018%	0,003%	0,015%	0,012%
	C	0,029%	0,013%	0,042%	0,028%
	D	0,024%	0,002%	0,022%	0,016%
	E	0,009%	0,000%	0,009%	0,006%
	F	0,001%	0,000%	0,001%	0,001%
	G a P	0,004%	0,000%	0,004%	0,003%
	AGGREGATE	0,010%	0,001%	0,009%	0,006%
SWE	A+B	0,038%	0,006%	0,044%	0,029%
	C	0,405%	0,395%	0,801%	0,534%
	D	0,076%	0,003%	0,073%	0,051%
	E	0,005%	0,006%	0,012%	0,008%
	F	0,004%	0,004%	0,008%	0,005%
	G a P	0,002%	0,001%	0,003%	0,002%
	AGGREGATE	0,021%	0,002%	0,022%	0,015%
	average AGGR.	0,019%	0,001%	0,018%	0,013%
	WHOLE ECON.	0,000%	0,000%	0,000%	0,000%

The following observations are suggested by the results contained in these two tables:

1) The GVA growth rate obtained for the whole economy is invariant: 2,34%. This is an expected result given that, as explained before, the only difference between the distinct Experiences relies on the distribution of intra-regional imports and exports among the 14 countries. Thus, despite the differences in the growth rates assigned to each individual country by the different Experiences, this doesn't seem to affect the growth rate for the whole economy.

2) We have considered that absolute differences between 0,005% and 0,05% are moderately relevant differences (these differences imply a deviation, for example, from a growth rate of 3,00% to a growth rate of 3,005%, or in round terms, of 3,01%). Below that reference value, we have considered the differences to be non-significant. Absolute differences above 0,05% are considered to be large. Using such boundaries, we may observe in Table A that almost half of the computed differences are moderately relevant (182 out of 392 differences lie between 0,005% and 0,05%); a significant part of those differences (169 out of 392) is below 0,005%, thus being considered insignificant; finally, a small amount of differences fall into the category of large deviations (41 in 392). It must be noted that none of the observed large differences occurs in the aggregate estimate for GVA growth rate, but rather when the product detail is taken into account. In Table B, the number of large differences is also small (43 out of 392); the remaining differences are all considered to be moderate (165) or non-significant (184). Yet, even being the minority, the cases of large differences should not be neglected. Using an example, they mean that, in practice, the use of product group C trade estimates from Experience 3 instead of the reference OECD-based values for interregional trade, makes the GVA growth rate for that product category to change from 4,68% to 5,40%, which is a considerable deviation (see Table A). Another example relates to the comparison among Experiences: the largest difference occurs in Sweden, also in product C, to which the predicted GVA growth rate is 4,60%, according to the data generated by Experience 2

and it is 5,40%, according to Experience 3. The previous examples illustrate the maximum differences observed in Table A and in Table B, respectively.

3) Looking at the average column of Table A, which indicates the mean deviation for each country generated by the use of interregional trade estimates, instead of the reference values, we conclude that for the majority of the countries (10 in 14) the mean deviation is moderately relevant. In the remaining cases, the mean deviation is non-significant. Looking at these data in more detail, we see that the average deviation against the real values reaches the highest value in the case of Irish growth estimates (0,041%), in which the difference is close to the upper limit of the "moderately relevant" interval.

4) The average row of Table A, computed as the mean, for each Experience, between the several differences found for each country as a whole, suggests that Experiences 1 to 3 generate GVA growth rate estimates which have comparable errors. Yet, among the three, Experience 2 is the one which evidences the best match against the reference growth rates – this Experience shows the minimum average deviation and also the minimum number of differences above 0,05%.

5) The average row of Table B (computed in a similar way as for Table A) makes evident that, as expected, the mean difference between Experience 3 and Experience 1, which are gravity-based Experiences, is non-significant and it is smaller than between each of those Experiences and Experience 2.

Given these observations, we may conclude that, in general, the multi-regional input-output model shows a moderate sensitivity to the insertion of different estimates for interregional trade. The results do not reject the reasonability of using indirect estimates for interregional trade, given that large deviations in the results of the model are an exception. Nevertheless, it can't be stated that the choice of one specific method among several alternatives is completely innocuous. Even recognizing that the "choice" is most frequently constrained by the availability of information (as it happened in the present study), the researcher must be aware that the results of the model will necessarily be affected, although only in a moderate manner.

2.6 Conclusions.

The main objective of present chapter was to study different interregional trade estimation methodologies and make an empirical comparison between them.

The literature review upon the several models proposed to estimate interregional trade, made in section 2.3, led us to conclude that: 1) the solutions of the several models show a considerable similitude among each other and 2) most of the alternatives suffer from problems of applicability when the objective is to generate undisclosed values of interregional trade. Those problems (affecting the entropy model, the model of minimization of information bias, and the behaviour-based models), jointly with the advocated strengths of the gravity model, caused the focus of the remaining chapter to be put at several gravity-based methodologies.

Our first approach consisted in attempting to attest the good performance usually attributed to the gravity model, when used as an explanatory model to trade flow behaviour. To do so, an econometric application was carried out using bilateral trade flows between 14 European countries as the database. Five versions of the gravity equation were tested, including one non-spatial basic equation and four different spatial models. The results have demonstrated the overall adequacy of the gravity-based model to explain trade flow behaviour, especially when spatial dependence – revealed by spatial autocorrelation of errors – is recognized. All the experienced models were separately applied to distinct products, revealing a great variability among the estimated coefficients to each different traded product.

The next step consisted in examining distinct formulations of the gravity model as a generator of undisclosed values of interregional trade. Three different methods of generating the initial matrices of flows were applied and two different procedures were used to adjust the row and column totals of those initial matrices to previously known values, leading to a total of six different Experiences. Two out of the three different methods used to obtain the initial matrices were gravity-based. The third method, consisting in evenly distributing the amount of imports by each supplier country, was applied with the aim of assessing the impact of using an ad-hoc fulfillment of the initial matrix, instead of using a widely investigated model, as the gravity model. Concerning the adjustment procedures, the two methods applied in this study were: 1) the RAS method and 2) a linear programming model using minimization of information bias as the objective function and employing an entropy constraint (besides the usual additivity constraints).

The comparison between the results provided by the six different methodologies allowed us to infer four main conclusions. First, all the methodologies based upon a linear programming model and involving the use of additional information on real trade flows have provided exactly the same results, which were also equal to the ones provided by Experience 2, the one that relied upon an ad-hoc method to generate the initial matrix and used RAS as the adjustment procedure. This means that the use of superior information about the real matrix of flows (in this case, the entropy level), jointly with the resource to a more sophisticated model doesn't seem critical to enhance the results. Second, among the first three Experiences, the gravity-based model with an independent estimate of the distance decay parameter seems to originate the most accurate matrix (although the differences between aggregate errors obtained from Experiences 1 to 3 are small). Third, the initial matrix seems to have an effective influence on the final results. In fact, when comparing the different Experiences among each other, we have concluded that the only case which is not gravity-based – Experience 2 – generates more outlying results, demonstrating that the way by which initial estimates are obtained is not innocuous. Finally, the mean errors generated by the three Experiences are not very high in the products which are most representative in international trade (around 28%). Thus, we may state that the non-survey methods proposed here produce quite reasonable results.

Finally, an analytical comparison among the different methodologies was made. This implied the construction of a simplified multi-regional input-output system, involving the 14 European countries of the sample, using a dataset composed by the individual Make and Use tables and bilateral trade data. The objective was to assess the sensitivity of the input-output model to the insertion on the input-output system of the Origin-Destination matrices obtained from the several interregional trade estimation Experiences. The impact on the model results was measured through the different GVA growth rates estimated as a consequence of an exogenous change in final demand. We have concluded that the results of the input-output model were not greatly affected by the consideration of different trade flow values, since large deviations between the obtained growth rates were the exception and not the rule.

The main practical contribution of this Chapter consists precisely in the conclusions that can be drawn from the absolute and analytical comparison between the different trade estimation methods, namely: 1) among the several Experiences applied, the one that generated the most accurate matrix corresponded to a gravity-based model, with independent estimation of the distance decay parameter and using RAS as the adjusting procedure; 2) the introduction of superior complexity in the models as well as the use of additional information about the real trade flows, such as the degree of Entropy of the real trade matrix, may not originate better results, as it happened it this case; 3) the impacts on the input-output model of using differently estimated trade flows are only moderate – thus, the results do not reject the reasonability of using indirect estimates for interregional trade.

Although it is not advisable to generalize these results, given that they were obtained from a particular set of data and using a specific set of hypothesis, we consider that these practical contributions are most relevant to regional input-output researchers, especially to those who intend to assemble an input-output model in a context of absent information on interregional trade flows (which is the most frequent situation at the sub-national level). In fact, our conclusions may be useful as important arguments to estimate those inexistent data through the use of gravity-based non-survey methods.

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CHAPTER 3 – INPUT-OUTPUT MODELLING BASED ON TOTAL USE RECTANGULAR TABLES.

3.1 Introduction.

Input-output tables, at national or at regional level, can be classified according to three main criteria:

1) symmetric or rectangular format.
2) total use or domestic use flows.
3) valuation of goods and services.

As it has been previously referred, in Chapter 1, the historical format of the input-output table is symmetric, which means that the inner part of the table depicts product-by-product or industry-by-industry relations. Yet, since the end of the 1960's, when the United Nations introduced the 1968 System of National Accounts, countries are recommended to compile and publish the input-output tables on a rectangular or Make and Use format[107]. In this case, two tables are combined to depict supply and use product-by-industry relationships. Since the number of products may be higher than the number of industries, this format is called rectangular.

The second criterion is defined according to the type of flows represented in the intermediate transactions part of the Use table and also in the several components of final demand. Intermediate consumption of products (made by industries) and final use (made by households, government, firms and foreign countries) involves the use of products which are not only domestically produced, but are also imported. A total-flow Use table records the whole amount of inputs used, whether these have been produced within the country (or the region, depending on whether we are dealing with a national or a regional model) or imported. Conversely, if intermediate and final use flows are expurgated from the value of imported products, then we are facing a domestic (or intra-regional) flow table.

[107] This format has briefly been presented in Chapter 1, section 5.1.

Finally, the third criterion is related to the different prices at which goods and services may be evaluated. Current input-output tables involve two different price systems: basic prices, the closest to the value of production factor costs, and purchasers' prices, which include taxes on the products (deducted from subsidies) and trade and transport margins.

Combining these criteria in several manners, many different input-output tables can be constructed. However, in practice, the starting point to the construction of these tables is as a rule the total-flow rectangular table at purchasers' prices, since this is the format in which statistical information is gathered and published (at least in EU countries).

Whenever the researcher intends to implement any input-output model, starting from a total-flow rectangular table at purchasers' prices, there are two alternative procedures:

1. convert the starting table into a domestic-flow symmetric table at basic prices, and then, implement the classical input-output model or
2. perform the direct modelling of the total-flow rectangular table at purchasers' prices, *i.e.*, implementing the model on the basis of the table as it is published.

Many authors have thought the first procedure as the most adequate for input-output model applications. For example, in what respects the symmetric feature of the table, the EUROSTAT Input-output manual advocates that "For analytical purposes a relationship is needed between the inputs and the outputs irrespective of whether the products have been produced by the primary industry or by other industries as their secondary output" (EUROSTAT, 2002, p. 23); as a consequence, symmetric input-output tables "are compiled mainly to be used in input-output analysis" (p. 230). Concerning the content of the intermediate and final use flows, the same manual states that "the separation of domestically-produced and imported goods and services is of great importance for analytical purposes" (p. 145), leading to the option for domestic flow tables. A similar view can be found in other papers, as for example Lopes and Dias (2003), who sustain the extreme importance, for input-output impact analysis, of having an import matrix which allows for the computing of a domestic flow table. At the regional level, the symmetric domestic flow table is usually presented as preferable to conduct regional input-output analysis. For this reason, most of the regional input-output table building involves two stages. This is the case, for example, of the compilation of the Azores input-output table described in ISEG/CIRU (2004), which was carried out in the following steps: 1) regionalizing the M&U national table and 2) transforming it into a symmetric format, with domestic flows and at basic prices.

The methodology used to obtain a domestic-flow symmetric table at basic prices from the starting table (total-flow rectangular table at purchasers' prices) generally involves the use of a set of hypotheses needed to perform the following operations:

1) To expurgate the import content from the intermediate and final use flows, in order to get a domestic (or intra-regional) flow table. A constant proportion of imports is usually assumed, which is independent from the type of industry (or final user) the input goes into.

2) To transform purchasers' prices into basic prices. In the absence of direct information, which allows for expurgating margins and net taxes from the different uses of inputs, the same proportionality hypotheses are applied, similarly to what is done regarding imports.

3) To convert the rectangular matrix into a symmetric one. In this operation, one of two alternative hypotheses is usually adopted, to relate industry's output with commodity's output: the first hypothesis implies that all products produced by an industry are produced with the same input structure, meaning that there is one technology assigned to each industry; this is, therefore, called the Industry-based technology assumption (ITA). In opposition, the second hypothesis implies that a given product has the same input structure in whichever industry it is produced, meaning that there is one technology assigned to each product (UN, 1993); thus, it is named Commodity-based technology assumption (CTA). As it will be further demonstrated, the first hypothesis implies that each product is produced always in the same fixed proportions, while the second involves

the assumption that each industry produces its different products always in the same fixed proportions.

The theoretical and practical consequences of each of the above-referred hypotheses, as well as their reasonability, will be discussed in the present Chapter. Although controversial, these hypotheses are adopted in many cases, either purely, or complemented with some direct information, even when the domestic-flow symmetric table is assembled by the official entities. In what concerns to the estimation of the imports matrix, for example, even OECD recognizes that this happens in the official statistics, stating that "Techniques used to construct the import matrix data vary between countries, but every country in the OECD database made, to some extent, use of the import proportionality assumption in the construction of their import matrices" (OECD, 2000, p.12). Moreover, the Input-output database provided by OECD (consisting of symmetric industry-by-industry tables) is compiled using this kind of assumptions, whenever supplementary information is not available (Yamano and Ahmad, 2006).

The alternative procedure, consisting of the direct modelling of the rectangular table in its original format is the main theme of this chapter. The rectangular model will be developed using the same hypotheses adopted when obtaining the symmetric table. The objective is to demonstrate that this procedure leads to a result, which is precisely the same than the obtained by the first procedure. To do so, a practical example will be presented, in which we perform both procedures starting from a common initial M&U table. In both cases, exactly the same results are achieved. More precisely, the multipliers obtainable from in the inverse matrix comprised in the final equation of the input-output model are the same, whether the model has been developed on the basis of a symmetric table (derived from the rectangular one), or whether it has been directly implemented from the total use rectangular table at purchasers' prices. It cannot therefore be argued, that one of the alternatives is better than the other, nor, that it is incorrect to use either of them. As stated in UN (2002), "(...) there is theoretically no need to force the separate input and output matrices included in the SUT framework of the 1993 SNA into the traditional input-output straightjacket"[108] (paragraph 3.44).

In fact, the preference for the symmetric framework as the unique valid basis for input-output modelling is being questioned by other authors, such as Madsen and Jensen-Butler (1999), Kauppila (1999) and Piispala (1998), which suggest that the direct use of the M&U format has considerable advantages, namely:

- In the assembling process of the tables, since M&U tables are exempt of additional hypotheses (conversely to product-by-product or industry-by-industry tables), being more directly connected to the data collected by official statistical agencies. In fact, industries are able to inform how much of each commodity they produce (information comprised in the Make matrix) and how much of each commodity they consume for intermediate purposes (Use matrix). Conversely, the fulfilment of an industry-by-industry or a product-by-product table requires some transformation to the originally surveyed data. For this reason, when the objective is to build regional tables using non-survey methods, it is more advantageous to depart directly from the "cleaner" national M&U tables, rather than from national symmetrized tables.
- Make and Use tables are more easily intelligible for potential users of the model, since it closer resembles reality. In effect, in the real world, each industry produces a growing diversity of products, one of these being the primary product and the others the secondary products. M&U tables basically tell what commodities are produced by each industry and what commodities are consumed by industries and final consumers in the economy.
- M&U format is more suitable for application in certain fields of research which deal specifically with spatial interaction flows of commodities such as: environmental modelling (for example, when flows of products to be used in different industries are attached with flows of polluting elements, such as CO_2) and trade modelling (given that it becomes easier to incorporate data of trade

[108] The term SUT stands for Supply and Use tables and is equivalent to the expression Make and Use tables, used in the present work.

statistics, which report trade taking place with products and not with the output of industries, in broad terms).

- Concerning specifically to the regional-level analysis, the straight modelling of the rectangular table allows for the direct use of officially available Regional Accounts, which are industry-related data (these data comprise: regional value added by industry, regional production by industry and regional intermediate consumption by industry). In order to use such information directly, with the minimum imposition of hypothesis, the option should fall upon a Make and Use format or, eventually, upon an industry-by-industry symmetric format, which is however considered a second best option for input-output analysis, given the high heterogeneity of products involved in each element of such tables (EUROSTAT, 2002).
- Finally, as it will be demonstrated further on this chapter, the direct modelling of the rectangular table is a more timesaving procedure, which can be considered as an advantage of this alternative over the first one (involving the previous transformation into a symmetric table).

In this context, the research developed in this essay is guided towards the following questions:

- What procedures and related hypotheses may be used to perform input-output modelling when the base data consists of a total-use rectangular table at purchasers' prices, with no available import matrix?
- Is it advantageous to perform a previous transformation of the original tables into the symmetric format and a previous calculation of domestic flows, before implementing the model?

All the theoretical and practical development will be made using the National input-output tables as a reference, since there is no survey-based regional information to do so in Portugal. The conclusions are, however, also valid for regional and multi-regional input-output tables. Some adaptations have obviously to be made: for example, in a regional context, the procedure proposed to deal with international imports has to be extended to imports from other regions (however, the problem here is more complex, since not even the total amount of regional imports by product is known in advance).

The essay is divided into six sections, including this Introduction. In the next section, the three main criteria used to classify input-output tables are examined in detail. Section 3.3 is dedicated to a description of the Portuguese input-output tables. Section 3.4 is the nuclear part of this essay: the algebraic development of the input-output model based on the M&U framework will be presented, as well as the meaning and implications of all the assumed hypotheses. A practical application will be carried out in Section 3.5, aiming to compare the results obtained from both above mentioned types of procedures. The last section presents a summary of the main conclusions.

3.2 The three fundamental criteria for the construction of input-output tables: definitions and implications.

To better understand the implications of each of the three criteria previously referred to, we begin this section by illustrating the structure of the total use M&U tables at purchasers' prices (Figure 3. 1) and of the domestic use symmetric table at basic prices (Figure 3. 2).

Figure 3. 1 illustrates the M&U framework. Matrix [(1),(2)][109] is the (intermediate) Use matrix; each of its columns indicates the total amount of each product used by the industry, irrespective of the origin of those products (total use: domestic plus imported use flows); this matrix is valuated at purchasers' prices. Vector [(1),(3)] is the final demand vector and each of its elements result from the aggregation of the different types of final demand (final consumption, gross capital formation and exports); also, the flows

[109] In this notation, sub-matrices at rows are identified by the first number in brackets, and at columns by the second; in this case, [(1),(2)] means the sub-matrix located at the quadrant at the first row and second column, in Figure 3.1 scheme ahead..

included in this vector include both domestic and imported products and are valuated at purchasers' prices.

Figure 3. 1 – Structure of the total use M&U tables at purchasers' prices.

		(1)	*(2)*	*(3)*	*(4)*
		Products	Industries	Final Uses	Total = (2) + (3)
(1)	Products	---	Intermediate consumption of domestic and imported products, by industries, at p.p.	Final Uses of domestic and imported products, at p.p.	**Total uses of domestic and imported products, at p.p.**
(2)	Industries	Production by industry and by product, at b.p.	---	---	***Total industries' domestic output at b.p.***
(3)	Value Added	---	Components of Value added by industry		
(4)	Domestic output	**Total products' domestic output at b.p.**	***Total industries' domestic output at b.p.***		
(5)	Imports	Imports by product at cif prices	---		
(6)	(4)+(5)	**Total supply at b.p.**			
(7)	Trade and Transport margins	trade and transport margins on products			
(8)	Taxes less subsidies on products	taxes less subsidies on products			
(9)	(6)+(7)+(8)	**Total Supply at p.p.**			

p.p. = purchasers' prices
b.p. = basic prices

Matrix [(2),(1)] is the Make matrix; each column depicts how the various domestic industries contribute to the output of that column's product; reading along the rows, it gives us the distribution of each industry's output over the several products: one, which is the primary product and the various secondary products. The Make matrix is valuated at basic prices. Row-vector [(4),(1)] corresponds to the column sum of the Make matrix and it gives us the total domestic output of each product, valuated at basic prices. Adding the imported products, depicted in vector [(5),(1)], we obtain total supply of each product ([(6),(1)]). Finally, by adding trade and transport margins and net taxes on products[110], we get total supply by product, valuated at purchasers' prices ([(9),(1)]). The balance between product supply and use is made at purchasers' prices (this is illustrated by the grey shadowed vectors, which must be equal). Looking at the Industry dimension, we have row vector [(3),(2)], which represents value added by industry; in fact each element of this row is the sum of the several components of value added in each industry. Adding this to total intermediate consumption, by industry, we achieve row [(4),(2)]: total industries' domestic output at basic prices. These same values may be found, in the transposed form, in column [(2),(4)], which is the result of adding all the columns of the Make matrix.

In Figure 3. 2, we can find a product-by-product framework. However, an industry-by-industry symmetric table may also be constructed. For this section's purposes, it is sufficient to look at one of the two possible symmetric tables by choosing the most common type of relation: product-by-product. Matrix [(1),(1)] comprises the symmetric intermediate consumption flows: each column indicates the amount of the various products consumed as inputs in the production of that column's product, regardless of the industry where that product is produced. These flows, as well as the final use flows, include only domestically produced goods and services (domestic flows). Imported products used as intermediate products, margins and taxes are added in order to obtain total intermediate consumption, by product, at purchasers' prices. The same is made in [(2),(2)] and [(3),(2)], getting total final uses at purchasers' prices. Adding Value Added by product, we get total supply of domestic products. The balance between domestic product's supply and use occurs at basic prices.

[110] It is assumed that these taxes also include duties on imported products.

Figure 3. 2 - Structure of the domestic flow symmetric table at basic prices.

		(1)	*(2)*	*(3)*
		Products	Final Uses	Total = (1) + (2)
(1)	Products	Intermediate consumption of domestic products, at b.p.	Final Uses of domestic products, at b.p.	**Total uses of domestic products, at b.p.**
(2)	Imports	Imports for intermediate consumption at cif	Imports for final uses at cif prices by type of	**Total Imports**
(3)	Margins and Taxes less subsidies on products	falling upon intermediate consumption, by product	falling upon final users, by type	**Total margins and taxes less subsidies**
(4)	(1)+(2)+(3)	Total intermediate consumption, by product, at p.p.	Total Final Uses, at p.p.	**Total uses, at p.p.**
(5)	Value Added	Components of value added by product at b.p.		
(6)	(4)+(5)	**Total supply of domestic products at b.p.**		

3.2.1 Symmetric and rectangular input-output tables revisited.

The simplified hypothesis inherent in the traditional symmetric input-output table is that each product is produced by one single industry and each industry produces one single product. However, in reality, the most common situation is that each industry produces a growing diversity of products, one of these being the primary product and the others the secondary products. These secondary products can be divided into two categories: subsidiary products and by-products (EUROSTAT, 2002); subsidiary products are those secondary products which are technologically dissociated from the primary product; by-products are outputs that unavoidably result from the primary product production process, therefore being technologically related to it. Due to the presence of secondary products, the M&U framework is the one which better depicts reality, especially through its Make matrix, which provides all the details of this varied production. Yet, current National Make tables, following the SNA (System of National Accounts) recommendations, involve some partial refining in the Industry classification. This is due to the fact that industries are grouped according to the concept of kind-of-activity unit, and not according to the concept of enterprise. The term kind-of-activity unit (KAU) is used to denote a part of an institutional unit in which only one particular type of economic activity is carried out (Jackson, 2000). Thus, as a rule, enterprises "must be partitioned into smaller and more homogeneous units, with regard to the kind of production" (ESA, 1995, p. 35). So, in the National Accounts' Industry classification, each Industry consists of a group of KAUs which are "engaged in the same or a similar kind of activity" (ESA, 1995, p. 35). This means that most of the subsidiary products produced in each enterprise are classified under a different Industry heading, the one that produces those products as its main activity. Exceptions to this procedure occur whenever it is not possible to separate the secondary from the primary activity, either because secondary production is of by-product nature, or because the available information obtained from enterprises does not allow for separation (this being the case with most small firms, which have no accounting documents which allow for partitioning into different KAUs). As a result, the values of production recorded outside the main diagonal in the Make matrix are, at least in the majority of the European countries, mostly by-products, along with some residual subsidiary products that could not be separated from the main activity in the firms in which they were produced.

To achieve a symmetric input-output table (SIOT), some hypotheses have to be assumed, in order to calculate the product-by-product (or industry-by-industry) intermediate consumption flows[111]. Therefore, the SIOT is a derivative table, built upon the M&U tables, using some hypotheses which will be discussed in section 3.4.3.

[111] As well as to compute the value added by products, or, in industry-by-industry tables, the final demand by industries.

3.2.2 Total use flows versus domestic use flows.

Another major difference between Figure 3. 1 and Figure 3. 2 is in the treatment of imported products. In the total use table, all the use flows (intermediate and final) also include imported products. This means that the intermediate use matrix reflects true technical relationships: each of its elements indicates the total amount of a certain input used to produce a certain output. Data collected by means of surveys to firms can be directly used to prepare these types of tables. The same does not apply to domestic flow tables. In this case, a Use matrix of imported products is needed in order to deduce its value from the total Use table[112]. Direct information to construct such an Imports matrix is very rare. It is in fact very difficult for firms to know the origin (imported or domestically produced) of many of their inputs. In the majority of cases, firms buy inputs from wholesale traders, hence ignoring their origin. Besides that, even if we could obtain the percentage of inputs, which some firms import, it would be complex to know the specific use of those inputs, mainly to distinguish between intermediate consumption and capital formation. For similar reasons, the computation of final demand domestic flows based on direct information is also very complicated (or even more complex, since the number of intermediate traders between the importing firm and the final user is usually greater). Being so, Import matrices are always built under some plausible assumptions, sometimes complemented by direct information on some particular products.

3.2.3 Basic versus purchasers' prices.

Different concepts can be used in the valuation of input-output flows of goods and services, ranging from the factor cost price to the purchasers' prices. The valuation at factor cost price represents the production price and it better reflects the production function of each product (Martins, 2004). At the opposite extremity of the distribution process we find the purchasers' price, representing the amount paid to obtain "a unit of a good or service at the time and place required by the purchaser" (EUROSTAT, 2002, p. 121). In spite of this multiplicity of concepts, in practice SNA input-output tables use

[112] Details will be given in section 4.1.

only two price concepts: basic prices and purchasers' prices. Basic prices are similar to factor costs, except for the fact that basic prices include other taxes and subsidies on production, which are not possible to allocate to specific products[113]. Basic prices (*bp*) can be obtained from purchasers' prices (*pp*), through the following calculations[114]:

$$bp = pp - \text{taxes on products} + \text{subsidies on products - trade and transport margins}$$

In the particular case of M&U tables, data is valuated as follows:

- Production in the Make matrix is at *bp*;
- Intermediate consumption in the Use matrix and the final use vectors are at *pp*;
- Imports are valuated at *cif* prices[115]; *cif* prices are *bp* in the sense that they do not include any taxes or margins to be paid in the importing country;
- Exports are valuated at *fob* prices[116], which, in practice, correspond to *pp*, since they are comprised of all the taxes and margins to be paid in the exporting country.

There are two possible ways to balance supply and use in M&U tables: 1) transform supply flows into *pp* in order to allow balance with *pp* use flows or 2) transform use flows into *bp* in order to match with *bp* supply flows. The first option is illustrated in Figure 3. 1: row vectors *(7)* and *(8)* of margins and taxes less subsidies, respectively, are

---

[113] Taxes (subsidies) on products are those that "are payable per unit for some goods or services produced or transacted"(EUROSTAT, 2002, p. 200); examples: Value added taxes, import duties or tobacco product tax. Taxes (or subsidies) on production are those paid (or received) by firms as a direct result of their production activity, "independently of the quantity or value of the goods and services produced or sold" (*idem*, p. 200).

[114] Trade and transport margins are necessary to convert basic prices into purchasers' prices of each specific product. Thus, in column (7) of Figure 1, trade and transport margins appear as a positive entry in most products, but are imputed as negative entries in Trade and Transport services, to avoid double counting in the output of these activities. Therefore, when the aggregate level is being considered, margins do not appear in the transformation of *bp* into *pp*.

[115] *cif* price – "price of a good delivered at the frontier of the importing country, or the price of a service delivered to a resident, before the payment of any import duties or other taxes on imports or trade and transport margins within the country" (EUROSTAT, 2002, p. 123).

[116] *fob* price – "price of a good at the frontier of the exporting country, or the price of a service delivered to a non-resident, including transport charges and trade margins up to the point of the border, and including any taxes less subsidies on the goods exported" (EUROSTAT, 2002, p. 123).

added to total supply at *bp* in order to obtain total supply at *pp*. The second option is more data demanding. In fact, the adjustment of use flows from *pp* to *bp* involves the deduction, to each *pp* flow, of the value of margins and taxes comprised in that particular flow. To do so, valuation matrices are needed. Valuation matrices (of margins, for example) are tables of the same dimension as the intermediate and final use table, which "tell us how many margins are included in the *pp* or, in other words, which amounts need to be deducted from the purchasers' price in order to achieve the valuation of basic prices, if similar product taxes less subsidies are also deducted" (EUROSTAT, 2002, p. 127).

The published M&U tables usually employ *pp* concept to balance supply and use. It is however, sometimes argued, that this valuation is not sufficiently homogeneous to be used for input-output analytical purposes; for example, the ESA's Input-Output Manual states that "a valuation at purchasers' prices is a less homogeneous option as the shares of trade and transport margins differ from industry to industry and also from and between the final uses; the same is true for the shares of product taxes less subsidies" (EUROSTAT, 2002, p.124). It is also true that basic prices are closer to the concept of production costs involved in the technical relations used in input-output analysis. These relations assume that a certain amount of an input represents the same physical unit irrespective of the production process in which it is used (EUROSTAT, 2002). Hence, it is desirable that prices are cleared from margins and taxes which differently affect the diverse uses of the products. The problem lies in the compilation of the valuation matrices required to transform *pp* into *bp*, since direct information on the value of margins and taxes comprised in each use flow is very scarce. In fact, when someone buys a certain item, he/she doesn't know the amount of margins comprised in the price that has to be paid. Whenever valuation matrices are compiled, some hypotheses are used, whose plausibility will later be discussed (section 3.4.2). It will also be shown that similar hypotheses may be directly used to model a rectangular input-output table valuated at *pp*.

3.3 Input-output tables in the Portuguese National Accounts.

Every year, since 1995, the Portuguese National Institute of Statistics provides a set of National Accounts, which includes a M&U table, structured as in Figure 3. 1[117].

Products and industries are usually published in a 60 by 60 disaggregate level (ESA95 - A60 classification) although products can be further disaggregated and provided under request, at a specific National Accounts classification with 137 groups of products.

The Portuguese Make matrices are heavily diagonal, meaning that most of the production has been allocated to its primary producing industry, in the process of partial refining of Industries' classification, as it has been previously explained.

Intermediate and final uses of goods and services are composed of both domestically produced and imported products, but no import matrices are regularly compiled. Additionally, these use flows are valuated at *pp*; in this case, the table is complemented with valuation matrices (which are published on a regular basis) that allow transformation into *bp*. Finally, there is no regular production and publication of any symmetric tables (product by product or industry by industry). Thus, whenever the researcher wants to make use of input-output tables with a format similar to Figure 3. 2, he/she has to assemble the import matrix and the symmetric input-output table, under a series of different hypotheses.

It must be noted, however, that domestic flow symmetric input-output tables at basic prices were officially provided, with reference to the period 1995-1999, and more recently, to the period 2000-2004. The compiling work was not directly done by the National Institute of Statistics, but by a partnership between it and the Ministry of Finance's Planning and Prospective Department. The description of these tables'

[117] Except for the fact that the Use and the Make matrices are provided in separate tables, rather than in a combined one.

assembling, as well as the resulting matrices, is available at Martins (2004), Lopes (2007) and Lopes (2008).

3.4 Input-output modelling based on the total use M&U matrix, at purchasers' prices.

The direct implementation of the input-output model from the rectangular table, with total use flows and at purchasers' prices, will be dealt with in this section. As in any theoretical model, we will have to assume hypotheses, bearing in mind that, in some cases, they may be somewhat limiting. The assumed assumptions will be explained and their reasonability will be discussed, insofar they have to be incorporated in the model.

The relationships involved in the table of Figure 3. 1 can be translated into algebraic terms. Consider: g_i^{bp} denote total domestic production of industry *i*, at basic prices; v_{ij}^{bp}, the domestic production of product *j* by industry *i*, at basic prices (elements of the Make matrix); u_{ji}^{pp}, the amount of product *j* used as an input in the production of industry *i*'s output, at purchasers' prices (elements of the Use matrix) and w_i^{bp}, the value added in the production of *i*, at basic prices. The industry balance may be expressed by:

$$g_i^{bp} = \sum_j v_{ij}^{bp} = \sum_j u_{ji}^{pp} + w_i^{bp} \tag{3. 1}$$

At product level, the balance can be expressed as:

$$p_j^{pp} = \sum_i v_{ij}^{bp} + m_j + d_j + l_j = \sum_i u_{ji}^{pp} + y_j^{pp} \tag{3. 2}$$

in which: p_j^{pp} represents total supply of product *j*; m_j, total imports of product *j*; d_j, margins falling upon product *j*; l_i, taxes (less subsidies) falling upon product *j* and, finally, y_j^{pp}, final use of product *j* (both domestically produced and imported).

These expressions are applied to all products *j* and all industries *i*. Thus, the balance can be written using matrices and vectors. Let us use: 1) vector *i* as a column vector composed by ones that computes the column sum of the correspondent matrix; 2) the sign $\mathbf{A}'$ to indicate a transpose of a matrix $\mathbf{A}$. Then, equations (3. 1) and (3. 2) can be expressed as[118]:

$$\mathbf{g}^{\mathbf{bp}} = \mathbf{V}^{\mathbf{bp}}\mathbf{i} = \left(\mathbf{U}^{\mathbf{pp}}\right)'\mathbf{i} + \left(\mathbf{w}^{\mathbf{bp}}\right)' \tag{3. 3}$$

$$\mathbf{p}^{\mathbf{pp}} = \left(\mathbf{V}^{\mathbf{bp}}\right)'\mathbf{i} + \mathbf{m}' + \mathbf{d}' + \mathbf{l}' = \mathbf{U}^{\mathbf{pp}}\mathbf{i} + \mathbf{y}^{\mathbf{pp}} \tag{3. 4}$$

These two equations are mere algebraic specifications of the required balances at industry and at product level. To develop a model it is necessary to assume some hypotheses, which will be discussed in the following sections.

3.4.1 The proposed hypothesis to deal with intermediate and final use of imported products.

As seen in section 3.2.2, direct information to construct import matrices is scarce or even non-existent. Therefore, we intend to suggest a hypothesis which avoids the construction of such a matrix and which can be incorporated by directly implementing the rectangular model. The proposed assumption, named import proportionality assumption, establishes that for each product the share of imports in any type of use (intermediate or final) of that

[118] Matrices and vectors are presented in bold, while variables are in italic.

product is the same and is given by the proportion of imports on total supply of the same product. For example, if 40% of steel's total supply is imported, and 60% is provided by domestic production (corresponding to the self-sufficiency ratio) it is assumed that, in every industry which uses steel, 40% is imported and the same applies to any type of final use. This means that imports shares are differentiated by type of product but not by type of use. The implicit reasoning behind this assumption is that every industry and every final user directs its demand to a common pool of resources, having no preference for imported or domestically produced goods. The composition of that common pool (divided into imported and domestic products) will then determine the composition of the several uses.

In analytical terms, this hypothesis can be expressed as follows: let $c_j = \frac{m_j}{p_j^{bp}}$ be the import coefficient, representing the share of imports in the total supply of a certain product *j*, valuated at basic prices. On the supply side, this means that:

$$
\begin{aligned}
&p_j^{bp} = v_j^{bp} + m_j \Leftrightarrow \\
&p_j^{bp} = v_j^{bp} + c_j p_j^{bp} \Leftrightarrow \\
&(1 - c_j) p_j^{bp} = v_j^{bp} \Leftrightarrow \\
&p_j^{bp} = (1 - c_j)^{-1} v_j^{bp}
\end{aligned}
\tag{3. 5}
$$

The import proportionality assumption takes place, on the demand side, as follows:

$$
\begin{aligned}
&u_{ji}^{bp} = \left(u_{ji}{}^{N}\right)^{bp} + c_j u_{ji}^{bp} \Leftrightarrow \\
&(1 - c_j) u_{ji}^{bp} = \left(u_{ji}{}^{N}\right)^{bp} \Leftrightarrow , \qquad \text{for all user industries } i \text{ and} \\
&u_{ji}^{bp} = (1 - c_j)^{-1} \left(u_{ji}{}^{N}\right)^{bp}
\end{aligned}
\tag{3. 6}
$$

$$\begin{aligned} y_j^{bp} &= \left(y_j^N\right)^{bp} + c_j y_j^{bp} \Leftrightarrow \\ (1-c_j)\, y_j^{bp} &= \left(y_j^N\right)^{bp} \Leftrightarrow \ , \\ y_j^{bp} &= (1-c_j)^{-1}\left(y_j^N\right)^{bp} \end{aligned} \qquad \textbf{(3. 7)}$$

for all types of final uses, in which superior index ^{N} indicates domestic origin of products.

In spite of being very simple and logically sound, this assumption is not free from limitations in what concerns its application. One crucial point is the disaggregation level of the product (EUROSTAT, 2002). If the import coefficients are calculated at a much aggregated level, the assumed hypothesis may not be acceptable. Let us take the "Wood and wood products" product group as an example; this group comprises different types of wood, in different stages of transformation. By calculating the import coefficient of this product group and applying it to all its different uses, this will cause a serious bias in the results. This is because final users like families may use almost no imported wood transformed products, while industries will use a great share of imported wood raw materials (e.g. exotic woods). This leads to concluding that: in applying such a hypothesis, the most detailed level of disaggregation available on import data should be used. This usually does not cause a great deal of trouble since import data is available at a very detailed product level. The problem is that import data must be combined with the data in the Use matrix, which is usually more aggregated, thus limiting the level of disaggregation used in the calculations. Another criticism pointed out to this assumption goes directly to the definition of it: the fact that it assumes an invariant import proportion, irrespectively of the type of intermediate or final use the imported product goes to. It is clearly recognized that some final uses, like exports, for example, have less incorporation of imported products than others, like investment. In order to take this differentiation into account, some authors have proposed to exclude exports from the import proportionality assumption, assuming that there are no re-exports. This is done, for example, in Miller and Blair (1985), and Jackson (1998). As emphasized by Lahr (2001), this approach should be preferred only in those cases in which the researcher knows that the export vector has no (or almost no) re-exports[119]. For this essay's purposes, however, the proposed methodology should be suited to be applied on regional tables, in which re-exports may be more the rule than the exception. Hence, the import proportionality assumption will be taken uniformly throughout the various types of intermediate and final uses. Still, we recognize that the assumption must be applied in a conscious manner, and the researcher must be aware of the possible bias in the results. The magnitude of the errors coming from such an assumption, however, can only be accounted for when there is a benchmark survey-based import matrix against which the estimated one can be compared. This is done in Oosterhaven and Stelder (2007), in their comparison between four alternative non-survey intercountry input-output table construction methods, for nine Asian countries and the USA. In one of the non-survey input-output tables, they assume that there is no import matrix and use the import proportionality assumption to indirectly estimate it. The comparison between this table and the benchmark (which is a semi-survey based intercountry table) allow the authors to conclude that in general, "The tests show that the impact of using self-sufficiency ratios to estimate the domestic flows is small (…)" (Oosterhaven and Stelder, 2007, p. 258).

3.4.2 The proposed hypothesis to deal with margins and taxes less subsidies on products.

Margins and taxes less subsidies comprised in the Use table may be treated in a similar manner as imports. In the absence of direct information to construct valuation matrices and obtain a basic price valuated table, the proposal here is to assume the following: for each product, the margin (net taxes) rate comprised in any type of use (intermediate or final) of that product is the same and is given by the proportion of margins on total supply of the same product. In analytical terms, let:

[119] We have used this approach in Chapter 2, when assembling the multi-national system, composed of 14 countries.

- $f_j = \dfrac{d_j}{p_j^{pp}}$ be the margins coefficient: it tells us the percentage of margins on total supply of a certain product, valuated at *pp*[120];

- $n_j = \dfrac{l_j}{p_j^{pp}}$ be the taxes (less subsidies) coefficient: it tells us the percentage of net taxes on total supply of a certain product, valuated at *pp*;

Hence, total supply valuated at *pp* may be written as a function of total supply valuated at *bp*:

$$\begin{aligned}
&p_j^{pp} = p_j^{bp} + d_j + l_j \Leftrightarrow \\
&p_j^{pp} = p_j^{bp} + f_j p_j^{pp} + n_j p_j^{pp} \Leftrightarrow \\
&(1 - f_j - n_j) p_j^{pp} = p_j^{bp} \Leftrightarrow \\
&p_j^{pp} = (1 - f_j - n_j)^{-1} p_j^{bp}
\end{aligned} \tag{3. 8}$$

The proposed assumption consists in horizontally applying these coefficients to all the different uses of the product:

$$\begin{aligned}
&u_{ji}^{pp} = u_{ji}^{bp} + f_j u_{ji}^{pp} + n_j u_{ji}^{pp} \Leftrightarrow \\
&(1 - f_j - n_j) u_{ji}^{pp} = u_{ji}^{bp} \Leftrightarrow \qquad \text{, for all user industries } i \text{ and} \\
&u_{ji}^{pp} = (1 - f_j - n_j)^{-1} u_{ji}^{bp}
\end{aligned} \tag{3. 9}$$

$$\begin{aligned}
&y_j^{pp} = y_j^{bp} + f_j y_j^{pp} + n_j y_j^{pp} \Leftrightarrow \\
&(1 - f_j - n_j) y_j^{pp} = y_j^{bp} \Leftrightarrow \qquad \text{, for all types of final users.} \\
&y_j^{pp} = (1 - f_j - n_j)^{-1} y_j^{bp}
\end{aligned} \tag{3. 10}$$

[120] It is considered that margins and taxes (less subsidies) fall upon total supply, including imported goods and services.

What is the plausibility of such an assumption? In this case, it is useful to look at each of the following items separately: Value Added Tax (VAT), margins, other taxes on products and subsidies on products.

In what concerns non-deductible VAT[121], the problem is quite complex. Ideally, direct information should be available in order to:

1. Identify the type of users who support non-deductible VAT. Non-deductible VAT is, in fact, supported mainly by households and, in some exceptional cases, by firms, either falling upon intermediate consumption or GFCF (e.g. firms exempt from VAT and therefore not allowed to deduct it from their purchases).
2. Perform the linkage between the different VAT taxes and the product classification in the Use matrix; if the level of aggregation is high, some problems can arise because groups of products may well involve different VAT taxes (EUROSTAT, 2002).

Due to the specific feature of non-deductible VAT, the proportionality assumption is not the most suitable to deal with it. In addition, direct information, in some cases, can be obtained relating to non-deductible VAT – which is the case in Portugal, where the National Institute of Statistics provides a non-deductible VAT matrix under request. If such information is available, it is advisable to subtract non-deductible VAT from the Use table before applying the model. In our theoretical exposition, VAT will be treated jointly with the remaining taxes on products; for this reason, we will assume the proportionality assumption for this kind of tax, as well as for the other taxes on products. This is however only done in this purely theoretical model deduction, and it should be avoided as far as it is possible in practical exercises.

[121] Deductible VAT is not included in the *pp* valuation.

Treating margins on a proportional assumption basis is also not completely realistic. In fact, it has to be recognized that different users of a product pay different margins on it. For example, an industrial enterprise will certainly pay a smaller amount of margins on stationery materials than the final consumer[122].

Finally, the use of the proportional assumption in the case of other taxes and subsidies is less controversial. These taxes and subsidies fall upon specific products and any type of user has to support them. For example, taxes on gasoline have to be paid equally by any type of user of this product.

In any of the above mentioned items, the proportionality assumption must be applied at the most disaggregated level of product classification. This is important in order to avoid situations in which groups of products are heterogeneous in respect to margins or tax rates.

3.4.3 Two alternative hypotheses to connect the products' output with the industries' output on the rectangular table: Industry technology assumption (ITA) and Commodity technology assumption (CTA).

Two dimensions are considered in rectangular tables: products and industries. In order to write the structural equations and achieve the final impact model, it is necessary to assume some correspondences between industries' output and products' output. The following equations allow for a better understandment of this issue. Let us begin by assuming the traditional starting hypothesis in input-output models: there is a fixed technical relationship between the product input and the industry output. We will use the inputs and the outputs directly as they are provided in the M&U table (see Figure 3. 1). Thus, the technical coefficient in the rectangular model is given by:

[122] This distinction may, to some extent, be carried out using separate types of margins: wholesale and retail trade margins, and assuming that the former is supported by intermediate consumers and in GFCF and the latter or both are borne by final consumers. Although this distinction is not always available, this level of detail goes beyond the purpose of the present exposition.

$$q_{ji} = \frac{u_{ji}^{pp}}{g_i^{bp}}$$

(3. 11)

The above mentioned equation indicates the amount of product *j* used as input, irrespective of its origin (domestic or imported), directly necessary to produce one unit of industry *i* output. This allows us to write $u_{ji}^{pp} = q_{ji} g_i^{bp}$ or, in matrix terms, $\mathbf{U^{pp} i = Q g^{bp}}$, in which $\mathbf{Q}$ represents the technical coefficient matrix and $\mathbf{g}$ represents the industry output vector. Please remember that **U** is the Use matrix in the M&U frame, and $\mathbf{p^{pp}}$ is the vector of the products output. The superscript **pp** means that both, the matrix and the vector, are valuated at purchasers' prices. Equation (3. 4) can, therefore, be developed as follows:

$$\mathbf{p^{pp}} = \mathbf{U^{pp} i} + \mathbf{y^{pp}} \Leftrightarrow$$
$$\mathbf{p^{pp}} = \mathbf{Q g^{bp}} + \mathbf{y^{pp}}$$

(3. 12)

To further develop this model, it is, at this stage, necessary to establish some assumption about the kind of technological link between $\mathbf{p}$ and $\mathbf{g}$. Two alternative assumptions may be considered.

One possible option is to assume that each product is produced in fixed proportions by the several industries, implying that the structure implicit in each column of $\mathbf{V^{bp}}$ (the Make matrix) is assumed invariant; assuming also constant import, margins and taxes coefficients, industry's output and commodity's output is linked through the use of the following ratio: $s_{ij} = \frac{v_{ij}^{bp}}{p_j^{pp}}$. This gives us the market share of industry *i* in total supply of product *j* (including imports, margins and taxes). From this market share, we can write: $v_{ij}^{bp} = s_{ij} p_j^{pp}$. In matrix terms, this corresponds to: $\mathbf{V^{bp}} = \mathbf{S \hat{p}^{pp}}$, in which $\mathbf{S}$ represents the matrix of elements s_{ij}, of equal dimension as matrix $\mathbf{V}$, and the sign ^ is used to denote a diagonal matrix. Taking into account that the sum of the columns of $\mathbf{V}$ is the vector of industries' output, $\mathbf{g^{bp}} = \mathbf{V^{bp}i}$, this produces:

$$\mathbf{V^{bp}} = \mathbf{S\hat{p}^{pp}} \Leftrightarrow \mathbf{V^{bp}i} = \mathbf{S\hat{p}^{pp}i} \Leftrightarrow \mathbf{g^{bp}} = \mathbf{Sp^{pp}} \tag{3. 13}$$

This equation provides a way of relating the output of industries with the output of products. The use of such a relation in model developing, starting from equation (3. 12), implies the employment of Industry-based technology assumption (ITA), as it will be seen in Section 3.4.3.1. The crucial supposition under ITA is that each industry has its own technology, which is common to all the commodities it produces. Thus, the technology assigned to each product depends on the industry where it is produced.

Another option is to assume that each industry produces different products in fixed proportions, involving the hypothesis that the structure implicit in each row of $\mathbf{V^{bp}}$ is invariant; in this case, industry's output and commodity's output is linked through: $h_{ij} = \dfrac{v_{ij}^{bp}}{g_i^{bp}}$ (or $h_{ij} g_i^{bp} = v_{ij}^{bp}$). This indicates the percentage of industry *i*'s output that consists of the output of product *j*. In matrix terms, this is equivalent to:

$$\begin{aligned}
&\mathbf{H\hat{g}^{bp}} = \left(\mathbf{V^{bp}}\right)' \\
&\mathbf{\hat{g}^{bp}} = \mathbf{H}^{-1}\left(\mathbf{V^{bp}}\right)' \\
&\mathbf{\hat{g}^{bp}i} = \mathbf{H}^{-1}\left(\mathbf{V^{bp}}\right)'\mathbf{i} \\
&\mathbf{g^{bp}} = \mathbf{H}^{-1}\mathbf{v^{bp}}
\end{aligned} \tag{3. 14}$$

in which $\mathbf{v^{bp}}' = \left(\mathbf{V^{bp}}\right)'\mathbf{i}$, is the vector of products' domestic production, at basic prices.

There are, however, two problems with this equation: firstly, to be mathematically possible, this equation requires the existence of $\mathbf{H}^{-1}$. A necessary condition (but not sufficient) to the existence of such an inverse is that it must be square, which means, in practice, that the number of industries has to be equal to the number of products. This is, in fact, one of the drawbacks of this assumption, which will be later discussed. Secondly, this equation gives us the relation between $\mathbf{g^{bp}}$ and $\mathbf{v^{bp}}$, and not between $\mathbf{g^{bp}}$ and $\mathbf{p^{pp}}$, the required relation to substitute in equation (3. 12). This problem can, however, be easily solved by making use of equations (3. 5) and (3. 8), in matrix terms:

$$\begin{aligned}\mathbf{v^{bp}} &= (\mathbf{I}-\hat{\mathbf{c}})\mathbf{p^{bp}} \\ \mathbf{v^{bp}} &= (\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\mathbf{p^{pp}}\end{aligned} \tag{3. 15}$$

Substituting this equation into (3. 14), it yields:

$$\mathbf{g^{bp}} = \mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\mathbf{p^{pp}} \tag{3. 16}$$

The development of the input-output model on the basis of this relationship between product output and industry output involves the use of the so-called Commodity-based technology assumption (CTA). It assumes that each product is always produced by the same technology, regardless of the industry in which it is produced.

At this stage, when all the relevant hypotheses have been explained, it is possible to go through the development of the input-output model, based on the total use rectangular table, at purchasers' prices. Since there are two alternatives to relate industry output with commodity output (ITA and CTA), this will originate two different models, analytically presented in the subsequent section.

3.4.3.1 Model development under each of the hypotheses.

INDUSTRY-BASED TECHNOLOGY ASSUMPTION

Substituting equation (3. 13) (industry technology hypothesis) into (3. 12), we can derive a product-by-product relationship:

$$\begin{aligned}
\mathbf{p}^{\mathbf{pp}} &= \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
\mathbf{p}^{\mathbf{pp}} &= \mathbf{Q}\mathbf{S}\mathbf{p}^{\mathbf{pp}} + \mathbf{y}^{\mathbf{pp}} \\
(\mathbf{I} - \mathbf{QS})\mathbf{p}^{\mathbf{pp}} &= \mathbf{y}^{\mathbf{pp}} \\
\mathbf{p}^{\mathbf{pp}} &= (\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}
\end{aligned}$$

(3. 17)

This equation allows the assessment of the impact on total product supply originated by changes in final demand, both valuated at *pp* and on a total use basis. In other words, the *ij*th element of its inverse represents the amount of total output of product *i* directly and indirectly needed to deliver an additional unit[123] of product *j* to final demand. As usual in input-output, this kind of impact analysis implies that the inverse elements are fixed, which, in turn, requires that the direct coefficients are also constant. In this case, the direct coefficients are the elements of the product matrix $\mathbf{QS}$, denoted by α_{ij}. Let us pay some attention to the meaning of these elements, exploring the way in which they are obtained: the specific element in position [1,2] (first row and second column) will be the result of the matrix product between the first row of matrix $\mathbf{Q}$ (technical coefficients) and the second column of matrix $\mathbf{S}$ (market shares). Considering, for example, that there are only 3 products and 3 industries, we have:

$$\alpha_{12} = q_{11}s_{12} + q_{12}s_{22} + q_{13}s_{32}$$

(3. 18)

[123] All reasoning is obviously made in monetary units, but for the sake of simplicity in exposition, we will dispense with the constant use of the term “monetary unit”.

This means that the amount of product 1 needed to produce one unit of product 2 is the sum of three portions, each corresponding to the share of each industry in producing product 2. Each industry that contributes to the production of product 2 uses its own technology, as established by the ITA. In this case, we have a combination of three different technologies involved in the production of product 2. Since it is required that direct coefficients are fixed, this implies that not only industries' technologies be fixed (expressed by the *u* elements), but also that the shares of each industry in the product's output also be constant. This is the main theoretical implication deriving from ITA: the technology associated to each product is a fixed linear combination of different industries' technologies.

The impact measured by equation (3. 17) comprises the effect on domestic production, on imported products and also on margins, taxes and subsidies because all these elements are included in $\mathbf{p}^{\mathbf{pp}}$. If one wants to isolate the effect on domestic production, valuated at *bp*, this can be achieved by making use of the hypotheses defined in sections 3.4.1 and 3.4.2. Substituting equation (3. 8) into (3. 17), we have:

$$\begin{aligned}
&\mathbf{p}^{\mathbf{pp}} = (\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}} \\
&(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}})^{-1}\mathbf{p}^{\mathbf{bp}} = (\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}} \\
&\mathbf{p}^{\mathbf{bp}} = (\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}})(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}
\end{aligned}$$

(3. 19)

Then, using equation (3. 5), it leads to:

$$\begin{aligned}
&(\mathbf{I} - \hat{\mathbf{c}})^{-1}\mathbf{v}^{\mathbf{bp}} = (\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}})(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}} \\
&\mathbf{v}^{\mathbf{bp}} = (\mathbf{I} - \hat{\mathbf{c}})(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}})(\mathbf{I} - \mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}
\end{aligned}$$

(3. 20)

It is still possible to arrange this equation in order to express the impact of demand directed towards only domestically produced products, valuated at *bp*. Using equations (3. 10) and (3. 7), it yields:

$$\mathbf{v^{bp}} = (\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})(\mathbf{I}-\mathbf{QS})^{-1}(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}(\mathbf{y^N})^{\mathbf{bp}}$$

(3. 21)

This expression represents the impact on domestic production valuated at *bp* caused by changes in demand for domestic products, also at *bp*. It shows that, by using some hypotheses, the rectangular model answers the same kind of questions as a product-by-product domestic flow symmetric model valuated at *bp*, built upon a symmetric table constructed under similar hypotheses. Moreover, it will be shown (in section 3.5) that the result of matrix multiplication $(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})(\mathbf{I}-\mathbf{QS})^{-1}(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}$ is exactly equal to the domestic flow symmetric model inverse, leading to equal multipliers.

All the previously derived equations depict product-by-product relationships. However, the rectangular model also permits us to achieve industry-by-industry relations. Taking equation (3. 12) again, we have:

$$\mathbf{p^{pp}} = \mathbf{Qg^{bp}} + \mathbf{y^{pp}}$$
$$\mathbf{Sp^{pp}} = \mathbf{SQg^{bp}} + \mathbf{Sy^{pp}}, \qquad \text{by (3.13)},$$
$$\mathbf{g^{bp}} = \mathbf{SQg^{bp}} + \mathbf{Sy^{pp}}$$
$$(\mathbf{I}-\mathbf{SQ})\mathbf{g^{bp}} = \mathbf{Sy^{pp}}$$
$$\mathbf{g^{bp}} = (\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{Sy^{pp}}$$

(3. 22)

Factor $\mathbf{Sy^{pp}}$ represents the final demand directed at domestic industries. In fact, if vector $\mathbf{y^{pp}}$ is the final demand for products and matrix *S* indicates the market shares of each industry in providing the several commodities, the product between them is the final demand directed at industries[124]. Thus, equation (3. 22) can be understood as an industry-

[124] The values comprised in vector Sy^{pp} are necessarily valuated at basic prices and exclude imported products, since matrix S is obtained from matrix *V*, which is a domestic production matrix, valuated at b.p..

by-industry relationship. The inverse matrix $(\mathbf{I}-\mathbf{SQ})^{-1}$ gives us the direct and indirect needs of industry output to answer increases in final demand directed at industries. The total requirements matrix involved in this equation ($(\mathbf{I}-\mathbf{SQ})^{-1}$) should be equal to the inverse matrix derived from a domestic flow industry-by-industry symmetric table (valuated at *bp*), constructed from the rectangular table, using similar hypotheses. Again, we postpone the practical demonstration to section 3.5.

Besides product-by-product and industry-by-industry relations, another type of connection may be of the researcher's interest: what is the effect on industries resulting from a change in final demand for products? This is answered by an industry-by-product relation, which can be extracted from equation (3. 22). In fact, post-multiplying the previous inverse matrix $(\mathbf{I}-\mathbf{SQ})^{-1}$ by *S*, we obtain the total requirements matrix that relates industry's output with final demand for products: $(\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{S}$. Furthermore, using equations (3. 7) and (3. 10) it is still possible to obtain an expression that reflects the impact on industries resulting from changes in domestic flow *bp* final demand:

$$\mathbf{g}^{\mathbf{bp}} = (\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{S}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \tag{3. 23}$$

An alternative way of deriving an industry-by-product inverse matrix starts from equation (3. 17), as follows[125]:

$$\begin{aligned}
\mathbf{p}^{\mathbf{pp}} &= (\mathbf{I}-\mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}} \\
\mathbf{Sp}^{\mathbf{pp}} &= \mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}} \\
\mathbf{g}^{\mathbf{bp}} &= \mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}
\end{aligned} \tag{3. 24}$$

[125] Provided that matrix **S** has an inverse, $\mathbf{S}^{-1}$, it can be easily demonstrated that equation (3.24) is equivalent to (3.22). In fact, $\mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1} = \left[(\mathbf{I}-\mathbf{QS})\mathbf{S}^{-1}\right]^{-1} = \left(\mathbf{S}^{-1}-\mathbf{Q}\right)^{-1} = \left(\mathbf{S}^{-1}-\mathbf{S}^{-1}\mathbf{SQ}\right)^{-1} = \left[\mathbf{S}^{-1}(\mathbf{I}-\mathbf{SQ})\right]^{-1} = (\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{S}$.

COMMODITY-BASED TECHNOLOGY ASSUMPTION

The same type of relationships can be derived using the CTA to connect industry output with product output. The obtained inverse matrices will, naturally, differ from the correspondent matrices derived under ITA, since technology assumptions, crucial in input-output analysis, are completely diverse. Let's again take the starting equation: (3. 12) and substitute (3. 16) into it:

$$\begin{aligned}
\mathbf{p^{pp}} &= \mathbf{Qg^{bp}} + \mathbf{y^{pp}} \\
\mathbf{p^{pp}} &= \left[\mathbf{QH^{-1}}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\right]\mathbf{p^{pp}} + \mathbf{y^{pp}} \\
\left[\mathbf{I}-\mathbf{QH^{-1}}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\right]\mathbf{p^{pp}} &= \mathbf{y^{pp}} \\
\mathbf{p^{pp}} &= \left[\mathbf{I}-\mathbf{QH^{-1}}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\right]^{-1}\mathbf{y^{pp}}
\end{aligned}$$

(3. 25)

This equation shows the effect on total product supply of changes in final demand, at *pp* and on a total use basis. Using CTA in this kind of exercise implies that, besides constant technical coefficients[126], we are assuming constant *H* elements, which forces the product composition of each industry's output to be fixed. Thus, each industry's technology is determined by a fixed linear combination of several technologies, one for each different product.

From this initial equation, several other relationships can be derived, similar to what has been done in the ITA development. Using equation (3. 14), which postulates CTA in an alternative way, we can substitute it into (3. 12) to achieve:

[126] In this particular case it is also necessary to assume constant import, margins and taxes coefficients.

$$
\begin{aligned}
&\mathbf{p}^{\mathbf{pp}} = \mathbf{Q}\mathbf{H}^{-1}\mathbf{v}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)^{-1}(\mathbf{I} - \hat{\mathbf{c}})^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{Q}\mathbf{H}^{-1}\mathbf{v}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{Q}\mathbf{H}^{-1}\mathbf{v}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\mathbf{H}^{-1}\mathbf{v}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}, \text{ by (3.15).} \\
&\left[\mathbf{I} - (\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{Q}\mathbf{H}^{-1}\right]\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left[\mathbf{I} - (\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{Q}\mathbf{H}^{-1}\right]^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}
\end{aligned}
$$

(3. 26)

This expression shows the impact of final demand for domestic products on domestically-produced products. It therefore gives us the same information which is contained in the inverse of a product-by-product symmetric domestic flow input-output table, if obtained by using similar hypotheses.

An industry-by-industry equation may also be derived from equation (3. 12), as follows:

$$
\begin{aligned}
&\mathbf{p}^{\mathbf{pp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)^{-1}(\mathbf{I} - \hat{\mathbf{c}})^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}, \textbf{ by (3.15).} \\
&\mathbf{H}^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{g}^{\mathbf{bp}} = \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}, \text{ by (3.14).} \\
&\left[\mathbf{I} - \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\right]\mathbf{g}^{\mathbf{bp}} = \mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{g}^{\mathbf{bp}} = \left[\mathbf{I} - \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\right]^{-1}\mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}
\end{aligned}
$$

(3. 27)

Being $\mathbf{H}^{-1}$ the matrix which allows the conversion from product dimension to industry dimension, the product $\mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}$ represents final demand directed at industries. Therefore, the inverse matrix $\left[\mathbf{I} - \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\right]^{-1}$ gives us the same kind of information as an inverse matrix of an industry-by-industry symmetric model. It will also be shown that these two inverse matrices are equal, if the same hypotheses are used to derive the symmetric model.

Evidently, equation (3. 27) also embraces an industry-by-product total requirements matrix: $\left[\mathbf{I}-\mathbf{H}^{-1}\left[(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\right]\mathbf{Q}\right]^{-1}\mathbf{H}^{-1}$ expresses the impact on industries' output caused by changes in domestic final demand for products.

Referring back to equation (3. 12), we can still work out an equation which relates industry's output with total final demand, valuated at *pp*:

$$
\begin{aligned}
&\mathbf{p}^{\mathbf{pp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}\mathbf{v}^{\mathbf{bp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}}, \text{ by (3.15)} \\
&\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}\mathbf{H}\mathbf{g}^{\mathbf{bp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}}, \text{ by (3.14)} \\
&\left[\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}\mathbf{H}-\mathbf{Q}\right]\mathbf{g}^{\mathbf{bp}} = \mathbf{y}^{\mathbf{pp}} \\
&\mathbf{g}^{\mathbf{bp}} = \left[\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}(\mathbf{I}-\hat{\mathbf{c}})^{-1}\mathbf{H}-\mathbf{Q}\right]^{-1}\mathbf{y}^{\mathbf{pp}}
\end{aligned}
$$

(3. 28)

In this section it has been shown that the data directly available in the rectangular table is suitable to model equations that allow for the assessment of product-by-product, industry-by-industry or mixed impacts. The underlying hypotheses have been previously presented and their plausibility has been discussed, except in what concerns CTA versus ITA; this will be done in section 3.4.3.3. In the next section it will be demonstrated that the previously derived equations may be obtained in a one-step procedure, which consists of the simultaneous inversion of the M&U coefficient matrices.

3.4.3.2 Partitioned matrix inverse: calculation and analysis.

<u>*INDUSTRY-BASED TECHNOLOGY ASSUMPTION*</u>

Let us start with the ITA based model. The nuclear part of the M&U table is represented by the shadowed quadrants in Figure 3. 3 (this is a simplified version of Figure 3. 1):

Figure 3. 3 – Make and Use matrix – simplified structure.

	Products	Industries	Final Uses	Total
Products	*0*	U^{pp}	y^{pp}	$\boldsymbol{p}^{pp}$
Industries	V^{bp}	*0*	---	$\boldsymbol{g}^{bp}$
Value Added	*0*	*w*		
Imports	*m*	*0*		
Margins	*d*	*0*		
Taxes less subsidies	*l*	*0*		
Total	$\boldsymbol{p}^{pp}$	$\boldsymbol{g}^{bp}$		

It should be noted that even if the matrices $\mathbf{U^{pp}}$ and $\mathbf{V^{bp}}$ are not square, the shadowed (and partitioned) matrix composed of these two (and of zero matrices of the appropriate dimension) will be square. Consider, for example, that there are 30 industries and 50 products. In this case, the matrix $\mathbf{U^{pp}}$ will have a dimension of 50*30 and $\mathbf{V^{bp}}$ will be a 30*50 matrix. Consequently, the shadowed matrix will have a dimension of 80*80 and it can be inverted. Hence, we can not agree with the following statement of Koronczi (2004): "A symmetric or square I-O matrix is required for I-O analysis, as only a square matrix can be inverted to obtain the Leontief inverse" (p. 24), presented as an argument to prevent the rectangular format to be used directly to input-output modeling.

Dividing all the elements of $\mathbf{U^{pp}}$ and $\mathbf{V^{bp}}$ by the correspondent column totals, we obtain the following partitioned matrix, composed by two matrices of zeros and the previously defined matrices *Q* and *S*:

$$\mathbf{D} = \begin{bmatrix} \mathbf{0} & \mathbf{Q} \\ \mathbf{S} & \mathbf{0} \end{bmatrix}$$

Using matrix *D*, we can write the matrix system:

$$\begin{bmatrix} \mathbf{0} & \mathbf{Q} \\ \mathbf{S} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}+\begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}=\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}$$

(3. 29)

From this system, we may derive equations (3. 12) and (3. 13) that were our starting point in previous section 3.4.3. In fact, if we multiply these partitioned matrices, we obtain: $\mathbf{Qg}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} = \mathbf{p}^{\mathbf{pp}}$ and $\mathbf{Sp}^{\mathbf{pp}} = \mathbf{g}^{\mathbf{bp}}$.

The matrix system in (3. 22) may be handled in order to isolate the outputs (products and industries) vector:

$$(\mathbf{I}-\mathbf{D})\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}=\begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}=(\mathbf{I}-\mathbf{D})^{-1}\begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}=\begin{bmatrix} \mathbf{I} & -\mathbf{Q} \\ -\mathbf{S} & \mathbf{I} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

(3. 30)

Applying the general formula for computing the inverse of a partitioned matrix[127], we obtain:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{Q} \\ -\mathbf{S} & \mathbf{I} \end{bmatrix}^{-1}=\begin{bmatrix} \mathbf{I}+\mathbf{Q}(\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{S} & \mathbf{Q}(\mathbf{I}-\mathbf{SQ})^{-1} \\ (\mathbf{I}-\mathbf{SQ})^{-1}\mathbf{S} & (\mathbf{I}-\mathbf{SQ})^{-1} \end{bmatrix}$$

---

[127] This formula can be found, for example, in Barnett (1990), pp. 71-72, and states that:

$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1}=\begin{bmatrix} \mathbf{A}^{-1}+\mathbf{A}^{-1}\mathbf{BF}^{-1}\mathbf{CA}^{-1} & -\mathbf{A}^{-1}\mathbf{BF}^{-1} \\ -\mathbf{F}^{-1}\mathbf{CA}^{-1} & \mathbf{F}^{-1} \end{bmatrix}$, in which $\mathbf{F}=\mathbf{D}-\mathbf{CA}^{-1}\mathbf{B}$, or equivalently,

$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1}=\begin{bmatrix} \mathbf{G}^{-1} & -\mathbf{G}^{-1}\mathbf{BD}^{-1} \\ -\mathbf{D}^{-1}\mathbf{CG}^{-1} & \mathbf{D}^{-1}+\mathbf{D}^{-1}\mathbf{CG}^{-1}\mathbf{BD}^{-1} \end{bmatrix}$, in which $\mathbf{G}=\mathbf{A}-\mathbf{BD}^{-1}\mathbf{C}$.

(3. 31)

or

$$\begin{bmatrix} \mathbf{I} & -\mathbf{Q} \\ -\mathbf{S} & \mathbf{I} \end{bmatrix}^{-1} = \begin{bmatrix} (\mathbf{I}-\mathbf{QS})^{-1} & (\mathbf{I}-\mathbf{QS})^{-1}\mathbf{Q} \\ \mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1} & \mathbf{I}+\mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1}\mathbf{Q} \end{bmatrix}$$

(3. 32)

Inserting equation (3. 32) into (3. 30), and multiplying these partitioned matrices, we get the equations: $\mathbf{p}^{\mathbf{pp}} = (\mathbf{I}-\mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}$ and $\mathbf{g}^{\mathbf{bp}} = \mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1}\mathbf{y}^{\mathbf{pp}}$, which are, precisely, the previously derived equations (3. 17) and (3. 24), respectively. The first equation gives us the impact on total product supply originated by changes in final demand for products ($\frac{\partial \mathbf{p}^{\mathbf{pp}}}{\partial \mathbf{y}^{\mathbf{pp}}}$). Therefore, this is a product-by-product relation. The second is an industry-by-product relation; it shows the impact on industry's supply caused by changes in final demand for products ($\frac{\partial \mathbf{g}^{\mathbf{bp}}}{\partial \mathbf{y}^{\mathbf{pp}}}$).

However, the right hand blocks are also susceptible to some interpretation. The lower right hand corner of (3. 31) is, in fact, precisely equation (3. 22), which depicts an industry-by-industry relation; it gives us $\frac{\partial \mathbf{g}^{\mathbf{bp}}}{\partial (\mathbf{S}\mathbf{y}^{\mathbf{pp}})}$. The upper right hand corner, $\mathbf{Q}(\mathbf{I}-\mathbf{SQ})^{-1}$ or $(\mathbf{I}-\mathbf{QS})^{-1}\mathbf{Q}$, accounts for the impact on product demand, including imports, margins and taxes, created by changes in the demand directed at domestic industries ($\frac{\partial \mathbf{p}^{\mathbf{pp}}}{\partial (\mathbf{S}\mathbf{y}^{\mathbf{pp}})}$). Hence, it is a product-by-industry relation. It should, however, be noted that a different type of product-by-industry relation can be established, yet not comprised in this rectangular inverse.

Starting from these equations, several others (at basic prices and domestic flows) may be derived through the application of similar hypotheses as in section 3.4.3.1.

The previous exercise allowed us to conclude that all the equations mathematically obtained in section 3.4.3.1 for ITA (product-by-product, industry-by-industry and industry-by-product) can be directly achieved through the inversion of the so-called rectangular matrix, with stable s coefficients derived from $\mathbf{V}$, according to ITA. Furthermore, this approach has several considerable advantages over symmetric models. For instance, in the symmetric model we have to build a different model for each type of relationship: the product-by-product symmetric model generates only a product-by-product impact equation; if we want to quantify an industry-by-industry impact, we will need to construct an industry-by-industry symmetric table and develop the corresponding model. On the contrary, in the rectangular approach the both kind of relationships result from the unique model.

<u>*COMMODITY-BASED TECHNOLOGY ASSUMPTION*</u>

Alternatively, a similar reasoning can be made using the CTA. In this case, in system (3. 29), equation (3. 13), which defines ITA, is substituted by equation (3. 16), which corresponds to CTA; by doing this, we obtain system (3. 33):

$$\begin{bmatrix} \mathbf{0} & \mathbf{Q} \\ \mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix} + \begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix}$$

(3. 33)

Let $\mathbf{E}$ designate the partitioned matrix. Then, the system leads to:

$$(\mathbf{I}-\mathbf{E}) \begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix} = \begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix} = (\mathbf{I}-\mathbf{E})^{-1} \begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{p}^{\mathbf{pp}} \\ \mathbf{g}^{\mathbf{bp}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{Q} \\ -\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}) & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{y}^{\mathbf{pp}} \\ \mathbf{0} \end{bmatrix}$$

(3. 34)

Applying the formula for inverting partitioned matrices, we get:

$$\begin{bmatrix} \mathbf{I}+\mathbf{Q}\left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1}\left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1} & \mathbf{Q}\left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1} \\ \left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1}\left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1} & \left[\mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\mathbf{Q}\right]^{-1} \end{bmatrix}$$

(3. 35)

or

$$\begin{bmatrix} \left[\mathbf{I}-\mathbf{Q}\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\right]^{-1} & \left[\mathbf{I}-\mathbf{Q}\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\right]^{-1}\mathbf{Q} \\ \mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\left[\mathbf{I}-\mathbf{Q}\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\right]^{-1} & \mathbf{I}-\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\left[\mathbf{I}-\mathbf{Q}\mathbf{H}^{-1}(\mathbf{I}-\hat{\mathbf{c}})\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)\right]^{-1}\mathbf{Q} \end{bmatrix}$$

(3. 36)

Inserting (3. 36) into (3. 34), we get the equations that represent the product-by-product and industry-by-product relations, expressed in equations (3. 25) and (3. 27), respectively. The lower right-hand corner of the first form of the inverse is exactly the inverse implicit in the industry-by-industry relation, deduced in equation (3. 27). Finally, the upper right-hand corner comprises a product-by-industry relation, having the same meaning as explained previously (in the ITA inverse matrix). Thus, using CTA, the same type of relations can be derived directly from the rectangular model, leading to different multipliers, since the technology assumption is diverse.

3.4.3.3 The present and past literature debate on ITA versus CTA.

In spite of the longstanding debate over the plausibility of the alternative assumptions, ITA and CTA, there is still no definite consensus. This debate started when most countries began to publish their input-output tables in the rectangular format. Some technological assumption is required, either to develop the input-output model directly from the rectangular table (as illustrated in the previous section) or to obtain a symmetric matrix, starting from M&U tables.

The subsequent literature review aims to systematize the main arguments in favour of each of these technological assumptions. Additionally, attention will be targeted at the two major existing assumptions: ITA and CTA, still recognizing that other hypotheses exist[128].

Summarizing what has been previously said, ITA states that each industry has its own input structure, regardless of the mix of products it produces, while CTA admits that each product has its own technology, irrespective of the industry in which it is produced (ten Raa and Rueda-Cantuche, 2007). Hence, when ITA is applied to obtain a symmetric table from the M&U matrices, the process consists of transferring the secondary production of each industry to the one in which the product is a primary production, using the input structure of the first industry. As a result, in the intermediate part of a product-by-product symmetric table derived through ITA, each column involves a combination of several input structures. Conversely, when the symmetric table is assembled on the basis of CTA, the products are transferred to the industry in which they are primary ones, using this latter industry's technology. Thus, in the intermediate part of a product-by-product symmetric table derived through CTA, each column comprises only one technology. The main problem with this methodology is the generation of negative entries in the symmetric intermediate consumption table. It is easy to understand why this happens: the fact that product *i* is transferred from industry *j* to industry *k* using the technology of industry *k*, implies that some inputs that are to be subtracted from the input structure of industry *j* may not be used in the production of *i* in this industry (or may be used in a lesser amount), originating meaningless negative flows. This problem will be further discussed in this section.

As it has been explained in section 3.4.3.1, when used in input-output impact analysis, the assumption of ITA corresponds to the supposition, to every industry, of fixed shares in each market product, while CTA implies considering that each industry has a fixed

[128] The fact is that these complementary hypotheses are all based on ITA, CTA or a mixture of both. The "Almon algorithm" for example, is derived from CTA (for a description of the method see, for example, Vollebregt and van Dalen, 2002) and the "Mixed-technology model" applies a mixture of CTA and ITA, according to the classification of the secondary production into subsidiary products or by-products, respectively (Jansen and ten Raa, 1990).

mix of products in its output. It should be stated that these implications only take place when input-output analysis is carried out; EUROSTAT (2002) emphasises this topic by stating: “input-output analysis with a product-by-product table based on the industry technology assumption implicitly requires the assumption of fixed market shares (but not in the process of compiling the matrix, as is sometimes argued).” (p. 228). The same applies to CTA. Then, whenever one chooses one or another technological assumption with the aim of performing input-output analysis, one should be aware of these implications and fully assume them. On this basis, we cannot agree with the argument used by Vollebregt and van Dalen (2002) to reject ITA: they refuse to consider ITA as a plausible assumption because it presumably “conflicts with the assumption of homogeneous production that is the basis for input-output analysis” (p. 3). The facts are: 1) the homogeneous production assumption, according to which every industry produces exactly one product and every product is produced by exactly one industry is a simplifying hypothesis used to apply the traditional Leontief input-output model; 2) on the contrary, the truth is that each industry produces more than one product and each product may be produced by more than one industry; 3) according to ITA, each product is, in fact, produced by one single, fixed input structure, given by a fixed linear combination of several industries’ technologies.

It has been referred to in section 3.2.1 that secondary production may be classified into: subsidiary products, technologically unrelated to the primary production, and by-products, which result from the application of the same technology as the primary product. It seems then, that ITA is best suited when secondary products have a by-product nature and CTA is more adequate to treat subsidiary production (Miller and Blair, 1985). Following this reasoning, some authors propose a mixed model, which consists of dividing the make matrix in two (excluding the primary production, in the diagonal): one with only subsidiary products and another with by-products, and then apply the best suited assumption to each of these sub-matrices. The problem, however, is that, in practice, this requires some additional information which is seldom available: the split of secondary production into subsidiary and by-products (ten Raa, et al., 1984).

Both ITA and CTA have known advantages and disadvantages. Guo et al. (2003) provide a comprehensive overview of the major contributions on this debate. The main critiques pointed out to CTA, by opposition to ITA, are:

- The fact that it doesn't allow the use of non-square Make and Use tables. The expressions deduced in section 3.4.3.1, according to which H^{-1} has to be computed, are proof of this drawback. On the contrary, ITA does not require the computation of any inverse, thus allowing the direct use of tables in which the number of products differs from the number of industries.
- It generates negative coefficients in the direct requirements matrix (as explained before). In ITA it never happens.
- According to de Mesnard (2004 and 2002), CTA is not logically compatible with a demand-driven input-output model. We will review this approach further on.

On the other hand, ITA is also criticized because it does not fulfill three of the four axioms established by ten Raa (ten Raa et al. 1984 and Jansen and ten Raa, 1990) as desirable properties of what these authors name matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$. Matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$ represents a product-by-product direct requirements matrix, i.e., it tells us the necessary amount of product input i in order to produce one unit of product output j[129]. The arguments $\mathbf{U}$ and $\mathbf{V}$ mean that the direct requirements matrix is based on the information given by the M&U tables (Jansen and ten Raa, 1990). According to these authors, matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$ should fulfil the following axioms: material balance, financial balance, price invariance and scale invariance. The first, material balance, states that the total intermediate use of each product must be equal in the product-by-product table and in the Use (product-by-industry) table. Financial balance establishes that, "for each commodity unit, revenue equals material cost plus value added" (Jansen and ten Raa, 1990, p. 217). Price invariance implies that matrix $\mathbf{A}(\mathbf{U},\mathbf{V})$ must remain the same, regardless of the base year price chosen to compute constant prices $\mathbf{U}$ and $\mathbf{V}$ tables.

[129] This type of matrix is also named technical coefficients matrix and is usually labeled as matrix $\mathbf{A}$ with elements a_{ij}. Such technical coefficients matrix was already presented in the first Chapter of this Dissertation, when developing the traditional symmetric input-output model.

Finally, scale invariance requires that, whenever all inputs and outputs of some industry are multiplied by the same scaling factor, the $\mathbf{A}(\mathbf{U},\mathbf{V})$ coefficients remain the same. In these two papers, these authors demonstrate that CTA is compatible with all four axioms, while ITA only fulfills the first one[130].

Several solutions have been suggested in order to overcome the shortcomings of CTA. The problem of rectangular tables is simply solved by making the tables square; this is done by merging "rows into aggregates with products that are alike" (Vollebregt and van Dalen, 2002, p. 15). Some information is obviously lost in this process, eliminating, to some extent, one of the advantages of the rectangular model: the detail on production provided in the Make matrix.

In what concerns the negative entries, the case is more problematic. Firstly, there is no consensus as to the causes which originate these negative flows. Yet, the following sources are frequently pointed out: 1) the assumption underlying CTA is incorrect; this happens, for example, whenever secondary production is of by-product nature; 2) a high level of aggregation is used in product classification, such that CTA becomes "there is a single technology to each group of products" and not, as originally suggested, "there is a single technology to each product"; given that heterogeneity is unavoidable, due to the need for aggregation in the M&U data, it is recommended to use the most disaggregate version available (EUROSTAT, 2002); 3) errors in the supply and use tables (Vollebregt and van Dalen, 2002); this last source of errors is evidently more difficult to overcome, unless the original M&U tables are changed. The correction of all these causes of errors requires "human expert knowledge about the [supply and use] tables" (Vollebregt and van Dalen, 2002, p.18). Several "tricks" are enumerated in these authors' paper, as well as in the Input-output Manual (EUROSTAT, 2002) in order to avoid the negatives in the $A(U,V)$ matrix. These comprise: industry merging, changing the main producer and creating new products among others. The usual procedure consists of: 1) applying CTA to the original M&U tables; 2) analyzing the results and identifying the causes of the negatives; 3) Making changes to the original M&U tables; 4) applying CTA again; 5)

130 All demonstrations can be found in the referred papers.

repeating the process until only small negatives remain; 6) setting those small negatives to zero, and then, applying RAS to balance column and row totals[131]. This procedure has two main drawbacks: first, it requires a lot of manual work and an inside knowledge of M&U tables; secondly, it modifies the original M&U tables, making these inconsistent with the final symmetric input-output table (this will be consistent only with the adjusted M&U tables) (EUROSTAT, 2002).

In a recent paper, Bohlin and Widell (2006) propose a different model to create symmetric input-output tables, based on ITA and CTA. They demonstrate that "both the ITA and CTA can be transformed into problems of minimizing the variance of a variable b_{ijk}, which is defined as the quantity of commodity *i* that is used for producing one unit of commodity *j* in industry *k*." (p. 206). The reasoning which underlies this minimizing approach is as follows. In respect to ITA, the assumption states that each industry used the same input structure for all the products it produces; then, the previously defined coefficient b_{ijk}, for a certain input *i*, will be the same for all outputs *j* of industry *k*. This means that all the coefficients are equal to their mean: $b_{ijk} = \overline{b}_{ik}$, or, in other words, they have a null variance referring to *j*. The authors propose to relax this equality and instead minimize the variance of b_{ik}. Conversely, the CTA states that each product uses the same mix of inputs irrespective of the industry in which it is produced. Using coefficient b_{ijk}, means that "the *b*-coefficients for a specific input in the production of a specific output are the same in all industries, and therefore the same as the economy-wide technical coefficient, a_{ij}" (Bohlin and Widell (2006), p.208). This implies that $b_{ijk} = \overline{b}_{ij} = a_{ij}$. Once again, the approach used in this paper is to minimize the variance of b_{ij}, instead of setting it equal to zero. Additionally, it must always hold that $u_{ik} = \sum_j b_{ijk} v_{jk}$, i.e., the amount of input *i* consumed by industry *k* must be equal to the sum of the amounts of input *i* consumed in the production of all the outputs of industry *k* (here v_{jk} stands for the

[131] If the Almon algorithm is applied instead of CTA, RAS will be dispensable, since the algorithm is applied in such a way that no negatives occur – they are eliminated during the running of the algorithm.

quantity of product *j* produced in industry *k*). Using these assumptions and constraints, the method proposed by these authors is expressed by:

$$Min \quad \left(\varpi \sum_k \mathrm{var}(b_{ik}) + \mu \sum_j \mathrm{var}(b_{ij}) \right)$$
$$S.T.$$
$$b_{ijk} \geq 0 \quad and \quad u_{ik} = \sum_j b_{ijk} v_{jk}$$

(3. 37)

This model considers the use of a combination of ITA and CTA, with weights ϖ and μ, expressing the relative importance of each assumption. Obviously, when we set $\omega = 0$, we fall on the case of CTA; when $\mu = 0$, we fall on ITA[132]. Another advantage is the constraint on non-null *b*-coefficients, which avoids the generation of negative coefficients caused by CTA. The other limitation of CTA – the incapacity of dealing with rectangular tables – is also solved by this approach, since no inverse matrix needs to be computed.

In two recent papers, de Mesnard (2002) and de Mesnard (2004), this author uses an economic-circuit approach, to show that CTA suffers from a lack of logic when applied to a demand-driven input-output model, being adequate only to supply-driven models. In demand-driven models, the multiplier effect starts with an impact on demand: using the author's approach, let's consider an increase in final demand for product *i*. This will force an increase in the production of several industries that produce that product. According to ITA, this link is given by matrix *S*, which indicates, through its market shares, what the increase in the production of each industry is. This additional industry output will create the necessity for new products to be used as inputs, and the process is repeated in a circular manner, which is precisely the multiplier effect. Now, if CTA is used, this circuit will be broken. This model cannot establish the link between the initial product increase in final demand and the increase in the industries' output. If production of product *i*

[132] A clear disadvantage of this model is the fact that weights ϖ and μ are both unknown. Besides, the existence of the non-negativity restriction can prevent the objective function to get the value zero, making the results different from the ones obtained through the use of CTA or ITA in their pure form.

increase, what will the augment be in the output of each industry? Matrix H tells us how each industry's output is composed; thus, it answers to the following question: "When the output of an industry increases, what is the augment of each product's output?", which is a different question. Thus, the economic circuit is broken. This lack of economical logic leads the author to reject CTA in demand-driven models; he proves that it is only suited for Ghosh or supply-driven models.

In spite of the theoretical relevance of the previous debate concerning ITA versus CTA, the practical consequence of the choice between these two assumptions (or using a combination of both) depends on the weight and type of the production located outside the main diagonal in the Make matrix (Koronczi, 2004). On the one hand, it is quite obvious that the more diffuse the Make matrix is, the larger the difference will be between symmetric input-output tables (and the corresponding multipliers) calculated by ITA and by CTA (Guo, *et al.* 2003). In this context, if we look at the Portuguese Make and Use tables and take, for example, the current prices tables for the year 2002, with 30 industries and 30 products, we can conclude that the secondary production is around 5% of the total industries' output. Therefore, the choice between ITA and CTA may not have such great relevance. This will be empirically examined in the following sections, in which multipliers obtained from ITA and CTA will be compared. On the other hand, as was previously referred to, the nature of the production outside the main diagonal is important when it comes to choosing between ITA and CTA. More precisely, ITA is more suitable for situations in which this production is mainly comprised of by-products and CTA is more adequate when dealing mainly with subsidiary products. Then, considering the partial refining done by the National Accounts when classifying industries according to the notion of KAU, the result is that the off-diagonal values of production are mainly by-products and residual subsidiary products. Thus, it seems that ITA is a more adequate assumption to use in Portugal and other countries that follow the same procedure (recommended by SNA and ESA). Yet, the choice between one and the other technological assumption is not essential in this essay, since we will compare the rectangular with the symmetric model under both assumptions.

3.5 Practical application.

In this section, it will be shown that the direct modelling of the starting table (rectangular M&U matrices, with total use flows and at *pp*), is exactly equivalent to the modelling of a domestic flow symmetric table (at *bp*), derived from the starting table, using similar assumptions. To do so, we will begin by presenting the multipliers obtained through the direct modelling of the rectangular table, making use of the hypotheses explained in the previous section. Then, we will explain how product-by-product and industry-by-industry symmetric tables can be obtained from the starting table, and present the correspondent inverse matrices. Finally, we will justify the fact that these two alternative procedures originate the same result, showing the equivalence that exists between the direct requirements matrices in both the rectangular and the symmetric model.

3.5.1 Calculation of different multipliers on the basis of the rectangular model, using the Portuguese M&U table.

In order to demonstrate the application of the rectangular model inverse matrices obtained in sections 3.4.3.1 and 3.4.3.2, a practical example is used. The basic data are the Portuguese Make and Use tables for the year 2002, at current prices, provided by INE[133]. The level of aggregation contains 30 products and 30 industries. However, in the CTA applications, this had to be reduced to 29 products and 29 industries. The reason for this supplementary aggregation was the fact that industry "CA - Mining and quarrying of energy producing materials" had no production, and consequently, no intermediate consumption in Portugal. This caused the $\mathbf{H}$ matrix to have a null determinant and, therefore, to be non-invertible, preventing the application of CTA (see the expressions deduced in sections 3.4.3.1 and 3.4.3.2). On the other hand, one could not opt for eliminating only industry CA and leave the imported products included in group CA, since CTA does not admit a different number of industries and products (this has been

[133] This is the Portuguese Statistics National Institute. We are thankful to INE, for its kindness in providing the author with the Make table for the reference year. The remaining information is available at the INE's official website: www.ine.pt.

previously referred to as one of the disadvantages of this assumption). Thus, the option was to merge industry CA with industry “CB – Mining and quarrying except energy producing materials and product”, as well as the correspondent products. The resulting Make and Use matrix, used as a basis for this empirical application, is presented in Annex A.3.1[134].

We begin by presenting the inverse matrices and correspondent multipliers obtained when ITA is the underlying assumption. The results of partitioned matrix inversion ($(\mathbf{I}-\mathbf{D})^{-1}$) are displayed in Annex A.3.2[135].

The upper left-hand block of this inverse matrix corresponds to $(\mathbf{I}-\mathbf{QS})^{-1}$ and it computes the impact of changes in $\mathbf{y}^{\mathbf{pp}}$ over $\mathbf{p}^{\mathbf{pp}}$. For example, value 0,0217, located at [EE, DJ] in this product-by-product block has the following meaning: when final demand for “DJ – Basic metals and fabricated metal products” valuated at p.p. suffers a unitary increase, the direct and indirect extra demand (at p.p.) for product “EE - Electricity, gas and water supply” increases 0,0217 units. This increase also includes the increase for imported “EE” products, since the effect evaluated here is on $\mathbf{p}^{\mathbf{pp}}$. Looking at the column sum of this block, we can see that the total supply of products (domestic and imported) has to increase 1,6033 in order to satisfy the direct and indirect needs created by the additional demand of “DJ” products. If we want to obtain the effect of demand for domestic products on the domestic supply of products, we may apply equation (3. 21), from which we obtain the impact of $(\mathbf{y}^{\mathrm{N}})^{\mathbf{bp}}$ on $\mathbf{v}^{\mathbf{bp}}$. The matrix derived from this equation is displayed in Annex A.3.3. This matrix is the one which should correspond to the inverse matrix derived from a domestic flow product-by-product symmetric table (valuated at *bp*). It can, therefore, be seen that a unitary increase in the demand for “DJ” domestically produced products creates direct and indirect extra needs in the production of all the domestic production by the amount of 1,7240.

134 All the Annexes referred in this Chapter are made available as Excel files at the CD-rom attached to this Dissertation.
135 For a better understanding of all the results displayed, different colors are used to distinguish industries from products: industries are in light green and products in light yellow.

The lower left-hand block of $(\mathbf{I}-\mathbf{D})^{-1}$ corresponds to $\mathbf{S}(\mathbf{I}-\mathbf{QS})^{-1}$ and it measures the impact on industry's domestic supply ($\mathbf{g}^{\mathbf{bp}}$) caused by changes in final demand for products ($\mathbf{y}^{\mathbf{pp}}$). From this matrix we can say, for instance, that a unitary change in final demand for "DJ" products creates an amount of 0,9327 direct and indirect additional demand for the output of all industries in the economy. It should be stressed that the total multiplier is less than one, because part of the effect goes into imported products, which are not accounted for in $\mathbf{g}^{\mathbf{bp}}$.

The lower right-hand block of $(\mathbf{I}-\mathbf{D})^{-1}$ quantifies the industry-by-industry relationships. It corresponds to $(\mathbf{I}-\mathbf{SQ})^{-1}$. From this matrix, one can evaluate the effect in each industry and in the domestic economy-wide caused by changes in the demand directed at some industry. For example, if the demand directed to the output of industry "DJ" increases unitarily, this same industry will have to increase 1,2983 (through direct and indirect effects) and the whole economy will be increased by 1,7246. As referred to before, the values of this matrix should be equal to the values of the inverse matrix derived from a domestic flow industry-by-industry symmetric table (valuated at *bp*), constructed from the rectangular table, using similar hypotheses.

As expected, other matrices can be computed according to the equations derived in section 3.4.3.1. Yet, for this essay's purpose, the relevant matrices are the product-by-product and the industry-by-industry total requirements matrix, since these will be further compared with the corresponding symmetric total requirements matrices.

The same type of matrices can be derived using CTA as the technology assumption. The partitioned inverse $(\mathbf{I}-\mathbf{E})^{-1}$, shown in Annex A.3.4, allows us to extract different types of total requirements matrices. The upper left-hand block corresponds to $\left[\mathbf{I}-\mathbf{QH}^{-1}(\mathbf{I}-\hat{\mathbf{c}})(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}})\right]^{-1}$ and expresses the effect of $\mathbf{y}^{\mathbf{pp}}$ on $\mathbf{p}^{\mathbf{pp}}$. From this, we can derive the total requirements matrix that relates $(\mathbf{y}^{\mathbf{N}})^{\mathbf{bp}}$ with $\mathbf{v}^{\mathbf{bp}}$ (applying equation (3.

26)). The resultant matrix is exhibited in Annex A.3.5. This is the matrix which corresponds to the inverse matrix derived from a product-by-product symmetric input-output table, constructed under similar hypotheses. We can see, for example, that a unitary increase in the demand for "DJ" domestic products creates direct and indirect extra needs in the production of all the domestic production by the amount of 1,7286. The lower right-hand block comprises the industry-by-industry total requirements matrix – it corresponds to the inverse matrix of a symmetric industry-by-industry table.

3.5.2 Obtaining the symmetric input-output table (SIOT) from the rectangular one.

In order to allow for comparison between the multipliers obtained directly from the rectangular table and the ones obtained from the constructed symmetric tables, the methodology used to derive the SIOTs followed the exact hypotheses established to model the rectangular tables. The objective was to compute the subsequent SIOTs: a) Product-by-product domestic flow symmetric table valuated at b.p.; b) Industry-by-industry domestic flow symmetric table valuated at b.p.;

The method involved three stages:

1) Computing use matrices for margins and for taxes (less subsidies), in order to subtract them from the purchasers' prices Use table and obtain the basic prices Use table. This was done applying the proportionality assumptions established in equations (3. 9) and (3. 10).

2) Computing the use matrix of imported products, in order to subtract it from the basic prices Use table and thus obtain the domestic flow basic prices Use table. To do this, the proportionality hypotheses expressed in (3. 6) and (3. 7) were used. In practice, most of the countries that construct an official import matrix also support their work in this kind of hypothesis (OECD, 2000).

3) Obtaining the product-by-product and industry-by-industry symmetric tables, using both the alternative technology assumptions.

The result of stage 1) in addition to stage 2), this is, the domestic flow basic prices Use tables, can be observed in Annexes A.3.6 and A.3.7[136]. From these, 4 SIOTs were derived: product by product using ITA, industry by industry using ITA, product by product using CTA and industry by industry using CTA.

3.5.2.1 Product-by-product symmetric tables.

Let's begin with the product-by-product table and consider the case of industry "AA – Agriculture, hunting and forestry" as an example. The objective is to convert the first column of the domestic flow basic prices Use table ($\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$), which gives us the amount of different products used by industry "AA", into the first column of the intermediate symmetric table ($\left(\mathbf{Z}^{\mathrm{N}}\right)^{\mathbf{bp}}$), which will indicate the amount of different products used to produce products "AA". Firstly, this will be done by using ITA, and in a second step by applying CTA. By looking at matrix $\mathbf{V}^{\mathbf{bp}}$ (first row) we can see that industry "AA" produces several products: "AA", "DA", "DJ", "FF", "GG" and "KK". Then, according to ITA, the technology of industry "AA", implicit in the correspondent column of the technical coefficients matrix $\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$, will be used to produce all these products, primary as well as secondary.

The intermediate consumptions associated with the secondary products of industry "AA", according to ITA, are displayed in columns 2 to 6 of Table 3.1. Each of these columns are obtained by multiplying the first column of matrix $\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$ (technology of industry "AA") by the correspondent production of industry "AA", given in the first row of matrix *V*. Product "AA" is, however, also produced by other industries. In this particular case,

[136] In principle, there should be only one domestic flow b.p. Use table, since the initial stages have nothing to do with the technology assumption. However, as explained before, a merge between two industries and two products in the original M&U tables had to be done in order to apply CTA. This led to different starting tables in the third stage.

product "AA" is produced only by its main producer and in industry "LL – Public administration and defence; compulsory social security" (this information is extracted from the first column of matrix *V*). This intermediate consumption, to be transferred to product "AA", is given in column 7 of Table 3.1. This column is obtained by multiplying the "LL" column of matrix $\left(\mathbf{Q}^{N}\right)^{\mathbf{bp}}$ by the production of product "AA" in industry "LL" (5 units). The result of subtracting the intermediate consumptions relative to secondary products, and summing up the intermediate consumption associated to the production of "AA" in other industries, is the first column of matrix $\left(\mathbf{Z}^{N}\right)^{\mathbf{bp}}$ (in the column (8) of Table 3.1).

Table 3. 1 – Calculation of the first column of matrix $\left(\mathbf{Z}^{N}\right)^{\mathbf{bp}}$ under ITA.

	(1) Ind. AA		*(2)* DA		*(3)* DJ		*(4)* FF		*(5)* GG		*(6)* KK		*(7)* AA (LL)		*(8)* Prod. AA
AA	661	-	38,8	-	0,1	-	1,9	-	1,5	-	2,6	+	0,0	=	**615,8**
BB	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
CA	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
CB	1		0,0		0,0		0,0		0,0		0,0		0,0		**0,7**
DA	614		36,0		0,1		1,8		1,4		2,5		0,0		**571,8**
DB	18		1,1		0,0		0,1		0,0		0,1		0,0		**17,0**
DC	1		0,0		0,0		0,0		0,0		0,0		0,0		**0,6**
DD	14		0,8		0,0		0,0		0,0		0,1		0,0		**13,1**
DE	17		1,0		0,0		0,0		0,0		0,1		0,0		**15,5**
DF	45		2,7		0,0		0,1		0,1		0,2		0,0		**42,1**
DG	86		5,1		0,0		0,2		0,2		0,3		0,0		**80,4**
DH	4		0,2		0,0		0,0		0,0		0,0		0,0		**3,7**
DI	51		3,0		0,0		0,1		0,1		0,2		0,0		**47,6**
DJ	15		0,9		0,0		0,0		0,0		0,1		0,0		**13,6**
DK	7		0,4		0,0		0,0		0,0		0,0		0,0		**6,2**
DL	1		0,0		0,0		0,0		0,0		0,0		0,0		**0,8**
DM	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
DN	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
EE	98		5,8		0,0		0,3		0,2		0,4		0,1		**91,4**
FF	71		4,2		0,0		0,2		0,2		0,3		0,0		**66,1**
GG	333		19,6		0,0		1,0		0,8		1,3		0,1		**310,6**
HH	19		1,1		0,0		0,1		0,0		0,1		0,1		**17,5**
II	49		2,9		0,0		0,1		0,1		0,2		0,1		**45,4**
JJ	158		9,3		0,0		0,5		0,4		0,6		0,0		**147,7**
KK	180		10,5		0,0		0,5		0,4		0,7		0,3		**167,7**
LL	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
MM	1		0,1		0,0		0,0		0,0		0,0		0,0		**0,9**
NN	22		1,3		0,0		0,1		0,1		0,1		0,0		**20,4**
OO	4		0,2		0,0		0,0		0,0		0,0		0,0		**3,4**
PP	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**

This may, obviously, be done in a more straightforward manner, using a specific matrix product. Let us begin by asking: "Which industries have to be considered in the production of product "AA"? The answer is in the first column of matrix *V*: industries "AA" and "LL". Thus, using ITA, two technologies are implicit in the production of product "AA". This means that the columns to be considered in $(\mathbf{Q}^{\mathrm{N}})^{\mathbf{bp}}$ are "AA" and "LL". This, however, corresponds to multiplying matrix $(\mathbf{Q}^{\mathrm{N}})^{\mathbf{bp}}$ by the first column of matrix *V* (which has non-null elements in rows "AA" and "LL"). Then, the first column of $(\mathbf{Z}^{\mathrm{N}})^{\mathbf{bp}}$ can be obtained by multiplying $(\mathbf{Q}^{\mathrm{N}})^{\mathbf{bp}}$ by the first column of matrix *V*. This operation does, in fact, make sense, given the meaning of these two matrices: for example, the first row of $(\mathbf{Q}^{\mathrm{N}})^{\mathbf{bp}}$ gives us the amount of product "AA" necessary to produce one unit of output of each industry *j*; the first column of *V* gives us the distribution of product "AA" throughout the several industries; then, by multiplying one by another, we obtain the amount of product "AA" used in the production of product "AA", by all the industries that produce it, which is precisely the first element of $(\mathbf{Z}^{\mathrm{N}})^{\mathbf{bp}}$. Generalizing, matrix $(\mathbf{Z}^{\mathrm{N}})^{\mathbf{bp}}$ can be obtained through:

$$(\mathbf{Z}^{\mathrm{N}})^{\mathbf{bp}} = (\mathbf{Q}^{\mathrm{N}})^{\mathbf{bp}}\,\mathbf{V} \tag{3. 38}$$

The intermediate consumption part of the symmetric product by product domestic flow table (at basic prices) obtained using ITA is presented in Annex A.3.8.

If the other technology assumption, CTA, is considered, the reasoning is very similar, except for the fact that in the transferences of the secondary production, the technology used is the one which is associated with the product being transferred. Then, the intermediate consumptions to be subtracted from and added to the first column of $\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$ are given in Table 3.2[137].

Table 3. 2 – Calculation of the first column of matrix $\left(\mathbf{Z}^{\mathrm{N}}\right)^{\mathbf{bp}}$ under CTA.

	(1) Ind. AA		*(2)* DA		*(3)* DJ		*(4)* FF		*(5)* GG		*(6)* KK		*(7)* AA (LL)		*(8)* Prod. AA
AA	661	-	85,4	-	0,0	-	0,0	-	0,0	-	0,0	+	0,5	=	**575,8**
BB	0		0,5		0,0		0,0		0,0		0,0		0,0		**-0,5**
CACB	0		0,1		0,0		0,0		0,0		0,0		0,0		**0,3**
DA	614		48,1		0,0		0,0		0,0		0,0		0,4		**565,9**
DB	18		0,1		0,0		0,0		0,0		0,0		0,0		**18,2**
DC	1		0,0		0,0		0,0		0,0		0,0		0,0		**0,6**
DD	14		0,9		0,0		0,4		0,0		0,0		0,0		**12,7**
DE	17		8,9		0,0		0,0		0,2		0,4		0,0		**7,1**
DF	45		0,7		0,0		0,2		0,1		0,0		0,0		**44,3**
DG	86		1,1		0,0		0,1		0,1		0,0		0,1		**85,0**
DH	4		4,1		0,0		0,1		0,1		0,1		0,0		**-0,4**
DI	51		4,2		0,0		1,9		0,0		0,0		0,0		**44,9**
DJ	15		3,8		0,2		0,7		0,2		0,1		0,0		**9,6**
DK	7		0,4		0,0		0,1		0,0		0,0		0,0		**6,1**
DL	1		0,0		0,0		0,1		0,1		0,1		0,0		**0,5**
DM	0		0,0		0,0		0,0		0,1		0,0		0,0		**-0,1**
DN	0		0,0		0,0		0,1		0,0		0,0		0,0		**-0,1**
EE	98		3,6		0,0		0,1		0,2		0,1		0,1		**94,1**
FF	71		2,5		0,0		5,2		0,2		0,9		0,1		**62,2**
GG	333		5,3		0,0		0,5		1,7		1,2		0,2		**324,7**
HH	19		1,5		0,0		0,0		0,3		0,2		0,0		**16,8**
II	49		5,4		0,0		0,1		1,0		0,5		0,0		**41,6**
JJ	158		7,1		0,0		0,4		0,6		1,9		0,1		**148,6**
KK	180		21,7		0,0		0,5		2,3		4,6		0,1		**150,5**
LL	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**
MM	1		0,4		0,0		0,0		0,0		0,0		0,0		**0,6**
NN	22		0,3		0,0		0,0		0,0		0,0		0,0		**21,7**
OO	4		0,4		0,0		0,0		0,1		0,4		0,0		**2,7**
PP	0		0,0		0,0		0,0		0,0		0,0		0,0		**0,0**

As explained before, this assumption generates negative flows. This example clarifies why this happens: for example, industry "AA" uses no product "BB" inputs. However, one of the secondary productions of industry "AA" is product "DA", of which the main producer uses some "BB" inputs. Thus, the result is a negative flow which has no economic sense. Yet, to allow for comparison between the multipliers achieved by SIOTs and the ones obtained directly from the M&U tables, the original M&U tables should not

[137] This table is presented with an illustrative purpose, for better comprehension of the CTA procedure. However, it involves a problem of simultaneity which makes the result of column (8) a little different from the result of matrix calculation. The fact is that the technologies underlying columns (2) to (6) are not really the technologies of products "DA", "DJ",... and "KK"; they are, instead, the technologies of their main producing industries, since these columns have not yet been converted into the symmetric form. This problem of simultaneity can only be solved making use of the product matrix in equation (3.42).

be changed in any way. This means that the negative values generated by CTA were not subject in this essay to any kind of correction.

Let's now derive the general formula to obtain $\left(\mathbf{Z}^{\mathbf{N}}\right)^{\mathbf{bp}}$. The supposition underlying CTA, is that the amount of product i used in the production of product j is the same in all industries that produce j. Therefore, taking the first element of matrix $\left(\mathbf{U}^{\mathbf{N}}\right)^{\mathbf{bp}}$ as an example, we can state that:

$$\left(u_{11}^{N}\right)^{bp} = a_{11}^{N} v'_{11} + a_{12}^{N} v'_{21} + \cdots + a_{1\,30}^{N} v'_{30\,1}$$

(3. 39)

In other words, this means that:

The amount of product 1 used to produce output of industry 1 =

Amount of (domestically produced) product 1 used to produce one unit of product 1 * amount of product 1 produced by industry 1 +

Amount of (domestically produced) product 1 used to produce one unit of product 2 * amount of product 2 produced by industry 1 +

…+

Amount of (domestically produced) product 1 used to produce one unit of product 30 * amount of product 30 produced by industry 1

We may write, in matrix terms:

$$\left(\mathbf{U}^{\mathbf{N}}\right)^{\mathbf{bp}} = \mathbf{A}^{\mathbf{N}}\mathbf{V}' \Leftrightarrow \mathbf{A}^{\mathbf{N}} = \left(\mathbf{U}^{\mathbf{N}}\right)^{\mathbf{bp}}\left(\mathbf{V}'\right)^{-1}$$

(3. 40)

From matrix $\mathbf{A}^{\mathbf{N}}$ (domestic input coefficient matrix in the product by product symmetric model), we can compute matrix $\left(\mathbf{Z}^{\mathbf{N}}\right)^{\mathbf{bp}}$ (domestic flow product by product intermediate consumption matrix) simply by multiplying by the corresponding values of domestic production:

$$\left(\mathbf{Z}^{\mathbf{N}}\right)_{\mathbf{CTA}}^{\mathbf{bp}} = \mathbf{A}^{\mathbf{N}}\hat{\mathbf{v}}^{\mathbf{bp}}$$

(3. 41)

The result is presented in Annex A.3.9.

3.5.2.2 Industry-by-industry symmetric tables.

The objective now is to obtain matrix $\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}}$, of which elements *ij* depict intermediate consumption of industry *i*'s output to produce industry *j*'s output. Taking, as an example, the first row of $\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}}$, it gives us the amount of industry "AA"'s output consumed in the production process of the several industries in the economy. How can we derive this row from the known matrices $\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$ and $\mathbf{V}$? Firstly, we have to answer the following question: "Which products are produced by industry "AA"?". As seen before, these are: "AA", "DA", "DJ", "FF", "GG" and "KK". The intermediate use of these products is given in matrix $\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$. Extracting the rows corresponding to these products, we obtain:

Table 3. 3 – Intermediate use of products produced in industry "AA".

	AA	BB	CA	CB	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL	DM	DN	EE	FF	GG	HH	II	JJ	KK	LL	MM	NN	OO	PP
AA	661	0	0	0	2538	136	1	474	154	1	8	7	0	1	0	0	0	2	1	0	62	263	1	0	24	9	2	88	6	0
DA	614	2	0	0	1431	5	41	0	15	0	16	1	0	1	0	0	0	2	0	1	28	1519	1	0	6	36	18	246	1	0
DJ	15	1	0	3	112	23	21	21	15	1	31	58	80	1303	260	281	311	139	12	999	286	41	15	0	69	18	4	14	14	0
FF	71	2	0	12	73	43	17	24	37	17	28	8	69	118	44	24	17	19	86	7026	287	40	335	52	887	82	36	20	121	0
GG	333	76	0	76	158	158	23	53	111	12	111	35	205	76	58	123	70	58	70	660	3098	351	1572	53	1134	368	94	304	158	0
KK	180	12	0	50	646	277	63	49	222	2	312	73	146	158	95	242	156	68	347	633	4213	585	1131	2126	4491	906	425	684	891	0

The aim is to transform this into the intermediate use of the output of industry "AA". To do so, we have to assess the percentage in which industry "AA" contributes to the production of each one of these products. This is shown in the first row of matrix $\mathbf{S}^{\mathbf{N}}$ (matrix with generic elements $s_{ij}^{N} = \dfrac{v_{ij}^{bp}}{v_{j}^{bp}}$: industry *i*'s market share in product *j*'s domestic supply). Then, by multiplying the first row of matrix $\mathbf{S}^{\mathbf{N}}$ by matrix $\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$, we obtain the first row of matrix $\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}}$. Generalizing, matrix $\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}}$ can be obtained as follows:

$$\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}} = \mathbf{S}^{\mathbf{N}}\left(\mathbf{U}^{\mathbf{N}}\right)^{\mathbf{bp}}$$

(3. 42)

The result is displayed in Annex A.3.10.

The application of CTA to operate the transformation of the use of products, depicted in Table 3.4, into the use of industry's output, implies the use of $\mathbf{H}^{-1}$, instead of $\mathbf{S}$. Thus, we may write:

$$\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)_{\mathbf{CTA}}^{\mathbf{bp}} = \mathbf{H}^{-1}\left(\mathbf{U}^{\mathbf{N}}\right)^{\mathbf{bp}}$$

(3. 43)

The result is displayed in Annex A.3.11.

3.5.3 Computation of the symmetric input-output table's Leontief inverse and the correspondent multipliers.

It is possible to obtain the Leontief inverse matrix and the corresponding multipliers from the previous $\mathbf{Z}$ matrices. Firstly, one must compute the matrices of domestic input coefficients. In the case of product-by-product symmetric tables, this is done simply by dividing the intermediate flow table by the total output of the corresponding product:

$$a_{ij}^{N} = \frac{\left(z_{ij}^{N}\right)^{bp}}{v_{j}^{bp}}$$

(3. 44)

In industry-by-industry symmetric tables, the intermediate consumption flow is divided by the total industry's output, as follows:

$$\left(a_{ij}^{N}\right)_{I} = \frac{\left(z_{ij}^{N}\right)_{I}^{bp}}{g_{j}^{bp}}$$

(3. 45)

Taking matrix $\mathbf{A}^{\mathbf{N}}$ as the domestic input coefficients matrix, the total requirements or Leontief inverse matrix is given by the well known $\left(\mathbf{I}-\mathbf{A}^{\mathbf{N}}\right)^{-1}$[138]. Inverse matrices derived from the symmetric input-output tables are shown in Annexes A.3.12 (product-by-product, based on ITA), A.3.13 (product-by-product, based on CTA), A.3.14 (industry-by-industry, based on ITA) and A.3.15 (industry-by-industry, based on CTA).

It can be seen that these matrices are exactly the same as the correspondent inverse matrices derived directly from the rectangular model, as intended to be demonstrated (these matrices may be observed in the lower right-hand block of the partitioned inverses for industry-by-industry relationship – in Annexes A.3.2 (for ITA) and A.3.4 (for CTA) and, for product-by-product relationships, in Annexes A.3.3 (ITA) and A.3.5 (CTA). The output multipliers are, consequently, also the same.

In spite of providing the same range of total requirements matrices, with identical meanings, ITA and CTA obviously originate different multipliers, since the technology assumption is diverse. In this context, it is worth investigating how different these values are, i.e., what is the effective consequence on the values of the technical coefficients and of the correspondent multipliers of choosing one or another technology assumption. Bohlin and Widell (2006) made a similar investigation, using Swedish Make and Use tables. Their results will be compared to ours. In order to evaluate the difference between matrices generated by different technology assumptions, the following measure will be used (following one of the measures used in the above mentioned paper):

[138] This has been proven in Chapter 1, where the symmetric input-output model was developed.

$$\frac{\sum_i \sum_j \left| a_{ij}^{N\,CTA} - a_{ij}^{N\,ITA} \right|}{\sum_i \sum_j 0{,}5\left(a_{ij}^{N\,CTA} + a_{ij}^{N\,ITA} \right)}. \tag{3. 46}$$

This measure is used to assess the difference between the coefficients obtained from ITA and CTA and also to compare the corresponding elements from the inverse matrix. It relates the sum of the deviations, in absolute values, with the sum of the average coefficients. The following table sums up the result of this exercise:

Table 3. 4 – Differences in domestic input coefficients and in the inverse elements caused by the technology assumption (ITA vs. CTA).

	Domestic input coefficients (elements of matrix A^N)	Total requirement coefficients (elements of the inverse)
Product by Product	10,16%	3,67%
Industry by Industry	12,82%	4,97%

We can conclude that the choice of one or another technology assumption does not greatly influence the multipliers. This is consistent with the previously mentioned feature of the Portuguese tables: there is a small relative importance of secondary production. In the analogous investigation carried out by Bohlin and Widell (2006), the deviation between the Swedish technical coefficients matrices derived from ITA and CTA is 46%, which is much higher than the difference obtained herein[139]. It is, however, difficult to discover the reason behind these different results since: firstly, the Swedish tables were used in a more disaggregated manner (55 products and industries); secondly, as explained before, these authors did not use CTA and ITA in their pure form; a minimization approach, based on these assumptions, was used instead. In the case of CTA, for

[139] This difference is computed by applying equation (3. 37) in a recursive manner, considering first $\varpi = 0$ (falling into ITA) and then $\mu = 0$ (CTA).

example, the inclusion of a non-negativity restriction on the *b*-coefficientes (see equation (3. 37)), makes the results to be different from the pure CTA case.

3.5.4 Direct requirements matrices in the rectangular and in the symmetric model.

The use of the same hypotheses in the rectangular model development and in the construction of the symmetric tables makes the direct requirements matrices identical, thus leading to equal inverse matrices and multipliers. This has already been shown in the previous sections, computing the multipliers themselves, but it can also be demonstrated as well as follows.

From the previously derived symmetric matrices $\mathbf{Z}$, we may deduce the correspondent expressions for the domestic input coefficient matrices. Thus, from equation (3. 38), we obtain:

$$\begin{aligned}
&\left(\mathbf{Z}^{\mathrm{N}}\right)^{\mathbf{bp}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{V} \\
&\left(\mathbf{Z}^{\mathrm{N}}\right)^{\mathbf{bp}} \left(\hat{\mathbf{v}}^{\mathbf{bp}}\right)^{-1} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{V}\left(\hat{\mathbf{v}}^{\mathbf{bp}}\right)^{-1} \\
&\mathbf{A}^{\mathrm{N}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{S}^{\mathrm{N}} \Rightarrow \left(\mathbf{I} - \mathbf{A}^{\mathrm{N}}\right)^{-1} = (\mathbf{I} - \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{S}^{\mathrm{N}})^{-1}
\end{aligned} \tag{3. 47}$$

The same domestic input coefficient matrix may be derived from the rectangular table applying the same hypotheses used to construct the symmetric table. Referring back to equation (3. 12), we get:

$$
\begin{aligned}
&\mathbf{p}^{\mathbf{pp}} = \mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \mathbf{y}^{\mathbf{pp}} \\
&(\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}\mathbf{p}^{\mathbf{pp}} = (\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}\mathbf{Q}\mathbf{g}^{\mathbf{bp}} + (\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}\mathbf{y}^{\mathbf{pp}} \\
&\mathbf{v}^{\mathbf{bp}} = (\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}\mathbf{Q}\mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = (\mathbf{I}-\hat{\mathbf{c}})^{-1}\left(\mathbf{I}-\hat{\mathbf{f}}-\hat{\mathbf{n}}\right)^{-1}\mathbf{U}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1}\mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1}\mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{S}^{\mathrm{N}}\mathbf{v}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\left(\mathbf{I}-\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{S}^{\mathrm{N}}\right)\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{I}-\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{S}^{\mathrm{N}}\right)^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}
\end{aligned}
$$

(3. 48)

In conclusion, the correspondent product-by-product output multipliers are necessarily the same.

The same reasoning applies to the CTA product-by-product table. Taking equation (3. 40), this is equivalent to:

$$
\begin{aligned}
&\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}} = \left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}(\mathbf{V}')^{-1} \\
&\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}} = \left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)(\mathbf{V}')^{-1} \\
&\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)(\mathbf{V}')^{-1} \\
&\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\left[(\mathbf{V}')\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)\right]^{-1} \\
&\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{H}^{-1} \Rightarrow \left(\mathbf{I}-\mathbf{A}^{\mathbf{N}}_{\mathbf{CTA}}\right)^{-1} = (\mathbf{I}-\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}\mathbf{H}^{-1})^{-1}
\end{aligned}
$$

(3. 49)

The same inverse matrix is, in fact, obtained from the rectangular model. By using the deduction carried out in (3. 48), and applying CTA, it yields:

…

$$
\begin{aligned}
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{g}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{H}^{-1} \mathbf{v}^{\mathbf{bp}} + \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\left(\mathbf{I} - \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{H}^{-1}\right)\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\mathbf{v}^{\mathbf{bp}} = \left(\mathbf{I} - \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} \mathbf{H}^{-1}\right)^{-1} \left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}
\end{aligned}
\tag{3. 50}
$$

Concerning industry-by-industry symmetric tables, and taking the ITA-based one, we have:

$$
\begin{aligned}
&\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}} = \mathbf{S}^{\mathrm{N}} \left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}} \\
&\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)^{\mathbf{bp}} \left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1} = \mathbf{S}^{\mathrm{N}} \left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}} \left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1} \\
&\mathbf{A}_{\mathbf{I}}^{\mathbf{N}} = \mathbf{S}^{\mathrm{N}} \left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}
\end{aligned}
\tag{3. 51}
$$

From equation (3. 22): $\mathbf{g}^{\mathbf{bp}} = (\mathbf{I} - \mathbf{SQ})^{-1}\mathbf{S}\mathbf{y}^{\mathbf{pp}}$, we can verify that the ITA-based industry-by-industry technical coefficient matrix derived from the rectangular model is $\mathbf{SQ}$. The correspondent technical coefficient matrix derived from the symmetric model, implicit in (3. 51), is $\mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$. We, therefore, only need to demonstrate that $\mathbf{SQ} = \mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$, as follows:

$$
\begin{aligned}
&\mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} = \mathbf{V}\left(\hat{\mathbf{v}}^{\mathbf{bp}}\right)^{-1}\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1} \\
&\mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} = \mathbf{V}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{p}^{\mathbf{pp}}\right]^{-1}(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{U}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1} \\
&\mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} = \mathbf{V}\left(\mathbf{p}^{\mathbf{pp}}\right)^{-1}\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)^{-1}(\mathbf{I} - \hat{\mathbf{c}})^{-1}(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{Q} \\
&\mathbf{S}^{\mathrm{N}}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}} = \mathbf{SQ}
\end{aligned}
\tag{3. 52}
$$

Finally, from the CTA industry-by-industry intermediate symmetric table we can derive the correspondent $\mathbf{A}$ matrix:

$$\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)_{\mathbf{CTA}}^{\mathbf{bp}} = \mathbf{H}^{-1}\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}$$
$$\left(\mathbf{Z}_{\mathbf{I}}^{\mathbf{N}}\right)_{\mathbf{CTA}}^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1} = \mathbf{H}^{-1}\left(\mathbf{U}^{\mathrm{N}}\right)^{\mathbf{bp}}\left(\hat{\mathbf{g}}^{\mathbf{bp}}\right)^{-1}$$
$$\left(\mathbf{A}_{\mathbf{I}}^{\mathbf{N}}\right)_{\mathbf{CTA}} = \mathbf{H}^{-1}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$$

(3. 53)

Equation (3. 27): $\mathbf{g}^{\mathbf{bp}} = \left[\mathbf{I} - \mathbf{H}^{-1}\left[(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\right]\mathbf{Q}\right]^{-1}\mathbf{H}^{-1}\left(\mathbf{y}^{\mathrm{N}}\right)^{\mathbf{bp}}$, obtained before, includes matrix $\mathbf{H}^{-1}(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{Q}$ as the CTA-based industry-by-industry technical coefficient matrix derived from the rectangular model. The correspondent technical coefficient matrix derived from the symmetric model, expressed in (3. 53), is $\mathbf{H}^{-1}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$, which is precisely the same $\mathbf{A}$ matrix. In fact:

$$\mathbf{H}^{-1}(\mathbf{I} - \hat{\mathbf{c}})\left(\mathbf{I} - \hat{\mathbf{f}} - \hat{\mathbf{n}}\right)\mathbf{Q} = \mathbf{H}^{-1}\left(\mathbf{Q}^{\mathrm{N}}\right)^{\mathbf{bp}}$$

(3. 54)

3.6 Conclusions.

The main issue of the present essay fell upon input-output modelling when the starting matrix is a total-use rectangular table at purchasers' prices. Two alternative procedures have been analyzed, both theoretically and also through a practical application: 1) to convert the initial matrix into a domestic-flow symmetric table at basic prices and then implement the classical Leontief-type input-output model; 2) perform the direct modelling of the total-flow rectangular table at purchasers' prices. It has been demonstrated that, when the hypotheses used to make the table symmetric and to operate the conversion from total use to domestic use flows (and from purchasers' prices to basic prices) are also used in the direct modelling of the starting matrix, the obtained results are exactly the same.

The equivalence between the results of both alternative procedures has been attested through a numerical example as well as algebraically. The numerical example consisted in using the Portuguese M&U table as a starting point (which is a total-use rectangular table at purchasers' prices) and implementing the input-output model, applying both previously referred procedures. As intended to be demonstrated, the final equation of the model comprised the same inverse matrices (product-by-product and industry-by-industry), either by one procedure or by the other. It was also shown that the direct requirements matrices obtained by either alternative procedures are equivalent, which algebraically reinforces the argument that the two methods are indifferent (as long as the hypotheses are the same).

Furthermore, it has been argued that the direct use of the rectangular format has a considerable advantage over the use of symmetric tables: in the rectangular framework, the simple inversion of a partitioned matrix generates a set of four different inverse matrices; conversely, the symmetric table approach originates only one type of inverse matrix. We must choose from the very beginning, in principle, between a product-by-product or industry-by-industry matrix.

The development of the input-output model directly from the total-flow rectangular table at purchasers' prices, involved the use of proportionality hypotheses concerning imports, margins and taxes comprised in the intermediate and final use flows. Additionally, the model was developed in two versions – one using ITA and another using CTA – originating two different sets of results, which were subject to comparison. The proportionality and the technology hypotheses adopted are of course controversial. This doesn't however jeopardize the validity of the conclusions, given that the same hypotheses have been used either in the direct modelling of the starting matrix, or in the conversion of this matrix into a domestic-flow symmetric table at basic prices. Besides, in many cases, even the official organisms of statistics use these kinds of hypotheses (or similar procedures) when assembling symmetric tables. These hypotheses are sometimes complemented or substituted by the inclusion of direct information. For example, if a true import matrix is available, it is obviously better to use such information than to use the proportionality hypothesis, even though the assemblage of direct information involves high costs and, in many cases, originates only a marginal improvement in the results. For this reason, even in symmetric tables assembled by official entities like the Portuguese Department for prospective and planning (Martins, 2004), the applied procedures involved the use of almost no direct information. Yet, all the available direct information can also be incorporated in the rectangular model. As mentioned before, for instance the valuation matrix for VAT, can and should be used instead of the proportionality hypothesis; to do so, the mere subtraction of the known VAT values must be done prior to the rectangular model application.

In what pertains to the alternative technological hypotheses, even recognizing and assuming the advantages and disadvantages of each one, the choice of ITA or CTA doesn't seem to be extremely relevant in the case of the particular table which was used. In fact, the results obtained from either hypothesis are not very different. Besides, the equivalence of results between direct modelling and modelling after the conversion of the starting matrix has been demonstrated for both technological hypotheses. Still, bearing in mind the type of methodology used by National Accounts in the Make matrix assemblage, ITA seems to be the most adequate hypothesis. As explained before, the partial refining done by the National Accounts when classifying industries according to the notion of "kind-of-activity unit" (in Portugal and in other countries that follow the same procedure recommended by SNA and ESA), makes the values of production located outside the main diagonal of the Make matrix to be mainly by-products and some residual subsidiary products. In such a case, as explained in section 3.4.3.3, the hypothesis underlying ITA is more appropriate.

3.7 Notation

Variables:

z_{ji}^{N} - Amount of domestically produced input j used in the production of output i (symmetric model);

y_{j}^{N} - Final use of domestically produced product j (includes: final consumption, gross capital formation and exports);

y_{j} - Final use of product j (domestically produced + imported);

y^{m} - final use of imported products;

m_{j} - total imports of product j;

w_{i} - value added in the production of i;

u_{ji} - the amount of product j used as an input in the production of industry i's output (elements of the Use matrix – rectangular model);

v_{ij} - domestic production of product j by industry i (elements of the Make matrix – rectangular model);

v_{j} - domestic production of product j (sum of the columns of the Make matrix);

g_{i} - domestic production of industry i (sum of the rows of the Make matrix);

p_{j} - total supply of product j;

d_{j} - margins falling upon product j;

l_{j} - taxes (less subsidies) falling upon product j;

i - column vector appropriately dimensioned, composed by 1's.

superscript ^{bp} – basic prices;

superscript ^{pp} – purchasers' prices;

superscript ^{N} – related to domestically produced goods and services (domestic flows).

^ - diagonal matrix.

subscript $_{I}$ – industry by industry matrix / coefficient (by default: product by product);

subscript $_{CTA}$ – commodity-technology assumption (by default: industry-technology assumption);

Coefficients:

$a_{ji}^{N} = \dfrac{z_{ji}^{N}}{v_i}$ - domestic input coefficient (amount of domestic input *j* per unit of output *i*);

output *j*);

$q_{ji} = \dfrac{u_{ji}}{g_i}$ - Technical coefficient in the rectangular model (amount of product *j* used as input in the production of one unit of industry *i*'s output);

$q_{ji}^{N} = \dfrac{u_{ji}^{N}}{g_i}$ - domestic input coefficient in the rectangular model (amount of domestically produced product *j* used as input in the production of one unit of industry *i*'s output);.

$c_j = \dfrac{m_j}{p_j^{bp}}$ - imports coefficient (share of imports in *j*'s total supply valuated at basic prices);

$f_j = \dfrac{d_j}{p_j^{pp}}$ - margins coefficient;

$n_j = \dfrac{l_j}{p_j^{pp}}$ - the taxes (less subsidies) coefficient;

$s_{ij} = \dfrac{v_{ij}}{p_j}$ - industry *i*'s market share in product *j*'s total supply.

$s_{ij}^{N} = \dfrac{v_{ij}}{v_j}$ - industry *i*'s market share in product *j*'s domestic supply.

$h_{ij} = \dfrac{v_{ij}^{bp}}{g_i^{bp}}$ - percentage of industry *i*'s output that is attributable to output of product *j*.

Matrices and vectors:

$\mathbf{I}$ - identity matrix;

$\mathbf{y}$ - final use vector;

$\mathbf{y}^{\mathbf{N}}$ - domestic flow final use vector;

$\mathbf{U}$ - intermediate consumption matrix (rectangular model);

$\mathbf{V}$ - Make matrix.

$\mathbf{Q}$ - technical coefficient matrix;

$\mathbf{Q^N}$ - domestic input coefficient matrix (rectangular model).

$\mathbf{g}$ - vector of industries' internal production;

$\mathbf{p}$ - Vector of products' total supply;

$\hat{\mathbf{c}}$ - diagonal matrix with import coefficients on the main diagonal;

$\mathbf{S}$ - matrix of market shares s_{ij}; (industry-based technology assumption);

$\mathbf{S^N}$ - matrix of domestic market shares s_{ij}^N; (industry-based technology assumption);

$\mathbf{H}$ - matrix of elements h_{ij} (commodity-based technology assumption);

$\mathbf{D}$ - partitioned matrix in ITA;

$\mathbf{E}$ - partitioned matrix in CTA;

$\mathbf{Z^N}$ - domestic flow product by product intermediate consumption matrix, derived by ITA (symmetric model);

$(\mathbf{Z^N})_{\mathbf{CTA}}$ - domestic flow product by product intermediate consumption matrix, derived by CTA (symmetric model);

$(\mathbf{Z_I^N})$ - domestic flow industry by industry intermediate consumption matrix, derived by ITA (symmetric model);

$(\mathbf{Z_I^N})_{\mathbf{CTA}}$ - domestic flow industry by industry intermediate consumption matrix, derived by CTA (symmetric model);

$\mathbf{A^N}$ - domestic input coefficient matrix (symmetric model – product by product).

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CONCLUSIONS

The present dissertation had the following main objectives:

- To make a broad review of the state of knowledge regarding input-output modelling and input-output table construction at the regional level, highlighting the quantitative and qualitative disagreement between the data requirements implicit in the traditional input-output models and the usually available data.

- To study and test methodologies to overcome the above mentioned mismatch between data requirements and data availability, focusing on two specific issues:

    - Interregional trade indirect estimation, as a viable alternative to solve the common difficulty in regional table construction – the inexistence of survey-based interregional trade data.

    - Input-output modelling based on total use rectangular input-output tables – this implies the adaptation of the traditional input-output model to the format in which the input-output database (published on a regular basis for the national level) is currently provided.

The dissertation was structured according to these main objectives, leading to the elaboration of three distinct essays.

The literature review that was made on Chapter 1, allowed us to conclude that:

- The input-output framework continues to be intensively studied and empirically applied, both at the national and at the regional level. In spite of its limitations, related to the set of hypotheses underpinning the model, the input-output framework is indeed receiving an increasing interest by the research community, due to new issues associated with the globalization and integration of economies, such as: production fragmentation, pollution flows, technology flows, regional disparities, and so on.

- The application of the input-output model is directly related to the construction of the input-output matrix, or system of matrices, in the case of many-region models. The efforts are guided, on the one hand, towards the construction of regional input-output tables, involving a combination of survey and non-survey regionalization methods of the national counterparts. On the other hand, researchers have been and continue to be concerned in linking the individual tables through trade matrices, which involves the choice for the suited model among the alternative many-region input-output models and, when dealing with regions, the estimation of the necessary interregional trade matrices. Even when the simpler model is used to represent the interactions between the system of regions (using simplifying hypotheses), a certain amount of interregional trade data is always required to the implementation of such model. Namely, it requires a complete origin-destination matrix for each commodity, comprised of flow shipments from all possible origins to all possible destinations.
- The classical input-output models were proposed having in mind a specific data arrangement which is different from the one in which information is currently published. On the one hand, the traditional models are suited for the symmetric tables (with an industry-by-industry or product-by-product configuration) whilst the input-output data is currently provided on a Make and Use basis, implying the combination of both product and industry dimensions. On the other hand, in many cases, the national Use tables are composed by total use flows (those which include imported and regionally produced products), and there are no available import matrices *a priori*, which implies the use of techniques to estimate intra-regional flows from total use flows.

The main contribution of the first Chapter consisted in the systematization of the basic notions and techniques involved in regional input-output modelling and table construction, which was made in a very critical manner, questioning the leading stream about some specific topics.

The second Chapter was dedicated to the study and comparison of alternative methodologies for interregional trade estimation. The main motivations for this study were: 1) the fact that, in most countries, there are no survey-based data on interregional flows, which however are indispensable for the construction and implementation of regional input-output models; 2) the fact that there is little experience concerning the accuracy evaluation of each of the proposed methodologies for trade estimation and the comparison between them and 3) the fact that there is insufficient knowledge about the impact felt on the input-output model solution caused by the consideration of different interregional trade values (corresponding to different methodologies).

The theoretical and empirical testing of different trade estimating methodologies led to the following main conclusions:

- The theoretical solutions of the several models show a considerable similitude among each other. However, these models are quite different in what concerns to their practical applicability to trade flow estimation, especially when there is no *a priory* matrix of flows. Thus, one of the determinants to consider when choosing between alternative methodologies consists of a very pragmatic one: the possibility of application given the set of data available. The simple nature and the low data requirements of the gravity model, jointly with its advocated strengths in explaining trade behaviour, caused the focus of the Chapter to be put at several gravity-based methodologies. A total of six distinct Experiences for interregional trade estimation were tested.

- The absolute and analytical comparison made between the different trade estimation methods, allowed us to conclude that:

    - Among the several Experiences applied, the one that generated the most accurate matrix corresponded to a gravity-based model, with independent estimation of the distance decay parameter and using RAS, the most common method, as the adjusting procedure.

- The starting matrix seems to have an effective influence on the final results. In fact, when comparing the different Experiences among each other, we have concluded that the only case which is not gravity-based generates more outlying results, demonstrating that the way by which initial estimates are obtained is not innocuous.

- The introduction of superior complexity in the models (in our case, the use of a linear programming model instead of RAS to make the adjustment of the starting matrix) as well as the use of additional information about the real trade flows, such as the degree of Entropy of the real trade matrix, may not originate improved results, as it happened it this case.

- The results of the input-output model are not greatly affected by the insertion of different trade flow values, since large deviations between the obtained growth rates were the exception and not the rule. Thus, the results obtained in our case do not reject the reasonability of using indirect estimates for interregional trade.

Although it is not advisable to generalize these results, given that they were obtained from a particular set of data and using a specific set of hypothesis, we consider that these practical contributions are most relevant to regional input-output researchers, especially to those who intend to assemble an input-output model in a context of absent information on interregional trade flows (which is the most frequent situation at the sub-national level). In fact, our conclusions may serve as important arguments to estimate those inexistent data through the use of gravity-based non-survey methods.

The third essay fell upon input-output modelling when the starting matrix is a total-use rectangular table at purchasers' prices. This study was driven by the following motivations: 1) Since the end of the 1960's, when the United Nations introduced the 1968 System of National Accounts, countries are recommended to compile and publish the input-output tables on a rectangular or Make and Use format; accordingly, this is the format in which the Portuguese National Accounts (as well as the other European

Accounts) provide the database for input-output modelling at the national level; 2) The Make and Use format presents unquestionable advantages associated with the detail on the linkages between the industry and the product dimension, making it more suited for the current applications of input-output modelling (for instance, environmental and trade modelling) and 3) In spite of the practical advantages of the Make and Use format, most of the researchers continue to develop their models on the basis of the symmetric tables, forcing the original data to be converted to a symmetric arrangement, under a series of hypotheses; 4) In many countries, such as in Portugal, there is no provision of an import matrix that allows the computation of an intra-national or domestic use table. Thus, the starting matrix is a total use one (in which both imported and regionally produced inputs are included); 5) Concerning specifically to the regional level analysis, we must keep in mind that Regional Accounts produced by the official statistics organisms are composed of industry data, such as: regional value added by industry, regional production by industry and regional intermediate consumption by industry. In order to use such available information directly, with the minimum imposition of hypothesis, the option should fall upon a Make and Use format or, eventually, upon an industry-by-industry symmetric format (which is however considered a second best option for input-output analysis, given the high heterogeneity of products involved in each element of such tables).

The objective of the study was to demonstrate the equivalence in the results of the input-output model between two alternative procedures: 1) to convert the initial matrix into a domestic-flow symmetric table at basic prices and then implement the input-output model; 2) to perform the direct modelling of the total-flow rectangular table at purchasers' prices.

It has been concluded that:

- When the hypotheses used to make the table symmetric and to operate the conversion from total flows to domestic flows (and from purchasers' prices to basic prices) are also used in the direct modelling of the starting matrix, the obtained results are exactly the same. The equivalence between the results of both alternative procedures has been attested through a numerical example as well as algebraically. The numerical example was based upon the Portuguese M&U table as a starting point, since there are no survey-based regional M&U tables to serve as a reference. The conclusions are, however, also valid for any regional version of the national table. Hence, the main contribution of this Chapter is that, when the same set of hypotheses is to be used, there is no advantage in making a previous transformation of the original tables into the symmetric format and a previous calculation of domestic flows.

- Furthermore, it has been proven that the direct use of the rectangular format has a considerable advantage over the use of symmetric tables: in the rectangular framework, the simple inversion of a partitioned matrix generates a set of four different inverse matrices; conversely, the symmetric table originates only one type of inverse matrix (product-by-product or industry-by-industry).

- The proportionality assumptions (namely, concerning intermediate imports, margins and net taxes) and technology-related hypotheses adopted in the both the above-mentioned procedures are controversial. Nevertheless, the soundness of the conclusions is independent from the validity of the hypotheses assumed, given that the same hypotheses have been used either in the direct modelling of the starting matrix, or in the conversion of this matrix into a domestic-flow symmetric table at basic prices. Besides, in many cases, even the official organisms of statistics use these kinds of hypotheses (or similar procedures) when assembling symmetric tables.

In what concerns to the expected developments of the research, the conclusions drawn by the three essays which compose the present work give support for the construction of a multi-regional model for the Portuguese economy. In face of the obtained results it seems reasonable to use a gravity-based non-survey method to overcome the main obstacle to achieve such a model – the assessment of interregional trade flows. Besides, the regionalization of the tables can be made maintaining the original structure of the national table – rectangular and with total use flows – and using the adequate assumptions in the modelling process. The subsequent natural step will be the upgrading of the input-output model into a General Equilibrium Model for the Portuguese economy.

ACKNOWLEDGEMENTS

The accomplishment of the present dissertation's objectives would not be possible without the help of some people and institutions to whom I especially wish to thank:

- First of all, I want to thank my guide professors: Professor Pedro Ramos and Professor Geoffrey Hewings for their unmeasurable support from the very beginning, in the definition of the research area and objectives, and during the whole research period. A special recognition is due to Professor Pedro Ramos who patiently guided me through numerous discussion meetings and made precious suggestions and corrections during my whole study. I also owe a special word of acknowledgement to Professor Hewings, for his motivation to work in this field, for having received me at the REAL (Regional Economics Applications Laboratory) – Illinois – and for his important suggestions during the research.

- To all the colleagues at the Polytechnic Institute of Leiria - School of Technology and Management, for the priceless conditions and support given to my research.

- To INE, the Portuguese National Institute of Statistics, for its kindness in providing a significant amount of data, including some unpublished information.

- To the Calouste Gulbenkian Foundation, for having given me financial support to work at REAL, in the United States, for one month.

- To all the anonymous researchers who made valuable comments to my work during the several conferences I had the opportunity to attend.

- Last, but not least, I would like to thank to the most important people in my life: firstly, my husband Henrique and my children – Leonor and Laura, who give me the strength and motivation to go on, in the good and the difficult moments; to my parents, brothers, sisters, grandparents, and the rest of the family, for their concern and for having supported me by taking care of my children in some working stages; to my longstanding friends, for their invaluable support, with a special word to Sónia and Eduarda.

To all, I wish to express my profound and sincere acknowledgment.